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CREDIT RATINGS-BASED MULTIPLE HORIZON DEFAULT PREDICTION

By

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Abstract

We estimate a model of natural default probabilities conditional on credit ratings and macroeconomic drivers. The output is an issuer-specific expected default rate at variable horizons, which can be combined to form an expected default rate for a given portfolio of rated credits. This permits us, for the first time, to associate an expected default rate with a rating category conditional on the future path of the economy. In fact, the model outputs an entire issuer-specific transition matrix, of which default is only one possible state. The other states include the entire credit rating scale, as well as withdrawal. We are thus able to assign probabilities to the complete future path of a credit rating conditional on the economy.

<I> Introduction

Probabilities associated with ratings transitions, in particular to the *default* state, are often key inputs in credit risk models. Common practice is to collect historical frequencies for a given horizon in a transition matrix of the type presented in Tables I.1 and I.2 below. This is certainly a reasonable thing to do, as any more elaborate statistical model is going to be centered around the same averages, but its limitations as a predictor of future rating migrations are fairly obvious: actual transitions are pro-cyclical and generally non-Markovian. As evidence of the first point, Figure I.1 compares the one year forward default with the one year forward change in the U.S. unemployment rate.[1,2] As evidence of the second point, Figure I.2 compares Kaplan-Meier estimates of the cumulative probability of downgrading for newly issued single-B issuers, those just downgraded, and those just upgraded The probability of downgrading further is substantially higher for those credits which were just downgraded themselves, and substantially lower for upgraded issuers.

More sophisticated models of rating transition in general and default in particular have been advanced which address these concerns. Most share the drawback that they are *horizon dependent*, such that obtaining transition probabilities for different horizons requires different models. This is at least statistically inefficient, and at worst may yield contradictory forecasts.

In this paper we present a model of rating transition which may be applied at multiple horizons. We study the complete set of transitions, including to the states *default* and *withdrawal*.

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[1] Throughout this paper, the *default rate* for a given pool of issuers over a given horizon is the share of those issuers which are *observed* to enter default at some point within that horizon. No adjustment for withdrawal is made. Consequently, the default rate statistics presented herein will not generally correspond to those presented in other Moody's publications. See Cantor and Hamilton (2007).

[2] The unemployment rate has been smoothed with a centered 3 quarter tent filter. See Section III.

The result is a transition matrix which is conditional on issuer-specific characteristics and macroeconomic drivers. One can thus quantify for a particular rated credit (and hence any portfolio of credits) the probability of upgrading to a particular rating, or downgrading, or defaulting, or withdrawing, all over given horizons of interest. The model itself is of the familiar proportional hazards type.

The issuer-specific factors the model conditions on include the current rating, whether the issuer was upgraded or downgraded into its current rating, how long the issuer has maintained its current rating, and how long the issuer has consecutively maintained *any* credit rating. We restrict ourselves to a parsimonious function of the macroeconomy and consider just two drivers: the unemployment rate and the high yield spread over treasuries.[3]

The structure of the model is similar to Figlewski, Frydman and Liang (2006). However, where they are interested in the explanatory power of various macroeconomic drivers, our primary interest is in constructing a forecasting tool. Thus we parameterize the baseline transition intensities and perform full maximum likelihood estimation.[4] Another model of similar structure is Duffie and Wang (2003) in which they condition on a Merton variable rather than credit ratings. This almost certainly offers improved discrimination in considering the transition to default, which is the focus of their study. But our focus is really *all* rating transitions.[5] We do not include a Merton variable because it would require us to model not only the dynamics of the Merton variable in addition to those of ratings, but to ensure at each point that a reasonable correspondence between the two was maintained.[6] Nesting the two remains an avenue of interesting future research.

Finally, a word about frailty. Some interesting recent research has been conducted modeling the dynamics of an unobserved factor in the context of proportional hazards models, see for example Duffie et al (2006) and Koopman et al (2006). This is motivated from the observation that most macroeconomic specifications are unable to account fully for correlation across defaults, and it is well known that failing to account for unobserved heterogeneity can significantly bias subsequent parameter estimates. While we certainly do not dispute this, we would point out that many default models are constructed at the monthly frequency, and we would not expect any survey of the macroeconomy to explain the timing of defaults at such a high frequency.[7] Earlier versions of this research allowed for a time-invariant unobserved mixing factor, but in recent iterations this factor has not been significant and has subsequently been omitted. Future efforts may include incorporating a richer, dynamic frailty factor, but of course it does to some extent defeat the purpose of a forecasting model to attribute too much explanatory power to an unobservable.

This paper is organized as follows. Section II describes the data, and Section III sketches the model. Examples of model output are presented in Section IV, while Section V is devoted to a brief discussion of parameter and economic uncertainty in the forecasts. Select parameter estimates are presented in Section VI. Section VII considers the performance of the model at multiple horizons conditional on perfect foresight of the economy. We recognize that in

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[3] We found that including other candidates did not sufficiently improve the model performance relative to the costs of accurately forecasting them, e.g. stock returns.

[4] For forecasting transitions, the baseline temporal pattern is often more important than the economic factors especially over short horizons. To take just one (extreme) example, an issuer that was assigned a rating yesterday is unlikely to change its rating tomorrow, no matter what the state of the business cycle.

[5] There are many applications, such as CDO valuation, in which understanding the upgrade and downgrade dynamics of credit ratings is important in and of itself.

[6] There is also the added advantage that our model is applicable to entities that don't otherwise have a Merton variable.

[7] Over half of the default events we study are missed interest or principal payments, which means their timing at the monthly frequency is due to the vagaries of their covenant documents and may have nothing to do with the "state of the economy" in that particular month.

applications, that is not possible; our intention is to separate the performance of the transition model on the one hand from the performance (or lack thereof) of any particular economic forecasting model on the other. Section VIII concludes.

Table I.1 4 Quarter Transition Matrix

											4 Quarters Later											
Current Rating	**Aaa**	**Aa1**	**Aa2**	**Aa3**	**A1**	**A2**	**A3**	**Baa1**	**Baa2**	**Baa3**	**Ba1**	**Ba2**	**Ba3**	**B1**	**B2**	**B3**	**Caa1**	**Caa2**	**Caa3**	**Ca**	**C**	**WR**	**DEF**
Aaa	**89**	3	3	0		0																5	
Aa1	3	**82**	5	5	0	0	0	0														5	
Aa2	1	3	**79**	8	2	1	0		0	0												7	
Aa3	0	1	3	**79**	7	2	1	0	0		0			0								6	
A1	0	0	0	5	**80**	7	2	1	0	0	0	0	0									5	
A2	0	0	0	1	5	**79**	7	3	1	0	0	0	0		0			0	0			4	0
A3	0	0	0	0	1	8	**74**	7	3	1	0	0	0	0	0	0		0	0	0		4	0
Baa1	0	0	0	0	0	2	6	**75**	8	3	1	0	0	0	0	0	0	0		0		4	0
Baa2	0	0	0	0	0	1	2	6	**76**	7	1	1	1	1	0	0	0	0	0	0		5	0
Baa3	0	0		0	0	0	1	2	8	**73**	5	3	1	1	0	0	0	0	0	0	0	5	0
Ba1			0	0	0	0	0	1	2	9	**65**	5	4	1	1	1	0	0	0	0	0	8	0
Ba2			0	0	0	0	0	0	1	3	8	**63**	6	4	2	1	1	0	0	0		9	1
Ba3			0	0	0	0	0	0	0	1	3	7	**65**	5	5	2	0	0	0	0		10	2
B1	0	0		0	0	0	0	0	0	0	1	2	6	**66**	6	4	1	1	0	0		9	3
B2	0		0	0	0	0	0	0	0	0	0	0	2	5	**67**	7	3	1	1	0	0	9	4
B3		0	0		0		0	0		0	0	0	0	2	5	**61**	5	4	1	1	0	11	9
Caa1						0				0		0	0	1	2	5	**59**	5	4	2	1	11	10
Caa2						0			0	0	0	0	0	1	1	2	3	**54**	3	3	1	13	18
Caa3											0		0	1	1	2	3	3	**45**	3	3	13	25
Ca													1		1	1	1	2	2	**43**	3	19	28
C															0	1		0	2	2	**45**	24	25

Table I.2 20 Quarter Transition Matrix

											20 Quarters Later											
Current Rating	**Aaa**	**Aa1**	**Aa2**	**Aa3**	**A1**	**A2**	**A3**	**Baa1**	**Baa2**	**Baa3**	**Ba1**	**Ba2**	**Ba3**	**B1**	**B2**	**B3**	**Caa1**	**Caa2**	**Caa3**	**Ca**	**C**	**WR**	**DEF**
Aaa	**56**	7	10	3	1	1	0		0	0	0											20	
Aa1	9	**46**	10	9	3	3	1	2	1	0			0									17	
Aa2	4	6	**32**	16	5	6	3	2	0	0	0	0										26	
Aa3	1	4	8	**35**	16	9	4	2	1	0			0	0								20	
A1	1	2	3	9	**33**	15	8	4	2	1	1	1	0	0		0		0				20	0
A2	0	1	1	3	11	**34**	14	8	4	2	1	1	1	0	0	0	0			0		20	0
A3	0	0	0	2	4	19	**24**	13	8	5	2	2	1	1	0	0	0	0		0		17	1
Baa1	1	1	0	1	2	6	12	**29**	13	7	2	2	1	1	0	0	0	0	0	0	0	19	1
Baa2	0	0	0	1	1	3	6	11	**31**	13	3	2	2	2	1	1	1	0	0	0		20	1
Baa3	1	0	0	0	1	2	3	6	15	**26**	7	4	4	2	2	1	1	0	0	0	0	23	3
Ba1	0	0	0	0	1	2	3	3	7	11	**18**	5	4	5	4	1	1	0	0	0	0	31	4
Ba2		0		0	0	0	1	2	3	8	9	**13**	7	7	5	2	1	0	0	0	0	36	5
Ba3	0		0	0	0	0	1	1	2	3	4	6	**13**	7	6	3	1	1	0	0	0	39	11
B1	0	0	0	0	0	0	0	0	0	2	2	3	6	**13**	7	5	2	1	1	1	0	39	17
B2	0			0		0	0	0	0	0	1	1	2	6	**15**	8	4	2	1	1	0	35	23
B3						0	0	0	0	0	0	1	1	4	4	**10**	3	2	1	1	0	38	32
Caa1									0	0	1	1	1	3	3	4	**8**	3	1	1	1	36	39
Caa2								0	0	0	0	1	1	2	1	2	2	**5**	1	2	1	41	40
Caa3											1	1		1	1		1	0	**2**			31	62
Caa3											0			0	2	2	2	1	1	**4**	1	46	41
Ca											1				2						**2**	34	62

Figure I.1 One Year Default Rates and the Change in the Unemployment Rate: Defaults are Pro-Cyclical

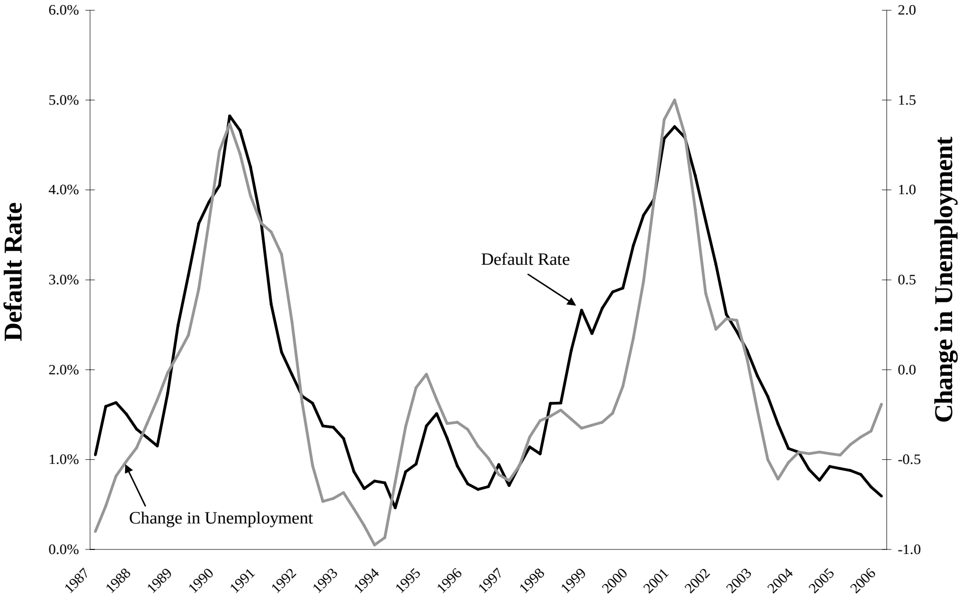
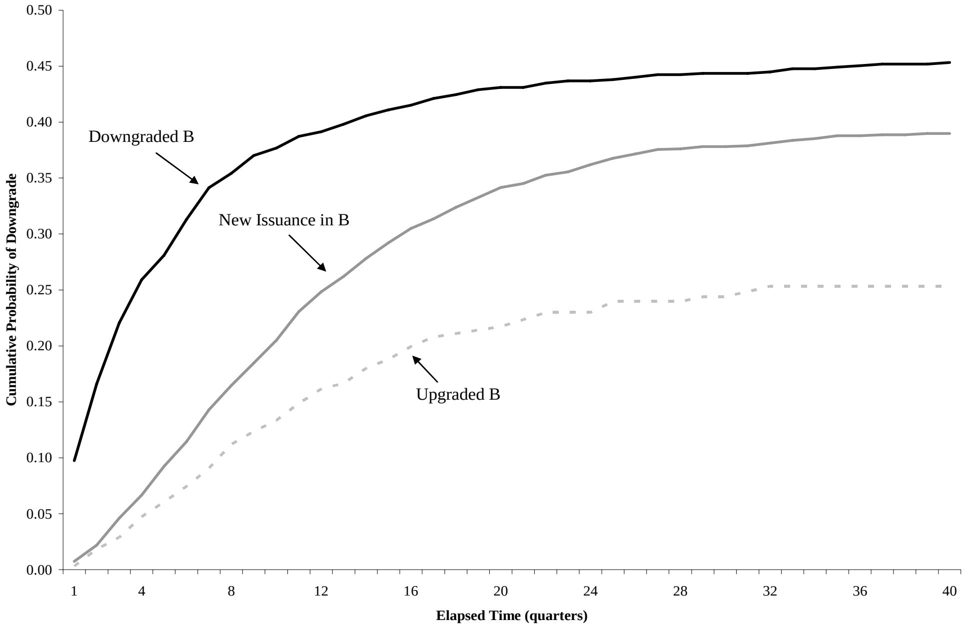


Figure I.2 Cumulative Probability of Downgrade for Single B Issuers: Much Higher for Recently Downgraded Issuers, Lower for Recently Upgraded Issuers



<II> Data

The ratings data for this study are Moody's estimated senior unsecured ratings. For a detailed discussion, please see Hamilton (2005). We collect rating transitions beginning July 1, 1982 and use data beginning January 1, 1987 for estimation. As such, there is no left censoring. The data are right censored as of January 1, 2007 and we treat this as random. The issuer universe is all North American issuers, including financial institutions, utilities and corporates.[8] There are 7,181 issuers with 17,992 observed transitions or initial assignments.

