Credit risk and parent-subsidiary links

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Abstract

Structural models of credit risk consider a company as a stand-alone unit. However, companies often own a subsidiary, which issues debt in its own name. This paper builds a structural model of a parent and a subsidiary, and contrasts their credit quality with that of two stand alone units with endogenous capital structure. We find that their joint default probability is lower than that of stand alone units, despite their higher debt capacity. This is due to a shift of the debt burden onto the subsidiary, which optimally exploits the tax shield of debt while minimizing default costs. Default probability and spreads of the subsidiary are thus higher than for a stand alone with similar size and volatility.

However, the opposite holds when the subsidiary is constrained to a debt equal to the optimal stand alone level. Interestingly, the subsidiary recovery rate is lower than the stand alone, because of the parent ability to support it.

In all cases we derive implicit ratings.

Keywords: business groups, credit risk, structural models, optimal leverage, default

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1 Introduction

Structural models of credit risk consider a company as a stand-alone unit. However, companies often own - at least partially - a subsidiary unit, which issues debt in its own name. These parent-subsidiary links characterize joint ventures, project financing, business groups and multinationals. A considerable amount of empirical evidence points to credit quality relationships within these organizations, but no model investigates rating links in a structural model of credit risk. This paper takes a step in this direction.

The parent-subsidiary link may imply very different relationships depending on legal covenants, informal support agreements, ownership levels and shared names (Samson, 2001). At one extreme, the two firms can be treated like an integrated business. This is the case when either the subsidiary is fully owned, and/or there

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is a legally binding guarantee issued by the parent. There is thus no difference between parent and subsidiary debt, because the parent is fully responsible for its subsidiary. At the other extreme the subsidiary is run independently, and the parent - which is totally unresponsible for its subsidiary's debt obligations - only receives dividends. These two cases respectively resemble the conglomerate merger and the stand-alone units which are analyzed in Leland (2007). We model the in-between case, where the parent provides support to bail out its insolvent subsidiary when it can afford to, but leverage in the two units is determined so as to maximize their joint value. We can thus contrast the credit quality of the two extreme situations with the intermediate one, that we identify with the case of business groups. In each case, the owner chooses firm capital structure in order to maximize the total value of assets, trading off bankruptcy costs with the tax advantage of debt. As a result, we can analyze how group structure affects default probabilities, recovery rates, the associated spreads and ratins. Furthermore, we are able to predict when there will be joint default of both units as opposed to selective default of one of the two.

Parent-subsidiary links have been studied in related papers. Boot et al. (1993) investigate why parent companies write comfort letters assuring subsidiaries' lenders that they would assist them in distress. These are seemingly useless papers, as they are legally unenforceable. However, precisely this feature allows parent companies to disregard them ex-post in states when supporting the insolvent subsidiary would undermine their own integrity. Thus the parent can choose whether to support its subsidiary or let it selectively default.

We thus assume that the parent is not legally responsible for its subsidiary debt obligations.[1] A few theoretical models build already on this assumption (Cestone and Fumagalli, 2005; Bianco and Nicodano, 2006) – although none provides a structural model of credit risk. They study how tranfers from the parent impact on the spread charged to a subsidiary from its outside financiers, when either managerial effort or investment risk cannot be observed. Higher credit quality for the subsidiary is associated to either increased managerial effort or reduced risk shifting, which are ruled out in the case we examine.

A preview of our main results is as follows. Debt financing - and the associated tax shield - is larger for group-affiliated than stand alone firms. Despite this, the probability of joint default for group companies is very low when compared to stand alone units. In terms of historical default frequencies, the improvement in their rating is from Baa3 as stand alone to an almost riskless Aaa group, when calibrated to the same exogenous parameters. This paradox is due to the optimal capital structure of the group, entailing a complete shift of the debt burden onto the subsidiary. The parent, which is instead unlevered, and the group default only when the parent cash flow turns negative because of operational losses.

The shift of debt onto the subsidiary optimally exploits the tax shield of debt while minimizing default costs, as the parent rescues its subsidiary provided their cash flows are less than perfectly correlated. The rescue probability is quite large (90% when correlation is -.8), falling to a still high 67% when correlation moves to .8. Despite such rescues, group affiliation dramatically worsens the credit quality of the subsidiary, with respect to their stand alone situation. Most of the time, the subsidiary default is selective, in the sense that the parent survives - a circumstance that is precluded in mergers (Sarig, 1985). Its risk neutral default probability increases for all cash flow correlations, reaching almost five times the stand alone level when the correlation is high.

[1] This is the case in major jurisdictions, including the U.S., the U.K., Germany and France (Hadden, 1996; Blumberg, 1989).

At the same time, a reduction in recovery rate occurs. Intuitively, the parent is more likely to be unable to provide support when subsidiary losses are very large - leaving these cases to lenders. Its spread increases in cash flow correlation as the parent ability to rescue its subsidiary falls and becomes up to ten times the stand alone level. In this case, the subsidiary rating reaches B2.

Well known models of credit risk used in the financial industry adopt a structural view at the multi firm level. However, their reference to asset correlation does not explicitly incorporate any consideration of ownership structure, with associated selective default features. A partial exception is Giesecke (2004), which allows for links between default boundaries without modelling the structural relationship of firm. On the contrary empirical analyses argue that multinational affiliation and intra-group support mechanisms affect the size, location and default risk of members' debt obligations (Gopalan et. Al., 2004; Emery and Cantor, 2005; Dewaelheyns and Van Hulle, 2006; Bianco and Nicodano, 2006; Huizinga et al., 2007), which is what we model.

The paper is organized as follows. Section 2 presents the set up, and three organizational modes for two activities - stand alone, group and conglomerate. Section 3 analyzes these cases through a numerical example, so as to understand their respective properties. Section 4 compares optimal leverage, default probabilities, recovery rates and credit spreads across the three modes, as the correlation between their cash flows varies. Section 5 extends the analysis to a subsidiary that is not allowed to raise more debt than a stand alone, for regulatory constraints. It also examines the case where units differ in bankrupcty costs, size and percentage volatility. The last section concludes.

2 The common set up

In this section we review Leland set up and his analysis of stand alone firms. We then extend it to the group case. We consider a no arbitrage environment with two dates $t = \{0, T\}$. There are two activities, and each activity i generates a random future operational (net) cash flow value X_i at time $t = T$. X_i is a continuous random variable. The riskfree interest rate over the time period T is r_T. No arbitrage implies that the value of the operational cash flow at $t = 0$ is its discounted expected value:

$$X_{0i} = (1 + r_T)^{-1} \mathrm{E} X_i \tag{1}$$

where $\mathrm{E}X_i$ is evaluated under the risk neutral measure. The owners can "walk away" from negative cash flows thanks to limited liability. Thus the (pre-tax) value of each activity with limited liability is

$$H_{0i} = (1 + r_T)^{-1} \mathrm{E} X_i^+ \tag{2}$$

where $X_i^+ = \max(X_i, 0)$, and the pre-tax value of limited liability is

$$L_{0i} = H_{0i} - X_{0i} \geq 0 \tag{3}$$

Now consider a tax rate on future cash flows equal to τ_i. The aftertax value of the unlevered firm is

$$V_{0i} = (1 - \tau_i) H_{0i} \tag{4}$$

and the present value of taxes paid by the firm (with no debt) is

$$T_{0i}(0) = \tau_i H_{0i} \tag{5}$$

Firms can issue non-collateralized zero-coupon bonds at time $t = 0$ with principal value P_i due, with absolute priority, at $t = T$. They have an incentive to do so as interest on debt is a deductible expense. However, debt will also increase the probability of default, which is assumed to cost a fraction α_i of (positive) cash flows. The assumption implies that bankruptcy through liquidation causes a loss proportional to the firm value. In our model firm value and cash flow X_i coincide at maturity T, since no continuation value exists. Otherwise stated, we model bankruptcy through liquidation and not through restructuring.

Let us examine separately the effects of taxes and bankruptcy costs.

Let $D_{0i}(P_i)$ denote the value, at $t = 0$, of debt. The promised interest payment is

$$P_i - D_{0i}(P_i) \tag{6}$$

Taxable income is the operational one net of interest payment:

$$X_i - (P_i - D_{0i}(P_i)) \tag{7}$$

The zero-tax level of cash flow or tax shield, X_i^Z, is then

$$X_i^Z(P_i) = P_i - D_{0i}(P_i) \tag{8}$$

Hereafter the argument P_i of D_{0i} and X_i^Z is often suppressed.

We assume that no tax refunds are paid to the firm when $X_i < X_i^Z$. It follows that operational cash flows, net of tax payments, are

$$X_i^n = X_i^+ - \tau_i(X_i - X_i^Z)^+ = \begin{cases} 0 & X_i < 0 \\ X_i & 0 < X_i < X_i^Z \\ X_i(1-\tau) + \tau X_i^Z & X_i > X_i^Z \end{cases} \tag{9}$$

Similarly to Merton (1974), default occurs when net operational cash flow at T is smaller than the face value of the debt:

$$X_i^n < P_i \tag{10}$$

Having defined the default threshold X_i^d as

$$X_i^d(P_i) = P_i + \frac{\tau_i}{1-\tau_i} D_{0i}(P_i) = \frac{P_i - \tau_i X_i^Z}{1-\tau_i} \tag{11}$$

the default triggering condition (10) can be written in terms of the pre tax cash flows as $X_i < X_i^d$. In the event of default, we assume that bondholders will receive a fraction $(1 - \alpha_i)$ of operational cash flow, X_i, when this is positive.They will however pay taxes out of this fraction, whenever operational cash flows are greater than the tax shield.

