Predicting Recession Probabilities with Financial Variables over Multiple Horizons *

Fabio Fornari[†] Wolfgang Lemke[‡]

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[**Figures are best printed in colour**]

Abstract

We forecast recession probabilities for the United States, Germany and Japan. The predictions are based on the widely-used probit approach, but the dynamics of regressors are endogenized using a VAR. The combined model is called a 'ProbVAR'. At any point in time, the ProbVAR allows to generate conditional recession probabilities for any sequence of forecast horizons. Standard probit models, in contrast, would have to be re-estimated separately for each forecast horizon. At the same time, the ProbVAR is as easy to implement as traditional probit regressions. This contrasts with the more general specification in Dueker (2005), which requires simulation-based methods. The slope of the yield curve turns out to be a successful predictor, but forecasts can be improved by adding other financial variables such as the short-term interest rate, stock returns or corporate bond spreads. The in- and out-of-sample performance is very good for the United States, somewhat less satisfactory for Germany, and considerably inferior for Japan.

JEL classification: C25, C32, E32, E37.

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[†] European Central Bank, fabio.fornari@ecb.int

[‡] Corresponding author, European Central Bank, wolfgang.lemke@ecb.int

1 Introduction

Business cycle forecasting has attracted a great deal of applied econometric work. This comes as no surprise, as the expected future level of economic activity is an essential piece of information for individual decision makers as well as for policy institutions. Taking, for instance, an investor's standpoint, investment decisions and consumption plans today are contingent on the expected path of economic activity. Forecasting the future state of the economy is also essential from a financial stability perspective, since periods of contraction are those in which financial institutions or financial markets can become extremely fragile. Moreover, the probabilities of default for industrial firms are highest in or around periods of decline in economic activity. As a final example, monetary policy makers are important stakeholders in econometric models aimed at business cycle forecasting, being interested in the outlook for economic activity and its impact on future inflation developments.

The literature on forecasting economic activity can be broadly divided into two strands, one dealing with forecasting the growth rates of GDP or industrial production, the other trying to predict recession and expansion phases or turning points. Econometric approaches for forecasting recessions usually employ discrete choice (e.g. probit) models: the probability of recession prevailing at a certain future horizon is modeled as a linear combination of selected predictors. Among these, the slope of the yield curve (long-term minus short-term interest rate) has turned out to be particularly successful, see Estrella, Rodriguez, and Schich (2003), Estrella and Hardouvelis (1991), Estrella and Mishkin (1997), and Estrella (2007) among others. As shown by King, Levin, and Perli (2007), the corporate bond spread also carries predictive power, although its role seems to be more relevant when predicting the rate of change of monthly industrial production, see Chan-Lau and Ivaschenko (2002).

However, the typical econometric approach to predict recessions seems to suffer from some limitations. First, analyses tend to use one regressor only, often the slope of the yield curve of the respective country. However, the forecasting power may be increased by including other indicators besides the domestic term spread. In particular, the use of multiple indicators may help to correct wrong signals (high probability of recession when no recession prevails or vice versa) which can be induced by an idiosyncratic move of one specific indicator. Beyond other domestic financial variables, regressors capturing the international environment may serve as additionally useful predictors. In fact, the literature has evidenced the existence of significant spillovers across financial variables in main economic areas and, similarly, commonalities in their business cycles, see, e.g., Kose, Otrok, and Whiteman (2009) or Kose, Otrok, and Whiteman (2003).

Second, discrete choice models for quantifying recession probabilities are usually specified for one particular forecast horizon. However, when one is interested to predict recession probabilities over various, say H, horizons, one would need to estimate H models.

As the main drawback of such an approach, the implied time profile of forecast recession probabilities may be rather ‘bumpy’ as the separate models do not account for the tight relationship between forecast horizons; producing large swings in recession probabilities between similar forecast horizons is not a desirable model outcome, as it would stand in contrast to the evidence that recessionary months or quarters are contiguous.

In this paper we compute predictive recession probabilities using an econometric approach that addresses both points. At the same time, it as straightforward to implement as the traditional fixed-forecast-horizon probit model based on the term spread only. The model consists of two components. The first one is a standard probit relation linking the regressors to the recession probability in the subsequent quarter. Equipped with this component, we are able to predict recession odds as usual but only for the one-quarter-horizon. Multi-period forecasts are made feasible as the probit equation is linked to a VAR – the second component – endogenizing the dynamics of regressors. The combined model is referred to as a ProbVAR. It allows at each point in time to generate recession probabilities for any forecast horizon. In this way, forecasts are consistent with one another as they arise from one single model.

Concerning the other point raised above, i.e. the choice of predictors, we feed the ProbVAR with a number of financial variables. Besides the term spread, we consider the level of the short-term interest rate, the corporate bond spread, and stock returns. We constrain ourselves to financial variables as they are timely indicators that do not suffer from real-time problems. Moreover, we allow these additional variables to enter either in their domestic form or as an average taken over the G7 countries.

We estimate the ProbVAR for the United States, Germany and Japan, using quarterly data from 1960Q1 to 2008Q3. The in-sample and out-of-sample fit are satisfactory for Germany and especially for the United States, but less so for Japan. The latter result is probably related to the particular pattern, with which Japanese recessions occurred (clustered at the end of the sample and with long durations). Our results confirm the well-established finding that financial variables are useful in forecasting the occurrence of recessions or, more generally, for predicting business cycle phases at short to medium term horizons, with the 1-year horizon prediction typically having a good fit for actual recessions, see King et al. (2007). For all countries considered, including additional regressors besides the slope of the yield curve improves the predictive ability over all horizons. However, the preferred choice of additional regressors differs across countries.

We illustrate that the ProbVAR is well-suited for providing a ‘term structure of recession probabilities’, i.e. the sequence of such probabilities from one to any future quarter ahead. This is not only helpful in gauging the length of a potentially forthcoming recession, but also for tracing the probability of leaving a recession period when a recession is already prevailing. Regarding the most recent recessionary period, associated with the financial turmoil that started in summer 2007, certain model specifications detected it, but first predicted this recession to be rather short-lived. Taking the US example, this is related to the fact that the slope of the yield curve is still a prominent predictor in our ProbVAR models: due to the swift and strong monetary policy response to the crisis the yield curve quickly steepened, thus sending a strong signal against recession. Here, the relevance of other financial variables (in this case the corporate bond spread) counteracting this effect, became apparent.

Finally, we present impulse responses of recession probabilities to shocks to the term spread. Unlike with plain linear VARs, impulse responses depend on initial conditions and are not scaling proportionally with the size of shocks. For instance, a decrease in the slope of the US yield curve by one percentage point leads to an increase of US recession probabilities which peaks after four quarters, amounting to about 25 percentage points. However, this effect is conditional on the shock hitting in a situation where regressors are on their sample averages. Tracing the same shock for a situation where the initial yield curve is steeper, leads to a much more subdued response.

There are several studies that likewise confirm the usefulness of simultaneously using several indicators (rather than one) for forecasting economic activity. Often the slope of the yield curve emerges as the most reliable predictor, but other variables have been found to support and sharpen its task. Ang, Piazzesi, and Wei (2006) conclude that the lagged short term interest rate should complement the slope, while Fornari and Mele (2009) evidence that a combination of the lagged term spread and lagged realized stock market volatility is hard to outperform. In addition, financial variables have also been complemented with combinations of economic variables, see Stock and Watson (1989), Stock and Watson (2003), Banerjee, Marcellino, and Masten (2005), Banerjee and Marcellino (2006), or with leading indicators such as business and consumer surveys. While the aforementioned papers were concerned with growth rates rather than recession probabilities, the study by Dovern and Ziegler (2008) uses a large set of variables, (survey indicators, composite indices, measures of real economic activity and financial indicators) to forecast both US economic growth and recession phases. They evidence that all these indicators improve the forecasting performance at short horizons, although they provide no help after one year. As for recession probabilities, they conclude that some of the indicators mentioned above are successful over some periods but not in others. In addition, they produce a significant number of false signals. Similarly, Haltmaier (2008) finds that the oil price, a leading indicator and a stock price index are significant predictors besides the term spread in a simple probit model for US recessions; the same (apart from the oil price) holds for Germany and Japan. Again, false signals are a major problem for out-of-sample predictions for the United States, but less so for Japan and Germany.

In our ProbVAR model, we do not include information contained in past values of the recession indicator (the dychotomic 0/1 variables) as done by Kauppi and Saikkonen (2005), or lagged values of the latent business cycle indicator in the Qual-VAR model by Dueker (2005). Hence, unlike these papers we are excluding direct feedback from the lagged business cycle variable itself. Compared to the Dueker (2005) specification, ours comes with the advantage that estimation and forecasting is straightforward, not requiring any computational burden. The Qual-VAR model, in contrast, requires computationally intensive simulation-based inference. Whether the simpler model that we propose leads to an economically significant loss in forecasting ability is largely an empirical question. At least for the out-of-sample forecast of the 2001 recession in the United States, for which a direct comparison is feasible, the two approaches give rise to similar results, therefore obviously supporting the simpler specification.

The paper is organized as follows. The next section presents the ProbVar model, its estimation and the derivation of recession probabilities over arbitrary horizons. We also explain the calculation of nonlinear impulse responses. The third section presents an application of the model to forecasting recession probabilities in the three countries. In addition to documenting the in-sample and out-of-sample fit we analyze how well the models capture the time profiles of the last two recession periods. The last section concludes.

2 A VAR-augmented probit model

The standard probit specification to quantifying recession probabilities for a fixed forecast horizon of k periods is of the form

$$P(y_{t+k} = 1|X_t) = \Phi(\beta_0 + \beta' X_t), \tag{2.1}$$

where y_{t+k} is a binary variable equal to one if a recession prevails at time $t + k$ and zero otherwise; X_t is a vector containing predictors observed at t (i.e. possibly including variables lagged further); β_0 and β_1 are a scalar and a vector of parameters, respectively; and $\Phi(\cdot)$ denotes the cumulative distribution function of a standard normal. Given estimated (β_0, β_1) and observed X_t, (2.1) delivers the recession probability for time $t + k$. However, in order to obtain recession probabilities for other forecast horizons $h \neq k$, the above equation has to be re-estimated. Noteworthy, the estimated β_0 and β would vary freely across the different horizons, so there is no 'smoothness constraint' on the set of probability forecasts as a function of h.

In the following, we propose a model, that produces recession probabilities for an arbitrary set of forecast horizons with one set of parameters. This is achieved by endogenizing the dynamics of the explanatory variables using a VAR.

2.1 Model structure and estimation

The first ingredient of the model we propose is a VAR(p) specifying the dynamics of the regressors. Let x_t denote an $N \times 1$ vector of variables that are potentially useful in explaining the future recession probability. The evolution of x_t is assumed to follow a homoscedastic Gaussian VAR(p) with serially uncorrelated errors,

$$x_t = c_0 + A_1 x_{t-1} + \dots A_p x_{t-p} + v_t, \quad v_t \sim N(0, S). \tag{2.2}$$

For the following, it is convenient to work with some arbitrary factorization of the residual variance-covariance matrix: $v_t := \Sigma u_t$, with $\Sigma\Sigma' = S$ and $u_t \sim N(0, I)$. As usual, we can represent the VAR(p) (2.2) in companion form,

$$X_t^{(p-1)} = c + B X_{t-1}^{(p-1)} + R u_t, \tag{2.3}$$

where here and in the following $X_t^{(n)} \equiv (x_t', \dots, x_{t-n}')'$ and with obvious definitions of c, B and R.

The second ingredient of the model is the standard probit relation, in which a latent variable y_t^* is specified as a linear function of explanatory variables:

$$y_t^* = \beta_0 + \beta' X_{t-k}^{(l)} + \epsilon_t, \quad \epsilon_t \sim N(0, 1), \quad \text{and } \epsilon_t, u_s \text{ independent for all } s,\, t. \tag{2.4}$$

where ${X_{t-k}^{(l)}}' = (x_{t-k}', x_{t-k-1}', \dots, x_{t-k-l}')$ with $k > 0, \quad l \geq 0$.

For the following, it will be assumed throughout that for at least one of the elements of x_{t-k}, the corresponding entry in β is nonzero, and likewise for x_{t-k-l}. Thus, k defines at which point in the past the vector of predictors is positioned, and l determines the number of lags relative to k.

Finally, a recession prevails at t if the binary indicator variable y_t equals unity,

$$y_t = 1, \text{ if } y_t^* > 0, \text{ and } y_t = 0, \text{ otherwise.} \tag{2.5}$$

The complete model comprises the VAR relation (2.3), the linear relation between the latent recession variable and the regressors (2.4), and the mapping (2.5) of y_t^* into the observable binary indicator y_t. The system will be referred to as a ProbVAR(p, k, l) model.

As each parameter of the ProbVAR system either appears only in (2.3) or only in (2.4) and the innovations ϵ and u are assumed to be independent, the parameters can be estimated consistently by separately estimating the VAR part (e.g. by OLS) and the probit part (by Maximum Likelihood).

2.2 Representing the model with a single state vector

The vector $X_t^{(p-1)}$ of the VAR in companion form can differ in length from $X_{t-k}^{(l)}$ used in the probit relation. For instance, it may be the case that there are no further lags beyond k in the probit relation, while the VAR comprises several lags, hence making $X_{t-k}^{(p-1)}$ longer than $X_{t-k}^{(l)}$. Likewise, there may be several lags required in (2.4) while in the VAR only one lag may suffice. However, for the derivation of recession probabilities over different horizons (see next subsection) it is convenient that $X_{t-k}^{(p-1)}$ and $X_t^{(l)}$ have the same length. We now describe how to represent both the probit regressors and its (Markovian) dynamics using the same state vector.

There are three cases to consider. If $l = p-1$, then $X_t^{(p-1)}$ entering the VAR companion form and $X_{t-k}^{(l)}$ in the probit relation have the same length and the same relative lag structure.

If $p - 1 > l$, then the vector of the VAR in companion form is larger. To represent the system in terms of this vector, the vector β in (2.4) has to be augmented with zeros. That is, we re-write (2.4) as

$$y_t^* = \beta_0 + \tilde{\beta}' X_{t-k}^{p-1} + \epsilon_t \tag{2.6}$$

where $\tilde{\beta}' = (\beta', 0_{N\cdot(p-1-l)})$.

If $p - 1 < l$, the probit relation makes use of a richer lag structure than the VAR. In this case, a VAR(1) representation of this longer state vector will be employed. Denote the number of additional lags to be accommodated by $p^\Delta = l - (p-1)$, then the dynamics of $X_t^{(l)} = (X_t^{(p-1)}, X_{t-p}^{(p^\Delta-1)})$ are given by[1]

$$\begin{aligned} &\begin{pmatrix} X_t^{(p-1)} \\ X_{t-p}^{(p^\Delta-1)} \end{pmatrix} \\ = &\begin{pmatrix} c \\ 0_{(N\cdot p^\Delta)\times 1} \end{pmatrix} + \begin{pmatrix} [B \mid 0_{(N\cdot p)\times(N\cdot p^\Delta)}] \\ \left[0_{(N\cdot p^\Delta)\times(N\cdot(p-1))} \mid I_{N\cdot p^\Delta} \mid 0_{(N\cdot p^\Delta)\times N}\right] \end{pmatrix} \begin{pmatrix} X_{t-1}^{(p-1)} \\ X_{t-p-1}^{(p^\Delta-1)} \end{pmatrix} \\ + &\begin{pmatrix} R \\ 0_{(p^\Delta\cdot N)\times N} \end{pmatrix} u_t \end{aligned} \tag{2.7}$$

Unless otherwise noted, we will always work with the canonical state vector X_t (without superscript) that is defined as $X_t \equiv X_t^{(\max\{l,p-1\})}$. We will write

$$X_t = c + BX_{t-1} + Ru_t, \tag{2.8}$$

for the VAR dynamics of regressors, hence without distinguishing explicitly, for instance, whether B is just the companion-form matrix of the VAR(p) in (2.3) or the augmented version appearing in (2.7).

2.3 Computing probabilities at the h-period horizon

Given the parameters $(c, B, R, \beta_0, \beta)$ of the ProbVAR, the objective is to compute probabilities of a recession occurring h periods ahead, i.e. $Pr(y_{t+h} = 1|X_t)$. For $h = k$, this is simply

$$Pr(y_{t+k} = 1|X_t) = \Phi(\beta_0 + \beta' X_t).$$

[1] If $p = 1$, the sub-matrix $0_{(N\cdot p^\Delta)\times(N\cdot(p-1))}$ vanishes.

