Default correlation: empirical evidence

Arnaud de Servigny[1]

Olivier Renault[2]

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Abstract:

The aim of this paper is to provide new empirical evidence on default correlation, using Standard & Poor's rating database, and to benchmark some popular market practices. Some of our findings confirm what previous research had already established; some also clearly tend to challenge several common practices that have limited empirical support. We advocate the use of empirical correlation as a benchmark for current credit portfolio model specifications. We then study the impact of the business cycle and of the horizon on correlation and on the credit value at risk of a portfolio. We also give directions for further quantitative modelling.

[1] Standard and Poors, Risk Solutions, 18 Finsbury Circus, London EC2M 7NJ.
arnaud_deservigny@standardandpoors.com
[2] olivier_renault@standardandpoors.com

I. Introduction

A lot of work has been carried out over the last years regarding the diversification effect within a portfolio of credit instruments. Most of this research has considered correlation as a good proxy for dependence. The purpose of this paper is not to question this choice but rather to provide some empirical results on the behaviour of correlation.

Several methodologies have been developed to proxy for transition correlation, but very little has yet been done to benchmark their output with empirical findings. In this paper, we investigate the properties of default correlation using Standard & Poor's historical database.

We first review various ways of estimating bivariate transition probabilities and correlations from actual default data, without relying on any specific default generating model. We study the performance of these measures on a fictitious bond portfolio and show that no single measure outperforms the others for all levels of correlation and sample size. In the remainder of the paper, we use the measure we think best suited to the size of our database to calculate empirical bivariate transition and correlation matrices per class of risk and per industry.

We then test the hypothesis that default correlation can efficiently be extracted from equity return correlation. Indeed over the past few years, it has become market practice to use a *factor model of credit risk* relying on equity correlation as proxy for asset correlation. This approach assumes that latent factors (such as firms' assets) drive the individual obligors' probabilities of default and, ultimately, default correlations among obligors.

By merging an equity return database and an empirical default database, we study the relationship between equity-driven default correlation and empirical default correlation. We use a Gaussian factor model and also test a Student-t bivariate copula. Furthermore, we investigate the idea that asset correlation should not only reflect equity correlation but should also integrate the correlation between the risk-less components of bonds (interest rate risk).

We then provide various insights in the behaviour of these matrices through the business cycle and analyse correlation matrices over longer holding horizons. Finally, we try to shed some light on open questions such as whether the intensity of the correlation effect depends on the quality of debt, or on a possible link between the size of the correlation within a portfolio and its maturity. The latter could be interpreted as a liquidity effect.

Our main source of data is the Standard & Poor's CreditPro 5.20 database. It features the last 21 years of default and transition experience for 9,769 companies rated by S&P since 1981. In this study we focus on the United States sub-sample. This comprises 6,907 firms and a total of 43,642 yearly observations. 764 defaults were recorded over the period 1981-2001. The restriction of the analysis to a single country may lead to slightly higher correlations than a study on a more diversified sample. However, ratings and in particular default data is very scarce outside the US.

II. Calculating one year correlation

We want to extract information about the joint behaviour of migrations and defaults directly from historical data, without relying on a specific model driving transitions. The first step is naturally to calculate the joint transition probabilities and correlation. We review the standard technique used by Lucas (1995) and Bahar and Nagpal (2001) to calculate joint *default* probabilities and compare it to another specification retained for the remainder of the paper.

Joint probabilities for a given year

We first consider estimation of joint probabilities from a dataset corresponding to one year of transition observations. Our sample - 1981 to 2001 - enables us to calculate 21 such yearly correlations.

We then show how individual year's estimates can be aggregated into an average estimate for our entire sample, thereby smoothing out year-specific noise.

Joint probabilities of discrete events such as transitions to a given rating or defaults can be obtained by calculating the ratio of the number of pairs of bonds in a given starting group (e.g. BBB or "Automotive sector") which actually migrated to a given class (for example to default) to the total number of pairs of bonds.

Migrations from the same starting group to same end group.

We first consider the case where two obligors from the same group i, say the same rating, jointly migrate to the same rating k.

Recall that from a given group with N_i elements, one can create $N_i\ (N_i-1)/2$ different pairs. Therefore, if one denotes by $T_{i,k}$ the number of bonds migrating from this group to a given category k, one way to obtain the probability for a given year is to use:

$$\frac{T_{i,k}\left(T_{i,k}-1\right)}{N_i\left(N_i-1\right)}=\frac{number\ of\ pairs\ migrating}{total\ number\ of\ pairs}.$$

This is the estimator used by Lucas (1995) or Bahar and Nagpal (2001). Although intuitive, this estimator has the drawback that it can generate spurious negative correlation when defaults are rare. To illustrate this, assume that there is only one default (class D) in a sample of $N_i=10$ firms. We have $T_{i,D}=1$ and $T_{i,D}(T_{i,D}-1)=0$. Therefore the joint default probability is estimated to be zero.
The marginal probability is however estimated to be $p_{i,D} = T_{i,D}/ N_i = 0.1$ and the joint probability, under zero correlation, would be $(p_{i,D})^2=0.01$. Thus the estimated joint probability is lower than that obtained under the assumption of independence, which implies negative correlation. This negative correlation is spurious as we only observed one default event.

We therefore propose to use

$$\left(T_{i,k}\right)^2/\left(N_i\right)^2,$$

as an estimator of joint probability which ensures that the correlation is zero in the above example.

This estimator corresponds to drawing pairs of firms with replacement. Although in very large samples, both estimators would yield very similar estimates, in samples of the size of a typical credit portfolio, the difference may be substantial. Appendix B illustrates this fact.

Migrations from the same starting group to different end groups.

Following a similar reasoning, an extension of the method of Lucas (1995) to migrations from the same rating i to two different ratings k and l would give the following estimator:

$$\frac{T_{i,k}\,T_{i,l}}{N_i\,(N_i-1)},$$

while the estimator corresponding to the alternate specification above would be:

$$\frac{T_{i,k}\,T_{i,l}}{(N_i)^2}.$$

Migrations from different starting groups

Finally, if we are interested in pairs of bonds from different groups i and j (e.g. a BBB and a B.) migrating respectively to category k and l (with k possibly equal to l), then the number of possible pairs is simply $N_i\ N_j$, where N_i and N_j are the number of bonds in each class. The probability is thus:

$$\frac{T_{i,k}T_{j,k}}{N_i N_j},$$

where $T_{i,k}$ and $T_{j,l}$ are the numbers of bonds from each group i and j which migrated to the target categories k and l.

Average joint probabilities

In practice, transition data is available on a yearly basis and probabilities are calculated as averages of yearly transition frequencies.

Once the yearly estimator of joint probabilities has been chosen, one needs to aggregate the individual yearly probabilities in an average probability over the observation period (in our case n=21 years), assuming that each year is an independent dataset. This is exactly the same process as building an average transition matrix from a set of yearly transition matrices. We now add the superscript t to denote the specific observation year.

We weight each year by its relative size, i.e. by the number of firms present in the sample each year. This is consistent with the approach retained by Standard and Poor's to calculate its standard univariate transition matrices.

Using this, we get for the case of two migrations from and to the same rating:

$$p_{i,i}^{k,k} = \sum_{t=1}^{n} \frac{N_i^t}{\sum_{s=1}^{n} N_i^s} \frac{T_{i,k}^t \left(T_{i,k}^t - 1\right)}{N_i^t \left(N_i^t - 1\right)}, \; or \; p_{i,i}^{k,k} = \sum_{t=1}^{n} \frac{N_i^t}{\sum_{s=1}^{n} N_i^s} \frac{\left(T_{i,k}^t\right)^2}{\left(N_i^t\right)^2},$$

Similar formulae can be derived for transitions to and from different classes.

Table 1a,b reports the bivariate transition probability matrix per rating categories obtained using the two formulae above. As mentioned above, our sample contains all US firms rated by Standard and Poor's from 1981 to 2001 (about 43,000 firm-year observations).

The full matrices have dimension 64x64 and are not reported here. Table 1a,b reports the reduced 36x36 matrix obtained by assuming that a pair (i,j) is equivalent to a pair (j,i). Joint default probabilities range from 0 for investment grade pairs to 10% or 12% for CCC pairs, depending on the estimator.

The impact of correlation is evident here as the joint CCC default probability would only be 8.14% (the square of the marginal default probability in the CCC class) under independence. Table 1c uses the second estimator but over broader rating categories, namely investment grade (IG), non investment grade (NIG) and default (D).

Correlation

Now that we have obtained estimates for joint probabilities, we can readily calculate default correlations. To do so, we use[3]:

$$\rho_{i,j}^{k,l} = \frac{p_{i,j}^{k,l} - p_i^k p_j^l}{\sqrt{p_i^k (1 - p_i^k) p_j^l (1 - p_j^l)}}. \qquad (1)$$

Default correlations for non investment grade bonds by sectors are provided in Table 2. As expected, most large positive correlation is found within a given industry (diagonal) but one can also see a 6.5% correlation between "telecom" and "building". The telecom sector has been particularly affected by a wave of defaults in 2001.

Table 3 reports default correlations per broad rating category for all industries. The use of broad ratings is necessary when we calculate correlations per industry as most sectors do not have a sufficient number of defaults to estimate correlations with reasonable accuracy. We will discuss the potential impact of this restriction below.

[3] see appendix A.

III. Correlation in a factor-based model of credit risk

One of the most popular classes of credit risk models is the so called factor-based approach. This approach specifies the rating transition process as the outcome of the realisation of systematic[4] and idiosyncratic factors.

One way to grasp the main idea behind factor models is to assume the driving factor to be the value of the firm's assets. When this value falls below some critical threshold often linked to the value of the firm's liabilities, default is triggered.

We briefly recall below the basic structure of a one factor model. Further details and extensions are provided in Lucas et al. (2001) or Schönbucher (2000).

Let us denote by A_j the latent variable driving default and migration for firm j.[5] It is assumed to be normally distributed and can be broken down into a systematic common component C and an idiosyncratic component ε_j, both also normally distributed with mean zero. ε_j is uncorrelated with the common component and with other firms' idiosyncratic components ε_i, $i\neq j$. We write:

$$A_j = \rho_j C + \sqrt{1-\rho_j^2}\,\varepsilon_j \qquad (2)$$

All variables are standardised such that $Var(C) = Var(\varepsilon_j) = Var(A_j) = 1$ for all j. We thus have $corr(A_i,A_j) = cov(A_i,A_j) = \rho_i\rho_j \equiv \rho_{i,j}$, $i\neq j$.

In the sequel, we will refer to $\rho_{i,j}$ as *asset correlation*.

The latent variable A_j is used as the driver of ratings transitions. A set of k cut-off levels $K_{j,u}$, u=1...k, (such that $K_{j,u+1} > K_{j,u}$) is introduced, thereby breaking down the space $(-\infty, +\infty)$ of possible values for A_j into k+1 buckets. These cut-off levels are identical for all firms with same initial rating as j.