The level of debt determines both the probability of default, PRD_i, and the (undiscounted) expected loss. The latter can be computed as the difference between the full repayment, P_i, and the expected recovery, $D_{0i}(1 + r_T)$.The percentage expected loss is then

$$\frac{P_i - D_{0i}(1 + r_T)}{P_i}$$

By taking the ratio of each company expected loss to the corresponding default probability, we get the loss given default, and therefore the recovery rate

$$R = 1 - \frac{P_i - D_{0i}(1 + r_T)}{P_i \times PRD_i} \tag{12}$$

Last but not least, the endogenous spread y can be determined from the ratio between the face and present value of debt [2]:

$$y = (P_i/D_{0i})^{1/T} - 1 - r_T \tag{13}$$

The value of equity and debt is the expected present value of cash flows accruing to shareholders and lenders respectively, evaluated under the risk neutral measure. We assume that the leverage policy of the firm aims at maximizing ν_{0i}, the sum of equity and debt, which in turn pairs the after-tax asset value of the firm.

The cash flows accruing to debt and equity holders vary with organizational structure, which we analyze below.

2.1 Stand alone companies

We now assume that the two activities, $i = 1, 2$, are separately incorporated, as in Leland (2007). Thus the face value of debt in firm i maximizes the value of firm i:

$$\nu_{0i}(P_i) = E_{0i} + D_{0i} \tag{14}$$

The payoff to shareholders, E_i, is operational cash flow less taxes and the repayment of principal, when the difference is positive:

$$E_i(P_i) = (X_i^n - P_i)^+ \tag{15}$$

By no arbitrage the value of equity is simply

$$E_{0i}(P) = (1 + r_T)^{-1}\mathrm{E}(X_i^n - P_i)^+ \tag{16}$$

The payoff D_i to lenders at time $t = T$ will equal P_i when $X_i > X_i^d$ and the firm is solvent. Recalling that the government has priority for tax payments before lenders, the latter will absorb a tax liability $\tau_i(X_i - X_i^Z)$ in default when $X_i^Z < X_i < X_i^d$. The payoff to lenders is therefore

$$D_i(P_i) = \begin{cases} (1-\alpha_i)X_i & 0 < X_i < X_i^Z \\ (1-\alpha_i)X_i - \tau_i(X_i - X_i^Z) & X_i^Z < X_i < X_i^d \\ P_i & X_i > X_i^d \end{cases}$$

In figure 1 we represent such payoff for both stand alone companies. When cash flow is below (above) X_i^d for both units, there is joint default (survival). Otherwise there is selective default.

Insert here figure 1

Debt present value $D_{0i}(P_i)$, the value of zero-coupon debt given the principal P_i, can be written as

$$D_{0i}(P_i) =$$

$$(1 + r_T)^{-1}\mathrm{E}\left[\begin{array}{c} (1-\alpha_i)X_i \ \mathbf{1}_{\{0<X_i<X_i^Z\}} + \\ \left[(1-\alpha_i)X_i - \tau_i(X_i - X_i^Z)\right] \mathbf{1}_{\{X_i^Z<X_i<X_i^d\}} + \\ +P_i \ \mathbf{1}_{\{X_i>X_i^d\}} \end{array}\right] \tag{17}$$

[2] This is the spread over r which makes the principal P_i^* the compound amount of D_i^*, over the specified horizon T.

where $\mathbf{1}_{\{\bullet\}}$ is the usual indicator function.The value of debt is negatively affected by taxes and bankruptcy costs paid in default states, given P_i.This feeds back on both the spread in equation *(13)* and on equity value *(16)* primarily through its effect on net income *(7)*.

Note that (17) is an implicit equation, since X_i^Z and X_i^d are themselves function of D_{0i} through (8) and (11). Numerical methods are required for its solution. Since D_{0i} determines the thresholds and the latter enter the equity value, the solution approach for finding firm value ν_{0i} consists in finding a fixed point for D_{0i} and then determine X_i^Z, X_i^d and E_{0i}.

2.2 Groups

We now depart from Leland (2007) and analyze the case where the two activities are still separately incorporated, but one of the two - the parent company - transfers cash flows to the subsidiary in order to honour debt obligations when this allows the survival of both. This is consistent with the evidence in Dewaelheyns and Van Hulle (2006), who report that "private business groups support struggling subsidiaries [..]. However, once groups profitability turns negative, groups tend to terminate support to weak subsidiaries". It is also consistent with "liquidity smoothing" by Indian business groups (Khanna and Palepu, 2000; Khanna and Yafeh, 2005).

Following the legal literature, we assume that the parent company enjoys limited liability if the subsidiary defaults, being not responsible for the subsidiary's debt obligations. The initial owner is assumed to choose the face value of debt in the parent ($i = h$) and in the subsidiary ($i = s$) so as to maximize levered group ($i = g$) value:

$$\nu_{0g} = v_0(P_h, P_s) = E_{0h} + D_{0h} + (1 - \omega)E_{0s} + D_{0s} \tag{18}$$

where ω is the ownership share of the parent in the subsidiary: $\omega \in [0, 1]$. The maximization is subject to the state contingent payoffs which we now characterize. We first posit that the parent controls the subsidiary with an infinitesimal equity share: $\omega = 0$ [3]. We later remove this simplifying assumption, so as to study what happens when the parent receives dividends from its subsidiary.

2.2.1 Control without dividends

Let us denote with X_s and X_h the pretax operational cash flows of two units. A necessary condition for the transfer is that the subsidiary is in default and the parent is not. Moreover, the parent limited liability implies that there is no rescue if the operational cash flows of the subsidiary are negative, as the parent would otherwise bear an operational loss that it could have avoided:

$$\begin{cases} 0 < X_s < X_s^d, \\ X_h > X_h^d \end{cases} \tag{19}$$

where the default thresholds X_i^d, $i = h, s$ and the tax shield X_i^Z correspond to (11) and (8).

A condition ensuring that the transfer honours the subsidiary debt obligations is that the after-tax parent cash flow, net of debt repayment, exceeds the corresponding difference for the subsidiary:

[3] It is well known that separation of ownership from control is a possibility in business groups. A parent may directly control a subsidiary with 50% of its voting equity, and may indirectly control a second layers subsidiary with 25% and so on. This feature lies at the basis of several models of group behavior, such as Bebchuk, et al. (2000).

$$X_h^n - P_h > P_s - X_s^n \tag{20}$$

Overall, a state-contingent transfer will occur if and only if (19) and (20) hold:

$$\begin{cases} 0 < X_s < X_s^d, \\ X_h > X_h^d \\ X_h^n - P_h > P_s - X_s^n \end{cases} \tag{21}$$

In what follows, we denote the occurrence of these conditions as event A and the amount of the transfer as $(P_s - X_s^n)\mathbf{1}_{\{A\}}$.

The cash-flow accruing to shareholders of the parent company is equal to the stand alone one, $(X_h^n - P_h)^+$, less the transfer amount:

$$E_h(P_h, P_s) = (X_h^n - P_h)^+ - (P_s - X_s^n)\mathbf{1}_{\{A\}} \tag{22}$$

It follows that the equity value, which now depends on both principals P_h and P_s, is

$$E_{0h}(P_h, P_s) = (1 + r_T)^{-1}\mathrm{E}\left[(X_h^n - P_h)^+ - (P_s - X_s^n)\mathbf{1}_{\{A\}}\right] \tag{23}$$

The payoff to subsidiary lenders is the same as in the stand alone case, outside the states when a transfer takes place.[4] It must instead be augmented by the transfer in the transfer area, as shown in figure 2.

Insert here figure 2

Selective default of the subsidiary (i.e., default of the subsidiary and survival of the parent) takes place when its own cash flow falls below its default threshold, the parent cash flow exceeds its own default threshold but is not sufficient to support the subsidiary. Figure 2 depicts also the combinations of cash flow realizations leading to either joint default ($X_i < X_i^d, i = h, s$), or selective default of the parent, or joint survival without rescue. Comparison with figure 1 allows to appreciate the potentially positive effect of group structure on the subsidiary credit quality keeping the threshold fixed: the area of selective default shrinks as a consequence of the parent transfer.

Since debt is the present expected value of these final payoffs, it becomes:

$$D_{0s}(P_s, P_h) =$$

$$(1 + r_T)^{-1}\,\mathrm{E}\left[\begin{array}{c} X_s(1-\alpha)\mathbf{1}_{\{B\}} + \\ + \left[X_s(1-\alpha) - \tau(X_s - X_s^Z)\right]\mathbf{1}_{\{C\}} + \\ + P_s\left[\mathbf{1}_{\{A\}} + \mathbf{1}_{\{X_s > X_s^d\}}\right] \end{array}\right] \tag{24}$$

where the event B occurs when there is no transfer and the subsidiary tax shield is exploited $(0 < X_s < X_s^Z)$, while C occurs when there is no transfer and taxes are paid: $X_s^Z < X_s < X_s^d$.In both cases we have selective default of the subsidiary.[5]

As in the stand alone case, a fixed point of the debt function determines its value. The reader can notice that it depends on the principals of both subsidiary and parent companies, since the transfer, tax shields and default thresholds do.

[4] We are assuming that there is no consolidation of assets in the event of default - which seems consistent with what happens in most real-world cases (Samson, 2001).

[5] For the sake of simplicity, we assume that the tax rate and default costs do not differ across the two units. Therefore, $\alpha_i = \alpha$ and $\tau_i = \tau$. This assumption is removed in the numerical analysis of asymmetric cases.

Thus both E_{0h} and D_{0s} depend on principals P_h and P_s, which must be simultaneously chosen.

