# Counterparty Risk in Derivatives and Collateral Policies: The Replicating Portfolio Approach

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**Abstract**

In this paper we develop the replicating portfolio approach to the evaluation and hedging of counterparty risk in derivatives. We show that counterparty risk in a linear contract may be represented by a sequence of vulnerable options of the call type for the long position and of the put type for the short end of the contract. Replicating portfolios are also designed for risk mitigating techniques which require exchange of collateral. We focus on two different policies. The first, requiring periodic deposit of the collateral, leads to represent counterparty risk as a ratchet option. The second, requiring deposits of the collateral for exposures beyond a given threshold, leads to a representation of counterparty risk in terms of a stream of ladder options. A pricing example shows that both weekly collateral or contingent collateral may lead to a substantial counterparty risk reduction, and both may represent competitive tools in the market for derivatives transactions with corporate counterparties.

**Preliminary draft**

## 1 Introduction

In this paper we address the problem of counterparty risk evaluation and management in derivative transactions using a replicating portfolio approach. We show how to unbundle a very simple OTC transaction in a replicating portfolio that takes into account the credit risk involved. The idea was first proposed by Sorensen and Bollier (1994), that suggested to decompose the counterparty risk in a swap transaction in terms of a sequence of vulnerable swaptions. Cherubini (2004) took over the idea allowing for dependence between counterparty risk and interest rate risk. Representing counterparty risk in terms of a replicating portfolio is an effective way of evaluating the risk in the position and to gauge

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the effects of counterparty risk on the corresponding hedging strategy. Indeed, overlooking counterparty risk induces two kinds of mistake: the first is that one forgets about a relevant source of risk; the second is that one is led to misprice the contract and apply the wrong hedging policy. So, it happens, as we will show, that linear contracts may fail to be linear, and be actually gamma-negative. Along the same line, the value of a linear contract may turn out to be sensitive to changes in volatility because of credit risk, an exposure that one would not be aware of unless he allowed for default of the counterparty. These effects are actually straightforward if one represents credit risk by a replicating portfolio: this would result in short call option positions for the long end of the contract and short put options for the short end. Using this financial engineering tool is also an easy way to gauge the effect of risk mitigating techniques. The typical risk mistigating technique, that is netting, is only a viable choice for the management of counterparty risk among financial intermediaries: it is obvious that the netting provision naturally leads to represent counterparty risk as a position in a basket option. For derivative transactions with corporate counterparties, the most effective risk mitigating technique appears to be the use of collateral. The counterparty risk in a derivative transaction with collateral may also be represented by a replicating portfolio, namely a portfolio of exotic options. We will provide two practical examples. In the first, we will allow for deposit of the collateral at intermediate dates of the transaction, and we will show that this results in introducing a ratchet option structure in the counterparty risk representation. In the second, we design a collateral policy under which the deposit is contingent on the value of the exposure beyond a given threshold: we will show that this will result in a ladder option representation of counterparty risk.

The plan of the paper is as follows. In section 2 we introduce the problem of counterparty risk in a linear derivative contract, assuming independence between credit risk and market risk. In section 3 we review possible approaches to allow for dependence between the underlying asset of the contract and default od the counterparty. In section 4 we introduce risk-mitigating techniques, with particular focus on periodic and contingent collateral policies. Section 5 will conclude.

## 2 Counterparty risk in linear contracts

In this section we lay out the basic set up of the model. We describe the loss the would be incurred if one of the two parties were to default during the life of a linear contract. Of course, the loss for each of the two parties will depend on the joint event of default of the counterparty and a positive market value of the contract. The aim of this section is to pinpoint the risk factors involved in OTC transactions. For the time being we assume that such risk factors be orthogonal, and we extract the replicating portfolio of a linear contract. Extension to dependency among the factors would be quite natural in the following sections.

## 2.1 Credit risk in linear OTC contracts: the pay-off

Consider a simple forward contract, written on the underlying asset $S$ for delivery at time $T$. The contract is stipulated at time $t = 0$ and according to the market practice the delivery price is assumed to be set equal to the forward price. In a default-free world the pay-off would be

$$CF(T) = S(T) - F(S, 0, T) \tag{1}$$

Given the risk-free discount function $v(t, T)$ the forward price at time $t$ for delivery at time $T$ is defined as $F(S, t, T) \equiv S(t)/v(t, T)$.

