Adjusting Corporate Default Rates for Rating Withdrawals

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The authors would like to thank Edward Altman, Lea Carty, Jerry Fons, Martin Fridson, Gus Harris, David Lando, and Til Schuermann for helpful discussions and comments on earlier drafts of this paper. The views expressed herein are solely those of the authors and Moody's Investors Service.

In press, *The Journal of Credit Risk*

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3 May 2007

Abstract

Many market practitioners base their parameter estimates on results reported in rating agency default studies. Although the comparability of default rates reported by the agencies has increased in recent years, many differences in default rate calculation methodologies remain. One important and poorly understood methodological difference is whether default rate estimates are (or should be) statistically adjusted for issuer rating withdrawals, which occur when borrowers shift from rated public to unrated private debt finance or when all their debts are extinguished outright. In this paper we review the mechanics and rationale behind the unadjusted and withdrawal-adjusted default rate calculation methodologies. We discuss the relative merits of adjusting or not adjusting for rating withdrawals and the importance of the assumptions underlying each method. We demonstrate that the assumption of random data censoring posited by the withdrawal-adjusted method is supported by the available data. We conclude that withdrawal-adjusted default rates are the appropriate estimates of expected default rates for obligations with specific expected tenors and provide common yardsticks for comparing default risk for credit exposures across different sectors, regardless of differences in realized rating withdrawal rates.

Keywords: default probability, credit ratings, rating agencies, data censoring, risk management

1. Introduction

The measurement of the probability of default for a corporate exposure is often the first step in credit risk modeling, management, and pricing. Rating agency default studies are widely-used sources for estimates of these important parameter values. The default statistics reported in rating agency studies are based on rich source data sets, containing a large number of corporate rating histories and credit events. It is frequently assumed that the default statistics reported by the rating agencies are calculated using more or less the same methodology and may, therefore, be used interchangeably, compared, and interpreted more or less consistently. Furthermore, it is often taken for granted that the default statistics reported by the rating agencies are equally appropriate measures of risk for a given purpose.

In the past decade there has indeed been a convergence in the methodologies used by the agencies to calculate cumulative default rates, and their methodologies currently share many similarities. Most rating agencies emphasize issuer-based default statistics rather than dollar-volume based statistics; average default rate estimates for an historical time period are calculated using a cohort-based approach; and, long-term multi-year default rates are derived using a discrete-time hazard rate method.[1] Despite these similarities, the default rates for corresponding rating categories reported by the rating agencies often differ significantly. While variations in default rates by rating category across agencies are to be expected due to differences in rating methodologies, discongruities in the distributions of the underlying rated populations, variations in the agencies' definitions of default, the historical time periods under study and the periodicity of observation, an additional reason to expect differences is that each rating agency's default rate calculation methodology differs in its statistical treatment of rating withdrawals.

Rating agency default rate calculation methodologies generally take one of two approaches to dealing with rating withdrawals when calculating default rates: ignore them and make no adjustment; or adjust for rating withdrawals by treating them as randomly censored data. Under the *no adjustment for withdrawals* method, issuers whose ratings are withdrawn are treated as if they remained in the data sample over the entire measurement horizon. An attempt is made to monitor their default status over the entire measurement horizon. If no default is observed, the firm is assumed not to have defaulted. Hence, the no adjustment method takes a relatively simple view of the evolution of credit risk in that there are only two possible outcomes, default or non-default.

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[1] Rating agencies also report default rates derived by calculating multi-period rating transition matrices. Although we do not discuss this method in this paper, transition matrix-derived default rates – which generally report rating withdrawals as a distinct state – are very close to those derived using the unadjusted method discussed later in this study.

Under the *withdrawal-adjusted* method, issuers whose ratings are withdrawn are treated as randomly censored data, meaning that it is assumed that firms whose ratings are withdrawn would have faced the same risk of default as other similarly rated issuers if they had stayed in the sample. The withdrawal-adjusted method recognizes that there are three possible end-of-period outcomes: default, survival, and rating withdrawal. Rating withdrawals represent losses from the data sample before the final outcome of interest (default or survival) is observed. Most rating agencies calculate and report default rates using both methods, but each tends to emphasize different sets of estimates. Moody's default statistics, for example, are most often reported using the adjusted-for-withdrawals method (Hamilton (2006), whereas Standard and Poor's has historically emphasized its unadjusted statistics (Vazza, Aurora, and Schneck (2006)).

Both calculation methods have legitimate uses under the appropriate assumptions, but each method makes a different statement about default risk. As they are derived from historical corporate rating histories and default data, the default rate estimates generated by each method represent a view of the "actual" default experience of a given data sample. However, empirical default rates are frequently used as proxies for expected default probabilities, and it is for this purpose that the treatment of rating withdrawals becomes an important concern. Unadjusted default rates may be useful benchmarks for the expected likelihood of default for obligations that have fixed maximum potential tenors and expected rating withdrawal rates similar to those exhibited by issuers in the empirical sample on which default rates were estimated. In contrast, withdrawal-adjusted default rates are the appropriate estimates of expected default rates for obligations with specific expected tenors. Withdrawal-adjusted default rates therefore provide common yardsticks for comparing default risk for credit exposures across different sectors, regardless of differences in realized rating withdrawal rates.

In many respects, the issue is similar to that studied in Altman (1989). Altman (1989) maintained that prevailing methods for calculating multi-year bond default rates were biased estimates of expected default risk because they failed to account for maturities, calls, and other early redemptions that occur prior to the end of a given measurement horizon. Altman's mortality rate estimator recognized that calculating default rates based on the surviving population was the relevant measure of *expected* default risk. Coming to a similar conclusion, Asquith, et. al. (1989) showed that bond default probability estimates are materially affected by early bond redemptions, as nearly two-thirds of high yield bonds in their data sample had been called, defaulted, or exchanged within 10 years of issuance. The adjustments advocated by Altman (1989) and Asquith et. al. (1989) therefore amounted to adjusting their bond default rates for survival bias. More recently, Mählmann (2005) showed that default probability estimates that do not adjust for missing ratings are downwardly biased.[2]

However, rating agency default rates are typically *issuer* (or corporate family) based, adjusting for withdrawals depends critically on the assumption of random data censoring. An issuer's rating may be withdrawn for a variety of reasons. One common reason is that a company has extinguished all of its rated public debt due to scheduled maturities, company-initiated calls, investor-initiated puts, or mergers and acquisitions. In many cases, the issuer is no longer at risk of default after a rating withdrawal because the withdrawal event corresponds to the extinguishment all of its debt obligations. However, in many other cases an issuer remains at risk of default after its rating has been withdrawn because it has replaced all of its public, rated debt with unrated, typically private, debt. The relevant question is whether issuer rating withdrawals are uninformative events or are correlated with changes in credit quality.

