The Credit Rating Process and Estimation of Transition Probabilities: A Bayesian Approach

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Abstract

This paper introduces a new methodology for modeling and estimating transition probabilities between credit classes. The paper makes three contributions. First, we develop a new statistical model that describes the typical credit rating process that most major banks employ. Second, we describe a Bayesian hierarchical framework for model calibration, using Markov Chain Monte Carlo techniques implemented through Gibbs sampling. This approach allows us to address the technical issues related to the estimation of default probabilities from low default portfolios. Third, we apply this methodology to the analysis of an extended rating transitions data set from Standard and Poor’s between 1981–2004, and we examine both the in-sample and out-of-sample performance of the credit rating process model relative to that of the traditional latent factor approach. The results of this paper provide a framework that banks and other financial institutions can use to show that their internal rating systems produce estimates of rating transition probabilities that are “reasonable” from a regulatory perspective. This is particularly relevant in light of the New Basel Capital Accord that allows banks to use their own estimates of the probability of default for capital budgeting decisions, subject to regulatory approval.

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1 Introduction

The internal ratings based (IRB) approach in the New Basel Capital Accord (Basel II) allows banks to develop their own internal credit ratings. The probabilities of default and rating transitions associated with each credit class must be chosen so that they reflect the nature of a bank’s portfolio, and are subject to supervisory review to ensure that they are “reasonable”. The Accord stresses (paragraph 444) that these probabilities are not only to be used for calculating regulatory capital, but must also play an essential role in the credit approval, risk management, internal capital allocation, and corporate governance functions of banks (BCBS, 2006). This provides banks with a strong incentive to ensure that probability estimates are up to date and accurate. This paper provides a new methodology for modeling and estimating probabilities of default and transition probabilities between credit classes, and can be used by bankers and regulators to assess whether internal credit rating systems are sound and appropriate.1

The paper makes three contributions. First, we develop a new model that describes the typical credit rating process that most major banks employ. In general, an obligor is assigned a credit rating based on an assessment of its current credit worthiness, which depends on many systematic and firm specific factors. Changes in a credit rating depend on changes in the underlying factors and on the type of rating methodology employed by the institution. Our model takes into account the heterogeneity in the credit worthiness of obligors in the same credit class. This heterogeneity can have significant effects on credit risk diversification (Hanson, Pesaran and Schuermann, 2006), yet it is neglected in traditional latent factor models used for risk management and pricing (Schonbucher, 2003; Gagliardini and Gourieroux, 2005).

Second, we propose a methodology for calibrating models for rating transition probabilities using historical data. Model calibration for this type of application is difficult in a classical frequentist estimation framework, because the sparsity of data often leads to unrealistic transition probabilities. This is a very important issue for many institutions that have a large number of high quality business lines for which extensive default data are not available. Low default portfolios typically include exposures to sovereigns, large corporations, or financial institutions such as banks

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1 The importance of the validation of internal rating systems, is stressed in the recent report issued by the BIS (May 2005).
(in developed nations) and insurance companies, where very few defaults have been observed over long horizons. Regulators expect that low default portfolios still follow minimum IRB standards for accuracy and conservatism of probabilities of default estimates, despite the data limitations. For low default portfolios, however, estimates of risk parameters based on simple historical averages or judgmental considerations alone, may underestimate capital requirements, raising the concern that financial institutions may not be able to apply the IRB approach for the many asset classes that have low number of defaults (The British Bankers Association et al., 2004). We address this issue by proposing a Bayesian hierarchical framework for model calibration, using Markov Chain Monte Carlo (MCMC) techniques. The MCMC approach produces the inferred distribution for all parameters of interest, including confidence intervals for the transition probabilities. This framework can also be used to incorporate further covariate data or expert opinion in a straightforward manner.

Third, the methodology can be applied to internal rating systems, or to rating data collected by external agencies, such as Moody’s or Standard and Poor’s. We use this methodology to analyze an extended rating transitions data set from Standard and Poor’s between 1981–2004. We compare the results of the credit rating process model to those of the traditional latent factor approach, examining both in-sample and out-of-sample performance. We find that the credit rating process model has better in-sample performance than the latent factor models, and better out-of-sample performance for non-investment grade obligors. For investment grade obligors, a comparison of out-of-sample performance is not relevant, due to the lack of default events in most years; however, both classes of models imply positive default probabilities, as desired. The results for the credit rating model also imply that the ratings transition matrix depends on the state of the economy and exhibits non-Markovian behavior. In addition, our results contribute to the policy debates around Basel II — the Accord imposes a floor of 3 basis points on any probability of default estimate (§285, BCBS 2005), yet there is little evidence on whether this floor is realistic or not. We show that, for the Standard and Poor’s data, this threshold falls between default probabilities for ratings A and AA, hence it is overly conservative for AAA and AA rated corporations.

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2 The scale of the issue is significant. In a joint industry survey of seven U.K. firms having nearly U.S. $3 trillion in total gross assets, over 50% of total wholesale exposures had insufficient default data (British Bankers Association et al., 2004).
The modeling and estimation of rating transition matrices is an important and intensely re-
searched area at the moment, due to the requirements of Basel II. Lando and Skodeberg (2002) give
a review of different approaches for estimating migration matrices, which are extensively compared
in Jafry and Schuermann (2004). There are many statistical issues in estimating transition prob-
abilities and assessing the precision of the estimates, and many of these challenges stem from the
frequent and inherent lack of event data. Lando and Skodeberg (2002), and Christensen, Hansen
and Lando (2004) address the issue of computing point and interval estimates for default prob-
abilities with rare events, using a continuous time homogeneous Markov chain transition matrix.
Christensen et al. (2004), Hanson and Schuermann (2006), and Truck and Rachev (2005) show
that bootstrapped intervals for duration based estimates are relatively tight, however they are
unable to distinguish probabilities of default for investment grades.\footnote{Truck and Rachev (2005) show that using confidence sets to estimate default probabilities, as suggested by Pluto and Tasche (2005), has a number of undesirable features.} Most of these papers rely
on the assumption of Markovian transitions between rating classes, however, there is considerable
evidence that the Markovian assumption is unrealistic for ratings transitions (Altman and Kao,
1992; Nickell, Perraudin and Varotto, 2000; Bangia et al., 2002; Frydman and Schuermann, 2005;
Chava, Stefanescu and Turnbull, 2006).

