Migration dependence among the U.S. business sectors

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Abstract

Based on the methodology of Jafry and Schuermann (2004) to summarize the rating transition matrix into a scalar (a mobility index), we build up time series of theses indices for each U.S. business sector. The database used consists on rating transitions reported by Moody's from the first quarter of 1980 to the first quarter of 2005. As a first step, we check if the mobility index used captures some stylized facts. For instance, we confirm the existence of rating momentum effects for the majority of U.S. business sectors. Then, we test for possibly crisis transmission phenomenon among sectors by estimating a Markov Switching Vector Autoregressions model. The results obtained provide evidence of high and low correlation regimes and prove default contagion among some sectors. For example, an increase of downgrades in the U.S. industrial sector during the high correlation regime implies more downgrades in the U.S. banking sector during the next three months.

Keywords: Credit rating transition matrices, mobility index, credit contagion.

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1 Introduction

The New Basel Accord (BCBS 2004) urges financial institutions to measure and evaluate credit risk by using credit rating matrices. These inputs are generally provided by external credit rating agencies such as Standard & Poor's and Moody's or obtained directly by internal ratings. Both methods are widely used in practice but we can mention the analysis of Treacy and Carey (2000) who documents that these internal credit ratings were adopted increasingly by banks during the last two decades.

Evaluating the total loss on a portfolio resulting from the default of a company's obligors should take into account the migration correlation. An omission or an inaccurate estimation of that correlation will induce a misestimation of the regulatory capital and, consequently, will embrittle the company's financial stability. Moreover, Zhou (2001) and Das, Duffie, Kapadia and Saita (2007) document that default correlation analysis is needed for asset pricing. Such data is fundamental when evaluating a basket default swap or a Collateralized Debt Obligation. Thus, an efficient estimation of the migration correlation is necessary to assess the credit risk.

Gagliardini and Gouriéroux (2005) provide an efficient way to estimate the joint migration probabilities and the migration correlations. However, two major limits are advanced to their methodology. First, they do not consider the contagion effects: the transmission of credit shocks from one firm or sector to the remaining firms. The studies of Giesecke and Weber (2004, 2006) evaluate the credit losses on portfolios by considering both the cross-sectional correlation (a consequence of the common factors affecting all firms) and the credit contagion (caused by the business links between firms). Das et al.(2007) show, empirically, the need to consider contagion effects when modelling default correlation. The second limit concerns the practical aspect: the number of migration correlations to report grows exponentially with the number of credit classes considered. Identifying K classes of credit risk, including the default state which is an absorbent state, will generate $[K(K-1)]^2$ migration correlations. Thus, the 8 classes of credit risk considered by major rating agencies will induce a calculus of 3 136 correlations. This dimensionality problem will be amplified once we distinguish between growth and recession cycles or between different business sectors.

In fact, the time variation of default correlation should be considered by pratictionners. The Das, Freed, Geng and Kapadia (2006) study documents the presence of two regimes of default correlation when focusing on the U.S. public non-financial firms from 1987 to 2000. Moreover, these authors explain the limits of the CreditMetrics methodology. They argue that "asset return correlations are relatively stable over time" when compared to the default correlation.

Thus, we propose the use of the mobility indices mentioned by Jafry and Schuermann (2004) to test for the existence of correlation migration and contagion effects between business sectors. Moreover, we will check if this dependence is cyclical. The mobility indices allow us to summarize the whole transition matrix by a scalar. We can, then, verify if the entire sector is, on average, in an upgrade or downgrade stage. This recapitulation will circumvent the dimensionality limit cited above. It will permit, also, the disentanglement of credit contagion and the cross-sectional correlation.

In addition, we test for some stylized facts belonging to the rating transition matrices literature to prove the usefulness of the mobility indices as a proxy for default risk. A well documented feature relates to the rating momentum: Carty and Fons (1994) study shows that prior rating changes may indicate the direction of future rating changes. In other words, a firm being upgraded (downgraded) during the last period is more susceptible to be upgraded (downgraded) during the following period. A second feature relates to linkage between rating transition probabilities and the business cycle. For example, the study of Nickell, Perraudin and Varotto (2000) reveals that the business cycle affects the rating transition matrix: higher downgrading probabilities are observed during recessions compared to expansion periods.

Based on quarterly rating transitions provided by Moody's for the period January 1980 to April 2005, we evaluate the mobility indices for corporate issuers belonging to various U.S. business sectors. The results obtained confirm the presence of rating momentum for the majority of the business sectors. They also prove the suspected linkage between macro economic factors and transition probabilities. A third result relates to the dependence between the various sectors. Generally, we are able to identify two Markov switching regimes for the migration correlation between two business sectors. Furthermore, the contagion effects vary according to each regime. These shock transmissions are measured by the impulse response functions.

The remainder of our paper is structured as follows: the next section introduces briefly the mobility indices and their applications. The third section describes the data: the transition matrices provided by Moody's and the macro economic variables. We will explain our choice of a specific mobility index in section 4. This chosen index will be used to test some stylized facts in the fifth section and to estimate migration dependence in the sixth section. Finally, concluding remarks will follow in section 7.

2 Mobility indices

Shorrocks (1978) was the first to introduce the concept of mobility indexes. Such a metric is produced to sum up the dynamic part of a transition matrix (off-diagonal probabilities). Therefore, it maps the whole matrix to a scalar and allows easier comparison among several transition matrices. For example, Jafry and Schuermann (2004) apply these metrics to compare the credit transition matrices obtained by various estimation techniques. Many mobility indices are cited in the literature. They can be classified into three categories.

2.1 Cell by cell distance metrics

These metrics are generally obtained via a simple addition of the off-diagonal elements of a transition matrix. Suppose the $(K * K)$ transition matrix P and the identity matrix $I(K)$, where K corresponds to the total number of credit classes. Also, let x_{ij} the transition probability from class i to class j.

$$P = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1K} \\ x_{21} & x_{22} & \cdots & x_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ x_{K-1,1} & x_{K-1,2} & & x_{K-1,K} \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

The most intuitive metrics belonging to this category can be defined as following:

$$DC1 = \sum_{i=1}^{K}\sum_{j=1}^{K} |\widetilde{x}_{ij}| \tag{1}$$

$$DC2 = \sum_{i=1}^{K}\sum_{j=1}^{K} \widetilde{x}_{ij}^{2} \tag{2}$$

$$DC3 = \sum_{i=1}^{K}\sum_{j=1}^{K} (i-j)\,\widetilde{x}_{ij} \tag{3}$$

where $\widetilde{x}_{ij}$ indicates the elements of the *mobility matrix* $\widetilde{P}$ defined in Jafry and Schuermann (2004) as the difference between P and I.

$$\widetilde{P} = P - I \tag{4}$$

2.2 Eigenvalue based metrics

Geweke, Marshall and Zarkin (1986) discussed various mobility indices based on the eigenvalues of P. Here are few examples of these metrics:

$$DEVA1 = 1 - |\det(P)| \tag{5}$$

$$DEVA2 = 1 - |\lambda_2(P)| \tag{6}$$

$$DEVA3 = \frac{\log(0.5)}{\log(\lambda_2)} \tag{7}$$

where det(...) denotes the *determinant*, and λ_i (...) denotes the ith eigenvalue (sorted from largest to smallest absolute value).

The third metric is known as the half-life measure. It allows the calculus of the time needed for the system to decay to 50% of the steady state (the default state when considering credit rating matrices).

2.3 Singular value based metrics

Jafry and Schuermann (2004) devised a new mobility index. They prove that the average of the singular values of the matrix $\widetilde{P}$ defined in (4) gives an idea about the dynamic part of a transition matrix. This new metric is obtained as following:

$$DSV = \frac{\sum_{i=1}^{K} \sqrt{\lambda_i \left(\widetilde{P'} \widetilde{P} \right)}}{K} \tag{8}$$

Finally, we can mention the metric proposed by Arvanitis, Gregory and Laurent (1999). They suggest that the similarity between two matrices (P and Q for example) could be detected according to the following ratio:

$$DEVE = \frac{\|PQ - QP\|}{\|P\| . \|Q\|} \tag{9}$$

where $\|...\|$ designates the norm of the matrix.

The ratio defined in (9) is delimited between zero and two: a nil ratio indicates that P and Q have the same eigenvectors. However, this eigenvector based metric presents a major limitation: we can only have an idea about the resemblance among two matrices. It is impossible, for example, to assess the mobility magnitude of a given matrix or to consider the identity matrix as a benchmark.