The ratings data provide essentially all of our issuer-specific information: the current rating, whether the issuer was upgraded or downgraded into this rating, how long the issuer has maintained this rating, and how long the issuer has maintained any rating. We will call this last factor the *Age* of the issuer, though that is something of a misnomer.[9]

Defaults are identified through Moody's proprietary default database which covers over 3,600 long-term bond defaults by both rated and unrated issuers. Defaults are defined to include missed or delayed interest or principal payments, bankruptcies, and distressed exchanges. In some cases, issuer ratings are withdrawn. We identify this as a separate exiting state but do not distinguish the reasons for withdrawal. Cantor & Hamilton (2007) presents a thorough discussion.

Tables II.1, II.2 and II.3 present summary statistics of the ratings transition data.[10] They distinguish those issuers that were newly assigned to a rating category, upgraded into a rating category, and downgraded into a category. Table II.1 reports that newly issued Caa2 issuers have a 10.1% chance of exiting *directly* to default,[11] while Table II.2 indicates that issuers upgraded to Caa2 have a lesser probability(5.0%) of direct default. These are dwarfed by the values of Table II.3 which reports a 45.1% probability of directly defaulting for those issuers downgraded into the Caa2 rating.

These summary statistics also indicate that newly assigned issuers almost always have longer average durations in a category than either upgraded or downgraded issuers to that category, which in fact have almost equal average durations. This is more pronounced in spec-grade ratings (e.g. the average duration for a newly assigned B2 rating is 10.5 quarters, while for issuers upgraded or downgraded to B2 the average duration is 7.3 and 7.0, respectively).

Conditional on exit (either upgrade, downgrade, withdraw or default), there isn't much difference in expected durations for newly assigned ratings, though there is some evidence that the longer one remains in a category, the more likely it is to withdraw. For the upgraded or downgraded issuers, there is strong evidence of momentum in the form of shorter durations conditional on exiting in the same direction and much longer durations conditional on reversing direction. This is more pronounced for investment-grade ratings.

Our economic data include the U.S. civilian unemployment rate (BLS series LNS14000000). The high yield spread over ten year treasuries is provided by the Lehman high yield index. We study transitions at the quarterly frequency. Specifically, we sample the ratings database on the first of January, April, July and October of each year to determine if a rating transition has occurred.[12] We average our monthly economic data to obtain the quarterly value.

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[8] Public and Structured Finance issuers are excluded, as are any Government Related Issuers.

[9] In a few cases, issuers have had their rating withdrawn (or defaulted) only to have another rating assigned later. In these cases, we reset the *Age* to 0.

[10] Additional statistics are presented in the Appendix.

[11] The probability of *ultimately* defaulting is, of course, higher.

[12] Any rating spells which occur entirely within the quarter will not be observed, but these are very few in number. What are most likely to be missed are those cases where an issuer rating transitions to C just prior to default.

Table II.1 Summary Statistics: Initial Rating Assignments

			Upgrade		Downgrade		Withdraw		Default	
Rating	**#**	**Duration**	**%**	**Duration**	**%**	**Duration**	**%**	**Duration**	**%**	**Duration**
Aaa	62	27.0			48.4	19.8	21.0	25.3		
Aa1	74	15.3	10.8	6.4	60.8	11.8	9.5	10.0		
Aa2	95	16.8	9.5	15.6	52.6	16.0	20.0	14.3		
Aa3	452	9.5	5.3	14.2	17.0	14.6	11.7	11.7		
A1	591	8.5	8.3	10.8	14.7	16.6	6.3	12.1		
A2	203	15.7	22.2	10.1	43.8	15.6	14.3	13.4		
A3	237	13.9	31.2	11.5	44.7	12.1	10.5	15.8		
Baa1	240	14.0	26.3	11.2	44.6	13.0	11.3	16.3		
Baa2	316	14.8	24.7	12.9	39.9	14.0	14.9	14.6		
Baa3	286	13.9	38.1	12.4	32.5	13.5	9.4	16.9	0.7	18
Ba1	257	12.1	33.9	10.6	37.0	10.6	20.6	13.2	0.4	2.0
Ba2	279	10.8	25.1	11.4	38.7	9.3	25.1	11.0	0.7	9.0
Ba3	497	11.1	25.8	10.8	36.2	10.5	26.0	11.6	2.4	11.5
B1	815	10.7	22.6	10.2	35.2	11.3	28.5	10.7	4.4	8.9
B2	881	10.5	19.2	10.8	35.2	10.6	22.2	11.2	4.3	9.7
B3	711	8.1	16.3	8.2	23.1	9.8	24.6	8.9	4.2	10.9
Caa1	283	7.8	12.7	6.7	30.0	8.6	19.8	8.8	4.2	14.3
Caa2	149	8.2	16.1	7.4	20.1	9.3	34.2	7.1	10.1	11.1
Caa3	28	7.0	14.3	4.8	28.6	5.1	21.4	11.7	7.1	7.0
Ca	26	8.0	23.1	8.7	3.8	6.0	50.0	8.2	23.1	7.2
C	3	6.0					66.7	1.0	33.3	16.0

Durations are means, measured in quarters
Percentages are shares of new issuances in rating category. Percentages do not sum to 1 because of right censoring.

Table II.2 Summary Statistics: Upgraded Assignments

			Upgrade		Downgrade		Withdraw		Default	
Rating	**#**	**Duration**	**%**	**Duration**	**%**	**Duration**	**%**	**Duration**	**%**	**Duration**
Aaa	67	21.4			23.9	25.5	31.3	15.5		
Aa1	86	16.4	15.1	13.2	15.1	16.2	26.7	13.3		
Aa2	135	13.7	28.9	9.2	19.3	15.6	31.9	14.4		
Aa3	247	12.4	26.7	8.8	24.7	13.2	22.3	10.0		
A1	269	16.0	32.3	11.5	24.9	20.7	21.2	12.4		
A2	412	15.8	29.1	12.5	31.3	17.2	18.4	12.2	0.2	15
A3	344	13.0	32.3	11.3	30.5	15.4	19.2	9.0		
Baa1	356	12.2	36.2	8.7	26.4	15.9	15.7	9.0	0.6	7
Baa2	395	11.1	34.7	10.4	21.5	13.5	23.0	9.7	0.3	17
Baa3	339	10.9	38.6	11.1	20.1	10.8	19.2	9.2	0.6	25.5
Ba1	336	7.9	43.2	7.5	16.7	10.9	26.8	6.4		
Ba2	352	7.3	42.0	6.4	15.6	9.7	26.7	7.0		
Ba3	357	7.7	37.0	6.3	19.6	11.1	26.3	7.3	0.3	8.0
B1	328	8.0	33.2	6.5	20.7	10.4	20.4	8.7	1.8	8.3
B2	186	7.3	41.4	6.5	13.4	10.9	15.6	8.6	1.6	6.0
B3	102	6.3	30.4	6.5	15.7	5.8	23.5	5.6	2.9	9.0
Caa1	43	5.5	34.9	5.8	11.6	5.0	18.6	5.3	4.7	6.0
Caa2	20	5.0	40.0	5.3	25.0	6.0	5.0	8.0	5.0	5.0
Caa3	9	4.2	66.7	3.8	11.1	1.0	11.1	12.0		
Ca	1	4.0			100.0	4.0				
C										

Durations are means, measured in quarters
Percentages are shares of upgrades to rating category. Percentages do not sum to 1 because of right censoring.

Table II.3 Summary Statistics: Downgraded Assignments

			Upgrade		Downgrade		Withdraw		Default	
Rating	**#**	**Duration**	**%**	**Duration**	**%**	**Duration**	**%**	**Duration**	**%**	**Duration**
Aaa										
Aa1	51	12.8	13.7	11.3	64.7	7.7	9.8	6.4		
Aa2	109	10.7	5.5	22.8	62.4	9.6	19.3	8.7		
Aa3	185	12.1	7.0	18.6	55.1	10.8	19.5	8.4		
A1	284	12.5	12.0	17.4	65.5	10.1	15.5	12.3		
A2	381	12.7	14.2	15.3	54.6	11.3	14.7	10.7		
A3	505	10.3	24.4	14.1	49.3	7.8	10.1	9.2		
Baa1	475	10.4	18.3	12.7	52.2	7.8	11.6	10.4	0.2	9
Baa2	526	10.1	19.6	13.1	50.2	6.8	10.5	12.2		
Baa3	523	8.9	24.7	11.7	47.0	5.8	10.3	11.1	0.4	1
Ba1	358	6.8	22.1	9.8	51.4	4.4	12.3	6.9	0.3	1.0
Ba2	356	6.5	19.9	10.4	56.5	4.5	9.0	7.3	2.8	1.7
Ba3	425	6.8	20.0	9.4	53.2	4.9	13.2	10.2	4.0	4.8
B1	391	7.8	23.5	9.6	49.1	6.2	12.5	9.9	2.8	5.8
B2	622	7.0	16.1	8.3	45.5	5.0	17.7	10.2	9.2	5.6
B3	720	5.8	13.8	7.7	33.1	4.8	15.7	8.3	25.1	3.7
Caa1	414	5.2	14.0	7.6	34.3	3.7	14.5	8.9	20.8	3.3
Caa2	375	4.7	11.2	8.1	17.9	3.7	13.1	9.8	45.1	2.9
Caa3	187	4.4	12.8	6.0	18.7	3.2	15.5	5.7	48.1	3.7
Ca	178	3.6	5.6	6.2	5.6	4.0	20.2	5.3	61.2	2.5
C	58	4.5	6.9	8.3			31.0	5.7	55.2	2.8

Durations are means, measured in quarters
Percentages are shares of downgrades to rating category. Percentages do not sum to 1 because of right censoring.

<III> The Model

Notation

We begin by establishing basic notational conventions. Consider the density governing the probability of transitioning from a rating category *r* to an exiting state *s* after elapsed time *u* conditional on no prior exit from *r*. This density is a function of elapsed time *u* in the rating category *r*,[13] observed (time-varying) covariates x_t,[14] and an unobserved (time-invariant) mixing factor *v*:[15]

$$h_s^r(u \mid x, v) \equiv \psi_s^r(u)\theta_s^r(x)v \qquad \text{(III.1)}$$

where $\psi_s^r(u)$ denotes the baseline transition intensity for rating category *r* to exit state *s* and $\theta_s^r(x)$ is a strictly positive function of the observed covariates for rating category *r* to exit state *s*. These transition intensities are used, per usual, to obtain the densities governing the probability of exiting from rating *r* to state *s* at time *u*:

$$f_s^r(u \mid x, v) = h_s^r(u \mid x, v) \cdot \overline{F}(u \mid x, v) \qquad \text{(III.2)}$$

$$\overline{F}(u \mid x_t, v) = \exp\left\{-\sum_{s=1}^{S}\int_{t-u}^{t} h_s^r(\tau \mid x_{t+\tau}, v)d\tau\right\} \qquad \text{(III.3)}$$

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[13] We denote elapsed time by *u* to distinguish it from calendar time *t*.

[14] Generally we will suppress the time-dependence of the covariates unless it is necessary for the exposition.

[15] The unobserved heterogeneity could, in general, be rating and state specific.

Finally, we integrate over the marginal distribution of *v* to obtain the empirical density:

$$f_s^r(u \mid x) = \int f_s^r(u \mid x, v) dG_s^r(v) \tag{III.4}$$

This will be familiar as a standard application of the multiple-destination mixed proportional hazards model. Also familiar is the assumption of conditional independence across issuers.[16] In our application, time is measured discretely, thus we use (III.4) to obtain the probability of an exit to a particular state occurring within a window of time (a, b] as:

$$\int_a^b f_s^r(u \mid x) du \tag{III.5}$$

Owing to the parameterizations discussed below, this simplifies to:

$$\frac{h_s^r(b \mid x)}{\sum_{i=1}^{S} h_i^r(b \mid x)} \cdot \left(1 - \exp\left\{-\sum_{i=1}^{S} h_i^r(b \mid x)\right\}\right) \cdot \overline{F}(a \mid x) \tag{III.6}$$

Restrictions

In practice, we do not estimate a free transition intensity from every rating category to every viable exit state, but instead impose certain restrictions across categories. In particular, we define three classes of ratings, the investment-grade *IG* process, the non-*C* speculative-grade *SG* process, and the *C* process. Individual rating categories within these larger processes are distinguished only as scalar transformations of the underlying process.

