Macroeconomic Conditions, Growth Opportunities and the Cross-Section of Credit Risk

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Abstract

This paper develops a dynamic trade-off model of optimal capital structure that takes into account the fact that most firms have both invested assets and growth opportunities. These two sources of value react quite differently to business cycle risk. In particular, growth options are more sensitive to regime changes than invested assets. "Growth firms" are, therefore, endogenously more likely to default in recession, when doing so is expensive. This in turn raises their costs of debt. The model quantitatively matches average stylized facts regarding credit spreads, leverage, default and investment clustering. Importantly, it also makes predictions about the cross-section of all these features.

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1. Introduction

This paper examines the effects of changing macroeconomic conditions on firm value and corporate policy choices of firms with both assets in place and growth opportunities. The central thesis we develop is that these two sources of value react differently to business cycle risk. We show that firms with lots of growth options ("growth firms") are more likely to default in recession. Therefore, these firms have higher costs of debt than those with mainly invested assets.

Our study is motivated by three puzzling empirical facts. First, standard structural models of default significantly underestimate credit spreads for corporate debt; this is the credit spread puzzle (see, for example, Elton, Gruber, Agrawal, and Mann (2001), Huang and Huang (2003), and Chen, Collin-Dufresne, and Goldstein (2009)). Second, corporations hold less debt than predicted from trading off tax advantages of debt against costs of financial distress; this is the under-leverage puzzle (see, for example, Myers (1977) and Graham (2000).) Third, the puzzles are particularly strong for growth firms. Davydenko and Strebulaev (2007) show that, after controlling for standard credit risk factors, proxies of growth opportunities are all positively and significantly related to credit spreads. Molina (2005) finds that firms with a higher ratio of fixed assets to total assets have lower yield spreads and higher ratings. Similarly, firms with more growth options typically have lower leverage (Smith and Watts 1992, Fama and French 2002, Frank and Goyal 2009).

Our model matches all these facts. It explains the credit spread and under-leverage puzzles on average, not only qualitatively, but quantitatively. Moreover, this paper shows that understanding the nature of the assets of a firm can go a long way towards explaining cross-sectional variation of credit spreads and leverage. Our model is also consistent with observed default clustering, investment behavior of firms, and empirical recovery rates. Additionally, the model makes cross-sectional predictions about all these features by allowing firms to be heterogenous in their asset composition.

To develop our analysis, we employ a structural model of financing decisions. Optimal leverage is determined by trading off tax benefits of debt against default costs. In a novel contribution, we simultaneously incorporate into this model both changing macroeconomic conditions (building on work by Hackbarth, Miao, and Morellec (2006)) as well as expansion options. Macroeconomic shocks to volatility, default costs, and asset values arise due to switches between two regimes, boom and recession. Growth opportunities are converted into invested assets when the underlying reaches the endogenously derived investment boundary. Shareholders maximize the value of equity by simultaneously choosing the optimal default and expansion option exercise policy. We pinpoint the effect of the asset composition of a firm on credit risk and leverage by assuming, in the main analysis, that the exercise price of the growth option is financed through the sale of some assets in place, i.e., without additional funds being injected into the company. We also study equity financing later.

Like other macroeconomic models, ours leads to countercyclical default boundaries, i.e., shareholders default earlier in recession than in boom. Thus, default is more likely in recession which, together with countercyclical default costs, raises the costs of debt for all firms compared to a benchmark model without business cycle risk.

The central new feature of our model is that debt is particularly costly for firms with a high portion of expansion opportunities in their assets' value. There are two reasons for this result. First, because options represent leveraged claims, firms with valuable growth options are more sensitive to the underlying uncertainty than firms which consist of only invested assets. The volatility of the underlying profit or cash flow process alone, for example, would consequently underestimate the true asset value risk. While the literature discusses this basic idea within equity financed firms (Berk, Green, and Naik 1999, Carlson, Fisher, and Giammarino 2004), little is known about its impact on debt prices. Our structural model allows us to jointly analyze expansion policy and financial leverage in the presence of macroeconomic risk, and to rigorously explore the quantitative implications of the riskiness of growth opportunities on debt prices and credit spreads.

The second driving force is that option values are more sensitive to regime changes than are assets in place. This higher sensitivity also arises because options are leveraged claims. Importantly, an additional effect derives from the fact that the optimal exercise level of growth opportunities increases in recession and decreases in boom. Intuitively, it is optimal to defer the exercise of an expansion opportunity when the economy switches to recession, i.e., to wait for better times. Because the moneyness of growth opportunities is regime-dependent, and because they represent leveraged claims, expansion options' values are more exposed to the state of the macro-economy than invested assets. Moreover, the changing moneyness causes expansion options to be less sensitive to the underlying development of asset values in recession than in boom, which reduces shareholders' value of their option to defer default during bad times. Together, these effects amplify the countercyclicity of default thresholds for firms with a high portion of growth opportunities, which, in turn, increases their costs of debt compared to those with only invested assets.

The model performs quantitatively well. The literature suggests that an average Baa-rated firm has a credit spread in the range of of 111-145.5 bps. (We arrive at this range by starting from the average bond yield of 148 bps in Duffee (1998) and the 194 bps reported in Huang and Huang (2003) and taking into account that around 25% of bond yields are often estimated to be due to non-default components). For a firm with only invested assets, a model without business cycle risk produces a mere 71 bps spread. The standard macroeconomic model in the spirit of Hackbarth, Miao, and Morellec (2006) includes business cycle risk by allowing shocks to asset values when the regime switches, but keeps volatility constant across regimes. This leads to a spread of 94.2 bps. When we incorporate the notion that volatility is lower while recovery rates are higher in booms (Frye 2000, Ang and Bekaert 2004), we obtain a credit spread of around 105 bps for a firm with only invested assets.

A reasonable estimate for the average US firm's asset composition is that total firm value is about 70% higher than the value of invested assets, which corresponds (approximately) to a Tobin's Q of 1.7.[1] For such a firm, we obtain a credit spread of about 126.8 bps. Note that less than half of the difference between the 71 bps in the one-regime model and the 126.8 bps in our full model is due to including macroeconomic asset value risk (94.2 bps - 71 bps). The larger part (126.8 bps - 94.2 bps) is due to the two novel features in this paper, namely due to taking into account the asset composition and due to including changing volatility and recovery rates (with the latter being relatively unimportant). This calculation applies to the average firm. For a firm with significant growth opportunities, the extra credit spread due to incorporating the asset composition and regime-dependent volatility can each be quantitatively as important as basic macroeconomic risk. A growth firm whose total asset value is about 2.4 times the value of its assets in place is predicted to have a credit spread of about 137.6 bps, about 32.6 bps more than a firm with only invested assets. Note that this large effect arises even though asset values are kept constant; we only vary the characteristics of the assets themselves.[2]

Of course, the results depend on the assumptions about underlying parameters; indeed, the literature uses a range of assumptions in their calibrations of structural credit spread models (see, for example, Huang and Huang (2003), or Collin-Dufresne and Goldstein (2001)). What is important

[1] Market values can be higher than book values also because of off-balance sheet assets, so there is, of course, a range for the asset composition of the "typical" firm.

[2] When the exercise of the option is financed with equity, this has minor quantitative effects for a broad range of asset composition. Intuitively, in the typical cases, debt is virtually risk-less when the asset value rises sufficiently far that it pays to exercise the growth option. Thus, debt would not benefit much from this alternative financing mode ex-ante and would command a similar credit spread as when the option is financed by selling some assets in place. Only when the option is such that exercise at particularly low asset values is optimal will debtholders' anticipation of additional equity funding induce them to accept signficantly lower credit spreads.

to keep in mind in this context is that credit spread *levels* can easily be increased by raising, for example, the size of the firm-specific shock, the loss rate, or the payout ratio. To solve the credit spread *puzzle*, however, a model needs to explain larger costs of debt than standard models would predict while holding calibrated parameters constant at historically observed levels. In our model, this non-equivalent variation is achieved by (a) the notion that firms exhibit inherently higher risk because their growth opportunities represent leveraged claims, and (b) through macroeconomic shocks. Specifically, we derive high default rates in recession because asset values have a lower drift, a higher volatility, and because shareholders optimally raise the default threshold. Moreover, loss rates are higher during recession because recovery rates and asset values are lower in those periods. Risk-neutral asset pricing then directly implies higher credit spreads for a given set of historically observed parameters.