Introduce now counterparty risk into the picture. For the time being we assume for the sake of simplicity that default could only occur at the end of contract, time $T$. We assume that if at that date the contract has positive value (*in-the-money*) for the defaulted counterparty, the other party is required to honour its obligation. If instead the defaulted counterparty is *out-of-the-money*, the other party's claim is assumed to have the same seniority as other debt issues.

The long end of the contract is denoted $A$, and the contract is *in-the-money* for her if $S(T) > F(0)$. If instead we have $S(T) \leq F(0)$ the contract is *in-the-money* for the short end of the contract, denoted $B$. Notation is completed by denoting $\mathbf{1}_i$ the indicator function taking value 1 in case of default of the counterparty $i = A, B$ by time $T$. By the same token, $RR_i$ denotes the recovery rate of each counterparty and $Lgd_i = 1 - RR_i$ the loss-given-default figure.

It is easy to check that allowing for counterparty risk changes the pay-off, as seen from each of the counterparty. For the long end of the contract $A$ we have in fact

$$\begin{aligned} CF_A(T) &= \max[S(T) - F(S, 0, T), 0](1 - \mathbf{1}_B) + \\ &\quad + \max[S(T) - F(S, 0, T), 0] RR_B \mathbf{1}_B \\ &\quad - \max[F(S, 0, T) - S(T), 0] \\ &= CF(T) - Lgd_B \mathbf{1}_B \max[S(T) - F(S, 0, T), 0] \end{aligned} \tag{2}$$

For the short end of the contract we have instead that

$$\begin{aligned} CF_B(T) &= \max[F(S, 0, T) - S(T), 0](1 - \mathbf{1}_A) + \\ &\quad + \max[F(S, 0, T) - S(T), 0] RR_A \mathbf{1}_A \\ &\quad - \max[S(T) - F(S, 0, T), 0] \\ &= CF(T) - Lgd_A \mathbf{1}_A \max[F(S, 0, T) - S(T), 0] \end{aligned} \tag{3}$$

Counterparty risk is then represented by a short position in options. The option is of call type for the long end of the contract and of put type from the viewpoint of the short counterparty.

## 2.2 Evaluation

As it is well known, in a default free world we have that

$$CF(t) = v(t,T)\left[E_Q(S(T)) - F(S,0,T)\right] = S(t) - v(t,T)\,F(S,0,T) \quad (4)$$

where $E_Q(.)$ is the expected value computed under the martingale measure $Q$ (in the case of interest rate derivatives, that is assumed to be the *forward martingale measure*). By the property of the risk-neutral measure we may also write $CF(t) = v(t,T)\left[F(S,t,T) - F(S,0,T)\right]$ and it is immediate to verify the market convention $CF(0) = 0$.

It is clear that counterparty risk introduces non-linearity in the pay-off, in the form of a short position in option. From the viewpoint of the long end of the contract we have in fact that

$$CF_A(t) = CF(t) - v(t,T)\,E_Q\left[Lgd_B \mathbf{1}_B \max\left[S(T) - F(S,0,T), 0\right]\right] \quad (5)$$

and from that of the short position

$$CF_B(t) = -CF(t) - v(t,T)\,E_Q\left[Lgd_A \mathbf{1}_A \max\left[F(S,0,T) - S(T), 0\right]\right] \quad (6)$$

In general then we have that the impact of counterparty risk on the value of the forward contract is given by

$$v(t,T)\,E_Q\left[Lgd_i \mathbf{1}_i \max\left[\omega\left(S(T) - F(S,0,T)\right), 0\right]\right] \quad (7)$$

whith $\omega$ equal to 1 or $-1$ if the position is long or short respectively.

Notice that counterparty risk results from the combination of risk factors that may actually be dependent on each other. We have discount factor risk, linked to the fluctuation in the term structure, market risk linked to fluctuations of the underlying asset and credit risk due to changes in the credit standing of the counterparty. If for the time being we make the simplifying assumption that all these risk factors be independent, we may compute the value of counterparty risk as

$$v(t,T)\,E_Q\left[Lgd_i \mathbf{1}_i\right] E_Q\left[\max\left[\omega\left(S(T) - F(S,0,T)\right), 0\right]\right] \quad (8)$$