Calibrating transition probabilities using factor model

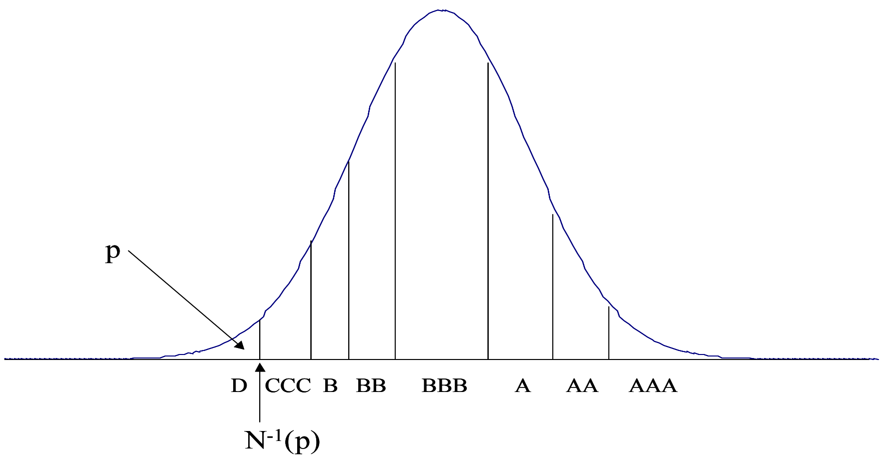

Figure 1

[4] such as industry, country etc.

[5] The model of Merton (1974) can be seen as a special case of the specification presented below. If the firm's assets follow $dV_t=\mu V_t dt + \sigma V_t\, dW_t$, then the factor A_j would be $(\log(V_t/V_0)-(\mu-\sigma^2/2)t)/(\sigma\sqrt{t})$.

The terminal rating of the obligor is determined by the value of its latent factor A_j in the following manner: if $A_j \leq K_{j,1}$, the obligor defaults, if $K_{j,1} < A_j \leq K_{j,2}$, the rating is that of the worst non-default state, ..., if $A_j > K_{j,k}$, the obligor ends up in the best rating class. Figure 1 summarises how the distribution of the latent factor is mapped to rating categories.

The thresholds are calibrated such that the probability between two thresholds corresponds to the actual probability of ending up in a given class, the worst one being default. For example, let us denote $p_{j,D}$ (resp. $p_{j,CCC}$) the probability of a firm with same initial rating as firm *j* of defaulting (resp. of having a terminal rating of CCC). $K_{j,1}$ is calculated as $N^{-1}(p_{j,D})$ and $K_{j,2} = N^{-1}(p_{j,D}+p_{j,CCC})$, where N(.) is the cumulative standard normal distribution and $N^{-1}(.)$ its inverse. Conversely $p_{j,D}= N(K_{j,1})$ and $p_{j,CCC}=N(K_{j,2})-N(K_{j,1})$.

Joint default probabilities are obtained as follows: from the default thresholds $K_{i,1}$ and $K_{j,1}$ and the asset correlation $\rho_{i,j}=\rho_i\rho_j$, we can obtain the joint probability of default of firms i and j as $p_{i,j}=N_2(K_{i,1}, K_{j,1}, \rho_{i,j})$, where $N_2(.)$ denotes the cumulative bivariate normal distribution. Default correlation then follows immediately from equation (1) above.

Alternatively, from an estimate of the joint probability $\hat{p}_{i,j}$ and the thresholds $K_{i,1}$ as obtained in Section 2, we can infer the "implied asset correlation" $\rho^a_{i,j}$ by inverting the joint distribution:

$$\rho^a_{i,j} = N_2^{-1}(\hat{p}_{i,j}, K_{i,1}, K_{j,1}).$$

Note that, as mentioned before, we only calculate the correlation over broad classes of ratings (investment grade and non investment grade) due to data constraint. Gordy and Heitfield (2002) show that the slight positive relationship between credit quality and asset implied correlation is not statistically significant and that there is no real value in terms of accrual precision to reject the hypothesis of constant implied correlation across ratings.

Is equity correlation a good proxy for asset correlation?

In a factor driven model, it has become common practice to use equity correlation as a proxy for factor correlation. Although intuitive, this approach has, to our knowledge, not really been supported by strong empirical analysis of the link between equity correlation and default correlation. In this section we aim at filling this gap by comparing average equity correlation and default correlation for the past twenty-one years[6].

We have gathered equity returns data from the Top Research 2000 database in Datastream. This database contains about 1600 stocks of which 1101 could be identified to fit in one of S&P's 12 industry categories. We first calculated monthly returns for all stocks for which more than 60 monthly observations were available over the period 1980-2001. We then computed the return correlation between each

[6] per industry

pair of equities. Then, we calculated average correlations both within an industry and across industries. To do so, we computed the average correlation of the returns on equity of firms in a given industry with those of firms in another industry.

The average equity correlation was 6% across industry and 12% within the same industry[7]. Note that we consider only one country, the US, our estimates of correlations are arguably higher than those we would obtain from a geographically diversified portfolio.

From equity correlations and the marginal probabilities of default, we then calculated the average joint probabilities of default via a Gaussian factor model. More precisely, we calculated the joint probability as:

$$p_{i,j}=N_2(K_{i,1}, K_{j,1}, \rho'_{i,j}), \qquad (3)$$

where $\rho'_{i,j}$ is their equity correlation coefficient and K_i, K_j, their respective default thresholds as described above. Table 4 reports the default correlations derived from equities.

Figure 2 is a scatter plot of empirical default correlations vs. default correlations calculated using the above equity correlation method (eq. (3)). Each point corresponds to a correlation between 2 of our 12 industry sectors.

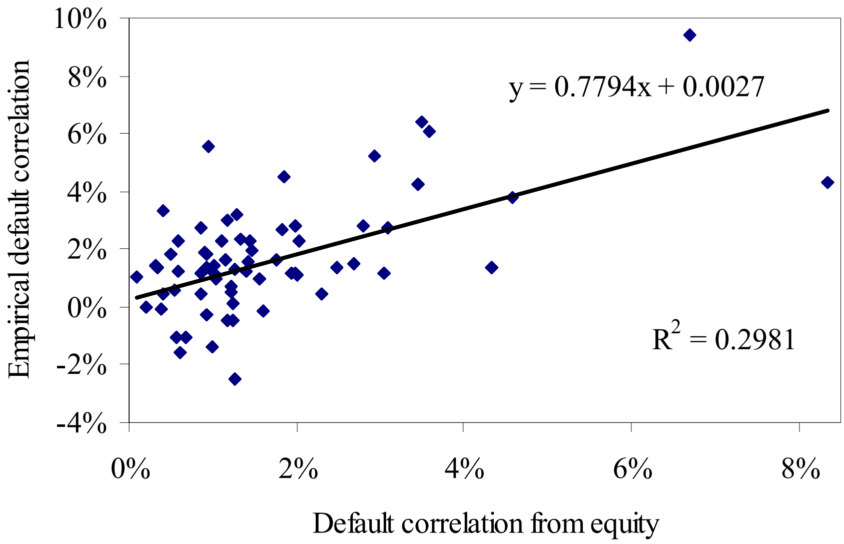

Figure 2: Empirical vs. equity based default correlations

Let us first consider the levels of correlations: default correlations extracted from equities tend to be slightly higher on average than empirical default correlations, as shown by the regression coefficients of 0.78 (the intercept is almost 0). But it is not obvious that default correlation implied by equity can serve as a good proxy for actual

[7] An alternative way to proceed would be to calculate correlations with a sector index. This tends to generate substantially higher "equity" correlations.

default correlation, as the level of R^2 is rather low and even lower if the impact of two large "outliers" is excluded.

For most points, especially for medium or low correlation levels, there does not seem to be an obvious link between equity correlation and asset correlation and this casts some doubt about the value of equities as precise indicators of default correlations.

This is somewhat disappointing as equity returns are available for many firms and would be appealing candidates for calculating asset correlations. Equity prices and returns however reflect many factors such as changes in risk aversion in the equity markets and liquidity effects. A substantial proportion of these changes may not be related to the financial stability of the firms.

As an alternative to equation (3) above, one might be interested in fitting another joint distribution to the data. This is possible while keeping Gaussian marginal distributions through the use of copula functions. The copula is the function that couples the marginals into a multivariate distribution. It therefore captures the entire dependency structure of the variables and is much more general than the linear correlation.

One choice could be to fit a bivariate t-distribution with ν degrees of freedom while keeping Gaussian marginals:

$$p_{i,j}=t_2(K_{i,1}, K_{j,1}, \nu, \rho'_{i,j}). \qquad (4)$$

where ν determines the fatness of tails in the t-distribution: the smaller the ν, the fatter the tails. In the case where $\nu=\infty$, the t-distribution coincides with the normal distribution.

We performed such a calibration on our data and found that in most cases ν was very large. In almost 50% of cases, ν converged to ∞ and therefore the t-distribution converged to the limiting Gaussian distribution. 88% of joint probabilities were optimally calibrated with $\nu>50$. The value added of the copula is therefore very limited in our case.

This is not surprising as equity correlations $\rho'_{i,j}$, as mentioned above, are of the same order of magnitude as asset implied correlations $\rho^a_{i,j}$. Therefore, the probabilities generated by (3) are also of the same order of magnitude as the empirical probabilities and there is no need for a low ν to compensate for a too low equity correlation[8].

Asset covariance defined as mix of equity covariance and variance of the risk-less rate

In a recent paper, Zeng and Zhang (2002) argue that equity correlation is insufficient to proxy for asset correlation. Under their specification the covariance between the

[8] If equity correlations were too low compared to asset implied correlations, the joint probability generated by equation (3) would be lower than the empirical joint default probability. Using equation (4), one could obtain a better fit by choosing a lower ν which would imply fat tails and therefore more probability of a joint default.

assets of two obligors can be explained primarily by the covariance between their equities plus the covariance between their default risk-less components, i.e. the variance of risk-less interest rates.[9]

From the results discussed above, the levels of implied asset correlations are comparable to the levels of equity correlation and we find no evidence of a downward bias in equity correlations compared to asset implied correlations.

Nonetheless, in order to further investigate a potential link between asset correlation and the correlation between the default risk-less component of corporate debt, we calculated default correlation matrices on subsamples corresponding to high (resp low) interest rate volatility periods.

Our results show (see Table 5) that more than half of the asset correlations calculated over periods with high interest rate volatility (and thus presumably high correlation between the default risk-free components of debt) are actually lower than during periods of low interest rate volatility.

We therefore find little support for the claim that equity correlations underestimate asset correlation or that risk-less interest rate volatility directly impacts on asset correlations.

IV. Calculating correlation over longer horizons.

In this section, we consider transitions over longer horizons of three and five years. These are calculated in the same fashion as one-year probabilities and correlations: ratings at the beginning and at the end of the horizon are collected and the formulae in Section 2 are applied. All intermediate transitions are ignored. For example, a AA obligor which is upgraded to AAA within the first year and downgraded back to AA in the second year will be recorded as "no move" for the calculation of three-year transitions.

As can be seen in Table 6, default correlations tend to increase with horizon. This has also been reported in Lucas (1995) and Bahar and Nagpal (2001) for example and is often explained by the fact that over a one year period, most defaults are firm specific (idiosyncratic default events) while horizons of three to five years may include industry-wide crises (correlated default events).