For $h > k$, the VAR dynamics has to be employed. Let $d \equiv h - k$. From (2.8) it follows that

$$X_{t+d} = B^d X_t + (I + B + \ldots + B^{d-1})c + \sum_{i=1}^{d} B^{d-i} R u_{t+i}$$

The distribution of X_{t+d} conditional on X_t is normal with conditional expectation

$$\mu_d(X_t) = B^d X_t + (I + B + \ldots + B^{d-1})c \tag{2.9}$$

and conditional variance-covariance matrix

$$V_d = \sum_{i=1}^{d} B^{d-i} R R' (B^{d-i})'. \tag{2.10}$$

Thus, the distribution of y^*_{t+h} conditional on X_t is also normal with conditional expectation

$$m_d(X_t) = \beta_0 + \beta' \mu_d(X_t)$$

and conditional variance

$$v_d^2 = \beta' V_d \beta + 1.$$

Accordingly, the probability of interest is given by[2]

$$\Pr(y_{t+h} = 1 | X_t) = 1 - \Phi(0; m_d(X_t), v_d) = \Phi\left(\frac{m_d(X_t)}{v_d}\right). \tag{2.11}$$

Before examining the empirical performance of the ProbVAR model, it is worth noting that the 'Qual VAR' model by Dueker (2005) is of a similar nature. In fact, it is more general then the specification chosen here, as it directly specifies the joint dynamics of y^*_t and X_t as a VAR. Hence, compared to our recursive approach, Dueker allows for richer dynamics as both X_t and y^*_t can depend on lagged y^*_t. Owing to the enlarged interaction with the latent variable, estimating the model and conducting predictions has to rely on simulation-based filtering techniques, whereas both tasks are less computationally demanding for the ProbVAR model that we propose. Whether the additional complexity of the Dueker approach adds value for a given forecast exercise remains largely an empirical issue.

2.4 Impulse response analysis

Similar to a linear VAR, the ProbVAR can be used to trace the effect of an unexpected change in one state variable at time t on the time profile of subsequent recession probabilities. Formally, we can define an impulse response function as

$$g^{IR}(h; X_t, \delta) := \Pr(y_{t+h} = 1 | X_t + \delta) - \Pr(y_{t+h} = 1 | X_t), \quad h \geq k. \tag{2.12}$$

[2] The function $\Phi(x)$ with one argument denotes the cdf of a standard normal evaluated at x, and the function $\Phi(x; a, b)$ with three arguments is the cdf of $N(a, b^2)$ evaluated at x.

As a crucial difference to standard VAR analysis, the sequence of probability responses will not only depend on the shock vector δ but also on the current state X_t. This is an immediate implication of the probabilities being a non-linear function of the state vector X_t.

For the choice of δ, we face the same considerations as with shock identification in linear VAR analysis. In the empirical analysis below, we do not attempt to provide a structural shock identification and use the concept of generalized impulse responses as in Pesaran and Shin (1998). That is, we fix the shock for the ith equation in (2.2), say δ_i and set the complete shock vector as the conditional expectation,

$$(\delta_1, \dots, \delta_N)' = E(v_t | v_{i,t} = \delta_i).$$

The computation of this conditional expectation employs the normality assumption $v_t \sim N(0, S)$ in (2.2), where S is replaced by its OLS estimate $\hat{S}$.[3]

3 Recession probabilities for the United States, Germany and Japan

We apply the ProbVAR models to estimate recession probabilities in the United States, Germany and Japan since the 1960s. We first describe the dependent variables, i.e. the binary recession indices, and the pool of potential regressors. Thereafter, we report the goodness of fit of various specifications. For each country, we also conduct an in-depth analysis of the current recession (triggered by the financial crisis that took its origin in the US sub-prime mortgage market) and the previous one. The last sub-section illustrates the applicability of the ProbVAR model to conduct impulse response analysis.

3.1 Data on recessions and explanatory variables

The data set consists of time series of recession indicators and financial variables to be employed as potential predictors. Data are quarterly and span the period from 1960Q1 to 2008Q3.

The business cycle dating for the United States is taken from the NBER, while the recession classifications for Germany and Japan are from the Economic Cycle Research Institute (ECRI).[4] A given quarter is taken as a recession if at least one month in that quarter was identified to be a recession.

We consider the following group of potential regressors: the slope of the yield curve (ten-year government bond yield minus short-term interest rate), sl, the short-term interest rate itself, i, the year-on-year return of a broad-based stock price index, sr, and the

[3] The remaining elements of δ are filled with zeros such that it complies with the overall length of the canonical state vector X_t in (2.8).

[4] http://www.businesscycle.com.

corporate bond spread, cs. All these time series are taken from Global Financial Data. For the stock return and the corporate bond spread, we try both the variable of the respective country and the average of that variable taken over the G7 countries (indicated by a bar, i.e. $\bar{sr}$ and $\bar{cs}$). This is done because many papers find the presence of commonalities in the business cycles of main economic area. Therefore, to the aim of forecasting the probability of recession in one country, variables that capture the international environment potentially matter beyond domestic indicators.

3.2 Model specification

For forecasting recession probabilities in the three countries, we run the ProbVAR model with different combinations of regressors and different lag specifications. For comparison, we also estimate sets of simple probit models, i.e. equation (2.1) for each specific horizon, without endogenizing the regressors. Recall that in contrast to the ProbVAR models, the simple probit models have to be estimated separately for each forecast horizon.

For both model types (ProbVAR and simple probit), we consider specifications with one to three regressors. We always include the slope sl of the domestic yield curve since this variable has been shown to be key in forecasting recessions in the literature. Besides the slope-only specification, we consider specifications with one or two additional regressors from the following pool of five variables: $\{i,\ sr,\ \bar{sr},\ cs,\ \bar{cs}\}$. To restrict the total number of examined combinations, we do not consider set-ups that contain sr together with $\bar{sr}$, or cs together with $\bar{cs}$. This leaves us with a total set of 14 regressor combinations.

Concerning the lag structure of the ProbVAR, we fix the lag k in the probit relation (2.4) to one, in order to have one quarter as shortest possible forecast horizon. As additional lags in the probit relation (2.4), we consider $l = 0$, i.e. no additional lags beyond one, and $l = 3$, i.e. the probit relation contains regressors lagged by between one to four periods. The lag structure of the VAR is chosen to be either one or four. Hence, using the terminology introduced in section 2.1, we consider four dynamic specifications for each set of regressors: a ProbVAR($p = 1$,$k = 1$,$l = 0$), a ProbVAR(1,1,3), a ProbVAR(4,1,0) and a ProbVAR(4,1,3).

For the simple probit models, the lag length of the regressor is determined by the forecast horizon. Thus, for instance, to the aim of forecasting recessions six quarters ahead using the slope of the yield curve, the simple probit model has to be estimated on data pairs $(y_t,\ sl_{t-6})$ possibly enhanced with additional lags, i.e. $(y_t,\ sl_{t-6}, sl_{t-6-1}, \ldots, sl_{t-6-l})$. In analogy to the ProbVAR case we consider the cases $l = 0$ and $l = 3$.

We compare models by looking at recession forecasts for prediction between one and six quarters ahead. We consider in-sample and out-of-sample forecasts. For the in-sample version, model parameters of the ProbVAR and simple probit models are estimated based on data from 1960Q1 to 2008Q3.[5] For the out-of-sample exercises, the models are estimated using data such that the last y_t is recorded for time $t = T_0^*$=1994Q4. Given the estimated parameters based on this sample, the respective state vector is plugged into the estimated model and recession probability forecasts for times $T_0^* + h$, $h = 1, 2, 3, \ldots$ are computed and stored. Then the model is estimated on a window expanded by one quarter, and so forth.

3.3 Goodness of fit

Tables 1, 3 and 5 show the fit of the 14 (identified by the regressor combination) times 4 (identified by the lag specifications) different ProbVAR models for the United States, Germany and Japan, respectively. The upper part of the tables documents the in-sample fit, while the lower part shows the fit out-of-sample. The tables report the so-called R^2_{count} measure of fit, i.e. the weighted sum of fractions of correctly identified recession and non-recession periods,

$$R^2_{count} = \frac{n_{11}}{n_1} \cdot \frac{n_1}{n} + \frac{n_{00}}{n_0} \cdot \frac{n_0}{n} = \frac{n_{11} + n_{00}}{n}, \tag{3.13}$$

where n is the number of forecasted quarters in the sample, n_1 is the number of recessions, $n_0 = n - n_1$ is the number of non-recession quarters, n_{11} is the number of correctly identified recessions and n_{00} is the number of correctly identified non-recessions.

It is important to note that the R^2_{count} fit measure as well as the single 'hit ratios' of correctly identified recessions, $\frac{n_{11}}{n_1}$, or correctly identified non-recessions, $\frac{n_{00}}{n_0}$, are all relying on a conversion of the model-implied recession probabilities into a binary variable. That is, given an estimated probability of recession $\widehat{\Pr}(y_t = 1)$ for some time t, a mapping $D : [0, 1] \mapsto \{0, 1\}$, $\hat{y}_t = D\left(\widehat{\Pr}(y_t = 1)\right)$ has to be applied that converts probabilities into alleged recessions or non-recessions. Typically, such decision rule is of the sort $\hat{y}_t = 1$, if $\widehat{\Pr}(y_t = 1) > \Pr^*$ and $\hat{y}_t = 0$ otherwise. Hence, the choice of the threshold $\Pr^*$ is essential. Setting $\Pr^* = 0.5$ is often too conservative a criterion, especially when the overall fraction of recessions in the sample is relatively small.[6] A natural choice is to set $\Pr^*$ equal to the frequency of recessions measured over a long period. We set $\Pr^* = 0.20$

[5] More precisely, the first data tuple used for estimation always includes the recession indicator of 1962Q2. This is done in order to guarantee that all specifications will provide forecasts for the same sequence of quarters. For instance, for the simple probit model with $l = 0$ and a forecast horizon of six quarters, the probability of recession in 1961Q3 could be forecast with variables dated 1960Q1, but for the specification with $l = 3$ (richer lag structure of probit), the required regressors lie before the beginning of the data set. For the richest specifications (simple probit with $l = 3$, and ProbVAR with $l = 3$ and/or $p = 4$), 1962Q2 is the first date for which a recession probability can be forecast over the six-quarter horizon.

[6] See the discussion in the chapter on discrete choice models in Greene (2003).

for the United States and Japan and to $\Pr^* = 0.25$ for Germany.[7] Strictly speaking, in out-of-sample exercises, this fraction could be adjusted period by period. However, in order to have results independent of such time variation, we treat it as constant. Since the computation of the R^2_{count} measure comes with the ad-hoc element of choosing $\Pr^*$, we will occasionally also refer to an alternative measure, which does not rely on the choice of such a threshold. It is given by the mean absolute difference between the model-implied probabilities and the actual outcome of the binary recession variable,

$$MAE = \frac{1}{T}\sum_{t=1}^{T}\left(\left|y_t - \widehat{\Pr}(y_t = 1)\right|\right). \tag{3.14}$$

Tables 2, 4 and 6 provide the same type of results as tables 1, 3 and 5 but for the simple probit models. Recall that unlike for the ProbVAR approach, the simple probit specifications are estimated separately for each of the indicated forecast horizon.

In the out-of-sample panel of the US tables, some lines are left blank. These refer to combinations of explanatory variables, for which the probit equations could not be estimated for at least one lag combination (or at least one horizon for the simple Probit models). As is well-known in the standard probit literature, this problem arises when the regressor and lag structure is too rich, so that it would provide a perfect fit.[8] These variable specifications are discarded.

3.3.1 United States

What should be considered the preferred specification for the United States? For the ProbVAR model variants, the R^2_{count} measure (averaged over prediction horizons 1 to 6) for the out-of-sample results is highest for the specification with the term spread sl, the short-term interest rate i, and the international average of corporate bond spreads $\bar{cs}$ (table 1, lower panel). Regarding the dynamics, four lags in the probit part of the model are preferred to only one lag, whereas for the VAR dynamics one lag rather than four lags seems to be slightly preferred. The fit measure indicates that the model manages to predict correctly 93% of recession and non-recession periods on average. Moreover, the *MAE* criterion would select the same specification, and the *in*-sample R^2_{count} measure is likewise highest for the (sl, i, $\bar{cs}$)-specification.

The R^2_{count} is a global measure of fit, obtained as a weighted average of the fraction of correctly predicted recession quarters and correctly predicted non-recession quarters. Taking a more detailed look on the model performance with respect to different horizons and with respect to recession vs. non-recession quarters, one observes that for prediction

[7] Computed over the whole sample, the fraction of recessions amounts to 0.18 for the United States, to 0.28 for Germany, and to 0.21 for Japan

[8] See the discussion in Greene (2003), p. 683-4.

horizons of 1, 4 and 6 quarters, the (sl, i, $\bar{cs}$)-model correctly predicts (i.e. the model-implied probability exceeds 0.20) 7 out of 8, 6 out of 8, and 3 out of 8 recession quarters, respectively, over the out-of-sample period. For the same forecast horizons, it gives false signals 2%, 2% and 5% of all periods, respectively. The specification with the slope as the only regressor, predicts 7 out of 8, 7 out of 8 and 6 out of 8 recession quarters, respectively, but false signals are emitted 26%, 26% and 31% of all time periods. Hence, the (sl, i, $\bar{cs}$)-specification greatly improves on the number of false signals, while performing only slightly worse in identifying recessions up to the four-quarter horizon. By contrast, it does not do well on the six-quarter horizon. Importantly, as there are only eight recession quarters (belonging to the 2001Q1-Q4 and 2007Q4-2008Q3 recession periods) in the out-of-sample period, the fraction of correctly identified recessions is somewhat sensitive to the choice of the threshold probability $\Pr^*$, here set to equal 0.20. For instance, lowering this ratio does not change the overall ranking of models according to the R^2_{count} measure, but increases the number of correctly predicted recession quarters for the preferred model, while slightly raising its percentage of false signals.

Figure 1 shows the in-sample fit for the preferred (sl, i, $\bar{cs}$) specification together with the benchmark approach that uses the slope only. For the one-quarter and four-quarter horizons, the fit appears to be fairly satisfactory. Even with a forecast horizon of six quarters the model captures the starts of three recessions (1970Q1, 1980Q1, and 1990Q4) and for all recession periods at least one quarter is correctly identified on the ground of the chosen regressors observed six quarters earlier. Overall, the preferred specification (sl, i, $\bar{cs}$) generates fewer false signals than the slope-only model.

Figure 3 shows the out-of-sample probabilities together with shaded recession areas from 1995Q1 to 2008Q3. Besides the slope-only specification ('Spec 1'), and the preferred (sl, i, $\bar{cs}$)-specification ('Spec 2'), we included the (sl, $\bar{cs}$) specification ('Spec 3'), which does not include the short-term interest rate: it improves upon specifications 1 and 2 in predicting all recession quarters for the one- and four-quarter horizon, and predicts 6 out of 8 recession quarters at the six-quarter horizon. It also beats the slope-only specification in emitting fewer false signals (17%, 17% and 26%), but it is still worse overall compared to the preferred specification 2. As the figures show, all the models perform fairly disappointingly on the six-quarter horizon, but do a fine job on the four- and one-quarter horizon. As a drawback, specifications 1 and 3 have typically been anticipating a recession for each following quarter since 2006, and have been doing so with increasing strength, mainly on account of the slope of the yield curve, which was flat or even negative during the period. The preferred specification 2, in contrast, managed to counteract this effect, by taking into account the relatively low level of the short-term interest rate. A more detailed analysis on how well the ProbVAR models manage to trace the time profile of the last two recessions is conducted in sub-section 3.4.

Turning to the simple-probit specifications, the out-of-sample results come out in favor of the same regressor group as identified for the ProbVAR models (see Table 2). Again, also the *MAE* is lowest for the (sl, i, $\bar{cs}$)-specification. The same specification with the corporate bond spread replaced by its domestic counterpart gives rise to similar fit statistics, but we will continue to use the same regressors as for the ProbVAR models for the sake of comparability.

It is remarkable that the goodness of fit is sightly better for the ProbVAR model (R^2_{count}=0.93) than for the corresponding simple probit specifications (R^2_{count}=0.88), which are tailored to the specific horizons. Comparing the out-of-sample fit of the ProbVAR and the simple probit approach (figure 3 vs. figure 4), the first thing to be noted is that the one-quarter ahead forecasts are identical by construction. For the four-quarter horizon, the ProbVAR specifications appear to be superior in predicting the four quarters of the 2001 recession and its end. The six-quarter predictions are both unsatisfactory to a similar extent, but the simple probit approach produces more erratic sequences of predicted probabilities.

Moreover, for the in-sample exercise – where, as already mentioned, the simple-probit approach, fitting a separate model for each prediction horizon, may be expected to be favoured – the fit measures turn out to be similar, see also figure 2 compared to figure 1. Again, the one-quarter prediction are identical by construction, the four-quarter fit is similar, and the six-quarter forecasts once more constitute a more ragged time series for the simple probit models.