We propose the use of a Bayesian framework for calibrating the credit rating models, as an
test a number of threshold Bayesian models with fixed and random effects using the traditional
latent factor formulation for estimating default probabilities, a framework extended in Wendin and
McNeil (2006) to the estimation of rating transition matrices. Bayesian MCMC techniques have
been applied in other areas of finance, such as volatility estimation and portfolio selection; see, for

The paper is structured as follows. In Section 2 we develop two classes of models for rating
transitions and we outline the Bayesian estimation framework. Section 3 contains a description
of the Standard and Poor’s rating transitions data set, and presents the results of our analyses.
Section 4 concludes the paper with a discussion and summary of the results.
2 Model Specification and Estimation

In this section, we present two classes of models for rating transitions. First, we develop a new class of models that directly describe the credit rating process. Second, for purposes of comparison we briefly discuss a class of models arising from the traditional latent factor approach. We then show how the parameters of both these classes of models are estimated in a Bayesian framework using Markov chain Monte Carlo techniques.

Let \( \{1, \ldots, K\} \) be the set of non default rating classes represented in ascending order of credit worthiness. We assume that there exists a latent variable representing the credit worthiness of each obligor at time \( t \), such that the obligor is assigned to a particular credit class if the latent variable lies within a certain interval. Let \( \gamma_{k,0} < \gamma_{k,1} < \ldots < \gamma_{k,K-1} < \gamma_{k,K} = \infty \) denote unobserved critical thresholds specific to each credit class.\(^4\) An obligor who is in credit class \( k \) at time \( t \) will default at time \( t + 1 \) if the latent variable representing its credit worthiness is less than \( \gamma_{k,0} \). The obligor will move to credit class \( l \) at time \( t + 1 \) if the latent variable lies in the interval \( [\gamma_{k,l-1}, \gamma_{k,l}) \), for \( k, l = 1, \ldots, K \). We assume that \( \gamma_{k,0} = 0 \) for all \( k \), to ensure identifiability. The lengths of the risk category intervals need not be equal, and it is expected that the obligors in a given risk category will exhibit roughly the same default risk.

The two classes of models described in this section differ in terms of the specification of the credit worthiness latent variable.

2.1 The Credit Rating Process Model

In a typical internal credit rating system, many quantitative and qualitative factors are combined during the rating process in order to form an assessment of the credit worthiness of an obligor over some defined horizon, which may be one year or through the business cycle. Examples of quantitative factors include cash flow variables, liquidity measures, and leverage, while the qualitative factors include competitive advantages and disadvantages, industry risk and trends, management

\(^4\)The assumption that the critical thresholds \( \gamma_{k,l} \) are credit class specific is induced by the aim of accommodating non-Markovian behavior in the credit rating process. In practice, it is well known that the rating process is not Markovian; the loan officers assign the obligor to a particular credit class based not only on its current state and on expectations about its future credit worthiness, but also on its rating history.
quality, legal and financial structure. For each obligor, the loan officer assigns a score to the different factors, and exercises professional judgment in reaching an assessment. The final outcome of this process is represented by the credit rating assigned to the obligor, which is then reported to senior management in the bank. In many cases, quantitative models may be used as a check on the relative accuracy and consistency of the loan officer’s assessment.

The outside regulators are mandated to assess the underlying methodology and the reasonableness of the ratings assigned to different obligors. In practice, the regulators often observe only the final outcome given by the assigned credit rating, rather than the relative importance attached to each factor and the obligor specific information used by the loan officers which is usually confidential. The credit ratings correspond to certain probabilities of default in each credit class, as well as transition probabilities between credit classes. The task facing a risk manager assessing the properties of the internal rating process, is to justify that the assigned default and transition probabilities are reasonable, and to present this evidence to the regulators.

In this section, we develop a new model that captures the dynamics of the credit rating process, and can therefore be used to show that the implied default and transition probabilities are reasonable from a regulatory perspective. The methodology that we develop here can be used to model the credit assessment changes for any individual obligor, provided that obligor specific data are available. In the absence of obligor specific data, we focus on the representative obligor in each credit class, and formally describe the process through which its credit assessment changes from one period to the next.

The bank’s rating methodology combines systematic factors and obligor specific factors to reach an assessment of the credit worthiness of the obligor. Formally, we model this process by a continuous latent random variable denoted by $D_{kt}$, which represents the credit assessment for the representative obligor for credit class $k$ at time $t$, as evaluated by the bank’s rating methodology. Over the next period, both the systematic and obligor specific state variables will change, and this

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5 A summary of the factors considered in a typical rating process is given in Crouhy, Galai and Mark (2000).
6 For certain asset classes, many institutions have been reluctant to rely solely on quantitative models, preferring the judgment of experienced loan officers.
7 A bank is required to fully document the rating methodology and to describe its actual use. Regulators can request to see the raw obligor scores. However, given the number of obligors in a typical loan portfolio (usually measured in thousands), it is rare for regulators to see all the scores.
will affect the bank’s assessment of the credit worthiness of the obligor. Some of these changes will be expected, while others will be unanticipated. We assume that the credit assessment for the representative obligor for credit class \( k \) at time \( t + 1 \) is given by

\[
D_{kt+1} = \mu_{kt} + D_{kt} + \beta'_k z_{t+1}.
\]

(1)

Here \( \mu_{kt} \) is a drift term representing the change in the credit assessment caused by expected changes in the systematic factors. We assume that \( \mu_{kt} \) depends on current macroeconomic conditions, so that \( \mu_{kt} = \alpha_k' X_t \), where \( X_t \) is a \( p \)-dimensional vector of observable macroeconomic covariates (for example, credit spread, term spread, or percentage change in GDP), and \( \alpha_k \in \mathbb{R}^p \) is a vector of coefficients. The term \( \beta'_k z_{t+1} \) in (1) represents the change in the credit assessment caused by unanticipated changes in the economy. Here \( z_{t+1} \) is a \( n \)-dimensional vector of zero mean systematic random factors, and \( \beta_k \in \mathbb{R}^n \) is a vector of coefficients. The latent factors \( z_t \) account for unobserved systematic risk, and therefore model heterogeneity beyond that which can be captured with the observed covariates \( X_t \).