The payoffs to lenders of the parent do not change with respect to the stand alone case, as the transfer to the subsidiary occurs only after the service of the parent debt. Similarly, equity holders of the subsidiary are unaffected, as the transfer occurs for the sake of servicing debt. As a consequence equations (17) and (15) still hold for $i = h$ and $i = s$ respectively, and $D_{0h} = D_{0h}(P_h), E_{0s} = E_{0s}(P_s)$.

It should be noted that nothing prevents, so far, the switch of the labels "subsidiary" and "parent". In other words, rescue goes in one direction only - from one company to the other - but the two companies are otherwise symmetric. In the next section we instead allow only the parent to receive dividends.[6]

2.2.2 Control with dividend flows

We now consider the general case of non-zero dividend flows from the subsidiary to the parent, $0 \leq \omega \leq 1$. Dividends are another type of state- contingent transfer: they are not distributed when the subsidiary is in default and are proportional to its profit after interest and taxes otherwise. Thus, cash flows received by stakeholders do not change as long as the subsidiary defaults, namely when $X_s < X_s^d$, or, equivalently, $X_s^n < P_s$. In the opposite case, the parent cash flows include both operational earnings and dividends.

If we exclude double taxation of subsidiary income, the cash flows of the parent are

$$X_h^n + \omega(X_s^n - P_s)^+ \tag{25}$$

These cash flows are negative when the parent has operational losses and either no dividend is distributed or dividends are smaller than losses:

$X_h < -\omega \left[X_s - \tau(X_s - X_s^Z) - P_s\right]$. Positive cash flows first repay debt, up to its face value P_h, by absolute priority, then the equity holders.

The payoff to lenders, which by a fixed point argument determines its current value, is then equal to:

$$\begin{cases} 0 & X_h^n + \omega(X_s^n - P_s)^+ < 0 \\ (1-\alpha)\left[X_h^n + \omega(X_s^n - P_s)^+\right] & 0 < X_h^n + \omega(X_s^n - P_s)^+ < P_h \\ P_h & X_h^n + \omega(X_s^n - P_s)^+ > P_h \end{cases} \tag{26}$$

It follows from the previous expression that X_h^d, the parent default threshold with infinitesimal ownership, remains the default threshold under the new ownership structure only until dividends are not received. When the subsidiary pays out dividends, the default threshold is the level of operational cash flows, net of taxes but gross of dividends, that equals P_h: this new default threshold then depends on the subsidiary cash flow X_s. It can be shown to be greater than the parent tax shield, and therefore to entail tax payments, as long as $X_s < X_s^\circ$. The latter is a known function of the default thresholds, the parent tax shield and the ownership share ω. We visualize the default and non default events in figure 3.

Insert here figure 3

The subsidiary may now rescue its parent whenever its dividends cover the parent operational losses. As long as no dividends are paid ($X_s < X_s^d$), the occurrence of

[6] While nothing logically prevents the subsidiary from receiving dividends, cross-holdings are often prohibited in the real world.

joint and selective default do not differ across figures 2 and 3. [7] The subsidiary dividends are able to rescue the parent from default when they are "large enough", namely if $X_h < X_h^d$ and $\omega\left(X_s^n - P_s\right) > -\left(X_h^n - P_h\right)$. This happens in the zone which we label "dividend rescue". The boundaries of this zone are again a known function of the levels X_i^Z,X_i^d, and of the principals P_s, P_h.

The equity and debt value in the parent obtain by discounting the expectation of cash flows to shareholders and lenders, respectively. The problem is complicated by the fact that now they both depend also on the face value of the subsidiary debt, i.e. $D_{0h} = D_{0h}(P_h, P_s), E_{0h} = E_{0h}(P_h, P_s)$. As a matter of fact parent equity holders receive no dividends when the subsidiary is in default, an occurrence that depend on the face value of the subsidiary debt. When the subsidiary pays out dividends, parent equityholders have right to them once any operational loss is offset and parent lenders are reimbursed, an occurrence which depends on the parent face value of debt.

The value of subsidiary debt is unaffected by dividend payment, and remains equal to $D_{0s} = D_{0s}(P_s, P_h)$. The value of total equity, $E_{0s} = E_{0s}(P_s)$,is also unchanged. They can be represented respectively as in (24) and as a fraction $(1-\omega)$ of (15).

Given the above payoffs, parent and subsidiary principals are chosen so as to maximize group value[8]

$$(P_h^*, P_s^*) = \arg\max v_{0g} \tag{27}$$

Once the optimal principals are determined, the current values of both debt, D_{0h}^*, D_{0s}^*, and the equity, E_{0h}^*, E_{0s}^*, can be computed. The optimal tax shields (X_i^{Z*}) and default thresholds (X_i^{d*}) follow.

The probabilities corresponding to the different events in figures 2 and 3 can be computed once the optimal tax shields, default thresholds and principal values are known. In what follows we will denote the selective default probabilities as $PRDS_i, i = h, s$, the joint default probability as $PRDJ$ and the marginal default probabilities $(PRDS_i + PRDJ)$ as $PRD_i, i = h, s$. We will also be interested in the rescue probability, PRR, and in the joint survival (with or without rescue), $PRND$. Finally, we will obtain recovery rates and spreads over Treasury.

2.3 Conglomerates

The conglomerate merger case - introduced in Leland (2007) - obtains when the two activities are either incorporated as one company, or when a parent company deliberately becomes legally responsible for its subsidiary debt obligations. Its cash flow X_m is the sum of the cash flows of the original activities:

$$X_m = X_1 + X_2 \tag{28}$$

[7] The optimal threshold levels (X_i^{Z*},X_i^{d*}, the rescue/no rescue lines) do differ, since firm values - the parent equity in particular- do.

[8] There is no closed formula for the group value, even with infinitesimal ownership. As evident in the text, at least the current value of debt and the thresholds (tax shield and default) of each name depend on the principals. We study the maximization problem numerically, in a base case - whose parameter values are drawn from Leland (2007) - and under some alternative parameter combinations. In the base case without dividends group value is concave in the subsidiary principal, for given holding principal, and decreasing in the latter, for given subsidiary principal. In the alternative cases too we explored the monotonicity properties of the group value in order to present the global maximum and not a local one.

The unique choice variable is the face value of debt, P_m, which maximizes [9]

$$\nu_{0m} = \nu_0(P_m) = E_0(P_m) + D_0(P_m) \tag{29}$$

where $E_0(P_m)$ and $D_0(P_m)$ are computed according to (15) and (17) with $i = m$. Debt is again a fixed point, while X_m^Z and X_m^d are defined as in (8) and (11).

After having selected the optimal debt value for the merger, $P_m^* = \arg\max \nu_{0m}$, one obtains via (15) and (17) the current value of optimized debt and equity, $E_0(P_m^*)$ and $D_0(P_m^*)$. The optimal leverage ratio follows, together with the optimal tax shield and default threshold, X_m^{Z*} and X_m^{d*}.

In the conglomerate case the probability of selective default is zero. Thus the probability of joint default $PRDJ$ coincides with the default probability of the merger, PRD_m. The recovery rate and spread are determined using (12) and (13) in the common set up.

3 Credit risk: a base case

We will numerically study the credit risk implications of the organizational structures outlined above assuming that - for each company - annual cash flows are Normal i.i.d.

We start from a so called base case, whose parameters are borrowed from Leland (2007), in which companies have identically - although not independently - distributed cash flows. We will refer to them as being symmetric. The parameters which characterize the symmetric case are calibrated to those of firms issuing BBB-rated unsecured debt. In particular, the debt maturity is assumed to be five years, consistent with investment grade evidence. Given an annual riskless rate interest rate of 5%, expected operational cash flow for each activity, $Mu = 127.6$, is chosen such that its present value is $X_0 = 100$. Operational cash flow at the end of 5 years has a standard deviation (Std) of 49.2. Given that annual cash flows are independent in time, this is consistent with an annual standard deviation of 22.0 ($= 49.2/\sqrt{5}$). Henceforth we express volatility σ as an annual percent of initial activity value X_0, e.g. $\sigma = 22\%$. The tax rate $\tau = 20\%$ and the default cost parameter $\alpha = 23\%$ are chosen so as to generate optimal leverage and recovery rates consistent with the BBB choice (see Leland (2007)).

Insert here table 1

We obtain the optimal capital structure and credit risk indicators of table 2 when the correlation coefficient between the units cash flows is equal to 0.2, as in Leland (2007). The first column reports values for a stand alone. The second and the third refer to parent and subsidiary respectively. The fourth refers to the overall group, while the last column to a conglomerate .

Insert here table 2

Let us analyze the stand alone versus group situation (columns one to four) first. The group versus conglomerate comparison (columns four and five) will follow.

[9] In the conglomerate case -as well as in the stand alone - the firm value is not a monotonic function of the principal value of debt. However, in this case too we will present the global maximum and not a local one.

3.1 Parent and subsidiary versus stand alone

The first, important feature of table 2 is that the overall group debt is on the subsidiary's shoulders. Given the possibility of supporting an insolvent subsidiary, the best way to solve the bankrupcty costs versus tax savings trade off is to raise capital via the subsidiary. The latter can indeed be saved from bankruptcy, exploiting diversification and avoiding the proportional loss of value inherent in it. It can also be left alone when transferring money to it would deplete the overall group value without avoiding bankruptcy costs. [10]

The resulting face value of debt for the subsidiary is higher than that of two stand alone companies (219 versus 114.4). This characteristic of the optimal solution is consistent with the empirical evidence in Dewaelheyns and Van Hulle (2006), who notice that the "decreased potential costs of financial distress allow group members to ex ante take on more debt, thus realizing more tax gains".