For example, the transition intensity for a Aaa rating is given by:

$$h_s^{Aaa}(u \mid x, v) \equiv \alpha_s^{Aaa} \cdot h_s^{IG}(u \mid x, v) = \alpha_s^{Aaa} \cdot \psi_s^{IG}(u) \theta_s^{IG}(x) v_s^{IG} \tag{III.5}$$

Of course, by including the appropriate indicator variables within *x*, we can write that as:

$$h_s^{Aaa}(u \mid x, v) = \psi_s^{IG}(u) \theta_s^{Aaa}(x) v_s^{IG} \tag{III.6}$$

with the understanding that $\theta_s^{Aaa} = \alpha_s^{Aaa} \theta_s^{IG}$.[17]

It is worth noting that there are virtually no restrictions imposed across processes. The parameters of the *IG, SG* and *C* processes are independent of each other. The only exceptions are those parameters related to the *Age* of an issuer which are constrained to be equal across processes but are independent across exit states.

---

[16] For an interesting discussion of the conditional independence assumption, see Das et al (2005).

[17] The scale effect is allowed to change over calendar time. In particular, we allow for a structural break in all means pre- and post-1997, corresponding to the introduction of additional C rating modifiers.

Exiting States

One could define exiting states to be all other rating categories as well as the absorbing states *default* and *withdrawal*. This strikes us as impractical, and we instead distinguish the states *upgrade*, *downgrade above C*, *downgrade to C*, *default* and *withdraw*.

Not every rating process transitions to every exiting state. In particular from *IG*, one can *upgrade*, *downgrade above C* or *withdraw*.[18] All exiting states are viable from *SG*. Obviously from *C*, *downgrade above C* is undefined, but all other exit states are viable.

These exiting states apply to individual rating categories, not just whole aggregate processes. In other words, a particular *IG* rating can "exit to *downgrade*" to a different *IG* rating – not just downgrade from one aggregate process to another. Our use of aggregate processes simply imposes some structural discipline on our estimates: up to scale, all *IG* ratings are identical, having the same baseline transition shapes and the same betas to the macroeconomic drivers. But we track movements within the broader processes as upgrades and downgrades.

Of course, this implies certain rating category-specific restrictions beyond those described above. The Aaa category cannot exit to *upgrade*, even though the broader *IG* process has a defined *upgrade* transition. Similarly, the B3 category cannot exit to *downgrade*, but only to *downgrade to C*.

Conditioning Information

The issuer-specific conditioning information includes, in addition obviously to elapsed time *u*:

- *Rating Category*
  As discussed above, the specific rating category determines the scale of the underlying process.

- *Rating History*
  We condition on whether (and when) an issuer was upgraded or downgraded into its current rating. We allow a flexible dynamic response for up to 12 quarters following the upgrade or downgrade.

- *Age*
  Age is here defined as how long the issuer has continuously maintained *any* Moody's rating. In those few cases where an issuer defaults or withdraws and later re-enters the data set, its *Age* is reset to 0. *Age* enters as a quadratic function, and is capped at 40 quarters.

- *Cumulative Change in Unemployment*
  We condition on the cumulative change in unemployment since the issuer entered its current rating. The notion is that having endured a period of increasing unemployment should increase the probability of *default* and *downgrade* even if the current economic state is relatively strong.

The macroeconomic conditioning information is more elaborate. We apply a three quarter centered tent smoothing filter to both the unemployment rate and the high yield spread to filter

---

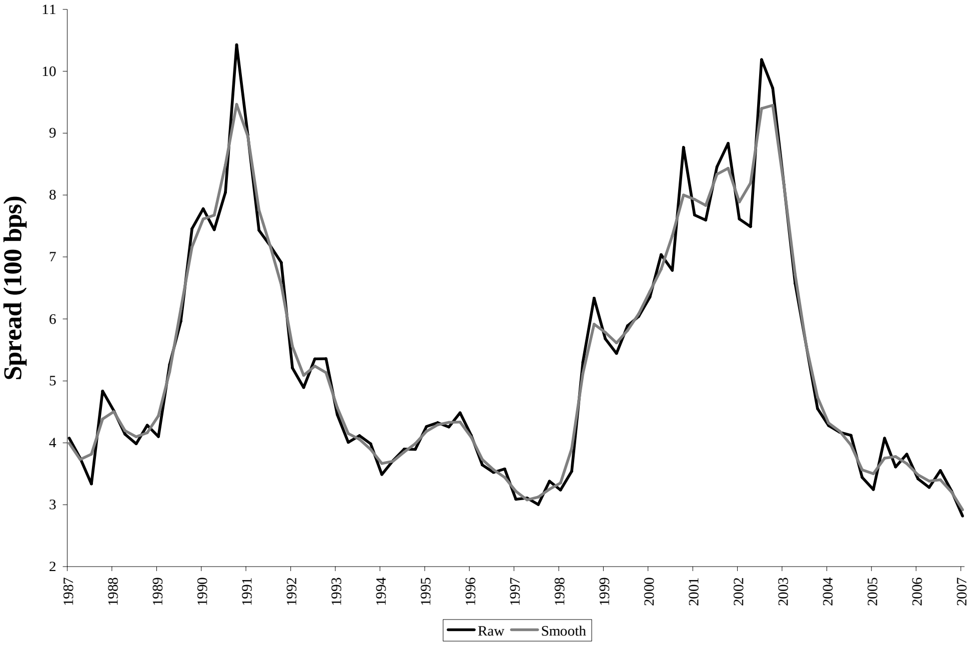
[18] In our data set, which covers the last 20 years, there have been only 11 instances of transitioning from an *IG* rating to default when transitions are measured at the quarterly frequency. Similarly, there are only 15 cases of transitioning from *IG* to *C*. In each case there are too few observations to separately estimate a transition from IG to *default* or *downgrade to C*. Instead, we score all of these events as *downgrades*, and for the purposes of forecast simulations, we include a non-zero probability that when downgrading from an *IG* rating, an issuer could transition directly to *C* or to *default*.

out the highest frequency noise in each series.[19] The results are presented in Figures III.1 and III.2.

For the high yield spread, we simply condition on this smoothed series. Our use of the unemployment rate is less straightforward. Time-series analysis indicates different frequency responses between the default rate and the unemployment rate; we therefore extract three different frequencies and condition on those. First, we first difference the (smoothed) unemployment rate to extract and amplify the high frequencies. Figure III.3 presents this from 1987. Second, we apply the Hodrick-Prescott filter (λ = 160000) to extract the cycle and long-run trend. These are presented in Figures III.4 and III.5.

The effect of this latter decomposition is evident in the sample period 1987-2007, shown in Figures III.6 and III.7. The stochastic trend in unemployment is decreasing even as recessions cycle around it. Separating this downward trend from the cycle is critical for default analysis, since default rates by rating category are positively related to the trend but negatively related to the cycle. While this last result may seem counter-intuitive, in fact it makes sense: default rates are highest at the start of a downturn (when unemployment rates are low but increasing) as marginal companies are first hit with negative shocks. By the time the recovery starts (when unemployment rates are high but decreasing), these marginal firms have disappeared and default rates are least.

Figure III.1 Raw and Smoothed High Yield Spread



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[19] We use the entire history of the unemployment rate since January 1948. The High Yield Spread series dates from January 1987.

Figure III.2 Raw and Smoothed Unemployment Rate

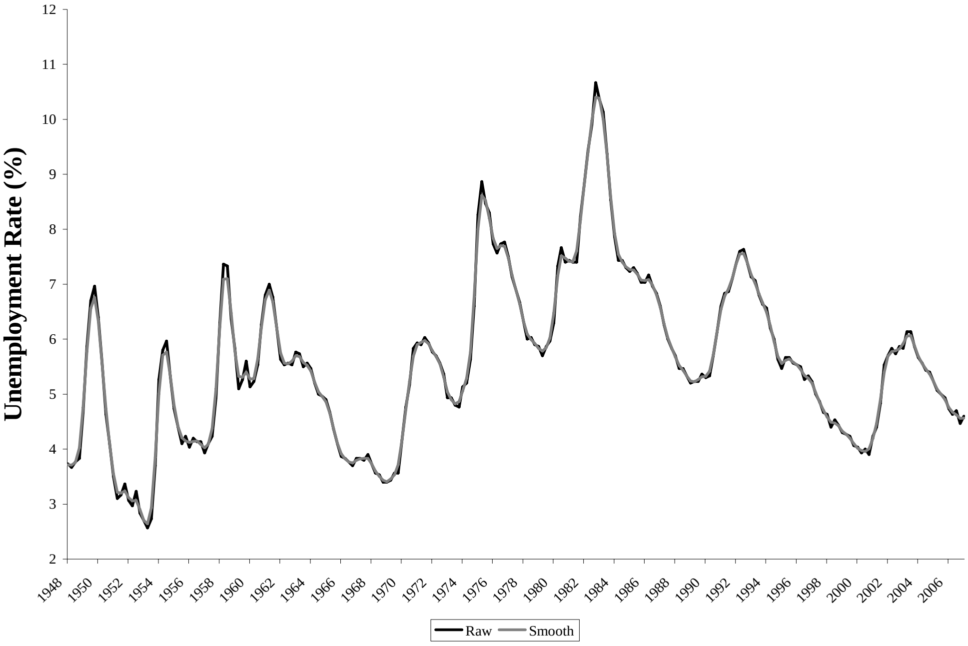

Figure III.3 First Difference of Smoothed Unemployment Since 1987

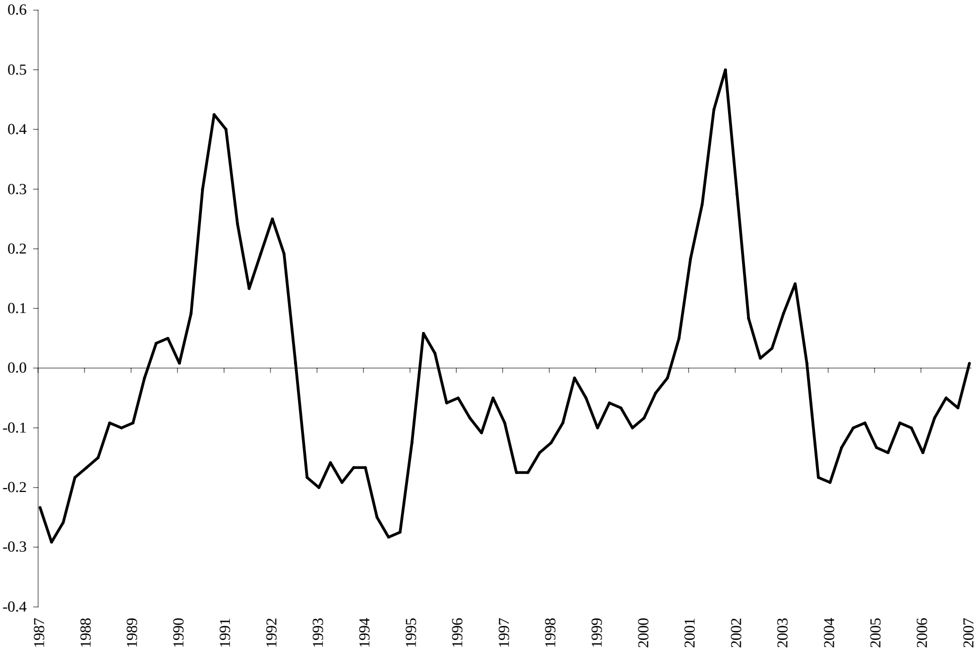

Figure III.4 Hodrick-Prescott (λ = 160000) Estimate of Stochastic Trend in Unemployment