The term $E_Q\left[Lgd_i \mathbf{1}_i\right]$ is the so-called *expected loss* on name $i$: $EL_i$.Notice that this is nothing but an extension of the corporate bond evaluation problem. A defaultable zero-coupon-bond issued by counterparty $i$ for maturity $T$, $D(t,T)$, is simply obtained by setting the pay-off of the derivative equal to 1, and the price turns out to be

$$D(t,T) = v(t,T) - v(t,T)\,E_Q\left[Lgd_i \mathbf{1}_i\right] \quad (9)$$

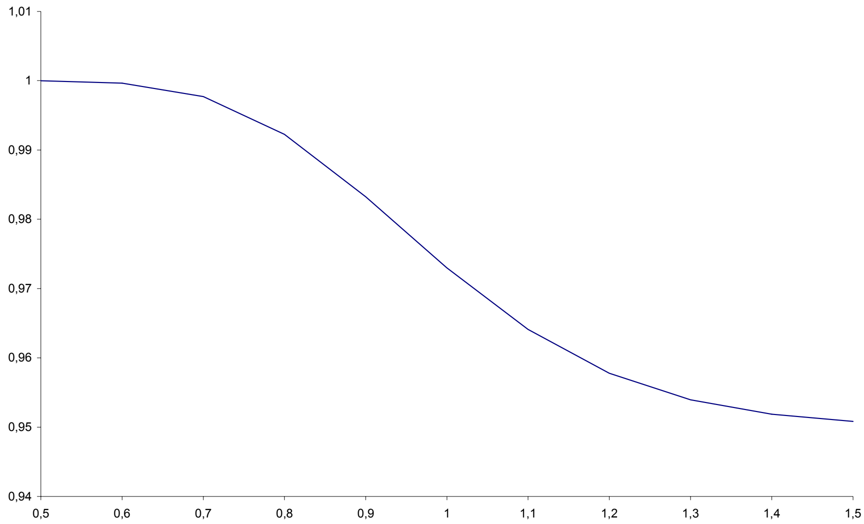


Figure 1: Delta of a long position in a forward contract, accounting for counterparty risk

## 2.3 Greek letters

An important effect of counterparty risk is to transform linear contracts into non-linear ones. So, if one overlooks counterparty risk not only he fails to recognise a relevant source of risk which is in the contract, but he is also induced to mis-represent the main market risk which is built-in the contract. The delta of a contract is

$$\Delta_A = 1 - EL_B N(\alpha) \quad \Delta_B = -1 + EL_A N(-\alpha) \tag{10}$$

with $N(.)$ the standard normal distribution

$$\alpha \equiv \frac{\ln (S(t)/(v(t,T) F(S,0,T))) + \sigma^2 (T-t)/2}{\sigma \sqrt{T-t}}$$

In figure 1 we represent the delta for the long end of the contract, plotted against the value of the underlying asset, measured in terms of the delivery price in the contract. Notice that the value of the delta tends to decrease toward a lower bound given by the value of the expected loss (assumed to be 5% in the example) the higher the value of the underlying asset: the more the forward contract is in-the-money for the long end of the contract, the lower the delta.

Another point, straightforward, which is worth noting is that the delta is non linear around the at-the-money value. This means that counterparty risk implies

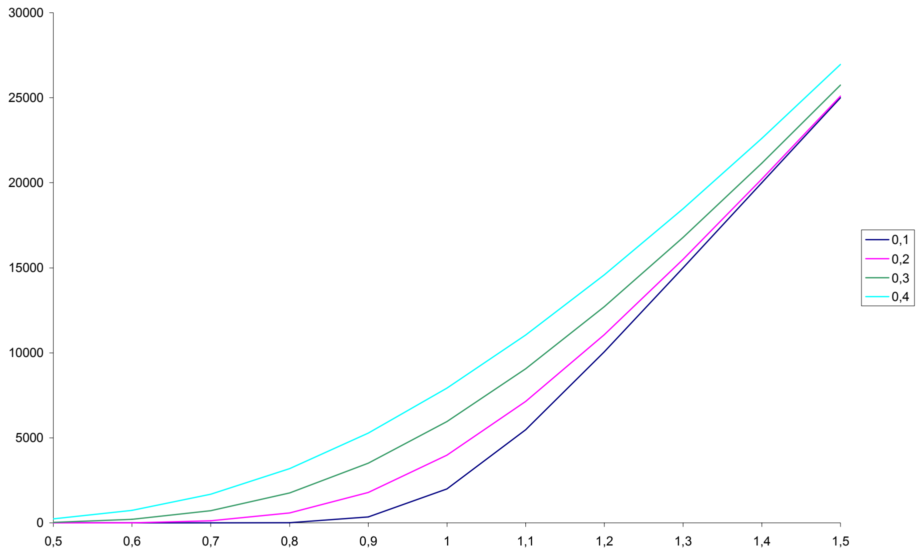


Figure 2: Counterparty risk of a long position in a forward contract

a gamma and vega negative position on the underlying asset. The gamma is in fact given by

$$\Gamma = -\frac{n(\alpha)}{\sigma\sqrt{T-t}} \tag{11}$$

with $n(.)$ the standard normal density. Figure 2 gives an idea of the relevance

of this factor for the evaluation of the forward contract. We plot the value of counterparty risk for the long end of a forward contract against the value of the underlying and for different values of volatility. The impact of an increase in volatility on counterparty risk is increasing and proportionally higher for the at-the-money contracts: this is obviously the case of contracts at the date of stipulation.