Expression (1) describes the general form of the credit rating process model. Different distributional assumptions, as well as different specifications for the structure of the latent factors \( z_t \) are possible, and greatly increase the versatility of the model. We test several such specifications in Section 3. In particular, the \( z_t \) factors induce dependence among rating transitions; this can be dependence among transitions taking place in the same time period (if \( z_t \) are independent), or dependence across time periods (if \( z_t \) are serially correlated and follow, for example, an autoregressive process). Within the multivariate structure of \( z_t \) it is also possible to account for sector, industry, sector, and obligor-specific factors. Under the assumption that these factors are described by diffusion processes, we have

\[
\begin{align*}
\frac{dF}{dt} & = \mu_F dt + \sigma_F dW_F, \\
\frac{df}{dt} & = \mu_f dt + \sigma_f dW_f,
\end{align*}
\]

where \( \mu_F = \mu_F(F, T), \sigma_F = \sigma_F(F, T), \mu_f = \mu_f(f, T), \sigma_f = \sigma_f(f, T) \), and \( W_F \) and \( W_f \) are vectors of independent Brownian motions. More complicated processes could also be assumed. Ito’s lemma then implies that

\[
\frac{dD}{dt} = \mu_D dt + \frac{\partial D}{\partial F} \sigma_F dW_F + \frac{\partial D}{\partial f} \sigma_f dW_f,
\]

using vector notation. Integrating the above expression, we derive an expression for the stochastic process describing the change in the credit assessment.

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8At the individual obligor level, expression (1) arises naturally from the assumption that the loan officer considers systematic factors, say \( F \), and obligor specific factors, say \( f \), for credit assessment purposes, so that \( D = D(F, f) \). Under the assumption that these factors are described by diffusion processes, we have

\[
\begin{align*}
\frac{dF}{dt} & = \mu_F dt + \sigma_F dW_F, \\
\frac{df}{dt} & = \mu_f dt + \sigma_f dW_f,
\end{align*}
\]

where \( \mu_F = \mu_F(F, T), \sigma_F = \sigma_F(F, T), \mu_f = \mu_f(f, T), \sigma_f = \sigma_f(f, T) \), and \( W_F \) and \( W_f \) are vectors of independent Brownian motions. More complicated processes could also be assumed. Ito’s lemma then implies that

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\frac{dD}{dt} = \mu_D dt + \frac{\partial D}{\partial F} \sigma_F dW_F + \frac{\partial D}{\partial f} \sigma_f dW_f,
\]

using vector notation. Integrating the above expression, we derive an expression for the stochastic process describing the change in the credit assessment.
or credit class effects by making some components of $z_t$ sector specific or credit class specific. This ability of capturing different patterns of heterogeneity and dependence is one of the most attractive features of the model.

In order to derive expressions for the default and transition probabilities implied by the credit rating process described in (1), we need to model the state at time $t + 1$ of an obligor $j$ who is in credit class $k$ at time $t$. We assume that the credit assessment $D_{jt+1}$ of obligor $j$ at time $t + 1$ depends upon changes in the credit assessment for the representative obligor in class $k$ between $t$ and $t + 1$, and on an idiosyncratic term $e_{jt+1}$. Thus

$$D_{jt+1} = D_{kt+1} + e_{jt+1},$$

where we assume that $D_{kt+1}$ and $e_{jt+1}$ are independent. Substituting expression (1), we have

$$D_{jt+1} = \mu_{kt} + D_{kt} + \beta'_k z_{t+1} + e_{jt+1}. \quad (2)$$

When available, obligor specific data can be incorporated in a linear form into the right side of equation (2). In the absence of obligor specific data, it is natural to make the assumption that all obligors in the same credit class have the same distribution. Obligor $j$ defaults at time $t + 1$ if $D_{jt+1} < \gamma_{k0}$. The conditional probability of default over the next period given $D_{kt}$ and $z_{t+1}$ is

$$P(D_{jt+1} < \gamma_{k0} \mid D_{kt}, z_{t+1}) = P(e_{jt+1} < \gamma_{k0} - \mu_{kt} - D_{kt} - \beta'_k z_{t+1} \mid D_{kt}, z_{t+1})$$

$$= g(\gamma_{k0} - \mu_{kt} - D_{kt} - \beta'_k z_{t+1}),$$

where $g(\cdot)$ is a link function that depends on the distribution of $e_{jt+1}$. For example, $g(\cdot)$ is the logit link if $e_{jt+1}$ has an Extreme Value distribution, and $g(\cdot)$ is the probit link if $e_{jt+1}$ has a Normal distribution.\(^9\) Note that we do not observe $D_{kt}$ and $z_{t+1}$. Recall that the term $D_{kt}$ represents the credit assessment of the representative obligor at time $t$. Over time this will change, as conditions within the economy change, inducing changes in the membership of credit class $k$. We model this form of heterogeneity by assuming a probability distribution for $D_{kt}$. The unconditional probability

\(^9\)The logit link is given by $g(x) = 1/(1 + \exp(-x))$, and the probit link is given by the standard normal cumulative distribution function $g(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp(-t^2/2) dt$.
of default for obligor $j$ over the next period is $P(D_{jt+1} < \gamma_{k0}) = E_t[P(D_{jt+1} < \gamma_{k0} \mid D_{kt}, z_{t+1})]$, where the expectation is taken with respect to the distributions of $D_{kt}$ and $z_{t+1}$.