3.3.2 Germany

Compared to the encouraging results for the United States, the out-of-sample results of the German ProbVAR specifications (lower panel of table 3) are inferior across all forecast horizons. Nonetheless, the highest average R^2_{count} measures for the ProbVar, which stands at around 0.78, is high enough to consider the behavior of the model as overall satisfactory . The best performing specification includes the slope of the yield curve, the level of the short-term interest rate and the international stock return measure (sl, i, $\bar{sr}$). Competing specifications that replace the latter variable by the domestic stock return measure or the corporate bond spread do only marginally worse. This underlines that combinations of predictors can strongly improve upon the standard slope-only specification. The latter reaches an average R^2_{count} of 0.37 only, which is less than half of what the (sl, i, $\bar{sr}$)-model achieves. For the in-sample results (upper panel of table 3), the improvement over the slope-only specification is not as strong but it is nonetheless remarkable. Table 3 also shows that the bulk of improvement stems from the inclusion of the short-term interest rate beyond the slope. Yet, the further inclusion of the stock return provides some additional fit both in-sample and out-of-sample.

The three panels of figure 5 show the in-sample fit of the preferred (sl, i, $\bar{sr}$) specification and the slope-only specification. The former is obviously better at discriminating between recession and non-recession periods, a result that holds for all the chosen forecast horizons. Figure 7 shows the out-of-sample performance of the two specifications ('Spec 1' and 'Spec 2'). For each forecast horizon, the slope-only specification does not generate enough variability in the probability forecasts,so that it attributes very small probabilities of recessions to periods which were actually recessions. Furthermore it provides a fairly large number of 'wrong signals' when the recession threshold is fixed at 0.25. The preferred (sl, i, $\bar{sr}$)-specification greatly reduces the number of false signals, which is the main reason for its better overall fit measure. However – in contrast to results for the United States – the ProbVAR does not manage to predict part of the last two recessions for the four- and six-quarter horizons. Regarding the one-period-ahead prediction, relevant for identifying recessions in real time, the model recognizes the 2001 recession somewhat belatedly. For the last recession, results are satisfactory as probabilities increase quickly as the recession approaches. At the same time, recession probabilities remained very low in 2005 and 2006, consistently with the positive GDP growth, a feature which is not shared by the slope-only specifications.

As we did for the United States, we include in figure 7 the out-of-sample performance of an additional specification ('Spec 3'), which includes the slope, the stock return and the corporate bond spread as regressors. This specification does not include the short-term interest rate as regressor. Compared to the (sl, i, $\bar{sr}$)-specification with the highest R^2_{count}, the (sl, sr, cs)-specification tracks the start of the 2001 recession quite well from the onset. It also manages to keep near-term recession probabilities low in 2005 and 2006, thereby improving on the slope-only specification. It also foresees the recession that started in December 2007, although the one-period-ahead recession probability begins to rise somewhat too early. The major drawback of this specification is the number of false signals provided at all forecast horizons during 1999 and 2000.

Regarding the comparison of the ProbVAR models with their simple-probit counterparts, the out-of-sample R^2_{count} tend to favor the latter only marginally, highlighting that the use of one single (ProbVAR) model for all forecast horizons, rather than a specific model for each horizon (as is the case for the simple-probits) does not lead to significant loss of forecasting ability. The in-sample measures of fit, by contrast, tend to be somewhat more supportive of the simple probit models.

Looking at the fitted recession probabilities rather than at overall measures of fit evidences that the in-sample fit of the simple probit models (figure 6) is somewhat more erratic compared to the analogous ProbVAR results for four- and six-quarter horizons.[9] When discussing the out-of-sample results, we added again the (sl, sr, cs)-specification for comparison. The results for the ProbVAR specifications and the simple probit variants

[9]Recall that the one-quarter forecasts are again identical by construction.

are similar for the four-quarter horizon. Looking at the six-quarter horizon, the (sl,i,$\bar{sr}$)-specification generates a more volatile series of probabilities when the simple-probit is used relative to the ProbVAR, leading to a higher number of recession quarters being correctly identified, but also to a larger number of false signals.

3.3.3 Japan

Figure 9 highlights that the Japanese economy has exhibited a pattern of recession periods which has significantly diverged from the experience of the United States and Germany. From the beginning of our sample in 1960 to 1991 there has been only one recession episode between 1973Q4 and 1975Q1. From 1992 until the end of the sample, in contrast, Japan has experienced a recession for about 50% of the time. This odd distribution of recession periods obviously poses a challenge for any forecast model.[10]

According to the R^2_{count} measure, the in-sample performance of both the best ProbVAR and the best simple probit specifications for Japan do not differ much from the results obtained for Germany. However, when judging the out-of-sample performance of the model, there is a significantly deterioration relative to the results obtained for the other two countries. The preferred specification (lower panel of table 5) contains the slope, the average corporate bond spread and the stock return as regressors. Note that unlike for the United States and Germany, this specification does *not* contain the short-term interest rate, while the best in-sample specification, (sl, i,sr), does contain it. The ranking of the various specifications differs more than for the other two countries depending on whether a out-of-sample or a in-sample standpoint is taken. This is possibly again due to the aforementioned distribution of the Japanese recession periods over the sample. However, as for Germany and the United States, the best in-sample and out-of-sample specifications of the ProbVAR greatly improve upon the slope-only specification. As figure 9 shows, the gain of the (sl, $\bar{cs}$, sr)-specification over that benchmark model occurs especially for the near-term horizon. In fact, when the out-of-sample exercise concerns the one-period horizon, the preferred ProbVAR model manages to detect with fairly good precision the start of the 2000Q3-2003Q2 recession and the most recent recession. In addition, the model-implied recession probabilities decreased quickly with the ending of the former recession. At the four- and six-quarter-ahead horizons, the slope-only and the (sl, $\bar{cs}$, sr)-specification are both disappointing, although the richer specification manages to achieve a somewhat higher fit measure.[11]

[10]Based on a sample from 1978 to 1997, Hirata and Ueda (1998) find that the term spread has some predictive power for Japanese recessions, but it is by far not as strong as in the case of the United States. They also find some predictive content of stock market data for longer forecast horizons, but this predictor produces fairly noisy signals.

[11]Unlike for the United States and Germany, we did not include a third specification for the out-of-sample results, as a similar trade-off (the third model predicts more recessions but at the cost of more false

Comparing also for Japan the ProbVAR with the simple probit results, the former are on average superior to their simple-probit counterparts regarding the out-of-sample measures of fit. Moreover, even in sample, the specifications that deliver the best results for the ProbVAR model are not worse than the best simple-probit specifications.

3.4 Zooming in on the last two recession periods

As argued above, the ProbVAR model is ideally suited to generate at a given point in time the whole sequence of recession probabilities for the quarters ahead. Such a 'term structure of recession probabilities' can likewise be generated with the simple probit model. However, a separate set of parameters has to be estimated for each horizon, which lacks the consistency imposed when using the ProbVAR model. The following shows these time profiles of recession probabilities for the most recent and the previous recession in the respective country.

3.4.1 United States

In order to see to what extent the ProbVAR model traces the recession profile of the US 2001 recession, we estimate the model parameters (both the VAR part and the probit part) using data up to and including 2000Q3, i.e. we use only information dated two quarters before the start of the recession. We then use the regressors and their required lags prevailing at that date to compute the recession probability for 2000Q4, 2001Q1, ... , 2003Q3. We employ the same model specifications as discussed in section 3.3.1: the slope sl as only regressor (Spec. 1); the preferred specification with the slope, the short-term interest rate and the international corporate bonds spread (sl, i, $\bar{cs}$) as regressors (Spec. 2); and the specification with the same regressors but without the short-term interest rate, (sl, $\bar{cs}$) (Spec. 3), which turned out to be an excellent predictor of recessions albeit at the cost of producing a higher number of false signals.

The left panel of figure 13 plots the three term structures of recession probabilities based on 2000Q3 information. Specifications 1 and 2 trace the time path of the recession very well. They initially assign a small recession probability to the subsequent quarter (2000Q4), in which the recession in fact did not occur, to then produce probabilities higher than 0.2 and rising towards 0.5 for the quarters which are within the recession. Further, the estimated probabilities for the quarters subsequent to the end of the recession tend to decrease at a very quick pace. Compared to the slope-only specification 1, the preferred specification 2 shows a faster decay of recession probabilities for the periods after the recession. Specification 3 gives a similar time profile for the probabilities but it also assigns a markedly higher probability to 2000Q4, which was classified an expansion, while giving higher odds (exceeding 0.6) to the recessionary periods. Overall, it shows the same

signals) did not arise for Japan.

speed of decay for the predicted probabilities as specification 2 once the recession is over. Overall, the ProbVAR model traces rather well the whole profile of the recession, when using information dated two quarters before its start.

The right panel of figure 13 also contains the results of Dueker (2005) based on his QualVAR. Recall that this model is more general as our ProbVAR since it also allows for feedback from the lagged latent business cycle indicator y_t^* to the current y_t^* and regressors X_t. In Dueker's study, X_t comprises real quarterly GDP growth, quarterly CPI inflation, the slope of the yield curve as well as the federal funds rate. He claims that "a recession probability above 50% for 2001:Q3 and 2001:Q4 is a rather strong signal of recession. This is especially true in light of the difficulties that professional forecasters and the leading indicators had in anticipating the 2001 recession." Against this background, the ProbVAR specifications do very well in this exercise and turn out to be similarly discriminative as the probabilities obtained by Dueker. As a difference, his recession probability for 2001Q1 is somewhat lower than all our ProbVAR specification. However, as the begin of the recession is dated as March 2001 (i.e. only the last third of 2001Q1 was in recession), it is not clear, which of the results should be considered superior for this quarter. In the last recession quarter 2001Q4, in contrast, our probabilities are somewhat lower than Dueker's, but still indicative of a recession. For the subsequent out-of-recession quarters, our recession probabilities show a quicker decay (which is desirable).

How do our ProbVAR results compare to the application of twelve (the number of horizons) simple probit models? The left panel of figure 14 shows that the corresponding sequences of probabilities do not match the recession profile as nicely as the different specifications of the ProbVAR do. Specifications 2 and 3 start with increasing probabilities through the recession, although thereafter probabilities suddenly decrease while the recession is still ongoing. Moreover, rather than converging towards the unconditional frequency of recession at longer horizons, as occurs with the ProbVAR specifications, the simple probit models predict another significant increase in probabilities nine quarters after the end of the recession. The slope-only specification correctly identifies a path of increasing probabilities over the length of the recession, but it also keeps them markedly elevated for some time after the recession's end.

Another exercise to assess a model's ability at fitting recession probabilities is to look how well it predicts the exit from a recession given that the forecaster stands at a point in time which is still within the recession. With the same set of parameters as before, but using state variables dated 2001Q3, i.e. standing at the third recession quarter, we produced again the recession probabilities for the twelve subsequent quarters. As shown by the right panel of figure 13, all three ProbVAR specifications do a very good job in assigning a high probability of recession to the following quarter (which was the last recession quarter), while dropping below 0.1 thereafter (when recession in fact was over) to finally slowly converge towards the long-run recession frequency (around 0.2).

Although the simple probit models performed similarly well in this experiment (right panel of figure 14), still the ProbVAR specifications appear to be somewhat superior, especially as concerns the post-recession quarters, when probabilities are smoothly returning to the unconditional average value.

Turning to the most recent US recession, the NBER dated its start as of December 2007, so that 2007Q4 is treated as the first recession quarter. Our sample ends in 2008Q3, a period which should still have been a recession, considering that the sharpest drops in output have been recorded in the last quarter of 2008 and in the first of 2009.

Unfortunately, the models seem to have some problems in fitting the most recent recession. The out-of-sample forecasting exercise is based on parameters estimated on data up to 2006Q4 and on state variables up to 2007Q3. Through these ingredients we produce recession probabilities for the twelve quarters after 2007Q3. Figure 15, left panel, plots the results for our three ProbVAR model specifications.[12] All three models correctly foresee the recession in 2007Q4 and assign a high probability to it, as estimated probabilities are all in excess of 0.6. The slope-only specification 1 as well as specification 3 with the slope and the corporate bond spread produce probabilities being larger than 0.2 also for 2008Q1 and 2008Q2 but quickly receding thereafter. Our preferred specification 2 (with the slope, the corporate bond spread and the short-term interest rate), in contrast, sees recession probabilities being negligible beyond 2007Q4. Repeating the exercise for the following quarters, always keeping the parameters fixed at the values based on estimation up to 2006Q4, it turns out that only specification 3 manages to capture the presence of a recession. For example, standing at the edge of our sample in 2008Q3 (see the right panel of figure 15), specification 3 forecasts additional three quarters of recession (i.e. until 2009Q2), while the other two specifications see the end of the recession already in 2008Q4.

Why do specifications 1 and 2 fail to see a protracted recession, given the intensity of the initial financial turmoil? The reason most likely has to be found in the very strong reaction of the US monetary policy, which led to a sudden rise of the slope of the yield curve as the short term interest rate was aggressively lowered since the start of the crisis. In fact, the slope of the yield curve turned from values ranging between -25 and approximately 90 basis points in 2007 to a steep-sloping configuration, between 210 and 300 basis points, in 2008, and the model could not but interpret it as a strong signal against recession. Specification 3, by contrast, complements the information of the slope with the corporate bond spread (international average) and this has been strongly increasing since 2007Q3, counteracting the expansionary signal coming from the yield curve. Specification 2 features instead the short term interest rate as a third regressor: its ability to reduce false signals in previous episodes comes as a drawback in the current recession since the Treasury Bill rate dropped from 3.7 % at the end of 2007Q3 to 0.9 % at the end of 2008Q3, thereby

[12] We do not present the corresponding results from the simple probit models, being qualitatively similar.

strongly reducing the model-implied recession probability.

3.4.2 Germany

Similarly to what occurred in the United States, the year 2001 marked the beginning of a recession also for the German economy. However, with nearly three year, it lasted significantly longer than the US one. As for the US case, we estimated the model until 2000Q3 and with this set of parameters we produced forecasts for recession probabilities for up to twelve quarters ahead, i.e. for the period ranging between 2000Q4 and 2003Q3. Figure 16, left panel, shows the result of this exercise for the slope-only specification (Spec 1), the (sl, i, $\bar{sr}$)-specification (Spec 2) – preferred on the basis of the R^2_{count} measure – and the (sl, sr, cs)-specification (Spec 3).

Using only the slope, elevated recession probabilities are generated for the whole prediction horizon. Also placing the forecaster in 2001Q3 (right panel of figure 16), an almost flat line of recession probabilities of around 0.35 would be obtained for all the subsequent twelve quarters (i.e. also for the subsequent expansion). Hence, the slope-only specification does not adequately trace the time profile of this recession. The predictions of the (sl, i, $\bar{sr}$)-specification are also disappointing, but in a different way: neither standing before the recession nor being inside it, the model seems to be able to forecast the recessionary periods ahead. The (sl, sr, cs)-specification is doing better. Conditional on 2000Q3 information, it predicts a small recession probability for the subsequent quarter, a probability near the threshold for 2001Q1, the start of the recession, and probabilities of around 0.4 for the remaining quarters. Standing in 2001Q3, the model fully recognizes the recession with odds close to one for the subsequent year. Subsequently it forecasts recession probabilities to go down monotonically.

Overall, the simple probit models perform roughly similar for this exercise, but exhibit the following differences: most strikingly, besides failing again to predict the recession at both points in time (i.e. in 2000Q3 and in 2001Q1), the (sl, i, $\bar{sr}$)-specification predicts (conditional on 2000Q3) recession probabilities to increase markedly *after* the recession is over. That is, compared to the corresponding ProbVAR, it worsens the performance of this combination of variables by adding a false signal at the end of the forecast horizon. The (sl, sr, cs)-specification produces a profile of recession probabilities in the twelve quarters after 2000Q3 that is more hump-shaped compared to its ProbVAR counterpart: it predicts higher probabilities for the horizons ranging between one half and two years but strongly decreasing probabilities, eventually going below 0.2, thereafter. Standing in 2001Q3, the (sl, sr, cs)-specification correctly anticipates that the recession will continue in the coming year, as the same combination of variables did in the corresponding ProbVAR version of the model, but after that, probabilities decrease at a faster rate.