The nature of assets, thus, has a powerful impact on costs of debt. Not surprisingly, it also affects optimal leverage ratios. We find that our model produces an optimal leverage of about 42.8% for the firm with the average asset composition. This ratio closely reflects the 43.3% observed average leverage ratio reported in Huang and Huang (2003). It is also significantly closer to this target than the 49.7% that come out of a model without business cycle risk, or the 45.9% that come out of the standard macroeconomic model (that includes neither regime-dependent volatilities and recovery rates nor growth opportunities).

We also derive additional testable predictions. Consistent with empirical evidence, the model predicts countercyclical credit spreads and lower recovery rates for growth firms. For most specifications, the model yields weakly procyclical leverage when asset volatilities are higher in recession than in boom (as is suggested by empirical evidence, see Ang and Bekaert (2004)). If volatility is constant, we obtain the same anticyclical leverage property as Hackbarth, Miao, and Morellec (2006), but it becomes somewhat harder to explain the credit spread puzzle on average. Furthermore, we predict that default clustering should be more prevalent for growth firms. The model additionally has implications for aggregate investment. In particular, it predicts procyclical investment patterns as reported in Barro (1990).

Our paper contributes to two streams of literature. First, the fact that growth opportunities are empirically strongly associated with observed leverage has, of course, also prompted other explanations. The most prominent of these additional explanations, agency, comes in two primary forms: a shareholder-bondholder conflict and a manager-shareholder conflict. Appealing to the former, Smith and Watts (1992) and Rajan and Zingales (1995) suggest that debt costs associated with shareholder-bondholder conflicts typically increase with the number of growth options available to the firm due to underinvestment (Myers 1977) and overinvestment by way of asset substitution (Jensen 1986); see also Sundaresan and Wang (2007). According to Leland (1998), however, optimal leverage even *increases* when firms can engage in asset substitution. Similarly, Parrino and Weisbach (1999) conclude that stockholder-bondholder conflicts are too limited to explain the cross-sectional variation in capital structure. Childs, Mauer, and Ott (2005) show that short-term debt can reduce the agency costs. Hackbarth and Mauer (2010) show that the joint choice of debt priority structure and capital structure can virtually eliminate the suboptimal investment incentives of equityholders. Neither of the papers incorporates macroeconomic risk.[3]

As for manager-shareholder conflicts, Morellec (2004) shows that agency costs of free cash flow can explain the low debt levels observed in practice, and the negative relationship between debt levels and the number of growth options; see also Barclay, Smith, and Morellec (2006). Morellec, Nikolov, and Schürhoff (2009) show that even small costs of control challenges are sufficient to explain the low-leverage puzzles. It is still a matter of debate to what extent conflicts of interest between managers and stockholders cause the empirically observed patterns. Graham (2000), for example, tests a wide set of managerial entrenchment variables and finds "at best weak evidence that managerial entrenchment permits debt conservatism." In any case, our model is not inconsistent with either of these views. But it does offer a quantitatively important reason for the cross-sectional variation in leverage and credit spreads that derives solely from the nature of assets of firms.[4]

Second, at the core of our model is the idea that macroeconomic (business cycle) risk matters in powerful ways for the optimal financing of firms. We share this idea with other work that has utilized the insight that macroeconomic risk increases the costs of corporate debt because firms are more likely to default when doing so is costly (see, for example, Almeida and Philippon (2007), Chen (2010), and Demchuk and Gibson (2006).) What we add to this literature is the idea that the impact of business cycle risk depends on the asset base of a firm.

The paper proceeds as follows. In section 2, we set up the model. In section 3, we solve the model. Section 4 discusses the optimal default policy and expansion policy as well as the implications of the model with a realistic parameter calibration. In section 5, we turn to the quantitative implications. Section 6 concludes.

[3] See Lyandres and Zhdanov (2010) for an explanation for accelerated investment that does not rely on agency.

[4] An alternative explanation for why low leverage may be optimal in the high-tech sectors is offered in Miao (2005). In his model, when a sector experiences technological growth, more competitors enter, leading to falling prices and possibly to a greater probability of default. Yet other explanations appeal to the fact that firms have the option to issue additional debt (Collin-Dufresne and Goldstein 2001).

2. The Model

Our model setup is based on a standard continuous time model of capital structure decisions in the spirit of Mello and Parsons (1992), as extended by Hackbarth, Miao, and Morellec (2006) for business cycle fluctuations. We generalize the model to incorporate growth opportunities.

We consider an infinitely-lived firm with assets in place and a growth opportunity. The economy is characterized by two possible regimes, boom (B) and recession (R). Agents are risk-neutral and discount at a constant interest rate r. Time is continuous and uncertainty is modeled by a complete filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{\{t\geq 0\}}, \mathbb{P})$. Management acts in the best interest of shareholders. Corporate taxes are paid at a constant rate τ, and full offsets of corporate losses are allowed.

In this framework, a firm is levered because debt allows it to shield part of its income from taxation. We consider infinite-maturity debt. Once debt has been issued, the firm pays a total coupon c at each moment in time. Following the standard in the literature, we assume that the firm finances coupons by issuing additional equity. At any point in time, shareholders have the option to default on their debt obligations, as well as the option to exercise an expansion option. If default occurs, the firm is immediately liquidated and bondholders receive the unlevered asset value less default costs, reflecting the 'absolute priority' of debt claims. The default costs in regime i are assumed to be a fraction $1-\alpha_i$ of the unlevered asset value at default, where $\alpha_i \in [0,1]$. Thus, leverage is limited due to the possibility of costly financial distress.

Managers face the following decisions: First, once debt has been issued, they select the default and expansion policies that maximize equity value. Hence, both expansion and default are initiated endogenously. In particular, as in Leland (1998), default is triggered when shareholders are no longer willing to inject additional equity capital to meet net debt service requirements. Second, managers determine the optimal capital structure by choosing the coupon level which maximizes the value of the firm.

At any point in time, the value of the assets in place satisfies

$$V_t = X_t Y_t. \tag{1}$$

$(X_t)_{t\geq 0}$ is the idiosyncratic asset value evolving according to the dynamics

$$\frac{\mathrm{d}X_t}{X_t} = \mu_i \mathrm{d}t + \sigma_i \mathrm{d}W(t), \quad i = B, R \tag{2}$$

where μ_i are the regime-dependent drifts, $\sigma_i > 0$, the regime-dependent volatilities, all of which are constant and known given a regime. The drifts μ_i include the impact of a payout ratio δ on X. W_t is a standard Brownian motion on $(\Omega, \mathcal{F}, \mathbb{P})$.

$(Y_t)_{t \geq 0}$ represents an aggregate shock process dependent on the state of the economy, with

$$Y_t = y_i \tag{3}$$

where $0 < y_i < \infty$ are macroeconomic variables dependent on regime i. The factors y_i are constant and known. In our main analysis, we assume that the aggregate shock variable is higher in boom, $y_B > y_R$. As suggested by the literature, we further posit that $\sigma_B < \sigma_R$ and $\alpha_R < \alpha_B$ (Ang and Bekaert 2004, Frye 2000).

The expansion opportunity of the firm is modeled as an American call option on the value of the underlying asset. Specifically, at any time t, the firm can pay exercise costs K to install additional assets of value sV_t for some factor $s > 0$.[5] We assume that if the firm exercises its expansion option, the option is converted into assets in place, such that the firm consists of only invested assets. The exercise of the growth option is assumed to be irreversible. It is assumed that, at default, bondholders recover not only a fraction of the assets in place, but also a fraction of the option's value. Thus, the option can be exercised independently of the considered firm. Later, we also consider firm-specific expansion options which are completely lost upon default.