The remainder of this paper is organized into five sections. In the next section we review the general cumulative default rate calculation methodology followed by the rating agencies. We also identify certain features of Moody's default rates that distinguish them from other approaches. In Section 3 we explain the mechanics of the adjustment for rating withdrawals under the assumption of uninformative censoring and discuss the rationale underlying the unadjusted and withdrawal-adjusted methods. Following a long line of academic research, we argue that withdrawal-adjusted default rates have the most general use for applications requiring estimates of *expected* future default risk. In Section 4 we analyze the hypothesis of the neutrality of issuer rating withdrawals. We demonstrate that the available evidence suggests the assumption of random censoring is reasonable. Section 5 offers some concluding thoughts.

2. Cumulative default rate methodology

The cumulative default rate calculation methodology used by the major rating agencies is a discrete-time approximation of the nonparametric continuous-time hazard rate approach.[3] A pool of issuers, called a *cohort*, is formed on the basis of the rating held on a given calendar date (or set of dates), and the default/survival status of the members of the cohort is tracked over some stated time horizon. The time horizon T for which we desire to measure a default rate is divided into evenly spaced time intervals (e.g. months, years) of length t. Hence, the data is discrete in that the time to default is not measured continuously. In each time

[2] Moreover, Mählmann (2005) discusses appropriate adjustments to PD estimates for different types of nonrandom censoring.

[3] The method is essentially that of Cutler and Ederer (1958). This approach is sometimes referred to as the *life-table* or *actuarial* method.

interval, some fraction of the cohort that has survived up to that time may default. The *marginal default rate* is the probability that an issuer that has survived in the cohort up to the beginning of a particular interval t will default by the end of the time interval.[4] The T-horizon *cumulative default rate* is defined as the probability of default from the time of cohort formation up to and including time horizon T.

Cohorts of issuers can be formed on the basis of their original ratings or on the ratings held as of the cohort formation date. The original rating method, studied by Altman (1989), groups issuers into pools based on the first rating that was assigned to the issuer (or one of its obligations); such pools consist only of first-time issuers that were rated as of the cohort formation date(s).[5] In contrast, the cohort rating method on which rating agencies' corporate default studies often rely are based on pools of issuers holding a given rating on the cohort date regardless of original rating or time since issuance. Because long-term corporate ratings address the likelihood of default over multiple time horizons, regardless of age or time to maturity, agency corporate default studies usually report default rates based on issuers' ratings held on the cohort date rather than on original ratings.[6]

Mathematically, the marginal default rate in time interval t, $d(t)$, for a cohort of issuers formed on date y holding rating z is defined as the number of defaults $x(t)$ from the cohort that occur in the time interval t divided by the effective size of the cohort, $n(t)$, at the start of time t:

$$d_y^z(t) = \frac{x_y^z(t)}{n_y^z(t)} \tag{2.1}$$

Initially, $n(t)$ is equal to the number of issuers in the pool holding rating z on the cohort formation date. As time from the initial cohort date passes the size of the denominator falls because some issuers in the cohort fail to survive to the next time interval. As we discuss in detail in the next section, differences in the default rates reported by the rating agencies arise to a large extent because each rating agency models the default/survival process differently.

[4] The term marginal default rate is particular to rating agencies and is usually referred to as the hazard rate.
[5] The original rating method captures the impact of the now well-known aging or seasoning effect (i.e. the term structure of default risk for a given issuance year and rating category). Marginal default (hazard) rates based on original ratings exhibit more pronounced "humps" relative to the cohort rating method.
[6] The major rating agencies assign both obligation-level and issuer-level credit ratings. When a suitable issuer-level rating does not exist, one is often inferred from existing obligation-level ratings. For Moody's approach see Hamilton (2005). Standard and Poor's methodology is described in Vazza, Aurora, and Schneck (2006).

Cumulative default rates for investment horizons of length T, denoted $D(T)$, are built up from the marginal default rates, and are found by subtracting the product of the fraction of surviving cohort members in each of the t time intervals from unity:

$$D_y^z(T) = 1 - \prod_{t=1}^{T} [1 - d_y^z(t)] \tag{2.2}$$

Or, expanding equation 2.2 (and dropping indices for brevity):

$$D(T) = d(1) + d(2)[1 - d(1)] + d(3)[(1 - d(1))(1 - d(2))] + ... + d(T)(\prod_{t=1}^{T-1}[1 - d(t)]) \tag{2.3}$$

Equation 2.3 highlights the fact that a cumulative default rate is a conditional probability. In the first time period, a fraction of the credit exposures in the cohort either defaults or survives. The credit exposures that survive period one may then go on to default or survive in period two; those that survive period two may go on to default or survive in period three, etc. Because the time periods are non-overlapping and the probability of default in each period is assumed to be independent, the T-period cumulative default rate is defined as one minus the product of the T marginal survival rates.

Issuer-based default rates receive particular emphasis in the rating process because the expected *likelihood* of default of a bond issuer holding a given rating is expected be the same regardless of differences in the nominal sizes of the exposures.[7] For example, the expected likelihood of default for a B-rated corporate issuer should be the same whether the size of the exposure is $200 million or $2 billion, everything else equal. Issuer-based default rates give equal weight to all issuers in the default rate calculation. Dollar volume based default rates, which weight each exposure by the total face (or market) value of its outstanding bonds, are useful statistics for portfolio benchmarking, but they are less useful for forming expectations about future default *probabilities* for ratings.[8]

The frequency with which cohorts are formed also impacts the accuracy of the average default probability estimates for a given rating category. The higher the sampling frequency – equivalently, the shorter the time interval between cohorts – the more accurate

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[7] When a firm defaults on one bond it usually defaults on all its bonds due to cross-default clauses in bond indentures. Additionally, in some bankruptcy codes (e.g. U.S. Chapter 11 and France's "sauvegarde" procedure) an automatic stay provision triggered upon a bankruptcy filing creates perfect cross default, causing all debt to default at the same time (unless the bankruptcy court grants a waiver). This approach is also consistent with the structural view of credit risk (e.g. Merton (1974)) which regards default as an issuer-level phenomenon that is primarily a function of firm-level characteristics, such as its operating performance and liability structure.