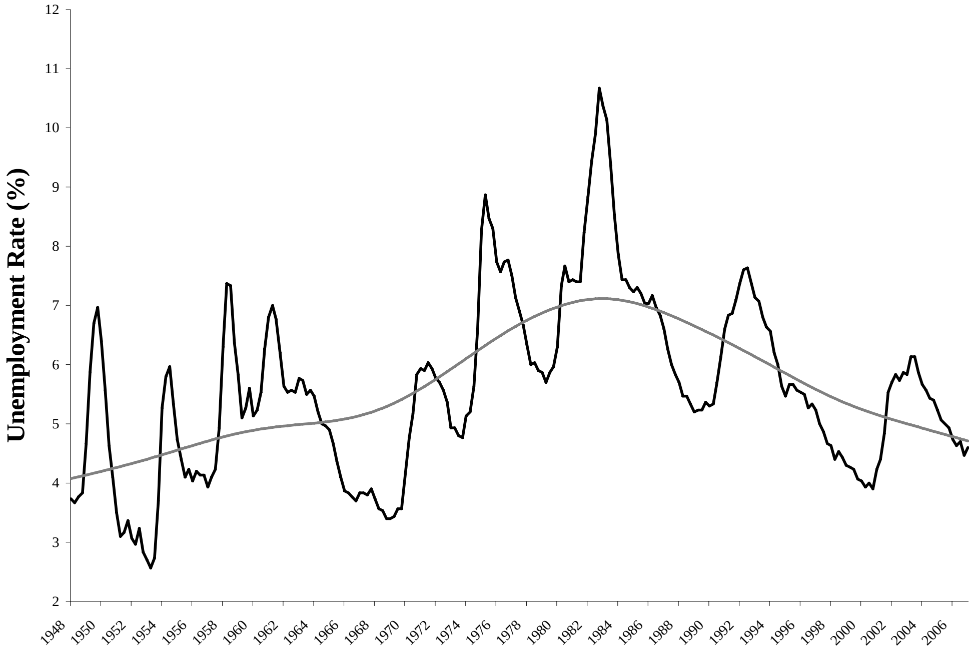

Figure III.5 Hodrick-Prescott (λ = 160000) Estimate of Cycle in Unemployment

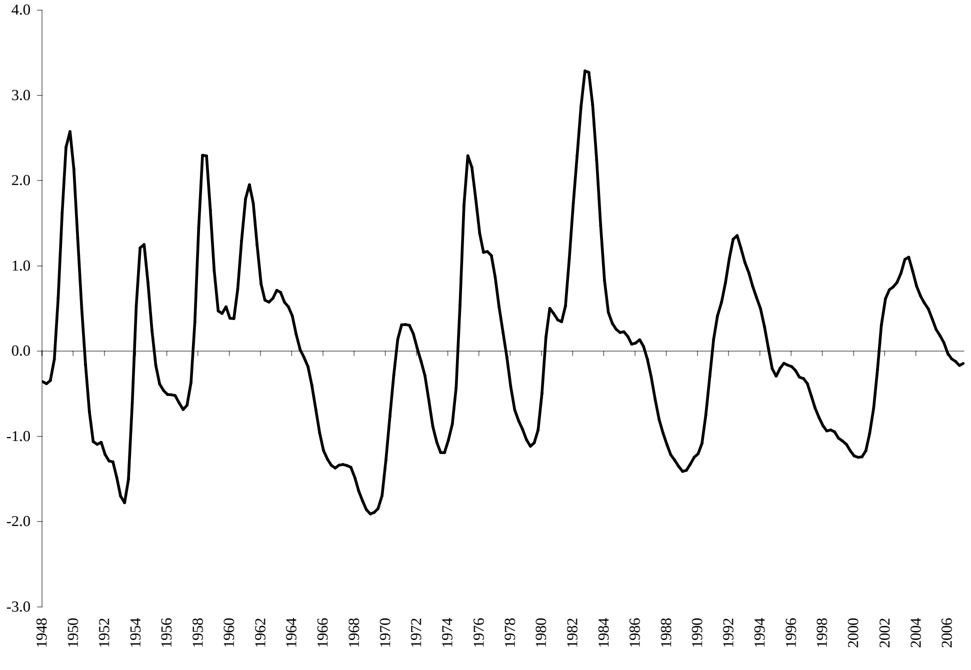

Figure III.6 Unemployment Has Negative Trend Since 1987

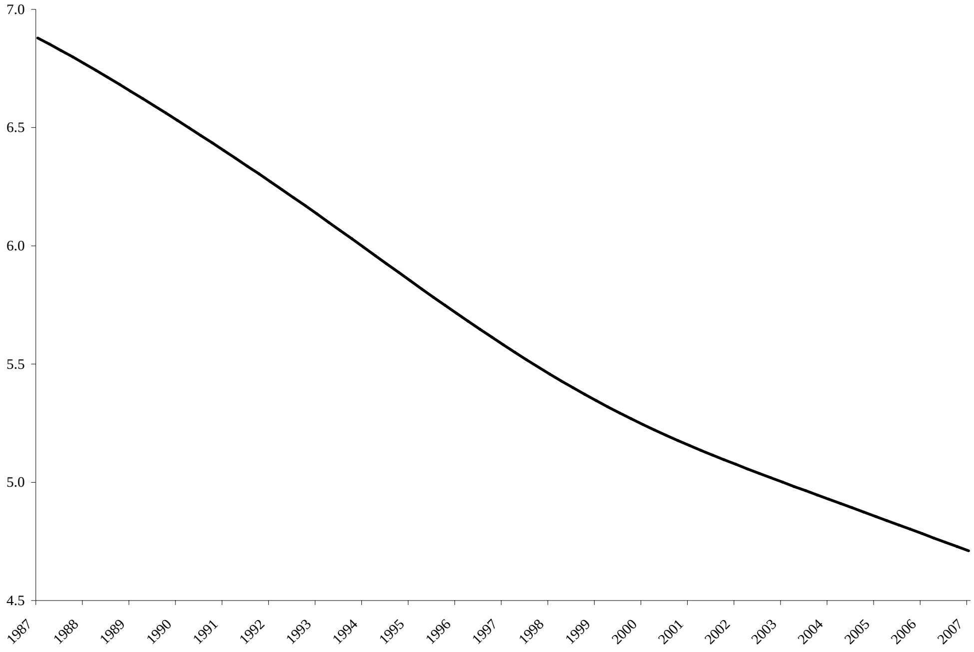

Figure III.7 De-trended Unemployment Cycles Since 1987

Parameterization

In this section we discuss details of our parameterization of the baseline intensities and covariate functions. We also discuss a structural shift in rating definitions that occurred in 1997.

- Baseline transition intensities $\psi_s^p(u)$:

  We specify step-function baseline intensities where the values are given by a piecewise linear process. We estimate break points of the linear process at quarters *u* = {1,2,3,4,5,6,8,12,16} and impose the scale normalization that $\psi_s^p(u) = 1$ for $u \geq 20$ quarters.

- Covariate functions $\theta_s^r(x)$:

  We follow standard practice and assume $\theta_s^r(x) = \exp\{x\beta_s^p\}$.

- Pre-1997 structural changes

  1997 saw the introduction of Caa rating modifiers and the subsequent expansion in the use of all C rating categories. Some issuers which previously might have been rated B3 were moved into one of the new C categories. This, of course, changed the upgrade, downgrade and default dynamics of several rating categories. We capture this by allowing all means – the scale effects of rating categories in all transitions – to change post-1997.

Forecasts

We have defined exiting states *upgrade* and *downgrade*, but by themselves these do not specify to which specific rating category the issuer upgrades or downgrades. To make that determination, we use historical transition frequencies. These are presented in Tables III.1 through III.3. To be clear, these matrices are conditional on the exit states *upgrade, downgrade above C,* and *downgrade to C.* For example, if an issuer transitions to the state *upgrade*, we consult Table III.1 to determine the probability of transitioning from its current rating to each specific rating; trivially, an issuer currently rated Aa1 can only transition to Aaa conditional on exiting to *upgrade*. If an issuer transitions to the state *downgrade to C*, we consult Table III.3.

The macroeconomy (and issuer-specific characteristics) change the absolute (and hence relative) probabilities of upgrade and downgrade, but not the probability of a *d*-notch change conditional on being changed. This is a binding restriction if we believe that large upgrades and downgrades are more likely in extremely good or bad economic times.[20]

---

[20] There is some evidence that this is true. Our in-sample fits to the net drift (upgrade notches – downgrade notches) are fairly good, but our fits to volatility (upgrade notches + downgrade notches) are less good. Since net drift is more important for default forecasting, this may not be a material drawback to the model. One possible extension would be to define the exiting states *Upgrade/Downgrade 1 Notch* and *Upgrade/Downgrade 2+ Notches* to allow the macroeconomic environment to have some impact on volatility.

Table III.1 Transition Matrix: Conditional on Upgrading

To Rating

From Rating	**Aaa**	**Aa1**	**Aa2**	**Aa3**	**A1**	**A2**	**A3**	**Baa1**	**Baa2**	**Baa3**	**Ba1**	**Ba2**	**Ba3**	**B1**	**B2**	**B3**	**Caa1**	**Caa2**	**Caa3**	**Ca**	**C**
Aaa																					
Aa1	100																				
Aa2	19	81																			
Aa3	2	17	81																		
A1	1	1	5	93																	
A2		0	4	15	81																
A3	1	1	1	2	9	86															
Baa1	1	1	1	1	1	19	75														
Baa2	1	1	1	2	1	8	19	68													
Baa3	1	0		0	2	4	4	17	72												
Ba1			0	0	2	2	2	5	18	71											
Ba2			0	0	1	1	1	2	4	21	69										
Ba3			1	0		2	1	1	3	5	22	66									
B1	0				1	1	0	2	1	2	5	19	68								
B2		0	0	0			1	1	2	1	3	5	18	68							
B3		0	1		0		1	1	0	1	1	2	3	27	63						
Caa1						1				1		2	2	6	22	67					
Caa2									1	4	3		5	5	7	31	43				
Caa3											3			12	3	9	29	44			
Ca													6		6	19	6	19	44		
C																		25	50	25	

Rating-specific transition probabilities conditional on upgrading.
Applies to ratings Aa1 and below.
Rows sum to 100 or 0.

Table III.2 Transition Matrix: Conditional on Downgrading Above *C* Ratings

To Rating

From Rating	**Aaa**	**Aa1**	**Aa2**	**Aa3**	**A1**	**A2**	**A3**	**Baa1**	**Baa2**	**Baa3**	**Ba1**	**Ba2**	**Ba3**	**B1**	**B2**	**B3**	**Caa1**	**Caa2**	**Caa3**	**Ca**	**C**	***def***
Aaa		63	35			2																
Aa1			71	25	3																	
Aa2				82	13	3			1	1												
Aa3					72	23	5	0						0								
A1						67	25	6	1			0	1									
A2							71	21	5	2	0		0		0			*0.5*				*0.2*
A3								63	26	7	2	1	0		0				*0.2*	*0.4*		
Baa1									72	19	5	1	1	1	0	0				*0.2*		*0.7*
Baa2										70	15	7	3	2	1	0	*0.4*			*0.2*		*0.2*
Baa3											54	28	11	3	1	1	*0.2*	*0.5*	*0.5*	*0.2*		*1.5*
Ba1												45	36	9	5	3	*1.2*				*0.3*	*0.6*
Ba2													51	26	16	8						
Ba3														44	42	14						
B1															60	40						
B2																100						
B3																						
Caa1																						
Caa2																						
Caa3																						
Ca																						
C																						

Rating-specific transition probabilities conditional on downgrading above C ratings.
Applies to ratings B2 and above.
For IG ratings and Ba1, this includes non-zero transition probabilities to C ratings and to default.
Rows sum to 100 or 0.

Table III.3 Transition Matrix: Conditional on Downgrading to *C* Ratings

	To Rating																			
From Rating	**Aaa**	**Aa1**	**Aa2**	**Aa3**	**A1**	**A2**	**A3**	**Baa1**	**Baa2**	**Baa3**	**Ba1**	**Ba2**	**Ba3**	**B1**	**B2**	**B3**	**Caa1**	**Caa2**	**Caa3**	**Ca**	**C**
Aaa																					
Aa1																					
Aa2																					
Aa3																					
A1																					
A2																					
A3																					
Baa1																					
Baa2																					
Baa3																					
Ba1																					
Ba2																	59	6	12	24	
Ba3																	59	24	12	6	
B1																	61	29	5	5	
B2																	61	26	8	4	
B3																	44	39	9	9	0
Caa1																		44	36	16	4
Caa2																			35	48	17
Caa3																				61	39
Ca																					100
C																					

Rating-specific transition probabilities conditional on downgrading to a C rating.
Applies to ratings between Ba2 and Ca, inclusive.
Rows sum to 100 or 0.

<IV> Model Output

We now present some examples of the model output. Our baseline forecasts will condition on a static economy with unemployment trend constant at 5.6% and the high yield spread constant at 530 bps, their sample averages. Table IV.1 presents the model output for a newly issued B2 credit. These are the probabilities that, conditional on this economic scenario, we would observe this credit in these states at the indicated horizons. For example, one quarter from now, there is a 96% probability that this issuer would still be rated B2. Twenty quarters from now, there is a 15% chance the issuer would have entered default and a 36% chance the rating would have otherwise been withdrawn. There is, therefore, only a 48% chance that the rating would still be outstanding, and this is distributed across the rating scale as indicated. There is only a 2.5% chance that this credit will transition to investment-grade over the next 20 quarters conditional on this economy.

Table IV.1 Forecasted Probabilities for a Newly Issued B2 Rating

	Quarters Ahead					
	1	**4**	**8**	**12**	**16**	**20**
Aaa	0	0	0	0	0	0
Aa1	0	0	0	0	0	0
Aa2	0	0	0	0	0	0
Aa3	0	0	0	0	0	0
A1	0	0	0	0	0	0
A2	0	0	0	0	0	0
A3	0	0	0	0	0	0
Baa1	0	0	0	0	0	0
Baa2	0	0	0	0	0	1
Baa3	0	0	0	0	1	1
Ba1	0	0	0	1	1	1
Ba2	0	0	1	1	2	2
Ba3	0	1	2	3	4	4
B1	0	3	6	9	8	7
B2	96	82	59	40	27	19
B3	1	3	6	7	7	6
Caa1	0	1	3	4	4	3
Caa2	0	1	2	2	2	2
Caa3	0	0	1	1	1	1
Ca	0	0	0	1	1	1
C	0	0	0	0	0	0
WR	2	7	14	21	29	36
DEF	0	2	5	9	12	15

Probabilities of observing this credit in these states after elapsed time.
Columns sum to 100.

This highlights the importance of considering the withdrawal state. In many, arguably most, applications, the relevant question would be, "assuming the rating otherwise remains outstanding, what is the probability it would have defaulted within 20 quarters?" In short, it is usually appropriate to adjust the probabilities for withdrawal. This, of course, is straightforward to do,[21] and the results are presented in Table IV.2. Now, with no possibility of the rating otherwise withdrawing over the next 20 quarters, we see that there is a 19% chance it would have defaulted by then, and a 32% chance that it would still be rated B2, and a 4% chance it would have transitioned to investment-grade.