3 Data

3.1 Transition matrices

The key input of our study is the rating transition matrices provided by Moody's. We focus on the corporate issuers domiciled in the United States between January 1980 and April 2005. By observing quarterly the rating of each issuer, considering only the 8 major categories (Aaa, Aa, ..., Caa-C, default) and excluding the "withdrawn rating", we obtain 208361 issuer ratings. We notice that the total number of withdrawals stands at 3340. A brief investigation of the database shows that the "withdrawn rating" is not synonymous with high default risk.

Table 1 : Distribution of withdrawals, defaults and new issues

	Aaa	**Aa**	**A**	**Baa**	**Ba**	**B**	**Caa-C**	**Total**
Withdrawals	54	261	550	561	851	834	229	**3340**
Defaults	0	0	2	8	86	454	523	**1073**
New issues	108	481	921	902	1433	1913	341	**6099**

As shown in table 1, some Aaa rated issues are withdrawn during the following quarter (as a result of debt expiration, for example). Thus, we will follow Carty's(1997) methodology and exclude the "withdrawn rating" when estimating the migration matrices.

We remark also that the total number of rating increases over time: less than 1300 ratings per quarter are observed during the eighties, versus more than 2500 ratings per quarter since 1997. This situation is explained by the new issues (6099) exceeding the sum of withdrawals and defaults (standing at 3340 and 1073 respectively).

In the present study, we are dealing with the correlation between business sectors. Consequently, we must consider each sector separately in order to assess its default risk and to test its dependence with the remaining sectors. To do so, we build up quarterly rating transition matrices for each business sector. The descriptive statistics relative to the quarterly ratings per sector are displayed in table 2.

Table 2 : Distribution of quarterly ratings with reference to business sectors

Business sector	**Mean**	**Stand. dev**	**Maximum**	**Minimum**
Banking	179.14	79.88	285	48
Industrial	434.05	132.75	686	246
Financial non bank	294.89	133.61	629	65
Energy	122.64	37.21	189	72
Consumer products	136.65	37.40	209	78
Technology	182.94	71.68	336	84
Transportation	100.31	11.80	123	77
Utilities	323.55	28.65	394	293
Miscellaneous*	288.80	98.26	437	125
All sectors	**2062.98**	**590.69**	**3017**	**1125**

*The Miscellaneous category includes Hotel gaming and leisure, Retail and Media sectors.

As noticed for pooled data, we observe an increase over time in the number of the ratings per quarter for each sector. However, some sectors are characterized by a relatively low rate of increase (the number of ratings nearly doubles for the transportation sector) and other sectors display a high rate of increase (the maximal quarterly ratings is nearly 9 times larger than the minimal ratings for the Financial non bank sector).

A second remark concerns the "Miscellaneous sector". It includes all business sectors defined by Moody's with an average ratings per quarter less than 100. We establish such a rule to avoid dealing with estimated transition probabilities characterized by large confidence intervals.

3.2 Macro economic variables

Several past studies have reported the linkage between default risk and the business cycle. In the present study, we will test if the evaluation of default risk via the mobility metrics allows us to assess this linkage. To do so, we will consider some macroeconomic variables already cited in the literature. The first data consists on the classification of historical periods of the U.S. economy. The National Bureau of Economic Research (NBER) lists 16 quarters of recession and 85 quarters of expansion from January 1980 to April 2005. This business cycle classification was used, for instance, by Bangia, Diebold, Kronimus, Schagen and Schuermann (2002). By distinguishing expansion periods from recession periods, they conclude a statistically significant variation of the estimated migration matrices across the two cycles.

The real GDP growth is the most common variable used by past studies to measure the link between default risk and the business cycle. For instance, Koopman and Lucas (2005) use the real GDP growth as a proxy of the economy growth. This data is available from the U.S. Bureau of Economic Analysis (www.bea.gov).

A third group of variables is provided by the U.S. Bureau of Labor Statistics (www.bls.org): we obtain time series of the unemployment rate and the Consumer Price Index (CPI) from the first quarter of 1980 to the first quarter of 2005. The CPI time series is used to assess the inflation rate. Both time series (unemployment and inflation rates) should be determinants of the strength of the economy: low unemployment and inflation rates are usually the targets of economic policy.

Another macroeconomic variable worth reporting is the real interest rate. A high real interest rate generally induces difficulties in serving the debt. Therefore, we will investigate the link between our mobility metrics and the real interest rate. This variable is derived from the U.S. 3 months T-bill rate provided by the International Monetary Fund and the inflation rate cited above.

Finally, we will consider a macroeconomic index previously used by Figlewski, Frydman and Liang (2006). These authors test whether the Chicago Fed National Activity Index (CFNAI hereafter) is a determinant of default risk. This index is provided monthly by the Chicago Federal Reserve and summarizes 85 existing monthly indicators of U.S. economic activity. A positive value of the CFNAI indicates a growth above the trend whereas a negative value corresponds to a growth below the trend.

4 Mobility measure selection

In order to deal with the dimensionality problem mentioned in the introduction, we have to summarize the quarterly observed migration matrix of each sector into one scalar. Thus, we will estimate, in a first step. the transition matrix for each sector and for each quarter. The cohort method will be used to estimate such a matrix[1]. The second step consists on calculating the mobility index for the estimated transition matrix. We will obtain, then, a time series of the mobility index for each U.S. business sector.

However, a large number of mobility measures are cited in the literature. Therefore, it would be judicious to select a unique and "informative" metric for our analysis. If we focus on the cell by cell distance metrics, we easily observe that the $DC1$ and $DC2$ measures, defined in (1) and (2) respectively, should be discarded. These two metrics take into account all the elements of the *mobility matrix* defined in (4) without distinguishing between upgrades and downgrades. Thus, we can not conclude if the considered business sector is mainly upgrading or downgrading during a specific quarter: this is a crucial information when dealing with default risk.

Also, the $DEVE$ measure, defined in (9), can not be useful for our study. This measure permits only the comparison between two matrices and does not provide information about the direction of migrations. Thus, we will keep, as a first selection, the $DC3$ metric, the eigenvalue based metrics (namely, $DEVA1$, $DEVA2$ and $DEVA3$) and the singular value based metric (DSV) defined by Jafry and Schuermann (2004).

It is worth noting that the three eigenvalue based metrics should lead to the same conclusions since they are based on the second largest eigenvalue of the transition matrix. Therefore, we will keep only one of these metrics for analysis in the following selection. Retaining the $DEVA3$ metric from this category looks the most judicious. Our choice is just explained by the easy interpretation of the scalar obtained via such a metric. For our case, a $DEVA3$ standing at 20 indicates that we need 5 years (20 quarters) to observe 50% of the initial firms of the sector defaulting (if we assume a Markov behavior of the rating migration).

[1] By excluding the withdrawn issues, as mentioned above.

Finally, we have to select one of the remaining metrics ($DC3$, $DEVA3$ and DSV). To do so, we will test the performance of these metrics via three transitions matrices, namely $P1$, $P2$ and $P3$ defined as following:

$$P1 = \begin{bmatrix} 0.85 & 0.09 & 0.04 & 0.02 \\ 0.02 & 0.94 & 0.03 & 0.01 \\ 0.03 & 0.06 & 0.85 & 0.06 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P2 = \begin{bmatrix} 0.85 & 0.09 & 0.04 & 0.02 \\ 0.02 & 0.94 & 0.03 & 0.01 \\ 0.03 & \mathbf{0.09} & \mathbf{0.82} & 0.06 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P3 = \begin{bmatrix} 0.85 & 0.09 & 0.04 & 0.02 \\ 0.02 & \mathbf{0.90} & 0.03 & \mathbf{0.05} \\ 0.03 & 0.06 & 0.85 & 0.06 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where the values in bold indicate the modified transition probabilities with reference to those observed in $P1$. Thus, we notice that $P2$ (*respec.* $P3$) reveals more upgrades (*respec.* downgrades), if compared to $P1$.

Next, we evaluate the mobility of each transition matrix in three different ways. In table 3, we display the calculations of the mobility index of $P1$, $P2$ and $P3$ via the three remaining techniques.

Table 3 : Comparison of the mobility metrics

	$P1$	$P2$	$P3$
DSV	0.1005	0.1089	0.1144
$DEVA3$	32.036	34.835	14.420
$DC3$	-0.200	-0.170	-0.280

If we consider $P1$ as our reference, we remark that the DSV metric increases when applied to $P2$. This same metric increases also when applied to $P3$. In other words, the DSV metric does not allow us to distinguish between a matrix characterized by more upgraded firms ($P2$ compared to $P1$) and a matrix containing more downgraded firms ($P3$ compared to $P1$). Thus, the DSV metric should be discarded since it looks inappropriate to assess the default risk.