Such a high debt produces a considerable increase the no tax profit threshold in the subsidiary (102.32 versus 14.98), and an associated increase in asset value with respect to that of one stand alone (116.71 versus 81.23). The overall group value (165.91) is higher than twice that of a stand alone (162.46), even though the parent value - which does not raise debt and stands ready to rescue its subsidiary - falls with respect to the stand alone situation (from 81.23 to 49.2). Thus parent-subsidiary links that preserve limited liability while allowing for state-contingent support create value.

Leverage impacts on the marginal default probabilities. These go from $PRD_1 = PRD_2 = 11.2\%$ for each stand alone to $PRD_h = 0.34\%$ for the parent and to $PRD_s = 46.54\%$ for its subsidiary. Indeed, the parent defaults only when its cash flows turn out to be negative, therefore in fewer cases than a leveraged stand alone. The subsidiary defaults whenever it cannot pay back its loans and the parents does not find possible to provide support. Since it is more leveraged than a stand alone, its default probability is higher in spite of rescue. However, in the absence of rescue, its default probability would be even larger - actually close to one (99.3%) - due to extreme leverage.

The recovery rate falls from 48.1% for the stand alone to 31.2% for the subsidiary. Indeed, the parent is more likely to be unable to support its subsidiary when the latter losses are larger - leaving these cases to lenders.

As a consequence of high default probability and low recovery, the credit spread dramatically increases for the subsidiary with respect to the stand alone (8.4 versus 1.26% over five years).

The previous figures are better understood if we consider not only the marginal default probabilities, but also the selective, rescue and joint default probabilities. Under the group organization, selective default of the subsidiary occurs in $PRDS_s = 46.2\%$ of the possible cases, while rescue occurs in 52.35% of the occurrences. These are the two most likely scenarios: either the subsidiary defaults, because of the leverage ratio it has been charged with together with inability of the parent to rescue her, or it is indeed rescued. Joint default, given that the parent is unlevered, is very rare (0.34%).

[10] Since a symmetric manoeuvre is not possible, the subsidiary is leveraged while the parent is not. However all debt would be borne by the parent in a group arrangement where rescue transfers are possible from the subsidiary to the parent only. Indeed, in the case under analysis there is no difference between the companies - other than the direction of the rescue possibility and the label.

3.2 Group versus conglomerate

It is well known that divisions of a conglomerate diversify away some risk provided that their operational cash flows are less than perfectly correlated (Lewellen, 1971). This justifies the findings , that the conglomerate raises more debt than the two stand alones (Leland, 2007). Because of risk sharing, the higher tax advantages induced by higher debt are not completely offset by higher expected default costs, and such a leverage policy creates a merge value ν_m^* greater than twice the optimal value of two stand alones: merging is profitable. These results can be visualized by comparing the first with the last column of Table 2: for the conglomerate, the overall debt (117.4) and company value (163.15) are greater than for two stand alone companies (114.4 and 162.46 respectively).

A conglomerate is more levered than two stand alone units because its debt is issued against a diversified portfolio of assets. Its default probability is higher (6.5% instead of 2%, the joint default probability of two stand alone units) since one activity can drag the other, profitable one into default (Sarig, 1985). For the same reason, this is accompanied by higher recovery (56.5 instead of 48.1%). The increase in recovery outweighs that in default probability, thus reducing the spread (0.6% instead of 1.26%)

We now turn to the comparison between a conglomerate and a group. Our model shows that group debt capacity is greater than for conglomerates (219 versus 117.4 in terms of face value). The novel reason is that the group is able to implement state-dependent rescue, as opposed to the state independent one inherent in cash flow pooling of mergers[11]. As a consequence, the value of a group, $\nu_{0g}^* = 165.91$, exceeds that of a conglomerate, $\nu_{0m}^* = 163.15$. Since both have an internal capital market, the value increase is not due to diversification, but to the specificities of the group - namely limited liability and separate incorporation. These characteristics allow to tailor capital structure so as to increase the no tax profit level (to 102.32 from 14) and halve bankruptcy costs. Indeed, the probability of joint default falls to 0.34% from 6.5% even if the optimal leverage ratio is greater for groups than for conglomerates: 70% for the former and 54.8% for the latter.

Subsidiary lenders however face a much higher default probability (46.54% instead of 6.5% for the conglomerate) as well as lower recovery (31.2% versus 56.5). Consequently, the spread is much higher for them (8.4% instead of 0.6% over five years).

To sum up, parent-subsidiary relationships in groups are value enhancing with respect to mergers, even tough they considerably deplete the credit worthiness of subsidiaries. The latter have lower recovery, greater default probabilities and far higher spreads than mergers.

3.3 Group with dividend flows

When the parent not only exerts control, but also receives 100% of the subsidiary dividends ($\omega = 1$), the whole debt should be again borne by the subsidiary. The overall capital structure and credit risk implications of the model are invariant- as apparent from Table 3. Therefore the amount of expected dividends and the associated increase in the parent company equity value are negligible (0.037 when $\rho = .2$), given that the subsidiary is highly leveraged.

Insert here table 3

[11] It is also able to exploit the asymmetry of taxation, namely the different elasticity of the tax shield and of the default threshold with respect to leverage. Tax asymmetry is studied in a companion paper, Luciano and Nicodano (2007).

The lesson we draw is that the internal capital market, and more specifically state-contingent transfers targeted to rescue, determine the optimal capital structure. Non targeted ones, as dividends in our model, are not crucial as they do not affect the trade-off between the tax-shield and bankruptcy costs.

4 Credit risk as correlation changes

It is evident that correlation should play a role in the results obtained so far. Exploring Leland's model, we get the perhaps unsurprising result that the merger should raise as much debt as two stand alone units when correlation equals 1. This is because the distictive characteristic of conglomerates is diversification.

For the same reason, one may expect that the optimal face values of debt in groups will converge to the stand alone level as correlation among cash flow increases, since the transfers from the parent to the subsidiary will become less likely. This intuition is incorrect: debt in the parent continues to be zero, because this still allows to eliminate the parent bankruptcy costs. The tax shelter differential between raising debt in the parent and raising it in the subsidiary is evidently not strong enough, to move debt from the subsidiary to the parent - i.e. from the company which can be rescued to the other one. Figure 4, top left corner, reports the optimal leverage ratio for the three types of organizations. As correlation increases, the effect of diversification vanishes and the optimal leverage of a conglomerate converges to that of a stand alone as discussed above. On the contrary, the one of groups falls from over 80% to less than 70%, a figure which remains 30% higher than leverage of conglomerates and stand alones. As a consequence, the value differential with respect to the stand alone situation is increasing for groups, decreasing for the merger, since in the latter case the lack of diversification when correlation increases is not counterbalanced by the ability to raise the tax shield. This is shown in figure 4, top right corner.

The bottom plots of figure 4 represent the recovery rate (left) and spread (right): apart from the fact that the inequalities across organizational forms hold throughout, we notice that the spread sensitivity to correlation are higher in the group case.

Insert here figure 4

We can summarize the results from figure 4 as follows:

Proposition 1 *Assume positive bankruptcy costs, fiscal deductibility of interest and the ability of parent company to commit to state contingent transfers to its subsidiary. Then the leverage of a group, as well as the resulting firm value, exceeds the one of conglomerates and of the corresponding BBB stand alone companies. The recovery rate is lower, while the spread over Treasuries is larger than in other organizations.*

In the top panel of the figure 5 we represent the marginal default probability of both a stand alone and a subsidiary. At a correlation equal to -0.8, the default probabilities are equal, despite the much higher debt in the subsidiary. This is clearly due to the high support probability by the parent. For comparison, we also add to the picture the default probability of a stand alone with the optimal subsidiary leverage: this is close to 100% for all correlation coefficients, as the amazing leverage of the subsidiary becomes unsustainable when no rescue through an internal capital market takes place.

In the bottom panel of figure 5, we add to the merger and group joint probabilities of default the chances for two stand alone companies to default together, evaluated at their own optimal capital structure. They would default more often than the group, even if the latter is more leveraged, given that they cannot support each other. At the same time they would default less than the merger, since they would be far less leveraged and none of them would drag the other into default.

Insert here figure 5

The interplay of recovery rates, default probability, leverage and spreads deserves special attention. As correlation increases, nothing changes in the stand alone case as there are no links between firms. The activities in the merger case diversify less and their default probability increases. As a consequence, the trade off between higher expected bankruptcy costs and tax considerations induces the optimal leverage of the merger to fall, leading however to a reduced recovery rate together with mildly increasing spreads. In a group, as diversification vanishes, the chances that the parent will rescue its subsidiary also fall. One may expect the subsidiary face level of debt to similarly fall, but this is not the case. Indeed, for given face value of debt, the recovery rate increases in the subsidiary, implying a reduction in expected default costs. Thus the trade off between lower expected bankruptcy costs and tax considerations induces the optimal face value of debt to increase. At the optimal capital structure, the recovery rate is constant in correlation. With subsidiary risk neutral default probability increasing from below 0.2 to more than 0.5, it is unsurprising that the credit spread rises from below 2% to more than 10%.

4.1 Implicit ratings

The probabilities we have presented so far are risk neutral ones, as needed for recovery and spread computations. Given the structural nature of the underlying model it is however possible to compute the corresponding historical probabilities. For given default and tax shield thresholds, the switch from risk neutral to historical probabilities is obtained, by Girsanov theorem, by changing the asset drift. Following Bhandari (1988), we choose an asset risk premium of 4%, independently of the rating. This is consistent with an asset beta of approximately 60%, and an equity risk premium of 6%.The following table presents the results, for different correlation levels and organizational forms. For groups, we consider the control without dividends case.