---

[21] We use the transition probabilities conditional on not withdrawing.

Table IV.2 Conditional Forecasted Probabilities for a Newly Issued B2 Rating

	Quarters Ahead					
	1	**4**	**8**	**12**	**16**	**20**
Aaa	0	0	0	0	0	0
Aa1	0	0	0	0	0	0
Aa2	0	0	0	0	0	0
Aa3	0	0	0	0	0	0
A1	0	0	0	0	0	0
A2	0	0	0	0	0	0
A3	0	0	0	0	0	0
Baa1	0	0	0	0	1	1
Baa2	0	0	0	0	1	1
Baa3	0	0	0	0	1	1
Ba1	0	0	0	1	1	2
Ba2	0	0	1	2	3	3
Ba3	0	1	3	4	6	6
B1	0	3	7	11	13	12
B2	99	88	69	52	40	32
B3	1	4	7	10	10	11
Caa1	0	1	3	4	5	5
Caa2	0	1	2	3	3	3
Caa3	0	0	1	1	1	1
Ca	0	0	0	1	1	1
C	0	0	0	0	0	0
WR						
DEF	0	2	5	10	15	19

Probabilities of observing this credit in these states after elapsed time, conditional on not otherwise withdrawing over this horizon. Columns sum to 100.

Figure IV.1 compares the five year cumulative default forecast[22] for a newly issued B2 credit with a three year old credit just downgraded to B2 and a three year old credit just upgraded to B2. There is little difference between the new issuance and the upgraded credit, but the default probabilities for the downgraded issuer are substantially greater. Four quarters hence, there is a 1.6% default probability for the new issuer versus 1.4% for the upgraded and 9.8% for the downgraded.

A seasoning effect is evident in Figure IV.2. This presents the cumulative five year default probability for original issued B2 credits as a function of the issuer age and time elapsed in the B2 rating. We see a significant increase, peaking at 12 quarters after which it declines slowly.

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[22] Henceforth we will only report results that are adjusted for withdrawal unless otherwise indicated.

Figure IV.1 20 Quarter Cumulative Default Forecast

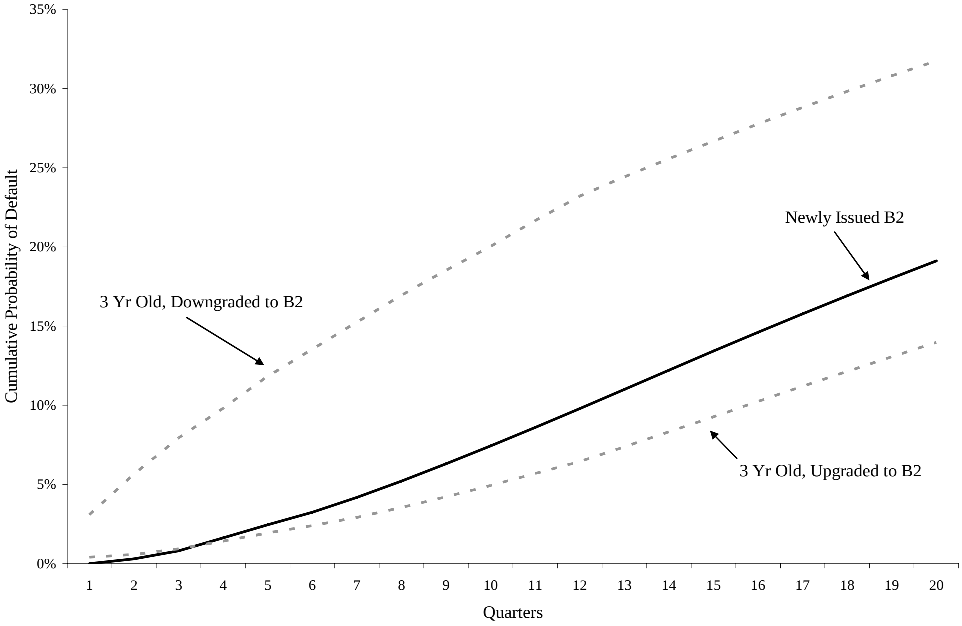
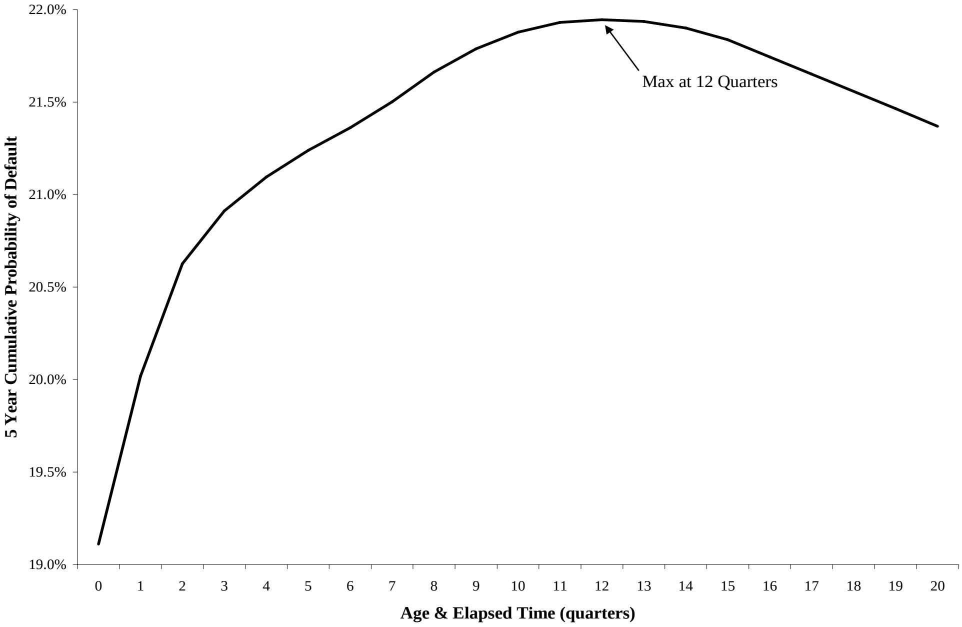
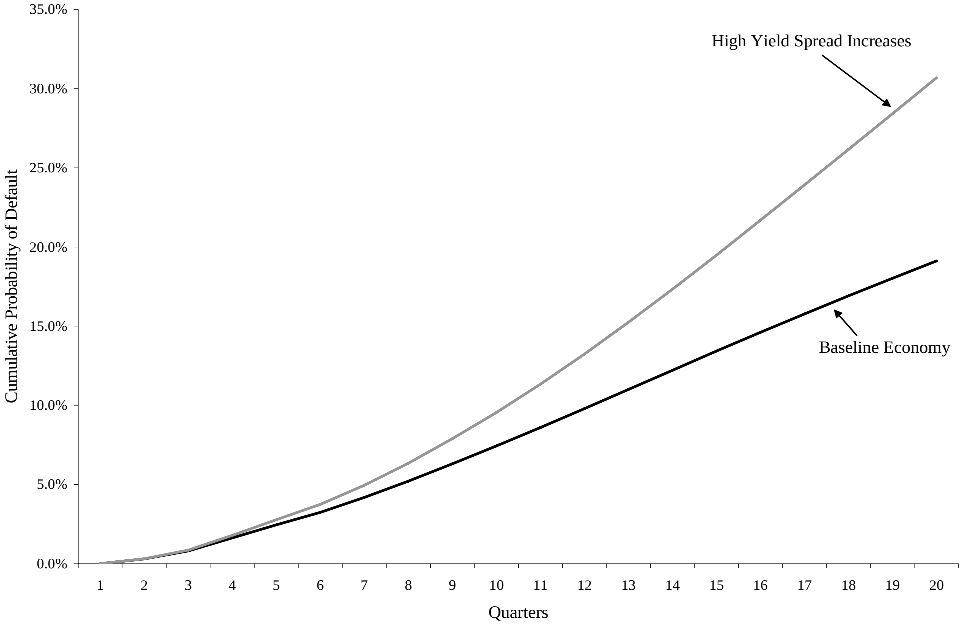


Figure IV.2 A Seasoning Effect: 20 Quarter Cumulative Default Rates for B2-Rated Issuers as a Function of Issuer Age



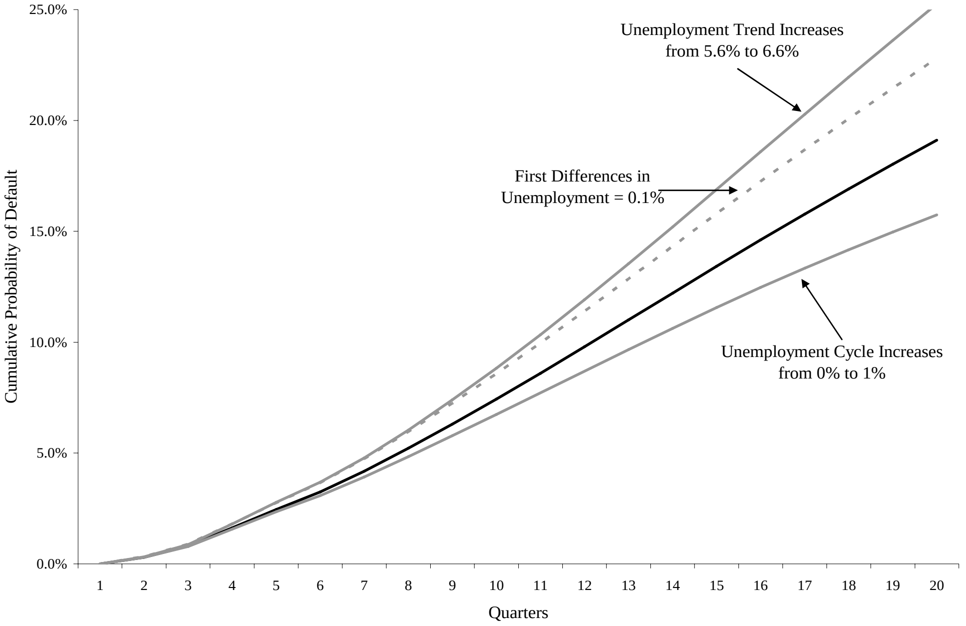
In Figure IV.3, we compare this baseline forecast against an economy where the unemployment rate continues to be static at 5.6%, but the high yield spread steadily increases from 530bps to 1,060bps over the forecast horizon. As expected, the forecast under the deteriorating economy is substantially worse.

Figure IV.3 The Effect of Spreads Increasing from 530 to 1060 bps for a New B2 Issuer



Default rates have very different responses to different frequencies of the unemployment rate. Figure IV.4 compares the response to increases in the underlying trend from 5.6% to 6.6%, increasing the cycle from 0% to 1%, and constant first differences of 0.1%. We see a positive relationship with the trend and with differences, but a negative association with the cycle.

Figure IV.4 Isolating the Frequency Responses to Unemployment for a New B2 Issuer



<V> Forecast Uncertainty

When generating transition probabilities, there are two sources of uncertainty: we have estimated model parameters, and we will generally be conditioning on an economic forecast. In this section we study these separately.

To incorporate parameter uncertainty into our forecasts, we can of course appeal to the Delta method. The results for our baseline forecast are presented in Figure V.I. The 95% confidence interval for the 20 quarter (adjusted) default rate ranges from 14.6% to 23.6%.

What about for a small portfolio? We will form a portfolio of one new Baa2, Ba2, B2, and Caa2 issuers. Figure V.II presents the 95% confidence interval for this portfolio. For the 20 quarter forecast, the standard errors for the individual issuers are 0.3%, 1.4%, 2.3% and 5.1% respectively. For the portfolio, it is 1.8%. Figure V.III plots the coefficient of variation for the individual issuers and for the portfolio. Perhaps surprisingly, they are monotonically *decreasing* with the forecast horizon. Beginning at the 7 quarter horizon, the coefficient of variation for the portfolio is less than any of its constituents. While there is a rather large degree of uncertainty for any individual issuer, we can be optimistic that it is decreasing as our portfolio size increases.