For the financing of investment, we present two variants. In the main analysis, we wish to isolate the effect of growth options in the value of firms' assets on corporate securities, and to abstract away from the effect of fund injections by debt- or equityholders to pay the exercise price. Therefore, we first assume that the firm installs additional assets with a value of $sXy_i - K$ upon exercise, i.e., the firm pays the exercise of the option by selling a part of the assets in place. In this case, the value of the additionally installed assets exactly corresponds to the value of the expansion option. Later, we also investigate the case where the firm finances the exercise price by issuing new equity.

The critical measure to capture the relative importance of a firm's expansion opportunity in the value of its assets is the *asset composition ratio.* We define it as the sum of the value of the

[5] We could include regime-dependency of the growth option parameters s and K. However, as the current setup already allows for a regime-dependent option payout through the economy-wide shock Y, and because we lack a clear economic motivation of including regime-dependent exercise costs, we forgo this extension. In any case, we expect a minor impact on the results.

expansion opportunity and invested assets, divided by the value of the invested assets.[6]

We assume that there are only two different macroeconomic states, namely boom (B) and recession (R). Formally, we define a time-homogeneous Markov chain $I_{t\geq 0}$ with state space $\{B, R\}$. $I_{t\geq 0}$ has generator $Q := \begin{bmatrix} 1-\lambda_R & \lambda_R \\ \lambda_B & 1-\lambda_B \end{bmatrix}$, where $\lambda_i \in (0,1)$ denotes the rate of leaving state i. In the main analysis, we consider $\lambda_B < \lambda_R$ (as in Hackbarth, Miao, and Morellec (2006)).

This Markov process exhibits the following well-known properties: First, the probability that the chain stays in state i longer than some time $t \geq 0$ is given by $e^{-\lambda_i t}$. Second, the probability that the regime shifts from i to j during an infinitesimal time interval Δt is given by $\lambda_i \Delta t$. Third, the expected duration of regime i is $\frac{1}{\lambda_i}$, and the expected fraction of time spent in that regime is $\frac{\lambda_j}{\lambda_i+\lambda_j}$.

Finally, we assume that both X and I are observable and that the Brownian motion W, driving the idiosyncratic asset value X, is independent of the Markov chain I.

As in Leland (1994), we postulate that at any point in time the expected rate of return must be equal to the risk-free rate:[7]

$$rX_t y_t = \delta X_t + \mathbb{E}\left[\mathrm{d}\left(X_t Y_t\right)|\mathcal{F}_t\right] \quad \forall t. \tag{4}$$

Using Ito's lemma for non-continuous semi-martingales yields the regime-dependent drifts[8]

$$\mu_i = r - \frac{1}{y_i}\delta + \lambda_i\left(1 - \frac{y_j}{y_i}\right) \quad \text{for } i = B, R. \tag{5}$$

The scaling factor of the payout ratio δ, $\frac{1}{y_i}$, accounts for the fact that δ is a payout ratio on X.

3. Model solution

The model is solved by backward induction. We start by calculating the value of corporate securities of a firm consisting of only invested assets, taking the capital structure, default and expansion policies as given at this point. Next, the value of the growth option, also for given capital structure

[6] Alternatively and similarly, we could use Tobin's Q defined as firm value divided by invested assets (or book assets) to measure asset composition. However, Tobin's Q not only contains the value of the expansion opportunity, but also the value of the tax shield and bankruptcy costs. The asset composition ratio allows us to abstract from those factors and to focus purely on the nature of the assets.

[7] This requires that the asset XY, or an asset perfectly correlated with XY, is traded, see Leland (1994) for a further discussion.

[8] Note that, consistent with an intuitive understanding of the regimes "boom" and "recession," the conditions $y_R < y_B$ and $\lambda_B < \lambda_R$ suffice to ensure that $\mu_R < \mu_B$.

and policies, is derived. We then proceed with the value of corporate securities of a firm which consists not only of assets in place, but also holds an expansion opportunity. Finally, we obtain the expansion and default policies which simultaneously maximize the value of equity, as well as the capital structure which maximizes the firm value.

We assume that the optimal strategies for both default and expansion are of regime-dependent threshold type in X, without formally proving optimality (cf. Hackbarth, Miao, and Morellec (2006) for the default strategy and Guo and Zhang (2004) for the expansion strategy). Precisely, suppose that $\hat{D}_i$ and D_i are the default thresholds in regime $i = B, R$ of a firm with only invested assets, and of a firm with both invested assets and a growth opportunity, respectively. X_i denotes the exercise boundary of the growth option in regime $i = B, R$. Here, we present the relevant case that $D_B < D_R$, $X_B < X_R$ and $\hat{D}_B < \hat{D}_R$, i.e., the boundaries are lower in boom for both expansion and default (before and after expansion).[9] Finally, we presume that $\max\left\{D_R, \hat{D}_R\right\} < X_B$, i.e., we are interested in firms which exercise their expansion option with a positive probability, and we exclude the possibility of immediate default after expansion.

3.1. Firms with only invested assets

The firm exercises its expansion option by converting it into assets in place. After exercise, the value of the corporate securities must be equal to the ones of a firm with only invested assets. Consequently, by backward induction, we first calculate the latter.

Consider a firm that consists of an asset $V_t = X_t Y_t$. For brevity, we will also use the notation $V_i(X)$ to denote the asset value in boom and recession, respectively, omitting the time variable. Let $\hat{d}_i(X), \hat{t}_i(X), \hat{b}_i(X)$ and $\hat{f}_i(X)$ denote the value of corporate debt, taxes, bankruptcy cost, and total firm value, respectively, in regime $i = B, R$. Hackbarth, Miao, and Morellec (2006) show how to solve a similar model.[10] The solution for our case can be found in Appendix A.1.

[9] Note that we can assume without loss of generality that $D_B < D_R$ (if not, interchange the names of the regimes). The case $D_B < D_R, \hat{D}_B < \hat{D}_R$, and $X_B > X_R$, (i.e., the exercise boundary in recession is lower than the one in boom) can be solved by similar techniques. The calculations are available upon request. We did not consider the case $D_B < D_R$ and $\hat{D}_B > \hat{D}_R$, which corresponds to the case that the order of the default boundary changes after exercising the option. The case that the default and exercise boundaries coincide, respectively, i.e., that there is only one regime, is presented in Appendix A.3, Case 2.

[10] Even though they start with the stochastic cash flow as the underlying state variable, and do not consider regime-dependency of volatility, the basic approach remains unchanged.

3.2. The value of the growth option

Firms in general consist of both assets in place and a growth option. In order to evaluate the value of corporate securities of firms, we need to calculate the value of a growth option under regime switches. Our approach follows the method of Guo and Zhang (2004), who derive a closed-form solution of an American put option in a regime switching model, whereas we solve for the value of an American call option. Early exercise is guaranteed by the positiveness of the payout ratio $\delta > 0$, which, in the context of the growth option, represents the opportunity costs of delaying its exercise (see Pindyck (1991)).

Denote the value functions of the growth option in regime B and R by $G_B(X)$ and $G_R(X)$, respectively. For each regime i, the option is exercised immediately whenever $X \geq X_i$ (option exercise region); otherwise it is optimal to wait (option continuation region). This structure results in the following system of ODEs for the value function, obtained by Ito's lemma for regime switches (see e.g. Yin, Song, and Zhang (2004)):

For $0 \leq X < X_B$:

$$\begin{cases} rG_B(X) &= \mu_B X G_B'(X) + \frac{\sigma 2_B}{2} X2 G_B''(X) + \lambda_B \left(G_R(X) - G_B(X)\right) \\ rG_R(X) &= \mu_R X G_R'(X) + \frac{\sigma 2_R}{2} X2 G_R''(X) + \lambda_R \left(G_B(X) - G_R(X)\right) \end{cases} \tag{6}$$

For $X_B \leq X < X_R$:

$$\begin{cases} G_B(X) &= sXy_B - K \\ rG_R(X) &= \mu_R X G_R'(X) + \frac{\sigma 2_R}{2} X2 G_R''(X) + \lambda_R \left(sXy_B - K - G_R(X)\right) \end{cases} \tag{7}$$

For $X \geq X_R$:

$$\begin{cases} G_B(X) &= sXy_B - K \\ G_R(X) &= sXy_R - K \end{cases} \tag{8}$$

Whenever the process is in the option continuation region, which corresponds to system (6) and the second equation of (7), the required rate of return r (left-hand side) must be equal to the realized rate of return as displayed on the right hand-side using Ito's lemma. Here, the last term accounts for a possible jump in the value of the growth option due to a shift in regime. It is calculated as the probability of a regime shift, λ_B or λ_R, times the change in the value of the option given a regime shift. The first equation of (7) and the system (8) states the payoff of the option at exercise, since the process is in the option exercise region in these cases.