However, $DC3$ and $DEVA3$ perform well when applied to different transition matrices. More upgrades should imply a longer time for firms to default and a higher $DEVA3$ metric. The results obtained in table 3 indicate a higher (*respec.* lower) $DEVA3$ if we compare the transition matrices $P1$ and $P2$ (*respec.*$P1$ and $P3$).The analysis of the results obtained through the $DC3$ metric allows for the same conclusions. In fact, this last metric, by its construction, should distinguish between more upgrades and more downgrades. An increase in upgrades will imply a higher $DC3$ measure whereas an increase of downgrades will imply a lower $DC3$. Moreover, this mobility index treats the observed migrations differently: an issue migrating from Aaa to B will have an impact on the $DC3$ metric five times greater than an issue migrating from Aaa to Aa. Table 3 demonstrates the increase in $DC3$ when we compare $P2$ to $P1$. The opposite result is obtained when considering $P3$ and $P1$. Thus, we conclude that $DC3$ is a useful metric to assess the default risk of a whole group, namely the firms belonging to one of the U.S. business sectors.

Theortically, both $DEVA3$ and $DC3$ would be appropriate metrics. However, when confronted to our data, the $DEVA3$ presents some important shortcomings. Here is a sample drawn from our database that will help us explaining our selection of the $DC3$ as the “best” metric and reasons compelling us to discard $DEVA3$.

The estimated transition matrix for the first quarter of 1989 and relative to the banking sector is the following:

	Aaa	Aa	A	Baa	Ba	B	Caa-C	D
Aaa	**1.00**	-	-	-	-	-	-	-
Aa	-	**1.00**	-	-	-	-	-	-
A	-	-	0.99	0.01	-	-	-	-
Baa	-	-	0.04	0.92	0.04	-	-	-
Ba	-	-	-	0.10	0.86	0.04	-	-
B	-	-	-	-	-	0.85	0.15	-
Caa-C	-	-	-	-	-	-	0.50	0.50
D	-	-	-	-	-	-	-	**1.00**

The eigenvalues of this matrix are:

0.82	0.95	0.99	**1.00**	**1.00**	0.85	0.5	**1.00**

We notice three absorbent states (Aaa, Aa and default) in this specific quarter and, consequently, three eigenvalues equal to unity. This statement implies a misleading interpretation of the $DEVA3$ metric. Recall that this index was built to gauge the convergence rate to the absorbent state. Then, this $DEVA3$ metric will give an idea about the convergence rate to all absorbent states. For our case, it will be impossible to assess the evolution of the default risk through this mobility metric.

Having selected the mobility metric for our study, we apply it to the estimated transitions matrices for each U.S. business sector. The descriptive statistics of the obtained time series are displayed in table 4.

Table 4 : Descriptive statistics of the selected mobility measure

Business sector	Mean	Std. dev	Min	Max	Skew	Kurt
Banking	-0.083	0.442	-1.890	0.863	-1.470	6.839
Industrial	-0.173	0.233	-1.025	1.074	0.556	11.565
Financial non bank	-0.138	0.341	-1.323	1.024	-0.325	5.959
Energy	-0.117	0.356	-1.194	1.889	1.047	12.282
Consumer products	-0.141	0.220	-0.933	0.871	-0.399	9.034
Technology	-0.175	0.316	-1.234	0.878	-0.804	5.725
Transportation	-0.146	0.368	-1.619	1.104	-0.952	7.839
Utilities	-0.022	0.432	-1.546	1.525	-0.294	6.148
Miscellaneous	-0.187	0.249	-1.365	0.525	-1.858	9.414
All sectors	-0.172	0.214	-1.151	0.592	-0.845	8.117

We notice a negative mean of the selected mobility metric for each business sector. We should not be surprised by such a result. Indeed, a negative $DC3$ indicates a larger proportion of firms migrating to lower ratings compared to the proportion of upgraded firms: a consequence of the unique absorbent state which corresponds to default. After a long horizon, all existing firms will default, even those rated Aaa in the present.

Figures 1.a and 1.b below display the evolution of the $DC3$ metric during the 101 quarters considered for each U.S. business sector.

Figure 1.a : Evolution of the mobility metric DC3 (panel 1)

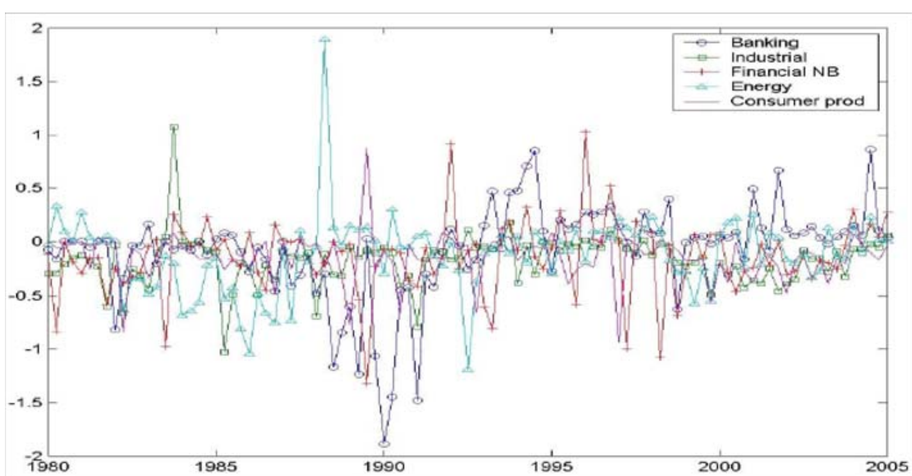
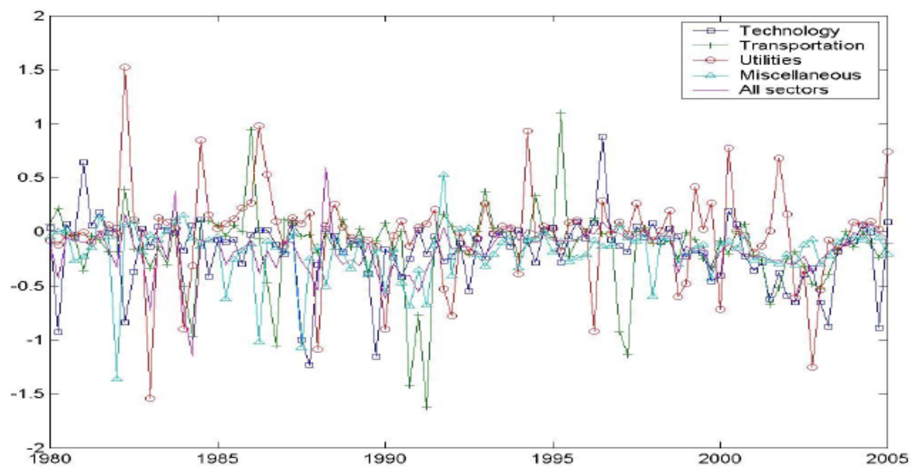


Figure 1.b : Evolution of the mobility metric DC3 (panel 2)



These figures show a spike for the energy sector during the second quarter of 1988: the $DC3$ metric stands at 1.889. This is mainly due to 2 firms migrating from the Caa-C class to the Baa class within the period[2]. A second remark concerns the lowest $DC3$ metric for the banking sector. Our data reveals 8 banks defaulting during the first quarter of 1990. Moreover, all banks rated Caa-C at the beginning of the quarter fall into default at the end of the quarter: that was a consequence of the *junk bonds* crisis observed during 1989-1990.

Finally, since we are analyzing the dependence among the different U.S. business sectors, it is judicious to evaluate the correlation coefficient between mobility metrics relative to these sectors. Table 5 exhibits the correlation coefficients obtained.

Table 5 : Mobility metrics correlation (in percentage)

Business sector	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**	**(9)**
Banking (1)	100	12	15	3	8	10	9	9	23
Industrial (2)	12	100	18	9	28	15	9	-3	22
Financial NB (3)	15	18	100	-11	-20	8	18	-1	2
Energy (4)	3	9	-11	100	10	8	-4	-4	-15
Consumer prod (5)	8	28	-20	10	100	17	4	-15	1
Technology (6)	10	15	8	8	17	100	2	9	8
Transportation (7)	9	9	18	-4	4	2	100	12	19
Utilities (8)	9	-3	-1	-4	-15	9	12	100	-7
Miscellaneous (9)	23	22	2	-15	1	8	19	-7	100

[2] There is a total of 3 firms belonging to the energy sector and rated Caa-C at the beginning of the considered quarter.