Figure V.I 95% Confidence Interval for Newly Issued B2 Security in Baseline Economy

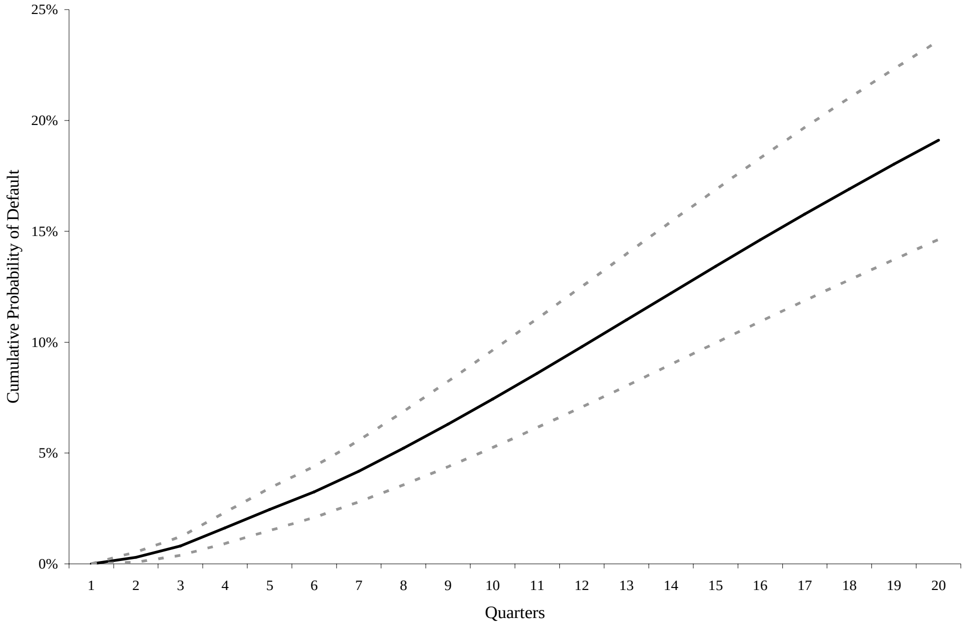


Figure V.II 95% Confidence Interval for Portfolio of New Baa2, Ba2, B2 and Caa2 in Baseline Economy

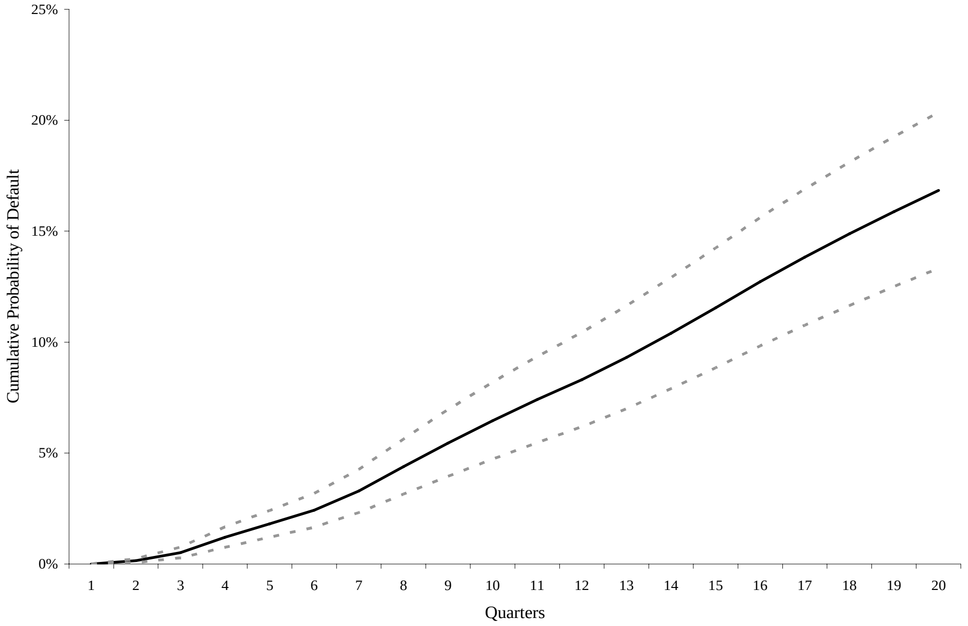
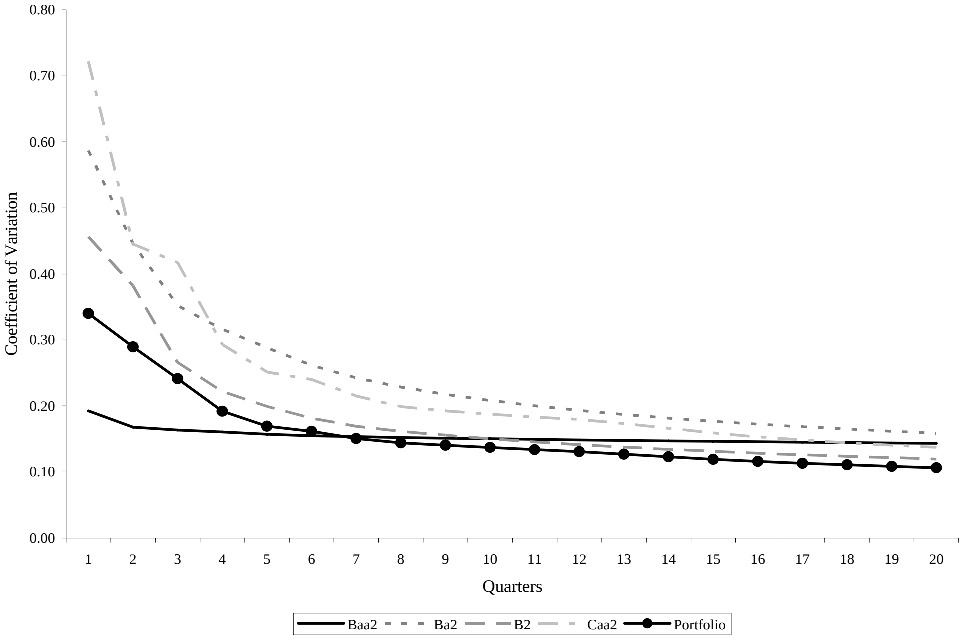


Figure V.III Coefficients of Variation for the Portfolio and Constituents



The case of economic uncertainty is a bit different. We cannot appeal to the Delta method *per se*, because there are no "asymptotics" driving our forecast uncertainty to zero. We can, of course, apply the Delta method mechanics – that is, take a first order linear approximation to our function and apply the known economic forecast covariance matrix. Ideally we would perform a Monte Carlo analysis of the expected default rate as a function of the economic forecast, but this is computationally unfeasible for large portfolios (it takes nearly 1 second to generate a 20 quarter forecast for a single issuer). Yet a third option is to take a second-order approximation to the model and perform a Monte Carlo simulation on that. In general, we have found that a first-order approximation is very good, and that a Monte Carlo simulation applied to the second-order approximation is almost indistinguishable from the function itself. Considering the computational time required, we would recommend the first-order approximation as the best choice.

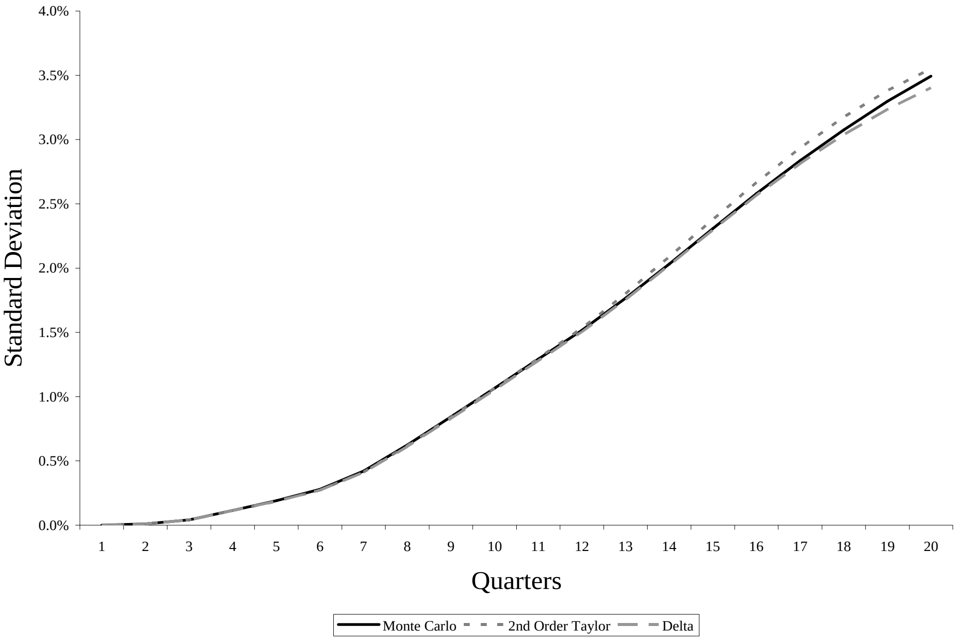
Figure V.IV compares the these three estimates of standard deviation for our small portfolio around a simple VAR economic forecast.[23] Given the known expected forecast and covariance matrix and assuming Gaussian innovations, our Monte Carlo simulation consists of drawing 2,000 times from that distribution and generating 2,000 different estimates of the portfolio's expected default rate over the next 20 quarters. Our second method consists of constructing a second-order Taylor approximation to the transition model, and then performing the same type of Monte Carlo simulation over that. Our third approach is to apply the Delta

---

[23] Since the details will change as the economic forecasting model changes, we will not bother describing our VAR specification. Our only purpose here is to compare these three methods.

method mechanics.[24] This, of course, is computationally the least costly. We see from Figure V.IV that all three approaches yield essentially the same answer.

Figure V.IV Comparing Estimates of the Standard Deviation of the Portfolio Default Rate as a Function of the Economic Forecast



There is, of course, more to the story than simply estimating the standard deviation. The distribution of default rates may not be symmetric, let alone Normal; a proper confidence interval would account for that. Our Monte Carlo simulation over the complete function indicates a 95% confidence interval for the 20th quarter default rate forecast of 16.4% to 30.5%. The second-order Taylor approximation comes close, yielding an interval of 16.0% to 29.8%. The "Delta method" approach is the least accurate, suggesting an interval of 16.0% to 29.5%. Nevertheless, considering the computational costs, we believe the "Delta method" is the best choice.

<VI> Econometric Estimates

In this section we present select details of our parameter estimates, organized by exiting state. Standard errors are estimated by the usual information matrix.

Default

We begin by examining our estimates of the baseline transition intensity from the non-C speculative grade ratings directly to the default state. Recall that the individual ratings Ba1 through B3 are just scale adjustments of the underlying aggregate process. Figure VI.I presents

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[24] Strictly speaking, we apply first- and second-order approximations to the log default probability, and then exponentiate.

this aggregate baseline up to scale (within the 50% confidence interval). Recall that we impose that all baselines are constant at the value "1" after 20 quarters, so the "units" of these figures are not well defined. We see that, aside from a slight bump at 4 quarters, the intensity is smoothly rising with elapsed time.

Figure VI.II presents the baseline intensity for the C aggregate process, again with the value normalized to 1 at 20 quarters. Unlike the non-C spec grade, the pattern here is much more erratic, with the intensity rising sharply after 12 quarters of elapsed time.

Figure VI.III compares the baseline intensity for new issuance with that of an upgraded or downgraded issuer for the SG process. The transition intensity is about half for the upgraded issuer, but the magnitude and pattern is significantly different for the downgraded issuer. Beyond the different magnitudes, the shape is different: default transition intensities peak in the first quarter following a downgrade, and then decay rapidly over the next 4 quarters until they are more or less constant thereafter.

Figure VI.IV is analogous for the C process. However, in our dataset there are too few cases of upgraded C issuers defaulting to estimate a separate process. As with the SG process, default intensities actually peak in the first quarter following a downgrade and decay from there.

Figure VI.I Baseline Default Transition Intensity and 50% Confidence Interval for the SG Aggregate

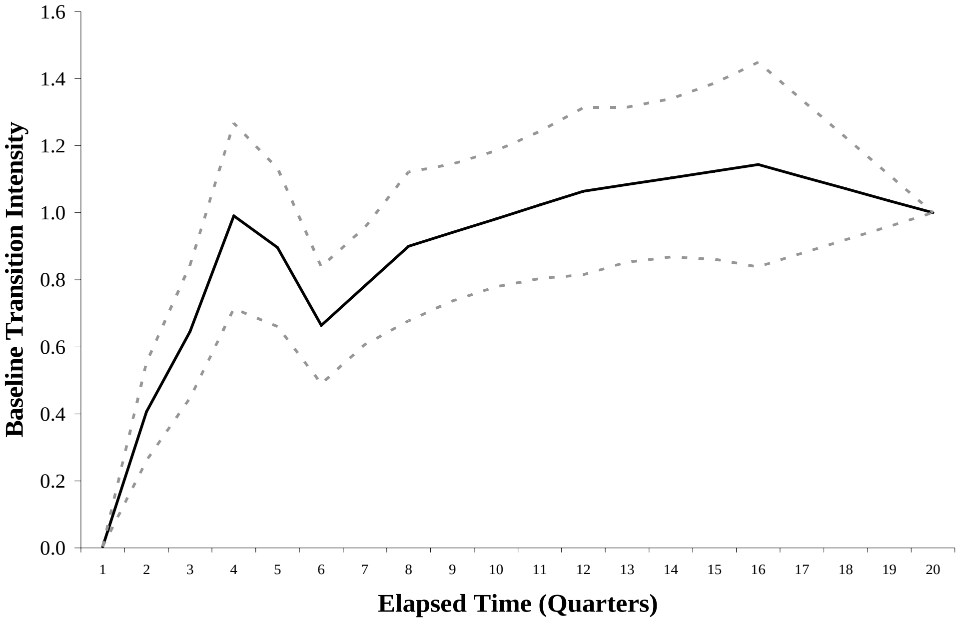


Figure VI.II Baseline Default Intensity and 50% Confidence Interval for C Aggregate