In the context of a structural model of credit risk, Zhou (1997) also finds a similar pattern for short horizons (1 to 5 years) and finds slowly decreasing correlations for longer horizons.

An increase in *default* correlations as the horizon lengthens may be compatible with a constant *asset* correlation, as in Zhou (1997), where it is solely due to an increase in

[9] In a firm-value based model of credit risk such as Merton (1974), equity E and debt D are seen as options on the value of the firm V. More specifically default risky debt can be broken down into a risk-less bond minus a put on the value of the firm.

default probabilities. To test this, we calculated three- and five-year default correlations using the following equation for the joint probability:

$$p^T_{i,j} = N_2(K^T_{i,1}, K^T_{j,1}, \rho'_{i,j}),$$

where T denotes the horizon (see Table 7). The default thresholds are calculated using the marginal probabilities of default at the horizon of 3 or 5 years, while the equity correlation is used as asset correlation.

Contrasting Tables 6 and 7, one can immediately observe that default correlations extracted from equities are always positive[10] while some empirical default correlations can be negative. Overall however, empirical default correlations tend to be higher than those calculated from equity as the horizon is extended. This is particularly true within a given industry (i.e. on the diagonal of the matrix). The average empirical intra-industry correlation increases from 4.8% at the one-year horizon to 7.2% and 12.5% at 3 and 5 years. Their counterparts extracted from equity are respectively 3.3%, 5.5% and 6.4%.

This indicates that a constant *asset* correlation does not seem to capture entirely the increase in *default* correlation. The data therefore implies an increasing asset correlation.

Recall that we showed earlier how one could extract asset implied correlations $\rho^a_{i,j}$ from default thresholds $K_{j,1}$ and the empirical default correlation. When we calculate inter- and intra-industry implied correlations from five-year default correlations, we find that they are twice as large as those implied from one-year default data. This has so far not been acknowledged in practical implementations.

V. Correlation and the business cycle

It is a well documented fact that probabilities of default and migration depend crucially on the business cycle. For instance, Nickell et al. (2000) use an ordered probit model to analyse the impact of industry, country and the business cycle on probabilities of migration and find that the latter has the largest explanatory power. Bangia et al. (2002) calculate the levels of economic capital necessary to cover credit losses at a given confidence level. They report that the economic capital at the 99% level is 30% higher in a recession year than in an expansion year.

Both these studies and several others focus primarily on standard *univariate* transition probabilities. Little is known however about the behaviour of *joint* probabilities and correlation matrices during economic growth or recession periods.

In this section we start by calculating bivariate default probability matrices as well as correlation matrices in times of expansion and recession. Not surprisingly, we find that joint default probabilities are substantially higher in recessions. This may be due either to an increase in the marginal probabilities of default (univariate) or to an

[10] because equity correlations are positive.

increase in correlation (bivariate) or to both. In order to investigate which factor is at play, we compute the credit values at risk of a fictitious portfolio for various confidence levels and separate the impacts of marginal probabilities from those of correlation.

Conditional correlation matrices

We define a recession year as a year in which at least one quarter was in recession according to the National Bureau of Economic Research (NBER). Over our sample, five years fall in this category: 1981, 1982, 1990, 1991 and 2001. Tables 8,9,10 report joint transition matrices and correlation for recession and growth years over the period 1981-2001.

From these tables we see that joint default probabilities increase very substantially during recession periods. This is partly due to the increase in marginal default probabilities (IG and NIG default probabilities increase from 0.08% and 4.32% to 0.23% and 8.88% respectively) but also to a doubling in NIG and cross-rating default correlation (default correlation within the IG category remain unchanged).

In the next subsection, we use these results as input in the simulation of a bond portfolio to extract the *relative* contribution of both drivers (correlation and marginal probabilities) of portfolio losses.

Impact on portfolio losses

We now want to determine how changes in correlation with the business cycle impact on a bond or loan portfolio. To do so, we would like to separate the part of the likely increase in losses during recessions which is due to changes in default probabilities from the part which is due to correlation.

We simulate a "typical" portfolio of 100 non-investment grade bonds with unit exposure. Each position in the portfolio is assumed to have an identical probability of default and correlation is assumed to be identical for all pairs of bonds. By "typical", we mean that the default probabilities and correlations are chosen equal to the empirical averages calculated for non investment grade bonds.

We perform 50,000 Monte-Carlo simulations of the portfolio with 100 positions. For each simulation run, 100 binary variables (default or no default) are drawn from a binomial distribution with appropriate mean and correlation.

The recovery associated to each default is drawn from a beta distribution with mean 0.507 and standard deviation 0.358. These figures correspond to the mean and standard deviation of the loss rate for senior unsecured bonds reported in Keisman and Van de Castle (1999). Portfolio losses are then calculated as the sum of losses on individual positions.

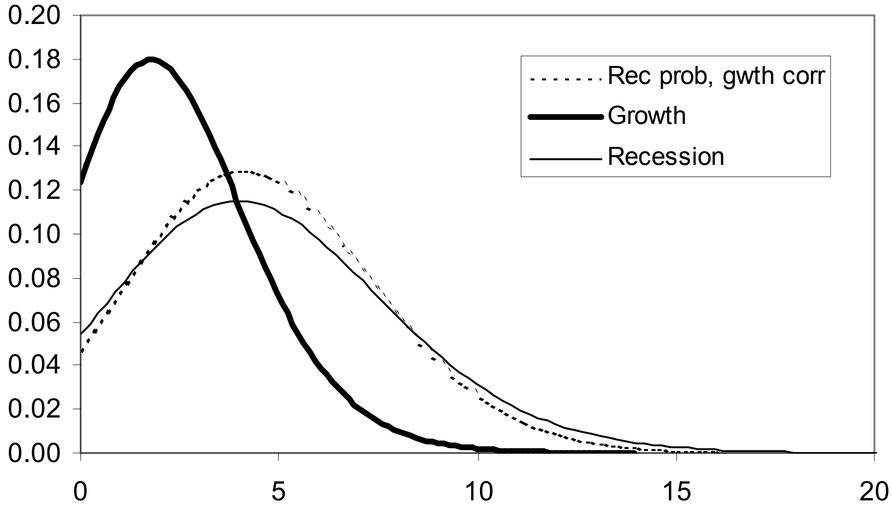

Figure 3: Portfolio losses in growth and recession

Figure 3 is a smoothed[11] plot of the distribution of losses in three scenarios: the first scenario is growth as defined above and the second is recession with respective probabilities of default of 4.31% and 8.88%. These numbers are the average figures over our sample from 1981 to 2001. Correlations are respectively set to be 0.77% and 1.5% during growth and recession (see our estimates in Table 10).

The third scenario is a hybrid case which enables us to extract the specific impact of correlation on losses: it corresponds to a case where the probability of default is 8.88% (recession) and the correlation is 0.77% (growth). The increase in portfolio losses from the first scenario to the hybrid scenario is therefore purely due to the increase in default probability. The further increase in loss associated to the move from the hybrid scenario to the recession case is purely attributable to correlation.

It is clear from the graph that the main impact on the centre of the distribution is linked to the substantial increase in default probability during recession periods. However, further in the right tail of the distribution, the relative impact of correlation becomes apparent.

To investigate further the impact of correlation on the tail of the loss distribution, we calculate the value at risk associated with default (CreditVaR) at various standard levels of confidence: 95%, 99%, 99.7% and 99.9% for our three scenarios. Table 11 shows that the further in the tail we look, the larger the relative impact of correlations.

The last row of the table is a measure of the contribution of correlations to the increase in credit VaR as the economy enters into recession. It is calculated as $(V_R-V_H)/(V_R-V_G)$ where V_G, V_R and V_H are the CreditVar in growth, recession and in the hybrid case respectively. At the 99.9% level, correlation become the major driver of the increase in CreditVaR.

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[11] We use a Gaussian kernel smoother.

Table 11: CreditVaR in growth and recession

	95% VaR	99% VaR	99.7% VaR	99.9% VaR
Growth	6.8	9.0	10.4	11.8
Rec prob, gwth corr	9.6	11.8	13.4	14.2
Recession	10.8	13.6	15.4	17.2
Correlation contribution	30%	39%	40%	56%

To conclude this section on the impact of the business cycle, we computed the eigenvalues and eigenvectors of the industry correlation matrix. The largest eigenvalues are reported in Figure 4, while the first eigenvector is plotted on Figure 5. No factor with high explanatory power seems to be emerging with the trough in the business cycle and eigenvectors exhibit no stability over time. This implies that *average default correlations across the business cycle do not provide a satisfactory hedging strategy for recession periods*.

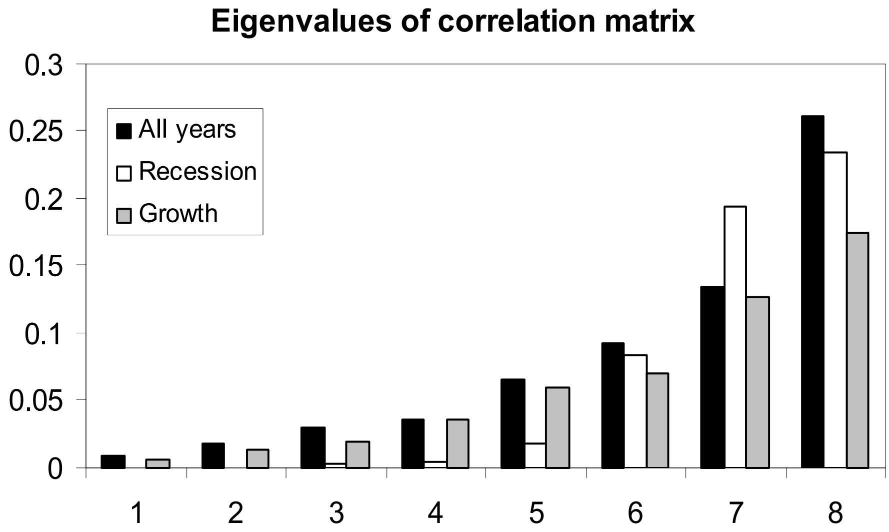

Figure 4

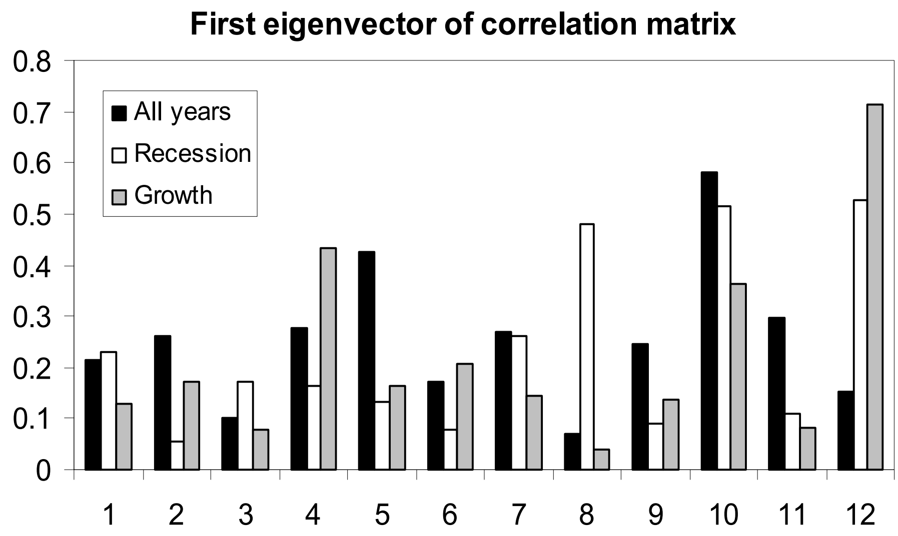

Figure 5

VI. Conclusion

This piece of empirical research raises many questions about how to account for correlation is a debt portfolio. Based on these observations we would like to suggest several directions for future research:

- Using interest rate information has been an interesting first step over the mere use of equity information to account for default correlation. As we saw, it has however not proved sufficient and conclusive. Should one wish to continue using equity correlation as a driver for default correlation, appropriate ways to filter out the noise in equity returns would need to be derived.