The boundary conditions are:

$$\lim_{X \searrow 0} G_i(X) = 0, \quad i = B, R \tag{9}$$

$$\lim_{X \searrow X_B} G_R(X) = \lim_{X \nearrow X_B} G_R(X) \tag{10}$$

$$\lim_{X \searrow X_B} G'_R(X) = \lim_{X \nearrow X_B} G'_R(X) \tag{11}$$

$$\lim_{X \nearrow X_R} G_R(X) = sX_R y_R - K \tag{12}$$

$$\lim_{X \nearrow X_B} G_B(X) = sX_B y_B - K \tag{13}$$

Condition (9) ensures that the option value goes to zero as the asset value approaches zero. Conditions (10) and (11) represent the value-matching and smooth-pasting conditions of the value function in recession at the optimal exercise boundary in boom, which ensure consistency of the value function in recession. The remaining conditions (12)-(13) are the value-matching conditions at the exercise boundaries in boom and recession, respectively. The solution of this system and its derivation are given in Appendix A.2.

We remark that similar to the occurrence of default, there are two possible ways of exercising the expansion option: Either the idiosyncratic shock X reaches the exercise boundary X_i in a given regime (system (7)), or the regime switches from recession to boom (system (8)) given that X lies between X_B and X_R.

We emphasize that the value of the option in the ultimate solution of the model indeed depends on the default policy of the firm. Equityholders choose default and expansion policies simultaneously. The resulting interdependence between the two policies affects the value of the growth option. This effect is not explicit in the above considerations due to the backward solution method.

3.3. Firms with invested assets and expansion opportunities

Using the previous results, we finally solve for the value of the corporate securities of a general firm. In detail, we derive the values of corporate debt and equity as well as the default thresholds selected by shareholders.

In each regime, the firm faces three different regions depending on the value of X: Below the default threshold, the firm is in the default region. This means that the firm defaults immediately, and debtholders receive a fraction α_i, the recovery rate, of its asset and option value. Next, the firm is in the continuation region, if X is between the default threshold and the exercise boundary.

Finally, the exercise region is reached if X is above the exercise boundary. After exercise, the firm consists of only invested assets, endowed with the initially determined optimal coupon level. The optimal default thresholds, however, now correspond to the ones of a firm with only invested assets, i.e., shareholders optimally adapt their default policy. Note that debtholders anticipate this change in the default policy.

3.3.1. The valuation of corporate debt

Let $d_i(X)$ denote the value of corporate debt in regime $i = B, R$. An investor investing in corporate debt requires an instantaneous return equal to the risk-free rate r. Again, an application of Ito's lemma with regime switches shows that debt satisfies the following system of ODEs:

For $0 \leq X \leq D_B$:

$$\begin{cases} d_B(X) &= \alpha_B(V_B(X) + G_B(X)) \\ d_R(X) &= \alpha_R(V_R(X) + G_R(X)) \end{cases} \tag{14}$$

For $D_B < X \leq D_R$:

$$\begin{cases} rd_B(X) &= c + \mu_B X d'_B(X) + \frac{\sigma 2_B}{2} X 2 d''_B(X) + \lambda_B(\alpha_R(V_R(X) + G_R(X)) - d_B(X)) \\ d_R(X) &= \alpha_R(V_R(X) + G_R(X)) \end{cases} \tag{15}$$

For $D_R < X < X_B$:

$$\begin{cases} rd_B(X) &= c + \mu_B X d'_B(X) + \frac{\sigma 2_B}{2} X 2 d''_B(X) + \lambda_B(d_R(X) - d_B(X)) \\ rd_R(X) &= c + \mu_R X d'_R(X) + \frac{\sigma 2_R}{2} X 2 d''_R(X) + \lambda_R(d_B(X) - d_R(X)) \end{cases} \tag{16}$$

For $X_B \leq X < X_R$:

$$\begin{cases} d_B(X) &= \hat{d}_B\left(\bar{s}X - \frac{K}{y_B}\right) \\ rd_R(X) &= c + \mu_R X d'_R(X) + \frac{\sigma 2_R}{2} X 2 d''_R(X) + \lambda_R\left(\hat{d}_B\left(\bar{s}X - \frac{K}{y_B}\right) - d_R(X)\right) \end{cases} \tag{17}$$

And, finally, for $X \geq X_R$:

$$\begin{cases} d_B(X) &= \hat{d}_B\left(\bar{s}X - \frac{K}{y_B}\right) \\ d_R(X) &= \hat{d}_R\left(\bar{s}X - \frac{K}{y_R}\right) \end{cases} \tag{18}$$

In system (14), the firm is in the default region in both boom and recession. Here, debtholders receive $\alpha_i(V_i(X) + G_i(X))$ at default. As the default boundary in boom is lower than the one in recession, system (15) corresponds to the firm being in the continuation region in boom, and default region in recession. For the continuation region in boom, we calculate the left-hand side of the first equation as the rate of return required by investors for holding a unit of corporate debt for one unit of time. The right-hand side is the realized rate of return, computed by Ito's lemma as the expected change in the value of debt plus the coupon payment c. Here, the last term captures the possible jump in the value of debt in case of a regime switch, which triggers immediate default in this region. Similarly, equations (16) describe the case that the firm is in the continuation region in both boom and recession. The next system, (17), deals with the case that the firm is in the exercise region in boom, and in the continuation region in recession. After exercising the option, the firm owns total assets in place of $Xy_i + sXy_i - K = \bar{s}Xy_i - K$, where $\bar{s} = s + 1$. The value of debt must then be equal to the value of debt of a firm with only invested assets, i.e., $d_B(X) = \hat{d}_B(\bar{s}X - \frac{K}{y_B})$, which is the first equation in (17). The second equation is obtained by the same approach as before. The last term captures the fact that in this case a regime switch from recession to boom triggers immediate exercise of the expansion option. Finally, equations (18) describe the case that the firm is in the exercise region in both boom and recession.

The boundary conditions for debt are as follows:

$$\lim_{X \searrow D_R} d_B(X) = \lim_{X \nearrow D_R} d_B(X) \tag{19}$$

$$\lim_{X \searrow D_R} d'_B(X) = \lim_{X \nearrow D_R} d'_B(X) \tag{20}$$

$$\lim_{X \searrow D_B} d_B(X) = \alpha_B \left(D_B y_B + G_B(D_B)\right) \tag{21}$$

$$\lim_{X \searrow D_R} d_R(X) = \alpha_R \left(D_R y_R + G_R(D_R)\right) \tag{22}$$

$$\lim_{X \searrow X_B} d_R(X) = \lim_{X \nearrow X_B} d_R(X) \tag{23}$$

$$\lim_{X \searrow X_B} d'_R(X) = \lim_{X \nearrow X_B} d'_R(X) \tag{24}$$

$$\lim_{X \nearrow X_B} d_B(X) = \hat{d}_B\left(\bar{s}X_B - \frac{K}{y_B}\right) \tag{25}$$

$$\lim_{X \nearrow X_R} d_R(X) = \hat{d}_R\left(\bar{s}X_R - \frac{K}{y_R}\right) \tag{26}$$

(19) and (20) represent the value-matching and smooth-pasting conditions for the debt value in boom at the default boundary in recession. Similarly, (23) and (24) are the corresponding conditions for the debt value in recession at the option exercise boundary in boom. (21) and (22) are the value-matching conditions at the default thresholds, and (25) and (26) are the value-matching conditions at the option exercise boundaries. The default thresholds and option exercise boundaries are chosen

by shareholders, and, hence, we do not have the corresponding smooth-pasting conditions for debt.