Based on table 5, we can conclude that the highest dependence is observed between the consumer products sector and industrial sector. A second remark concerns the banking sector: an increase of the default risk in this specified sector will induce an increase of the default risk in the remaining sectors. Thus, a lot of attention should be accorded by the economic policy to the banking sector since it insures the financing activity of the whole economy. Finally, we can notice that the majority of sectors show a positive dependence: only few sectors are negatively correlated and a coefficient nearing zero is generally obtained for these negative correlations. Thus, we should take an interest to analyze the possible default contagion between business sectors.

5 Stylized facts

In the present section, we will test if the selected mobility metric, namely the $DC3$ metric, captures some stylized facts cited by past studies such as the rating momentum and the links between the business cycle and the default cycle.

5.1 Rating momentum

In order to use the transition matrix to measure default risk, we must assume that the firms' ratings follow a Markov chain. However, this contradicts previous results by Carty and Fons (1994) and Duffie and Singleton (2003) that reveal a rating momentum. In other words, future credit rating changes are affected by historical rating transitions.

In order to verify that the $DC3$ metric picks up this rating momentum, we estimate the following model for each business sector:

$$DC3_t = \beta_0 + \beta_1 DC3_{t-1} + \epsilon_t \tag{R1}$$

where $DC3_t$ corresponds to the mobility metric for a specific business sector during the t-th quarter.

The results of the regression (R1) are displayed in table 6:

Table 6 : Rating momentum

	$\widehat{\beta}_0$ *(t-stat)*	$\widehat{\beta}_1$ *(t-stat)*	R^2
Banking	-0.036	0.557***	31%
	(-0.96)	*(6.64)*	
Industrial	-0.126***	0.262***	7%
	(-4.43)	*(2.68)*	
Financial non bank	-0.138***	0.002	0.1%
	(-3.69)	*(0.02)*	
Energy	-0.074**	0.375***	14%
	(-2.10)	*(4.00)*	
Consumer products	-0.149***	-0.046	0.2%
	(-5.66)	*(-0.46)*	
Technology	-0.131***	0.257***	6%
	(-3.71)	*(2.63)*	
Transportation	-0.109***	0.264***	7%
	(-2.84)	*(2.72)*	
Utilities	-0.020	0.065	0.4%
	(-0.45)	*(0.63)*	
Miscellaneous	-0.195***	-0.035	0.1%
	(-6.20)	*(-0.35)*	
All sectors	-0.144***	0.162	3%
	(-5.26)	*(1.62)*	

*** Significant at the 1% level

** Significant at the 5% level

A rating momentum means that the probability of observing a rating downgrade (*respec.* upgrade) increases if we have already noticed a downgrade (*respec.* upgrade) during the previous period. Thus, testing the presence of a rating momentum phenomena amounts to test if $\widehat{\beta}_1$is statistically positive. The results shown in table 6 indicate that 5 of the 9 business sectors display a statistically significant rating momentum and conforms to previous studies. Also, none of the remaining sectors display a significantly negative dependence between current and previous rating migration.

A second conclusion drawn by previous studies (e.g. Kavvathas (2001) and Carty and Fons (1994)) relates to the asymmetry of the rating momentum. Specifically, the momentum effects are more pronounced for the downgrade case, compared to the upgrade case. We propose to test if the selected mobility metric captures this asymmetry via a two step methodology. As a first step, we estimate the models (R2.1) and (R2.2) for two different cases: the specific business sector was mainly downgraded during the previous quarter ($DC3_{t-1} < 0$) or mainly upgraded ($DC3_{t-1} > 0$).

$$DC3_t = \beta_{0,1} + \beta_{1,1} DC3_{t-1} + \epsilon_{1t} \text{ if } DC3_{t-1} < 0 \tag{R2.1}$$

$$DC3_t = \beta_{0,2} + \beta_{1,2} DC3_{t-1} + \epsilon_{1t} \text{ if } DC3_{t-1} > 0 \tag{R2.2}$$

The results of both regressions are displayed in the table 7.

Table 7 : Asymmetric rating momentum

	$DC3_{t-1} < 0$ *(downgrade)*			$DC3_{t-1} > 0$ *(upgrade)*		
	$\widehat{\beta}_{0,1}$ *(t-stat)*	$\widehat{\beta}_{1,1}$ *(t-stat)*	*N. obs*	$\widehat{\beta}_{0,2}$ *(t-stat)*	$\widehat{\beta}_{1,2}$ *(t-stat)*	*N. obs*
Banking	0.001	0.619***	48	0.018	0.358*	44
	(0.02)	*(4.42)*		*(0.27)*	*(1.66)*	
Industrial	-0.141***	0.244**	91	0.054	-0.101	9
	(-4.55)	*(2.21)*		*(0.33)*	*(-0.23)*	
Financial NB	-0.143**	-0.039	71	-0.195**	0.234	28
	(-2.41)	*(-0.25)*		*(-2.41)*	*(0.90)*	
Energy	0.021	0.649***	56	-0.019	0.081	38
	(0.24)	*(3.06)*		*(-0.73)*	*(1.07)*	
Cons. Prod	-0.169***	-0.090	76	-0.077	-0.242	15
	(-5.85)	*(-0.88)*		*(-1.07)*	*(-0.78)*	
Technology	-0.114**	0.298**	72	-0.139*	0.188	28
	(-2.33)	*(2.46)*		*(-1.77)*	*(0.52)*	
Transportation	-0.070	0.372***	65	-0.065	-0.050	23
	(-1.18)	*(2.83)*		*(-1.54)*	*(-0.41)*	
Utilities	-0.027	0.022	48	-0.045	0.156	52
	(-0.29)	*(0.11)*		*(-0.68)*	*(0.89)*	
Miscellaneous	-0.232***	-0.116	87	-0.025	-0.610*	11
	(-6.04)	*(-0.99)*		*(-0.43)*	*(-1.91)*	
All sectors	-0.123***	0.263**	88	-0.042	-0.684**	12
	(-3.57)	*(2.16)*		*(-0.68)*	*(-2.32)*	

*** Significant at the 1% level
** Significant at the 5% level
* Significant at the 10% level

The second step for testing rating momentum asymmetry consists to check the following hypothesis:

$$\begin{aligned} H_0 &: \beta_{1,1} = \beta_{1,2} \\ H_1 &: \beta_{1,1} \neq \beta_{1,2} \end{aligned}$$

Chow (1960) demonstrates that testing the equality among two coefficients in two regressions needs the examination of the statistic F defined in (10).

$$F = \frac{Q_1 - Q_2}{\frac{Q_2}{m+n-2p}} \tag{10}$$

where Q_1 designates the sum of squares of the residuals (for both regressions R2.1 and R2.2) under H_0. Q_2corresponds to the sum of squares of the residuals under $H_1$for both regressions. m (*respec.* n) represents the number of observations of the regression R2.1 (*respec.* R2.2).Finally, p corresponds to the total number of explanatory variables of each regression.

Chow (1960) proves that, under H_0, the statistic F follows a $Fisher-Snedecor$ distribution with 1 and $(m+n-2p)$ degrees of freedom. The application of this Chow (1960) test to our sample leads to the following results:

Table 8 : Chow (1960) test of asymmetric rating momentum

	Deg. freedom	**F-statistic**	**p-value**
Banking	(1 , 88)	0.828	0.3655
Industrial	(1 , 96)	1.746	0.1895
Financial non bank	(1 , 95)	0.788	0.3770
Energy	(1 , 90)	5.185	0.0252
Consumer products	(1 , 87)	0.351	0.5549
Technology	(1 , 96)	0.105	0.7462
Transportation	(1 , 84)	2.468	0.1199
Utilities	(1 , 96)	0.258	0.6126
Miscellaneous	(1 , 94)	0.793	0.3755
All sectors	(1 , 96)	6.676	0.0113

The results show that the rating momentum asymmetry is statistically significant only if we consider all sectors pooled together or if we focus on the energy sector[3]. However, we notice that the previous studies testing for the rating momentum asymmetry did not adopt a classification of the firms according to their business sector affiliation.

Finally, we remark that the 5 business sectors displaying a rating momentum in table 6 correspond to the sectors with a statistically significant downgrade rating momentum in table 7. Also, none of them shows a statistically significant upgrade rating momentum (except the banking sector at 10% level).