- The dynamics of correlation over time seem to have been largely ignored up until now. On the other hand, a strong research trend emphasises a more systematic recourse to copulas as a way to reach a higher level of default correlation. Apparently these two approaches are disconnected but we think they should complement one another.
  Indeed copulas allow us to obtain a non-Gaussian joint distribution of assets while keeping Gaussian marginals[12]. They can therefore be very useful in capturing fat tails in joint events. One area of investigation could be to define an appropriate range of copulas which would capture the "term structure" of correlations reported in our paper. One could indeed imagine a constant asset correlation, for example set equal to equity correlation, and an adjustment variable (such as the number of degrees of freedom in the Student t-distribution) which would jointly match the increase in default correlation with the horizon.

- It is clear to us that portfolio models, based on a one year horizon and static default correlation should be used with great caution in active portfolio management, especially when it is highly illiquid.

- In order to have a better grasp of potential default losses, we believe that the traditional 2D portfolio loss distribution (severity, probability) should be extended to three dimensions (severity, horizon, probability) to account for the impact of the change in time horizon and the dynamics of correlation.

- Even if the amount of information available to generate empirical default correlation is smaller than that extracted from equities, using empirical correlation as a back-testing tool should become part of banks best practices.

- Understanding the impact of the macro-economic cycle on a portfolio is not easy, some effects are univariate, at the company level[13], some others are multivariate, impacting industries, portfolios[14]. Based on the experience of past cycles, defining stress-tests would help to understand the behaviour of credit portfolios under stress and enable capital manager to optimise capital management, not only to improve shareholder value in stable periods, but also to optimise the stability of performance, thus avoiding pro-cyclicality.

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[12] In the case of non-elliptical distributions, correlation is a very partial measure of dependence.
[13] Reflects on ratings, PDs, transition matrices
[14] impact correlations

Bibliography

Bahar R. and K. Nagpal (2001), "Measuring default correlation", *Risk*, March, 129-132.

Bangia A., F. Diebold, A. Kronimus, C. Schagen and T. Schuermann (2002), "Ratings migration and the business cycle, with application to credit portfolio stress testing", *Journal of Banking and Finance*, 26, 445-474.

Gordy M. and E. Heitfield (2002), "Estimating default correlations from short panels of credit rating performance data", Working Paper, Federal Reserve Board.

Keisman D. and K. VandeCastle (1999), "Recovering your money: Insights into losses from defaults", Standard and Poor's Commentary.

Lopez J. (2002), "The empirical relationship between average asset correlation, firm probability of default and asset size", Working Paper, Federal Reserve bank of San Francisco.

Lucas D. (1995), "Default correlation and credit analysis", *Journal of Fixed Income*, March, 76-87.

Lucas A., P. Klaassen, P. Spreij and S. Straetmans (2001) "An analytic approach to credit risk of large corporate bond and loan portfolios", Journal of Banking and Finance, 25, 1635-1664.

Merton R. (1974), "On the pricing of corporate debt : the risk structure of interest rates", Journal of Finance, 29, 449-470.

Nickell P., W. Perraudin and S. Varotto (2000), "Stability of rating transitions", *Journal of Banking and Finance*, 24, 203-227.

Schönbucher P. (2000), "Factor models for portfolio credit risk", Mimeo, Bonn University.

Zeng B. and J. Zhang (2002) "Measuring credit correlations: equity correlations are not enough !", Working Paper, KMV corporation.

Zhou C. (1997) "Default correlation: an analytical result", Working Paper, Federal Reserve Board.

Appendix A - Calculation of correlations

Consider the two binary events A and B occurring respectively with probability p_A and p_B and let us denote by 1_A (resp. 1_B) the indicator function taking the value 1 if A (resp. B) is true and 0 otherwise. We can easily calculate
$Var(1_A)= E[(1_A)^2] - (E[1_A])^2 = p_A-(p_A)^2$,
$Var(1_B)= E[(1_B)^2] - (E[1_B])^2 = p_B-(p_B)^2$,
and
$Cov(1_A,1_B)= E[1_A\ 1_B]- E[1_A]\ E[1_B] = p_{AB}- p_A\ p_B$.

Then, using the formula for the correlation

$$\mathrm{Corr}(1_A, 1_B) = \frac{\mathrm{Cov}(1_A, 1_B)}{\sqrt{\mathrm{Var}(1_A)\mathrm{Var}(1_B)}}$$

we obtain the stated formula.

Appendix B - Properties of correlation estimators.

In this appendix, we compare the properties of the two estimators of correlations described in Section 2. Estimator 1 uses the N(N-1) and T(T-1) formulae while estimator 2 uses $(N)^2$ and $(T)^2$.

We simulate 10,000 realisations of default or survival (indicator functions) experiences over 21 years (corresponding to our sample 1981-2001) with various levels of default correlation between 0 and 20%. In order to provide a realistic example, the actual number of observations each year and the actual default probability as in the Automotive sector.

For each simulation and each level of correlation, we apply both estimators in order to obtain estimates of default correlation. Figure 6 is a comparative plot of the average estimated value for both estimators together with the known “true” correlation used in the simulations. We can see that Estimator 1 performs better for very low levels of correlation (≤3%) and that both estimators are biased at high levels, although much less so for the second one.

Moreover, as shown in Section 2, typical defaults correlations are estimated to be in the region of 1% to 10% and we believe that the second estimator outperforms the first one over that range. We have therefore decided to focus on that estimator more specifically.

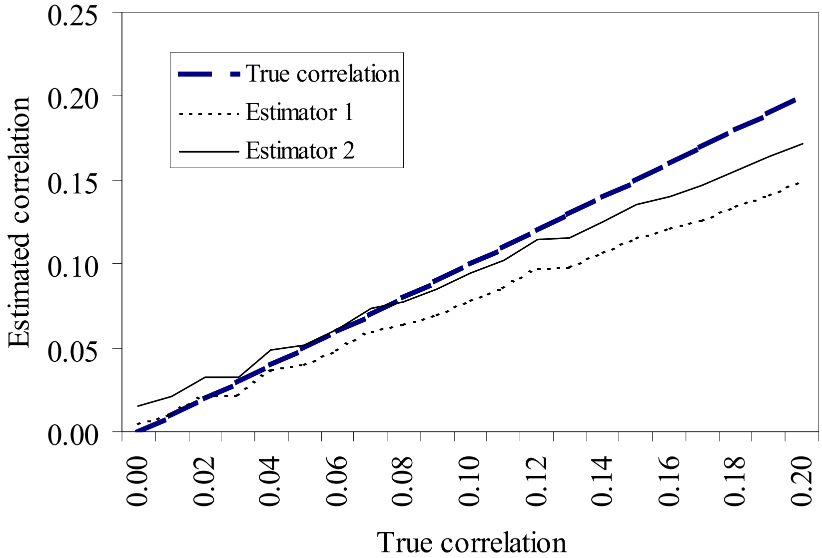

Figure 6: Performance of estimators : 21 years of data

The figure below concentrates on the second estimator and provides confidence bounds. These increase quite substantially with the level of correlations.

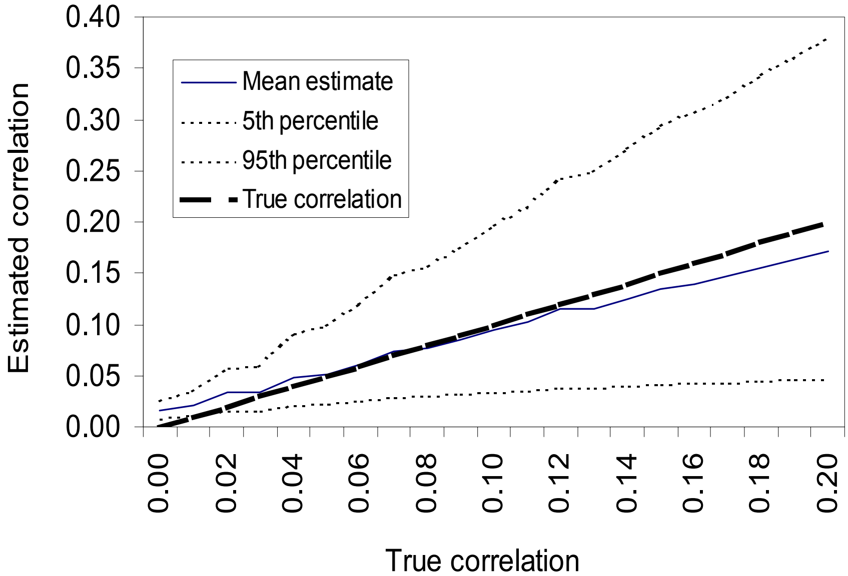

Figure 7: Performance of Estimator 2: 21 years of data

We believe that the downward bias at high values is probably due to the small size of the sample. We have thus decided to try a similar experiment but assuming that we have 50 years of data instead of 21. The 50 year experiment is designed to have years with the same average number of observations (115) as the 21 year experiment and same default probability (4.29%). Figure 8 shows that the bias is substantially reduced and that confidence bounds are slightly narrower, as expected.