[8] Fridson (1991) is an interesting discussion of the many different ways to measure default rates that addresses this and other topics.

the estimates of expected default rates for a given rating category become. Closer cohort spacing captures rating changes and default events that occur in small time intervals, an important consideration when an issuer's rating is undergoing rapid change. The effect of cohort spacing on default rate estimates becomes clear in the following example. Consider the senior unsecured rating history for LTV Steel Company up to its default on July 17, 1986:

Table 2.1 – LTV Steel Company Rating History

Rating Date	Rating	Credit Event
11/18/1970	A	First rating assigned
4/26/1982	A3	Alphanumeric rating assigned
5/5/1982	Baa2	Downgraded
10/18/1982	Baa3	Downgraded
11/18/1983	Ba1	Downgraded
3/20/1985	Ba3	Downgraded
8/9/1985	B3	Downgraded
7/17/1986	Caa	Defaulted

Using annual cohort spacing, LTV Steel Company's default is recorded for the A-rated cohorts from 1971-1982, the Baa3-rated 1983 cohort, the Ba1-rated cohorts in 1984 and 1985, and the B3 1986 cohort. If one instead formed cohorts at monthly intervals, the default event gets captured at the appropriate time horizon for every rating in its rating history, including its A3, Baa2, Ba3 and Caa ratings that are ignored under annual cohort spacing. Moody's has traditionally reported its average cumulative default rates calculated using annual cohort spacing (cohorts of issuers formed on January 1 of each year). In Moody's 2005 default study, Moody's moved to monthly cohort spacing in calculating its average cumulative default rates. Moody's believes that monthly cohort spacing strikes a reasonable balance between the competing goals of informational efficiency and tractability.[9]

While investors may be interested in the cumulative default experience of a particular cohort, averages taken over many cohort periods (which capture the effects of several macroeconomic and credit cycle peaks and troughs) are required to estimate *expected* cumulative default probabilities. The average cumulative default rate for a given historical time period is calculated by first averaging the period t marginal default rates across all

[9] There is a tradeoff between informational efficiency and tractability when calculating default rates using duration methods. Default/survival times are precisely measured using continuous-time methods, but the resulting output may be unwieldy. Making default event times discrete by arbitrarily choosing the width of the marginal time intervals the distance between cohort formation dates results in some loss of efficiency, but greatly facilitates the presentation and interpretation of cohort cumulative default rates. For example, investors are often interested in default rates for certain discrete time horizons (e.g. one, five, ten years). As the time interval t is allowed to shrink so as to be so small that at most one default occurs within an interval, the derived default rates approach the continuous time estimate (Kaplan and Meier (1958)). More importantly, the discrete-time cohort approach does not depend on the Markov assumption (see Lando and Skodeberg (2002)).

available cohort dates y in the historical data set Y, then calculating the cumulative rates using equation 2.2 or 2.3. Most often, average cumulative default rates are weighted averages, where each period's marginal default rate is weighted by the relative size of the cohort (proportion of issuers) in each time interval t.[10]

Equation 2.4 shows that the calculation of the average cumulative default rate for rating class z, $\overline{D}^z(T)$, is derived from the weighted average marginal default rates, $\overline{d}^z(t)$, calculated from all the available cohort marginal default rates in the historical data set Y:

$$\overline{D}^z(T) = 1 - \prod_{t=1}^{T} [1 - \overline{d}^z(t)] \tag{2.4}$$

where

$$\overline{d}^z(t) = \frac{\sum_{y \in Y} x_y^z(t)}{\sum_{y \in Y} n_y^z(t)} \tag{2.5}$$

If, for example, one were calculating the average three-year cumulative default rate for the 2003-2005 time period (with annual cohort spacing), one would first take the weighted average of the $d(1)$ from each of the three cohort years. The second year's average marginal default rate would consist of the weighted average of the two cohort years (2003 and 2004) with two years of exposure available, $d(2)$. The third year average marginal default rate would simply consist of the 2003 cohort's third year marginal default rate since it is the only cohort with three years of history available (i.e. it would receive 100% weight). The average cumulative default rate would then simply be calculated according to equation 2.4 using the weighted average marginal default rates derived using equation 2.5.

Note that this procedure for calculating average cumulative default rates maximizes the existing historical information by using *all* the available rating and marginal default rate data, not just issuers with rating histories that endure for a period of at least length T. While the third year's *marginal* default rate is calculated from just one cohort year (the 2005 cohort), the three-year *cumulative* default rate reflects information on conditional default/survival derived from *all three* cohort years. This approach has the great advantage of allowing one to utilize data for issuers with both long and short histories to calculate default probabilities for *any* time horizon.

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[10] Weighted averages place greater weight on more recent cohorts as both the number and total dollar volume of bond issuance has experienced secular growth over time. This is appealing from a statistical sampling point of view, but also because defaults tend to be correlated with periods of active bond issuance. Simple averaging may be appropriate in some circumstances; e.g. the impact of macroeconomic fluctuations on multi-year default rates.

What may seem like the simplest method – deriving average cumulative default rates directly from the cohort cumulative default rates – limits the estimated average to the set of cohorts with at least T full periods of data. For long-horizon default rate averages, this requirement throws away much useful data (as well as raises the noise of the estimate). More importantly, however, deriving average cumulative default rates directly from cohort cumulative default rates may result in seriously biased and inconsistent estimates of expected cumulative default risk. For example, average cumulative default rates could possibly be *decreasing* if the historical data sample consists of default rates that have been very high in recent cohorts but very low in past cohorts.

3. Adjusting for rating withdrawals

The calculation methodologies for cohort and average cumulative default rates described in the previous section are generally followed by all the major rating agencies (again, with minor variations). Agency default rate calculation methodologies diverge, however, on their assumptions about the default/survival process. Whereas unadjusted marginal default rates are incrementally adjusted for defaults that occurred in the past, the withdrawal-adjusted method also accounts for rating withdrawals that occur prior to the end of the measurement period. Rating withdrawals complicate the calculation of default rates because there will be some issuers initially included in a cohort that will be lost from the data sample before the final outcome of interest (default or survival) is observed. In practice, a rating agency's approach to modeling the survival process is reflected in its calculation of the effective cohort size in each time interval; i.e. the denominator of equation 2.1.

Under the unadjusted method, the effective size of a cohort of issuers rated z formed on date y in time interval t is equal to the initial size of the cohort less the total number of issuers that have defaulted prior to the current time interval (and therefore cannot default in the future).[11] The denominator of the cohort marginal default rate (equation 2.1) for the unadjusted method is therefore calculated:

$$n_y^z(t) = n_y^z(0) - \sum_{i=2}^{t-1} x_y^z(i) \tag{3.1}$$

[11] The unadjusted method is often referred to as the "static pool" method. However, the term has been subject to some confusion. Sometimes, the term static pool is meant to imply no adjustment for withdrawals. Other times, the term static pool has been used to refer to what we have defined (in Section 2) as the cohort approach to calculating multi-year default rates. Using our terminology, the static pool method can be defined as a cohort-based method that does not adjust for rating withdrawals.

In contrast, the withdrawal-adjusted method recognizes that there are three possible end-of-period outcomes: default, survival, and rating withdrawal.[12] The cohort size at time *t* is calculated as in equation 3.1, but with an additional adjustment for the number of issuers that have had their ratings withdrawn in periods prior to the current time interval. Additionally, a small adjustment is made for rating withdrawals that occur *within* the current time interval. Withdrawn ratings that occur within an interval are treated as if they were censored at the midpoint of the interval; i.e. were at risk for half the time.[13] Equation 3.2 shows the calculation of the denominator for the adjusted for withdrawals method. Note that equation 3.2 is simply 3.1 with two additional terms subtracted to account for rating withdrawals. It becomes clear, then, that the denominators of adjusted default rates are never larger than those using the unadjusted method. Hence, default rates using the adjusted method will be higher.