5.2 Mobility metrics and the business cycle

The linkage between rating transitions and the business cycle is well documented by previous studies. Nickell et al.(2000), among others, conclude that the transition probabilities differ from peak to trough periods. We propose the following regression to test if these linkages are still captured when we sum up the whole transition matrix by way of the selected mobility index:

$$DC3_t = \alpha_0 + \alpha_1 RGDPG_t + \epsilon_t \qquad \text{(R3)}$$

[3]In order to check if the rating momentum asymmetry for the all sectors is only due to the energy sector, we apply the Chow (1960) test to all sectors pooled together except the energy sector. The results show up the existence of the rating momentum asymmetry even after excluding the energy sector.

where $RGDPG$ corresponds to the real GDP growth. Table 9 summarizes the results of regression (R3):

Table 9 : Effects of the real GDP growth on the mobility index

	$\widehat{\alpha}_0$ *(t-stat)*	$\widehat{\alpha}_1$ *(t-stat)*	R^2
Banking	-0.148**	2.172	2%
	(-2.38)	*(1.48)*	
Industrial	-0.250***	2.545***	11%
	(-7.95)	*(3.43)*	
Financial non bank	-0.208***	2.315**	4%
	(-4.35)	*(2.05)*	
Energy	-0.060	-1.905	3%
	(-1.19)	*(-1.61)*	
Consumer products	-0.141***	-0.015	<0.1%
	(-4.49)	*(-0.02)*	
Technology	-0.245***	2.301**	5%
	(-5.54)	*(2.21)*	
Transportation	-0.153***	0.223	<0.1%
	(-2.90)	*(0.18)*	
Utilities	-0.023	0.034	<0.1%
	(-0.37)	*(0.02)*	
Miscellaneous	-0.239***	1.717**	4%
	(-6.85)	*(2.09)*	
All sectors	-0.200***	0.943	2%
	(-6.60)	*(1.32)*	

*** Significant at the 1% level

** Significant at the 5% level

We notice that the selected mobility metric conforms to the previous studies: an increase of the real GDP growth implies, usually, a lower default risk. The rating migration of four U.S. business sectors (from the 9 studied) is positively linked to the business cycle with a 95% confidence level. None of the remaining business sectors shows a statistically negative relation between the mobility index and the real GDP growth.

Adding three explanatory variables to the regression (R3) does not alter our previous conclusions. Indeed, if we take into account the real interest rate ($Rinter$), the unemployment rate ($Uempl$) and the inflation rate ($Infl$) to test the linkage between the business cycle and default risk, we can estimate the following model:

$$DC3_t = \alpha_0 + \alpha_1 Rinter_t + \alpha_2 Uempl_t + \alpha_3 RGDPG_t + \alpha_4 Infl_t + \epsilon_t \qquad \text{(R4)}$$

The (R4) estimation results are shown in table 10.

Table 10 : Effects of some macro variables on the mobility index

	$\widehat{\alpha}_0$ *(t-stat)*	$\widehat{\alpha}_1$ *(t-stat)*	$\widehat{\alpha}_2$ *(t-stat)*	$\widehat{\alpha}_3$ *(t-stat)*	$\widehat{\alpha}_4$ *(t-stat)*	R^2
Banking	-0.118	-5.863***	4.616	1.806	-4.882***	13%
	(-0.61)	*(-3.03)*	*(1.44)*	*(1.28)*	*(-2.85)*	
Industrial	-0.259**	-0.612	0.539	2.520***	-0.289	11%
	(-2.53)	*(-0.59)*	*(0.31)*	*(3.34)*	*(-0.32)*	
Financial NB	-0.183	-2.279	0.962	2.220**	-0.889	6%
	(-1.19)	*(-1.46)*	*(0.37)*	*(1.96)*	*(-0.65)*	
Energy	0.405***	0.354	-9.142***	-1.873*	2.615*	17%
	(2.68)	*(0.23)*	*(-3.60)*	*(-1.68)*	*(1.93)*	
Cons. prod.	-0.166	0.828	0.255	-0.004	-0.239	1%
	(-1.63)	*(0.81)*	*(0.15)*	*(-0.01)*	*(-0.26)*	
Technology	-0.323**	4.022***	-1.295	2.488**	1.801	13%
	(-2.35)	*(2.89)*	*(-0.56)*	*(2.45)*	*(1.47)*	
Transportation	-0.226	0.753	-0.066	0.339	1.569	1%
	(-1.33)	*(0.44)*	*(-0.02)*	*(0.27)*	*(1.03)*	
Utilities	-0.061	1.502	-1.012	0.159	1.742	1%
	(-0.30)	*(0.74)*	*(-0.30)*	*(0.11)*	*(0.97)*	
Miscellaneous	-0.353***	-1.915*	2.314	1.719**	0.292	9%
	(-3.17)	*(-1.71)*	*(1.24)*	*(2.10)*	*(0.29)*	
All sectors	-0.148	-1.148	-0.505	0.916	0.138	4%
	(-1.51)	*(-1.16)*	*(-0.31)*	*(1.27)*	*(0.16)*	

*** Significant at the 1% level
** Significant at the 5% level
* Significant at the 10% level

We conclude that the statistically positive link between $DC3$ and the real GDP growth is obtained for the same four U.S. business sectors. Again, none of the remaining business sectors becomes more risky in presence of a positive real GDP growth. An increase of the real interest rate will imply, notably, a decrease of the mobility index relative to the banking sector and thus an increase of its default risk. Finally, we remark the negative effects of inflation on the solvability of the banking sector. The same effect is not observable for the remaining sectors at the 95% confidence level.

In order to confirm our conclusions about the dependence between the business cycle and the default cycle, we propose another proxy for the state of the economy, namely the Chicago Fed National Activity Index (CFNAI).

$$DC3_t = \alpha_0 + \alpha_1 CFNAI_t + \epsilon_t \tag{R5}$$

The results of regression (R5) are the following:

Table 11 : Effects of the CFNAI on the mobility index

	$\widehat{\alpha}_0$ *(t-stat)*	$\widehat{\alpha}_1$ *(t-stat)*	R^2
Banking	-0.071	0.098*	3%
	(-1.62)	*(1.72)*	
Industrial	-0.158***	0.129***	18%
	(-7.42)	*(4.67)*	
Financial non bank	-0.123***	0.126***	8%
	(-3.72)	*(2.94)*	
Energy	-0.124***	-0.053	1%
	(-3.45)	*(-1.15)*	
Consumer products	-0.140***	0.010	0.1%
	(-6.31)	*(0.35)*	
Technology	-0.166***	0.082**	4%
	(-5.28)	*(2.03)*	
Transportation	-0.143***	0.026	0.3%
	(-3.84)	*(0.53)*	
Utilities	-0.027	-0.045	0.6%
	(-0.63)	*(-0.80)*	
Miscellaneous	-0.183***	0.036	1%
	(-7.29)	*(1.10)*	
All sectors	-0.168***	0.031	1%
	(-7.81)	*(1.10)*	

*** Significant at the 1% level

** Significant at the 5% level

* Significant at the 10% level

Again, the $DC3$ is positively related to the CFNAI for four business sectors. If compared to the (R3) results, the only difference concerns the banking and the miscellaneous sectors. The first becomes significantly dependent to the new proxy whereas the second becomes independent. Such similarity is mainly explained by the correlation among the two proxies (the real GDP growth and the CFNAI) standing at 78.5%.

Finally, we propose to follow Bangia et al.(2002) methodology to assess the evolution of our mobility metric through the contraction and expansion cycles defined by the National Bureau of Economic Research (NBER). The mean and the standard deviation of the $DC3$ metric are first estimated for both cycles. Then, we test if the mean differs statistically among the cycles through the following $t-stat$:

$$t-stat = \frac{M_c - M_e}{\sqrt{\frac{s_c^2}{N_c} + \frac{s_e^2}{N_e}}} \qquad (11)$$

where M_i, s_i^2 and N_i correspond, respectively, to the sample mean, the sample variance and the sample size of the cycle $i(i = c, e)$. Table 12 displays the results.

