

**VALUATION OF RISKY DEBT:
A MULTI-PERIOD BAYESIAN MODEL.**

Leonid V. Philosophov.

First draft: January 20, 2006

This version: July 06, 2009

Corresponding author

Leonid V. Philosophov, D. Sc., Professor, Independent,
33 – 1 – 147 Vilis Latsis Street, 125480, Moscow, Russia.

E-mail: lvphilosophov@gmail.com

VALUATION OF RISKY DEBT: A MULTI-PERIOD BAYESIAN MODEL.

Abstract.

The paper describes model of a new type for valuation of risky bonds and loans that we call Bayesian Multi-Period (BMP) model. BMP is neither structural model nor reduced form and not a Merton-type model at all.

BMP proceeds from concept of a risky bond (loan) value as Net Present Value (NPV) of a cash flow, generated by a bond. For a defaultable bond NPV is random value, and BMP identifies “fair” price of a risky bond as its mean NPV. Statistical properties of a (random) difference between NPV of a risky bond and NPV of risk-free bond with the same terms of issuance characterize riskness of a bond.

BMP supposes that a borrower (e.g. a firm) generally has several debt issues (bonds, loans) simultaneously - with different terms of issuance (interest rates, maturity horizons, payment schedules etc.) and calculates risk characteristics for each debt issue separately.

It considers exact contractual cash flow schedule of each specific debt issue and combines it with probabilities of a borrower’s default at all stages of cash flow process. Default prognosis in turn accounts for joint influence of all outstanding debt of a firm.

BMP uses multi-period default prognosis of Bayesian type based on indices of borrower’s current financial position with accounting for predictive abilities of repayment schedule of a firm’s long-term debt. This type prognosis can additionally incorporate other predictive variables like familiar market factor - “distance to default”.

BMP calculates “fair” interest rates for newly issued risky corporate bonds, “fair” prices and “fair” yield to maturity for risky bonds at intermediate moments of bond’s life. We compare them with observed market prices, rates and spreads.

The model explains on average about 70% of observed interest rates, credit spreads and market prices of a bond. That is much more, than usually explain Merton-type models

The paper discusses relation between multi-period default probabilities and credit ratings.

Keywords: Credit risk, Risky bonds, Default prediction, Bond valuation, Bayesian model, Credit rating.

GEL codes: C11, C13, C51, E58, G21, G33.

1. Introduction

The problem of valuation of a risky debt and assessing credit risk is one of the central problems of modern finance. Its correct and constructive decision is important for banks, lending money, firms issuing bonds, borrowers, investors, regulators that control banking activities. The problem is interesting and important for numerous scientists working in the area of finance.

In recent times interest to the problem became greatly stimulated by developing of the New Basel Capital Accord (Basel II) that establishes the new legal standards for measurement methodology and value of adequate bank capital with strong emphasis on improvement of bank capabilities to assess and manage credit risks.

World financial crisis of 2008-2009 is to some degree product of undervaluation of riskiness of mortgage credits in USA.

Modern financial theory of credit risk is dominated by structural approach that goes back to Black, Scholes (1973) model of pricing options and its extension on pricing of risky bonds proposed by Merton (1974). In this model default (bankruptcy) of a firm on its obligations (driving factor of credit risk) occurs if market value of assets in its random motion fall behind a book value of a firm's debt, consisting of a single type zero coupon bonds. The approach was very fruitful in the past times and created ground for many past and present financial studies. Among these studies one can find a number of extensions of Black-Scholes-Merton model (BSM) aimed on reduction of serious restrictive assumptions used in original BSM.

Among practical applications of structural approach most familiar is Moody's KMV EDF (Expected Default Frequency) model for calculation of probability (frequency) of firms' default. To do this EDF calculates the only predictive variable - distance to default (DD) at a horizon of prediction of default probability and then links DD with EDF using empirical database (see Crosbie, Bohn, 2003).

Basel II credit risk model can be viewed as a derivative of structural model. Firm's assets are considered at initial time moment and one year later. Normalized return on assets is viewed as a risk factor (with standard normal distribution). Default threshold for the factor is mapped to a borrower's one-year default probability. All other default-relevant characteristics of borrowers are omitted.

Other credit risk models for practical applications, known from literature, are developed mainly by major investment banks and rating agencies. Such models of J.P. Morgan (1997), Credit Suisse First Boston (1997) etc. are based on various formal indexes of a client's risk.

Notwithstanding many achievements in developing structural models they failed to overcome serious drawbacks.

Employed in the approach economic condition of default (market value assets fall behind book value of a borrower's debt) rarely holds in practice. Our empirical data shows that this condition is neither necessary nor sufficient. We observed a number of going concerns that have negative net worth (i.e. with debt exceeding total assets) for several years while other filed for bankruptcy with positive net worth.

Corporate debt is usually diversified and consists of many bonds (and loans) with different maturities and terms of issuance. This increases a firm's ability to maneuver and thus reduce its debt burden.

Structural approach has restricted abilities of mapping credit risk model to each specific firm, its financial position, characteristics of its bonds and loans. In particular the approach acknowledges the only prognostic factor – distance to default (DD) – and claims its exclusive and exhaustive character. At the same time many studies (Hillegeist, Cram, Keating, Lundstedt (2002), Bharath, Shumway (2004), Duffie, Saita, Wang (2005)) report finding other predictive variables that provide incremental information to DD. Structural credit risk model can hardly incorporate those additional predictive variables.

Structural models are mainly one-period while credit process in its nature is multi-period – includes many cash flows spread over the entire credit period. Credit loss depends on the stage at which default occurs. To cope with such credit processes one needs simultaneously and coherently assess a set of probabilities of borrower's default at all stages of cash flow process. Within structural models (that consider diffusion-type asset value processes) this can be done in simplest cases only with no hope on accounting for an individual propensity to default of each specific borrower.

As a result many authors (Jones, Mason, Rosenfeld (1984); Kim, Ramaswamy, Sundaresan (1993); Ogden (1987); Lyden, Saraniti (2000); Leland (2002); Eom, Helwege and Huang (2004)) report that structural models are imprecise and greatly undervalue credit risk.

According to Bohn (2000), "The conventional wisdom, while praising the theoretical insights gained from structural models, dismisses them as impractical for actual bond valuation".

To overcome above described drawbacks we propose an alternative - Bayesian Multi-period (BMP) model that *provides absolutely exact decision* of the problem of valuation of risky debt and has much more clear perspectives of practical applications.

BMP uses concept of bond value as Net Present Value (NPV) of future cash flows, generated by a bond. If a bond is risky, its NPV is random and BMP specifies bond "fair" price as mean NPV. If default risk reduces to zero, BMP continuously transfers into usual discount model of bond valuation.

In contrast, Structural approach infers value of a risky bond from option pricing theory, developed by Black and Scholes basing on considerations of market equilibrium. They consider value of option as a nonrandom function of price of an asset and time. Merton (1974), using option properties of a firm's equity and assets, obtains formula of the same type for valuation risky bond. Merton's approach (as mentioned above) employs rather specific model of a firm's default.

Both approaches – NPV and BSM - are widely acknowledged and very popular though their assessments of risky bond values and credit spreads are absolutely different.

Practice seemingly resolves this conflict in favor of NPV approach. BMP model explains on the average about 70% of observed credit spreads (as is shown below), while Merton-type models only 5 through 22 percents (according to Delianedis and Geske (2001)).

Principal features and distinctions of BMP model are as follows:

- BMP is discrete time model that considers credit process on discrete time intervals matched with actual cash flow schedule of a bond (or loan);
- BMP assesses credit loss for each bond or loan of a firm separately, considering at the same time total debt of a firm while assessing its propensity to default (bankruptcy);
- BMP is multi-period model. It determines in coherent and non-contradictory way probabilities of a firm's default at all future time intervals within active credit period and after its end;
- in contrast to BSM model BMP considers value of a risky bond to be random (function of random event and time of default). This makes applicable such probabilistic concepts as mean value, value variance, marginal value etc;
- BMP does not predict future values of a firm's assets, equity or predictive variables. It uses current (observed) values of predictive variables to calculate probabilities of default at various time intervals in future. After all only current values of predictive variables are observable and their predicted values can be useful if uncertainty of prediction is small. Being calculated within insufficiently substantiated assumptions, predicted values of prognostic variables can bring additional errors.
- The model can use all known default-informative characteristics of a borrower (including distance-to-default) and employs enhanced Bayesian methodology to account for their non-normality and mutual dependence. It provides detailed mapping of model's characteristics to each specific borrower and bond.
- BMP does not make maximum likelihood assessments of conditional hazard rates (ex-post default probabilities) with various assumptions about their dynamics and dependence on predictive variables. It calculates those probabilities using exact

probabilistic formulae within enhanced Bayesian methodology. Empirical data is used at intermediate stages to assess conditional distributions of predictive variables by means of widely known and well-established kernel and histogram estimators. This principal distinction of BMP model motivates us to call it “Bayesian”.