(3.2) $$n_y^z(t) = n_y^z(0) - \sum_{i=2}^{t-1} x_y^z(i) - \sum_{i=2}^{t-1} w_y^z(i) - \tfrac{1}{2} w_y^z(t)$$

Unadjusted default rates are highly intuitive. They report the share of issuers that were observed to have experienced a default over a particular time horizon. Unadjusted default rates are useful benchmarks for the likelihood of default for obligations that have fixed maximum tenors and expected rating withdrawal patterns similar to those of the empirical sample from which the default rate estimate was derived.[14] In contrast, withdrawal-adjusted default rates are more complex in both calculation and interpretation. Withdrawal-adjusted default rates are based in part on “hypothetical” data whose accuracy depends on the assumption that issuers whose ratings are withdrawn would have defaulted at the same average rates as other similarly-rated issuers. One might reasonably ask, therefore, why bother adjusting default rates for rating withdrawals? Unadjusted default

[12] The three possible end-of-period outcomes are usually considered to be mutually exclusive. Issuers that are downgraded and default in the same time interval are treated as defaults, for example; issuers that default and have their rating withdrawn in the same time interval are categorized as defaults, not withdrawals.

[13] The within-period adjustment is valid only if one is confident that defaults are observable after a rating withdrawal *within* the current time interval. The time interval must, therefore, be reasonably short (such as one year or less). Of course, for small enough time intervals the effect of the within period adjustment is immaterial and can be dropped from equation 3.2.

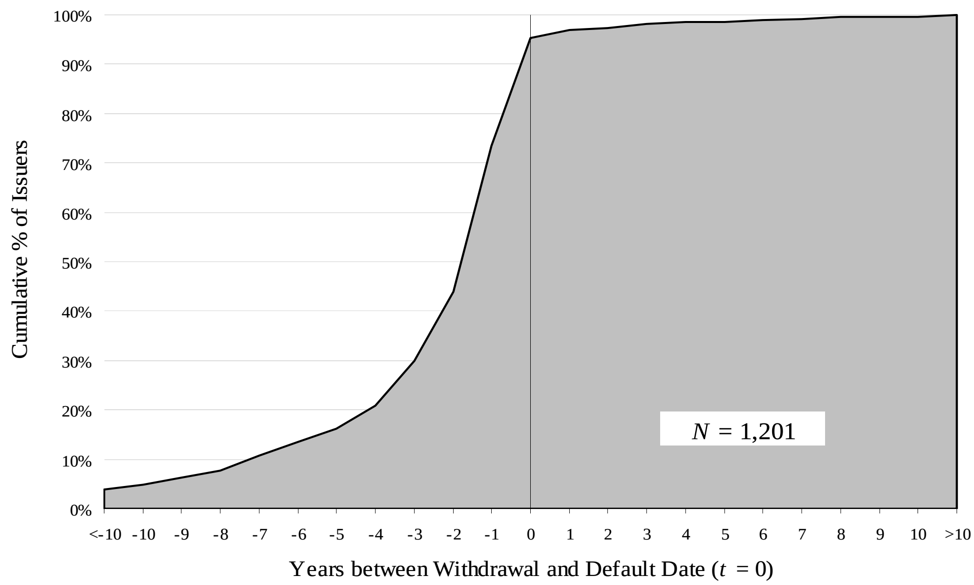
[14] A prime example is static synthetic corporate CDOs, which reference the debt obligations of a large number of corporations over a common and fixed maturity. In the event that all of the public debts and syndicated loans of a corporation are paid off, the risk in the CDO associated with that entity disappears. In such a structure, the historical average rating withdrawal pattern of the typical corporate issuer may be very relevant. For cash CDOs, on the other hand, the pattern of issuer rating withdrawals is less relevant, since it is unlikely to be closely related to the realized maturity patterns loans and bonds that comprise the structure’s collateral pool.

rates, it turns out, have three shortcomings – not shared by withdrawal-adjusted default rates – that limit their usefulness as measures of expected default risk for similarly rated obligations.

Firstly, unadjusted cumulative default rates are downwardly biased measures of default risk because one cannot observe all defaults experienced by issuers after their ratings are withdrawn.[15] Although rating agencies make every attempt to monitor default events for all formerly rated issuers, successfully doing so is exceptionally difficult. Figure 3.1 gives an indication of the magnitude of the problem. Of the 1,201 Moody's-rated corporate bond issuers since 1980 that have defaulted and had their ratings withdrawn, the percentage of defaults observed after the rating withdrawal date (5%) is relatively low compared to that before the withdrawal date (95%). As we show in the next section, many rating withdrawals occur when issuers retire their public debt with proceeds raised through private bank borrowings. These firms remain at risk of default but rating agencies cannot easily track their subsequent default experience. Hence, without adjustments for rating withdrawals, measured default rates are likely to understate true long-term default probabilities.

Figure 3.1 – Distribution of Distance between Withdrawal and Default Dates

[15] It has often been argued (e.g. DeRosa-Farag, et. al. (1999)) that withdrawal-adjusted cumulative default rates are "biased" too high due to the correction for data censoring. However, this perspective confuses concerns about sample size and statistical significance with issues of bias. Consider the following oft-used hypothetical example. Suppose that there were 10 bond issuers in a cohort, nine of which had their ratings withdrawn over a 10-year time span for benign reasons such as mergers or retirement of debt. If, in the 10th year, the one company that was still rated were to default, the withdrawal-adjusted marginal default rate would be 100%. This sample statistic is not biased. However, because only one issuer was at risk of default in its tenth year, the statistical reliability of the 100% point estimate is virtually nil. In order for the unadjusted default rate to reach 100%, all nine of the censored issuers would need to default after their ratings were withdrawn. The unadjusted method assumes that there were ten issuers at risk of default in the tenth year, lowering the empirical marginal default rate estimate to 10%. The small sample problem does not go away by changing the definition of the default rate.



Secondly, the relevance of unadjusted default statistics as guides of future expected default risk is limited to sets of issuers with similar rating withdrawal patterns. While all issuers that carry the same rating can reasonably be expected to have roughly the same mean withdrawal-adjusted default rates, their unadjusted default rates are likely to vary significantly. Borrowers from different industries within the non-financial corporate sector and across different sectors altogether (non-financial corporates, financial corporates, and sovereigns) have very different rating withdrawal patterns and should therefore be expected to have markedly different unadjusted default rates by rating category. Moreover, structured finance securities have rating withdrawal patterns that differ considerably from one another and from those of the broad universe of corporate issuers.[16] As a result, it is hard to imagine how one can realistically use unadjusted corporate issuer default rates to benchmark and compare the risks of such diverse obligations. Withdrawal-adjusted default rates, in contrast, facilitate comparisons of default rates across asset classes with markedly different rating withdrawal patterns.