Table 12 : Descriptive statistics of the mobility metric according to NBER cycles

	Contraction		Expansion		Two sample t-test	
	M_c	s_c^2	M_e	s_e^2	$M_c - M_e$	*t-stat*
Banking	-0.164	0.494	-0.067	0.432	-0.097	-0.731
Industrial	-0.366	0.189	-0.137	0.223	-0.229***	-4.322
Financial NB	-0.265	0.224	-0.114	0.355	-0.151**	-2.228
Energy	-0.048	0.236	-0.130	0.374	0.083	1.155
Cons. Prod	-0.181	0.248	-0.134	0.215	-0.047	-0.706
Technology	-0.256	0.334	-0.159	0.313	-0.097	-1.071
Transportation	-0.249	0.432	-0.126	0.354	-0.123	-1.076
Utilities	0.111	0.426	-0.047	0.431	0.158	1.365
Miscellaneous	-0.245	0.374	-0.176	0.219	-0.069	-0.715
All sectors	-0.225	0.182	-0.162	0.219	-0.063	-1.221

*** Significant at the 1% level

** Significant at the 5% level

Only two business sectors show a statistically different mean if we compare expansion and contraction periods. However, none of the remaining sectors displays a contradiction to the results of Bangia et al.(2002). In other words, we never observe more upgrades (higher $DC3$ indices) during the contraction periods.

6 Migration dependence

Having proved the utility of the $DC3$ as a measure of default risk for the U.S. business sectors, we will focus on the dependence that might exist between these entities. We suspect the existence of two patterns of linkage: a cross-sectional dependence explained by common factors affecting all the sectors at the same time (such as the economic policy, the expansion and contraction periods...) and a credit contagion phenomena, that is to say, a transmission of the shock observed in a particular business sector to one (or more) of the remaining sectors. This second feature of dependence is due to the business links between firms belonging to different sectors.

At first sight, it seems judicious to use a vector autoregressions (VAR) model to estimate these two dependences. Such a model is generally used to capture comovements among many variables. However, we will favor a more generalized model that allows for regime-switching. This generalization is needed as a consequence of our previous results and those obtained by past studies: the default risk increases during economic downturns. Thus, we propose to expose, as a first step, some regime-switching VAR models that are applicable to our context and discuss their estimations. The empirical results will follow.

6.1 Regime-switching VAR models

The common feature of regime-switching VAR models consists on a K dimensional time series vector $y_t = (y_{1t}, y_{2t}, ..., y_{Kt})$ defined conditional upon the regime $s_t \in \{1, 2, ..., M\}$. The general form of such processes is the following:

$$y_t = \nu(s_t) + A_1(s_t) y_{t-1} + \ldots + A_P(s_t) y_{t-p} + B_1(s_t) x_t + \ldots + B_q(s_t) x_{t-q+1} + \epsilon_t \tag{12}$$

where $\nu(s_t)$ is the vector of intercepts at regime s_t, $A_i(s_t)$ the matrix containing the autoregressive parameters at the same regime, ϵ_t an error term vector such that $\epsilon_t | s_t \to IID\left(0, \Sigma(s_t)\right)$, $x_t = (x_{1t}, x_{2t}, ..., x_{Nt})$ the vector containing the N exogenous variables, and $B_j(s_t)$ the matrix containing the non autoregressive parameters at regime s_t.

The regime variable s_t can be observable or unobservable: an example of observable regimes is relative to *structural change models*. In such processes, the regime is determined as following:

$$s_t = \begin{cases} 1 & \text{if } t \le \tau_1 \\ 2 & \text{if } \tau_1 < t \le \tau_2 \\ \vdots & \\ M & \text{if } t > \tau_{M-1} \end{cases} \tag{13}$$

A second example of observable regimes concerns the *threshold autoregressive models*. In this case, the regime shifts are triggered by one or more exogenous variables crossing a specified threshold (c_i). In other words,

$$s_t = \begin{cases} 1 & \text{if } g(x_t) \le c_1 \\ 2 & \text{if } c_1 < g(x_t) \le c_2 \\ \vdots & \\ M & \text{if } g(x_t) > c_{M-1} \end{cases} \tag{14}$$

Such process could be used in our context: the rating migration is linked to some macroeconomic variables such as the real GDP growth, NBER cycles, and the CFNAI. However, Amato and Furfine (2004) found "*no evidence that credit ratings are unduly influenced by the business cycle*". Thus, we will favor a process with an unobservable regime changes called Markov-Switching VAR model (MSVAR hereafter). In these models, first introduced by Krolzig (1997), the unobservable regime variable $s_t = \{1, 2, ..., M\}$ is generated by an ergodic Markov chain defined by the transition probabilities p_{ij} with:

$$\begin{aligned} p_{ij} &= \Pr(s_{t+1} = j | s_t = i) \\ \sum_{i=1}^{M} p_{ij} &= 1 \quad \triangledown j = \{1, 2, ..., M\} \end{aligned} \tag{15}$$

A major advantage of MSVAR models consists on their flexibility since we can allow for some non switching estimators. For example, we can estimate an

MSVAR model by assuming invariant autoregressive parameters and invariant intercepts of equation (12). Such particular specification is known as MSH VAR model. Also, we can consider the least constrained specification : the one allowing for varying intercepts, autoregressive parameters and conditional variance, usually known as the MSIAH VAR specification[4]. The Akaike Information or Schwarz criteria are generally used to select one of the available specifications and/or the number of lags to be considered. However, the optimal number of regimes (M) is obtained according to Davies (1987) procedure[5].

6.2 Model estimation

Once we select the MSVAR specification, the number of lags and the number of regimes, we can estimate the obtained model by maximizing its likelihood function. Such maximization entails the implementation of the Expectation-Maximization (EM hereafter) algorithm introduced by Dempster, Laird and Rubin (1977) as a consequence of the unobservable regime variable s_t. Each iteration of this EM algorithm is composed into two steps. The *expectation* step consists on computing the expected likelihood by replacing the unknown parameters by their estimated value obtained during the previous iteration. At the *maximization* step, an estimate of the parameters is derived by maximizing the expected likelihood function found during the *expectation* step. Then, these estimated parameters will be used at the beginning of the next iteration and so on.

At the end of iterations, we can derive the filtered regime probabilities which refer to inferences about the regime variable s_t conditional on information up to time t, the smoothed regime probabilities reporting the inference about s_t by using all the information of the sample and the one-step predicted regime probabilities exposing the inference about s_t conditional on information up to $t-1$.

6.3 Empirical results

We will start by focusing on the U.S. banking and industrial sectors in this empirical part. First, we will estimate an MSIAH VAR specification with one lag, two regimes and no exogenous variables (MSIAH(2) VAR(1)). We set a minimum percentage change of 10^{-6} of the likelihood function as condition to let the EM algorithm iterate again. Finally, we assume Gaussian error terms to derive the likelihood function. In other words, the model exhibited in (12) will be simplified to:

$$\begin{cases} y_{b,t} = \nu_{b,s} + a_{bb,s} y_{b,t-1} + a_{ib,s} y_{i,t-1} + \epsilon_{b,t} \\ y_{i,t} = \nu_{i,s} + a_{bi,s} y_{b,t-1} + a_{ii,s} y_{i,t-1} + \epsilon_{i,t} \end{cases} \tag{16}$$

[4] See Krolzig (1997) for more details on MSVAR specifications.
[5] Garcia and Perron (1996) describe concisely this procedure.

with $s = \{1, 2\}$ indicating the regime variable, $y_{b,t}$ and $y_{i,t}$ designate the observable $DC3$ metric for the banking and industrial sectors at the $t-$th quarter. The vector of error terms $(\epsilon_{b,t}; \epsilon_{i,t})'$ follows, conditionally, a multivariate centered Gaussian distribution:

$$\begin{gathered} \begin{bmatrix} \epsilon_{b,t}|s \\ \epsilon_{i,t}|s \end{bmatrix} \rightarrow NID\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \sum(s) \right) \\ \sum(s) = \begin{bmatrix} \sigma^2_{b,s} & \rho_{bi,s}\sigma_{b,s}\sigma_{i,s} \\ \rho_{bi,s}\sigma_{b,s}\sigma_{i,s} & \sigma^2_{i,s} \end{bmatrix} \end{gathered} \tag{17}$$

$\sum(s)$ denotes the variance covariance matrix of the error terms during a specified regime. The estimation results are displayed in table 13 following:

Table 13: MSIAH(2) VAR(1) estimation results for the banking and industrial sectors

	Regime 1	Regime 2
$\nu_{b,s}$	-0.0797	0.0526
$\nu_{i,s}$	-0.1716	-0.0631
$a_{bb,s}$	0.7403	0.1344
$a_{ib,s}$	0.0930	0.1130
$a_{bi,s}$	0.0561	-0.0591
$a_{ii,s}$	0.2081	0.3833
$\sigma_{b,s}$	0.4756	0.1097
$\sigma_{i,s}$	0.3067	0.0847
$\rho_{bi,s}$	0.0007	0.1558

log-likelihood : 12.4954
Schwarz criterion : 0.6711
Likelihood ratio linearity test : 91.9521
Davies test p-value : 0.0000

For our case, the Davies (1987) test shows a superiority of the MSIAH VAR model compared to the linear VAR model (no switching regime). The likelihood of these two models allows the calculus of the likelihood ratio linearity test. At a first sight, we can notice a regime of low correlation ($s = 1$) among both considered sectors and a regime of high correlation. The statistical significance of the autoregressive parameters will be discussed later in order to assess the contagion effects.