The BMP approach is based on studies Philosophov, Philosophov (2002) and Philosophov, Batten, Philosophov (2003), which provide multi-period bankruptcy prediction basing on borrower’s (a firm’s) current financial indices and schedule of repaying of long-term debt. To ensure possibility of independent reading of the current paper we shortly describe below main issues of those studies.

The rest of the paper is organized as follows:

Section 2 represents mathematical inference of BMP model for valuation of risky debt. The model combines a true cash flow schedule of a risky debt with a multi-period default prediction of a debtor at all stages of cash flow process. As a result the model calculates probabilistic distribution of a bond’s NPV, its mean value and other statistical characteristics, taking in account possible recoveries.

Section 3 represents short description of approach and principal issues in multi-period Bayesian default prediction that constitutes principal part of bond valuation model.

Section 4, basing on BMP model, calculates fair interest rates on a risky bond (at time of bond issuance), fair prices and fair yields on a bond at various times through the bond life cycle. Section compares assessed risky prices, yields and interest rates with true (observed) interest rates, market prices and yields of real corporate bonds. To make such comparison we simultaneously use accounting data of a firm, information concerning issues of a firm’s bonds, market data on bond trades.

In the section 5 we briefly discuss relation between bond rating – widespread characteristic of probability of bond default and multi-period default probabilities that represent full probabilistic description of the same event.

Section 6 concludes.

2. Bayesian multi-period model for valuation of risky bonds (loans).

Suppose that at time t_0 a firm issues a bond whose par value is U dollars. A bond pays interest at annual rate r_b at time moments $t_m (m = 1, \dots, M)$ and is redeemed after M years at time t_M for the same U dollars.

Alternatively one can consider a bank that at time t_0 lends to a client (a firm) U dollars. The loan must be returned after M years at time t_M and additionally client pays annual interest at rate r_b (at time moments t_m).

Consider first the value of a bond (loan) at time of issuance.

If the bond is risk-free its Net Present Value at time t_0 can be calculated as

$$V = \sum_{m=0}^M \frac{r_b U}{(1+r_f)^m} + \frac{U}{(1+r_f)^M}, \tag{1}$$

where r_f is risk-free discount rate.

If at time of bond issuance its emitter chooses $r_b = r_f$, he obtains $V = U$.

A firm that issues a bond is subject to default (bankruptcy), which can occur at a random time t_D . This time moment can lie within one of the following time intervals:

- time interval $T_1 \Rightarrow \{t_0 \div t_1\}$ with probability $P_D(T_1)$;
- time interval $T_2 \Rightarrow \{t_1 \div t_2\}$ with probability $P_D(T_2)$;
-
- time interval $T_M \Rightarrow \{t_{M-1} \div t_M\}$ with probability $P_D(T_M)$;
- time interval $T_{M+} \Rightarrow \{t_M \div t_\infty\}$ with probability $P_D(T_{M+})$.

All intervals except the last one are intervals between adjacent interest payments that can be equal to one year, half of a year etc. Other intervals (in case of more complex cash flow schedules) can be also considered if necessary. The last time interval corresponds to defaults, which occur after the debt is paid off. It includes situation when default does not occur; this situation is referred to as default at infinite time (t_∞).

Note that the described group of default events is full and $P_D(T_1) + \dots + P_D(T_M) + P_D(T_{M+}) = 1$.

Methods of calculation of probabilities $P_D(T_m)$ are discussed in section 3.

If a firm defaults before the debt is paid off, cash flows between firm and its lenders cease, and value (NPV) of a bond decreases.

Consider first that no recoveries are possible.

One can see from (1) that value of a bond V can take one of the discrete random values, namely:

- $V = V_1 = 0$ with probability $P_D(T_1)$;
-
- $V = V_m = \sum_{k=1}^{m-1} \frac{r_b U}{(1+r_f)^k}$ with probability $P_D(T_m)$; (2)
-
- $V = V_{M+} = \sum_{m=1}^M \frac{r_b U}{(1+r_f)^m} + \frac{U}{(1+r_f)^M}$ with probability $P_D(T_{M+})$.

Formulae (2) in combination describe probabilistic distribution of value of a bond. This distribution is of discrete type. Probability density distribution in equivalent continuous form can be written as:

$$P(V) = P_D(T_{M+}) \times \delta(V - V_{M+}) + \sum_{m=1}^M P_D(T_m) \times \delta(V - V_m), \quad (3)$$

where $\delta(\bullet)$ is (well known in mathematics) symbolic delta-function, which may be thought of as a normal probability density with very small (almost zero) variance. $\delta(V - V_m)$ is concentrated around $V = V_m$

Cumulative distribution function $F(V)$ of a bond value is a stepwise function. Steps occur at values $V = V_m$ and have magnitudes $P_D(T_m)$:

$$F(V) = P_D(T_{M+}) \times 1(V - V_{M+}) + \sum_{m=1}^M P_D(T_m) \times 1(V - V_m), \quad (4)$$

where indicator function $1(x)$ is defined as: $1(x) = 0$ if $x < 0$ and $1(x) = 1$ if $x \geq 0$.

From (3,4) one can easily calculate mean value of a risky bond that is equal to:

$$\bar{V} = V_{M+} \times P_D(T_{M+}) + \sum_{m=1}^M V_m \times P_D(T_m). \quad (5)$$

We identify \bar{V} with "fair" price of a risky bond.

Suppose now that after a defaulting firm's recovery procedures are over, bank receives back a portion β of lost amount. If β is fixed, each random bond value V_m must be increased to

$$V'_m = V_m + \beta(V_{M+} - V_m) = V_{M+} \beta + V_m(1 - \beta).$$

In this case instead of (3) one can obtain probability density of bond value conditional on recovery rate β in form:

$$P(V / \beta) = P_D(T_{M+}) \cdot \delta(V - V_{M+}) + \sum_{m=1}^M P_D(T_m) \cdot \delta(V - V_{M+} \cdot \beta - V_m \cdot (1 - \beta)), \quad (6)$$

and mean conditional bond value:

$$\bar{V}(\beta) = V_{M+} \times P_D(T_{M+}) + \sum_{m=1}^M (V_{M+}\beta + V_m(1-\beta)) \times P_D(T_m). \quad (7)$$

Expression (6) can be further averaged if recovery rate is random:

$$P(V) = \int P(V/\beta)P_r(\beta)d\beta,$$

where $P_r(\beta)$ is probability density of recovery rate ($P_r(\beta) = 0$ if $\beta < 0$ and $\beta > 1$), that is sometimes taken as beta-distribution.

The final (unconditional) probability density of a firm's value is:

$$P(V) = P_D(T_{M+}) \cdot \delta(V - V_{M+}) + \sum_{m=1}^M P_D(T_m) \cdot \frac{1}{V_{M+} - V_m} \cdot P_r\left(\frac{V - V_m}{V_{M+} - V_m}\right), \quad (8)$$

and its cumulative distribution

$$F(V) = P_D(T_{M+}) \cdot 1(V - V_{M+}) + \sum_{m=1}^M P_D(T_m) \cdot B\left(\frac{V - V_m}{V_{M+} - V_m}\right), \quad (9)$$

where $B(x)$ is cumulative distribution of recovery rate (β). One can see that probability distribution function $F(V)$ is of combined (discrete plus continuous) type.

A characteristic view of cumulative distributions (3) and (9) in case $r_b = r_f$ is represented in figure 1.