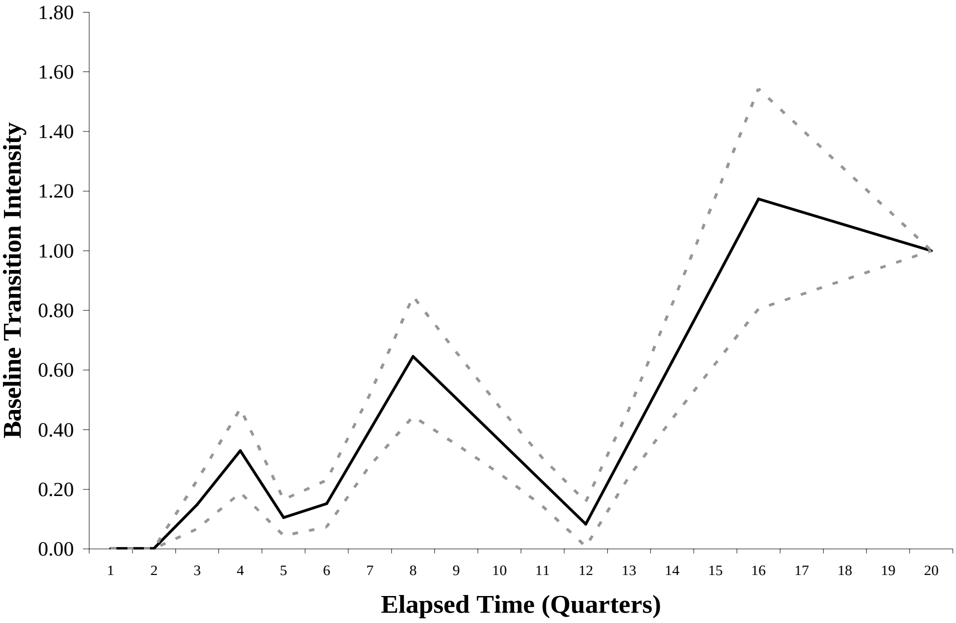


Figure VI.III Comparing Default Intensities: New Issuance, Just Downgraded and Just Upgraded for the SG Aggregate

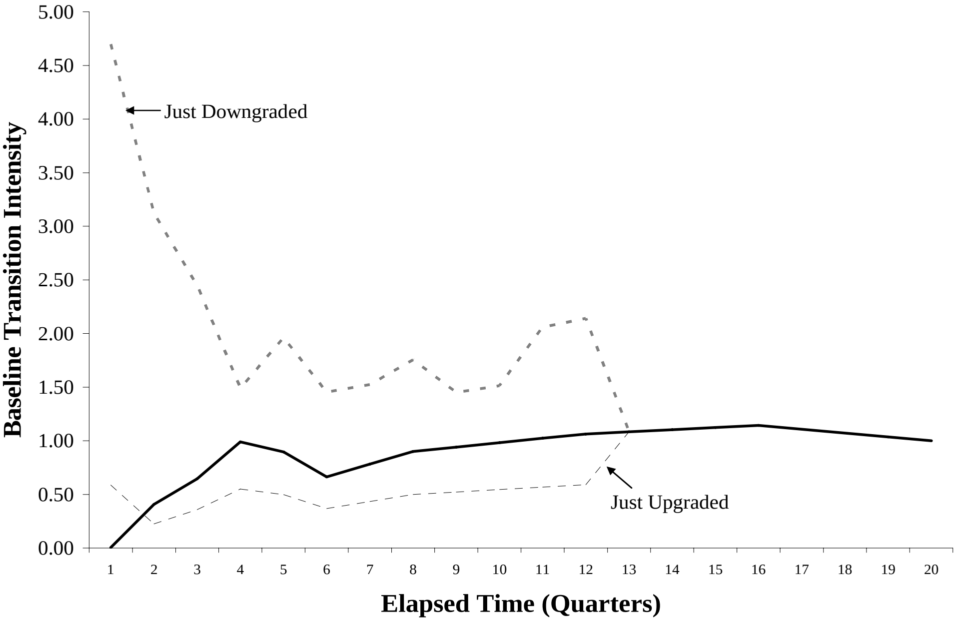
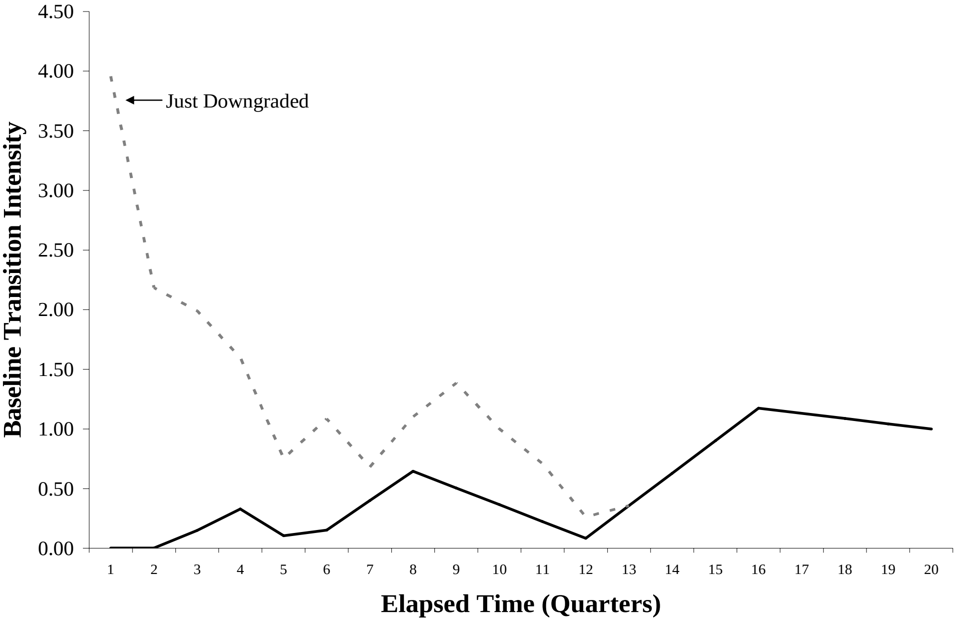


Figure VI.IV Comparing Default Transition Intensities: New Issuance and Just Downgraded for the C Aggregate



Before turning to the coefficient estimates on the economic covariates, a few words of caution are probably warranted. It is not generally possible to ascertain the full effect of an economic covariate simply by looking at its coefficient on an individual transition process. For example, suppose we want to know the impact of an increase in credit spreads on default rates. There are four avenues by which such an increase can impact the observed default rate: it can increase the probability of transitioning directly from a rating to default, it can increase the probability of downgrading (which subsequently increases default rates, as we saw above), it can decrease the probability of upgrading or it can decrease the probability of withdrawing.[25] Any of these will result in an increase in the observed default rate. Nevertheless, a study of the individual coefficients is useful in ascertaining partial effects. The difference is somewhat analogous to partial versus general equilibrium analysis.

With that in mind, we now examine the coefficients of the economic covariates on the default transition intensity. Not surprisingly, increases to the unemployment rate increase the transition intensity for both the *SG* aggregate and the *C* aggregate.[26] The *SG* aggregate is positively related to the trend in unemployment as well, though the *C* aggregate is nominally negative. What may be surprising is that higher levels of the unemployment cycle are associated with lower default rates for both processes. This combination suggests that, all else equal, default rates are higher at the start of a downturn (low but increasing unemployment) and lowest at the start of a recovery (high but decreasing unemployment).

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[25] If a rating is withdrawn, we will not *observe* a subsequent default. Research relating to the neutrality of rating withdrawals would suggest that they do not affect the *actual* default rate.

[26] Recall that we do not separately identify a transition from the *IG* aggregate directly to default.

The default transition intensity is positively related to the high yield spread for the spec-grade aggregate, but statistically and numerically it has no relation for the *C* aggregate. This would suggest that "bad credit environments" are already fully characterized by the distribution of *C* ratings. In other words, as credit conditions deteriorate and the spread widens, we would expect to see more *C* rated issuers, but given that an issuer already has a *C* rating, its probability of (direct) default is not changing with the spread.[27]

Table VI.I Coefficient Estimates and T-Statistics for Default Transition Intensity

	SG Aggregate	**C Aggregate**
Current Δ Unemployment Rate	0.66	0.51
	(1.45)	*(1.44)*
Cumulative Δ Unemployment Rate	0.13	0.30
	(1.34)	*(2.71)*
Unemployment Cycle	-0.21	-0.52
	(-2.10)	*(-6.91)*
Unemployment Trend	1.07	-0.18
	(3.75)	*(-0.70)*
High Yield Spread	0.15	0.05
	(2.93)	*(1.39)*

Withdrawal

Table VI.II presents the coefficient estimates for the withdrawal transition.[28] All three aggregates are positively if not always significantly related to the level of the unemployment cycle, and all three are, again to varying levels of significance, negatively related to credit spreads. Otherwise, the picture is somewhat mixed. For investment-grade ratings, withdrawal is positively related to the cumulative change in unemployment. For the *C* aggregate, withdrawals are negatively associated with the unemployment trend. Loosely speaking, the economic impact on the transition to withdrawal seems to be almost the mirror image of the transition to default.

Table VI.II Coefficient Estimates and T-Statistics for Withdrawal Transition Intensity

	IG Aggregate	**SG Aggregate**	**C Aggregate**
Current Δ Unemployment Rate	0.05	-0.03	0.84
	(0.18)	*(-0.13)*	*(1.50)*
Cumulative Δ Unemployment Rate	0.09	0.02	-0.03
	(2.13)	*(0.44)*	*(-0.31)*
Unemployment Cycle	0.24	0.09	0.15
	(4.35)	*(1.84)*	*(1.12)*

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[27] While credit spreads do not (statistically) increase the direct default transition or the downgrade to *C* transition, they do decrease the upgrade transition. Working together, therefore, we would expect to see default rates increase as credit spreads rise.

[28] Baseline transition intensities are presented in the Appendix.

Unemployment Trend	0.07 *(0.50)*	0.08 *(0.68)*	-1.61 *(-3.93)*
High Yield Spread	-0.05 *(-1.92)*	-0.06 *(-2.49)*	-0.05 *(-0.85)*

Upgrade

Table VI.III presents the coefficient estimates for the withdrawal transition.[29] All three aggregates are negatively (if not significantly) associated with the high yield spread and with the cumulative change in unemployment, as we would expect. Somewhat surprisingly, all three are positively, and usually significantly, related to the current change in unemployment. The effect of the unemployment cycle is mixed however. Higher levels of cyclical unemployment reduce upgraded transitions for the IG process, as one would expect, and this effect is more statistically important than the credit spread. However, the association is positive for the C aggregate. The unemployment trend is insignificant in all cases.

Table VI.III Coefficient Estimates and T-Statistics for the Upgrade Transition Intensity

	IG Aggregate	**SG Aggregate**	**C Aggregate**
Current Δ Unemployment Rate	0.59 *(2.74)*	0.81 *(3.41)*	0.97 *(1.50)*
Cumulative Δ Unemployment Rate	-0.16 *(-4.94)*	-0.07 *(-1.92)*	-0.22 *(-1.53)*
Unemployment Cycle	-0.11 *(-2.42)*	0.05 *(1.14)*	0.50 *(3.37)*
Unemployment Trend	-0.10 *(-0.96)*	-0.01 *(-0.11)*	0.24 *(0.87)*
High Yield Spread	-0.03 *(-1.54)*	-0.15 *(-6.52)*	-0.14 *(-2.01)*

Downgrade Above C

In Table VI.IV we report the coefficient estimates for the downgrade transition, excluding transitions to *C* level ratings.[30] The results are more intuitive than for the upgrade transition. Downgrades in both the *IG* and *SG* processes are positively related to the cumulative increase in unemployment (though the current increase warrants no additional weight) and the level of the high yield spread. Somewhat surprisingly however, *SG* downgrades are negatively impacted by the unemployment cycle and trend.

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[29] Baseline transition intensities are presented in the Appendix.

[30] Baseline transition intensities are presented in the Appendix.

Table VI.IV Coefficient Estimates and T-Statistics for the Downgrade Above C Transition Intensity

	IG Aggregate	SG Aggregate
Current Δ Unemployment Rate	0.13	0.10
	(0.92)	*(0.51)*
Cumulative Δ Unemployment Rate	0.08	0.10
	(3.19)	*(2.67)*
Unemployment Cycle	0.02	-0.09
	(0.59)	*(-2.00)*
Unemployment Trend	0.15	-0.33
	(1.58)	*(-2.80)*
High Yield Spread	0.14	0.10
	(9.91)	*(5.32)*

Downgrade to C
Table V.V presents results for downgrade transitions to *C* level ratings.[31] Downgrades are positively, and sometimes significantly, related to credit spreads and changes in unemployment. However, the cycle and trend are negatively related to downgrades, and for the *SG* aggregate significantly so.

Table VI.V Coefficient Estimates and T-Statistics for the Downgrade to C Transition Intensity

	SG Aggregate	C Aggregate
Current Δ Unemployment Rate	0.52	0.25
	(1.84)	*(0.60)*
Cumulative Δ Unemployment Rate	0.07	0.25
	(1.00)	*(2.20)*
Unemployment Cycle	-0.17	-0.12
	(-2.40)	*(-1.26)*
Unemployment Trend	-0.68	-0.37
	(-2.98)	*(-1.36)*
High Yield Spread	0.13	0.06
	(4.32)	*(1.46)*

<VII> Evaluating the Transition Model

The model will generate probabilities of transition to every rating category as well as *default* and *withdrawal*. These probabilities are conditional on the path of the economy, specifically the unemployment rate and high yield spread over Treasuries. We can thus separate two distinct

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[31] Baseline transition intensities are presented in the Appendix.

forecasting problems: forecasting the economy, and forecasting rating transitions conditional thereon. As it is well beyond the scope of this paper to discuss economic forecasting, we concentrate only on the latter problem. We thus present simulations to the default and withdrawal states at one and five year horizons given perfect foresight of the economic path.