The most recent recession is classified to have started in 2008Q2 for Germany. Using regressor information up to 2007Q4 and parameters estimated until 2006Q4, the ProbVAR specification featuring the slope as the single predictor forecasts a long period of recession probabilities (left panel of figure 18). However, the slope-only specification has been indicating relatively high recession probabilities since 2005, which makes this model relatively unreliable. The (sl, sr, cs)-specification also anticipates elevated recession probabilities, but the (sl, i, $\bar{sr}$)-specification does not. In 2008Q3 then, when data releases had made it rather clear that the German economy was in recession, the slope-only specification displayed again a rather flat and not much elevated profile of recession probabilities, compared to the predictions made three quarters before. For the two richer specifications, by contrast, the profile of forecast recession probabilities has been markedly changing beyond 2007Q4. Both specifications in fact predicted probabilities exceeding 0.8 when evaluated at the values of the explanatory variables in 2008Q4 and 2009Q1. Looking further, probabilities go down for the (sl, i, $\bar{sr}$)-specification, remaining above 0.25 for the whole 2009 and then dropping below 0.2 as of 2010Q3. The results of the (sl, sr, cs)-specification appear much more pessimistic: this model anticipates a decay of recession probabilities but less fast than specification 2, with probabilities still near 0.5 in 2011Q2.[13]

3.4.3 Japan

The first Japanese recession that we use to check the model's forecasting ability starts in 2000Q3 and ends in 2003Q2. Besides its extreme length, another peculiarity is given by the fact that it started only four quarters after the end of the previous recession (itself having a remarkable length of 11 quarters). Hence it comes as no surprise that the ProbVAR (left panel of figure 19) and the simple probit models (left panel of figure 20) have a hard time in assigning high probabilities to such an event. Once the recession has already elapsed for three quarters (right panels of the respective pictures), the preferred ProbVAR specification (sl, $\bar{cs}$, sr) predicts the recession to stay for at least one more year (probabilities exceeding 0.4) or even six quarters (when a threshold probability of 0.2 is applied). The simple-probit specification, in contrast, only foresees half a year of recession to follow.

The most recent and still ongoing recession started for Japan in 2008Q1. Again we use the same strategy as before, based on estimating the model's parameters using information until 2006Q4 and then using regressor information until 2007Q4. The preferred ProbVAR model predicts a recession in the two subsequent quarters with probabilities of around 0.7 and 0.5, respectively. The slope-only specification, in contrast, could not anticipate the occurrence of a recession. Placing the preferred model in 2008Q3, the length of the recession is perceived to be of around additional three quarters, with probabilities reaching respectively 1.00, 0.85 and 0.65. For 2009Q3, the predicted recession probability records

[13] As for the United States we omit to show the simple probit results.

a large drop (from 0.65 to 0.35), and slowly decreases thereafter.[14]

3.5 Illustrating impulse response analysis

In section 2.4, we explained how the ProbVAR model can be employed to generate impulse responses, i.e the changes in recession probabilities in reaction to unexpected changes of the explanatory variables. We exemplify this method empirically by showing how recession probabilities respond to an unexpected change in the slope of the yield curve, for the United States, Germany and Japan. This particular type of shock is chosen since the term spread is one of the most prominent variables used in previous studies on recession forecasting, and because it enters the preferred specification of all the countries considered.

Due to the nonlinearity of the ProbVAR specification, the impulse responses differ from those obtained from linear VARs in two important dimensions: first, scaling the impulse vector does not proportionately scale the impulse responses; second, impulse responses will depend on the initial state vector. To illustrate these points we show for each country six versions of the impulse responses, which comes from three different shock sizes, combined with two different initial conditions. All results are based on 'generalized' impulse responses in the sense of Pesaran and Shin (1998).[15]

For the United States, a decrease of the slope of the yield curve by one percentage point leads to an increase in subsequent recession probabilities as expected. For the scenario in which the slope shock hits in a situation when all variables are at their sample means, the recession probability shows its peak response in the fourth quarter after the shock with a magnitude of nearly 25 percentage points, see figure 24. Thereafter, it slowly peters out towards zero. If the same shock is taken to be of double size, the peak response occurs likewise after four quarters. However, the probability more than doubles compared to the half-sized shock and amounts to nearly 70 percentage points. As a third variant of the shock we consider a one-percentage-point *increase* in the term spread. While this shock mirrors the flattening shock considered first, the two responses of recession probabilities are not at all mirror images of each other: the maximum response does not even reach 10 percentage points in absolute magnitude; in addition, the peak response is recorded after six rather than four quarters.

The same three shocks are again considered for different initial conditions. This time, regressors are also set equal to their sample averages, but the initial slope of the yield curve is elevated by one sample standard deviation. Hence, we start from a situation characterized by lower recession probabilities for the quarters ahead. The results for

[14] Again, we omit to show results from the simple-probit version.

[15] That is, the initial shock vector is given as the expectation of shocks to the driving variables in the respective specification (e.g., slope, short-term interest rate and average corporate bond spread for the US) conditional on the shock to the slope assuming the specified magnitude.

the three shocks point out clearly the other effect of nonlinearity, namely that impulse responses are dependent on the initial condition. Here, the responses associated with an initially steeper yield curve are much more dampened than their counterparts based on the initial slope being at its sample mean.

Turning to the results for Germany, figure 23, the same asymmetry as for the United States is observable, but to a much lesser extent. In particular, the dependence on initial conditions is less distinct. The response to a one-percentage-point shock of the slope shows the same order of magnitude as in the US case (slightly over 20 percentage points), but the peak occurs already in the third quarter after the shock. The responses to the double-sized negative shocks are smaller than their US counterparts, the reactions to the positive slope shocks are slightly more distinct and peak earlier.

Finally, for Japan the response to the term spread shock is very small, see figure 24. The maximum reaction to a flattening of the yield curve of one percentage point occurs after two quarters already and is below 10 percentage points. The dependence on initial conditions is negligible.

4 Conclusion

This paper proposed a model, named ProbVAR, for forecasting recession probabilities. It retains the traditional features of the probit regressions approach of, e.g., Estrella and Mishkin (1997), but combines it with a VAR capturing the dynamics of regressors that enter the probit equation. At each point in time, the model can produce a smooth 'term structure of recession probabilities', which is consistent with the evidence that business cycle phases are persistent. As for the choice of regressors, we use the slope of the yield curve, but also other financial variables that turn out to markedly improve forecasting power relative to univariate probit regressions based on the term spread only. We apply the model to forecasting the recession phases of the United States, Germany and Japan since the 1960s. The in- and out-of-sample performance is very good for the United States, somewhat less satisfactory for Germany, and considerably inferior for Japan. As mentioned, our ProbVAR model produces recession probabilities for all forecasting horizons based on one single estimation. We compare the recession probabilities that originate from the ProbVar for horizons of one to six quarters to the comparable predictions stemming from standard probit models, which are separately estimated for each forecast horizon. It turns out that the quality of the fit is overall similar across the two approaches. However, the ProbVAR model tends to imply more plausible time profiles of recession probabilities, compared to the term structures of recession probabilities implied by separately estimated standard probit models.

The model that we propose is straightforward to estimate and to implement. It is suited to represent a useful tool in various fields of applications. For instance, from a macro-prudential perspective, it can help to identify threads of recession over short- and medium-term horizons. Forecasting information is quickly updatable as regressors are financial variables, which are available in real time. For central banks, it can likewise help to identify and predict periods of economic slack, but it also provides useful information on the expected additional length of a recession, once it has actually begun.

The results of this paper are encouraging and call for several extensions. For instance, the ProbVAR may be extended to a bi- or multivariate approach in the sense that it predicts the joint probability of recession for two or more countries. Second, while we employ averages of financial variables from various countries, it appears worthwhile to explore the forecasting power of financial factors drawn from a larger set of financial data. Third, in our approach, the same set of variables appeared in the VAR and in the probit relation. This one-to-one relation may be fruitfully relaxed: there may be variables in the VAR part that help forecasting, e.g., the term spread, but these variables may not show up in the probit relation. Finally, a more in-depth comparison (empirical or Monte-Carlo-simulation based) between our simple ProbVAR and the QualVAR of Dueker (2005) is in order.

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A Tables

The tables report the R^2_{count} measure of fit as defined in (3.13): The abbreviations for the regressors read as follows: *sl*: slope of the yield curve, *i*: short-term interest rate, *cs*: corporate bond spread, *sr*: year-on-year stock return. Variables without a bar denote the variables of the respective country, a bar on top of the variable denotes the average of the respective variable taken over the G7 countries.

		Forecast horizon: 1q				Forecast horizon: 4 q				Forecast horizon: 6 q				Av. over forec. hor. 1 to 6 q			
		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4	
	Specification	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)
1	sl	0.63	0.63	0.80	0.80	0.84	0.81	0.75	0.76	0.83	0.84	0.74	0.72	0.77	0.77	0.77	0.76
2	sl, i	0.78	0.78	0.88	0.88	0.82	0.84	0.82	0.79	0.77	0.79	0.75	0.76	0.80	0.82	0.81	0.81
3	sl, cs	0.74	0.74	0.86	0.86	0.81	0.81	0.76	0.79	0.81	0.81	0.71	0.70	0.79	0.78	0.78	0.78
4	sl, sr	0.81	0.81	0.89	0.89	0.71	0.74	0.76	0.76	0.71	0.71	0.75	0.70	0.74	0.75	0.80	0.78
5	sl, i, cs	0.79	0.79	0.88	0.88	0.79	0.81	0.81	0.81	0.73	0.74	0.73	0.74	0.78	0.79	0.81	0.82
6	sl, i, sr	0.82	0.82	0.95	0.95	0.77	0.79	0.82	0.78	0.74	0.72	0.72	0.73	0.77	0.78	0.83	0.81
7	sl, cs, sr	0.84	0.84	0.95	0.95	0.72	0.77	0.79	0.80	0.70	0.72	0.71	0.70	0.76	0.78	0.81	0.81
8	sl, $\bar{cs}$	0.75	0.75	0.84	0.84	0.74	0.76	0.82	0.82	0.71	0.76	0.75	0.70	0.74	0.75	0.80	0.79
9	sl, $\bar{sr}$	0.78	0.78	0.88	0.88	0.68	0.76	0.79	0.81	0.66	0.78	0.80	0.80	0.70	0.77	0.81	0.82
10	sl, i, $\bar{cs}$	0.80	0.80	0.91	0.91	0.76	0.81	0.86	0.85	0.68	0.76	0.84	0.81	0.76	0.80	0.87	0.86
11	sl, i, $\bar{sr}$	0.84	0.84	0.96	0.96	0.77	0.83	0.82	0.81	0.77	0.76	0.78	0.76	0.79	0.81	0.85	0.84
12	sl, cs, $\bar{sr}$	0.81	0.81	0.92	0.92	0.68	0.76	0.78	0.82	0.68	0.73	0.75	0.72	0.72	0.78	0.81	0.82
13	sl, $\bar{cs}$, sr	0.84	0.84	0.91	0.91	0.74	0.77	0.79	0.79	0.71	0.68	0.70	0.67	0.76	0.78	0.80	0.79
14	sl, $\bar{cs}$, $\bar{sr}$	0.86	0.86	0.92	0.92	0.74	0.78	0.79	0.82	0.66	0.72	0.78	0.75	0.76	0.80	0.83	0.82

		Forecast horizon: 1q				Forecast horizon: 4 q				Forecast horizon: 6 q				Av. over forec. hor. 1 to 6 q			
		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4	
	Specification	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)
1	sl	0.56	0.56	0.76	0.76	0.90	0.72	0.76	0.74	0.84	0.80	0.70	0.64	0.75	0.71	0.74	0.72
2	sl, i	0.84	0.84	0.94	0.94	0.84	0.84	0.90	0.90	0.84	0.84	0.72	0.74	0.84	0.84	0.84	0.85
3	sl, cs	0.62	0.62	0.82	0.82	0.84	0.74	0.76	0.74	0.82	0.78	0.76	0.68	0.73	0.70	0.77	0.74
4	sl, sr	0.84	0.84	0.94	0.94	0.68	0.66	0.82	0.70	0.76	0.64	0.82	0.62	0.74	0.71	0.85	0.74
5	sl, i, cs	0.84	0.84	0.94	0.94	0.84	0.84	0.88	0.86	0.84	0.84	0.86	0.78	0.84	0.85	0.87	0.85
8	sl, $\bar{cs}$	0.74	0.74	0.86	0.86	0.82	0.78	0.86	0.84	0.76	0.86	0.74	0.68	0.76	0.78	0.82	0.79
9	sl, $\bar{sr}$	0.86	0.86	0.90	0.90	0.70	0.78	0.76	0.74	0.70	0.72	0.72	0.70	0.74	0.77	0.78	0.77
10	sl, i, $\bar{cs}$	0.84	0.84	0.96	0.96	0.88	0.88	0.94	0.86	0.84	0.80	0.86	0.86	0.86	0.86	0.93	0.90
12	sl, cs, $\bar{sr}$	0.74	0.74	0.86	0.86	0.70	0.74	0.76	0.74	0.74	0.70	0.76	0.70	0.72	0.73	0.79	0.75
13	sl, $\bar{cs}$, sr	0.82	0.82	0.92	0.92	0.70	0.70	0.84	0.72	0.72	0.62	0.84	0.66	0.75	0.71	0.84	0.76
14	sl, $\bar{cs}$, $\bar{sr}$	0.82	0.82	0.84	0.84	0.70	0.72	0.82	0.74	0.64	0.70	0.76	0.70	0.73	0.76	0.79	0.76

Table 1: United States. ProbVAR. In-sample (top), and out-of-sample (bottom).

		Forecast horizon: 1q		Forecast horizon: 4 q		Forecast horizon: 6 q		Av. over forec. hor. 1 to 6 q	
		Additional probit lags:		Additional probit lags:		Additional probit lags:		Additional probit lags:	
	Specification	0	3	0	3	0	3	0	3
1	sl	0.63	0.80	0.76	0.77	0.75	0.74	0.72	0.77
2	sl, i	0.78	0.88	0.86	0.83	0.79	0.82	0.82	0.85
3	sl, cs	0.74	0.86	0.77	0.86	0.75	0.83	0.76	0.85
4	sl, sr	0.81	0.89	0.76	0.79	0.75	0.73	0.76	0.80
5	sl, i, cs	0.79	0.88	0.86	0.90	0.79	0.88	0.81	0.88
6	sl, i, sr	0.82	0.95	0.85	0.84	0.76	0.84	0.82	0.87
7	sl, cs, sr	0.84	0.95	0.78	0.87	0.74	0.81	0.79	0.87
8	sl, $\bar{cs}$	0.75	0.84	0.82	0.83	0.75	0.79	0.78	0.82
9	sl, $\bar{sr}$	0.78	0.88	0.78	0.86	0.82	0.83	0.79	0.86
10	sl, i, $\bar{cs}$	0.80	0.91	0.85	0.86	0.78	0.82	0.83	0.87
11	sl, i, $\bar{sr}$	0.84	0.96	0.85	0.88	0.84	0.88	0.84	0.89
12	sl, cs, $\bar{sr}$	0.81	0.92	0.81	0.91	0.83	0.84	0.81	0.89
13	sl, $\bar{cs}$, sr	0.84	0.91	0.82	0.84	0.73	0.78	0.80	0.85
14	sl, $\bar{cs}$, $\bar{sr}$	0.86	0.92	0.83	0.85	0.82	0.84	0.82	0.86

		Forecast horizon: 1q		Forecast horizon: 4 q		Forecast horizon: 6 q		Av. over forec. hor. 1 to 6 q	
		Additional probit lags:		Additional probit lags:		Additional probit lags:		Additional probit lags:	
	Specification	0	3	0	3	0	3	0	3
1	sl	0.56	0.76	0.76	0.76	0.74	0.66	0.69	0.75
2	sl, i	0.84	0.94	0.90	0.88	0.90	0.82	0.88	0.86
3	sl, cs	0.62	0.82	0.72	0.80	0.70	0.74	0.69	0.77
4	sl, sr	0.84	0.94	0.74	0.70	0.66	0.64	0.74	0.76
5	sl, i, cs	0.84	0.94	0.86	0.88	0.88	0.84	0.86	0.88
8	sl, $\bar{cs}$	0.74	0.86	0.86	0.78	0.72	0.74	0.78	0.79
9	sl, $\bar{sr}$	0.86	0.90	0.74	0.78	0.76	0.72	0.77	0.79
10	sl, i, $\bar{cs}$	0.84	0.96	0.94	0.88	0.82	0.82	0.88	0.87
12	sl, cs, $\bar{sr}$	0.74	0.86	0.74	0.82	0.78	0.82	0.75	0.82
13	sl, $\bar{cs}$, sr	0.82	0.92	0.82	0.72	0.66	0.74	0.77	0.78
14	sl, $\bar{cs}$, $\bar{sr}$	0.82	0.84	0.80	0.80	0.74	0.74	0.78	0.80

Table 2: United States. Simple Probit. In-sample (top), and out-of-sample (bottom).