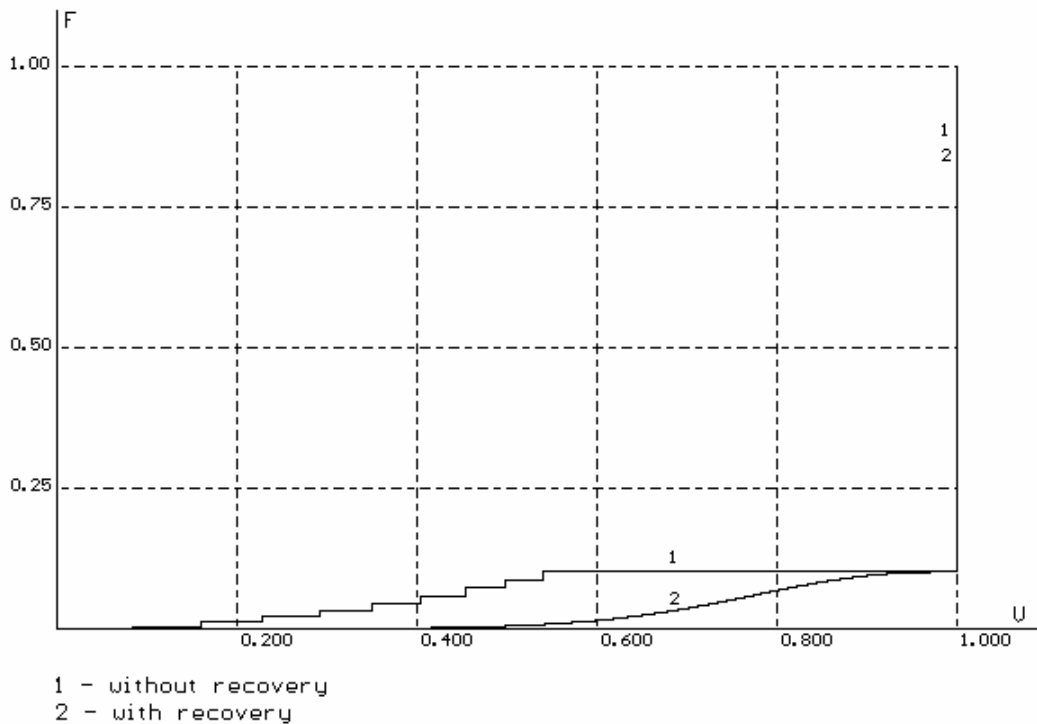


Figure 1. Cumulative distributions of NPV of a risky bond with $r_b = r_f$. Both distributions have big vertical steps at $V=1$ whose magnitude is probability that default did not occur within active bond

period. The curve has stepwise character if recovery is impossible (curve 1) and combined (continuous plus discrete) character in case of random continuously distributed recovery rate (curve 2).

For each value V plot on horizontal axis of the graph, ordinate $F(V)$ of cumulative distribution is probability of NPV of the bond be less than V . One can see that in this example probability of NPV be less than $V_{M+} = U$ is nearly $0,1$. This is probability of a bond's default within active period of its life.

Generally, if $r_b = r_f$, all possible random values V_m , except V_{M+} , are less than NPV of risk-free bond with the same parameters (risk-free bond corresponds to $P_D(T_m) = 0$, ($m = 1, ..M$) and $P_D(T_{M+}) = 1$). As a result mean NPV of risky bond will also be less than NPV of equivalent risk-free bond. In the current example $\bar{V} = 93,68\%$ of a bond's face value U .

To compensate investors for risky character of bond, issuer must increase interest rate r_b over risk-free rate r_f . For a real risky bond using the same formulae (2,3,9) and the same discount rate r_f one obtains distributions of bond value V represented in figure 2.

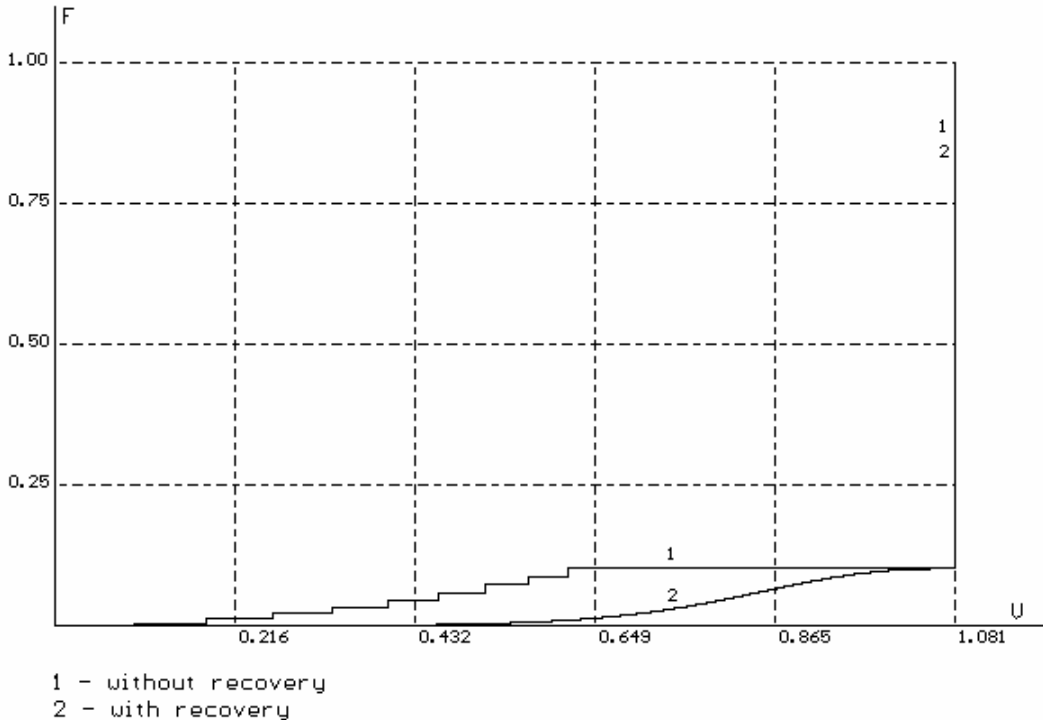


Figure 2. Cumulative distributions of NPV of 10¼% ten year bonds of Cabot Corporation (Rating Baa1) at time of issuance (December 15, 1987). Recovery parameters of the bond are hypothetical.

The graph is drawn for 10¼%, 10 - year bond of Cabot Corporation at time of its issuance (December 15, 1987). Bond rating is Baa1; risk-free interest rate on 10-year bonds in December 1987 was 8,99%. In this case *NPV* of the bond, if it does not default, is 108,1% of face value. Mean *NPV* is 101,48% of face value that is near to *NPV* of risk free bond (100%).

Note that above inference contrasts with Merton's theory of valuation of risky bonds. Remember that this theory considers a bond value as the deterministic (*non-random*) function of value of the firm and time (this is taken directly from option pricing theory), while in the current study bond value is random variable.

We do not also need to consider some increased risky discount rates (as is done some times in literature) and account instead for bond riskness by considering probabilities of borrower's default at various stages of credit process. This is more natural.

3. Assessing probabilities $P_D(T_k)$.

While assessing probabilities $P_D(T_m)$ one must distinguish between ex-ante (unconditional) and ex-post (conditional) probabilities.

If default probabilities $P_D(T_m)$ are determined on economy-wide (country-wide) level and averaged over the long time periods, one can refer to such probabilities as being *unconditional*.

Being based on default statistics collected from restricted subsets of borrowers, probabilities can reflect specific characteristic of a macroeconomic environment of time period, area, industry. Such probabilities are sometimes identified as conditional.

Truly conditional probabilities $P_D(T_m | \mathbf{f})$ can be obtained with accounting for individual financial position \mathbf{f} of each borrower. They are more preferable than unconditional probabilities, because they allow for assessing individual credit risk of a borrower.

To assess probabilities $P_D(T_m)$ one must explicitly include time component in formulation of a bankruptcy (default) prediction problem. To incorporate the time to bankruptcy, the bankruptcy prediction problem may be conceived as a multi-alternative problem of Statistical Decision Theory (SDT).

Consider the infinite set of basic hypotheses $H_1, H_2, \dots, H_i, \dots$, of firm bankruptcy during the first, second, ... i - th, ... time interval (e.g. half-year, year etc.), starting from the date of observation. Also consider the set of basic events of bankruptcy during the first, second, ..., i - th time interval, denoted by the same letters. The term "basic event (hypothesis)" emphasizes that some other, -derivative-events can also be considered. These

can consist, for example, of bankruptcy during the next two, three years or jointly during all years after maturity of a debt and may be derived as the conjunction of basic events.