Thus the probability of default assigned to credit class $k$ at time $t$ is

$$p_t(k, 0) = E_t[g(\gamma_{k0} - \mu_{kt} - D_{kt} - \beta'_k z_{t+1})].$$  \hspace{1cm} (3)

The conditional probability of a transition from credit class $k$ at time $t$ to credit class $l$ at time $t + 1$ is given by

$$P(\gamma_{k,l-1} \leq D_{jt+1} < \gamma_{k,l} \mid D_{kt}, z_{t+1})$$

$$= P(\gamma_{k,l-1} \leq \mu_{kt} + D_{kt} + \beta'_k z_{t+1} + e_{jt+1} < \gamma_{k,l} \mid D_{kt}, z_{t+1})$$

$$= g(\gamma_{k,l} - \mu_{kt} - D_{kt} - \beta'_k z_{t+1}) - g(\gamma_{k,l-1} - \mu_{kt} - D_{kt} - \beta'_k z_{t+1}),$$

and the unconditional transition probability is then

$$p_t(k, l) = E_t[g(\gamma_{k,l} - \mu_{kt} - D_{kt} - \beta'_k z_{t+1})] - E_t[g(\gamma_{k,l-1} - \mu_{kt} - D_{kt} - \beta'_k z_{t+1})].$$  \hspace{1cm} (4)

### 2.2 The Latent Factor Model

A second class of models for rating transitions relies on the traditional linear latent factor approach that is used both for risk management and for pricing credit derivatives (Frey and McNeil, 2003; McNeil and Wendin, 2005; Wendin and McNeil, 2006). We briefly describe this approach here.

Denote by $L_{kt}$ the latent variable representing the credit worthiness of the representative obligor in credit class $k$ at time $t$. We assume that $L_{kt+1}$ is related to a vector of common factors $z_{t+1}$ through the expression

$$L_{kt+1} = \mu_{kt} + \beta'_k z_{t+1} + e_{kt+1},$$ \hspace{1cm} (5)

where $\mu_{kt}$ is a drift term representing the effect of observed macroeconomic covariates, $\beta_k \in \mathbb{R}^n$ is the sensitivity to the common latent factors $z_t$, and $e_{kt+1}$ is an unobservable idiosyncratic random effect. As in the previous subsection, the model can be calibrated at individual obligor level rather
than credit class level, when obligor specific data are available.

The conditional probability of default over the next period assigned to credit class \( k \) at time \( t \) is given by

\[
P(L_{kt+1} < \gamma_{k0} | z_{t+1}) = P(\mu_{kt} + \beta'_k z_{t+1} + e_{kt+1} < \gamma_{k0} | z_{t+1})
\]

\[
= g(\gamma_{k0} - \mu_{kt} - \beta'_k z_{t+1}),
\]

where \( g(\cdot) \) is, as before, a link function that depends on the distribution of \( e_{kt+1} \). The unconditional probability of default is then

\[
p_t(k, 0) = E_t[g(\gamma_{k0} - \mu_{kt} - \beta'_k z_{t+1})],
\]

(6)

where the expectation is taken with respect to the distribution of \( z_{t+1} \). The probability of a transition from credit class \( k \) at time \( t \) to credit class \( l \) at time \( t + 1 \) is given by

\[
p_t(k, l) = E_t[g(\gamma_{k,l} - \mu_{kt} - \beta'_k z_{t+1})] - E_t[g(\gamma_{k,l-1} - \mu_{kt} - \beta'_k z_{t+1})].
\]

(7)

In the latent factor model, the value of the latent variable \( L_{kt+1} \) depends on a drift term \( \mu_{kt} \), on changes in the macroeconomic factors \( z_{t+1} \), and on the idiosyncratic random effect \( e_{kt+1} \). The credit rating process model represented by expression (2) contains similar terms, and in addition takes into account the uncertainty associated with the current latent credit value \( D_{kt} \). Both the credit rating process model and the latent factor model are parsimonious and allow modeling of joint defaults.

2.3 Bayesian Estimation

Standard likelihood based inference for the models specified in equations (1) and (5) is difficult to achieve, because the multivariate structure and serial dependence of the latent risk factors lead to joint migrations distributions in the form of multivariate integrals lacking closed form expressions. As an alternative to maximum likelihood inference, we propose a Bayesian approach to estimating
the parameters of the models from sample data, which can be implemented in a Markov chain Monte Carlo (MCMC) framework. An introduction to MCMC techniques is given by Gilks, Richardson and Spiegelhalter (1996).

Let us denote by $\theta$ the vector of parameters of the model, which includes $\{\alpha_k\}$, $\{\beta_k\}$, and the parameters of the distribution of $z_t$. A Bayesian specification requires prior distributions to be chosen for $\theta$. Let $p(\theta)$ be the probability density of the prior distribution of $\theta$, and let $\mathcal{Y}$ be the sample data. The joint posterior density $p(\theta \mid \mathcal{Y})$ of all parameters given the observed data is proportional to the product of the likelihood function $p(\mathcal{Y} \mid \theta)$ and the prior density:

$$p(\theta \mid \mathcal{Y}) \propto p(\mathcal{Y} \mid \theta) \cdot p(\theta). \quad (8)$$

We make the standard assumption that the model parameters are a priori independent, so that the prior density $p(\theta)$ is a product of prior densities for each parameter. With little external information available, we generally would like to specify non-informative priors $p(\cdot)$ for the components of $\theta$. For instance, in the application described in Section 3 we specify normal priors with large variances for the parameters $\alpha_k$ and $\beta_k$, and gamma priors with large variances for the threshold increments $\gamma_{k,l+1} - \gamma_{k,l}$. However, if expert opinion is available, it can be incorporated into the analysis by specifying more concentrated priors. For example, a downturn in the economy that may lead to a general increase in the defaults for all rating categories, can be accounted for in the model by changing the priors for $\beta_k$ to plausible ranges.

In a Bayesian framework, inference on model parameters is based on their marginal posterior density, obtained by integrating out the other parameters from the joint posterior density given by (8). This is difficult to achieve analytically, therefore we propose the use of Gibbs sampling (Geman and Geman, 1984) for generation of the marginal posterior distributions. The Gibbs sampler is an iterative algorithm for generation of samples from a multivariate distribution. It proceeds by updating each variable by sampling from its conditional distribution given current values of all other variables. After a sufficiently large number of iterations, under mild conditions it can be proven that the values of the updated variables so obtained form a sample from the joint posterior distribution.
After convergence of the Gibbs sampler, 95% credible intervals for all model parameters can be computed from the samples of observations generated from the posterior densities, and these can be then used to test specific hypotheses about the parameters. In the application described in Section 3, we shall use the Deviance Information Criterion (DIC) to choose among different models fitted to the same data set, following Spiegelhalter et al. (2002). This criterion is a Bayesian alternative to Akaike’s Information Criterion (AIC), and is an estimate for the expected predictive deviance which has been suggested as a measure of model fit when the goal is to choose a model with best out-of-sample predictive power. The model with the smallest DIC value is estimated to be the model that would best predict a data set of the same structure as the data actually observed.