Insert here table 4

Let us compare the entries of the previous tables with the five-year average default probabilities over the period 83-99, as resulting from Moody's statistics (see Falkenstein et al., 2000). The comparison allows us to assign an implicit rating. A proviso is in order: since the assignment is based on average values, it could be improved by considering the so called floors of Moody's, which at the moment we do not have. As it is, our rating assignment is generous and overstates credit worthiness.

We first recognize that a Standard& Poor BBB firm - our base case - corresponds to a Baa3 rating in Moody's metric. Analyzing the other entries, we get the following tentative conclusions.

- Two companies, that are separately rated Baa3, can form a group which - via an appropriate leverage choice - is almost riskless, and therefore deserves a Aaa rating. This is the result of reducing the credit quality of the subsidiary and improving the parent one. The subsidiary itself, depending on correlation, would deserve from Aa1 (.35% default probability in Moody's sample) to Caa (29%).

- Alternatively, if the companies merge and still optimize their capital structure, they may deserve from Aaa to Baa3 (3.28% default probability according to Moody's).
- A stand alone with the subsidiary leverage would default with probability greater than 90% over five years.

The previous statements are the results of a simple mapping from historical default probabilities into a rating. Such automatic mapping - based on observable parametrs such as cash flow correlation - may improve with respect to the qualitative or econometric methodology used by rating agencies in order to assign a credit standing to group affiliated companies.

5 Extensions

5.1 The case of a constrained subsidiary

The leverage structure of a group described so far can lead to even higher spreads if risk shifting is a possibility that lenders cannot monitor (Bianco and Nicodano, 2006). It may thus pay to reduce subsidiary leverage. There may also be regulatory constraints. Sometimes some subsidiaries shareholders do not have any stake in the holding. This situation may for instance occurs when only subsidiaries are listed on public exchanges. Several jurisdictions impose to subsidiary managers to act in the interest of the subsidiary shareholders, rather than implement what is optimal for the group (Hadden, 1996). In this circumstance, a leverage close to 100% and a value of equity close to zero in the subsidiary can easily be considered as a violation of the rule. We therefore investigate the effects of this policy by imposing a subsidiary debt level equal to the stand alone one (57.2), and optimize leverage in the holding.

Insert here table 5

Table 5 shows that the optimal debt in the parent ompany has a face value which is closer to the subsidiary one, and - as a consequence - the difference in the value of debt and equity is lower than in the unconstrained case. Let us focus for the moment on the case $\rho = 0.2$.The relative leverage of parent and subsidiaries (0.50 and 0.53) is now closer to the one observed in Belgian and Italian groups, where the former tends to exceed the latter (Bianco and Nicodano, 2006; Dewaelheyns and Van Hulle, 2007). Similarly, the implied optimal group leverage (0.51) is close to the stand alone level of Table 2 - and closer to evidence showing the group leverage tends to be smaller than stand alone leverage (Deloof and Verschueren, 2001; Dewaelheyns and Van Hulle, 2007).

Despite the similar debt burden, the parent recovery rate is much higher than that of a subsidiary, as it does not receive any support (0.46 versus 0.24). Similarly, its marginal default probability is higher (0.096 versus 0.036). The joint default probability reaches 2%, versus 0.34% in the unconstrained case. Conversely, the occurrence of selective defaults reduces to 0.096%, down from 0.46 in the unconstraind case. Although we do not calibrate observed historical probabilities at this stage, we observe that this rise in joint default occurrences for lower rated groups is also found in the data (Emery and Cantor, 2005).

Finally, the probability of selective default of the parent (7.8%) now by far exceeds that of the subsidiary (1.8%)- which has similar leverage but receives support from its parent.

Group value still exceeds that of stand alone firms, but the differential - obviously - shrinks as expected default costs for the parent are now positive. The Table also presents the recovery, marginal default probabilities and spreads for the group affiliated and for the corresponding stand alone units. All the endogenous credit evaluations for the subsidiary differ from those of a stand alone, despite their common face value of debt. In particular, its default probability is much smaller, since the holding can support it. Consequently the spread it deserves is also smaller even if its recovery continues to unfavourably compare to that of the stand alone. The probability of joint default for two stand alones now exceeds that for the group (0.02 versus 0.0181).

As far as correlation is concerned, we only observe that group debt decreases as diversification opportunities vanish, contrary to what happens in the unconstrained case. In fact, debt in the subsidiary cannot increase in order to counteract the rising recovery rate associated with reduced support - as was happening in the unconstrained case.

The following proposition summarizes these results:

Proposition 2 *Consider a subsidiary with face value of debt equal to that of a BBB stand alone company. Assume positive bankruptcy costs, fiscal deductibility of interests and the ability of the holding company to commit to state contingent transfers to its subsidiary. Then the default probability of and the spread charged to the subsidiary are lower than the stand alone ones.*

5.2 Asymmetric companies

In this section we consider the leverage and credit risk outcomes for non identically distributed activities. In particular, we will in turn analyze the cases of lower default costs (Table 6), higher volatility (Table 7) and smaller size (Table 8) for the subsidiary. These cases have been shown to be value enhancing with respect to the symmetric ones. That is, the expected value of the group would be lower if the subsidiary were less volatile, costlier and larger than its holding company, at their optimal unconstrained capital structure (see Luciano and Nicodano, 2007).

Insert here table 6

Higher default costs in the holding do not change the type of optimal capital structure, in the sense that the whole debt burden is still borne by the subsidiary only. Even if they are as high as 75%, exactly the same face value of debt obtains as in the symmetric case above. Setting to zero the holding company leverage is *a fortiori* optimal with larger default costs. These will never be incurred in, and hence cannot affect credit quality or spreads or value. Comparison across Table 6 and Table 3 reveals that both stand alones and conglomerates suffer from the increase in bankruptcy costs of one unit. This indicates that the capital structure flexibility allowed by group structure can be especially valuable when there are asymmetric bankruptcy costs.

Insert here table 7

A riskier subsidiary faces a reduced probability to both independently survive and to be rescued by the holding, for given leverage. As a consequence, the subsidiary turns out to have a slightly lower leverage. Consider the benchmark cash flow correlation of 0.2, and volatility equal to 44% and 22% for the subsidiary and the holding respectively. The optimal subsidiary leverage is 97.3% instead of 100%. Its default probability increases (48.7% instead of 46.5%); the corresponding recovery and spread are 20.2% and 10.9%, which are respectively smaller and higher than in the symmetric case (31.2 and 8.4)[12].

When risk increases, the stand alone spread jumps to 6.2% from 1.26. Thus the credit quality of a group appears to be less sensitive to highly volatile cash flows in one of its units. In conglomerates diversification opportunities help, but the spread still more than doubles (from 0.6% to 2%). This indicates that the capital structure flexibility in groups can help maintaining credit quality in situations of asymmetric risk.

Insert here table 8

Size asymmetry makes it profitable to shift some debt onto the holding. Let us explore the case in which the holding is five times as large as its subsidiary, for the $\rho = 0.2$ case. The unlevered parent asset value at time 0 in therefore 167, while the subsidiary one is equal to 37. The holding leverage ratio rises from zero to 51%, while that of the subsidiary is unchanged relative to the symmetric case. The holding cash flow is comparatively large enough to be able to rescue its subsidiary despite its positive debt committment, which reduce the tax burden. The credit quality of the holding drops, as its default probability increases to 0.04 from 0.34% in the symmetric case. However, its selective default probability is still zero.

Overall, we may conclude that the insights obtained in the symmetric case are robust to parametric changes.

6 Summary and concluding remarks.

Our paper contributes to the literature on credit risk by showing how state-contingent support by a parent company modifies optimal capital structure and the associated default probability in affiliated companies. Optimal capital structure entails a highly-leveraged subsidiary with a face value of debt that is almost four times the stand-alone one and the one of a conglomerate division. By contrast, dividends from the subsidiary to the parent company hardly affect optimal leverage and credit quality, because they leave the tax-bankruptcy cost trade-off unchanged.

Discretionary support by the parent company implies the possibility of a sudden jump in the subsidiary asset value, precisely when the subsidiary is close to insolvency. Recent empirical work shows that structural models with asset value jumps perform much better than diffusive models (see for instance Huang and Zhou, 2007)[13]. Given the pervasiveness of parent-subsidiary links in the form of joint ventures, multinationals and business groups, our model provides a rationale the presence of jumps in asset value.

[12] Note that, also in the group case, we find that as default probability grows for higher volatility the recovery rate falls. This is a stylized fact in the literature on credit risk (Altman et al., 2004).

[13] Indeed, strategic rescue is observationally equivalent to a positive asset jump. The jumps in Huang and Zhou (2007) can be either positive or negative: however, the jump model which Huang and Huang (2003) found to be of best fit on bond yield data had on average positive jumps.

Our theory also offers insights into how parent-subsidiary links affect default probabilities of the affiliated units. A given face value of debt is associated with an annual yield spread of 1.3% in a stand-alone as compared with a spread of 0.6% in a subsidiary. This rationalizes the discriminating assignments of ratings by most agencies when a parent subsidiary link is in place. They also explain why the prediction of default frequency conditional on firm debt improves when the credit standing of the other affiliated units is taken into account (Dewaelheyns and Van Hulle, 2006). Last but not least, we are able to identify some characteristics that should be related to selective defaults. These are the correlation between operating cash-flows, the size of the affiliate relative to the group and the relative risk of its operations.