Two additional informations are obtained during the previous estimation: the transition probabilities defined in (15) and the inferences about the regime variable. For the MSIAH(2) VAR(1) specification, the migration from one regime to the other is determined by the transition matrix P:

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = \begin{bmatrix} 0.57 & 0.43 \\ 0.37 & 0.63 \end{bmatrix}$$

The second set of information consists on the filtered, smoothed and predicted regime probabilities. We sum up these inferences in figure 2 :

Figure 2.a : Probabilities of regime 1

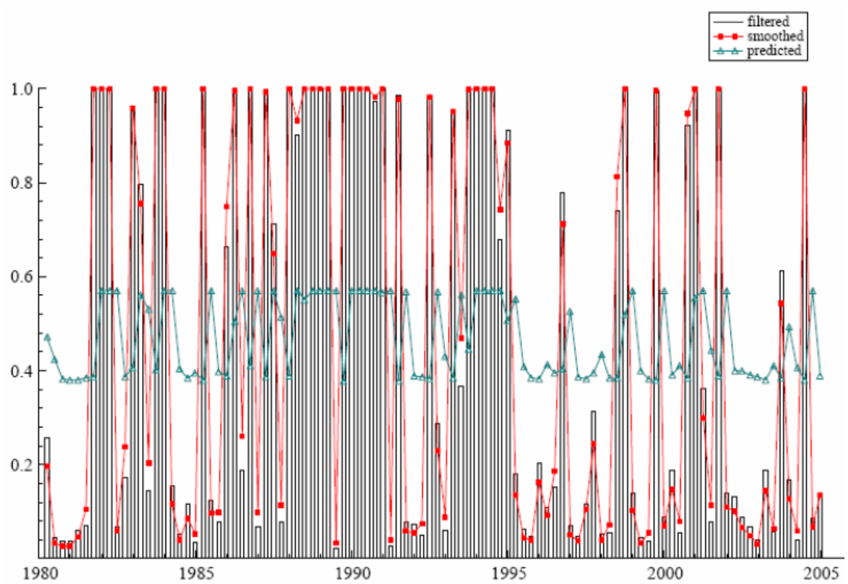
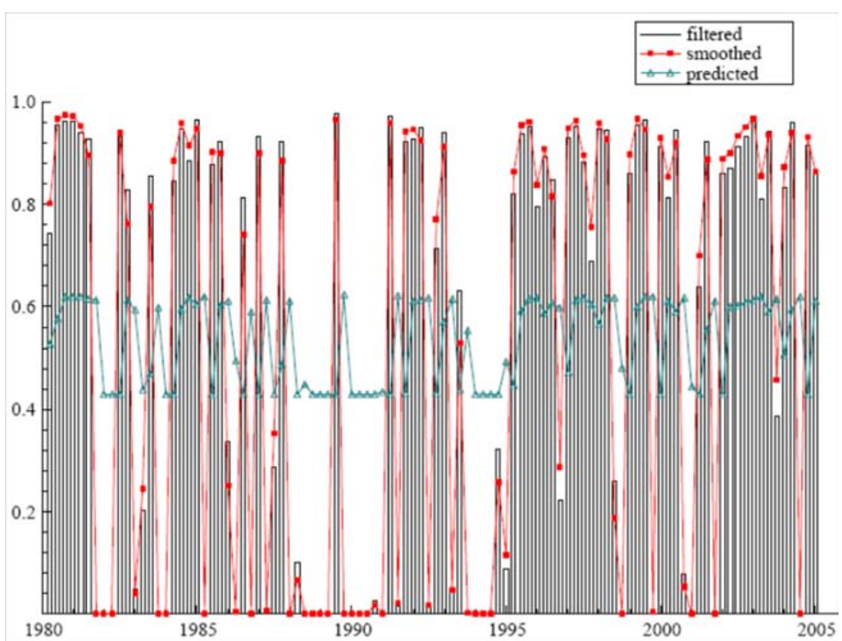


Figure 2.b : Probabilities of regime 2



Given these probabilities, we notice the difference between the default cycle and the business cycle: some contraction quarters (defined by the NBER) belong to the low correlation regime such as the period from the third quarter of 1990 to the first quarter of 1991. However, the recession of the first three quarters of 2001 belong to the high correlation regime. Thus, our results confirm the conclusions of Amato et al.(2004): the business cycle may differ from the default cycle.

A last point worth discussing relates to the significance of the autoregressive parameters or more generally the impact of a shock observed in a particular business sector to all sectors during the next periods. Such analysis is usually done by the Impulse Response Functions (IRF hereafter) analysis for the VAR models. However, testing the significance of the autoregressive parameters or the IRF needs the use of simulation techniques: the non normality[6] of error terms prevents us using the standard t-statistics to test the parameters significance.

Recall that the IRF conditional on regime s and based on the model (16) are the following:

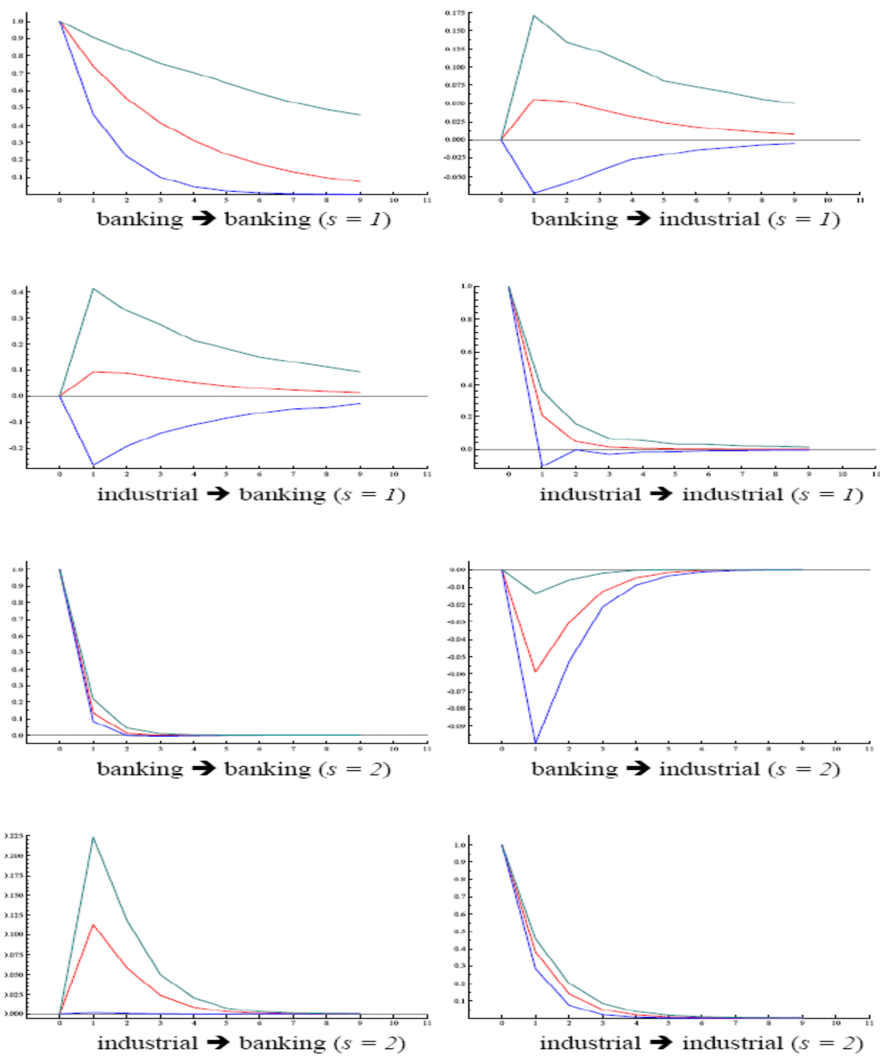
$$\begin{bmatrix} IRF_{bb,t,s} & IRF_{ib,t,s} \\ IRF_{bi,t,s} & IRF_{ii,t,s} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} * \begin{bmatrix} a_{bb,s} & a_{ib,s} \\ a_{bi,s} & a_{ii,s} \end{bmatrix}^t \tag{18}$$

where $IRF_{ib,t,s}$ corresponds to the effect after t quarters of a unitary shock in the industrial sector on the banking sector, given the regime s. By analogy, we define the remaining terms of (18).