3 Analysis of Standard and Poor’s Data

In this section, we test different specifications of the credit rating and latent factor models. We first describe the data set used in the study, then we list the models that we calibrate from the data, and finally we discuss the results and implications of our analysis.

3.1 Data Description

The source of the data set that we analyze here is the CreditPro® 7.0 database of long–term local currency issuer credit ratings. The data pooled the information from 11,150 companies that were rated by Standard and Poor’s as of 31 December 1980, or that were first rated between that date and 31 December 2004. These rated issuers include industrials, utilities, financial institutions, and insurance companies around the world. Public information ratings as well as ratings based on the guarantee of another company were not taken into consideration. The data also did not include structured finance vehicles, public–sector issuers, subsidiaries whose debt is fully guaranteed by a parent or whose default risk is considered identical to that of their parents, as well as sovereign issuers.

The data set contains information on seven rating categories: AAA, AA, A, BBB, BB, B, and CCC/C. The number of issuers and the number of transitions between each pair of rating
categories (including default) are available for every year during the 24 year horizon. In this data set, a default has been recorded by Standard and Poor’s upon the first occurrence of a payment default on any financial obligation, rated or unrated, other than a financial obligation subject to a bona fide commercial dispute. The classification of an issuer into a credit rating category reflects Standard and Poor’s opinion of a company’s overall capacity to pay its obligations, focusing on the obligor’s ability and willingness to meet its financial commitments on a timely basis. The rating generally indicates the likelihood of default regarding all financial obligations of the firm. Note, however, that a company may not have rated debt but it may still be assigned a credit rating. A Standard and Poor’s rating reflects a through-the-cycle assessment of the credit risk of an obligor. Moreover, the agencies assign ratings based on a “stress scenario” for the borrowers, therefore the estimate is close to the estimate of the borrower’s default probability at the time of rating assignment only if the borrower already is in the stress scenario (Carey and Hrycay, 2001). This implies that Standard and Poor’s will overestimate the credit risk of the obligor when the economy is in a good state, and underestimate it when the economy is in a bad state.

Standard and Poor’s report acknowledges that their ongoing enhancement of the CreditPro® database from which this data is extracted may lead to outcomes that differ to some extent from those reported in Standard and Poor’s previous studies. The data set that we analyze here is the latest version of Standard and Poor’s data.

In our analysis described in the following section we also use a macroeconomic covariate $X_t$ given by the credit spread, which is computed as the difference between AAA and BAA yields, and is a measure of the aggregate credit risk of the economy. An increase in this spread signals a decline in the average credit worthiness of obligors.

### 3.2 Model Delineation

We calibrate the models developed in Section 2 using the Standard and Poor’s data, under different specifications of the dynamics of the macroeconomic factors $z_t$. The candidate models that we consider are the following:

\footnote{This is not true when an interest payment missed on the due date is made within the grace period.}
Model R1

We assume that $\beta_k = 0$ for all $k$ in the credit rating process (CRP) model specified by equation (1), implying that there is no effect of the macroeconomic factor.

Model R2

We assume that the macroeconomic factors $z_t$ are independent and identically normally distributed in the CRP model specified by (1), so that $z_t \sim N(0, \sigma_z^2)$ with $\sigma_z^2 > 0$.

Model R3

We assume that the macroeconomic factors $z_t$ have an autoregressive AR(1) structure in the CRP model specified by (1), so that $z_{t+1} = a_z z_t + \varepsilon_{t+1}$, where $a_z \in (-1, 1)$, $\varepsilon_t \sim N(0, \sigma_z^2)$ are independent, and $\sigma_z^2 > 0$.

Model L1

We assume that the factors $z_t$ are independent and identically normally distributed in the latent factor (LF) model specified by equation (5), so that $z_t \sim N(0, \sigma_z^2)$, with $\sigma_z^2 > 0$.

Model L2

We assume that the factors $z_t$ have an autoregressive AR(1) structure in the LF model specified by (5), so that $z_{t+1} = a_z z_t + \varepsilon_{t+1}$, where $a_z \in (-1, 1)$, $\varepsilon_t \sim N(0, \sigma_z^2)$ are independent, and $\sigma_z^2 > 0$.

Our motivation for focusing on these models is twofold. First, we compare models R1–R3 that directly capture the credit rating process, with models L1–L2 inspired by the latent factor approach and lacking the credit rating process interpretation. Second, within both classes of models, we investigate several patterns of heterogeneity and migration dependence implied by the different specifications of the dynamics of the macroeconomic factors $z_t$. For example, model L1 can only capture dependence among rating transitions in the same time period, since the $z_t$ factors are independent. Note that model R2 also assumes that the $z_t$ factors are independent, but it models dependence among default probabilities via the dependence on the credit assessment $D_{kt}$. Models R3 and L2 capture dependence of rating transitions across time periods, induced by the serial correlation of the $z_t$ factors.
Throughout the study, we choose $g(\cdot)$ to be the logit link function corresponding to the logistic distribution for the idiosyncratic terms $e_{kt}$. Other link functions (e.g. probit or log-log) could be chosen, but our investigations showed that this would have a minimal impact on the results of the analysis.

### 3.3 Empirical Results

We estimate the models outlined in Section 3.2 from the Standard and Poor’s data using the Bayesian methodology described in Section 2.3. All models were calibrated using WinBUGS version 1.4 (Spiegelhalter et al., 2003). We specified diffuse but proper priors for all parameters, however other priors are also possible if specific prior information on some parameters is available. We chose $N(0, 10^3)$ priors for the parameters $\alpha_k$ and $\beta_k$ representing the effects of the macroeconomic covariate credit spread and of the macroeconomic latent factor $z_t$. To assess the impact of the choice of prior variance, we carried out a sensitivity analysis by investigating different prior normal distributions with variances ranging between $10^3$ and $10^6$. The results were broadly similar, hence here we report only the summary statistics based on the $N(0, 10^3)$ priors. We also specified a gamma prior with large variance $\Gamma(.001, .001)$ for the positive threshold increments $\gamma_{k,t+1} - \gamma_{k,t}$, and a diffuse inverse gamma prior $\Gamma^{-1}(1, 1)$ for the variance $\sigma^2_z$ of the macroeconomic latent factor $z_t$. For the autoregressive parameter $a_z$ in models R3 and L2 we chose a uniform prior on $(-1, 1)$. These are standard choices of non-informative prior distributions.