Our model predicts zero optimal leverage for supporting companies. We are thus suggesting one potential explanation for the presence of a large proportion of zero debt firms in the Compustat database (Strebulaev and Yang, 2006). The implied optimal capital structure of group firms may however be considered at odds with the less extreme leverage of many companies.

While both the constrained and asymmetric size case are already consistent with less extreme leverage structures, our model can be further extended to consider empirically relevant features that we overlooked. Thus, it is possible to include other fiscal parameters, such as double-taxation of intercorporate dividends, differential tax rates between affiliated firms and/or tax loss offset provisions. More importantly, it is possible to allow for a purely financial holding company that gathers dividends from its controlled subsidiaries, may raise external financing and lends to them through internal loans. We conjecture that this modification will be able to better match some empirical observations that point to groups having lower bank debt but higher internal debt than their stand alone counterparts (Deloof and Verschueren, 2001; Dewaelheyns and Van Hulle, 2007), as well as a much higher joint default probability (Emery and Cantor, 2005) than the one predicted by the current set up.

7 References

Altman, E., Resti, A., and A. Sironi, (2004) Default Recovery Rates in Credit Risk Modeling: A Review of the Literature and Empirical Evidence, *Economic Notes,* 33(2), 183-208

Bhandari, C., (1988), Debt Equity Ratio and Expected Common Stock Returns: Empirical Evidence, *Journal of Finance,* 43, 507-528

Bianco, M. and G. Nicodano, (2006), Business Groups and Debt, *European Economic Review*

Blumberg Phillip I., (1989), The Law of Corporate Groups, Boston: Little, Brown and Company

Boot, A.W.A., S.I. Greenbaum and A.V. Thakor, (1993), Reputation and Discretion in Financial Contracting, *The American Economic Review*, 83(5), 1165-1183

Cestone, G., and C. Fumagalli, (2005), The Strategic Impact of Resource Flexibility in Business Groups, *Rand Journal of Economics*, 36, 193-214

Deloof, M., and I. Verschueren, (2001), How Does Intragroup Financing Affect Leverage? Belgian Evidence, Journal of Accounting Auditing and Finance, 83-108

Dewaelheyns N. and C. Van Hulle, (2007), Internal Capital Markets and Capital Structure: Bank versus Internal Debt, Katholieke Universiteit Leuven, wp 0703

Dewaelheyns N. and C. Van Hulle, (2006), Corporate Failure Prediction Modeling: Distorted by Business Groups' Internal Capital Markets?, *Journal of Business*

Finance and Accounting, 33(5)(6), 909-931

Emery K.M., and R. Cantor, (2005), Default Correlation Among Non-Financial Corporate Affiliates, *The Journal of Fixed Income*, 87-96

Falkenstein E., A. Boral, and L.V.Carty, (2000) RiskCalc for Private Companies: Moody's Default Model, Moody's Investors Service

Giesecke, K., 2004, Correlated Default with Incomplete information, *Journal of Banking and Finance*, 28, 1521-1545

Gopalan R, Nanda V. and A. Seru, (2004), Bankruptcy and Business Groups, W.P. ASU

Hadden T., Liabilities in Corporate Groups, in Rossi Guido et al. eds., (1996), I Gruppi di Società, Proceedings of the International Conference on Business Groups, Venice, Milano: Giuffrè, 1361-1388

Huang J., and M., Huang, (2003), How much of the Corporate-Treasury Yield Spread is due to Credit Risk?, Penn State and Stanford University.

Huang J., and H., Zhou H, (2007), Specification Analysis of Structural Credit Risk Models, Working Paper, Penn State University and Moody's 2007 Credit Risk Conference

Huizinga H., Laeven L. and G. Nicodème, (2007), Capital Structure and International Debt Shifting, IMF

Khanna, T, and K. Palepu, (2000), Is Group Membership Profitable in Emerging Markets? An Analysis of Diversified Indian Business Groups, *Journal of Finance*, 55, 867-891

Khanna, T., and Y. Yafeh, (2006), Business Groups in Emerging Markets: Paragons or Parasites?, ECGI Working Paper.

Lamont, O.A. and R. H. Thaler, (2003), Can the market add and subtract? Mispricing in tech stock carve-outs, *Journal of Political Economy*, 111, 227-268

Leland, H., (2007), Purely financial sinergies and the optimal scope of the firm: implications for mergers, spin offs, and structured finance, *Journal of Finance*, 62, 765-807

Lewellen, W., (1971), A Pure Financial Rationale for the Conglomerate Mergers, *Journal of Finance*, 26, 521-537

Li L.X., (2000), On default correlation: a copula function approach, *Journal of Fixed Income*, 43-54.

Luciano, E., and G., Nicodano, (2007), Firm Scope, Debt, and Value: A Structural Model of Business Groups, Collegio Carlo Alberto and University of Turin

Merton, R.C., (1974), On the pricing of corporate debt: The risk structure of interest rates, *Journal of Finance*, 449-70

Samson B., (2001), Corporate Ratings Criteria, Standard & Poor's

Sarig, O., (1985), On mergers, divestments, and options: a note, *Journal of Financial and Quantitative Analysis*, 385-89

Strebulaev, I. A., and B.Yang, (2006), The Mystery of Zero-Leverage Firms, GSB Stanford University

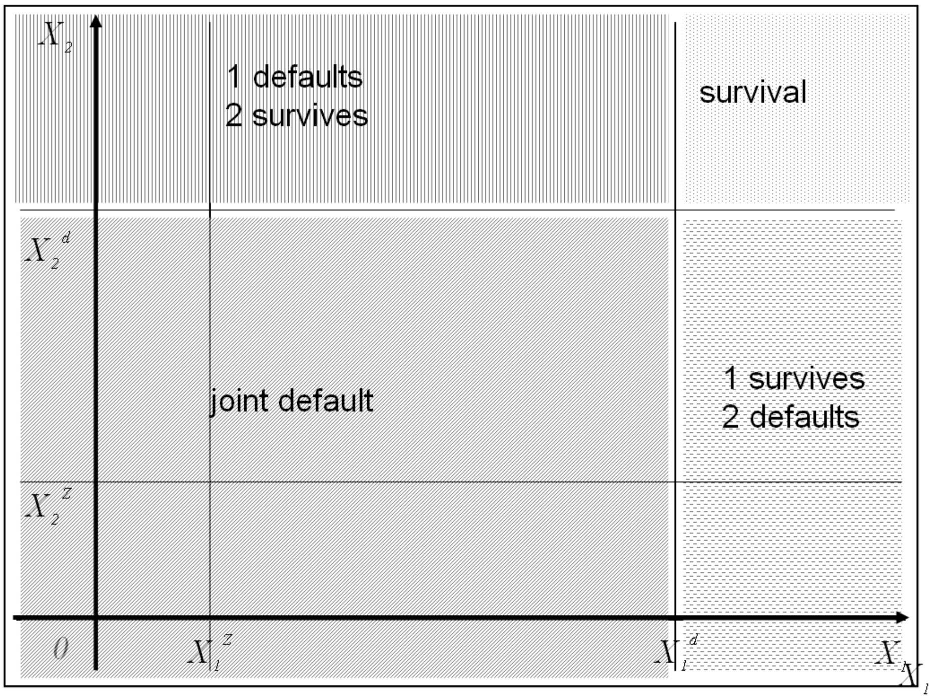

Figure 1: This figure displays combinations of cash flows such that either joint default or selective defaults or survival obtain. The cash flows of the stand alone firms 1 and 2 are on the horizontal and the vertical axis, respectively.

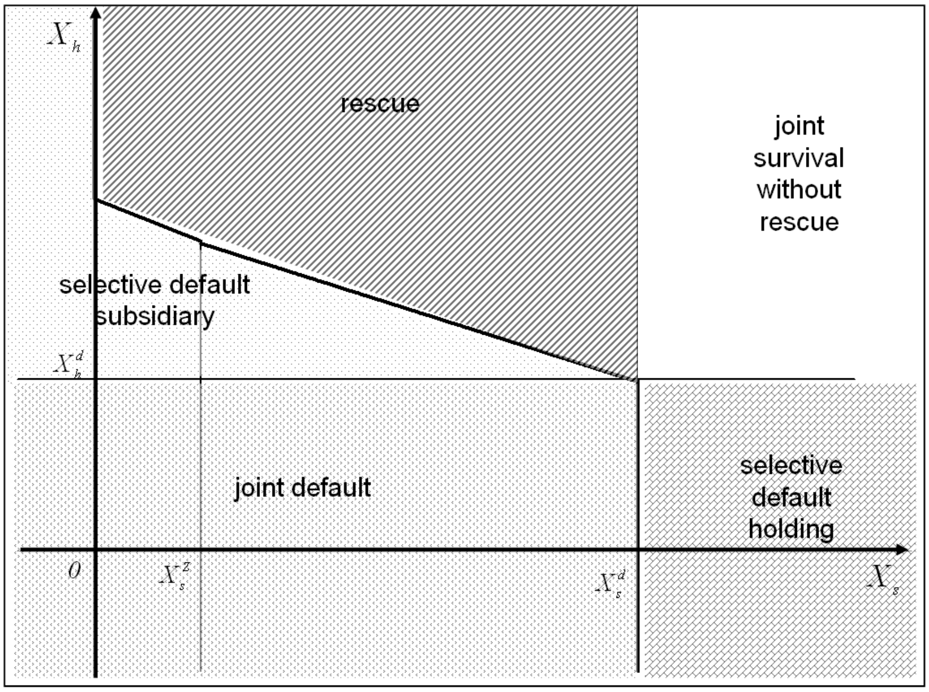

Figure 2: This figure shows the cash flows combinations ensuring the rescue of an insolvent subsidiary, as well as the areas of joint or selective defaults. It represents the cash flow of the subsidiary on the horizontal axis and of the holding on the vertical axis.