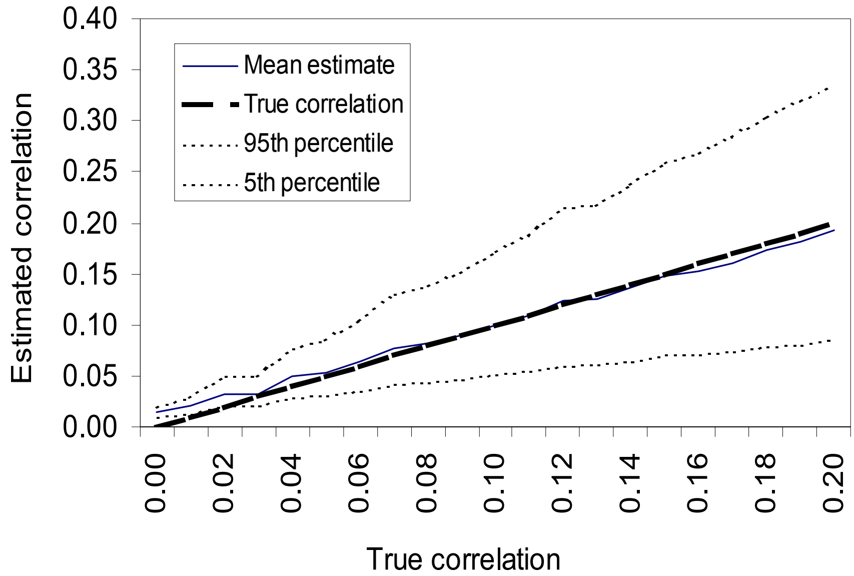

Figure 8: Performance of estimator : 50 years of data

Tables

Table 1a: One year bivariate transition matrix – US, all industries – 1981-2001 – First estimator - Percent

		AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AA	AA	AA	AA	AA	AA	AA	A	A	A	A	A	A	BBB	BBB	BBB	BBB	BBB	BB	BB	BB	BB	B	B	B	CCC	CCC	D
		AAA	AA	A	BBB	BB	B	CCC	D	AA	A	BBB	BB	B	CCC	D	A	BBB	BB	B	CCC	D	BBB	BB	B	CCC	D	BB	B	CCC	D	B	CCC	D	CCC	D	D
AAA	AAA	89.55	8.80	0.83	0.17	0.08	0.00	0.00	0.00	0.51	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AAA	AA	0.50	86.79	6.43	0.54	0.07	0.06	0.03	0.02	4.61	0.73	0.12	0.04	0.00	0.00	0.00	0.04	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AAA	A	0.06	1.91	87.35	4.74	0.42	0.15	0.05	0.05	0.11	4.33	0.24	0.03	0.01	0.00	0.00	0.38	0.10	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AAA	BBB	0.04	0.24	4.41	85.43	4.06	0.60	0.17	0.26	0.01	0.28	3.71	0.22	0.04	0.01	0.02	0.02	0.34	0.02	0.00	0.00	0.00	0.09	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AAA	BB	0.04	0.07	0.43	6.48	79.00	7.33	0.88	1.18	0.00	0.03	0.31	3.29	0.34	0.03	0.05	0.00	0.02	0.32	0.03	0.00	0.01	0.00	0.09	0.01	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AAA	B	0.00	0.10	0.26	0.34	5.23	79.32	4.04	6.28	0.00	0.01	0.02	0.22	3.27	0.14	0.24	0.00	0.00	0.02	0.33	0.01	0.02	0.00	0.00	0.11	0.00	0.01	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AAA	CCC	0.14	0.00	0.29	0.82	1.85	9.60	54.37	27.84	0.00	0.02	0.02	0.09	0.39	2.81	1.13	0.00	0.01	0.01	0.07	0.26	0.11	0.00	0.00	0.01	0.08	0.03	0.00	0.01	0.04	0.02	0.00	0.00	0.00	0.00	0.00	0.00
AAA	D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	93.96	0.00	0.00	0.00	0.00	0.00	0.00	5.43	0.00	0.00	0.00	0.00	0.00	0.48	0.00	0.00	0.00	0.00	0.09	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00
AA	AA	0.00	1.00	0.08	0.01	0.00	0.00	0.00	0.00	84.37	12.35	1.15	0.12	0.16	0.06	0.03	0.53	0.09	0.01	0.02	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AA	A	0.00	0.08	0.48	0.03	0.00	0.00	0.00	0.00	1.90	84.83	4.63	0.42	0.16	0.05	0.05	6.18	0.91	0.10	0.08	0.03	0.02	0.03	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AA	BBB	0.00	0.04	0.03	0.42	0.02	0.01	0.00	0.00	0.24	4.44	82.31	3.98	0.65	0.16	0.25	0.32	6.06	0.31	0.06	0.02	0.02	0.47	0.08	0.06	0.02	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AA	BB	0.00	0.03	0.00	0.04	0.40	0.05	0.00	0.00	0.07	0.43	6.25	76.14	7.00	0.83	1.11	0.03	0.45	5.67	0.55	0.08	0.11	0.04	0.47	0.04	0.01	0.01	0.06	0.06	0.02	0.02	0.01	0.00	0.00	0.00	0.00	0.00
AA	B	0.00	0.00	0.00	0.00	0.03	0.42	0.02	0.03	0.09	0.27	0.32	4.99	76.22	3.82	5.91	0.02	0.02	0.35	5.86	0.32	0.51	0.00	0.03	0.50	0.03	0.04	0.00	0.06	0.01	0.01	0.06	0.02	0.03	0.00	0.00	0.00
AA	CCC	0.00	0.12	0.02	0.01	0.01	0.04	0.36	0.14	0.00	0.28	0.80	1.72	9.40	52.84	26.01	0.02	0.06	0.15	0.68	4.25	2.31	0.01	0.01	0.07	0.37	0.14	0.00	0.01	0.05	0.03	0.01	0.04	0.01	0.01	0.01	0.01
AA	D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.58	0.00	0.00	0.00	0.00	0.00	0.00	91.68	0.00	0.00	0.00	0.00	0.00	6.88	0.00	0.00	0.00	0.00	0.64	0.00	0.00	0.00	0.08	0.00	0.00	0.09	0.00	0.03	0.02
A	A	0.00	0.00	0.12	0.01	0.00	0.00	0.00	0.00	0.05	3.81	0.22	0.02	0.01	0.00	0.00	84.97	9.14	0.76	0.27	0.10	0.11	0.28	0.06	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
A	BBB	0.00	0.00	0.04	0.06	0.00	0.00	0.00	0.00	0.00	0.33	1.83	0.09	0.01	0.00	0.01	4.23	83.20	3.90	0.58	0.16	0.25	4.39	0.54	0.15	0.08	0.09	0.02	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
A	BB	0.00	0.00	0.04	0.00	0.05	0.01	0.00	0.00	0.00	0.08	0.13	1.68	0.18	0.02	0.03	0.42	6.12	76.60	7.09	0.82	1.14	0.31	4.12	0.42	0.06	0.08	0.29	0.14	0.07	0.07	0.01	0.01	0.01	0.00	0.00	0.00
A	B	0.00	0.00	0.00	0.00	0.00	0.05	0.00	0.00	0.00	0.09	0.01	0.10	1.68	0.09	0.13	0.25	0.33	4.90	76.55	3.89	6.09	0.02	0.24	4.21	0.24	0.37	0.02	0.34	0.02	0.03	0.14	0.08	0.08	0.01	0.02	0.01
A	CCC	0.00	0.00	0.11	0.01	0.00	0.01	0.04	0.02	0.00	0.00	0.01	0.03	0.15	1.10	0.52	0.28	0.77	1.67	9.24	52.50	27.37	0.05	0.09	0.46	3.04	1.66	0.01	0.04	0.29	0.15	0.01	0.14	0.04	0.04	0.09	0.04
A	D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.00	0.00	0.00	0.00	0.00	0.00	2.16	0.00	0.00	0.00	0.00	0.00	91.78	0.00	0.00	0.00	0.00	5.21	0.00	0.00	0.00	0.49	0.00	0.00	0.18	0.00	0.05	0.06
BBB	BBB	0.00	0.00	0.00	0.08	0.00	0.00	0.00	0.00	0.00	0.02	0.45	0.02	0.00	0.00	0.00	0.22	7.61	0.36	0.07	0.01	0.02	81.68	7.34	0.95	0.33	0.52	0.19	0.06	0.02	0.03	0.01	0.00	0.00	0.00	0.00	0.00
BBB	BB	0.00	0.00	0.00	0.04	0.03	0.00	0.00	0.00	0.00	0.00	0.07	0.22	0.02	0.00	0.00	0.02	0.67	3.54	0.32	0.04	0.05	5.60	75.73	6.88	0.82	1.18	3.36	0.79	0.19	0.30	0.04	0.02	0.03	0.00	0.01	0.01
BBB	B	0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.07	0.02	0.22	0.01	0.02	0.01	0.24	0.24	3.52	0.17	0.26	0.28	4.51	74.86	4.02	6.32	0.20	3.43	0.21	0.32	0.49	0.19	0.29	0.01	0.04	0.03
BBB	CCC	0.00	0.00	0.00	0.09	0.01	0.00	0.02	0.01	0.00	0.00	0.00	0.00	0.02	0.15	0.07	0.01	0.32	0.10	0.44	2.62	1.22	0.72	1.48	8.46	49.60	28.84	0.07	0.45	2.55	1.48	0.07	0.46	0.19	0.13	0.27	0.13
BBB	D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.26	0.00	0.00	0.00	0.00	0.00	4.88	0.00	0.00	0.00	0.00	89.14	0.00	0.00	0.00	4.48	0.00	0.00	0.74	0.00	0.18	0.28
BB	BB	0.00	0.00	0.00	0.01	0.07	0.01	0.00	0.00	0.00	0.00	0.01	0.09	0.01	0.00	0.00	0.00	0.06	0.69	0.08	0.01	0.01	0.45	10.25	0.89	0.11	0.13	69.82	12.48	1.50	2.15	0.74	0.15	0.22	0.01	0.03	0.02
BB	B	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.00	0.01	0.06	0.05	0.00	0.00	0.00	0.02	0.23	0.36	0.02	0.03	0.02	0.61	5.00	0.25	0.37	4.11	69.91	3.77	5.90	6.11	1.14	1.66	0.05	0.16	0.12
BB	CCC	0.00	0.00	0.00	0.01	0.08	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.02	0.03	0.01	0.00	0.03	0.25	0.06	0.26	0.15	0.05	0.86	0.72	3.71	1.86	1.31	7.57	46.04	26.63	0.73	4.50	2.71	0.61	1.18	0.58
BB	D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00	0.00	0.00	0.00	0.00	0.49	0.00	0.00	0.00	0.00	6.73	0.00	0.00	0.00	82.34	0.00	0.00	8.11	0.00	0.93	1.29
B	B	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.13	0.01	0.01	0.00	0.00	0.02	0.39	0.02	0.03	0.00	0.04	0.46	0.02	0.03	0.27	7.80	0.37	0.57	68.83	7.55	11.79	0.26	0.77	0.61
B	CCC	0.00	0.00	0.00	0.00	0.00	0.09	0.01	0.01	0.00	0.00	0.00	0.00	0.01	0.06	0.04	0.00	0.00	0.03	0.30	0.16	0.07	0.00	0.05	0.88	0.20	0.15	0.11	1.84	2.84	1.52	6.82	45.92	27.65	2.49	5.85	2.91
B	D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10	0.00	0.00	0.00	0.00	0.00	0.29	0.00	0.00	0.00	0.00	0.35	0.00	0.00	0.00	5.32	0.00	0.00	83.17	0.00	4.22	6.56
CCC	CCC	0.00	0.00	0.00	0.00	0.01	0.01	0.17	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.23	0.19	0.01	0.05	0.14	1.17	0.57	0.03	0.42	2.32	1.07	1.33	9.72	4.64	31.33	34.26	12.19
CCC	D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.14	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.29	0.00	0.00	0.00	0.00	0.86	0.00	0.00	0.00	1.87	0.00	0.00	9.94	0.00	58.36	28.53
D	D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.0

Table 1b: One year bivariate transition matrix – US, all industries – 1981-2001 – New estimator - Percent

		AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AA	AA	AA	AA	AA	AA	AA	A	A	A	A	A	A	BBB	BBB	BBB	BBB	BBB	BB	BB	BB	BB	B	B	B	CCC	CCC	D
		AAA	AA	A	BBB	BB	B	CCC	D	AA	A	BBB	BB	B	CCC	D	A	BBB	BB	B	CCC	D	BBB	BB	B	CCC	D	BB	B	CCC	D	B	CCC	D	CCC	D	D
AAA	AAA	88.68	9.46	0.87	0.17	0.08	0.00	0.00	0.00	0.66	0.07	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AAA	AA	0.53	86.14	6.48	0.58	0.07	0.08	0.03	0.02	5.06	0.78	0.13	0.04	0.01	0.00	0.00	0.04	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AAA	A	0.07	2.01	86.48	4.90	0.46	0.17	0.05	0.05	0.13	4.80	0.27	0.04	0.01	0.00	0.00	0.41	0.11	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AAA	BBB	0.04	0.25	4.60	84.32	4.25	0.70	0.17	0.26	0.01	0.33	4.18	0.26	0.06	0.01	0.02	0.02	0.38	0.03	0.00	0.00	0.00	0.08	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AAA	BB	0.03	0.08	0.46	6.49	77.87	7.68	0.89	1.22	0.01	0.04	0.36	3.80	0.42	0.04	0.06	0.00	0.03	0.35	0.03	0.00	0.01	0.00	0.09	0.01	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AAA	B	0.00	0.10	0.29	0.35	5.22	78.88	3.94	6.11	0.00	0.01	0.02	0.25	3.84	0.15	0.26	0.00	0.00	0.02	0.36	0.01	0.02	0.00	0.00	0.10	0.00	0.01	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AAA	CCC	0.12	0.00	0.29	0.66	1.63	10.10	57.04	24.27	0.00	0.02	0.02	0.07	0.46	3.62	1.09	0.00	0.01	0.01	0.07	0.29	0.11	0.00	0.00	0.01	0.06	0.02	0.00	0.00	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00
AAA	D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	93.96	0.00	0.00	0.00	0.00	0.00	0.00	5.43	0.00	0.00	0.00	0.00	0.00	0.48	0.00	0.00	0.00	0.00	0.09	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00
AA	AA	0.01	1.05	0.08	0.01	0.00	0.00	0.00	0.00	84.17	12.44	1.15	0.14	0.16	0.06	0.03	0.56	0.09	0.01	0.02	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AA	A	0.00	0.08	0.52	0.03	0.00	0.00	0.00	0.00	1.97	84.40	4.77	0.46	0.17	0.04	0.05	6.25	0.94	0.11	0.09	0.03	0.02	0.04	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AA	BBB	0.00	0.04	0.04	0.47	0.03	0.01	0.00	0.00	0.24	4.70	81.48	4.21	0.75	0.16	0.25	0.34	6.11	0.33	0.07	0.02	0.02	0.50	0.10	0.07	0.03	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AA	BB	0.00	0.03	0.01	0.04	0.44	0.06	0.01	0.01	0.09	0.47	6.42	75.29	7.42	0.84	1.16	0.04	0.47	5.64	0.59	0.08	0.11	0.04	0.50	0.05	0.01	0.01	0.06	0.07	0.02	0.02	0.01	0.00	0.00	0.00	0.00	0.00
AA	B	0.00	0.00	0.00	0.00	0.03	0.46	0.02	0.03	0.09	0.31	0.36	5.18	76.35	3.65	5.67	0.02	0.03	0.36	5.83	0.30	0.49	0.00	0.03	0.52	0.03	0.04	0.00	0.07	0.01	0.01	0.07	0.03	0.02	0.00	0.00	0.00
AA	CCC	0.00	0.10	0.02	0.01	0.01	0.04	0.40	0.12	0.00	0.28	0.61	1.48	10.09	56.60	22.46	0.02	0.04	0.12	0.70	4.18	1.87	0.01	0.01	0.08	0.41	0.13	0.00	0.01	0.05	0.03	0.02	0.06	0.02	0.02	0.01	0.01
AA	D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.58	0.00	0.00	0.00	0.00	0.00	0.00	91.68	0.00	0.00	0.00	0.00	0.00	6.88	0.00	0.00	0.00	0.00	0.64	0.00	0.00	0.00	0.08	0.00	0.00	0.09	0.00	0.03	0.02
A	A	0.00	0.00	0.13	0.01	0.00	0.00	0.00	0.00	0.06	3.91	0.24	0.02	0.01	0.00	0.00	84.35	9.44	0.88	0.31	0.09	0.10	0.31	0.07	0.03	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
A	BBB	0.00	0.00	0.04	0.07	0.00	0.00	0.00	0.00	0.01	0.35	1.89	0.10	0.02	0.00	0.01	4.64	81.93	4.21	0.74	0.16	0.25	4.54	0.65	0.19	0.06	0.07	0.03	0.02	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
A	BB	0.00	0.00	0.03	0.01	0.06	0.01	0.00	0.00	0.00	0.10	0.14	1.74	0.20	0.02	0.03	0.47	6.42	75.37	7.49	0.85	1.17	0.34	4.25	0.47	0.06	0.08	0.37	0.18	0.05	0.06	0.01	0.01	0.01	0.00	0.00	0.00
A	B	0.00	0.00	0.00	0.00	0.00	0.06	0.00	0.00	0.00	0.10	0.01	0.11	1.78	0.09	0.12	0.30	0.38	5.24	76.57	3.60	5.63	0.02	0.26	4.33	0.24	0.35	0.03	0.41	0.03	0.03	0.15	0.06	0.07	0.00	0.01	0.01
A	CCC	0.00	0.00	0.09	0.01	0.00	0.01	0.05	0.01	0.00	0.01	0.01	0.03	0.20	1.42	0.47	0.30	0.60	1.46	10.08	56.33	22.94	0.04	0.07	0.50	3.23	1.37	0.01	0.04	0.31	0.13	0.01	0.13	0.04	0.02	0.06	0.02
A	D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.00	0.00	0.00	0.00	0.00	0.00	2.16	0.00	0.00	0.00	0.00	0.00	91.78	0.00	0.00	0.00	0.00	5.21	0.00	0.00	0.00	0.49	0.00	0.00	0.18	0.00	0.05	0.06
BBB	BBB	0.00	0.00	0.00	0.08	0.00	0.00	0.00	0.00	0.00	0.03	0.45	0.02	0.01	0.00	0.00	0.30	8.52	0.46	0.10	0.02	0.03	79.65	7.87	1.26	0.31	0.49	0.23	0.08	0.02	0.03	0.02	0.00	0.00	0.00	0.00	0.00
BBB	BB	0.00	0.00	0.00	0.03	0.04	0.00	0.00	0.00	0.00	0.00	0.10	0.22	0.02	0.00	0.00	0.03	0.79	4.00	0.40	0.05	0.06	5.99	73.82	7.21	0.83	1.16	3.63	1.00	0.19	0.29	0.06	0.02	0.03	0.00	0.01	0.01
BBB	B	0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.01	0.08	0.02	0.22	0.01	0.02	0.02	0.29	0.30	4.10	0.18	0.28	0.33	4.95	74.16	3.62	5.67	0.24	3.76	0.20	0.31	0.65	0.18	0.28	0.01	0.03	0.02
BBB	CCC	0.00	0.00	0.00	0.08	0.01	0.01	0.03	0.01	0.00	0.00	0.00	0.00	0.03	0.16	0.06	0.01	0.34	0.10	0.52	3.11	1.13	0.61	1.39	9.48	53.28	23.96	0.06	0.49	2.72	1.22	0.09	0.51	0.18	0.09	0.23	0.10
BBB	D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.26	0.00	0.00	0.00	0.00	0.00	4.88	0.00	0.00	0.00	0.00	89.14	0.00	0.00	0.00	4.48	0.00	0.00	0.74	0.00	0.18	0.28
BB	BB	0.00	0.00	0.00	0.00	0.05	0.00	0.00	0.00	0.00	0.00	0.01	0.13	0.02	0.00	0.00	0.00	0.07	0.78	0.09	0.01	0.01	0.54	11.01	1.01	0.13	0.15	68.09	12.92	1.50	2.10	0.91	0.15	0.22	0.02	0.03	0.03
BB	B	0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.01	0.08	0.07	0.00	0.00	0.00	0.02	0.28	0.43	0.02	0.03	0.03	0.72	5.58	0.27	0.40	4.48	69.22	3.44	5.33	6.56	1.10	1.58	0.05	0.15	0.11
BB	CCC	0.00	0.00	0.00	0.01	0.07	0.02	0.02	0.02	0.00	0.00	0.00	0.00	0.02	0.04	0.02	0.00	0.03	0.28	0.07	0.31	0.12	0.04	0.75	0.80	4.14	1.70	1.25	8.53	49.29	22.35	0.86	5.11	2.13	0.54	1.04	0.43
BB	D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00	0.00	0.00	0.00	0.00	0.49	0.00	0.00	0.00	0.00	6.73	0.00	0.00	0.00	82.34	0.00	0.00	8.11	0.00	0.93	1.29
B	B	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.16	0.01	0.02	0.00	0.00	0.03	0.49	0.02	0.03	0.00	0.05	0.57	0.02	0.04	0.37	8.82	0.39	0.60	69.37	6.86	10.69	0.24	0.66	0.55
B	CCC	0.00	0.00	0.00	0.00	0.00	0.09	0.01	0.01	0.00	0.00	0.00	0.00	0.01	0.06	0.04	0.00	0.00	0.03	0.36	0.20	0.07	0.00	0.04	0.79	0.23	0.15	0.11	1.90	3.22	1.46	7.97	49.41	24.12	2.40	5.12	2.21
B	D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10	0.00	0.00	0.00	0.00	0.00	0.29	0.00	0.00	0.00	0.00	0.35	0.00	0.00	0.00	5.32	0.00	0.00	83.17	0.00	4.22	6.56
CCC	CCC	0.00	0.00	0.00	0.00	0.01	0.01	0.16	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.06	0.27	0.22	0.03	0.04	0.11	1.00	0.51	0.08	0.41	2.20	0.93	1.91	11.01	4.44	35.77	30.51	10.17
CCC	D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.14	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.29	0.00	0.00	0.00	0.00	0.86	0.00	0.00	0.00	1.87	0.00	0.00	9.94	0.00	58.36	28.53
D	D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.0

Table 1c: One year bivariate transition matrix – US all industries – 1981-2001

	IG,IG	IG,NIG	IG,D	NIG,NIG	NIG,D	D,D
IG, IG	95.9%	3.8%	0.2%	0.0%	0.0%	0.0%
IG, NIG	3.8%	89.2%	5.0%	1.8%	0.2%	0.0%
IG, D	0.0%	0.0%	97.9%	0.0%	2.0%	0.1%
NIG, NIG	0.2%	6.7%	0.4%	82.8%	9.5%	0.4%
NIG, D	0.0%	0.0%	3.7%	0.0%	91.0%	5.3%
D, D	0.0%	0.0%	0.0%	0.0%	0.0%	100.0%