## 2.4 Default before maturity

We extend here the analysis to the case in which default may occur before maturity of the contract. We then assume that the contract were market-to-market at a set of intermediate dates, and that the counterparty could default in each period between these dates. We then to partition the time from $t$ through $T$ in sub-intervals by a set of dates $\{t_1, t_2, ..., t_n\}$. Denote by $\overline{G}_A(t_i)$ and $\overline{G}_B(t_i)$

the survival probability of counterparty $A$ and counterparty $B$ beyond time $t_i$. Then, the probability that one of the two counterparties, say $A$, defaulted between time $t_{i-1}$ and $t_i$ is $\overline{G}_A(t_{i-1}) - \overline{G}_A(t_i)$.

The loss incurred in the event of default at time $\tau < T$ is given by the pay-off

$$\max\left(S(\tau) - v(\tau, T) F(S, 0, T), 0\right) \tag{12}$$

for the long end of the contract and

$$\max\left(v(\tau, T) F(S, 0, T) - S(\tau), 0\right) \tag{13}$$

for the short end. We may assume for simplicity that if the counterparty defaults at time $\tau$, $t_{i-1} < \tau < t_i$ the position is replaced at time $t_i$. In this case, the value of counterparty risk is given by

$$Lgd_j \sum_{i=1}^{n} \left[\overline{G}_j(t_{i-1}) - \overline{G}_j(t_i)\right] Call\left(S, t; t_i, v(t_i, T) F(S, 0, T)\right) \tag{14}$$

where $Call(.)$ denotes a call option.

In the limit, in continuous time we would have

$$\int_t^T Lgd_j E_Q\left[\max\left(\omega\left(S(\tau) - v(\tau, T) F(S, 0, T), 0\right)\right)\right] g_j(\tau)\, d\tau \tag{15}$$

where $g_j(.)$ is the density of the default time.

Figure 3 describes counterparty risk for different marking to market periods, for a one year forward contract. We see that the reduction impact in credit risk is particularly relevant if we compare marking to market at the end of the contract with a weekly frequency. Moving from weekly to daily monitoring of the position does not improve much the counterparty risk figure.

# 3 Dependence between counterparty risk and market risk

The approach described above assumes independence between credit risk and the dynamics of the underlying asset. The vulnerability feature of the option relyies on the fact that its exercise is not only contingent on the contract ending up in the money, but also on the event of default of the counterparty. Of course, if the balance sheets of one or both the two parties in the contract are affected by changes in the value of the underlying asset, the two events can be correlated. This problem can be handled in two ways. The first is to resort to a change of measure, using the defaultable bond as a numeraire. The second is to use copula functions to represent dependence between the value of the derivative and default of the counteparty.

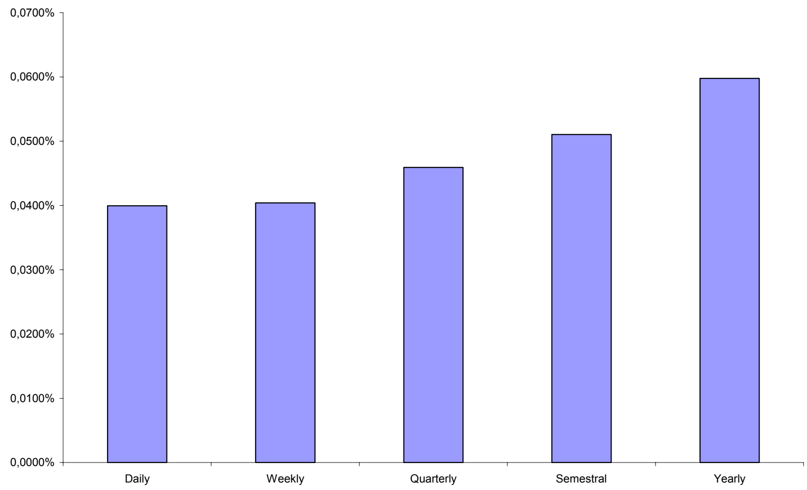


Figure 3: The effect of periodic marking to market of the contract on counterparty risk

## 3.1 The change of numeraire approach

The approach of the change in numeraire seems to be the most widespread approach in practical applications. In fact, it looks quite natural to apply the discount factor recovered from defaultable debt issued by the counterparty, or by similar entities, to the evaluation of the derivative contract. It should not be forgotten, though, that technically this approach would involve a change of measure, at least if one has to take into account the dependence problem.