		Forecast horizon: 1q				Forecast horizon: 4 q				Forecast horizon: 6 q				Av. over forec. hor. 1 to 6 q			
		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4	
	Specification	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)
1	sl	0.61	0.61	0.62	0.62	0.54	0.55	0.56	0.57	0.47	0.47	0.50	0.51	0.54	0.55	0.57	0.58
2	sl, i	0.66	0.66	0.69	0.69	0.65	0.65	0.68	0.69	0.59	0.63	0.59	0.62	0.65	0.66	0.67	0.68
3	sl, cs	0.71	0.71	0.74	0.74	0.58	0.58	0.66	0.65	0.51	0.50	0.55	0.55	0.61	0.61	0.66	0.66
4	sl, sr	0.78	0.78	0.83	0.83	0.59	0.57	0.68	0.66	0.51	0.46	0.54	0.50	0.64	0.61	0.69	0.67
5	sl, i, cs	0.81	0.81	0.83	0.83	0.65	0.68	0.74	0.73	0.59	0.59	0.58	0.61	0.69	0.70	0.74	0.75
6	sl, i, sr	0.82	0.82	0.84	0.84	0.70	0.70	0.71	0.73	0.61	0.58	0.61	0.64	0.71	0.70	0.74	0.75
7	sl, cs, sr	0.78	0.78	0.83	0.83	0.57	0.59	0.66	0.66	0.49	0.48	0.55	0.50	0.63	0.62	0.69	0.68
8	sl, $\bar{cs}$	0.66	0.66	0.70	0.70	0.57	0.59	0.57	0.58	0.51	0.51	0.50	0.50	0.59	0.60	0.60	0.61
9	sl, $\bar{sr}$	0.73	0.73	0.74	0.74	0.60	0.58	0.61	0.59	0.49	0.50	0.52	0.49	0.62	0.60	0.62	0.61
10	sl, i, $\bar{cs}$	0.75	0.75	0.76	0.76	0.68	0.74	0.69	0.74	0.61	0.65	0.59	0.66	0.69	0.72	0.69	0.72
11	sl, i, $\bar{sr}$	0.79	0.79	0.82	0.82	0.66	0.73	0.70	0.73	0.59	0.59	0.59	0.69	0.70	0.72	0.71	0.75
12	sl, cs, $\bar{sr}$	0.73	0.73	0.75	0.75	0.60	0.57	0.63	0.60	0.52	0.52	0.52	0.50	0.63	0.61	0.64	0.63
13	sl, $\bar{cs}$, sr	0.77	0.77	0.82	0.82	0.61	0.59	0.68	0.66	0.50	0.46	0.55	0.50	0.63	0.62	0.69	0.68
14	sl, $\bar{cs}$, $\bar{sr}$	0.75	0.75	0.74	0.74	0.60	0.59	0.61	0.61	0.49	0.50	0.52	0.48	0.62	0.61	0.63	0.61

		Forecast horizon: 1q				Forecast horizon: 4 q				Forecast horizon: 6 q				Av. over forec. hor. 1 to 6 q			
		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4	
	Specification	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)
1	sl	0.38	0.38	0.40	0.40	0.36	0.32	0.36	0.36	0.32	0.32	0.32	0.32	0.35	0.35	0.37	0.36
2	sl, i	0.68	0.68	0.72	0.72	0.74	0.72	0.74	0.72	0.72	0.76	0.72	0.76	0.72	0.71	0.72	0.72
3	sl, cs	0.48	0.48	0.52	0.52	0.48	0.44	0.52	0.52	0.38	0.36	0.46	0.42	0.45	0.43	0.51	0.50
4	sl, sr	0.76	0.76	0.90	0.90	0.48	0.50	0.60	0.62	0.44	0.36	0.46	0.44	0.57	0.53	0.67	0.64
5	sl, i, cs	0.82	0.82	0.86	0.86	0.76	0.80	0.76	0.78	0.74	0.70	0.72	0.66	0.78	0.79	0.79	0.79
6	sl, i, sr	0.92	0.92	0.90	0.90	0.70	0.62	0.76	0.76	0.68	0.58	0.66	0.60	0.77	0.72	0.78	0.76
7	sl, cs, sr	0.72	0.72	0.80	0.80	0.52	0.54	0.68	0.64	0.46	0.46	0.48	0.46	0.57	0.57	0.68	0.65
8	sl, $\bar{cs}$	0.50	0.50	0.52	0.52	0.34	0.40	0.24	0.28	0.28	0.30	0.24	0.24	0.38	0.40	0.34	0.36
9	sl, $\bar{sr}$	0.68	0.68	0.72	0.72	0.48	0.50	0.54	0.58	0.32	0.30	0.40	0.36	0.50	0.51	0.58	0.56
10	sl, i, $\bar{cs}$	0.74	0.74	0.74	0.74	0.74	0.78	0.72	0.72	0.68	0.72	0.68	0.72	0.73	0.74	0.73	0.74
11	sl, i, $\bar{sr}$	0.88	0.88	0.92	0.92	0.74	0.68	0.72	0.66	0.72	0.66	0.72	0.70	0.77	0.73	0.79	0.77
12	sl, cs, $\bar{sr}$	0.66	0.66	0.70	0.70	0.50	0.52	0.64	0.62	0.44	0.30	0.50	0.50	0.54	0.51	0.63	0.62
13	sl, $\bar{cs}$, sr	0.74	0.74	0.86	0.86	0.46	0.48	0.56	0.54	0.30	0.32	0.38	0.34	0.54	0.51	0.62	0.59
14	sl, $\bar{cs}$, $\bar{sr}$	0.70	0.70	0.74	0.74	0.36	0.38	0.38	0.38	0.26	0.28	0.28	0.32	0.46	0.46	0.50	0.49

Table 3: Germany. ProbVAR. In-sample (top), and out-of-sample (bottom).

		Forecast horizon: 1q		Forecast horizon: 4 q		Forecast horizon: 6 q		Av. over forec. hor. 1 to 6 q	
		Additional probit lags:		Additional probit lags:		Additional probit lags:		Additional probit lags:	
	Specification	0	3	0	3	0	3	0	3
1	sl	0.61	0.62	0.56	0.59	0.49	0.51	0.56	0.60
2	sl, i	0.66	0.69	0.68	0.75	0.62	0.77	0.66	0.74
3	sl, cs	0.71	0.74	0.66	0.69	0.58	0.58	0.67	0.68
4	sl, sr	0.78	0.83	0.66	0.66	0.56	0.55	0.69	0.69
5	sl, i, cs	0.81	0.83	0.77	0.78	0.63	0.77	0.75	0.80
6	sl, i, sr	0.82	0.84	0.72	0.76	0.62	0.76	0.74	0.80
7	sl, cs, sr	0.78	0.83	0.68	0.66	0.58	0.56	0.70	0.70
8	sl, $\bar{cs}$	0.66	0.70	0.56	0.61	0.52	0.55	0.59	0.63
9	sl, $\bar{sr}$	0.73	0.74	0.62	0.63	0.52	0.50	0.64	0.64
10	sl, i, $\bar{cs}$	0.75	0.76	0.67	0.76	0.59	0.77	0.68	0.76
11	sl, i, $\bar{sr}$	0.79	0.82	0.70	0.78	0.61	0.77	0.72	0.80
12	sl, cs, $\bar{sr}$	0.73	0.75	0.67	0.66	0.58	0.57	0.67	0.68
13	sl, $\bar{cs}$, sr	0.77	0.82	0.68	0.70	0.59	0.58	0.69	0.71
14	sl, $\bar{cs}$, $\bar{sr}$	0.75	0.74	0.59	0.63	0.54	0.55	0.64	0.65

		Forecast horizon: 1q		Forecast horizon: 4 q		Forecast horizon: 6 q		Av. over forec. hor. 1 to 6 q	
		Additional probit lags:		Additional probit lags:		Additional probit lags:		Additional probit lags:	
	Specification	0	3	0	3	0	3	0	3
1	sl	0.38	0.40	0.36	0.38	0.32	0.30	0.36	0.37
2	sl, i	0.68	0.72	0.74	0.68	0.70	0.62	0.72	0.67
3	sl, cs	0.48	0.52	0.52	0.50	0.52	0.46	0.50	0.51
4	sl, sr	0.76	0.90	0.60	0.62	0.46	0.44	0.64	0.65
5	sl, i, cs	0.82	0.86	0.76	0.80	0.66	0.68	0.77	0.79
6	sl, i, sr	0.92	0.90	0.74	0.72	0.62	0.64	0.77	0.76
7	sl, cs, sr	0.72	0.80	0.66	0.64	0.52	0.44	0.66	0.66
8	sl, $\bar{cs}$	0.50	0.52	0.26	0.28	0.28	0.28	0.33	0.37
9	sl, $\bar{sr}$	0.68	0.72	0.52	0.50	0.32	0.38	0.54	0.55
10	sl, i, $\bar{cs}$	0.74	0.74	0.72	0.68	0.66	0.60	0.73	0.68
11	sl, i, $\bar{sr}$	0.88	0.92	0.72	0.60	0.72	0.68	0.78	0.76
12	sl, cs, $\bar{sr}$	0.66	0.70	0.60	0.58	0.52	0.44	0.61	0.61
13	sl, $\bar{cs}$, sr	0.74	0.86	0.58	0.52	0.28	0.24	0.57	0.59
14	sl, $\bar{cs}$, $\bar{sr}$	0.70	0.74	0.38	0.40	0.24	0.30	0.47	0.49

Table 4: Germany. Simple Probit. In-sample (top), and out-of-sample (bottom).

		Forecast horizon: 1q				Forecast horizon: 4 q				Forecast horizon: 6 q				Av. over forec. hor. 1 to 6 q			
		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4	
	Specification	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)
1	sl	0.61	0.61	0.62	0.62	0.55	0.54	0.56	0.54	0.50	0.50	0.50	0.50	0.56	0.55	0.56	0.56
2	sl, i	0.70	0.70	0.72	0.72	0.70	0.72	0.70	0.71	0.69	0.71	0.69	0.71	0.70	0.71	0.71	0.71
3	sl, cs	0.65	0.65	0.63	0.63	0.59	0.58	0.58	0.58	0.55	0.55	0.56	0.54	0.59	0.60	0.60	0.59
4	sl, sr	0.81	0.81	0.82	0.82	0.66	0.59	0.67	0.69	0.56	0.44	0.58	0.49	0.68	0.63	0.68	0.67
5	sl, i, cs	0.70	0.70	0.71	0.71	0.71	0.71	0.70	0.71	0.70	0.71	0.69	0.70	0.71	0.71	0.71	0.71
6	sl, i, sr	0.83	0.83	0.84	0.84	0.77	0.74	0.77	0.75	0.78	0.71	0.75	0.71	0.79	0.76	0.78	0.77
7	sl, cs, sr	0.81	0.81	0.84	0.84	0.65	0.62	0.66	0.65	0.57	0.52	0.59	0.55	0.68	0.66	0.69	0.67
8	sl, $\bar{cs}$	0.64	0.64	0.69	0.69	0.49	0.49	0.51	0.52	0.44	0.43	0.47	0.47	0.52	0.52	0.54	0.55
9	sl, $\bar{sr}$	0.69	0.69	0.71	0.71	0.55	0.51	0.55	0.51	0.45	0.50	0.46	0.54	0.57	0.57	0.57	0.57
10	sl, i, $\bar{cs}$	0.76	0.76	0.77	0.77	0.72	0.72	0.70	0.70	0.70	0.69	0.69	0.70	0.73	0.72	0.72	0.71
11	sl, i, $\bar{sr}$	0.76	0.76	0.75	0.75	0.74	0.74	0.73	0.72	0.73	0.70	0.73	0.70	0.75	0.73	0.74	0.72
12	sl, cs, $\bar{sr}$	0.70	0.70	0.73	0.73	0.59	0.59	0.59	0.60	0.51	0.58	0.55	0.57	0.61	0.62	0.61	0.62
13	sl, $\bar{cs}$, sr	0.78	0.78	0.78	0.78	0.61	0.58	0.62	0.63	0.47	0.47	0.52	0.49	0.64	0.62	0.66	0.65
14	sl, $\bar{cs}$, $\bar{sr}$	0.72	0.72	0.73	0.73	0.51	0.51	0.51	0.51	0.44	0.44	0.46	0.51	0.57	0.56	0.57	0.58

		Forecast horizon: 1q				Forecast horizon: 4 q				Forecast horizon: 6 q				Av. over forec. hor. 1 to 6 q			
		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4	
	Specification	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)
1	sl	0.40	0.40	0.36	0.36	0.34	0.34	0.34	0.34	0.28	0.28	0.26	0.26	0.34	0.34	0.33	0.33
2	sl, i	0.56	0.56	0.52	0.52	0.44	0.50	0.46	0.46	0.48	0.54	0.44	0.46	0.50	0.51	0.48	0.48
3	sl, cs	0.42	0.42	0.38	0.38	0.34	0.32	0.34	0.34	0.32	0.32	0.28	0.30	0.34	0.34	0.34	0.35
4	sl, sr	0.82	0.82	0.84	0.84	0.52	0.48	0.58	0.52	0.38	0.32	0.36	0.44	0.60	0.55	0.59	0.58
5	sl, i, cs	0.56	0.56	0.52	0.52	0.44	0.50	0.44	0.46	0.50	0.54	0.44	0.46	0.50	0.52	0.47	0.47
6	sl, i, sr	0.66	0.66	0.66	0.66	0.46	0.48	0.48	0.48	0.46	0.50	0.46	0.48	0.53	0.53	0.53	0.53
7	sl, cs, sr	0.84	0.84	0.86	0.86	0.50	0.52	0.48	0.48	0.40	0.28	0.42	0.36	0.58	0.58	0.59	0.58
8	sl, $\bar{cs}$	0.54	0.54	0.58	0.58	0.36	0.40	0.24	0.24	0.26	0.28	0.30	0.30	0.39	0.41	0.38	0.36
9	sl, $\bar{sr}$	0.66	0.66	0.60	0.60	0.46	0.42	0.36	0.36	0.38	0.36	0.32	0.32	0.50	0.47	0.44	0.44
10	sl, i, $\bar{cs}$	0.52	0.52	0.48	0.48	0.48	0.46	0.52	0.46	0.44	0.42	0.52	0.48	0.48	0.47	0.50	0.47
11	sl, i, $\bar{sr}$	0.54	0.54	0.54	0.54	0.50	0.48	0.46	0.46	0.52	0.50	0.50	0.48	0.53	0.51	0.49	0.49
12	sl, cs, $\bar{sr}$	0.60	0.60	0.58	0.58	0.46	0.42	0.40	0.30	0.46	0.38	0.38	0.30	0.48	0.46	0.43	0.38
13	sl, $\bar{cs}$, sr	0.80	0.80	0.80	0.80	0.58	0.52	0.54	0.46	0.36	0.34	0.42	0.40	0.60	0.55	0.61	0.57
14	sl, $\bar{cs}$, $\bar{sr}$	0.66	0.66	0.64	0.64	0.46	0.36	0.32	0.32	0.40	0.30	0.30	0.38	0.51	0.45	0.42	0.43

Table 5: Japan. ProbVAR. In-sample (top), and out-of-sample (bottom).