Lastly, credit spreads are unlikely to be closely related with unadjusted default rates. For a risk neutral investor, the appropriate discount rate for debt obligations subject to default risk is, in theory, equivalent to the risk free rate $r(t)$ plus a spread to account for expected loss in default.[17] Hence, $R(t) = r(t) + d(t)L$, where $d(t)$ is the expected marginal default rate and L is the expected (and assumed constant) rate of loss given default. If we

[16] See Hu (2004).

[17] In addition to compensation for expected credit losses, credit spreads may also be influenced by tax effects, interest rate risk premia, and a variety of other potential sources of risk premia. The academic literature in this area is large. See Duffie and Singleton (2003) for an overview.

assume that investors recover nothing in the event of default (i.e. L=1), then the required spread s(t) for a risk-neutral investor is simply the expected marginal default rate: $s(t) = R(t)-r(t) = d(t)$. The appropriate measure of $d(t)$ is the withdrawal-adjusted marginal default rate, not the unadjusted marginal default rate. If an investor's expected investment horizon were, say, 10 years, then (s)he would only require compensation for default risk only on exposures expected to survive for at least 10 years. In fact, as long as withdrawal-adjusted marginal default rates are used in pricing, no further adjustments for realized rating withdrawals are required. In Appendix B we demonstrate this argument with an example.

As proxies for expected default probabilities, the advantages of withdrawal-adjusted default rates are consequently threefold: they avoid the downward bias that can arise from incomplete knowledge of defaults for firms whose ratings are withdrawn; they provide a common yardstick for measuring default risk for issuers and obligations across different sectors and asset classes regardless of differences in rating withdrawal rates and, thus, associate a single time profile of default rates for each rating category for all types of credit exposures. Lastly, they provide useful and relevant data for pricing a wide variety of debt obligations.

An example should make the differences between the two methods clear. Table 3.1 shows detailed calculation of the 1- through 10-year cumulative default rates for the January 1, 1996 cohort of B-rated corporate bond issuers using Moody's data. The table shows the number of defaults, $x(t)$ and rating withdrawals, $w(t)$, in each year after the cohort formation date, as well as the effective size of the denominator in each time interval, $n(t)$, for each of the two methods. The table also shows the marginal default rates and the resulting cumulative default rates for each method (calculated using equation 2.2).

Table 3.1 – 10-Year Cumulative Default Rates: Adjusted vs. Unadjusted Methods

January 1, 1996 Cohort of B-Rated Corporate Issuers

			Withdrawal-Adjusted Method			Unadjusted Method		
t	$x(t)$	$w(t)$	$n(t)$	$d(t)$	$D(t)$	$n(t)$	$d(t)$	$D(t)$
0	0	0	519	0.00%	0.00%	519	0.00%	0.00%
1	7	55	491.5	1.42%	1.42%	519	1.35%	1.35%
2	13	51	431.5	3.01%	4.39%	512	2.54%	3.85%
3	19	61	362.5	5.24%	9.41%	499	3.81%	7.51%
4	12	42	292	4.11%	13.13%	480	2.50%	9.83%
5	17	23	247.5	6.87%	19.10%	468	3.63%	13.10%
6	21	12	213	9.86%	27.07%	451	4.66%	17.15%
7	19	28	172	11.05%	35.13%	430	4.42%	20.81%
8	8	22	128	6.25%	39.18%	411	1.95%	22.35%
9	4	14	102	3.92%	41.57%	403	0.99%	23.12%
10	1	14	84	1.19%	42.26%	399	0.25%	23.31%

For each year t, $x(t)$ is the number of defaults, $w(t)$ is the number of issuer withdrawals, $n(t)$ is the effective denominator of the marginal default rate, $d(t)$, and $D(t)$ is the cumulative default rate.

Table 3.1 shows that, at any given measurement horizon, the withdrawal-adjusted method results in higher default rate estimates than the unadjusted method, with the difference growing larger as the time horizon lengthens. The unadjusted 10-year cumulative default rate shows that 23.31% of issuers originally in the cohort defaulted by the tenth year. The unadjusted method is calculated *as if* the 322 issuers whose ratings were withdrawn had remained in the cohort and did not default over the entire 10 year measurement period. In contrast, the withdrawal-adjusted 10-year cumulative default rate method yields an estimate of 42.26%. Default rates calculated using the withdrawal-adjusted method are based on the number of issuers that remain at risk (i.e. have not previous defaulted nor had their ratings withdrawn) of default in each time interval. For example, the marginal default rate in the tenth year is 25 basis points under the unadjusted method; under the withdrawal-adjusted method, it is nearly five times higher, 1.19%. The withdrawal-adjusted approach recognizes that at the start of the tenth year only 84 issuers actually remained at risk of default. Appendix A shows average cumulative unadjusted and withdrawal-adjusted default rates for a 20 year time horizon.

In addition to the three advantages of the withdrawal-adjusted method discussed above, the method also generates default probability estimates with intuitive and appealing statistical characteristics. The data in Table 3.1 illustrates that cumulative default rates calculated using the withdrawal-adjusted method will, at sufficiently long time horizons, approach 100%. This has a natural statistical interpretation: conditional on survival, all firms will likely eventually default. Cumulative default rates for a given cohort calculated using the unadjusted method, on the other hand, may never approach 100% over *any* measurement horizon. In order for the cumulative default rate to approach 100%, all the issuers whose ratings were withdrawn would need to be observed to ultimately default.

As we have mentioned several times in the preceding sections, the accuracy of the withdrawal-adjusted default rate measure depends critically upon the assumption of random data censoring. We analyze the validity of this assumption in the next section.

4. Assessing the neutrality of issuer rating withdrawals

An issuer rating withdrawal indicates that a rating agency has ceased to rate the issuer and/or all of its publicly rated bonds.[18] At the *bond* level, rating withdrawals are

[18] Rating withdrawal policies vary across agencies. At Moody's, an issuer rating withdrawal almost always indicates that all of an issuer's debt ratings have been withdrawn. Standard and Poor's issuer rating withdrawals, however, are not necessarily correlated with debt rating withdrawals. Moody's policy for rating withdrawals is described in Moody's (2004). To

overwhelmingly correlated with scheduled or anticipated redemptions. Table 5.1 shows the reasons for bond rating withdrawals organized into five categories using Moody's data. The table shows that of the 137,414 bond[19] rating withdrawals between 1980 and 2005, 97% were associated with maturity, calls, puts, conversions, or mergers. The business reasons category includes instances where the issuer chose to stop paying for a rating or the size of the bond issue was increased or decreased (and re-rated), or Moody's removed the rating because of lack of information from the issuer. The defaulted category includes cases where the bond rating was withdrawn on or shortly after the date of default.

the authors' knowledge, the other major rating agencies do not have publicly available methodologies describing their criteria for the removal of ratings.

[19] Includes coupon, discount, and convertible bonds.