The solution of this system is given in closed form in Appendix A.3.1.

3.3.2. The valuation of tax benefits

Let $t_i(X)$ denote the value of tax benefits in regime $i = B, R$. Debt coupon payments shield income from taxation. We assume full loss carry-forwards. Therefore, the value of tax benefits corresponds to the value of debt with recovery rates equal to zero and a coupon of $c\tau$. In detail, we obtain a system of equations akin to the system (14)-(18), and the same boundary conditions as in (19) - (26): Because $\alpha_B = \alpha_R = 0$, (21) - (22) translate into

$$\lim_{X \searrow D_i} t_i\left(X\right) = 0, \qquad i = B, R, \tag{27}$$

reflecting the loss of tax benefits at bankruptcy. At the option exercise boundary, we have that

$$\lim_{X \nearrow X_i} t_i\left(X\right) = \hat{t}_i\left(\bar{s}X_i - \frac{K}{y_i}\right), \qquad i = B, R, \tag{28}$$

corresponding to conditions (25) - (26). In words, if the option is exercised, the value of the tax shield is equal to the one of a firm with only invested assets.

3.3.3. The valuation of default costs

Let $b_i(X)$ denote the value of default (or bankruptcy) costs in regime $i = B, R$. $b_i(X)$ can be calculated as the value of a debt contract with recovery rates $1 - \alpha_B$ and $1 - \alpha_R$, respectively, and a coupon of zero, as there are no continuous cash-flows associated with default costs.

The value-matching boundary conditions at default (21) - (22) then correspond to

$$\lim_{X \searrow D_i} b_i\left(X\right) = \left(1 - \alpha_i\right)\left(D_i y_i + G_i(D_i)\right), \qquad i = B, R, \tag{29}$$

reflecting the fact that the value of default costs at the boundary must be 1 minus the recovery rate of the value of assets in place and the growth option. Conditions (25) - (26) are now

$$\lim_{X \nearrow X_i} b_i\left(X\right) = \hat{b}_i\left(\bar{s}X_i - \frac{K}{y_i}\right), \qquad i = B, R. \tag{30}$$

The intuition is that, at the exercise boundary of the option, default costs must be equal to the ones of a firm with only invested assets.

3.3.4. Firm value

Total firm value f_i in regime $i = B, R$ corresponds to the value of assets in place $y_i X$, plus the value of the expansion option $G_i(X)$ and the value of tax benefits from debt $t_i(X)$, less the value of potential default costs $b_i(X)$, i.e.,

$$f_i(X) = y_i X + G_i(X) + t_i(X) - b_i(X). \tag{31}$$

3.3.5. The valuation of equity

The levered firm value equals the sum of debt and equity values. Hence, equity value $e_i(X)$, $i = B, R$, can be written in a closed form expression as

$$e_i(X) = f_i(X) - d_i(X) = y_i X + G_i(X) + t_i(X) - b_i(X) - d_i(X). \tag{32}$$

3.3.6. Default and expansion policies

Managers select the default and investment policies that ex-post maximize the value of equity. Denote these policies by D_i^* and X_i^*, respectively. As in Leland (1998), default is triggered by shareholders' decision to cease injecting funds into the firm. Formally, the default policy which maximizes equity value is determined by postulating that the first derivative of the equity value has to be zero at the corresponding default boundary. Simultaneously, optimality of the investment thresholds is achieved by equating, for each regime, the first derivative of the equity value at the investment threshold with the first derivative of the equity value of a firm with only invested assets, evaluated at the corresponding asset value after expansion. These four optimality conditions are smooth-pasting conditions for equity at the corresponding boundaries:

$$\begin{cases} e'_B(D_B^*) & = & 0 \\ e'_R(D_R^*) & = & 0 \\ e'_B(X_B^*) & = & \hat{e}'_B(sX_B^* - \frac{K}{y_B}) \\ e'_R(X_R^*) & = & \hat{e}'_R(sX_R^* - \frac{K}{y_R}). \end{cases} \tag{33}$$

We then solve this system numerically.

3.3.7. Capital structure

For each coupon level c, debtholders evaluate debt at issuance anticipating the ex-post optimal default and expansion decisions of shareholders. As debt-issue proceeds accrue to shareholders, the latter do not only care about the value of equity, but also about the value of debt. Hence, the optimal capital structure is determined ex-ante by the coupon level c^* which maximizes the value of equity and debt, i.e., the value of the firm. Denote by $f_i^*(X)$ the firm value given optimal ex-post default and expansion thresholds as determined by the system (33). The ex-ante optimal coupon of this firm hence solves

$$c_i^* := \text{argmax}_c f_i^*(X). \tag{34}$$

As indicated in equation (34), the capital structure depends on the current regime.

4. Results

This section summarizes the results of our framework. We first calibrate the model to a typical firm to analyze its workings. Section 4.1 provides our choice of parameters. Next, Section 4.2 discusses the optimal default policy and introduces firm-specific heterogeneity by allowing the portion of the expansion options' value in the overall value of firms' assets to vary. Section 4.3 summarizes the intuition for how a firm's asset composition ratio affects the costs of debt.

4.1. Calibration of parameters

Table I summarizes our parameter choice. The baseline parameters are selected to roughly reflect a typical Baa-rated S&P 500 firm.[11]

INSERT TABLE I HERE

We set the initial value of the idiosyncratic asset value X to 100. While this value is arbitrary, neither credit spreads nor optimal leverage ratios depend on this parameter. As is standard in the literature, we choose the tax advantage of debt as $\tau = 0.15$, and a payout ratio equal to $\delta = 0.03$; see, for example, Hackbarth, Miao, and Morellec (2006) and Collin-Dufresne and Goldstein (2001), respectively.

[11] Our qualitative results do not depend on the ratings of firms.

Next, empirical work such as Ang and Bekaert (2004) suggests that asset volatility is lower in boom than in recession. We set $\sigma_B = 0.23$ and $\sigma_R = 0.28$, i.e., $\frac{\sigma_B}{\sigma_R} = 0.82$. The resulting unconditional asset volatility of 0.251, calculated by weighting the corresponding variances by the expected fraction of time spent in each regime and taking the square root of the sum, reflects the one of Baa-rated firms reported in Huang and Huang (2003). Conditional (regime-dependent) volatility is hard to measure exactly. As shown below, our results are qualitatively robust to alternative specifications of the ratio $\frac{\sigma_B}{\sigma_R}$.[12]

Following Acharya, Bharath, and Srinivasan (2007) we assume that recovery rates fall during recession. They report that recovery in a distressed state of the industry is lower than the recovery in a healthy state of the industry by up to 20 cents on a dollar. The reason can be a downward revision in the economic worth of firms' assets, financial constraints that industry peers of defaulted firms face as proposed by the fire-sales or the industry-equilibrium theory of Shleifer and Vishny (1992), or time varying market frictions such as adverse selection. We choose recovery rates as $\alpha_R = 0.5$ and $\alpha_B = 0.7$, respectively, which matches the 20 cents on a dollar difference in Acharya, Bharath, and Srinivasan (2007). It also closely reflects the mean recovery rate on assets of 0.6 used in Hackbarth, Miao, and Morellec (2006).[13] Our general results are insensitive to the choice of α_i as long as $\alpha_B > \alpha_R$.

For the expansion option we choose an exercise price of $K = 140$ and a scale parameter of $s = 1.2$. These parameters imply a ratio of firm value to invested assets of 1.7, which closely reflects the reported average Tobin's Q for Compustat firms (Bertrand and Schoar 2003). K is set to 140 in order to investigate firms with expansion opportunities which are not exercised immediately for a reasonable range of s. Varying the scale parameter s then allows us to change the expansion option's value, and, hence, to analyze firms with different portions of option value in the overall value of their assets.