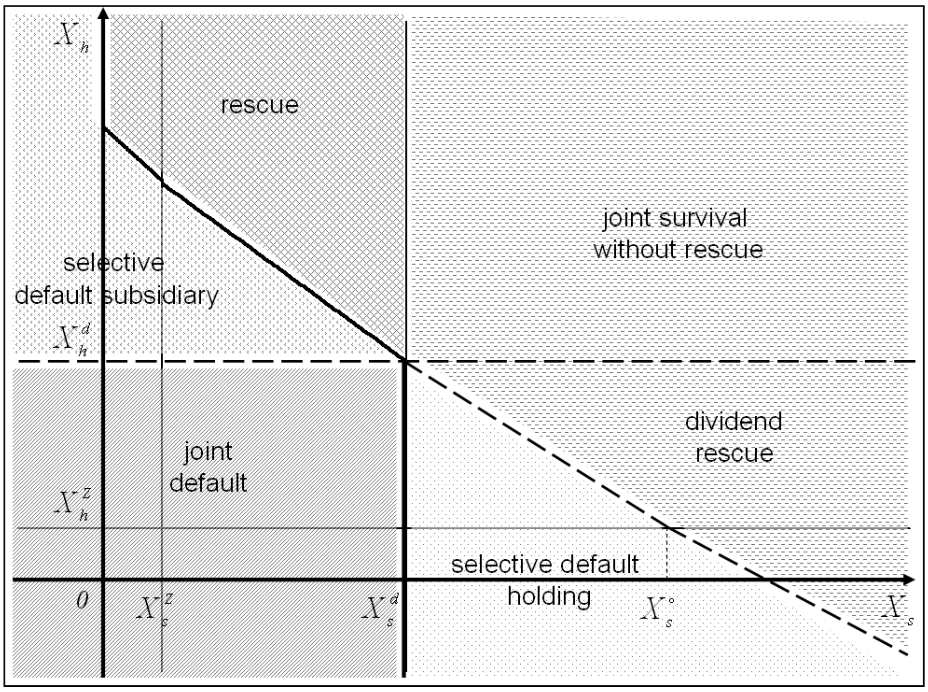

Figure 3: This figure shows when there is a subsidiary rescue, as well as the areas of joint or selective defaults. It can be seen that the area of selective holding default shrinks, with respect to figure 2, thanks to dividends. Cash flows of the subsidiary and of the parent, when ownership is non infinitesimal, are on the horizontal and the vertical axis, respectively.

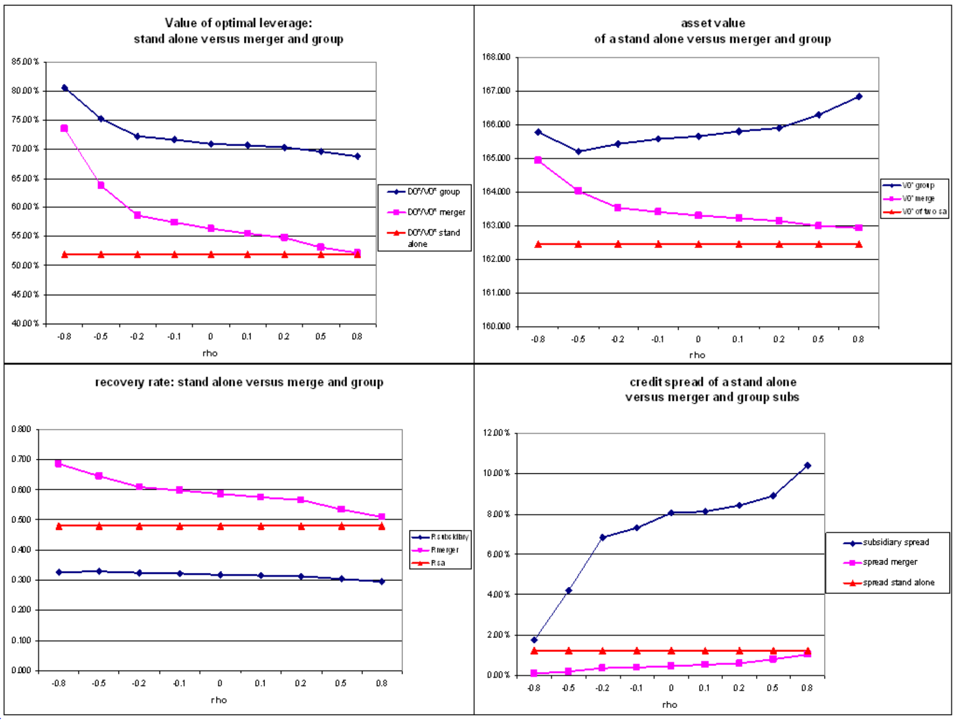
Value of optimal leverage:
stand alone versus merger and group
85.00%
80.00%
75.00%
70.00%
65.00%
60.00%
55.00%
50.00%
45.00%
40.00%
-0.8 -0.5 -0.2 -0.1 0 0.1 0.2 0.5 0.8
rho
D0*/V0* group
D0*/V0* merger
D0*/V0* stand alone
asset value
of a stand alone versus merger and group
168.000
167.000
166.000
165.000
164.000
163.000
162.000
161.000
160.000
-0.8 -0.5 -0.2 -0.1 0 0.1 0.2 0.5 0.8
rho
V0* group
V0* merge
V0* of two sa
recovery rate: stand alone versus merge and group
0.800
0.700
0.600
0.500
0.400
0.300
0.200
0.100
0.000
-0.8 -0.5 -0.2 -0.1 0 0.1 0.2 0.5 0.8
rho
R subsidiary
R merger
R sa
credit spread of a stand alone
versus merger and group subs
12.00%
10.00%
8.00%
6.00%
4.00%
2.00%
0.00%
-0.8 -0.5 -0.2 -0.1 0 0.1 0.2 0.5 0.8
rho
subsidiary spread
spread merger
spread stand alone

Figure 4

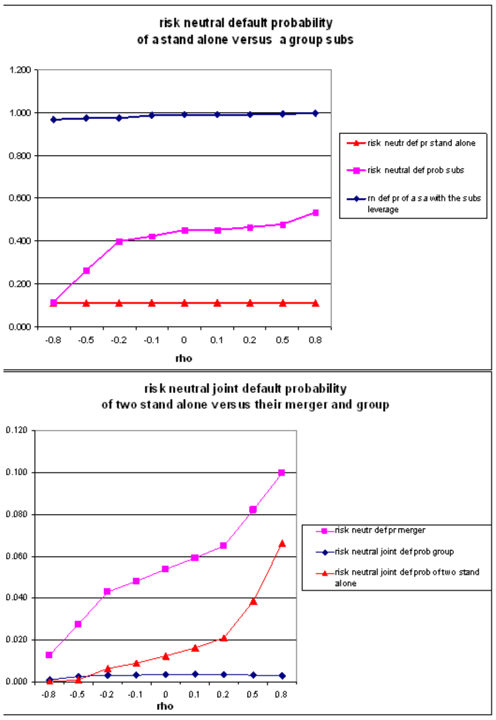
risk neutral default probability
of a stand alone versus a group subs
1.200
1.000
0.800
0.600
0.400
0.200
0.000
-0.8 -0.5 -0.2 -0.1 0 0.1 0.2 0.5 0.8
rho
risk neutr def pr stand alone
risk neutral def prob subs
rn def pr of a s a with the subs leverage
risk neutral joint default probability
of two stand alone versus their merger and group
0.120
0.100
0.080
0.060
0.040
0.020
0.000
-0.8 -0.5 -0.2 -0.1 0 0.1 0.2 0.5 0.8
rho
risk neutr def pr merger
risk neutral joint def prob group
risk neutral joint def prob of two stand alone

Figure 5

Table 1: Base Case Parameters

Variables	Symbols	Values
Annual Riskfree Rate	r	5.00%
Time Period/Debt Maturity (yrs)	T	5.00
T-period Riskfree Rate	$r_T = (1+r)^T - 1$	27.63%
Capitalization Factor	$Z = (1+r_T)/r_T$	4.62
Unlevered Firm Variables		
Expected Future Operational Cash Flow at T	Mu	127.63
Expected Operational Cash Flow Value (PV)	$X_0 = Mu/(1+r)^T$	100.00
Cash Flow Volatility at T	Std	49.19
Annualized Operational Cash Flow Volatility	$\sigma = Std/T^{0.5}$	22.00
Tax Rate	τ	20%
Value of Unlevered Firm w/Limited Liability	V_0	80.05
Value of Limited Liability	L_0	0.057

Table 2: Credit risk in the base case, different organizational forms, $\rho = 0.2$

Variables	Symbols	Values				
		Stand Alone	Holding	Subsidiary	Group	Conglomerate
Optimal Face Value of Debt	P^*	57.20	0	219	219	117. 4
Default Threshold	X^{d*}	67.75	0	248.169	-	69.87
Breakeven Profit Level	X^{Z*}	14.98	0	102.32	-	14.00
Value of Optimal Debt	D_0^*	42.22	0	116.68	116.68	44.70
Value of Optimal Equity	E_0^*	39.01	49.2	0.037	49.237	36.88
Optimal Levered Firm Value	$\nu_0^* = D_0^* + E_0^*$	81.23	49.2	116.71	165. 91	163.15
Optimal Leverage Ratio	D_0^*/ν_0^*	52%	0	99,9%	70.33%	55%
Annual Yield Spread of Debt (%)	y	1.26%	//	8.4%	-	0.6%
Value of Optimal Leveraging	$\nu_0^* - V_0$	1.18	-	-	6. 31	3.15
Recovery Rate	R	48.1%	//	31.2%	-	56.5%
Default Probability	PRD_i	0.112	0.0034	0.4654		0.065
Selective Default Probability	$PRDS_i$		0	0.462		0
Joint Default Probability	$PRDJ$				0.0034	0.065
Rescue Probability	PRR				0.5235	positive
Non Default Probability	$PRND$	0.888			0.5292	0.935