		Forecast horizon: 1q		Forecast horizon: 4 q		Forecast horizon: 6 q		Av. over forec. hor. 1 to 6 q	
		Additional probit lags:		Additional probit lags:		Additional probit lags:		Additional probit lags:	
	Specification	0	3	0	3	0	3	0	3
1	sl	0.61	0.62	0.54	0.55	0.50	0.50	0.56	0.55
2	sl, i	0.70	0.72	0.71	0.72	0.70	0.72	0.70	0.72
3	sl, cs	0.65	0.63	0.58	0.61	0.58	0.62	0.59	0.62
4	sl, sr	0.81	0.82	0.64	0.66	0.59	0.53	0.69	0.68
5	sl, i, cs	0.70	0.71	0.71	0.70	0.71	0.72	0.70	0.71
6	sl, i, sr	0.83	0.84	0.78	0.76	0.74	0.71	0.78	0.76
7	sl, cs, sr	0.81	0.84	0.68	0.68	0.62	0.63	0.70	0.71
8	sl, $\bar{cs}$	0.64	0.69	0.55	0.57	0.54	0.62	0.56	0.61
9	sl, $\bar{sr}$	0.69	0.71	0.55	0.56	0.50	0.51	0.58	0.58
10	sl, i, $\bar{cs}$	0.76	0.77	0.71	0.76	0.70	0.72	0.72	0.75
11	sl, i, $\bar{sr}$	0.76	0.75	0.72	0.73	0.70	0.75	0.73	0.74
12	sl, cs, $\bar{sr}$	0.70	0.73	0.59	0.60	0.59	0.63	0.62	0.65
13	sl, $\bar{cs}$, sr	0.78	0.78	0.69	0.69	0.63	0.63	0.71	0.71
14	sl, $\bar{cs}$, $\bar{sr}$	0.72	0.73	0.55	0.61	0.56	0.62	0.59	0.64

		Forecast horizon: 1q		Forecast horizon: 4 q		Forecast horizon: 6 q		Av. over forec. hor. 1 to 6 q	
		Additional probit lags:		Additional probit lags:		Additional probit lags:		Additional probit lags:	
	Specification	0	3	0	3	0	3	0	3
1	sl	0.40	0.36	0.34	0.30	0.26	0.28	0.34	0.33
2	sl, i	0.56	0.52	0.46	0.46	0.42	0.40	0.49	0.47
3	sl, cs	0.42	0.38	0.34	0.32	0.34	0.28	0.36	0.33
4	sl, sr	0.82	0.84	0.52	0.56	0.40	0.34	0.58	0.58
5	sl, i, cs	0.56	0.52	0.46	0.44	0.44	0.42	0.49	0.46
6	sl, i, sr	0.66	0.66	0.44	0.46	0.42	0.38	0.51	0.49
7	sl, cs, sr	0.84	0.86	0.50	0.46	0.42	0.30	0.58	0.54
8	sl, $\bar{cs}$	0.54	0.58	0.24	0.40	0.40	0.42	0.36	0.45
9	sl, $\bar{sr}$	0.66	0.60	0.26	0.30	0.32	0.32	0.40	0.41
10	sl, i, $\bar{cs}$	0.52	0.48	0.46	0.46	0.40	0.34	0.47	0.43
11	sl, i, $\bar{sr}$	0.54	0.54	0.44	0.40	0.42	0.40	0.47	0.45
12	sl, cs, $\bar{sr}$	0.60	0.58	0.26	0.30	0.30	0.26	0.37	0.37
13	sl, $\bar{cs}$, sr	0.80	0.80	0.44	0.50	0.52	0.36	0.57	0.57
14	sl, $\bar{cs}$, $\bar{sr}$	0.66	0.64	0.24	0.36	0.38	0.36	0.41	0.45

Table 6: Japan. Simple Probit. In-sample (top), and out-of-sample (bottom).

B Figures

B.1 In- and out-of-sample fit

B.1.1 United States

For ProbVAR results: Specification 1 is always a ProbVAR(1,1,3) with the slope of the yield curve. Specification 2 is a ProbVAR(1,1,3) with slope, short-term interest rate and average corporate bond spread. Specification 3 is a ProbVAR(1,1,3) with slope and average corporate bond spread.

For the simple probit results, the regressors of the three specifications are the same as for the ProbVAR models. All simple probit models contain 1 to 4 lags of the regressors.

Note that by construction, one-quarter-ahead forecasts are identical for ProbVar and Simple Probit models with the same regressor specification. Hence, the left panels of figures 1 and 2 are the same, and the left panels of figures 3 and 4 are the same.

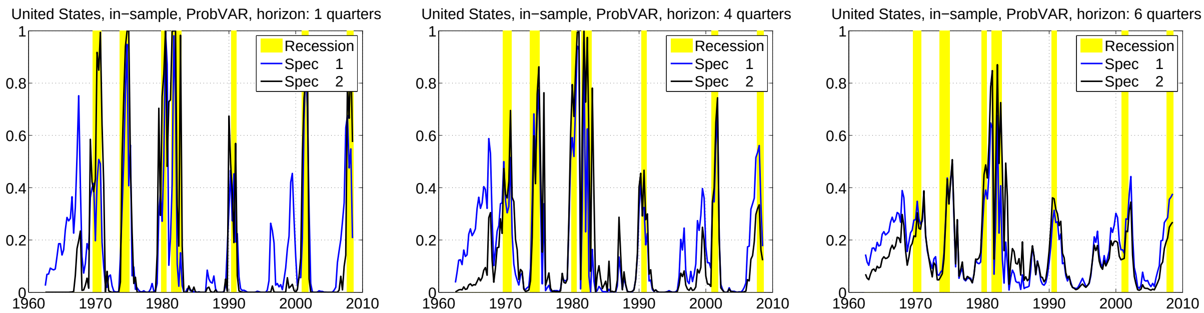

Figure 1: United States. ProbVAR. Recessions (yellow bars) and model-implied recession probabilities. In-sample fit. Probabilities for prediction horizons of 1, 4 and 6 quarters.

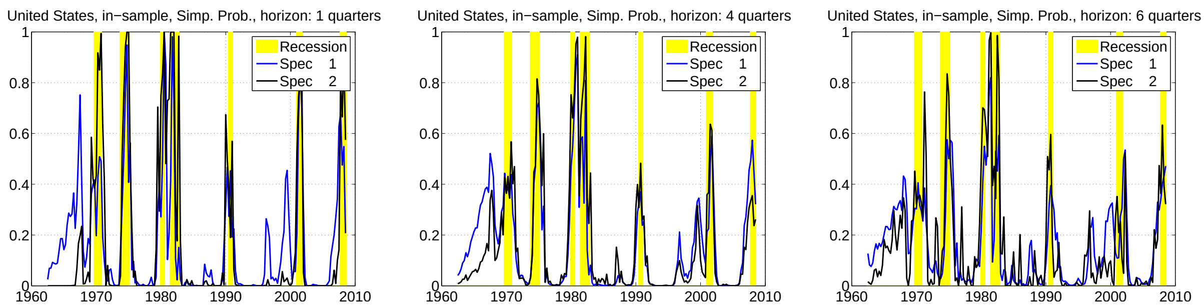

Figure 2: United States. Simple Probit. Recessions (yellow bars) and model-implied recession probabilities. In-sample fit. Probabilities for prediction horizons of 1, 4 and 6 quarters.

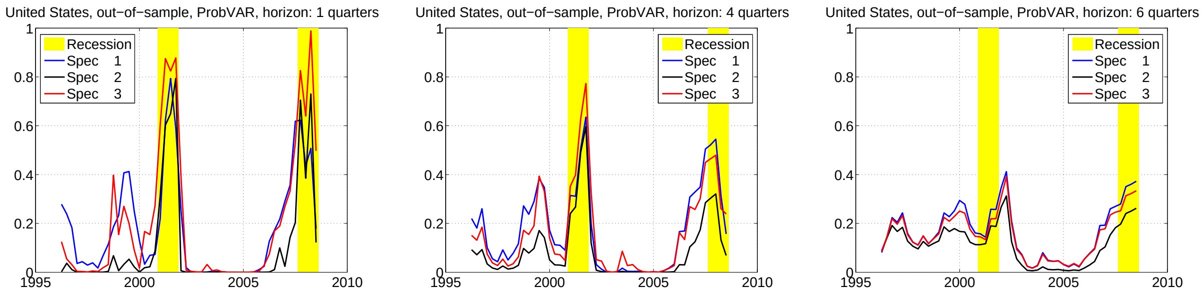

Figure 3: United States. ProbVAR. Recessions (yellow bars) and model-implied recession probabilities. Out-of-sample fit. Probabilities for prediction horizons of 1, 4 and 6 quarters.

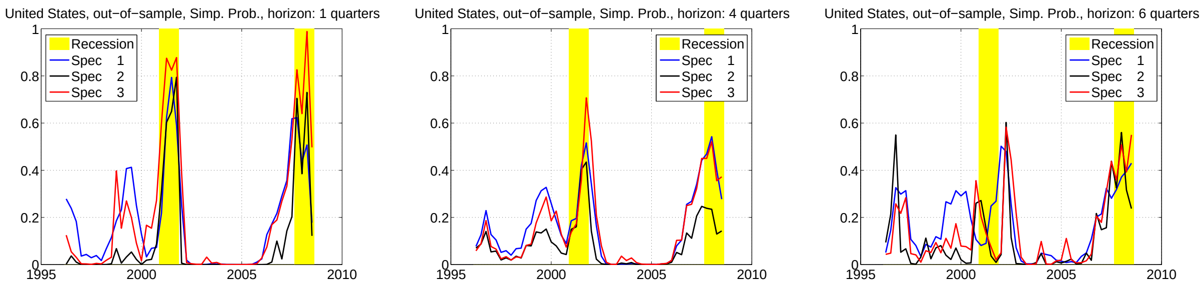

Figure 4: United States. Simple Probit. Recessions (yellow bars) and model-implied recession probabilities. Out-of-sample fit. Probabilities for prediction horizons of 1, 4 and 6 quarters.

B.1.2 Germany

For ProbVAR results: Specification 1 is always a ProbVAR(1,1,3) with the slope of the yield curve. Specification 2 is a ProbVAR(1,1,3) with slope, short-term interest rate and average stock return. Specification 3 is a ProbVAR(1,1,3) with slope, corporate bond spread and stock return.

For the simple probit results, the regressors of the three specifications are the same as for the ProbVAR models. All simple probit models contain 1 to 4 lags of the regressors.

Note that by construction, one-quarter-ahead forecasts are identical for ProbVar and Simple Probit models with the same regressor specification. Hence, the left panels of figures 5 and 6 are the same, and the left panels of figures 7 and 8 are the same.

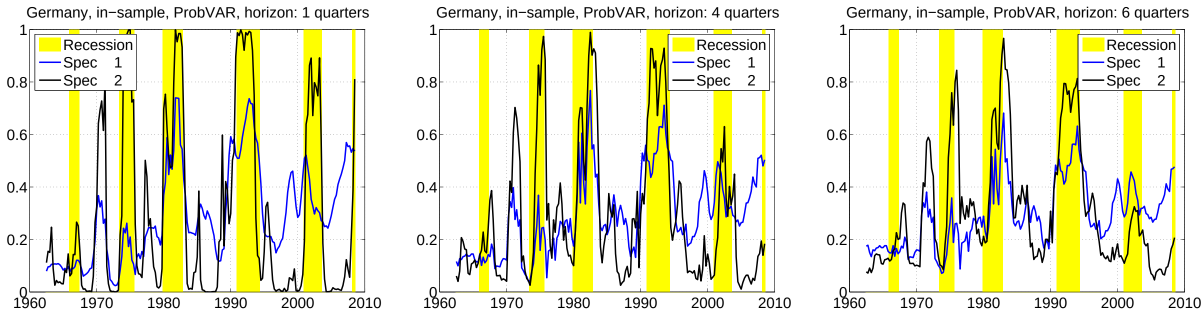

Figure 5: Germany. ProbVAR. Recessions (yellow bars) and model-implied recession probabilities. In-sample fit. Probabilities for prediction horizons of 1, 4 and 6 quarters.

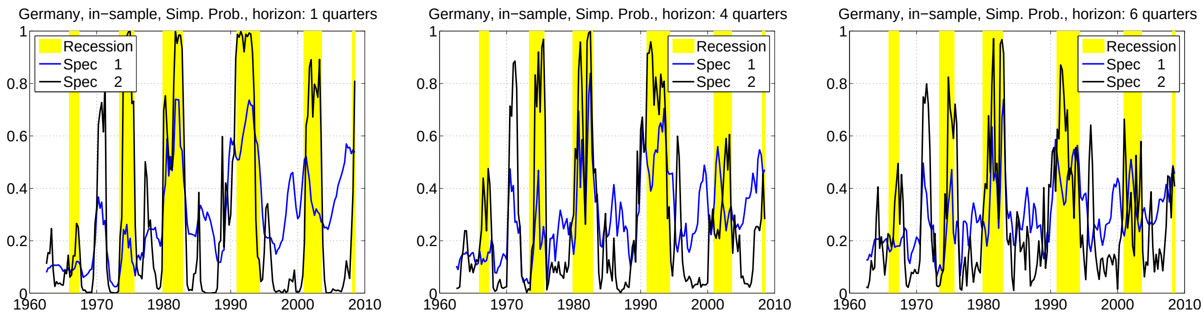

Figure 6: Germany. Simple Probit. Recessions (yellow bars) and model-implied recession probabilities. In-sample fit. Probabilities for prediction horizons of 1, 4 and 6 quarters.

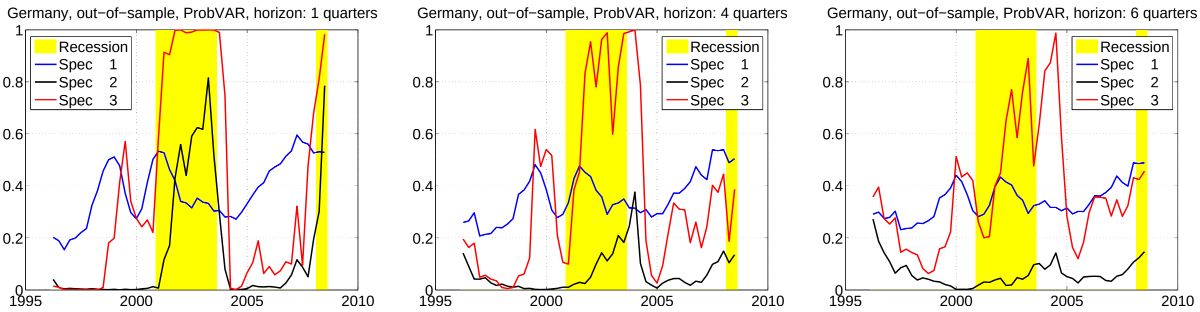

Figure 7: Germany. ProbVAR. Recessions (yellow bars) and model-implied recession probabilities. Out-of-sample fit. Probabilities for prediction horizons of 1, 4 and 6 quarters.

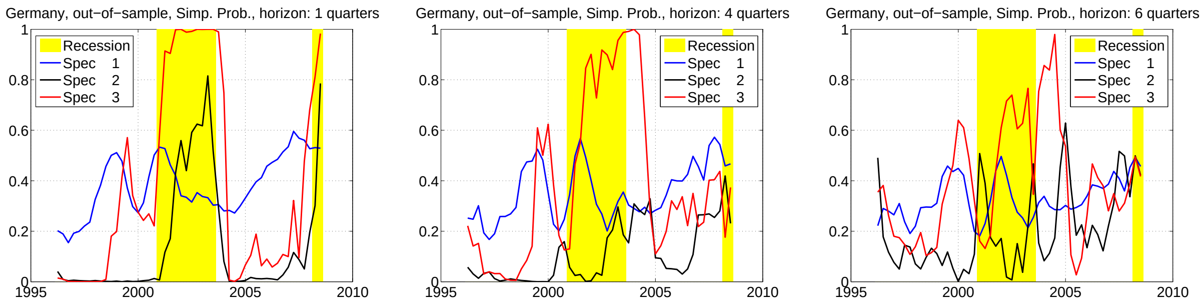

Figure 8: Germany. Simple Probit. Recessions (yellow bars) and model-implied recession probabilities. Out-of-sample fit. Probabilities for prediction horizons of 1, 4 and 6 quarters.

B.1.3 Japan

For ProbVAR results: Specification 1 is always a ProbVAR(1,1,3) with the slope of the yield curve. Specification 2 is a ProbVAR(1,1,3) with slope, average corporate bond spread and stock return.

For the simple probit results, the regressors of the three specifications are the same as for the ProbVAR models. All simple probit models contain 1 to 4 lags of the regressors.

Note that by construction, one-quarter-ahead forecasts are identical for ProbVar and Simple Probit models with the same regressor specification. Hence, the left panels of figures 9 and 10 are the same, and the left panels of figures 11 and 12 are the same.

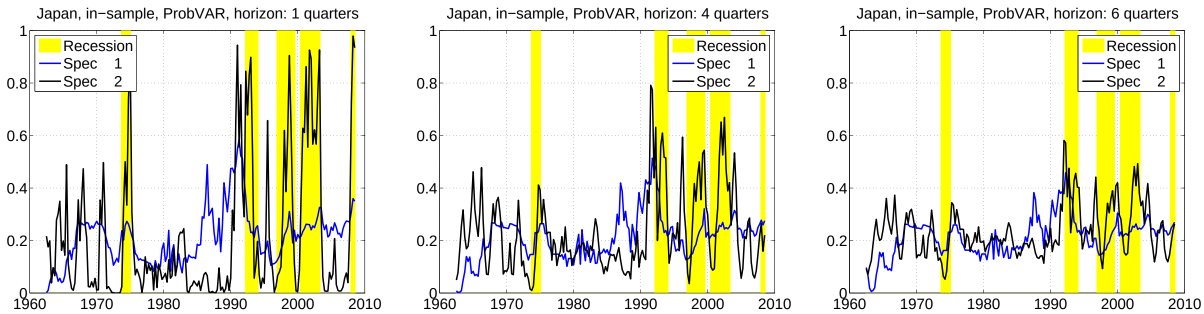

Figure 9: Japan. ProbVAR. Recessions (yellow bars) and model-implied recession probabilities. In-sample fit. Probabilities for prediction horizons of 1, 4 and 6 quarters.