Following Hackbarth, Miao, and Morellec (2006), we set $\lambda_R = 0.15$, and $\lambda_B = 0.1$. The expected duration of regime R (B) then corresponds to 6.67 (10) years, and the average fraction of time spent in regime R (B) is 0.4 (0.6). The risk free interest rate is assumed to be $r = 0.06$. Finally, we set

[12] To get a rough validation of the calibration used in the main analysis, consider mean stock return standard deviations for S&P 500 firms (Harris 1989). The average, annualized equity variance within official NBER recession years is 0.37, and the one in boom is 0.28. After taking the square root and deleveraging these numbers with the average leverage ratio of S&P 500 firms of around 0.433 (Huang and Huang 2003), this implies an annualized asset volatility of 0.20 for recession, and 0.16 for boom. The resulting ratio, $\frac{0.16}{0.2} = 0.8$, closely matches the chosen relation between $\sigma_B = 0.23$ and $\sigma_R = 0.28$.

[13] The exact mean recovery rate in our model depends on the propensity to default in each regime, which is affected by the parameter choice. It always lies between α_B and α_R, and, hence, close to 0.6.

the aggregate shock variable to $y_B = 1.15$ and $y_R = 0.85$ in the baseline specification. The relative increase in the value of the assets following a shift from recession to boom is, therefore, similar to the calibration in Hackbarth, Miao, and Morellec (2006).

4.2. Optimal default policy

In this section, our ultimate goal is to derive the optimal default policy of a firm with both growth opportunities and invested assets. To structure the discussion, we first discuss properties of the default policy of a firm with only invested assets. Then, we study the optimal option exercise policy of a firm with growth opportunities. Recall that this expansion policy is chosen simultaneously with the default policy of the firm. Equipped with knowledge about features of optimal growth option exercise, we are finally able to explain the properties of the default policy of a growth firm.

For both value and growth firms, the optimal default policy is determined by recognizing that, at any point, shareholders can either make coupon payments and retain their claim together with the option to default, or forfeit the firm in exchange for the waiver of debt obligations. It is helpful to dissect the effect of macroeconomic regime changes on optimal default into two parts. When the economy shifts from boom to recession, equityholders' position changes. On the one hand, asset values decline as y_B switches to y_R. Moreover, the asset value drift becomes lower which raises the probability of default. The value and drift decline both reduce the continuation value for equityholders in recession, making them default earlier. We will refer to this combined effect as the *value effect*. On the other hand, a high volatility in recession makes the option to default more valuable, which tends to defer default in bad times. This is the *volatility effect*. We will see that the value and volatility effects depend on the growth opportunities of a firm.

We begin with the default policy of a firm with only invested assets. This is presented in Figure 1. To separate the value and volatility effects and to identify their relative quantitative importance on the default policy, we first shut down the volatility's regime dependence by setting $\sigma_B = \sigma_R = 0.23$, which yields the pure value effect as in the model of Hackbarth, Miao, and Morellec (2006). The straight line plots the optimal default threshold for each coupon in boom, and the higher dashed line the one in recession. In the no-default region above the line corresponding to a given regime, the continuation value exceeds the default value and it is optimal for shareholders to inject funds into the firm. Note that the default policy of a levered firm is characterized by countercyclical default thresholds. Hence, equityholders will optimally default earlier (at higher X levels) in recession, which implies countercyclicality of the default probabilities consistent with the empirical literature (Chava and Jarrow 2004, Vassalou 2004).

INSERT FIGURE 1 HERE

Next, we illustrate the combination of the value effect and volatility effect for invested assets by setting $\sigma_B = 0.23 < 0.28 = \sigma_R$. Adding such regime-dependent volatility decreases the default threshold in recession. The reason is that during recession, the higher volatility makes the equityholders' default option more valuable, which tends to defer default. Hence, regime-dependent volatility dampens, but does not overturn the countercyclicity of default thresholds for realistic parameters. Figure 1 shows the combined value and volatility effect as the difference between the solid and the lower dashed lines.[14]

In reality, the value of a firm's assets is typically composed of both invested assets and growth opportunities. To understand the default policy of such a firm, it is instructive to first consider some features of the expansion opportunity. Figure 2 depicts the equity value maximizing exercise policy of the expansion option in the standard firm, which is simultaneously determined with the default policy.

INSERT FIGURE 2 HERE

The area above the dashed line is the exercise region in recession, and the area below the dashed line represents the continuation region. In boom, the regions are defined in the same way with respect to the solid line. As expected, the exercise boundaries decrease with s. (The graph is drawn for optimal leverage, but the same qualitative option value properties also hold at other leverage levels.) Importantly, the expansion opportunity is exercised at lower levels of the idiosyncratic asset value X in boom than in recession.

Figure 3 plots the value of the expansion option as a function of the idiosyncratic asset value X, using jointly optimal expansion and default policies.

INSERT FIGURE 3 HERE

Obviously, the option's value is affected by the current regime. When the asset value jumps due to a regime switch, so does the value of the option.

[14] The default threshold in boom only changes minimally, so we do not show this solid line separately.

Critically, as options represent leveraged claims, relative value changes of expansion options are higher than relative value changes of assets in place when the regime switches. Moreover, the endogenous exercise boundary is higher in recession than in boom (see Figure 2).[15] Together, these findings suggest that the value effect is stronger for firms with growth opportunities. This in turn implies that default thresholds are even more countercyclical for such firms than for firms with only assets in place.

As can be seen in Figure 3, both value functions are convex, but the value function in boom is steeper than the one in recession. Therefore, the expansion option's value is less sensitive to the underlying idiosyncratic asset value in recession than in boom. Intuitively, the exercise boundary increases in recession which drives options out-of-the money, and, simultaneously, potential current gains from exercising are lower. As a consequence, an expansion option represents a less leveraged claim in bad times. While in recession the volatility of X is high, the sensitivity of a growth option's value to the idiosyncratic asset value is low, and this lower sensitivity attenuates the increase in the equityholder's default option in recession. Hence, regime-dependent volatility does not narrow the distance between the optimal default thresholds to the same extent as in the case of invested assets. That is, the volatility effect, which decreases the distance between default thresholds, is lower for firms with expansion opportunities than for firms with only invested assets, implying more countercyclical default boundaries for growth firms.

Figure 4 finally compares the equity value-maximizing default policies of two firms, one with only invested assets and one with an asset composition ratio of 2.4, respectively. The latter firm has a higher value than the one consisting of only invested assets for each level of the idiosyncratic asset value X on the vertical axis. Therefore, we normalize coupon payments by firm value on the horizontal axis which allows us to compare the default thresholds for the same debt policy of each firm. The upper and lower solid lines represent the default thresholds of the firm consisting of only invested assets in recession and boom, respectively. The upper dashed line is the default threshold of a firm with an asset composition ratio of 2.4 in recession, while the lower dashed line shows the one in boom.

INSERT FIGURE 4 HERE

[15] Relative value changes are determined in Appendix A.2. In untabulated results, we confirm numerically that the relative value changes are indeed higher for expansion options than for the underlying assets in place for plausible parameter values.

The countercyclicity, given by the distance between the default thresholds in boom and recession, is always larger for the firm with the asset composition ratio of 2.4 than for the firm with only invested assets. Hence, while all firms are more likely to default in recession than during boom, this behavior is particularly pronounced for growth firms.

4.3. How the model addresses the credit spread puzzle

The main drivers of costs of debt in our model (as in any structural model of default) are the default probability and default costs. Our model shows that both elements are higher than predicted in a standard model, and particularly so for firms with growth opportunities. The intuition is as follows. First, due to the inherent leverage and the endogenous choice of exercise boundaries, firms with growth opportunities are more sensitive to the underlying stochastic processes than invested assets. This higher sensitivity drives up the default probability of firms with growth options compared to firms with only invested assets. Second, countercyclical default boundaries imply a higher default probability during recession. When recovery rates are lower during bad times, this covariation between the default policy and the recovery rate implies higher default costs than predicted in a model without regimes.[16] Due to the strong sensitivity of option values to regime switches, and because they are less volatile during recession, countercyclicality of the default boundaries is more pronounced for firms with growth opportunities, which particularly drives up their expected default costs.

Consequently, the costs of debt of a typical firm, consisting of both invested assets and growth opportunities, are potentially significantly higher than in existing models for the same common parameter values. As such, our model contributes to solving the aggregate credit spread *puzzle*. Moreover, it implies a positive relationship between the portion of growth opportunities in the value of a firm's assets and the costs of debt.