Table 5.1 – Reasons for Rating Withdrawals, 1980-2005

Reason	%Bonds	%Issuers
Matured	68.84%	32.00%
Called, put, converted, etc.	27.85%	43.97%
Reason unknown	3.15%	21.31%
Business reasons	0.11%	2.02%
Defaulted	0.05%	0.70%
N	137,414	3,275

Table 5.1 also shows that 76% of Moody's *issuer* rating withdrawals corresponded to the final maturity, call, etc. of its bonds. However, even if it were known that all the bonds of an issuer were withdrawn due to, say, maturity (therefore making default on those particular bonds impossible), the rationale behind the firm's decision exit the rated bond market might reveal information about its default risk. Bond rating withdrawals, which are closely associated with maturities and redemptions of specific bonds, and issuer rating withdrawals, which are related to a firm's decision to exit or re-issue in the rated public bond market, reflect different corporate finance choices of a firm.

An issuer rating withdrawal might signal heightened future credit risk. For example, a bond issuer experiencing financial distress may be forced into the private or short-term debt market. Contrarily, an issuer rating withdrawal might be negatively correlated with default risk if issuers experiencing improving credit quality choose to pay off their rated debt obligations or replace debt with equity. Moody's rating policy is that the rating outstanding immediately prior to a withdrawal is intended to reflect Moody's view of the credit at the time of the withdrawal.[20] Hence, for Moody's data, rating withdrawals are supposed to be neutral events that are not systematically correlated with changes in default risk.

The neutrality of issuer rating withdrawals is, ultimately, an empirical question, yet there is almost no published research on the subject. Carty (1997) is the only study that has attempted to assess whether treating rating withdrawals as randomly censored data is justified. Carty's analysis using Moody's data was, however, indirect because he analyzed the reasons for *bond* rating withdrawals and their correlation with *issuer* rating withdrawals. In this section we attempt to asses whether the assumption that firms whose ratings are withdrawn would have faced similar default risk as firms that did not withdraw if had they remained in the data sample is valid. We seek to answer two specific questions. Firstly, does default risk increase or decrease leading up to and/or shortly following the rating withdrawal date? Secondly, is the *level* of default risk correlated with rating withdrawal events? Establishing the neutrality of issuer rating withdrawals is not straightforward as there is no direct statistical test for random versus informative censoring. We therefore attempt to infer

[20] See Moody's (2004).

the neutrality of withdrawals by examining several indications of default risk near rating withdrawal dates. Collectively, the results we present in this section provide evidence that the assumption of random censoring is reasonable.

A large body of literature[21] has shown that default rates are correlated with past rating actions: default rates are relatively higher conditional on a past downgrade and relatively lower conditional on a past upgrade. If rating withdrawal rates exhibit the same dependence on past rating actions, then we might have cause to doubt the random censoring hypothesis. For example, if rating withdrawal rates are higher conditional on a past downgrade, then rating withdrawals might represent "hidden" defaults. Table 5.2 shows mean one-year issuer rating withdrawal rates conditional on rating upgrades, downgrades, and no changes in the prior year based on monthly cohorts of corporate issuers between 1983 and 2005. A rating action represents a change of one alphanumeric rating notch or more; e.g. Ba2 to Ba3. Standard deviations are shown in parentheses.

For investment-grade rated issuers, mean issuer rating withdrawal rates do not exhibit significant systematic differences when conditioned on rating changes in the year prior to the rating withdrawal ($F(2,794) = 2.20$, $p = 0.1114$). Among speculative-grade rated issuers there is no monotonic relationship between past rating changes and withdrawal rates: issuers whose ratings were unchanged in the past year exhibit the highest mean rates of rating withdrawal, while issuers that experienced a rating upgrade show the lowest mean rating withdrawal rates. The differences in the means for each sub-sample are statistically significant ($F(2,794) = 32.86$, $p < 0.001$). One interpretation of these results is that risky speculative-grade issuers that survive long enough (i.e. do not default) to experience a rating upgrade are more likely to pay off their debts at maturity, convert them to private debt or equity, and have their ratings withdrawn. Of course, this is not knowable ex ante. Overall the likelihood of a rating withdrawal does not appear to be strongly correlated with rating changes in the prior year.

Similar results are evident when issuer rating withdrawal frequencies are conditioned on rating outlook and review (Watchlist) status. Moody's rating outlooks and reviews provide indications of the likely direction and timing of future credit rating changes. Cantor and Hamilton (2004) and Cantor and Hamilton (2005) showed that default rates for similarly rated issuers differ when conditioned on rating outlook and Watchlist status. Similar to the analysis of rating actions, a positive correlation between outlooks/review and rating withdrawal rates would cast a doubt on the validity of the assumption of uninformative rating withdrawals. Table 5.3 shows mean one-year issuer rating withdrawal rates between 1995 and 2005 conditional on outlook and Watchlist status held at the start of

[21] See, for example, Altman (1991), Carty (1997), and Cantor and Hamilton (2004).

each monthly cohort.[22] One-year rating withdrawal rates are remarkably similar across the outlook categories, about 3.7% per annum for investment-grade issuers and 8.2% for speculative-grade issuers. Moreover, statistical tests of the equivalency of the means strongly indicates no differences by outlook status.[23]

Table 5.2 – Mean One-Year Rating Withdrawal Rates Conditional on Rating Changes in Prior Year

	Rating Action in Prior Year		
	Upgraded	Unchanged	Downgraded
Investment-Grade	4.540% (2.014%)	4.549% (1.463%)	4.832% (1.927%)
Speculative-Grade	8.069% (4.616%)	10.689% (3.814%)	9.082% (2.527%)
All Rated	5.302% (2.341%)	6.351% (1.701%)	6.566% (1.913%)

Sample period: 1983-2005

Standard deviations appear in parentheses

Table 5.3 – Mean One-Year Rating Withdrawal Rates Conditional on Outlook Status

	Outlook on Cohort Date				
	Watch Up	Positive	Stable	Negative	Watch Down
Investment-Grade	3.553% (3.166%)	3.592% (0.952%)	3.553% (1.591%)	3.739% (1.172%)	3.887% (2.602%)
Speculative-Grade	8.030% (8.859%)	8.132% (2.857%)	7.629% (4.544%)	8.267% (3.567%)	9.090% (7.653%)
All Rated	4.547% (3.376%)	4.471% (0.943%)	4.317% (1.519%)	4.553% (1.065%)	4.916% (2.689%)