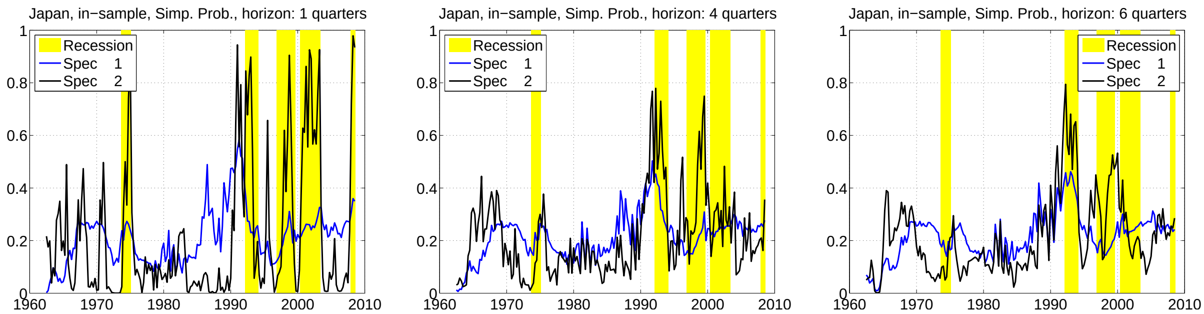

Figure 10: Japan. Simple Probit. Recessions (yellow bars) and model-implied recession probabilities. In-sample fit. Probabilities for prediction horizons of 1, 4 and 6 quarters.

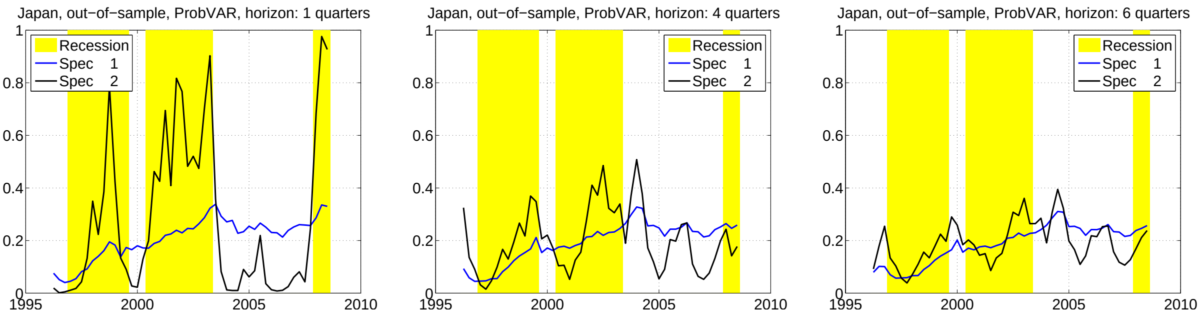

Figure 11: Japan. ProbVAR. Recessions (yellow bars) and model-implied recession probabilities. Out-of-sample fit. Probabilities for prediction horizons of 1, 4 and 6 quarters.

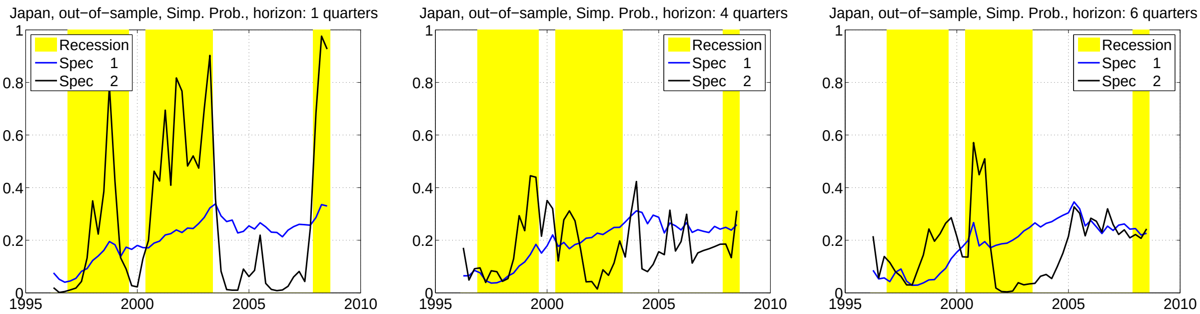

Figure 12: Japan. Simple Probit. Recessions (yellow bars) and model-implied recession probabilities. Out-of-sample fit. Probabilities for prediction horizons of 1, 4 and 6 quarters.

B.2 Zooming in on the last two recession periods

B.2.1 United States

For ProbVAR results: Specification 1 is always a ProbVAR(1,1,3) with the slope of the yield curve. Specification 2 is a ProbVAR(1,1,3) with slope, short-term interest rate and average corporate bond spread. Specification 3 is a ProbVAR(1,1,3) with slope and average corporate bond spread.

For the simple probit results, the regressors of the three specifications are the same as for the ProbVAR models. All simple probit models contain 1 to 4 lags of the regressors.

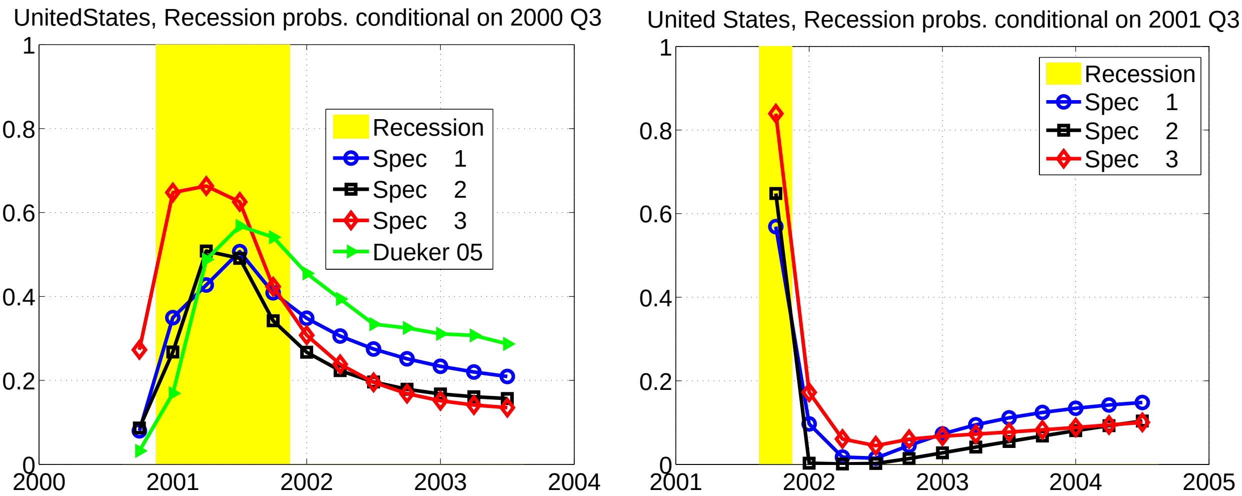

Figure 13: United States. ProbVAR. Model-implied recession probabilities for 1, 2, ..., 12 quarters ahead. Parameters estimated based on sample ending in 2000Q3. Prediction based on regressors observed in 2000Q3 (left) and 2001Q3 (right). Recessions are marked as yellow bars. The green line with triangles are the out-of-sample recession probabilities obtained by Dueker (2005), table 2, also based on 2000Q3 information.

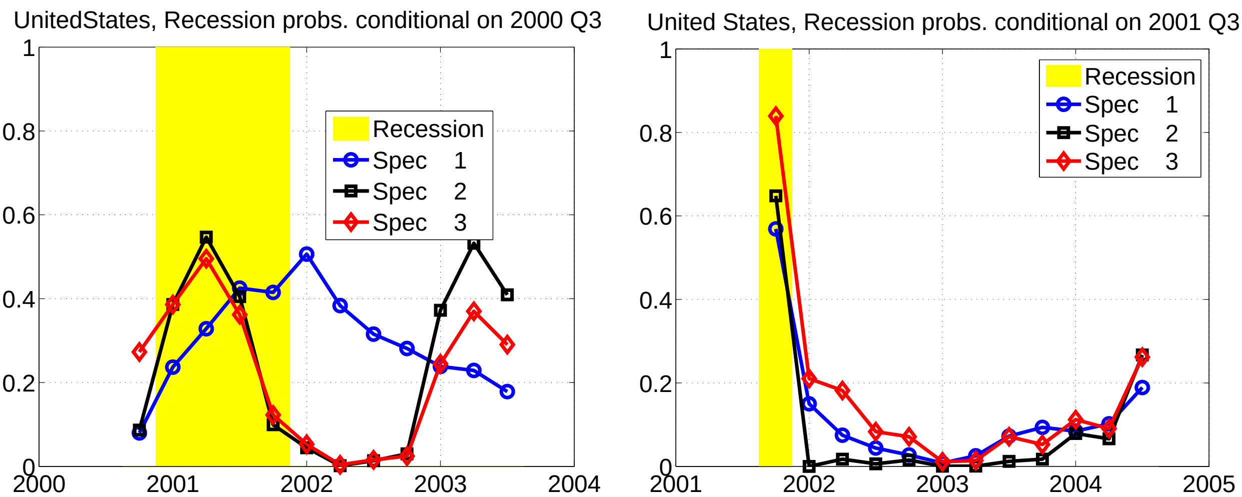

Figure 14: United States. Simple Probit. Model-implied recession probabilities for 1, 2, ..., 12 quarters ahead. Parameters estimated based on sample ending in 2000Q34. Prediction based on regressors observed in 2000Q3 (left) and 2001Q3 (right). Recessions marked as yellow bars.

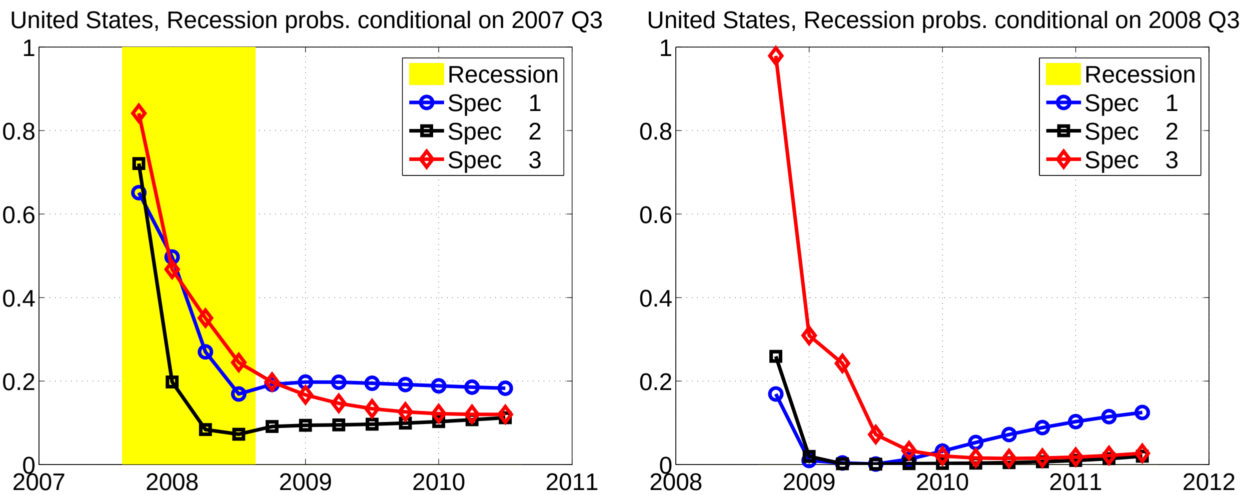

Figure 15: United States. ProbVAR. Model-implied recession probabilities for 1, 2, ..., 12 quarters ahead. Parameters estimated based on sample ending in 2006Q4. Prediction based on regressors observed in 2007Q3 (left) and 2008Q3 (right). Recessions marked as yellow bars.

B.2.2 Germany

For ProbVAR results: Specification 1 is always a ProbVAR(1,1,3) with the slope of the yield curve. Specification 2 is a ProbVAR(1,1,3) with slope, short-term interest rate and average stock return. Specification 3 is a ProbVAR(1,1,3) with slope, corporate bond spread and stock return.

For the simple probit results, the regressors of the three specifications are the same as for the ProbVAR models. All simple probit models contain 1 to 4 lags of the regressors.

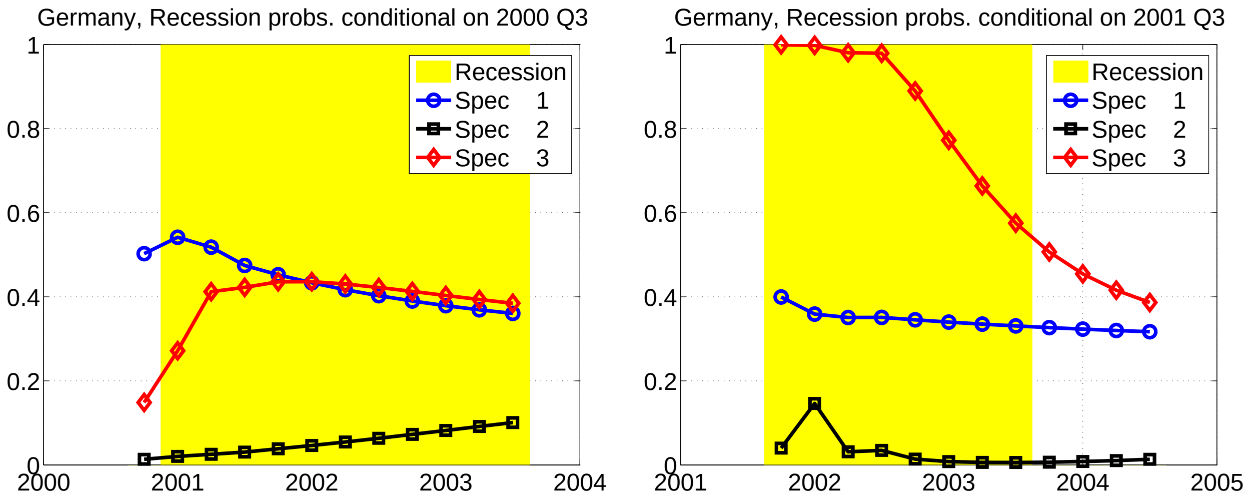

Figure 16: Germany. ProbVAR. Model-implied recession probabilities for 1, 2, ..., 12 quarters ahead. Parameters estimated based on sample ending in 2000Q3. Prediction based on regressors observed in 2000Q3 (left) and 2001Q3 (right). Recessions marked as yellow bars.

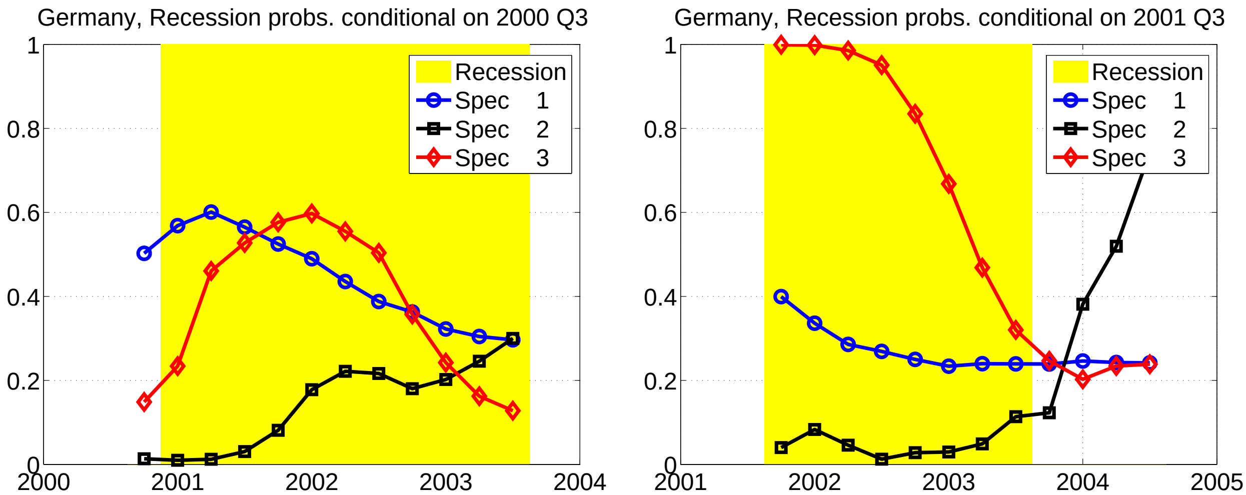

Figure 17: Germany. Simple Probit. Model-implied recession probabilities for 1, 2, ..., 12 quarters ahead. Parameters estimated based on sample ending in 2000Q3. Prediction based on regressors observed in 2000Q3 (left) and 2001Q3 (right). Recessions marked as yellow bars.