5. Quantitative implications and empirical predictions

In this section, we discuss the quantitative implications and empirical predictions of our model.

[16] Default costs are given by the difference between the face value of debt and the recovered firm value at default.

5.1. Credit spreads

We first investigate the credit spread on newly issued corporate debt, $(c/d(X)) - r$. To address the credit spread puzzle, i.e., to answer the question why standard models underestimate credit spreads for a given level of empirically observed leverage, we investigate a typical Baa-rated firm with a leverage ratio equal to the 43.3% observed in Huang and Huang (2003). Later, we show credit spreads with optimal leverage.

To determine target observed average credit spreads we start with the results in Duffee (1998). He estimates an average yield spread of 148 bps for Baa-rated bonds with 10 years to maturity in the industrial sector. Huang and Huang (2003) calculate bond yield spreads at 194 bps for Baa-rated firms. Their estimate is higher then the one in Duffee (1998) because of the embedded call options in the corporate bond sample and the inclusion of two recessions with high spreads. The 194 bps serve as an upper bound on bond yield spreads. Many studies such as Jones, Mason, and Rosenfeld (1984), Collin-Dufresne, Goldstein, and Spencer (2001), or Longstaff, Mithal, and Neis (2005) find evidence of large nondefault components such as liquidity spreads in corporate bond yields. In particular, Longstaff, Mithal, and Neis (2005) report a nondefault component between 6% and 32% for the Baa-rating class depending on the choice of the risk free interest rate and the model approach. Subtracting a nondefault component of 25% to reflect the results in these studies from the 148 bps reported in Duffee (1998), and from the 194 bps in Huang and Huang (2003), we arrive at a plausible target range of the pure default component between 111 and 145.5 bps.

Table II analyzes the ability of various models to explain empirically observed credit spreads. In the one regime model, we set all regime-dependent parameters equal to their unconditional mean, calculated by weighting their values in boom and recession by the average expected time spent in each regime ($\frac{\lambda_B}{\lambda_B+\lambda_R}$ and $\frac{\lambda_R}{\lambda_B+\lambda_R}$, respectively). The resulting credit spread of 71 bps for only invested assets explains around 55% (48.8%-64%) of observed average credit spreads - the credit spread puzzle.

The standard macroeconomic model for invested assets with constant (unconditional) recovery rate and asset volatility in row (3) yields a credit spread of 91 bps in boom and 99 bps in recession.[17] Weighted by the expected time spent in each regime, the resulting average credit spread is 94.2 bps. This result demonstrates that - consistent with previous studies - structural models with macroeconomic uncertainty yield higher credit spreads than one regime models due to the additional

[17] Hackbarth, Miao, and Morellec (2006) allow for regime-dependent recovery rates in their model but present their main results on credit spreads for constant recovery rates.

uncertainty introduced by regime switches. In particular, they explain around 73% of observed average credit spreads. Incorporating regime dependence of recovery rates and volatility in row (4) increases the average credit spread to 105 bps. The reason is that regime-dependent recovery rates elevate the countercyclicity of default costs, and regime-dependent volatility enhances the countercyclicity of default probabilities, which, together, increase the costs of debt.

INSERT TABLE II HERE

To tease out the effect of growth opportunities on credit spreads, we vary the asset composition ratio by altering s. As increasing s raises the value of the expansion option, we need to simultaneously lower the idiosyncratic asset value to maintain the same total asset value level. Row (5) in Table II reports the calculated credit spread for firms with an average asset composition ratio of 1.7. The credit spread in boom is 114 bps, while in recession it is 146 bps. Therefore, the average default component generated by our model corresponds to 126.8 bps. It reflects around 98% (87.1%-114.2%) of the historically observed default component of corporate bond yields. This result suggests that when accurately incorporating macroeconomic variation and expansion opportunities into standard structural models, aggregate credit spreads can, in fact, be very well explained.

Several aspects are noteworthy about these findings. First, our model increases average credit spreads by 79% (from 71 bps to 126.8 bps) compared to the one regime model.[18]

Second, the presented extension of the standard macroeconomic model is quantitatively important. In particular, the standard macroeconomic model only generates 32.7% additional bps compared to the one regime model. Our approach more than doubles the additional explanatory power for a *given* level of historically observed input parameters.

Finally, our model allows to go beyond the aggregate level by identifying the cross-sectional relationship between expansion opportunities and credit risk. Row (2) in Table III depicts credit spreads in the one regime model for firms with different asset composition ratios. A growth firm with an asset composition ratio of 2.4 exhibits a credit spread that is 27 bps higher than the one of a firm with only invested assets. This effect is remarkable given that we solely vary the assets' characteristics but keep the asset value constant. It arises because expansion opportunities are more sensitive to the underlying uncertainty, and, hence, more volatile. The standard macroeconomic

[18] The resulting credit spread of 126.8 bps is comparable to the one generated in Chen (2010). He calculates credit spreads around 140 bps for Baa-rated firms in his business cycle model calibrated to aggregate consumption where default is particularly costly in recession because marginal utilities are higher in these times.

model with a constant recovery rate and volatility in row (4) illustrates a similar effect of expansion opportunities on credit risk (+28.6 bps). Our model in row (6) shows that a growth firm with an asset composition ratio of 2.4 yields 32.6 bps, or 32%, more than a firm with only invested assets. The reason for the enhanced cross-sectional relationship between credit risk and expansion opportunities is that our model entails countercyclical default costs and countercyclical volatility. It, therefore, allows to fully capture the implications of the enhanced countercyclicity of the default probability induced by growth opportunities, yielding additional credit spreads for firms with a high asset composition ratio.

INSERT TABLE III HERE

Arguably, an average firm with valuable growth opportunities exhibits different parameters than a firm which only consists of invested assets. The tax advantage of debt, or the payout ratio, for example, may be lower for growth firms, which certainly affects explained credit spread levels. The empirical literature, however, finds that existing models are, particularly for growth firms, unable to explain credit spreads after controlling for historically observed parameters of value and growth firms, respectively. Hence, simple variation of input parameters does not contribute to the solution of the credit spreads *puzzle*. What is needed to address the puzzle and its cross-sectional evidence is a model like ours which generates higher explained credit risk than standard models for a *given* level of historically observed input parameters.

The qualitative predictions of our model are consistent with some empirical findings. For example, Davydenko and Strebulaev (2007) find that market-to-book asset values, the ratio of research and development expenses to total investment expenditure, and one minus the ratio of net property, plant, and equipment to total assets are all significantly and positively related to credit spreads (Table VI on p. 2652). Similarly, Molina (2005) documents that firms with a higher ratio of fixed assets to total assets have lower bond yield spreads and higher ratings (Table II on p. 1438). Those studies do not, however, control for all factors relevant to financing decisions in tradeoff models of capital structure.

5.2. Leverage ratios

The previous section shows that macroeconomic variation and a high asset composition ratio generate high credit risk. As firms trade off the tax shield against bankruptcy costs when they determine optimal debt levels, higher credit risk induces them to decrease the optimal leverage ratio. This section investigates the quantitative impact of macroeconomic risk and expansion opportunities on optimal leverage.

We start by addressing the aggregate under-leverage puzzle in Table IV.

INSERT TABLE IV HERE

The target leverage ratio is the average historical leverage ratio of 43.3% for Baa-rated firms reported in Huang and Huang (2003). The calculated ratio of 49.7% from the one regime model fails to conform to this target. As expected, the standard two regime model in row (3) generates a lower average leverage ratio of 45.9%, obtained by weighting optimal market leverages in boom and recession by the expected time spent in each regime. The reduction in the calibration to Baa-rated firms is, however, limited (-3.8 percentage points). In the next step, regime-dependent asset volatilities and recovery rates are introduced according to Table I, while the unconditional mean of each parameter is kept constant. Optimal leverage drops to 44.8%. Incorporating growth opportunities in row (5), i.e., an average asset composition ratio of 1.7, results in an optimal leverage ratio of 42.8% which closely reflects observed leverage ratios for Baa-rated firms. Overall, market leverage drops from 49.7% in the one regime model to 42.8% when we jointly incorporate macroeconomic risk and expansion opportunities. We are, therefore, able to explain the under-leverage puzzle on an aggregate level.