Sample period: 1995-2005

Standard deviations appear in parentheses

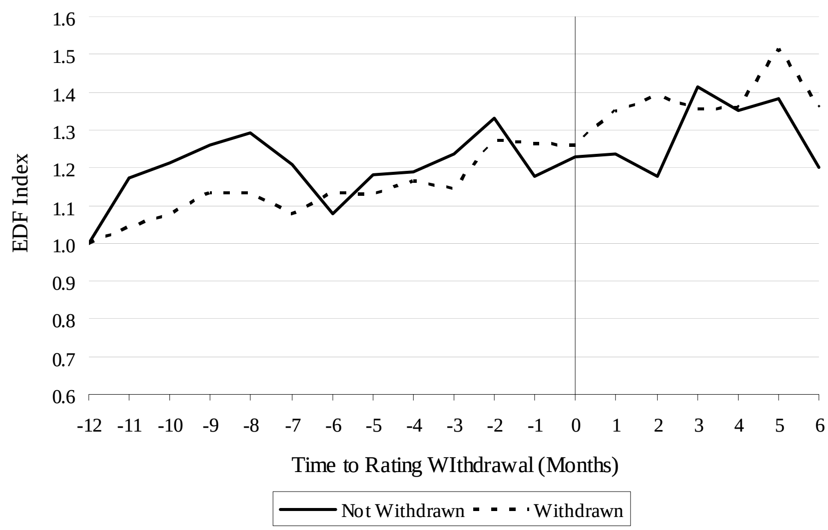
Moody's KMV EDFs offer a direct way to analyze changes in default risk both before and after an issuer rating withdrawal occurs. We analyzed monthly Moody's KMV EDFs in the 12 months before and the 6 months after withdrawal dates for 5,577 issuers between 1998 and 2005. The data was divided into two samples: 206 issuers with EDF data available that actually experienced a rating withdrawal, as well as a control group consisting of 5,371 issuers that did not experience a rating withdrawal. For the non-withdrawn subset, we measured EDFs around the same rating withdrawal dates as for the withdrawn subset. Because EDFs can vary between 0 and 0.20, and because we are interested in changes in

[22] Moody's introduced its Watchlist in 1991 and rating outlooks in 1995.

[23] For the investment-grade subset, $F(4,600) = 0.59$, $p = 0.6678$. For speculative-grade issuers, $F(4,600) = 0.98$, $p = 0.4205$.

EDFs around withdrawal dates, we normalized the EDFs so that they are equal to 1 in the twelfth month prior to the rating withdrawal date. Figure 5.1 shows the average EDF indices around rating withdrawal dates for the two groups. The graph shows that the changes in the average EDFs of the two subsets exhibit very similar patterns in the months leading up to and after the rating withdrawal date. Both series exhibit a slight upward trend leading up to the rating withdrawal date, but the magnitude of the change is relatively small.

Figure 5.1 – Average EDF Index around Rating Withdrawal Date



As a final assessment of the neutrality hypothesis we compared the weighted average *levels* of credit ratings for issuers that experienced a rating withdrawal to those that did not. Although rating withdrawal rates are inversely related to rating level[24] (as clearly demonstrated by agency default studies), the random censoring hypothesis would be challenged if the average rating levels for issuers initially rated the same moved far apart with the passage of time. For example, for two issuers rated Caa1 on a given cohort date, one of which experienced a rating withdrawal in the future and one which remained outstanding, what was the relative difference in their rating levels with the passage of time?

We formed annual cohorts for 4,833 corporate bond issuers over the 1983 to 2005 time period, and recorded their estimated senior unsecured ratings from the cohort year to cohort year plus 5 years. To calculate the average rating level at each point in time, we first

[24] There are several reasons to expect higher rating withdrawal rates for lower-rated issuers. Investors prefer to lend money to highly leveraged issuers at shorter maturities, so speculative-grade issuers must periodically raise new capital to replace short-maturity debt. At the same time, it may be optimal for highly leveraged firms to choose to issue debt with relatively shorter maturities in order to minimize adverse selection costs (Mitchell (1991)). Speculative-grade issuers also tend to be smaller relative to investment-grade issuers and have historically been more likely to refund rated public bonds with unrated private loans when it is economical to do so (for a related discussion see Titman and Wessels (1988)).

mapped the Aaa-Caa3 ratings to a linear numerical scale (1-19), then weighted the ratings using Moody's CDO rating factors.[25] Moody's CDO rating factors place greater weight on lower rating categories, so downgrades will have a relatively larger effect on weighted average ratings. Table 4.3 presents the results. The table shows the mean number of notches of difference between the ratings of the withdrawn sub-sample and the non-withdrawn sub-sample. By construction, the notch differences are zero on the cohort formation dates. Negative values in the table indicate the number of rating notches lower the withdrawn sub-sample is relative to the non-withdrawn sub-sample. The relevant question is whether the sub-sample that experienced a rating withdrawal exhibited significant rating improvement or degradation relative to the sub-sample that did not experience a rating withdrawal.

The data shows that 47% of the time the average ratings of the sub-samples is the same at year 5; 42% of the time the average ratings of the withdrawn sub-sample is lower than the not withdrawn sample by one rating notch at year 5. Hence, it appears that the average ratings of a sub-set of issuers whose ratings are ultimately withdrawn exhibit little difference compared to the average ratings of issuers whose ratings are not withdrawn over the same time period.

Table 5.4 – Withdrawn vs. Non-Withdrawn CDO Factor-Weighted Rating Notch Differences

Cohort Rating	+1 year	+2 years	+3 years	+4 years	+5 years
Aaa	0	0	0	-1	0
Aa1	0	0	-1	-1	0
Aa2	0	0	0	-1	0
Aa3	-1	-1	0	0	-1
A1	-1	-1	0	-1	-1
A2	0	0	0	-1	-1
A3	-1	-1	-1	-1	-1
Baa1	0	0	-1	-1	-1
Baa2	0	0	1	0	0
Baa3	0	-1	-1	-1	-2
Ba1	0	0	-1	-1	-1
Ba2	0	0	0	0	0
Ba3	0	0	0	-1	-1
B1	0	0	0	-1	-1
B2	0	0	0	0	0
B3	0	0	0	0	0
Caa1	0	0	-1	-1	-1
Caa2	0	0	0	0	0
Caa3	-1	0	0	0	0

Sample period: 1983-2005

[25] Moody's CDO rating factors are described in Yoshizawa and Witt (2003).

5. Conclusions

In this paper we reviewed the default rate calculation methodology that is now generally followed by most major credit rating agencies. Rating agency default rates are calculated using a non-parametric discrete-time approximation to the continuous-time estimator that either does or does not adjust for issuer rating withdrawals. We argued that unadjusted statistics, while useful statements of historical fact, are generally less useful as proxies for expected default probabilities as they rely on certain problematic assumptions. Moreover, we argued that unadjusted default rates are downwardly biased estimates of default risk. Withdrawal-adjusted default rates, on the other hand, are useful data for a variety of applications requiring expected default probabilities. We showed that the crucial assumption of the withdrawal-adjusted method, random data censoring, appears to be supported using Moody's data on rating withdrawals.

The results of this paper have implications for both investors and regulators. Investors need to be aware if data on which they are relying are adjusted for withdrawals; default rate assumptions and parameter values are, as we have shown, significantly affected by adjusting for rating withdrawals. It is also critical for making comparisons of default risk across different asset classes and for making comparisons across rating agency reports. The issue also has implications for ratings-based regulation and oversight of rating agencies. Default probabilities for rating grades are vital performance statistics. Ratings have been increasingly relied on by regulators to a large extent because of their ability to rank order default risk. Hence, agency reported historical default rates are not only viewed as backward-looking data but as *expected* default probabilities as well. For example, regulatory capital standards using the Standardized approach under Basle II were derived by assessing the empirical default probabilities for various ratings from different rating agencies. Additionally, the default rate data required for ECAI recognition is rather unclear with regard to calculation method. While a uniform reporting standard may be desirable, at a minimum agencies applying for ECAI recognition must be transparent about their calculation methods.