For each model we ran two parallel Markov chains started with different sets of initial values. The Gibbs sampler ran for 10,000 iterations, with the first 5,000 iterations discarded as a burn-in period. Gelman and Rubin’s diagnostic (Gelman et al., 1995) indicated satisfactory convergence of all chains. After convergence, inference on the parameters of interest was based on the pooled sample iterations of both chains.

Table 1 reports the values of the Deviance Information Criterion (DIC) for each model. The lowest DIC value corresponds to model R2, derived from the credit rating process and with independent macroeconomic latent factors $z_t$. This implies that model R2 would best predict a data set of the same structure as the Standard and Poor’s data. The fit of R2, as measured by DIC,
is significantly better\textsuperscript{11} than that of the latent factor model L1 with a similar specification for \( z_t \). The fit of R2 is also better than the fit of R3 and of the latent factor model L2. Both of these models include an AR(1) specification for the macroeconomic term \( z_t \). Model R2 has better predictive power because the time dependence of transition probabilities is inherently taken into account in the credit rating model, without the necessity of specifying an autoregressive structure for \( z_t \). Indeed, equation (1) includes both \( D_{kt} \) and \( D_{kt+1} \), the values at times \( t \) and \( t + 1 \) of the latent variable representing the credit worthiness. This is not the case for the latent factor model specified by (5), where the only solution for capturing time dependence of migrations is to include serial dependence in the latent factor \( z_t \). For the remaining discussion of the in-sample properties, we present the results only for model R2.

[Table 1 about here.]

Table 2 reports the posterior means and standard errors for model R2 parameters. For all credit classes except AAA, the credit spread effect parameters \( \alpha_k \) and the macroeconomic latent factor effect parameters \( \beta_k \) are negative and statistically significant. The negative signs of \( \alpha_k \) and \( \beta_k \) show that the probability of default for these credit classes does indeed increase both with increasing credit spread and with a worsening state of the economy, as expected. Also, notice that the effect of the credit spread covariate is generally increasing as we go down the credit scale. This is to be expected — the credit spread is a proxy for the state of the economy, and we would expect lower credits to be more sensitive to the economy.

For credit class AAA, the credit spread effect parameter \( \alpha_{\text{AAA}} \) is not statistically significant, while the macroeconomic latent factor effect parameter \( \beta_{\text{AAA}} \) is significant and positive. This finding that credit class AAA is different than the other rating classes and possibly counter-cyclical, is somewhat surprising. One possible explanation is that AAA-rated firms may benefit from expanding market share during a recession, while their lower rated competitors are performing poorly. This is consistent with findings in Jorion and Zhang (2006), who show that the default correlations implied by credit default swaps are not always positive; contagion and competition effects may

\textsuperscript{11}The difference in DIC values between models R2 and L1 is 34 (= 4,445 – 4,411). It is a difficult task to define what constitutes an important difference in DIC values between models. Spiegelhalter et al. (2002) propose the following rule of thumb: if the difference in DIC is greater than 10, then the model with larger DIC has considerably less support than the model with smaller DIC.
coexist with each other, and the sign of default correlation depends on industry concentration. A second possible explanation for the different results in credit class AAA is that there may be a flight to quality during recessions.

[Table 2 about here.]

Figure 1 gives the time series values of the latent macroeconomic factor $z_t$ estimated from model R2. The upper graph plots the posterior means of $z_t$ with 95% credible intervals. For comparison, the lower graph plots the posterior means of $z_t$ together with the annual standardized values of the credit spread covariate\(^{12}\) between 1981–2004. There is a certain degree of co-movement between the time series of the credit spread and the macroeconomic factor, but it is also apparent that the macroeconomic factor is able to capture some variability in transition rates that is left unexplained by the credit spread covariate.

[Figure 1 about here.]

Table 3 reports the posterior means and standard errors for the threshold values $\gamma_{k,t}$. Notice that, for each credit class $k$, the widest threshold interval is $(\gamma_{k,k-1}, \gamma_{k,k})$, corresponding to the probability of no transition from the credit class. This implies that there is potentially large heterogeneity among firms in the same credit class, which can have significant effects on credit risk diversification (Hanson, Pesaran and Schuermann, 2006).

[Table 3 about here.]

Transition probabilities between each pair of rating classes in every year are also estimated using model R2. As an illustration, Table 4 reports the matrix of posterior means for the estimated transition probabilities in 2004. As expected, the probability of default increases as the credit quality decreases. The transition matrix has high probability mass on the diagonal since obligors are most likely to maintain their current rating, but the probability of remaining within the current credit class overall decreases as the credit quality decreases. This is a phenomenon often observed in transition matrices (Standard and Poor’s, 2005). Conditional on the initial rating of an obligor, the second largest transition probabilities are in direct neighborhood to the diagonal, consistent with the monotonicity property.

\(^{12}\)Note that because the credit spread is standardized, some values are negative.
Figure 2 gives the time series plots of posterior means and 95% credible limits for default probabilities in the A and B rating classes between 1981–2004. The time series are comparable with the year-by-year estimates reported in Figure 3 from Hanson and Schuermann (2006). The plots emphasize the overall increase in the estimated default probabilities around 1990 and 2001, consistent with the observed high default rates for speculative grade bonds in these periods (Standard and Poor’s, 2005). The time series show some degree of co-movement, partially explained by the common effect of the state of the economy on both series of estimates. This is to be expected given the results in Table 2, where both the credit spread and macroeconomic coefficients have negative signs and are statistically significant.

For internal risk management and capital allocation purposes, banks require an assessment of the accuracy of estimates of the probabilities of default for different rating classes, and of the associated transition probabilities based on current information. A common test for accuracy is to examine the model’s ability to correctly identify out-of-sample obligors that default over the horizon (Shumway, 2001). It is possible to perform this type of test for low quality investment grade and speculative grade obligors, for which there exist enough default events. For high investment grade obligors, however, this type of test is not feasible due to the lack of default events. A specific model may predict a nonzero probability of default, as economic theory would require, although ex post there may be no defaults over many periods.