Is this merely an in-sample fit? The parameters are estimated in-sample, but to take just one example, a default forecast beyond one quarter requires generating probabilities of every possible transition in the interim (and every subsequent transition from every transition, and so on). Suppose a new issuer is assigned a B3 rating on January 1, 2000. On April 1, 2000 there is some probability that it will have been upgraded to one of several ratings, or downgraded to one of several ratings, or defaulted, or withdrawn. We must calculate all of those probabilities. From each one of those possible new ratings, the issuer may again upgrade or downgrade (or default or withdraw) on July 1, 2000. And so on. The "probability of default" within the next five years, given the five year path of the economy, requires good predictions of each of these transition probabilities. It is somewhat analogous to using a sample of data to estimate the parameters of a VAR, but then applying the VAR sequentially to see what would have been forecasted had we had the parameter values in hand at that time. We cannot say that the resulting forecasts are truly "out-of-sample," but neither are they trivially in-sample, as is the case when comparing data to a fit line from OLS, for instance.

Figure VII.1 compares the actual and predicted one year default rates. For example, we form a cohort of all North American rated issuers on January 1, 2000, and see what percent of them we observe to default by January 1, 2001. We compare that with the average probability of default over that year generated by our model. Figure VII.2 compares the actual with the predicted default rates over a five year horizon. In both cases, the correspondence is very tight: if we had known the path of the economy, and if we had had the model parameters in our hand at that time, our predictions would have been very accurate.

Figure VII.3 and VII.4 present analogous comparisons for the withdrawal forecast. While few users are interested in withdrawal in and of itself, an accurate forecast of withdrawal is necessary to construct a good forecast of the withdrawal-adjusted default rate.

These are predictions over the entire North American rated universe, but since the model is fundamentally an issuer model, we can generate predictions for arbitrary portfolios. As an example, we will consider the single-B rated universe. In other words, we form cohorts of all issuers that are rated B1, B2 or B3 on the start date, and compare their observed and predicted default rates over one and five years in Figures VII.5 and VII.6. We continue to see very close correspondence.

Figure VII.1 Actual and Predicted 1 Year Cohort Default Rates

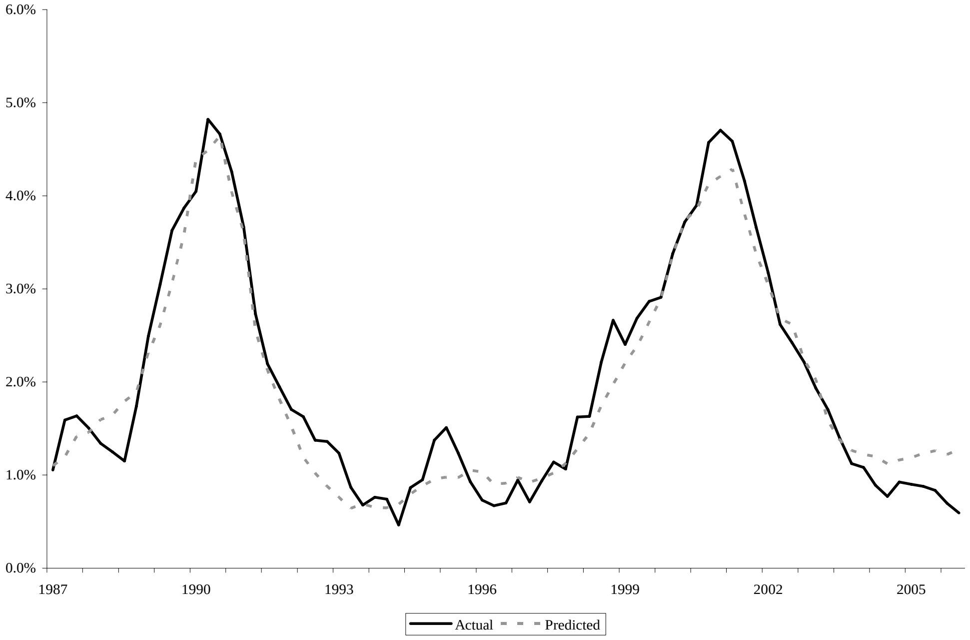


Figure VII.2 Actual and Predicted 5 Year Cohort Default Rates

Figure VII.3 Actual and Predicted 1 Year Cohort Withdrawal Rates

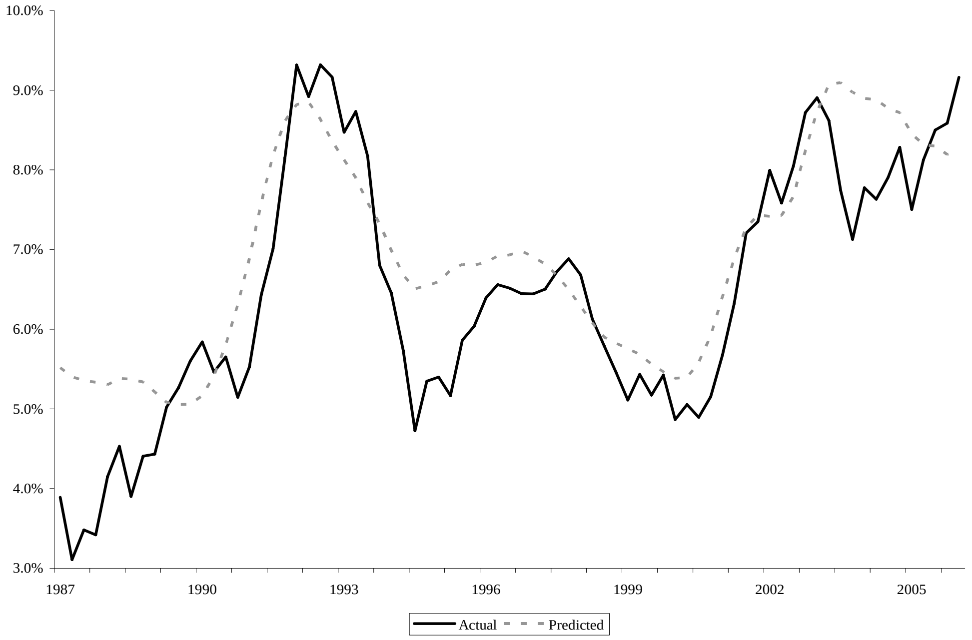


Figure VII.4 Actual and Predicted 5 Year Cohort Withdrawal Rates

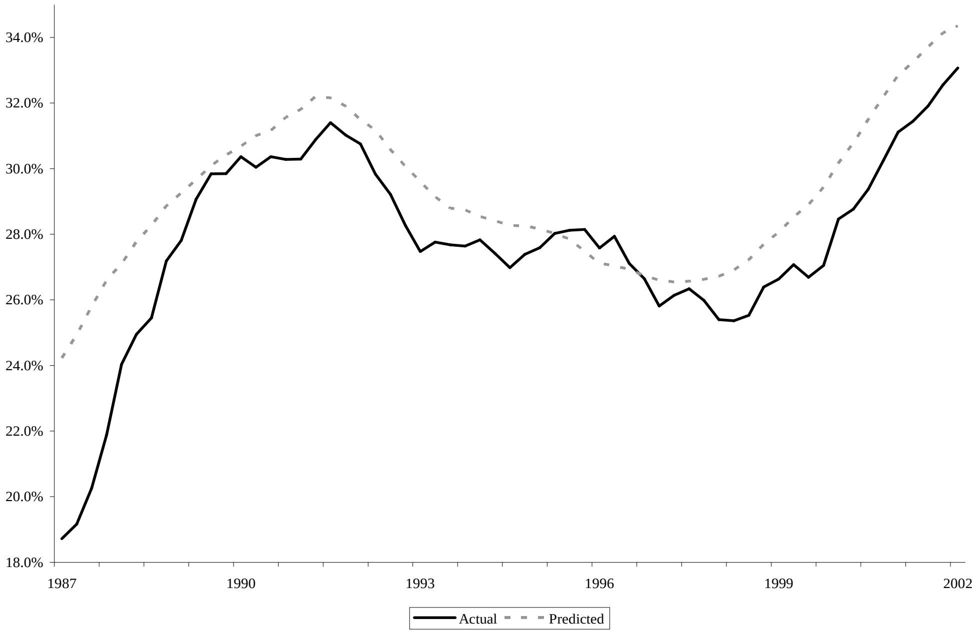


Figure VII.5 Actual and Predicted 1 Year Single-B Cohort Default Rates

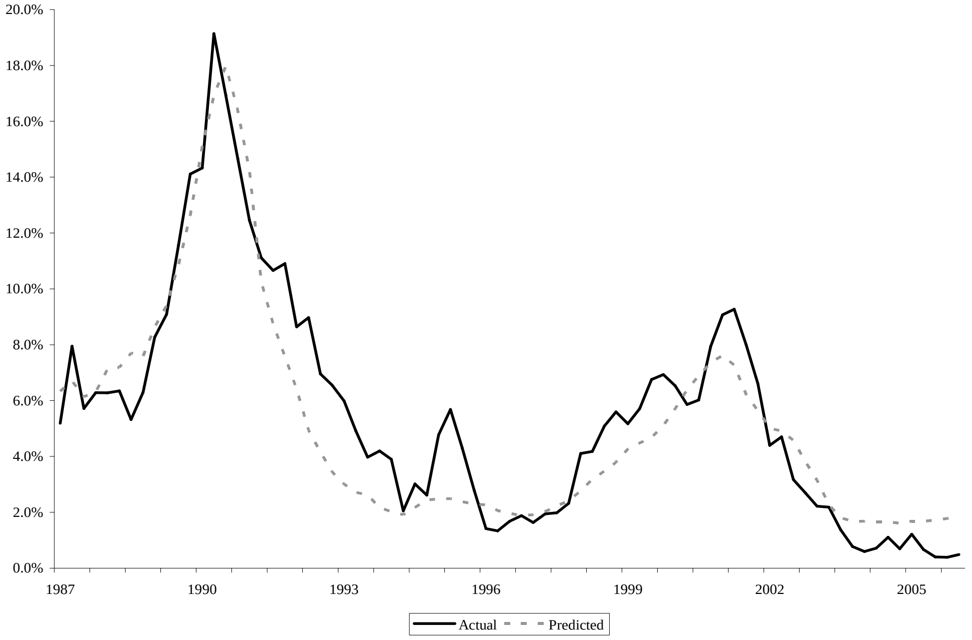
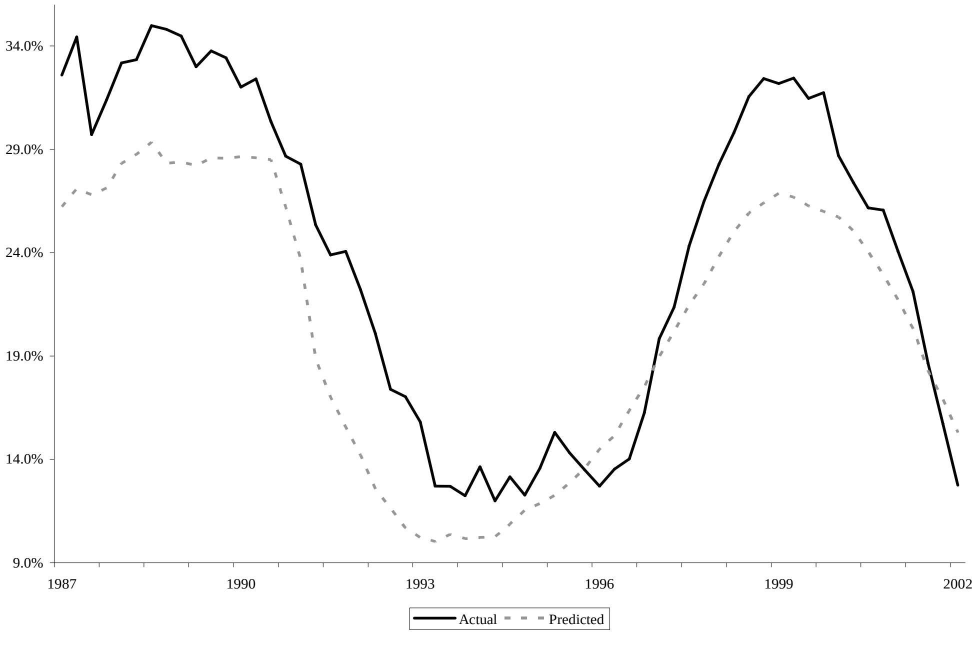


Figure VII.6 Actual and Predicted 5 Year Single-B Cohort Default Rates



<VIII> Conclusion

Moody's has constructed an issuer-level model of rating transitions which conditions on issuer facts and the future path of two economic drivers: the unemployment rate and the high yield spread over Treasuries. The model, an application of the standard multiple-destination proportional hazards type, separately identifies upgrades, downgrades, withdrawals and defaults. While the last transition is usually the subject of greatest interest, rating changes themselves are often important.

Generally speaking, the (partial) effect of the macroeconomic drivers conforms with expectations: downgrades and defaults are increasing with changes in the unemployment rate and the level of high yield spreads, upgrades are decreasing with spreads. One result which may appear anomalous is that defaults are decreasing with the level of the unemployment cycle. This may be explained by the timing of the business cycle: default rates are highest at the start of a downturn, which is characterized by low but increasing unemployment, and least at the start of the recovery, where unemployment is high but decreasing.

As a *partial* effect, recent rating actions generally dwarf changes in the economic environment when it comes to default transitions. Having been downgraded drastically increases the default transition for both the SG and C aggregates. However, since the economic environment impacts these rating actions, it remains true that the *general* impact of the economic drivers is significant.

Important avenues of research remain to be explored. In particular, it would seem useful to incorporate rating outlooks in any model of rating transitions. Perhaps some of the effects we currently attribute to recent rating actions are in fact due to their proxy for future rating actions, and conditioning on outlooks may reduce or even eliminate much of their importance. Further work might also include conditioning on more issuer-specific information, either financial data or a Merton variable of some sort. As discussed above, this does present added complications, but they are probably not insurmountable.

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