Allow $D_1, D_2, \dots, D_s, \dots$ to denote the infinite set of the basic predictive or prognostic decisions, which are forecasts of firm bankruptcy during the first, second, ..., s - th year. To choose a decision from $D_1, D_2, \dots, D_s, \dots$ it is necessary to consider values of n predictive factors f_1, f_2, \dots, f_n , and hence a decision must be some function of these factors $D_s = D_s(f_1, f_2, \dots, f_n) = D_s(\mathbf{f})$.

SDT introduces the loss function $W(H_i, D_s)$, which describes losses in the situation where decision D_s (a firm will become bankrupt during the s - th year) is made while the true event is H_i (the bankruptcy occurs during the i - th year). The Bayesian approach of SDT acknowledges as optimal the decision $D_s^*(\mathbf{f})$, which ensures a minimal value of the average risk $R(D)$.

$$R(D) = \sum_{H_i} \iint W(H_i, D(\mathbf{f})) \cdot P(H_i, \mathbf{f}) d\mathbf{f}, \quad (10)$$

where $P(H_i, \mathbf{f})$ is the joint probability distribution of bankruptcy events H_i and predictive factor values \mathbf{f} .

SDT proves that each optimal decision can be made by considering ex-post (conditional) probabilities $P(H_i | \mathbf{f})$ of hypotheses H_i given a set of predictive factors \mathbf{f} . The most convenient way to calculate ex-post probabilities $P(H_i | \mathbf{f})$ is by using Bayes' formula:

$$P(H_i | \mathbf{f}) = \frac{P(\mathbf{f} | H_i) \cdot P(H_i)}{\sum_i P(\mathbf{f} | H_i) \cdot P(H_i)}, \quad (11)$$

where $P(H_i)$ is the ex-ante probability of hypothesis H_i , and $P(\mathbf{f} | H_i)$ is the distribution of the vector of predictive factors \mathbf{f} given the hypothesis H_i (likelihood function of the hypotheses H_i).

Note that $P(H_i | \mathbf{f})$ is alternative notation of $P_D(T_i | \mathbf{f})$.

To calculate the ex-post probabilities by means of formula (11), one must set the system of multivariate probability densities of predictive factors $P(\mathbf{f} | H_i)$ for firms at different time horizons before their bankruptcy and the system of ex-ante (prior) bankruptcy probabilities $P(H_i)$ for the infinite set of hypotheses H_i .

The most suitable ex-ante probabilistic characteristic of default is the default rate π , which is defined as the conditional probability of a borrower's default prior to the end of the time interval of interest, given it was operating at the beginning of the time interval. In practice this probability may be determined approximately as the percentage of firms operating at the beginning of the time interval, which defaulted during that time interval

Having default rates π_1, \dots, π_M for time intervals T_1, \dots, T_M between successive interest payments, one can calculate the probabilities of a firm's default during each of these intervals.

The probability of a firm's default during the m -th interval can be calculated as

$$P_D(T_m) = (1 - \pi_1) \cdot \dots \cdot (1 - \pi_{m-1}) \cdot \pi_m, \quad (12)$$

and the probability of default after a debt is paid off is

$$P_D(T_{M+}) = (1 - \pi_1) \cdot \dots \cdot (1 - \pi_M). \quad (13)$$

If the default rate is a constant π , then

$$P_D(T_m) = (1 - \pi)^{m-1} \cdot \pi, \quad (14)$$

$$P_D(T_{M+}) = (1 - \pi)^M. \quad (15)$$

Models of this type are very popular in probability theory. Their underlying assumption of the independence of default events means that the distribution of time remaining until default does not (ex-ante) depend on a firm's history, i.e. on how long it was operating before (Feller, 1966).

The determination of the set of multivariate likelihood functions $P(\mathbf{f} | H_i)$ is a more complex problem; its solution may be based on theoretical considerations and past observational data of firms at different time intervals prior to their bankruptcy. Empirical data evidences that likelihood functions $P(\mathbf{f} | H_i)$ are stable (do not change from year to year) at far distances before bankruptcy when a firm is a healthy going concern and begin to change at distances 5,4...1 year before bankruptcy as a financial position of a firm deteriorates.

We represent likelihood function $P(\mathbf{f} | H_i)$ in form:

$$P(\mathbf{f} | H_i) = \sum_{l=1}^m p_l \cdot P(\mathbf{f} | \mathbf{f}^{(l)}(H_i)), \quad (16)$$

by introducing the concept of typical variants of the firm's financial position i years prior to bankruptcy. Each typical variant (identified by index l) is described by a combination (vector) of values of predictive factors $\mathbf{f}^{(l)}(H_i)$ that characterizes some particular case of the firm's financial position i years prior to bankruptcy. Alternatively, $\mathbf{f}^{(l)}(H_i)$ can be viewed as possible portraits (proxies) of a firm, drawn in terms of predictive variables, i years prior to its bankruptcy.

Past observations of a firm's indices represent examples of such portraits (typical variants) but those portraits can also be derived from economic considerations.

We allow small random variations of the factors around the basic value; these variations for the l -th portrait are described by the needle-type distribution $P(\mathbf{f} | \mathbf{f}^{(l)}(H_i))$.

The construction of the likelihood functions $P(\mathbf{f} | H_i)$ corresponds to the estimation of a probability density. Estimators of type (16), if they are based on past observations, are known in the literature. A distinction is made between kernel and histogram estimators, depending on the method of construction.

For kernel estimators, the basic combinations of predictive factors $\mathbf{f}^{(l)}(H_i)$ are those obtained from past observations of the factors. The weights p_i are taken to be equal for all combinations, as are forms and dispersions of partial distributions, which depend on the size of the sample. Kernel estimators give the fastest convergence (of all known estimators) of the estimator to the true factor probability density.

For histogram estimators, the basic combinations are chosen as a grid of certain characteristic values; the weights p_i are adjusted to match the observational data. Classically, the probability density $P(f_j | \mathbf{f}^{(l)}(H_i))$ is rectangular, but other shapes, say, Gaussian, may also be considered. Histogram estimators are useful when the sample size is small and appeal is made to economic theory or common sense considerations (smoothness, monotonousness, unimodality, etc.).

In more details methodology for assessing probabilities $P_D(T_m | \mathbf{f})$ (or $P(H_i | \mathbf{f})$) of a firm's default within any given time intervals (conditional on the set (vector) \mathbf{f} , of current financial indices of a borrower) was described in the studies Philosophov, Philosophov (2002), and Philosophov, Batten, Philosophov (2003). Numerical data in the studies relate to a firm's bankruptcy (identified as event of filing a bankruptcy petition). Default is slightly different concept but the methodology is fully applicable. Quantitative data seem to be applicable also, although some further specification is desirable. The difficulty is due to the fact that empirical data on firms' defaults are less publicly available.

The proper choice of financial indices \mathbf{f} used for prediction is of great importance because informative prognostic variables enhance default prediction and increase accuracy of assessing credit risks. The above cited studies propose two groups of predictive variables.

Group of indices of a firm's current financial position includes four financial ratios:

- f_1 - Working Capital / Total Assets;

- f_2 - Retained Earnings / Total Assets;
- f_3 - Earnings Before Interest and Tax / Total Assets;
- f_4 - Interest Payments / Total Assets.

Good predictive power of above ratios was confirmed by parts in many studies starting from Altman (1968).

Another group of indices is first proposed in Philosophov, Batten, Philosophov (2003). It is derived from a schedule of paying off a firm's long-term debt and includes:

- g_1 - portion of long-term debt due within the first year starting from the date of the firm's last financial statements;
- g_2 - portion of long-term debt due within the second year;
-
- g_m - portion of long-term debt due within the m -th year.

The set of factors g_1, \dots, g_n must include all available data on firm's long-term payments due in each future year, not only those from current credit.

Relevant literature studies also variables of market anticipation of bankruptcy of a publicly traded firm. They are not studied in above cited papers but can be easily incorporated in the proposed models.

4. Multi - period Bayesian valuation of risky debt: empirical testing.

In this section we define "fair" interest rates on risky bonds (at time of issuance), "fair" prices and yield spreads on those bonds. Then those values are calculated for some real bonds and compared with observed interest rates, prices and credit spreads. In contrast with many other studies we distinguish between fair interest rates and spreads. For BMP model is important that the former are established by emitters directly at time of bond issuance while the later are established by market via bond market prices. Procedures of determining fair interest rates and fair yield spreads are different.