We investigate the out-of-sample behavior of the credit rating model R2 and the latent factor model L2 for predicting annual probabilities of default. We choose model L2 for this comparative study, because it is the second best model according to the DIC values. We focus on the investment grade A and the speculative grade B, and report the results in Figure 3. For every year between 1996-2004, we estimate the parameters of models R2 and L2 using information available up to the beginning of that year, then predict the probability of default for credit classes A and B over the

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13We also investigated the out-of-sample predictions obtained from model L1, which is similar to model R2, given the assumption that there is no serial correlation among the latent macroeconomic factors $z_t$. The results were similar to those from model L2.
one-year regulatory horizon. These values, as well as the observed annual frequency of defaults, are plotted in Figure 3. Note that for investment grade A there are no observed defaults in years 1996-1998, 2003 and 2004, yet every obligor has default risk and therefore a good model should imply a positive probability of default. Both models generated positive probabilities of default, however, it is not possible to comment about the relative performance of the models, given the low frequency of defaults. In fact, the issue of methodological comparison is not a question of one model being superior to another, since no internal risk system or methodology can be considered best in a low default portfolio (Standard and Poor’s, 2005). In this situation, the only way to assess a model is by its performance across those credit classes that are considered to have sufficient default data. For the speculative grade B, the credit rating model R2 is closer to the observed number of defaults than model L2 for the first six years, and it overestimates the number of defaults relative to L2 for years 2002, 2003 and 2004.

[Figure 3 about here.]

The results reported here have implications for the current policy debate arising from Basel II. The Accord imposes a floor of 3 basis points on any probability of default estimate (§285, BCBS 2005), yet there is little evidence on whether this floor is realistic or not. From Table 4, the estimated default probabilities in 2004 for credit classes AAA, AA and A are respectively 0.3, 1.1 and 3.3 basis points. Our results show that the threshold of 3 basis points falls between ratings A and AA, hence it is overly conservative for AAA and AA rated corporations. The threshold implies that firms rated AA and AAA would be treated the same from a regulatory perspective, potentially distorting lending decisions by banks subject to Basel II regulations.

4 Conclusions

The credit rating process model developed in this paper describes a typical form of internal credit assessment used by financial institutions. The loan officer reaches an assessment of the credit worthiness of an obligor and assigns a certain credit rating based on the range within which this assessment falls. This implies that there is heterogeneity in the credit worthiness of the obligors within any credit class. The credit rating process model takes this heterogeneity into account, while
extant latent factor models completely ignore it. Our Bayesian estimation methodology jointly uses all available transition data, and thus overcomes the problems generated by the scarcity of observed defaults in low default portfolios. Consequently, this paper directly addresses some of the issues raised by regulators and industry groups pertaining to low default portfolios.

Our empirical study of the Standard and Poor’s ratings transition data shows that the in-sample performance of the credit rating process model R2 is superior to that of the latent factor models that we tested. For out-of-sample testing, we compared observed default rates during 1996–2004 with estimated default probabilities for two credit classes, the investment grade A and the speculative grade B. For the investment grade, there are no observed defaults in five out of the nine years, and it is not relevant to compare the out-of-sample performance of the credit rating models and of the latent factor models. However, both classes of models estimate positive default probabilities, as desired. For the speculative grade, we find that the estimates from the credit rating process model are closer to the observed default rates than the estimates from the latent factor model in six of the nine years.

The results of this paper have significant implications for both regulators and credit risk practitioners. Following the Basel II Accord, banks have an incentive to ensure that estimates of transition and default probabilities are up to date and reasonable from a regulatory perspective. Our paper makes three contributions towards this goal. First, the methodology that we develop here allows taking into account observable obligor and macroeconomic information. The model can also capture different patterns of obligor heterogeneity and ratings migration dependence, through the use of latent systematic risk factors. Second, we describe a Bayesian framework for calibrating the models from rating transition data, which specifically addresses the technical challenges raised by lack of events in low-default portfolios and allows us to also derive credible intervals for probability of default estimates. Third, the methodology can be applied to typical internal rating systems employed by most banks subject to Basel II regulations.
Acknowledgements

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References


Table 1: Values of the Deviance Information Criterion (DIC) for the credit rating process models R1, R2 and R3, and for the latent factor models L1 and L2 estimated from the Standard and Poor’s transition rating data. The model with the smallest Deviance Information Criterion is estimated to be the model that would best predict a replicate data set of the same structure as the data actually observed (Spiegelhalter et al., 2002).

<table>
<thead>
<tr>
<th>Model</th>
<th>R1</th>
<th>R2</th>
<th>R3</th>
<th>L1</th>
<th>L2</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIC</td>
<td>5134.120</td>
<td>4411.050</td>
<td>4455.480</td>
<td>4444.690</td>
<td>4434.510</td>
</tr>
</tbody>
</table>

Table 2: Bayesian estimates for the parameters of the credit rating process model R2. Posterior means with standard errors in parentheses. The effect of the observed macroeconomic covariate credit spread on credit class $k$ is given by $\alpha_k$, while the effect of the latent macroeconomic factor $z_t$ on credit class $k$ is given by $\beta_k$. The variance of the macroeconomic factor $z_t$ is $\sigma_z^2$.