To grasp the idea, notice that the value of derivative $g$, including counterparty risk is obtained as

$$g_A(t) = E_Q\left[v(t,T)\left(1 - Lgd_B \mathbf{1}_B\right) g(S,T)\right] \tag{16}$$

where $g(S,T)$ is the pay-off function. We have already seen that in the specific case $g(S,T) = 1$ we get the value of the defaultable bond

$$D(t,T) = E_Q\left[v(t,T)\left(1 - Lgd_B \mathbf{1}_B\right)\right] \tag{17}$$

Under standard regularity conditions we may assume that there exists a martingale measure $Q_A$ such that

$$g_A(t) = D(t,T)\, E_{Q_A}\left[g(S,T)\right] \tag{18}$$

It is clear, however, that $Q_A = Q$ if and only if underlying asset risk and credit risk of the counterparty are orthogonal. If they are not, the change of measure would call for a sort of "quanto adjustment", and the risk-neutral drift of the underlying asset should be changed by subtracting the covariance between the two risks.

## 3.2 The copula function approach

The copula function approach was proposed in Cherubini and Luciano (2003) and Cherubini (2004). The main idea is to derive the price of vulnerable call and put options as the integrals of vulnerable pricing kernels, represented by copula functions. The price of the vulnerable option representing credit risk for the long end of the contract is written in this case as

$$Lgd_B \sum_{i=1}^{n} v(t, t_i) \int_k^{\infty} Q(S(t_i) > u, t_{i-1} \leq \tau_B < t_i)\, du \tag{19}$$

where $\boldsymbol{\tau}_B$ is the default time of counterpart $B$ and $k = F(S, 0, T)$. Using copula functions the above relationship can be written as

$$Lgd_B \sum_{i=j}^{n-1} v(t, t_i) \int_k^{\infty} \widetilde{C}_B \left[\overline{Q}(u), \overline{G}_B(t_{i-1}) - \overline{G}_B(t_i)\right] du \tag{20}$$

where $\overline{Q}(u)$ denotes the risk-neutral probability of the event $S(t_i) > u$, and $\overline{G}_B(t)$ is the survival probability, under the same measure, of counterparty $B$ beyond time $t$. $\widetilde{C}_B[x, y]$ is a bivariate function fulfilling the requirements for a copula function. As it is well known, copula functions enable to represent whatever joint probability as a function of the marginal ones (see Cherubini et al. 2004 for financial applications). In this case, function $\widetilde{C}[x, y]$ is such that

$$Q(S(t_i) > u, t_{i-1} \leq \tau_B < t_i) = \widetilde{C}_B \left[\overline{Q}(u), \overline{G}_B(t_{i-1}) - \overline{G}_B(t_i)\right] \tag{21}$$

Following the same argument, we may compute counterpart risk for the short end of the contract:

$$Lgd_A \sum_{i=j}^{n} v(t, t_i) \int_0^{k} C_A \left[Q(u), \overline{G}_A(t_{i-1}) - \overline{G}_A(t_i)\right] du \tag{22}$$

where $Q(u) \equiv 1 - \overline{Q}(u)$ is the risk-neutral probability distribution of $S(t_i)$.

Notice that the copula function $C_A[x, y]$ is conceptually different from the function $\widetilde{C}_A[x, y]$ used above, beyond the obvious difference that they are referred to different counterparts. In fact, we have

$$Q(S(t_i) \leq u, t_{i-1} \leq \tau_A < t_i) = C_A \left[Q(u), \overline{G}_A(t_{i-1}) - \overline{G}_A(t_i)\right] \tag{23}$$