Table 3: Credit risk in the base case, different organizational forms, $\rho = 0.2, w = 1$

Variables	Symbols	Values			
		Stand Alone	Holding	Subsidiary	Group
Optimal Face Value of Debt	P^*	57.20	0	219	219
Default Threshold	X^{d*}	67.75	0	248.169	-
Breakeven Profit Level	X^{Z*}	14.98	0	102.32	-
Value of Optimal Debt	D_0^*	42.22	0	116.68	116.68
Value of Optimal Equity	E_0^*	39.01	49.2	0.037	49.237
Optimal Levered Firm Value	$\nu_0^* = D_0^* + E_0^*$	81.23	49.2	116.71	165. 91
Optimal Leverage Ratio	D_0^*/ν_0^*	52%	0	99,9%	70.33%
Annual Yield Spread of Debt (%)	y	1.3%	//	8.4%	8.4%
Value of Optimal Leveraging	$\nu_0^* - V_0$	1.43	-30.60	36.91	6. 31
Recovery Rate	R	0.481	//	0.0533	0.0533
Default Probability	PRD_i	0.112	0.034	0.4654	
Selective Default Probability	$PRDS_i$	0.112	0	0.462	
Joint Default Probability	$PRDJ$	0.02			0.034
Rescue Probability	PRR	0			0.5235
Non Default Probability	$PRND$	0.888			0.057

Table 4: Implicit ratings

ρ	-0.8	-0.5	-0.2	-0.1	0	0.1	0.2	0.5	0.8
hist def prob subsidiary									
	0.30%	4.62%	13.80%	16.23%	19.35%	20.39%	22.13%	25.54%	32.02%
hist def prob holding									
	0.08%	0.09%	0.08%	0.08%	0.08%	0.08%	0.08%	0.09%	0.08%
hist joint def prob group									
	0.00%	0.03%	0.07%	0.08%	0.08%	0.08%	0.08%	0.08%	0.08%
hist def prob merge									
	0.00%	0.14%	0.52%	0.70%	0.90%	1.13%	1.38%	2.23%	3.25%
hist def prob sa									
	4.00%	4.00%	4.00%	4.00%	4.00%	4.00%	4.00%	4.00%	4.00%
hist def pr of a sa with the subs leverage									
	90.51%	92.32%	92.32%	96.28%	97.00%	96.99%	97.24%	97.58%	98.72%

Table 5: Credit risk in the constrained case, different values of rho

Variables	Symbols	ρ			
		-0.8	0	0.2	0.8
Face Value of Subsidiary Debt	P_{0s}	57.2	57.2	57.2	57.2
Optimal Face Value of Parent Debt	P^*_{0h}	57	56	54	51
Optimal Value of Subsidiary Debt	D^*_{0s}	44.47	43.82	43.59	42.78
Optimal Value of Parent Debt	D^*_{0h}	42.09	41.44	40.12	38.11
Optimal Levered Group Value	ν^*_{0g}	162.95	162.83	162.79	162.68
Optimal Value of Two Levered Stand Alone	$\nu^*_{01} + \nu^*_{02}$	162.46	162.46	162.46	162.46
Optimal Group Leverage Ratio	D^*_{0g}/ν^*_{0g}	53.1%	52.4%	51.4%	49.7%
Optimal Stand Alone Leverage Ratio	D^*_{01}/ν^*_{01}	52%	52%	52%	52%
Subsidiary's Recovery Rate	R_s	-	17.6%	23.8%	37.2%
Holding's Recovery Rate	R_h	46.7%	46.8%	46.1%	.44.6%
Stand Alone's Recovery Rate	R_{sa}	48.1%	48.1%	48.1%	48.1%
Subsidiary's Default Prob	PRD_s	0.0031	0.0272	0.0362	0.0727
Holding's Default Prob	PRD_h	0.1084	0.1048	0.0963	0.0838
Stand Alone's Default Probability	PRD_{sa}	0.112	0.112	0.112	0.112
Joint Default Probability	$PRDJ$	0	0.0118	0.0181	0.0524
Annual Yield Spread (Subs)	y_s	0.2%	0.5%	0.6%	1.0%
Annual Yield Spread (Holding)	y_h	1.3%	1.2%	1.1%	1.0%
Annual Yield Spread (Stand Alone)	y_{sa}	1.3%	1.3%	1.3%	1.3%

Table 6: Credit risk different alfas, different organizational forms, $alfasubs = 0.23,\ alfahold = 0.75, \rho = 0.2$

Variables	Symbols	Values				
		Stand Alone (alfa=0.75)	Holding	Subsidiary	Group	Conglomerate
Optimal Face Value of Debt	P^*	33	0	219	219	93
Default Threshold	X^{d*}	39.247	0	248.17	-	110.86
Breakeven Profit Level	X^{Z*}	8.01	0	102.32	-	21.58
Value of Optimal Debt	D_0^*	24.99	0	116.68	116.68	71.42
Value of Optimal Equity	E_0^*	55.84	49.2	0.037	49.237	91.05
Optimal Levered Firm Value	$\nu_0^* = D_0^* + E_0^*$	80. 83	49.2	116.71	165. 91	162.47
Optimal Leverage Ratio	D_0^*/ν_0^*	30.9 2%	0	99,9%	70.33%	44%
Annual Yield Spread of Debt (%)	y	0.7%	//	8.4%	-	0.4%
Value of Optimal Leveraging	$\nu_0^* - V_0$	1.43	-	-	6.31	2. 87
Recovery Rate	R	6.2%	//	5.3%		
Default Probability	PRD_i	0.036	0.0034	0.4654		0.029
Selective Default Probability	$PRDS_i$		0	0.462		
Joint Default Probability	$PRDJ$				0.0034	
Rescue Probability	PRR				0.5235	
Non Default Probability	$PRND$	0.964			0.5293	0.971

Table 7: Credit risk different volatilities, different organizational forms,$\rho = 0.2, volsubs = 44\%, volhold = 22\%$

Variables	Symbols	Values				
		Stand Alone(vol 44%)	Holding	Subsidiary	Group	Conglomerate
Optimal Face Value of Debt	P^*	83	0	223	223	118
Default Threshold	X^{d*}	95-19	0	248.169	-	69.50
Breakeven Profit Level	X^{Z*}	34.25	0	102.32	-	17.01
Value of Optimal Debt	D_0^*	48.75	0	106.83	106.83	83.97
Value of Optimal Equity	E_0^*	36.10	60.29	3.01	63.30	79.28
Optimal Levered Firm Value	$\nu_0^* = D_0^* + E_0^*$	84.84	60.29	109.84	170.13	163.26
Optimal Leverage Ratio	D_0^*/ν_0^*	57.46%	0	97.3%	62.8%	51.4%
Annual Yield Spread of Debt (%)	y	6.2%	//	10.9%	-	2%
Value of Optimal Leveraging	$\nu_0^* - V_0$	4.79	-	-	10.53	3. 16
Recovery Rate	R	1.64%	//	20.2%	-	negative
Default Probability	PRD_i	0.2548	0.0034	0.4871		0.0635
Selective Default Probability	$PRDS_i$		0.0001	0.4839		
Joint Default Probability	$PRDJ$				0.0033	
Rescue Probability	PRR				0.4019	
Non Default Probability	$PRND$	0.7452			0.5074	0.9365

Table 8: Credit risk different size, different organizational forms, $\rho = 0.2,\ V_{h0} = 167, V_{s0} = 33.$

Variables	Symbols	Values					
		Stand Alone(1/3)	Stand Alone(5/3)	Holding	Subsidiary	Group	Conglomerate
Optimal Face Value of Debt	P^*	19	95	63.33	121.33	184.66	115
Default Threshold	X^{d*}	22.512	112.56	75.39	138.90	-	136.62
Breakeven Profit Level	X^{Z*}	4.951	24.77	15.08	51.07	-	28.53
Value of Optimal Debt	D_0^*	14.05	70.24	48.25	70.26	118.51	86.47
Value of Optimal Equity	E_0^*	13.11	65.54	47.15	0	47.15	76.51
Optimal Levered Firm Value	$\nu_0^* = D_0^* + E_0^*$	27.16	135.78	95.40	70.26	165.66	162.98
Optimal Leverage Ratio	D_0^*/ν_0^*	51.73%	51.73%	50.58%	100%	71.54%	53.06%
Annual Yield Spread of Debt (%)	y	1.2%	1.2%	0.6%	6.5%	-	0.9%
Value of Optimal Leveraging	$\nu_0^* - V_0$	0.48	2.36	-	-	3.6	2.88
Recovery Rate	R	/	85.2%	38.4%	20.96%	-	32.1%
Default Probability	PRD_i	0.0163	0.3797	0.0454	0.3304		0.0597
Selective Default Probability	$PRDS_i$			0.0000	0.2849		
Joint Default Probability	$PRDJ$					0.0455	
Rescue Probability	PRR					0.6656	
Non Default Probability	$PRND$	0.9837	0.6203	0.9546	0.7696	0.6656	0.9403