<table>
<thead>
<tr>
<th>Credit class $k$</th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC/C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_k$</td>
<td>0.057</td>
<td>-0.303</td>
<td>-0.278</td>
<td>-0.239</td>
<td>-0.298</td>
<td>-0.412</td>
<td>-0.438</td>
</tr>
<tr>
<td></td>
<td>(0.048)</td>
<td>(0.058)</td>
<td>(0.042)</td>
<td>(0.043)</td>
<td>(0.047)</td>
<td>(0.046)</td>
<td>(0.063)</td>
</tr>
<tr>
<td>$\beta_k$</td>
<td>0.058</td>
<td>-0.123</td>
<td>-0.099</td>
<td>-0.091</td>
<td>-0.100</td>
<td>-0.149</td>
<td>-0.175</td>
</tr>
<tr>
<td></td>
<td>(0.021)</td>
<td>(0.012)</td>
<td>(0.011)</td>
<td>(0.009)</td>
<td>(0.013)</td>
<td>(0.011)</td>
<td>(0.019)</td>
</tr>
<tr>
<td>$\sigma_z^2$</td>
<td>1.472</td>
<td>(0.669)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 1: Time series values of the latent macroeconomic factor $z_t$ estimated from model R2. The upper graph plots the posterior means of $z_t$ with 95% credible intervals. For comparison, the lower graph plots the posterior means of $z_t$ together with the annual standardized values of the credit spread covariate between 1981–2004.
Table 3: Threshold parameters $\gamma_{k,l}$ estimated from the credit rating process model R2. The rows represent the credit classes at the beginning of the year, and the columns represent the credit classes at the end of the year. Standard errors in parentheses. $\gamma_{k,0} = 0$ for any $k$.

<table>
<thead>
<tr>
<th></th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC/C</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>8.787</td>
<td>6.138</td>
<td>4.742</td>
<td>3.788</td>
<td>2.025</td>
<td>1.019</td>
</tr>
<tr>
<td></td>
<td>(1.875)</td>
<td>(1.861)</td>
<td>(1.837)</td>
<td>(1.763)</td>
<td>(1.372)</td>
<td>(0.973)</td>
</tr>
<tr>
<td>AA</td>
<td>14.350</td>
<td>6.871</td>
<td>4.373</td>
<td>2.986</td>
<td>2.591</td>
<td>1.106</td>
</tr>
<tr>
<td></td>
<td>(0.815)</td>
<td>(0.805)</td>
<td>(0.795)</td>
<td>(0.779)</td>
<td>(0.772)</td>
<td>(0.634)</td>
</tr>
<tr>
<td>A</td>
<td>15.390</td>
<td>11.640</td>
<td>5.099</td>
<td>2.798</td>
<td>1.754</td>
<td>0.612</td>
</tr>
<tr>
<td></td>
<td>(0.474)</td>
<td>(0.356)</td>
<td>(0.351)</td>
<td>(0.343)</td>
<td>(0.319)</td>
<td>(0.238)</td>
</tr>
<tr>
<td>BBB</td>
<td>14.200</td>
<td>11.860</td>
<td>8.936</td>
<td>3.036</td>
<td>1.480</td>
<td>0.508</td>
</tr>
<tr>
<td></td>
<td>(0.545)</td>
<td>(0.220)</td>
<td>(0.157)</td>
<td>(0.147)</td>
<td>(0.131)</td>
<td>(0.095)</td>
</tr>
<tr>
<td>BB</td>
<td>12.160</td>
<td>11.070</td>
<td>9.717</td>
<td>7.106</td>
<td>2.178</td>
<td>0.594</td>
</tr>
<tr>
<td></td>
<td>(0.497)</td>
<td>(0.301)</td>
<td>(0.169)</td>
<td>(0.097)</td>
<td>(0.084)</td>
<td>(0.059)</td>
</tr>
<tr>
<td>B</td>
<td>12.700</td>
<td>10.010</td>
<td>8.659</td>
<td>7.917</td>
<td>5.495</td>
<td>0.617</td>
</tr>
<tr>
<td></td>
<td>(1.347)</td>
<td>(0.372)</td>
<td>(0.192)</td>
<td>(0.139)</td>
<td>(0.062)</td>
<td>(0.030)</td>
</tr>
<tr>
<td>CCC/C</td>
<td>8.313</td>
<td>7.823</td>
<td>6.317</td>
<td>5.559</td>
<td>4.553</td>
<td>2.683</td>
</tr>
<tr>
<td></td>
<td>(0.986)</td>
<td>(0.846)</td>
<td>(0.432)</td>
<td>(0.314)</td>
<td>(0.201)</td>
<td>(0.096)</td>
</tr>
</tbody>
</table>

Table 4: Transition probabilities for 2004 (percentages), estimated from the credit rating process model R2. The rows represent the credit classes at the beginning of the year, and the columns represent the credit classes at the end of the year.

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC/C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>93.860</td>
<td>5.670</td>
<td>0.346</td>
<td>0.071</td>
<td>0.039</td>
<td>0.007</td>
<td>0.004</td>
<td>0.003</td>
</tr>
<tr>
<td>AA</td>
<td>0.712</td>
<td>91.880</td>
<td>6.750</td>
<td>0.491</td>
<td>0.053</td>
<td>0.086</td>
<td>0.017</td>
<td>0.011</td>
</tr>
<tr>
<td>A</td>
<td>0.071</td>
<td>2.739</td>
<td>92.400</td>
<td>4.286</td>
<td>0.324</td>
<td>0.121</td>
<td>0.026</td>
<td>0.033</td>
</tr>
<tr>
<td>BBB</td>
<td>0.032</td>
<td>0.267</td>
<td>4.934</td>
<td>90.023</td>
<td>3.702</td>
<td>0.644</td>
<td>0.157</td>
<td>0.241</td>
</tr>
<tr>
<td>BB</td>
<td>0.067</td>
<td>0.119</td>
<td>0.511</td>
<td>7.943</td>
<td>84.230</td>
<td>5.575</td>
<td>0.689</td>
<td>0.866</td>
</tr>
<tr>
<td>B</td>
<td>0.014</td>
<td>0.107</td>
<td>0.325</td>
<td>0.479</td>
<td>8.511</td>
<td>83.718</td>
<td>3.031</td>
<td>3.815</td>
</tr>
<tr>
<td>CCC/C</td>
<td>0.010</td>
<td>0.058</td>
<td>0.477</td>
<td>0.679</td>
<td>2.146</td>
<td>15.240</td>
<td>58.330</td>
<td>23.060</td>
</tr>
</tbody>
</table>
Figure 2: Posterior means with 95% credible intervals for annual default probabilities (percentages) in rating classes A and B between 1981–2004.
Figure 3: Observed annual default rates and out-of-sample estimated annual default probabilities (percentages) in rating classes A and B between 1996–2004. Out–of–sample estimates are obtained from the credit rating model R2 and the latent factor model L2.