Table 2: One year US default correlations - Non investment grade bonds 1981-2001

	Auto	Cons	Ener	Fin	Build	Chem	HiTec	Insur	Leis	Tele	Trans	Util
Auto	3.8%	1.3%	1.2%	0.4%	1.1%	1.6%	2.8%	-0.5%	1.0%	3.9%	1.3%	0.5%
Cons	1.3%	2.8%	-1.4%	1.2%	2.8%	1.6%	1.8%	1.1%	1.3%	3.2%	2.7%	1.9%
Ener	1.2%	-1.4%	**6.4%**	-2.5%	-0.5%	0.4%	-0.1%	-1.6%	-1.0%	-1.4%	-0.1%	0.7%
Fin	0.4%	1.2%	-2.5%	**5.2%**	2.6%	0.1%	0.4%	3.0%	1.6%	3.7%	1.5%	4.5%
Build	1.1%	2.8%	-0.5%	2.6%	**6.1%**	1.2%	2.3%	1.8%	2.3%	**6.5%**	4.2%	1.3%
Chem	1.6%	1.6%	0.4%	0.1%	1.2%	3.2%	1.4%	-1.1%	1.1%	2.8%	1.1%	1.0%
HiTec	2.8%	1.8%	-0.1%	0.4%	2.3%	1.4%	3.3%	0.0%	1.4%	4.7%	1.9%	1.0%
Insur	-0.5%	1.1%	-1.6%	3.0%	1.8%	-1.1%	0.0%	**5.6%**	1.2%	-2.6%	2.3%	1.4%
Leis	1.0%	1.3%	-1.0%	1.6%	2.3%	1.1%	1.4%	1.2%	2.3%	4.0%	2.3%	0.6%
Tele	3.9%	3.2%	-1.4%	3.7%	**6.5%**	2.8%	4.7%	-2.6%	4.0%	**10.7%**	3.2%	-0.8%
Trans	1.3%	2.7%	-0.1%	1.5%	4.2%	1.1%	1.9%	2.3%	2.3%	3.2%	4.3%	-0.2%
Util	0.5%	1.9%	0.7%	4.5%	1.3%	1.0%	1.0%	1.4%	0.6%	-0.8%	-0.2%	**9.4%**

Correlations above 5% are in bold.

S&P sectors:

Auto Aerospace / Automotive / Capital Goods /
Cons Consumer / Service Sector
Ener Energy and Natural Resources
Fin Financial Institutions
Build Forest and Building Products / Homebuild
Chem Healthcare / Chemicals
HiTec High Technology / Computers / Office Equ
Insur Insurance and Real Estate
Leis Leisure Time / MediaTele Telecommunications
Trans Transportation
Util Utility

Table 3: US Default correlations per industry 1981-2001.

All industries

	IG	NIG
IG	0.1%	0.3%
NIG	0.3%	1.7%

Automotive

	IG	NIG
IG	0.7%	0.8%
NIG	0.8%	3.8%

Consumer

	IG	NIG
IG	0.6%	0.2%
NIG	0.2%	2.8%

Energy

	IG	NIG
IG	3.0%	2.8%
NIG	2.8%	6.4%

Financial

	IG	NIG
IG	0.5%	0.4%
NIG	0.4%	5.2%

Building

	IG	NIG
IG	3.3%	1.0%
NIG	1.0%	6.1%

Chemicals

	IG	NIG
IG	1.2%	-0.4%
NIG	-0.4%	3.2%

High tech

	IG	NIG
IG	5.6%	-1.0%
NIG	-1.0%	3.3%

Insurance

	IG	NIG
IG	0.7%	0.3%
NIG	0.3%	5.6%

Leisure

	IG	NIG
IG	3.2%	1.5%
NIG	1.5%	2.3%

Telecom

	IG	NIG
IG	NA	NA
NIG	NA	10.7%

Transport

	IG	NIG
IG	NA	NA
NIG	NA	4.3%

Utility

	IG	NIG
IG	0.9%	0.0%
NIG	0.0%	9.4%

Table 4: US default correlations - Non investment grade bonds 1981-2001
Calculated from equity correlations using a factor model.

One year horizon

	Auto	Cons	Ener	Fin	Build	Chem	HiTec	Insur	Leis	Trans	Util
Auto	4.6%	2.5%	1.9%	2.3%	3.1%	1.8%	0.8%	1.2%	1.6%	4.3%	1.2%
Cons	2.5%	2.8%	1.0%	2.0%	2.0%	1.4%	0.5%	1.0%	1.3%	3.1%	1.5%
Ener	1.9%	1.0%	3.5%	1.3%	1.2%	0.9%	0.4%	0.6%	0.7%	1.6%	1.2%
Fin	2.3%	2.0%	1.3%	2.9%	1.8%	1.2%	0.4%	1.2%	1.1%	2.7%	1.8%
Build	3.1%	2.0%	1.2%	1.8%	3.6%	1.4%	0.6%	0.9%	1.1%	3.5%	0.9%
Chem	1.8%	1.4%	0.9%	1.2%	1.4%	1.3%	0.3%	0.6%	0.8%	2.0%	1.0%
HiTec	0.8%	0.5%	0.4%	0.4%	0.6%	0.3%	0.4%	0.2%	0.3%	0.9%	0.1%
Insur	1.2%	1.0%	0.6%	1.2%	0.9%	0.6%	0.2%	0.9%	0.6%	1.4%	1.0%
Leis	1.6%	1.3%	0.7%	1.1%	1.1%	0.8%	0.3%	0.6%	1.3%	2.0%	0.5%
Trans	4.3%	3.1%	1.6%	2.7%	3.5%	2.0%	0.9%	1.4%	2.0%	8.3%	0.9%
Util	1.2%	1.5%	1.2%	1.8%	0.9%	1.0%	0.1%	1.0%	0.5%	0.9%	6.7%

Table 5 : Default correlations in high and low interest rate volatility periods
US Non investment grade issuers – 1981-2001

High interest rate volatility years

	Auto	Cons.	Ener.	Fin.	Build.	Chem.	High t.	Ins.	Leis.	Tel.	Trans.	Util.
Automotive	4.7%	2.2%	1.5%	2.5%	2.3%	1.9%	4.0%	-0.3%	1.3%	3.2%	2.6%	-0.8%
Consumer	2.2%	3.3%	-2.5%	2.9%	1.8%	2.4%	2.7%	1.1%	2.0%	1.6%	2.4%	0.8%
Energy	1.5%	-2.5%	7.6%	-2.1%	0.0%	-1.7%	0.4%	-1.2%	-1.8%	-1.5%	0.2%	0.3%
Financial	2.5%	2.9%	-2.1%	4.9%	1.6%	1.9%	1.9%	0.7%	1.9%	4.5%	2.5%	-1.0%
Building	2.3%	1.8%	0.0%	1.6%	1.7%	1.7%	2.4%	1.3%	1.1%	2.1%	1.6%	1.0%
Chemicals	1.9%	2.4%	-1.7%	1.9%	1.7%	2.8%	2.6%	0.3%	1.5%	1.8%	1.9%	0.6%
High tech	4.0%	2.7%	0.4%	1.9%	2.4%	2.6%	4.2%	1.1%	1.9%	2.6%	2.6%	0.5%
Insurance	-0.3%	1.1%	-1.2%	0.7%	1.3%	0.3%	1.1%	5.0%	0.4%	-2.0%	1.6%	3.4%
Leisure	1.3%	2.0%	-1.8%	1.9%	1.1%	1.5%	1.9%	0.4%	1.2%	0.9%	1.8%	0.2%
Telecom	3.2%	1.6%	-1.5%	4.5%	2.1%	1.8%	2.6%	-2.0%	0.9%	7.1%	1.7%	-2.0%
Transport	2.6%	2.4%	0.2%	2.5%	1.6%	1.9%	2.6%	1.6%	1.8%	1.7%	2.8%	1.3%
Utility	-0.8%	0.8%	0.3%	-1.0%	1.0%	0.6%	0.5%	3.4%	0.2%	-2.0%	1.3%	4.4%

Low interest rate volatility years

	Auto	Cons.	Ener.	Fin.	Build.	Chem.	High t.	Ins.	Leis.	Tel.	Trans.	Util.
Automotive	1.0%	0.3%	0.3%	0.6%	0.8%	0.5%	0.5%	0.8%	1.1%	5.2%	0.5%	2.1%
Consumer	0.3%	2.3%	-0.3%	0.7%	3.5%	0.7%	0.9%	1.3%	0.9%	7.7%	3.0%	2.9%
Energy	0.3%	-0.3%	4.8%	-2.0%	-0.5%	2.8%	-0.8%	-1.3%	-0.2%	-0.8%	-0.2%	1.1%
Financial	0.6%	0.7%	-2.0%	4.6%	2.8%	-0.1%	0.3%	2.4%	1.5%	8.5%	1.0%	7.6%
Building	0.8%	3.5%	-0.5%	2.8%	8.2%	1.3%	2.5%	1.5%	2.9%	15.7%	5.7%	1.6%
Chemicals	0.5%	0.7%	2.8%	-0.1%	1.3%	3.3%	-0.4%	-1.1%	1.0%	4.9%	0.6%	1.4%
High tech	0.5%	0.9%	-0.8%	0.3%	2.5%	-0.4%	2.2%	-0.1%	1.2%	9.1%	1.4%	1.5%
Insurance	0.8%	1.3%	-1.3%	2.4%	1.5%	-1.1%	-0.1%	4.5%	1.3%	0.0%	2.3%	0.8%
Leisure	1.1%	0.9%	-0.2%	1.5%	2.9%	1.0%	1.2%	1.3%	3.0%	11.0%	2.5%	0.9%
Telecom	5.2%	7.7%	-0.8%	8.5%	15.7%	4.9%	9.1%	0.0%	11.0%	19.7%	8.2%	2.1%
Transport	0.5%	3.0%	-0.2%	1.0%	5.7%	0.6%	1.4%	2.3%	2.5%	8.2%	5.2%	-1.4%
Utility	2.1%	2.9%	1.1%	7.6%	1.6%	1.4%	1.5%	0.8%	0.9%	2.1%	-1.4%	13.7%

Notes: a "high (resp. low) interest rate volatility year" is defined as a year where the standard deviation of the monthly changes in the US 30 year Treasury bond yield is above (resp. below) its 1981-2001 average.