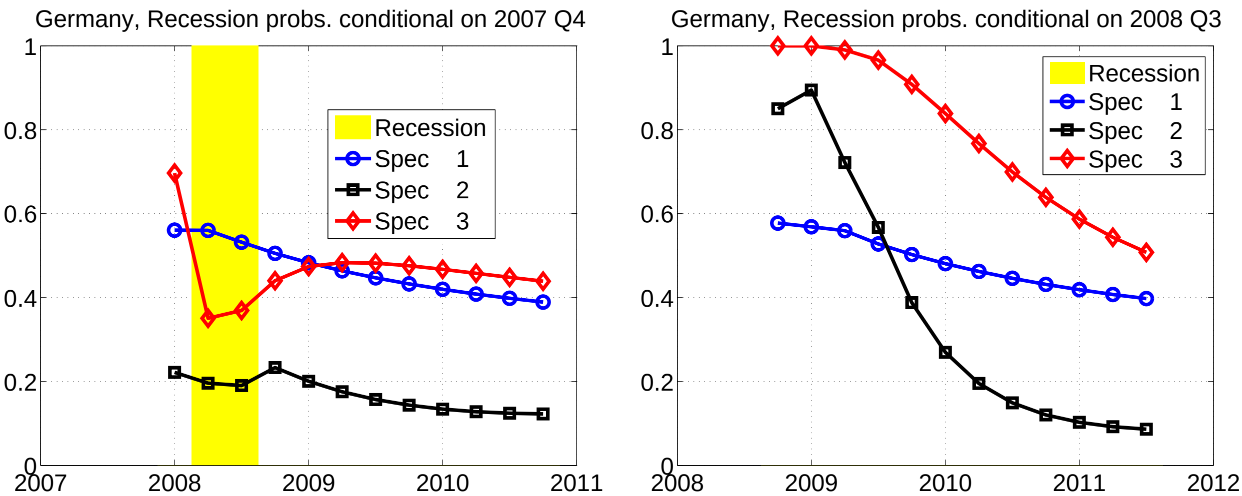

Figure 18: Germany. ProbVAR. Model-implied recession probabilities for 1, 2, ..., 12 quarters ahead. Parameters estimated based on sample ending in 2006Q4. Prediction based on regressors observed in 2007Q4 (left) and 2008Q3 (right). Recessions marked as yellow bars.

B.2.3 Japan

For ProbVAR results: Specification 1 is always a ProbVAR(1,1,3) with the slope of the yield curve. Specification 2 is a ProbVAR(1,1,3) with slope, average corporate bond spread and stock return.

For the simple probit results, the regressors of the three specifications are the same as for the ProbVAR models. All simple probit models contain 1 to 4 lags of the regressors.

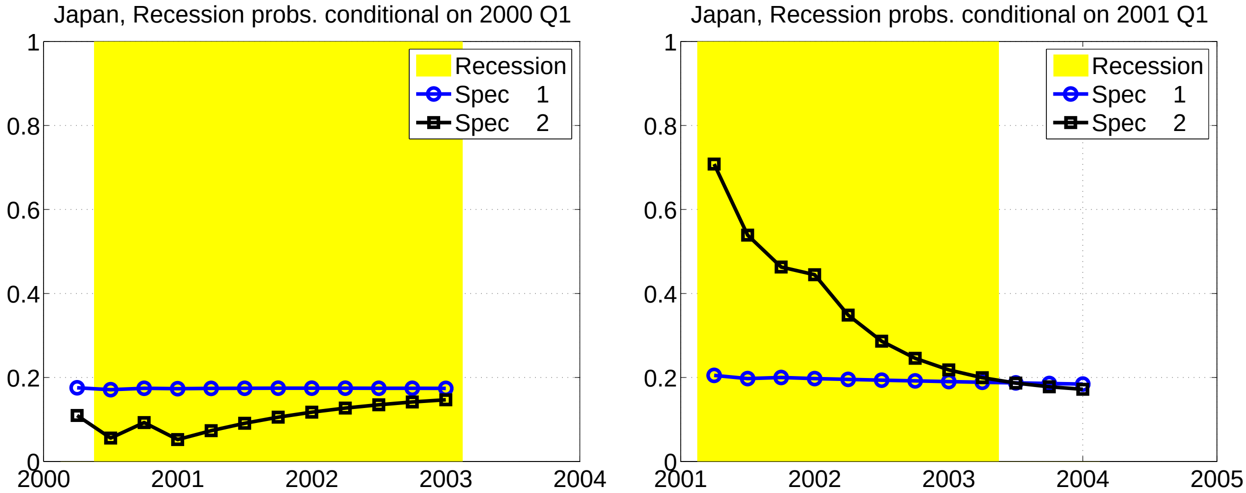

Figure 19: Japan. ProbVAR. Model-implied recession probabilities for 1, 2, ..., 12 quarters ahead. Parameters estimated based on sample ending in 2000Q1. Prediction based on regressors observed in 2000Q1 (left) and 2001Q1 (right). Recessions marked as yellow bars.

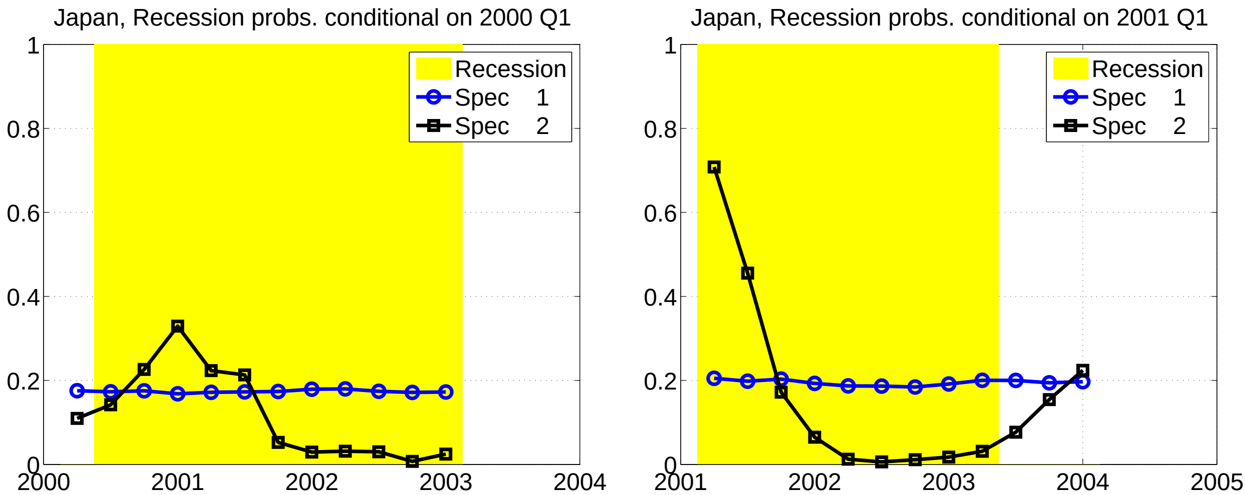

Figure 20: Japan. Simple Probit. Model-implied recession probabilities for 1, 2, ..., 12 quarters ahead. Parameters estimated based on sample ending in 2000Q1. Prediction based on regressors observed in 2000Q1 (left) and 2001Q1 (right). Recessions marked as yellow bars.

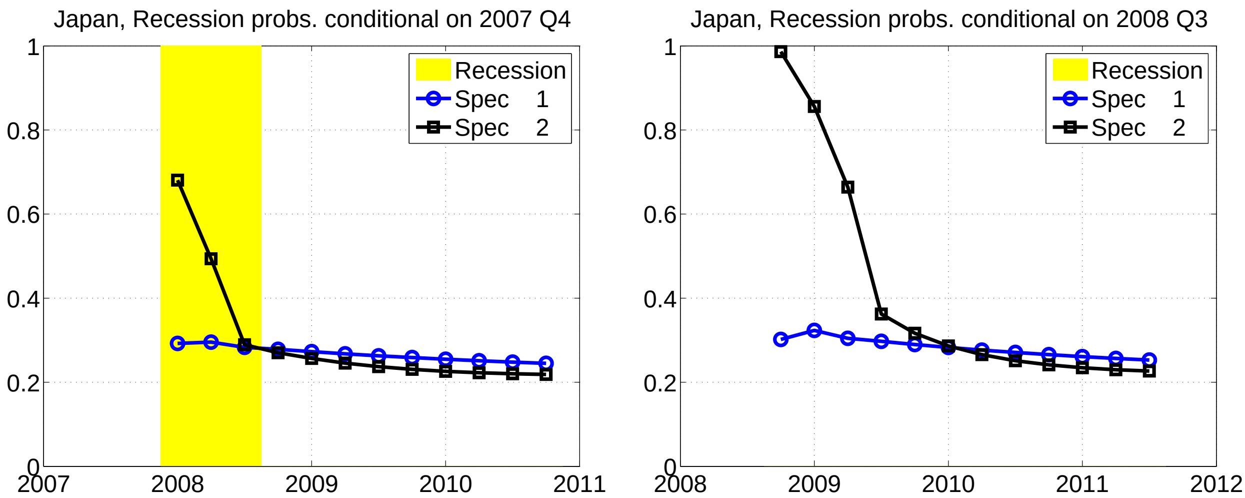

Figure 21: Japan. ProbVAR. Model-implied recession probabilities for 1, 2, ..., 12 quarters ahead. Parameters estimated based on sample ending in 2006Q4. Prediction based on regressors observed in 2007Q4 (left) and 2008Q3 (right). Recessions marked as yellow bars.

B.3 Impulse responses of recession probabilities to a slope shock

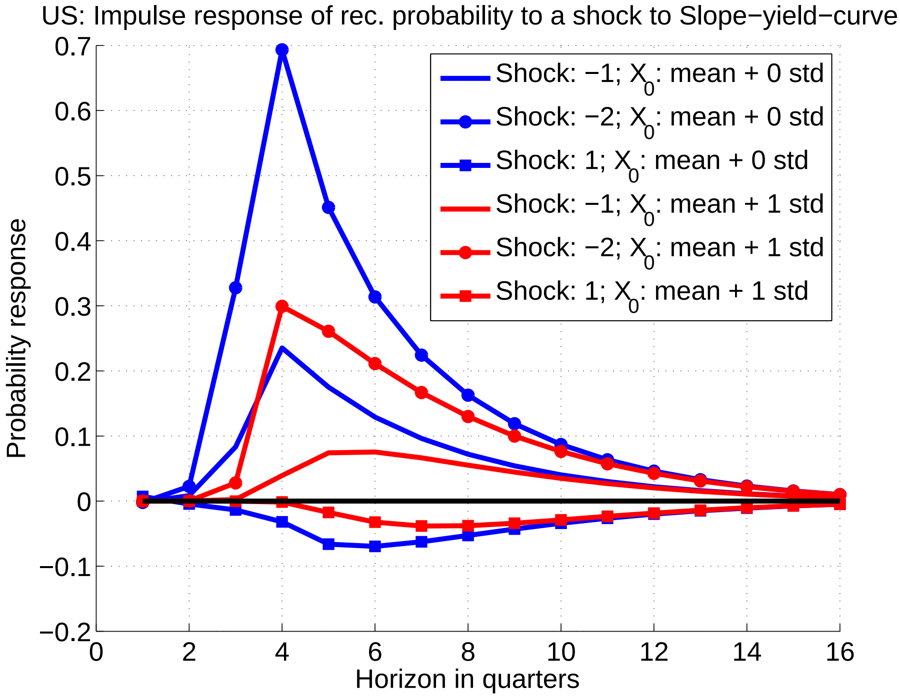

Figure 22: Impulse response of recession probabilities to a shock to the slope of the yield curve. – United States, ProbVAR(1,1,3) with slope, short-term interest rate and average corporate bond spread. Estimated using data until 2006Q4. Blue lines represent responses to a shock to the slope of the yield curve of -1 pp (plain line), -2 pp (line with circles), and +1 pp (line with squares). The initial regressors are at their sample averages. Red lines: the same, but for another initial condition of regressors: all variables and their lags are at their sample averages, but the slope equals the sample average plus one sample standard deviation.

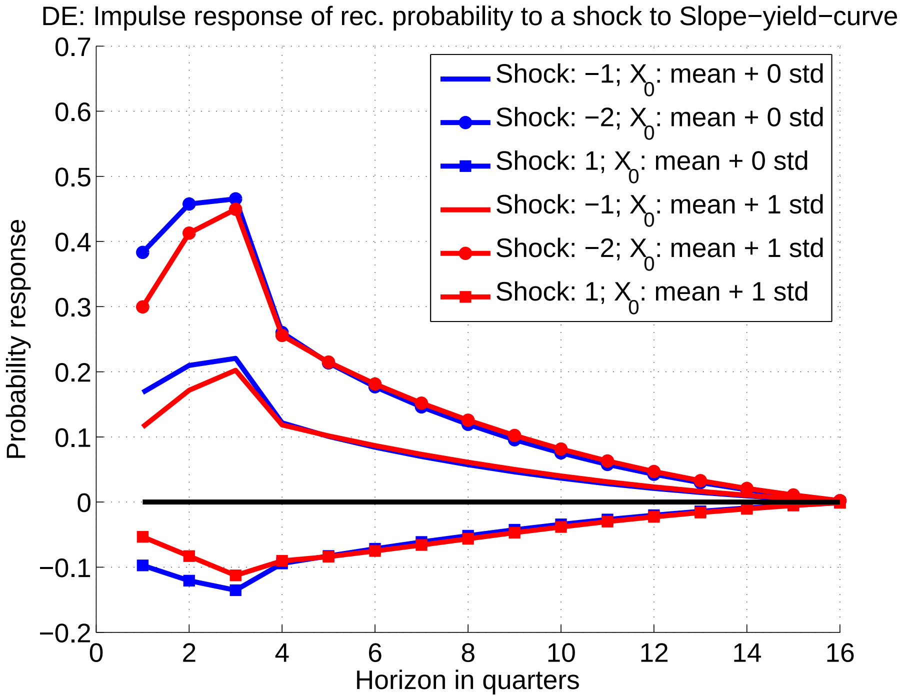

Figure 23: Impulse response of recession probabilities to a shock to the slope of the yield curve. – Germany, ProbVAR(1,1,3) with slope, short-term interest rate and average stock return. See figure .

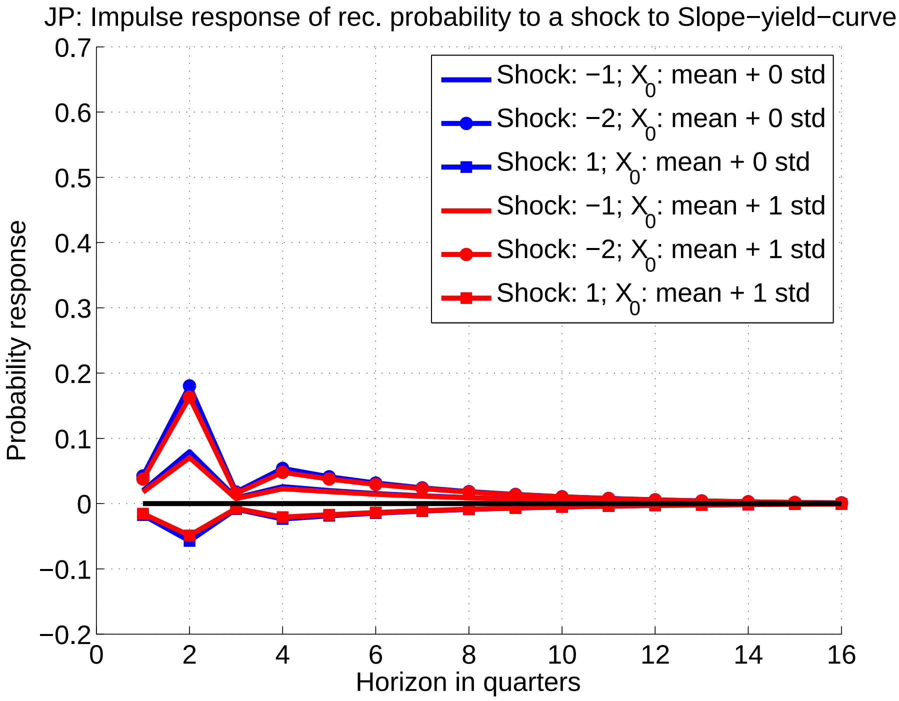

Figure 24: Impulse response of recession probabilities to a shock to the slope of the yield curve. Japan, ProbVAR(1,1,3) with slope, stock return and average corporate bond spread. See figure .