So, if we drop for a minute the subscripts $A$ and $B$, the copula function $C[x,y]$ represents dependence between the event that the underlying asset is **lower** than $u$ and default of the counterparty, while copula function $\widetilde{C}[x,y]$ determines dependence between the event that the underlying asset is **greater** than $u$ and default of the counterparty. It may be easily proved that the two copula functions are linked by the relationship

$$C\left[Q(u), \overline{G}(t_{i-1}) - \overline{G}(t_i)\right] = \overline{G}(t_{i-1}) - \overline{G}(t_i) - \widetilde{C}\left[\overline{Q}(u), \overline{G}(t_i) - \overline{G}(t_i)\right] \quad (24)$$

# 4 Risk mitigating techniques

In the recent developments of derivative markets mitigating techniques have been introduced to reduce credit risk. The most widespread tool used, mostly in transactions between financial intermediaries, is netting. Allowing for netting naturally leads to representing credit risk as a position in basket options. This provision is typically coupled with the use of collateral, and possibly with the reset of the price of the transactions. This tool, which is somewhat borrowed from the working of the futures market, may be also effectively employed in transactions with corporate customers. Here we show how to apply the replicating portfolio approach to the case of collateral.

## 4.1 Periodic collateral

Say the time to exercise is broken down into a sequence of dates $\{t_1, t_2, ..., t_n\}$. At each date the contract is marked to market and the counterparty which is out of the money deposits the amount lost in the contract as collateral with the other party. For example, the long end of the contract deposits

$$K_{i-1} = \max\left(S(t_{i-1}) - v(t_{i-1}, T) F(S, 0, T), 0\right) \quad (25)$$

We assume that the amount could be deposited in terms of the risk-free asset, so yielding $1/v(t_{i-1}, t_i)$ over the period. At the end of the period the collateral amount will be

$$\frac{K_{i-1}}{v(t_{i-1}, t_i)} = \max\left(\frac{S(t_{i-1})}{v(t_{i-1}, t_i)} - v(t_i, T) F(S, 0, T), 0\right) \quad (26)$$

where we assumed the interest rate to be deterministic (alternatively, the same result would be obtained if one worked under the forward martingale measure in a stochastic interest rate environment). If default occurs at time $\tau$, $t_{i-1} < \tau < t_i$, and we assume that the position be replaced at time $t_i$, we may rewrite the loss as

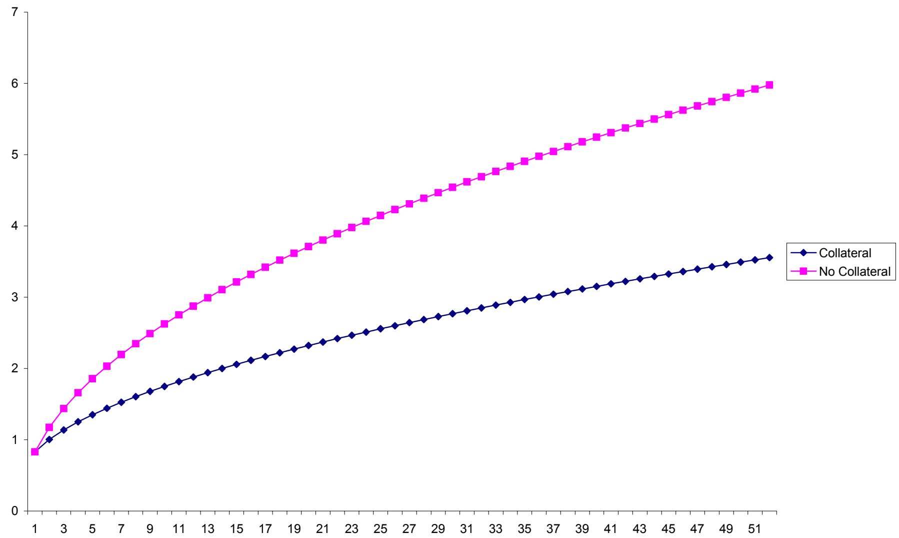


Figure 4: Stream of options representing counterparty risk, with and without collateral

$$\max\left(S\left(t_i\right)-v\left(t_i,T\right)F\left(S,0,T\right)-\frac{K_{i-1}}{v\left(t_{i-1},t_i\right)},0\right)=\max\left(S\left(t_i\right)-\frac{S\left(t_{i-1}\right)}{v\left(t_{i-1},t_i\right)},0\right) \tag{27}$$

if $K_{i-1}>0$ and

$$\max\left(S\left(t_i\right)-v\left(t_i,T\right)F\left(S,0,T\right),0\right) \tag{28}$$

otherwise.