4.1. A Risky Bond at Time of Issuance.

Modern financial theory and practice establish risk premium $\delta r_b = r_b - r_f$. in dependence on bond rating and basing on empirical analysis of current (at time of issuance) yields on traded bonds of the same maturity and risk.

We can now propose a method of theoretical assessment of fair risky interest rate r_b^* . In general one can consider as fair, rate that equalizes mean NPV of a risky bond and NPV

of risk-free bond with the same terms of issuance - date, maturity, dates of interest payments, etc. At the time of issuance both must be equal to a bond's par value (this is acknowledged point of view (see e.g. Hull (2003))).

Following this criterion one can determine fair rate r_b^* by resolving equation

$$U = V_{M+} \times P_D(T_{M+}) + \sum_{m=1}^M V_m \times P_D(T_m), \quad (17)$$

where V_m and V_{M+} are determined in accordance with (2) and depend on r_b^* .

As the example in the table 1 we represent true and fair (calculated) interest rates on various senior unsecured bonds of USA firms.

Firm	Bond rating	Date of issuance	Maturity date	Risk-free interest rate (%)	Stated interest on bond(%)	True spread	Fair interest rate (%)	Fair interest rate spread to observed spread (%)
1	2	3	4	5	6	7	8	9
Johnson & Johnson	Aaa	15.05.2003	15.05.2033	4,700	4,950	0,250	5,225	210,00
General Electric Company	Aaa	01.02.2003	01.02.1013	4,010	5,000	0,990	4,606	60,202
Procter & Gamble	Aa3	15.12.2003	15.12.2015	4,460	4,850	0,390	5,036	147,94
Procter & Gamble	Aa3	01.02.2004	01.02.2034	5,020	5,500	0,480	5,626	126,25
Abbott Laboratories	A1	15.03.2004	15.03.2011	3,230	3,750	0,520	3,716	93,462
Abbott Laboratories	A1	15.03.2004	15.03.2014	3,750	4,350	0,600	4,286	89,333
Anheuser Busch Companies .	A1	15.04.2004	15.04.2012	3,890	4,700	0,810	4,595	87,037
Anheuser Busch Companies	A1	15.10.2003	15.10.2016	4,545	5,050	0,505	5,051	100,198
Anheuser Busch Companies	A1	15.11.2005	15.11.2017	4,658	5,491	0,833	5,156	59,784
Anheuser Busch Companies	A1	01.03.2004	01.03.2019	4,465	5,000	0,535	4,953	91,215
Bristol Myers Squibb Co.	A1	01.10.2001	01.10.2011	4,660	5,750	1,090	5,146	44,587
Bristol Myers Squibb Co.	A1	15.11.1996	15.11.2026	6,450	6,800	0,350	7,001	157,43
Archer Daniels Midland Co.	A2	15.12.1997	15.12.2027	6,070	6,750	0,680	6,700	92,647
Mean A								90,632
AOL Time Warner Inc.	Baa1	15.04.2001	15.04.2011	5,080	6,750	1,670	5,931	50,958
AOL Time Warner Inc.	Baa1	15.04.2001	15.04.2031	5,590	7,625	2,035	6,346	37,150
AOL Time Warner Inc.	Baa1	01.05.2002	01.05.2012	5,130	6,875	1,745	6,049	52,665
AOL Time Warner Inc.	Baa1	01.05.2002	01.05.2032	5,790	7,700	1,910	6,585	41,623
International Paper Co.	Baa2	15.01.2004	15.01.2009	3,240	4,250	1,010	3,965	71,782
International Paper Co.	Baa2	01.04.2004	01.04.2010	2,975	4,000	1,025	3,722	72,878
International Paper Co.	Baa2	15.01.2004	15.01.2014	4,270	5,500	1,230	4,997	59,106
International Paper Co.	Baa2	01.04.2004	01.04.2016	3,940	5,250	1,310	4,666	55,488

Motorola Incorporated	Baa2	15.11.2000	15.11.2010	5,850	7,625	1,775	6,489	36,000
Goodrich Corporation	Baa3	15.04.1998	15.04.2008	5,550	6,450	0,900	6,126	64,000
Goodrich Corporation	Baa3	15.04.1998	15.04.2038	5,930	7,000	1,070	6,468	53,333
Goodrich Corporation	Baa3	15.05.1999	15.05.2009	5,530	6,600	1,070	6,232	65,607
Mean Baa								55,049

Table 1. True and fair interest rates on corporate bonds.

Risk-free interest rates in the table are weekly interest rates related to the last Friday before bond issuance date; data is taken from USA Federal Reserve statistical releases.

Mean annual ex-ante default rate $\pi = 1,26\%$ used in calculation of probabilities $P_D(T_m / \mathbf{f})$ and mean annual recovery rate $\beta = 44,9\%$ used in calculation of V_m are taken from extensive study of Moody's Investor Service, (2005). They cover all rated bonds observed in time interval 1970 – 2004.

According to the table 1 default risk (as assessed by BMP model) explains on the average about 90% of observed interest rate spreads for *A* rated bonds and about 55% for bonds rated *Baa*. We assess such coincidence of true and fair interest rates as good.

Note that majority of bonds in table 1 (and other tables below) are callable; call provision is unfavorable for bond holders and must be compensated by an increment in interest rate spread. Accounting for this additional "call spread" could ensure still better coincidence between calculated and observed interest rate spreads. Nevertheless waiting for absolute coincidence is problematic because of some indefiniteness that is present in setting risk-free interest rates, mean ex-ante default and recovery rates. This influences calculated fair interest rates (most sensitive to improperly set risk-free interest rates are bond with higher ratings *Aaa*, *Aa*).

Another source of discordance consists in fact that one can use various sets of probabilities $P_D(T_m)$ for calculating fair risky interest rates. Those can be ex-ante probabilities, calculated via annual default rates and several variants of ex-post probabilities $P_D(T_m / \mathbf{f})$, conditional on various sets \mathbf{f} of indices of a firm's current financial position, as described in section 3. Different sets of variables \mathbf{f} informative in predicting defaults, will lead to different estimators r_b^* . Coincidence with observed interest rates will be better if set \mathbf{f} is near to that implicitly used by market, rating agencies, investment banks.

Even after adjusting mean NPV of a risky bond, its instantaneous NPV stays to be random, and investors can demand additional compensation for this randomness.

Data of the Table 1 contrasts with numerous studies (Elton, Gruber, Agrawal, Mann (2001); Huang, Huang (2003); Longstaff, Mittal, Neis (2005); Driessen (2005); Chen, Collin-Dufresne, Goldstein (2005); Crermers, Driessen, Maenhout, Weinbaum (2005); Berndt, Douglas Duffie, Ferguson, Schranz (2005)) that used Merton-type bond valuation models and found that default risk explains only small part of observed credit spreads (5 through 22

percents according to Delianedis and Geske (2001)). The rest of spread those studies attribute to taxation differences between corporate and Treasury bonds, jumps in asset value process, liquidity and market effects.

Current study represents more simple explanation of above findings: this is the extreme imprecision of Merton-type models.

4.2. A Risky Bond at an arbitrary time t .

Consider now the same risky bond at an arbitrary time t of its life cycle. Suppose time t is within interval T_j , i.e. between t_{j-1} and t_j moments of interest payment.

The bond's NPV stays to be random but number of its (discrete) possible values decreases because some horizons of interest payments are already passed.

Those values can be described using expressions similar to that of formulae (2):

- $V^{(t)} = V_j^{(t)} = 0$ with probability $P_D^{(t)}(T_j^{(t)})$;
- $V_{j+1}^{(t)} = \frac{r_b^{(j)}U}{(1+r_f^{(j)})}$ with probability $P_D^{(t)}(T_{j+1})$;
-
- $V_m^{(t)} = \frac{r_b^{(j)}U}{(1+r_f^{(j)})} + \sum_{k=j+1}^{m-1} \frac{r_b U}{(1+r_f^{(j)})(1+r_f)^{k-j}}$ with probability $P_D^{(t)}(T_m)$; (18)
-
- $V_{M+}^{(t)} = \frac{r_b^{(j)}U}{(1+r_f^{(j)})} + \sum_{k=j+1}^M \frac{r_b U}{(1+r_f^{(j)})(1+r_f)^{k-j}} + \frac{U}{(1+r_f^{(j)})(1+r_f)^{M-j}}$ with probability $P_D^{(t)}(T_{M+})$.