The foregoing discussion also suggests that a rating agency's policy with regard to its withdrawal of ratings is an important part of its overall ratings management framework. Yet most rating agencies do not provide public information about their policies for the removal of ratings. Whether rating withdrawals represent hidden credit events is a concern that with important implications for investment decision making. Our analysis showed that the assumption of random data censoring holds for Moody's data, but the neutrality result may or may not hold for other agency ratings. The statistical properties of rating withdrawals therefore require further analysis and should be part of each agency's periodic statistical reporting process.

Appendix A. 20-Year cumulative default rate tables

Table A1 – Average Cumulative Default Rates by Whole Letter Rating, Unadjusted vs. Withdrawal-Adjusted

Unadjusted

	Years After Cohort Formation Date																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Aaa	0.00	0.00	0.00	0.02	0.08	0.14	0.19	0.25	0.30	0.36	0.41	0.47	0.52	0.55	0.57	0.60	0.63	0.64	0.64	0.64
Aa	0.01	0.02	0.04	0.09	0.15	0.20	0.25	0.29	0.32	0.34	0.37	0.40	0.45	0.49	0.51	0.54	0.57	0.60	0.62	0.65
A	0.02	0.10	0.21	0.31	0.41	0.51	0.60	0.70	0.79	0.87	0.95	1.02	1.08	1.13	1.21	1.28	1.36	1.43	1.50	1.55
Baa	0.18	0.49	0.86	1.25	1.60	1.91	2.18	2.41	2.64	2.87	3.08	3.28	3.47	3.64	3.80	3.92	4.03	4.12	4.20	4.27
Ba	1.18	2.98	4.85	6.52	7.86	8.90	9.68	10.34	10.90	11.40	11.84	12.25	12.59	12.89	13.12	13.34	13.52	13.64	13.75	13.84
B	5.41	11.04	15.41	18.46	20.66	22.17	23.21	23.85	24.29	24.59	24.78	24.91	25.00	25.08	25.14	25.18	25.19	25.20	25.20	25.20
Caa-C	19.90	29.37	34.86	37.81	39.32	40.16	40.58	40.88	41.06	41.18	41.23	41.23	41.23	41.23	41.23	41.23	41.23	41.23	41.23	41.23

Withdrawal-Adjusted

	Years After Cohort Formation Date																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Aaa	0.00	0.00	0.00	0.03	0.11	0.18	0.27	0.36	0.45	0.56	0.66	0.78	0.90	0.97	1.04	1.12	1.22	1.25	1.25	1.25
Aa	0.01	0.02	0.05	0.12	0.19	0.29	0.38	0.46	0.51	0.58	0.65	0.76	0.92	1.06	1.16	1.29	1.44	1.58	1.74	1.91
A	0.02	0.10	0.24	0.37	0.51	0.67	0.83	1.01	1.22	1.42	1.63	1.82	2.02	2.21	2.49	2.81	3.17	3.53	3.88	4.20
Baa	0.18	0.53	0.98	1.52	2.06	2.60	3.13	3.65	4.23	4.89	5.50	6.17	6.85	7.56	8.24	8.84	9.41	9.97	10.44	10.91
Ba	1.23	3.31	5.75	8.26	10.57	12.65	14.48	16.28	18.05	19.86	21.62	23.41	25.15	26.82	28.29	29.78	31.14	32.17	33.15	33.97
B	5.65	12.35	18.65	24.09	29.06	33.50	37.47	40.71	43.59	46.12	47.56	48.77	49.65	50.51	51.26	51.77	51.96	52.12	52.12	52.12
Caa-C	21.12	33.53	43.47	51.01	56.52	61.05	64.58	68.50	71.98	74.72	75.16	75.16	75.16	75.16	75.16	75.16	75.16	75.16	75.16	75.16

Sample Period: 1970-2005, monthly cohort spacing

Appendix B. Corporate bond pricing with rating withdrawals

Using a reduced form model similar to that of Fons (1994), we show that the credit spread required by a risk-neutral investor is independent of the rate withdrawal; i.e., the frequency that a bond prepays prior to maturity. The equation below represents the price a risk neutral investor would pay at time zero for a risky bond with face value of 1 payable at maturity date T. The bond pays an annual coupon $c(t)$. The bond is subject to three possible end-of-year outcomes: it may default, it may withdraw, or it may survive. In each year, the issuer may default with probability $d(t)$; if the issuer does not default, it may then pay off the bond early and withdraw with probability $w(t)$. For simplicity, we assume that loss-given-default for the bond is 100%. $r(t)$ denotes the risk-free rate.

$$P(0,T) = \sum_{i=1}^{T}\left[\left[\prod_{i=1}^{T}[1-d(i)][1-w(i-1)]w(i)\right]\left[\sum_{i=1}^{T}\left(\frac{c(i-1)}{\prod_{t=1}^{T}(1+r(i-1))}\right)+\frac{1+c(i)}{\prod_{t=1}^{T}(1+r(i))}\right]\right]$$

$$+\sum_{i=1}^{T}\left[\left[\prod_{i=1}^{T}[1-d(i-1)][1-w(i-1)]d(i)\right]\sum_{i=1}^{T}\left(\frac{c(i-1)}{\prod_{t=1}^{T}(1+r(i-1))}\right)\right]$$

$$+\left[\prod_{i=1}^{T}[1-d(i)][1-w(i)]\right]\left[\sum_{i=1}^{T}\left(\frac{c(i)}{\prod_{t=1}^{T}(1+r(i))}\right)+\frac{1}{\prod_{t=1}^{T}(1+r(i))}\right]$$

The three terms on the right-hand side of the equation are the probability weighted payoffs in the event of withdrawal, default, and survival to maturity, respectively. In the event of a withdrawal prior to maturity, an investor receives all the coupons and principal up to the withdrawal date. If default occurs in year t, an investor only receives coupons paid prior to t and loses the entire face value upon default. The last term in the equation is the payoff if the bond survives – i.e. does not withdraw and does not default – to maturity.

Suppose the coupon paid in each year is equal to the sum of the risk free rate and the marginal default probability adjusted for survival,

$$c(t) = \frac{r(t)+d(t)}{1-d(t)}.$$

Upon substituting this value of c(t) into the formula, it can easily be seen that the price of the bond at time 0 should be 1 and all the terms involving $w(t)$ drop out. That is, the bond will price at par if the coupon rate is set equal to the risk free rate plus an adjustment for default risk; however, the withdrawal rate has no impact on the required spread. Since credit spreads are compensation for bearing risk, only marginal default rates matter for pricing, not marginal withdrawal rates.

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