Denote $Q_{i-1}=\Pr\left(S\left(t_{i-1}\right)\leq v\left(t_{i-1},T\right)F\left(S,0,T\right)\right)$ the probability of the event $K_{i-1}=0$ under the risk-neutral measure. The value of the loss is then represented by

$$Lgd_i\sum_{i=1}^{n}\left[\overline{G}\left(t_{i-1}\right)-\overline{G}\left(t_i\right)\right]\left[\begin{array}{c}Q_{i-1}Call\left(S,t;t_i,v\left(t_i,T\right)F\left(S,0,T\right)\right)+\\+\left(1-Q_{i-1}\right)FSCall\left(S,t;t_i,\frac{S\left(t_{i-1}\right)}{v\left(t_{i-1},t_i\right)}\right)\end{array}\right] \tag{29}$$

where $FSCall\left(.\right)$ denotes a forward start call option.

Figure 4 describes the sequence of options representing counterparty risk with and without a periodic collateral policy. The reduction in countertparty risk, represented by the integral below the lines, looks sizable.

Figure 5 shows the impact of a periodic collateral policy on counterparty risk. Reduction of risk looks substantial particularly for the weekly and daily frequency.

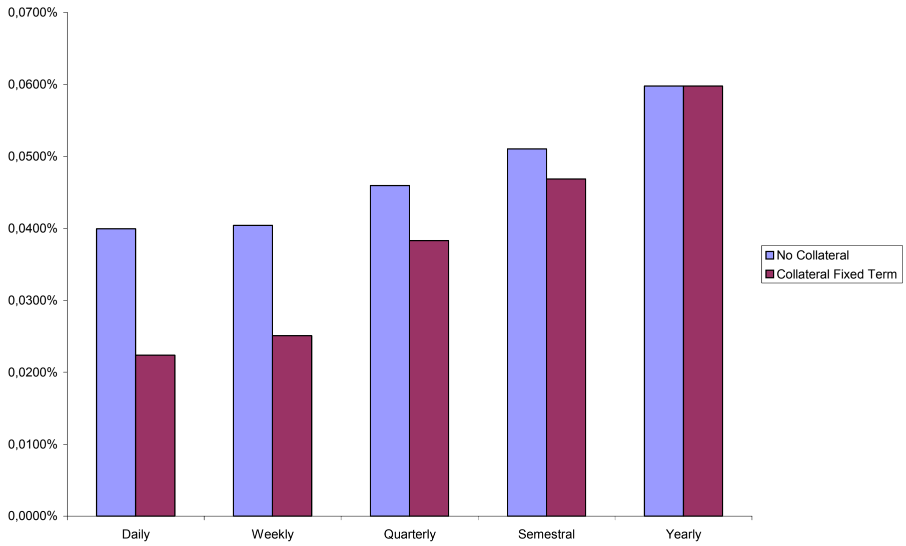


Figure 5: Counterparty risk reduction under a fixed term collateral policy

## 4.2 Contingent collateral

Requiring periodic posting of the collateral, particularly with a high frequency, may impose costs in the management of liquidity. While weekly collateral posting is the common rule in counterparty risk management among the financial intermediaries, it may be too costly to be imposed in derivative transactions with corporate counterparties. An alternative to the use of collateral is to make it contingent on the exposure: the counterparty will be required to post collateral only if the exposure to the other counterparty grows above a given threshold.

The possible loss on the contract in this case can be described as follows. Say default occurs at time $\tau$. The loss is going to be

$$\max\left(S\left(\tau\right)-v\left(\tau,T\right)F\left(S,0,T\right),0\right) \tag{30}$$

if

$$S\left(t\right)-v\left(t,T\right)F\left(S,0,T\right)<H,\forall t<\tau \tag{31}$$

and

$$\max\left(S\left(\tau\right)-v\left(\tau,T\right)\left(F\left(S,0,T\right)+H\right),0\right) \tag{32}$$

otherwise. Define *Ladder* $(S,t;T,K,K+H)$ the ladder option in which the strike is reset at level $K+H$ as soon as the underlying asset crosses that level. More precisely, in our application the replicating portfolio of this option is

$$Ladder\left(S,t;T,K,K+H\right) = UOC\left(S,t;T,K,K+H\right)+ \\ +UIC\left(S,t;T,K+H,K+H\right) \tag{33}$$

where $UOC\left(.\right)$ denotes an up-and-out option and $UIC\left(.\right)$ an up-and-in option. Using the well known simmetry relationship between in and out options

$$Call\left(S,t;T,K\right) = UOC\left(S,t;T,K,K+H\right)+ \\ +UIC\left(S,t;T,K,K+H\right) \tag{34}$$

we have

$$Ladder\left(S,t;T,K,K+H\right) = Call\left(S,t;T,K\right)+ \\ +\left[\begin{array}{c} UIC\left(.;T,K+H,K+H\right)- \\ UIC\left(.;T,K,K+H\right)\end{array}\right] \tag{35}$$

The difference between the ladder option and the corresponding plain vanilla option is then a spread position in barrier options.