Table 6: Empirical US default correlations - Non investment grade bonds 1981-2001

One year horizon

	Auto	Cons	Ener	Fin	Build	Chem	HiTec	Insur	Leis	Trans	Util
Auto	3.8%	1.3%	1.2%	0.4%	1.1%	1.6%	2.8%	-0.5%	1.0%	1.3%	0.5%
Cons	1.3%	2.8%	-1.4%	1.2%	2.8%	1.6%	1.8%	1.1%	1.3%	2.7%	1.9%
Ener	1.2%	-1.4%	6.4%	-2.5%	-0.5%	0.4%	-0.1%	-1.6%	-1.0%	-0.1%	0.7%
Fin	0.4%	1.2%	-2.5%	5.2%	2.6%	0.1%	0.4%	3.0%	1.6%	1.5%	4.5%
Build	1.1%	2.8%	-0.5%	2.6%	6.1%	1.2%	2.3%	1.8%	2.3%	4.2%	1.3%
Chem	1.6%	1.6%	0.4%	0.1%	1.2%	3.2%	1.4%	-1.1%	1.1%	1.1%	1.0%
HiTec	2.8%	1.8%	-0.1%	0.4%	2.3%	1.4%	3.3%	0.0%	1.4%	1.9%	1.0%
Insur	-0.5%	1.1%	-1.6%	3.0%	1.8%	-1.1%	0.0%	5.6%	1.2%	2.3%	1.4%
Leis	1.0%	1.3%	-1.0%	1.6%	2.3%	1.1%	1.4%	1.2%	2.3%	2.3%	0.6%
Trans	1.3%	2.7%	-0.1%	1.5%	4.2%	1.1%	1.9%	2.3%	2.3%	4.3%	-0.2%
Util	0.5%	1.9%	0.7%	4.5%	1.3%	1.0%	1.0%	1.4%	0.6%	-0.2%	9.4%

Three year horizon

	Auto	Cons	Ener	Fin	Build	Chem	HiTec	Insur	Leis	Trans	Util
Auto	6.1%	0.9%	5.1%	-1.4%	2.8%	6.4%	3.6%	-0.1%	2.3%	2.1%	3.0%
Cons	0.9%	3.7%	-4.1%	0.4%	3.5%	2.1%	2.4%	2.6%	3.1%	4.1%	3.1%
Ener	5.1%	-4.1%	13.0%	-7.0%	-1.5%	4.9%	0.9%	-3.5%	-3.2%	-2.3%	2.0%
Fin	-1.4%	0.4%	-7.0%	12.9%	8.3%	-1.2%	1.1%	7.9%	5.3%	5.5%	11.1%
Build	2.8%	3.5%	-1.5%	8.3%	10.7%	3.3%	4.1%	6.6%	6.7%	7.7%	4.6%
Chem	6.4%	2.1%	4.9%	-1.2%	3.3%	9.5%	4.8%	-1.1%	4.7%	2.4%	0.7%
HiTec	3.6%	2.4%	0.9%	1.1%	4.1%	4.8%	4.9%	1.0%	3.2%	3.8%	2.9%
Insur	-0.1%	2.6%	-3.5%	7.9%	6.6%	-1.1%	1.0%	6.5%	4.5%	5.1%	3.2%
Leis	2.3%	3.1%	-3.2%	5.3%	6.7%	4.7%	3.2%	4.5%	6.7%	6.4%	3.3%
Trans	2.1%	4.1%	-2.3%	5.5%	7.7%	2.4%	3.8%	5.1%	6.4%	7.2%	2.9%
Util	3.0%	3.1%	2.0%	11.1%	4.6%	0.7%	2.9%	3.2%	3.3%	2.9%	12.7%

Five year horizon

	Auto	Cons	Ener	Fin	Build	Chem	HiTec	Insur	Leis	Trans	Util
Auto	10.6%	2.1%	8.5%	-0.3%	3.1%	9.9%	5.7%	2.7%	3.4%	8.3%	3.7%
Cons	2.1%	7.1%	-7.8%	1.3%	5.3%	4.7%	3.2%	4.2%	7.0%	9.4%	5.0%
Ener	8.5%	-7.8%	21.8%	-9.5%	-6.3%	5.0%	4.5%	-1.2%	-7.2%	1.5%	5.2%
Fin	-0.3%	1.3%	-9.5%	19.3%	15.1%	1.8%	4.2%	9.1%	10.0%	14.8%	12.5%
Build	3.1%	5.3%	-6.3%	15.1%	14.3%	5.2%	4.5%	7.6%	11.7%	13.3%	8.0%
Chem	9.9%	4.7%	5.0%	1.8%	5.2%	14.6%	3.4%	1.9%	7.2%	6.5%	0.7%
HiTec	5.7%	3.2%	4.5%	4.2%	4.5%	3.4%	5.5%	3.8%	3.4%	6.0%	5.6%
Insur	2.7%	4.2%	-1.2%	9.1%	7.6%	1.9%	3.8%	5.8%	6.9%	7.3%	5.1%
Leis	3.4%	7.0%	-7.2%	10.0%	11.7%	7.2%	3.4%	6.9%	12.6%	15.1%	6.1%
Trans	8.3%	9.4%	1.5%	14.8%	13.3%	6.5%	6.0%	7.3%	15.1%	13.8%	6.9%
Util	3.7%	5.0%	5.2%	12.5%	8.0%	0.7%	5.6%	5.1%	6.1%	6.9%	12.1%

Table 7: US default correlations - Non investment grade bonds 1981-2001
Calculated from equity correlations.

One year horizon

	Auto	Cons	Ener	Fin	Build	Chem	HiTec	Insur	Leis	Trans	Util
Auto	4.6%	2.5%	1.9%	2.3%	3.1%	1.8%	0.8%	1.2%	1.6%	4.3%	1.2%
Cons	2.5%	2.8%	1.0%	2.0%	2.0%	1.4%	0.5%	1.0%	1.3%	3.1%	1.5%
Ener	1.9%	1.0%	3.5%	1.3%	1.2%	0.9%	0.4%	0.6%	0.7%	1.6%	1.2%
Fin	2.3%	2.0%	1.3%	2.9%	1.8%	1.2%	0.4%	1.2%	1.1%	2.7%	1.8%
Build	3.1%	2.0%	1.2%	1.8%	3.6%	1.4%	0.6%	0.9%	1.1%	3.5%	0.9%
Chem	1.8%	1.4%	0.9%	1.2%	1.4%	1.3%	0.3%	0.6%	0.8%	2.0%	1.0%
HiTec	0.8%	0.5%	0.4%	0.4%	0.6%	0.3%	0.4%	0.2%	0.3%	0.9%	0.1%
Insur	1.2%	1.0%	0.6%	1.2%	0.9%	0.6%	0.2%	0.9%	0.6%	1.4%	1.0%
Leis	1.6%	1.3%	0.7%	1.1%	1.1%	0.8%	0.3%	0.6%	1.3%	2.0%	0.5%
Trans	4.3%	3.1%	1.6%	2.7%	3.5%	2.0%	0.9%	1.4%	2.0%	8.3%	0.9%
Util	1.2%	1.5%	1.2%	1.8%	0.9%	1.0%	0.1%	1.0%	0.5%	0.9%	6.7%

Three year horizon

	Auto	Cons	Ener	Fin	Build	Chem	HiTec	Insur	Leis	Trans	Util
Auto	7.6%	4.3%	3.4%	4.0%	5.4%	3.2%	1.7%	2.2%	2.9%	7.3%	2.2%
Cons	4.3%	4.8%	1.8%	3.4%	3.6%	2.6%	1.0%	1.9%	2.3%	5.3%	2.5%
Ener	3.4%	1.8%	5.9%	2.3%	2.3%	1.6%	0.7%	1.2%	1.3%	2.9%	2.1%
Fin	4.0%	3.4%	2.3%	5.0%	3.3%	2.3%	0.8%	2.2%	2.1%	4.6%	3.2%
Build	5.4%	3.6%	2.3%	3.3%	6.4%	2.6%	1.2%	1.8%	2.1%	6.1%	1.7%
Chem	3.2%	2.6%	1.6%	2.3%	2.6%	2.4%	0.7%	1.1%	1.6%	3.6%	1.9%
HiTec	1.7%	1.0%	0.7%	0.8%	1.2%	0.7%	0.8%	0.4%	0.6%	1.7%	0.2%
Insur	2.2%	1.9%	1.2%	2.2%	1.8%	1.1%	0.4%	1.9%	1.1%	2.7%	1.9%
Leis	2.9%	2.3%	1.3%	2.1%	2.1%	1.6%	0.6%	1.1%	2.5%	3.7%	1.0%
Trans	7.3%	5.3%	2.9%	4.6%	6.1%	3.6%	1.7%	2.7%	3.7%	13.0%	1.6%
Util	2.2%	2.5%	2.1%	3.2%	1.7%	1.9%	0.2%	1.9%	1.0%	1.6%	10.1%

Five year horizon

	Auto	Cons	Ener	Fin	Build	Chem	HiTec	Insur	Leis	Trans	Util
Auto	9.1%	5.1%	4.1%	4.9%	6.5%	3.8%	2.0%	2.8%	3.4%	8.5%	2.6%
Cons	5.1%	5.5%	2.1%	4.1%	4.3%	3.0%	1.2%	2.4%	2.8%	6.1%	3.0%
Ener	4.1%	2.1%	7.0%	2.7%	2.8%	1.9%	0.9%	1.5%	1.5%	3.4%	2.6%
Fin	4.9%	4.1%	2.7%	6.0%	4.1%	2.7%	1.0%	2.7%	2.5%	5.4%	3.8%
Build	6.5%	4.3%	2.8%	4.1%	7.8%	3.1%	1.5%	2.3%	2.6%	7.1%	2.1%
Chem	3.8%	3.0%	1.9%	2.7%	3.1%	2.8%	0.8%	1.4%	1.9%	4.2%	2.2%
HiTec	2.0%	1.2%	0.9%	1.0%	1.5%	0.8%	1.1%	0.6%	0.8%	2.1%	0.2%
Insur	2.8%	2.4%	1.5%	2.7%	2.3%	1.4%	0.6%	2.4%	1.4%	3.3%	2.4%
Leis	3.4%	2.8%	1.5%	2.5%	2.6%	1.9%	0.8%	1.4%	3.0%	4.3%	1.2%
Trans	8.5%	6.1%	3.4%	5.4%	7.1%	4.2%	2.1%	3.3%	4.3%	14.5%	2.0%
Util	2.6%	3.0%	2.6%	3.8%	2.1%	2.2%	0.2%	2.4%	1.2%	2.0%	11.7%

Table 8: One year bivariate transition matrix – US all industries – Recessions

	IG,IG	IG,NIG	IG,D	NIG,NIG	NIG,D	D,D
IG, IG	95.0%	4.5%	0.4%	0.1%	0.0%	0.0%
IG, NIG	3.2%	86.3%	8.1%	2.1%	0.4%	0.0%
IG, D	0.0%	0.0%	97.4%	0.0%	2.3%	0.2%
NIG, NIG	0.1%	5.5%	0.5%	77.6%	15.4%	0.9%
NIG, D	0.0%	0.0%	3.1%	0.0%	88.0%	8.9%
D, D	0.0%	0.0%	0.0%	0.0%	0.0%	100.0%

Table 9: One year bivariate transition matrix – US all industries – Growth years

	IG,IG	IG,NIG	IG,D	NIG,NIG	NIG,D	D,D
IG, IG	96.2%	3.6%	0.2%	0.0%	0.0%	0.0%
IG, NIG	4.0%	90.1%	4.1%	1.7%	0.2%	0.0%
IG, D	0.0%	0.0%	98.1%	0.0%	1.9%	0.1%
NIG, NIG	0.2%	7.1%	0.3%	84.3%	7.9%	0.2%
NIG, D	0.0%	0.0%	3.9%	0.0%	91.8%	4.3%
D, D	0.0%	0.0%	0.0%	0.0%	0.0%	100.0%

Table 10: One year correlation matrices in growth and recession years

Recession

	IG	NIG
IG	0.09%	0.27%
NIG	0.27%	1.50%

Growth

	IG	NIG
IG	0.08%	0.13%
NIG	0.13%	0.77%