Placed separately in expressions (18) are addendums corresponding to the time interval $T_j^{(t)}$ between t and t_j that constitutes only part of standard time interval between interest payments. For this interval one need apply corrected interest and discount rates $r_b^{(j)}, r_f^{(j)}$ and corrected probability of default $P_D^{(t)}(T_j^{(t)})$. Generally $P_D^{(t)}(T_m)$ denote probabilities of a firm's default within time intervals T_m as they can be assessed at time t .

Risk-free interest rate r_f is also considered for time t .

Cumulative distribution of a bond value is now:

$$F(V) = P_D^{(t)}(T_{M+}) \times 1(V - V_{M+}^{(t)}) + \sum_{m=j}^M P_D^{(t)}(T_m) \times 1(V - V_m^{(t)}), \quad (19)$$

and mean value of a bond can be determined using formula similar to formula (5):

$$\bar{V}^{(t)} = V_{M+}^{(t)} \times P_D^{(t)}(T_{M+}) + \sum_{m=j}^M V_m^{(t)} \times P_D^{(t)}(T_m). \quad (20)$$

Formulae (6,7,8) related to non-zero recovery rates can be rewritten for time t_j by analogy.

Considering risky bonds at an arbitrary time t one can try to explain observed market prices and observed yields on those bonds and determine what influences their values. It is interesting to establish if risk of default is the only or at least principal factor that determines observed credit spreads.

We can do this by introducing concept of “fair prices” and “fair yields” on risky bonds.

Fair price of bond V^* can be considered to be equal to bond’s mean NPV - $\bar{V}^{(t)}$ as determined by (20).

Yield to maturity Y is usually determined as a decision of equation, which in our notation can be written as:

$$V^{(mr)} = \frac{r_b^{(j)}U}{(1+Y^{(j)})} + \sum_{k=j+1}^M \frac{r_b U}{(1+Y^{(j)})(1+Y)^{k-j}} + \frac{U}{(1+Y^{(j)})(1+Y)^{M-j}} \quad (21)$$

where $V^{(mr)}$ is market price of the bond at time moment t , $r_b^{(j)}$ and $Y^{(j)}$ are interest rate r_b and yield Y recalculated to partial time interval.

For a risky bond equation (21) implies that a bond survives until maturity and brings increased (risky) yield relatively to risk-free yield (interest rate r_f) as compensation for possibility of default.

Really, possibility of default within one of future time intervals T_m leads to random bond value $V_m^{(t)}$, as described by formulae (18), and randomness of true yield (yield to default). Possible values of random yield can be found by deciding the set of equations $V^{(mr)} = V_m^{(t)}(Y)$, for $m = j, \dots, M, M+$, where $V_m^{(t)}(Y)$ are taken from (18) with Y used as substitute for r_f . Minimally possible yield is equal -100% and corresponds to situation when investment is fully lost, i.e. a firm defaults before any interest is paid and there is no recovery.

To clarify, in table 2 we represent data of Abbott Laboratories (ABT) whose senior unsecured bond issued on the March 15, 2004 is traded on the December 22, 2006. The bond pays semiannual interest at annual rate 4,35% and matures on the March 15, 2014. Moody’s rating of the bond is A1, market price as of December 22, 2006 is 96,680% of par and stated yield to maturity is 4,901%.

Rows of the table correspond to semiannual intervals between successive interest payments. Columns 2, 3 represent beginning and ending dates of each interval; observation

date (December 22, 2006) is within the sixth interval. The last – 21-th interval covers all dates after the end of bonds' life period.

Probabilities of default for Abbott Laboratories for each time interval are represented in column 4. They were calculated as described in section 3 basing on Abbott Laboratories accounting data of as of December 31, 2005. As before, annual ex-ante default rate was taken equal to 1,26% and recovery rate to 44,9%.

The column 5 represents yields to default calculated by means of formulae (18) for situations of possible ABT default within each time interval. All defaults within active bond period lead to loss of principal and as a consequence to negative yield. If default does not occur or occurs after bond's active period, yield to default is the same as yield to maturity and is equal to 4,901%. This amount is exactly the same as was achieved in actual trades. Table 3 evidences that such yield can be obtained with probability 0,9261.

Interval #	Interval start date	Interval end date	Probability. of default	Yield to default (%)
1	2	3	4	5
6	22.12.2006	15.03.2007	0,0003	-7,422
7	15.03.2007	15.09.2007	0,0008	-7,339
8	15.09.2007	15.03.2008	0,0018	-7,146
9	15.03.2008	15.09.2008	0,0033	-6,944
10	15.09.2008	15.03.2009	0,0040	-6,733
11	15.03.2009	15.09.2009	0,0051	-6,512
12	15.09.2009	15.03.2010	0,0054	-6,283
13	15.03.2010	15.09.2010	0,0061	-6,046
14	15.09.2010	15.03.2011	0,0064	-5,801
15	15.03.2011	15.09.2011	0,0071	-5,549
16	15.09.2011	15.03.2012	0,0073	-5,291
17	15.03.2012	15.09.2012	0,0077	-5,028
18	15.09.2012	15.03.2013	0,0068	-4,761
19	15.03.2013	15.09.2013	0,0060	-4,491
20	15.09.2013	15.03.2014	0,0058	-4,219
21	15.03.2014	all later dates	0,9261	4,901

Table 2. Yields on bond in dependence on time interval of its default. The data corresponds to 4,35% 10-year senior unsecured bond of Abbott Laboratories due March 15, 2014. Bond is traded on the December 22, 2006 at price 96,680%. Its stated yield to maturity is 4,901%.

In general data of the table 2 describes probabilistic distribution of random yield of Abbott Laboratories; the distribution is of discrete type.

To explain theoretically observed market yields to maturity on risky bonds (i.e. to calculate “fair yields”) one can act in two different ways.

1. Determine fair price of risky bond by means of formula (20) and then use it in equation (21) instead of market price to determine “fair” risky yield.

2. Try to directly infer “fair” risky yields from risk-free yield and probabilities of default. There are several variants of such inference; the most simple and transparent is to suppose that average of risky market yields (like those represented in table 2) must be near to risk-free yield.

$$Y_{M+}^{(t)} \times P_D^{(t)}(T_{M+}) + \sum_{m=j}^M Y_m^{(t)} \times P_D^{(t)}(T_m) \approx Y_{rf}^{(t)}.$$

Considering further that all negative yields are inappropriate and have restricted economic sense we can equalize them to zero. As a result we obtain for yield to maturity:

$$Y_{M+}^{(t)} \times P_D^{(t)}(T_{M+}) = Y_{rf}^{(t)} \quad \text{and} \quad Y_{M+}^{(t)} = Y_{rf}^{(t)} / P_D^{(t)}(T_{M+}). \quad (22)$$

The resulting formula is simple and rather transparent. Risky yield to maturity must exceed risk-free yield the more, the less is probability $P_D^{(t)}(T_{M+})$ (i.e. the more is probability $1 - P_D^{(t)}(T_{M+})$ that default does occur within active bond period).

Table 3 represents examples of calculation of fair prices and fair yields to maturity for some issues of senior unsecured bonds of USA industrial firms. Data on bond issues and issuers’ accounting information is taken from SEC filings and annual corporate reports. Data on bond market prices and yields to maturity is NASD data on OTC trades.

Represented in the table (four right columns) are Fair Prices, Fair Yields, calculated by two just described methods and ratio of fair to observed yield spreads (for fair yields calculated via fair price). Risk-free interest rates in the table are weekly interest rates related to the last Friday before bond trade date; data is taken from USA Federal Reserve statistical releases.

Mean annual default rate ($\pi = 1,26\%$) and mean annual recovery rate ($\beta = 44,9\%$) used in calculation are taken as before from extensive study of Moody’s Investor Service, (2005).

Table 3 evidences that, like interest rate spreads in table 1, default yield spreads constitute significant part of total credit yield spreads, on average nearly 72%. This again contrasts with what can be obtained from structural models. Note also that fair yields, calculated via fair price are nearer to observed risky yields to maturity than directly calculated fair yields.