Going back to our problem, it is easy to check that the value of counterparty risk is represented by

$$Lgd_j\sum_{i=1}^{n}\left[\overline{G}\left(t_{i-1}\right)-\overline{G}\left(t_i\right)\right]Ladder\left(.;t_i,v\left(t_i,T\right)F\left(S,0,T\right),v\left(t_i,T\right)\left(F\left(S,0,T\right)+H\right)\right) \tag{36}$$

The impact of the contingent collateral provision can be then readily reecovered as

$$Lgd_j\sum_{i=1}^{n}\left[\overline{G}\left(t_{i-1}\right)-\overline{G}\left(t_i\right)\right]\left[\begin{array}{c} UIC\left(.;t_i,v\left(t_i,T\right)F\left(S,0,T\right),v\left(t_i,T\right)\left(F\left(S,0,T\right)+H\right)\right)- \\ -UIC\left(.;t_i,v\left(t_i,T\right)\left(F\left(S,0,T\right)+H\right),v\left(t_i,T\right)\left(F\left(S,0,T\right)+H\right)\right)\end{array}\right] \tag{37}$$

In figure 6 we report the impact of contingent collateral provisions for a threshold of 8% of the notional value of the contract. Notice that contingent collateral provisions may be an effective alernative to periodic collateral. Actually, the efficiency of the contingent collateral policy is comparable with that of a weekly collateral policy.

# 5 Conclusions

Counterparty risk in derivatives transactions can be represented by replicating portfolios. We show that counterparty risk in linear contracts may be represented by a sequence of vulnerable options of the call type for the long position

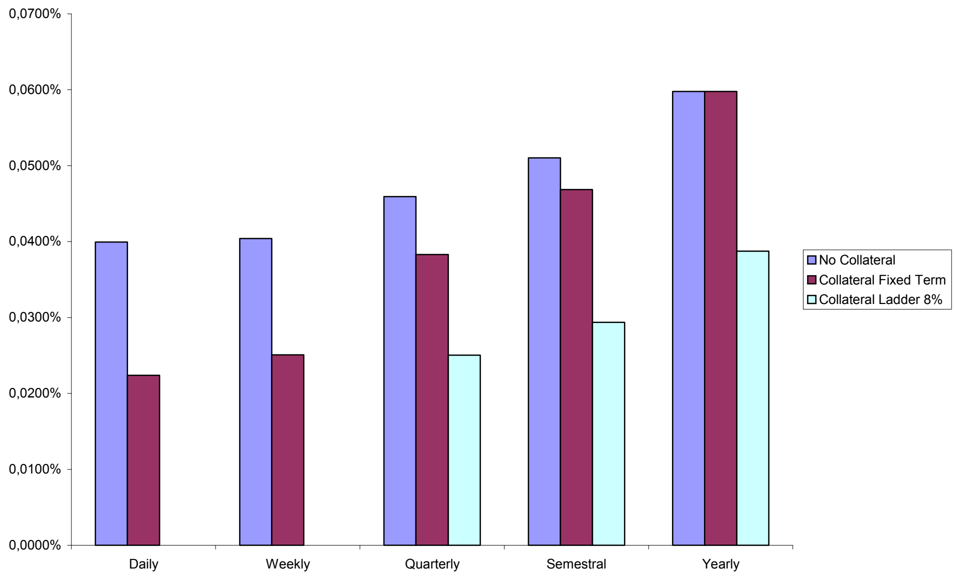


Figure 6: Counterparty risk without collateral, and with periodic or contingent collateral (8% threshold)

and of the put type for the short end of the contract. Replicating portfolios were also designed for risk mitigating techniques which require exchange of collateral. We focus on two different policies. The first, requiring periodic deposit of the collateral, leads to a counterparty risk representation in terms of a ratchet option. The second, requiring deposits of the collateral for exposures beyond a given threshold, leads to represent counterparty risk as a stream of ladder options. A pricing example shows that both weekly collateral or contingent collateral may lead to a substantial counterparty risk reduction, and both the alternatives may represent competitive tools in the market for derivatives transactions with corporate counterparties.

# References

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