Then, by applying the simulation methodology developed by Ehrmann, Ellison and Valla (2003), where we consider 500 simulations and 68% confidence intervals, we obtain the IRF for the next 9 quarters. As shown in figure 3 below, we notice that a positive shock in the banking sector will imply a significant increase of the $DC3$ metric during the next 6 quarters conditional on being in the low correlation regime. However, the effects of the same shock will vanish quickly if we consider the high correlation regime. Moreover, we notice the presence of significant contagion effects during the high correlation regime: an increase of default risk in the industrial sector will induce a significant increase of the default risk relative to the banking sector. Finally, our results confirm again the rating momentum hypothesis: a positive shock in one sector never implies a decrease of the $DC3$ metric relative to the same sector for any regime and any next period.

[6] In our case, we have a mixture of normals since we assume that each error term follows a Gaussian distribution, conditional on regime s.

Figure 3 : Impulse Response Functions for the banking and industrial sectors



A key assumption used in this empirical part concerns the use of a particular specification of MSVAR models, namely the MSIAH(2) VAR(1). In others words, we consider a MSVAR model with one lag, two regimes and intercepts, autoregressive parameters and standard errors varying with the endogenously defined regimes. We propose to test, by use of Schwarz criterion, if this selected specification is the best to represent the observed mobility metrics of the U.S.

banking and industrial sectors. Table 14 following sums up the results relative to alternative specifications:

Table 14: Schwarz criterion for the banking and industrial sectors

	$lag = 1$	$lag = 2$	$lag = 3$
$MSIAH$	0.6711	0.8736	0.8475
$Linear$	1.0841	1.1600	1.2501
MSI	1.1790	1.0343	1.4073
MSA	1.1109	1.2643	1.3477
MSH	0.7747	0.7418	0.8082
$MSIA$	1.1127	1.0906	1.4388
$MSIH$	0.6804	0.6980	0.8527
$MSAH$	0.7237	0.9899	1.2294

The notation used to specify the various Markov Switching models follows this rule:

* **I** denotes a varying intercept term
* **A** denotes a varying autoregressive parameters
* **H** denotes heteroskedasticity (varying variances and covariances)

Thus, we notice the superiority of the MSIAH(2) VAR(1) specification if compared to alternative MSVAR models since it shows the minimal Schwarz criterion. Adding exogenous variables (such as the real GDP change) does not improve our results[7].

The same analysis held above could be applied to other couples of sectors. For instance, the estimation of various MSVAR specifications for the U.S. energy and industrial sectors supports the superiority of the MSIH(2) VAR(1) model. As shown in table 15, the minimal Schwarz criterion is obtained by assuming constant autoregressives parameters, varying intercept terms, heteroskedasticity and a lag equal to one.

Table 15: Schwarz criterion for the energy and industrial sectors

	$lag = 1$	$lag = 2$	$lag = 3$
$MSIAH$	0.7413	0.8648	0.7568
$Linear$	0.8294	0.8800	1.0443
MSI	0.9246	1.0370	1.2213
MSA	0.7559	0.8902	0.7451
MSH	0.7585	0.7434	0.7041
$MSIA$	0.6547	0.6918	0.5650
$MSIH$	0.5433	0.6550	0.7443
$MSAH$	0.6865	0.6260	0.7732

[7]For brevity, we did not include the complete results in our study. They could be provided upon request.

Recall that the MSIH(2) VAR(1) specification applied to the U.S energy and industrial sectors is expressed as following:

$$\begin{cases} y_{e,t} = \nu_{e,s} + a_{ee} y_{e,t-1} + a_{ie} y_{i,t-1} + \epsilon_{e,t} \\ y_{i,t} = \nu_{i,s} + a_{ei} y_{e,t-1} + a_{ii} y_{i,t-1} + \epsilon_{i,t} \end{cases} \tag{19}$$

where $s = \{1, 2\}$ indicates the regime variable, $y_{e,t}$ and $y_{i,t}$ designate the observable $DC3$ metric for the U.S. energy and industrial sectors at the $t-$th quarter. The vector of error terms $(\epsilon_{e,t}; \epsilon_{i,t})'$ is defined by analogy to (17). The estimation results of (20) are the following:

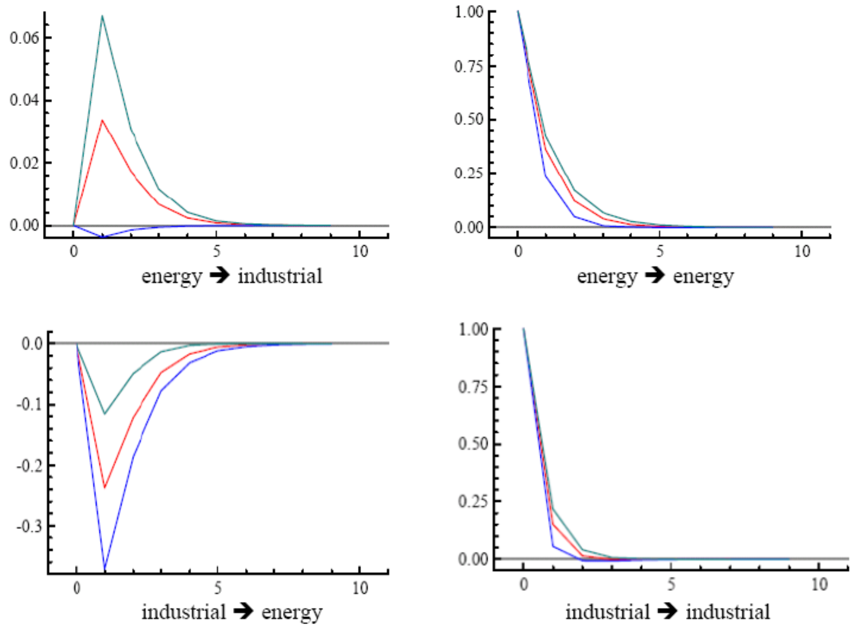
Table 16: MSIH(2) VAR(1) estimation results for the enery and industrial sectors

	Regime 1	Regime 2
$\nu_{e,s}$	-0.1630	-0.0729
$\nu_{i,s}$	-0.2277	-0.0611
a_{ee}	0.3610	
a_{ie}	-0.2346	
a_{ei}	0.0347	
a_{ii}	0.1469	
$\sigma_{e,s}$	0.2335	0.3866
$\sigma_{i,s}$	0.2937	0.0693
$\rho_{ei,s}$	0.1837	0.0373

log-likelihood : 4.6766
Schwarz criterion : 0.5433
Likelihood ratio linearity test : 50.8418
Davies test p-value : 0.0000

The IRF analysis for the both U.S.business sectors confirms that an increase of default risk in the industrial sector will induce a significant decrease of the default risk relative to the energy sector during the following year. Moreover, the effects of a positive shock in the energy sector will statistically vanish after one semester. Figure 4 below displays in details this IRF analysis.

Figure 4 : Impulse Response Functions for the energy and industrial sectors



7 Conclusions

In this paper, we proposed a technique allowing to measure simultaneously the migration correlation and the default contagion among several groups of issuers. Furthermore, this technique avoids the dimensionality problem since it sums up the whole transition matrix to a scalar, namely the mobility metric. As a first step, we tested if the selected mobility metric conforms with previous studies. The results support the rating momentum effect and the linkage between the business and default cycles for several U.S. business sectors during the period 1980-2005. Then, we estimate the dependence among some of these business sectors by use of a regime switching VAR model. For instance, we identified two regimes when focusing on the U.S. banking and industrial sectors: a regime of low correlation (7%) between both sectors and a second regime of higher correlation (15.58%). Moreover, we noticed a difference between the business cycles defined by the NBER and the default cycles identified endogenously by the MSIAH(2) VAR(1) model. Finally, the analysis by impulse response functions allows us to assess the persistence of a shock in a particular sector to the two considered sectors. For example, we observed that an increase of the default risk in the industrial sector will imply a significant increase of the default risk in the banking sector during the next quarter, conditional on being at the high correlation regime. Thus, a lot of attention should be accorded by regulators and investors to the financial institutions if they remark more frequent downgrades of the industrial issuers.

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