Firm	Bond rating	Bond matures	Bond interest rate	Trade Date	Risk-free Rate (weekly)	Closing Price	Closing Yield	Fair price	Fair yield (as determined by fair price)	Fair yield (as determined directly)	Fair yield spread to observed yield spread (%)
Abbott Laboratories	A1	15.03.2011	3,750	15.09.2006	4,735	93,863	5,305	94,466	5,145	4,898	71,928
Abbott Laboratories	A1	15.03.2011	3,750	22.12.2006	4,5793	94,680	5,200	95,230	5,017	4,737	74,337
Abbott Laboratories	A1	15.03.2014	4,350	14.09.2006	4,748	93,100	5,486	94,217	5,295	5,128	74,108
Abbott Laboratories	A1	15.03.2014	4,350	22.12.2006	4,572	96,680	4,901	95,270	5,142	4,937	173,420
Anheuser Busch Companies Inc.	A1	15.04.2012	4,700	22.12.2006	4,562	98,322	5,064	98,939	5,053	4,699	97,889
Anheuser Busch Companies Inc.	A1	15.10.2016	5,050	15.09.2006	4,792	96,345	5,528	98,501	5,244	5,236	61,455
Anheuser Busch Companies Inc.	A1	15.10.2016	5,050	22.12.2006	4,598	96,850	5,468	99,894	5,064	5,023	53,555
Anheuser Busch Companies Inc.	A1	01.03.2019	5,000	15.09.2006	4,844	96,889	5,345	97,030	5,330	5,454	96,986
Anheuser Busch Companies Inc.	A1	01.03.2019	5,000	21.12.2006	6,648	94,397	5,641	98,686	5,147	5,233	50,251
Bristol Myers Squibb Co.	A1	01.10.2011	5,750	15.09.2006	4,730	102,705	5,133	103,395	4,981	4,844	62,263
Bristol Myers Squibb Co.	A1	01.10.2011	5,750	21.12.2006	4,566	101,250	5,446	103,884	4,830	4,674	30,035
Bristol Myers Squibb Co.	A1	15.11.2026	6,800	13.09.2006	5,009	107,790	6,121	115,312	5,531	6,216	46,948
Bristol Myers Squibb Co.	A1	15.11.2026	6,800	22.12.2006	4,818	110,744	5,876	117,591	5,353	5,977	50,583
Mean for A1 bonds											67,662
International Paper Co.	Baa2	15.01.2009	4,250	15.09.2006	4,790	97,725	5,302	97,381	5,460	4,926	130,775
International Paper Co.	Baa2	15.01.2009	4,250	22.12.2006	4,694	96,547	6,068	97,734	5,425	4,822	53,219
International Paper Co.	Baa2	01.04.2010	4,000	15.09.2006	4,745	94,125	5,866	95,224	5,502	4,980	67,541
International Paper Co.	Baa2	01.04.2010	4,000	22.12.2006	4,603	95,845	5,406	95,830	5,407	4,826	100,175
International Paper Co.	Baa2	01.09.2011	6,750	21.12.2006	4,568	106,759	5,110	105,789	5,340	4,876	142,432
International Paper Co.	Baa2	15.01.2014	5,500	15.09.2006	4,746	98,088	5,823	100,046	5,493	5,243	69,371
International Paper Co.	Baa2	15.01.2014	5,500	21.12.2006	4,571	99,712	5,549	100,928	5,341	5,045	78,731
International Paper Co.	Baa2	01.04.2016	5,250	14.09.2006	4,782	93,970	6,093	98,041	5,517	5,434	56,049
International Paper Co.	Baa2	01.04.2016	5,250	20.12.2006	4,593	96,188	5,786	99,311	5,346	5,213	63,124
Goodrich Corporation	Baa3	15.04.2008	6,450	14.09.2006	4,897	99,600	6,718	101,648	5,353	4,961	25,037
Goodrich Corporation	Baa3	15.04.2038	7,000	28.08.2006	4,950	103,650	6,719	120,322	5,619	7,399	37,818
Goodrich Corporation	Baa3	15.05.2009	6,600	15.09.2006	4,770	103,109	5,324	103,014	5,373	4,911	108,847
Mean for Baa bonds											77,760

Table 3. True and fair prices and yields to maturity of some issues of US corporate bonds.

5. Multi-Period Default Probabilities and Credit Ratings

Credit rating is old, well established, world - wide acknowledged and used characteristic of credit quality of bonds, firms, other borrowers and loans. Looking at any manual on financial management, e.g. Brigham (1991), one can read that credit rating characterizes probability of borrower's default.

The statement is rather uncertain because the next arising question is what probability specifically (of various possible) is characterized by credit rating and how it is characterized.

Consider, for example, a common case of coupon bond. As was described in sections 2, full probabilistic description of its default characteristics can be obtained by specifying the set of default probabilities $P_D(T_1), P_D(T_2) \dots P_D(T_M), P_D(T_{M+})$.

As was described in section 3 probabilities $P_D(T_i)$ are described by complex formulae, depend on many variables, and It is rather obvious that a single value of categorical type – a bond rating can not adequately represent them. The picture, represented by default probabilities is much richer.

One can try to simplify probabilistic description of default by reducing above set to only one probability, which, no doubt, must be probability $P_D(T_{M+})$. This probability characterizes whether or not debt principal can be returned, that usually constitutes main part of debt's NPV.

Unfortunately, debt rating can't represent even this single probability, because probability $P_D(T_{M+})$ is highly dependent on bond maturity horizon, while bond rating does not depend. All senior unsecured bonds of the same firm have usually the same credit rating independently on when they mature.

One can also take in account that default probabilities depend on a firm's current financial position. Bond rating, especially for senior unsecured bonds, also is mainly determined by a firm's current financial position, but this dependence is less sharp. Financial position (as determined, for example, by financial ratios) changes with each new financial account while credit rating is more conservative and alters only when changes in financial position are significant.

Considering all above properties of credit ratings and default probabilities, one can suppose that rating must characterize some local probabilistic characteristic of default. In light of very popular now Merton-type models of default risk and Basel II documents most used characteristic of such type is probability of borrower's default within the first year (PD). In our notation this probability is $P_D(T_1)$.

In Table 4 some real USA corporate bonds are viewed at time of their issuance. The bonds are the same as in Table 1. For each bond we represent bond rating, probabilities of bond default within the first year $P_D(T_1)$ and within entire period of its life $1 - P_D(T_{M+})$ and other relevant information.

Firm	Bond rating	Offered	Due	Stated interest on bond (%)	Fair interest rate (%)	Fair spread to true (established) spread (%)	Probability of Default within the First Year	Probability of Default before bond Maturity
1	2	3	4	6	10	11		
Johnson & Johnson	Aaa	15.05.2003	15.05.2033	4,950	5,225	210,00	0,00032	0,28346
General Electric Co.	Aaa	01.02.2003	01.02.1013	5,000	4,606	60,202	0,00111	0,10643
Procter & Gamble	Aa3	15.12.2003	15.12.2015	4,850	5,036	147,94	0,00111	0,12369
Procter & Gamble	Aa3	01.02.2004	01.02.2034	5,500	5,626	126,25	0,00134	0,30366
							0,00097	0,20431
Abbott Laboratories	A1	15.03.2004	15.03.2011	3,750	3,716	93,462	0,00052	0,06225
Abbott Laboratories	A1	15.03.2004	15.03.2014	4,350	4,286	89,333	0,00052	0,09726
Anheuser Busch Comp..	A1	15.04.2004	15.04.2012	4,700	4,595	87,037	0,00107	0,05670
Anheuser Busch Comp..	A1	15.10.2003	15.10.2016	5,050	5,051	100,198	0,00081	0,12105
Anheuser Busch Comp..	A1	15.11.2005	15.11.2017	5,491	5,156	59,784	0,00083	0,11031
Anheuser Busch Comp..	A1	01.03.2004	01.03.2019	5,000	4,953	91,215	0,00059	0,13633
Bristol Myers Squibb Co.	A1	01.10.2001	01.10.2011	5,750	5,146	44,587	0,00040	0,09020
Bristol Myers Squibb Co.	A1	15.11.1996	15.11.2026	6,800	7,001	157,43	0,00088	0,29218
Archer Daniels Midland.	A2	15.12.1997	15.12.2027	6,750	6,700	92,647	0,00307	0,30828
Mean A						90,632	0,00097	0,14162
AOL Time Warner Inc.	Baa1	15.04.2001	15.04.2011	6,750	5,931	50,958	0,01320	0,14210
AOL Time Warner Inc.	Baa1	15.04.2001	15.04.2031	7,625	6,346	37,150	0,01320	0,33430
AOL Time Warner Inc.	Baa1	01.05.2002	01.05.2012	6,875	6,049	52,665	0,01398	0,15156
AOL Time Warner Inc.	Baa1	01.05.2002	01.05.2032	7,700	6,585	41,623	0,01398	0,34161
International Paper Co.	Baa2	15.01.2004	15.01.2009	4,250	3,965	71,782	0,00532	0,06432
International Paper Co.	Baa2	01.04.2004	01.04.2010	4,000	3,722	72,878	0,00675	0,07890
International Paper Co.	Baa2	15.01.2004	15.01.2014	5,500	4,997	59,106	0,00532	0,12522
International Paper Co.	Baa2	01.04.2004	01.04.2016	5,250	4,666	55,488	0,00675	0,14849
Motorola Incorporated	Baa2	15.11.2000	15.11.2010	7,625	6,489	36,000	0,00494	0,11145
Goodrich Corporation	Baa3	15.04.1998	15.04.2008	6,450	6,126	64,000	0,00217	0,10330
Goodrich Corporation	Baa3	15.04.1998	15.04.2038	7,000	6,468	53,333	0,00217	0,38700
Goodrich Corporation	Baa3	15.05.1999	15.05.2009	6,600	6,232	65,607	0,00474	0,12215
Mean Baa						55,049	0,00771	0,17587

Table 4. Bond ratings and probabilities of default for some USA corporate bonds.

The Table evidences that on the average there is noticeable difference between one-year default probabilities of A and Baa rated bonds. The probability is about 0,1% for A rated bonds and 0,77% for bonds rated Baa. The difference between average probabilities of default before maturity is less noticeable. These probabilities are highly dependent on a maturity horizon.

For bonds of highest ratings Aaa and Aa calculations are less accurate. These bonds have very low credit spreads that are more easily distorted by errors and noise.

Because of small sample size we can not reveal distinctions in default probabilities within more detailed rating grades.

6. Conclusion.

The study represents the new-type model (Bayesian Multi-Period model - BMP) for assessing credit risk of bonds and loans.

BMP considers value of a risky bond as net present value of expected cash flows, generated by a bond (interest payments, return of principal etc.), and combines them with the detailed multi-period prediction of a bond default at all stages of cash flow process.

BMP fairly predicts observed bond values, interest rates and yields spreads.

This contrasts with currently most popular Merton-type models that are based on option pricing theory and highly undervalue credit risks.

The model does not use serious restrictive assumptions.

It can consider simultaneously coupon as well as zero-coupon bonds of different types and maturities issued by the same firm. It considers realistic causes and symptoms of default. It does not suppose normality of predictive variables or their mutual independence. It does not use any predetermined model of ex-post default probabilities (conditional hazard rates) or of future dynamics of predictive variables.

We calculate ex-post probabilities within enhanced Bayesian methodology; empirical data is used at intermediate stages of the methodology to assess likelihood functions (conditional distributions of predictive variables) by well established and acknowledged methods.

Described model for valuation risky bond and loans can be most conveniently realized as a PC program. The model has many facilities for natural mapping it to each specific bond, loan, firm, macroeconomic situation. We do not see any drawbacks or obstacles that can impede its practical application, though much wider testing must precede this application.

Literature.

Altman E., 1982. Corporate Financial Distress, a Complete Guide to Predicting, Avoiding and Dealing with Bankruptcy, (John Wiley, New York.).

Berndt A., Douglas R., Duffie D., Ferguson M., Schranz D., 2005, Measuring Default Risk Premia from Default Swap Rates and EDF's, Working Paper.

Bharath S., Shumway T., 2004, Forecasting Default with the KMV-Merton Model, Working paper, University of Michigan.

Black F., Scholes M. 1973, The Pricing of Options and Corporate Liabilities, Journal of Political Economy, 81, 637-654.

Bohn J., 2000, A Survey of Contingent-Claims Approaches to Risky Debt Valuation, Institutional Investor Inc.

Brigham E.F., 1991, Fundamentals of Financial Management, Dryden Press.

Chen L., Collin-Dufresne P., Goldstein R., 2005, On the relation Between Credit Spread Puzzles and Equity Premium Puzzle, Working Paper, Michigan State University , U.C. Berkeley, and University of Minnesota.

Collin-Dufresne P., Goldstein R., Martin S., 2001, The determinants of Credit Spread Changes, Journal of Finance, 56, 2177-2208.

Credit Suisse First Boston 1997, *CreditRisk +TM* 1997, A Credit Risk Management Framework,

Cremers M., Driessen J., Maenhout P., Weinbaum D., 2005, Individual Stock Option Prices and Credit Spreads, Working Paper, Yale University, University of Amsterdam, INSEAD, Cornell University.

Crosbie P., Bohn J., 2003, Modelling Default Risk, Moody's KMV document.

Delianedis G., Geske R., 2001, The Components of Corporate Credit Spreads: Default, Recovery, Tax, Jumps, Liquidity, and Market Factors,

Driessen J., (2005), Is Default Event Risk Priced in Corporate Bonds?, Journal of Finance, 18, 165 - 195.

Duffie D., Saita L., Wang K., 2005, Multi-period Corporate Default Prediction with Stochastic Covariates, Working paper, Stanford University.

Elton E., Gruber M., Agrawal D., Mann C., 2001, Explaining the Rate Spread on Corporate Bonds, Journal of Finance, 56, 247-278.

Eom Y., J. Helwege, J-Z Huang, 2004, Structural Models of Corporate Bond Pricing: An Empirical Analysis, The Review of Financial Studies, vol. 17, pp499-544.

Feller W., 1966, An Introduction to Probability Theory and its Applications, (John Wiley & Sons, Inc., New York, London, Sydney).

Hillegeist S.A., Cram D.P., Keating E.K. Lundstedt K.G., 2002, Assessing the Probability of Bankruptcy, Working paper, Northwestern University.

Huang J.Z., Huang M., 2003, How Much of Corporate- Treasury Yield Spread is Due to Credit Risk?, Working paper, Penn State University and Stanford University.

Jarrow R., Turnbull S. 1995, Pricing Derivatives on Financial Securities Subject to Credit Risk, *Journal of Finance*, 50, 53-86.

Jones E., Mason S., Rosenfeld E., 1984, Contingent Claims Analysis of Corporate Capital Structures: An Empirical Investigation, *Journal of Finance*, 39, 611-625.

JP Morgan 1997, *CreditMetricsTM*, Technical document,.

Kim I., Ramaswamy K., Sundaresan S., 1993, Does Default Risk in Coupons Affect the Valuation of Corporate Bond?, A Contingent Claims Model, *Financial Management*, 22, 117-131.

Leland H., 2002, Prediction of Expected Default Frequencies in Structural Models of Debt, working paper.

Longstaff F., Mittal S., Neis E., 2005, Corporate Yield Spreads: Default Risk or Liquidity? New Evidence from the Credit Default Swap Market, *Journal of Finance*, 60, 2213-2253.

Lyden S., Saraniti D., 2000, An Empirical Investigation of the Classical Theory of Corporate Security Valuation, Barclays Global investors.

Merton R. 1974, On the Pricing of Corporate Debt: The Risk Structure of Interest Rates, *Journal of Finance*, 29, 449-470.

Moody's Investor Service, 2005, Default & Recovery Rates of Corporate Bond Issuers, 1920-2004.

Ogden J., 1987, Determinants of Credit Ratings and Yields on Corporate Bonds: Test of Contingent Claims Model, *Journal of Financial Research*, 10, 329-339.

Philosophov L., Philosophov V., 2002, Corporate bankruptcy prognosis: An attempt at a combined prediction of the bankruptcy event and time interval of its occurrence, *International Review of Financial Analysis*, 11, 375-406.

Philosophov L., Batten J., Philosophov V., 2003, Predicting the Event and Time Horizon of Bankruptcy Using Financial Ratios and the Maturity Schedule of Long-Term Debt, <http://ssrn.com/abstract=461460> - November 2003, EFA-2005 (Moscow), *Mathematics and Financial Economics*, 1(2008), 181-212. .