Table V investigates cross-sectional predictions for optimal leverage within the baseline parameter specification from Table I. In our model (row (2)), increasing the asset composition ratio from 1 to 2.4 decreases leverage from 44.8% to 42.6%. A higher asset composition ratio causes higher asset value volatility and more exposure to macroeconomic risk which increases the costs of debt. Higher costs of debt, in turn, induce shareholders to reduce optimal leverage.

We also calculate credit spreads at optimal leverage in row (3). This yields 110.2 bps for a firm with only invested assets, 124.6 bps for an average firm, and 129.2 bps for a firm with an asset composition ratio of 2.4. Hence, even though growth firms optimally reduce leverage, their higher costs of debt dominate, resulting in a positive relation between growth opportunities and credit spreads.

Our qualitative finding is widely accepted in the empirical literature (Bradley, Jarrell, and Kim 1984, Barclay, Smith, and Morellec 2006, Rajan and Zingales 1995). Moreover, the quantitative size of the negative relation derived in our model proposes that increasing the asset composition ratio by one results in an optimal leverage reduction of more than two percentage points. Controlling for all factors relevant to leverage choice in tradeoff models (asset volatility, profitability, nondebt tax shield, payout, and depreciation), Fama and French (2002) obtain coefficients between -0.061 and -0.096 in their regression of market leverage on a similar ratio of asset composition. Hence, our model suggests that a large portion of the empirically observed negative relationship between growth opportunities and market leverage can be explained by accurately capturing the asset risk of growth opportunities. This also leaves room for additional features – for example, agency costs and variation in marginal tax rates – to contribute explanatory power.

5.3. Robustness

We consider alternative parameters and two modifications of our basic model setup.

5.3.1. Alternative parameter choice

In each row in Table VI, we vary a particular parameter of interest while keeping all other parameters constant at their baseline level from Table I. Rows (2) and (3) depict average credit spreads for a scenario of low ($y_B = 1.1$, $y_R = 0.9$) and high ($y_B = 1.2$, $y_R = 0.8$) macroeconomic variation, respectively. The resulting credit spreads show that our qualitative implications remain unchanged. However, both the level of credit spreads and their dependence on the asset composition ratio are sensitive to the specification of the underlying macroeconomic risk. Next, we decrease the expected duration of recessions to two years in row (4), and increase it to ten years in row (5). The results suggest that while macroeconomic variation drives our results, the relative length of regimes seems fairly unimportant.

INSERT TABLE VI HERE

In the last row of Table VI, we increase the degree of regime dependence of the volatility. In particular, we set $\sigma_B = 0.21 < \sigma_R = 0.303$, which leaves the unconditional volatility unchanged at 0.251. Note that average credit spread levels increase compared to the baseline scenario with $\sigma_B = 0.23 < \sigma_R = 0.28$. This result indicates that the degree of regime dependence of a given unconditional volatility also affects credit spreads in macroeconomic structural models. The size of this effect suggests that regime dependence of volatility and recovery rates are jointly necessary to explain the credit spread puzzle.

As macroeconomic variation and regime dependence of volatility are important determinants of credit risk, we also report optimal leverage ratios in the scenario with high macroeconomic variation ($y_B = 1.2$, $y_R = 0.8$) in row (3) of Table VII, and with high regime dependence of volatility $\sigma_B = 0.21$, $\sigma_R = 0.303$) in row (4). The negative relationship between a firm's asset composition ratio and its leverage implied by our model is robust across alternative parameter specifications.

INSERT TABLE VII HERE

5.3.2. Model modifications

Two modifications of our basic model are investigated in this section. First, we analyze the case where the exercise price of the expansion opportunity is financed by issuing additional equity instead of selling assets. Appendix A.5.1 presents the resulting system of ODEs for corporate debt. The possibility that the investment is financed by issuing both additional equity and debt is not presented, because a solution for this case is not available in closed-form.[19] Even though additional debt used to finance the option's exercise can induce equityholders to exercise the opportunity too early, Hackbarth and Mauer (2010) show in the one-regime case that an appropriate debt priority structure can virtually resolve this issue.

Second, we consider firm-specific expansion options. We relax the assumption that assets in place and growth options have identical recovery rates. In the extreme, options might be completely lost upon default. The resulting system of ODEs for the value of corporate debt is stated in Appendix A.5.2.

Again, we first analyze credit risk for a given leverage. Row (3) of Table VIII shows credit spreads for the setting with equity financed exercise costs K. New equity decreases leverage and, hence, lowers credit risk. As firms with a high asset composition ratio are closer to the endogenous exercise boundary where new equity financing occurs, credit spreads are slightly reduced for growth firms compared to the benchmark case. Next, row (4) of Table VIII shows the results for the case where expansion options have a zero recovery rate. As expected, credit spread levels increase for

[19] The reason is that due to the presence of more than one regime, the asset value at option exercise is unknown at time zero, but will influence the choice of the coupon for the new debt. Consequently, the backward induction approach fails in this case.

growth firms. However, in this setting the implied recovery rate of the firm decreases by construction as we increase the asset composition ratio. This result, therefore, merely speaks to credit spread levels, and not to the credit spread puzzle, which requires to generate higher explained credit spreads for a given level of the recovery rate.

INSERT TABLE VIII HERE

Next, we show how leverage ratios are affected by the modifications. While the optimal leverage of firms consisting only of invested assets naturally remains unchanged in the setting with equity financed lump-sum costs, row (3) of Table IX reveals that the anticipated equity financing induces shareholders to optimally choose higher leverage for firms with growth opportunities compared to our baseline setting. There are two reasons for this result. First, the costs of debt slightly decrease for firms with growth opportunities due to the new equity financing at investment. Second, leverage will be lower than induced by an optimal trade-off between bankruptcy costs and the tax shield after the issue of new equity financing. Before investment, shareholders consequently hold higher leverage than in our baseline setting without equity financing to partially counteract the anticipated suboptimal leverage after investment. In untabulated results, we find that for our base case parameters the latter effect becomes particularly strong for firms with an asset composition ratio beyond four, resulting in an optimal leverage around 44%. Close to firms' exercise boundaries, credit spreads and leverage are mainly driven by the expected new financing upon investment, and do not primarily reflect the nature of assets. This validates our focus on asset-financing rather than on equity-financing of growth option exercises to understand the impact of the asset composition on corporate policy choices.

Finally, row (4) in Table IX shows that optimal leverage decreases when the expansion option has zero recovery value, i.e., when it is firm-specific. This is due to the higher costs of debt. As in the case of credit spreads, this finding is explained by the fact that the assumed recovery rate of the firm decreases by construction when the asset composition ratio increases.

INSERT TABLE IX HERE

5.4. Additional implications

Besides the credit spreads and under-leverage puzzles, the model also speaks to additional stylized facts and makes further predictions. These are summarized in Table X.

INSERT TABLE X HERE

The model implies that credit spreads are countercyclical, consistent with, for example, Fama and French (1989).

Next, it delivers a refinement of the default waves prediction generated by standard macroeconomic models: When the aggregate shock can shift between discrete states at random times, defaults by firms in a common market or industry arise simultaneously, namely when the aggregate shock shifts from y_B to y_R. Our results indicate that default clustering should be particularly pronounced for firms with high expansion opportunities, because the size of the discrete jump in the default threshold is positively related to the asset composition ratio.

It is widely accepted that, on average, growth firms have lower recovery rates than value firms (Cantor and Varma (2005)). One explanation of this fact is offered by Shleifer and Vishny (1992). They argue that industry buyers of growth firms have little cash relative to the value of assets. Hence, growth firms are likely to be themselves severely credit constrained when owners have troubles meeting debt payments in recession and need to sell assets. As a consequence, they are poor candidates for debt finance. Similar to our model, the authors assume that shocks which cause the sellers' distress are industry- or economy-wide. While this argument is plausible in a static framework, it breaks down in a dynamic model: When one accounts for the fact that equityholders are allowed to contribute cash to the firm whenever it is worthwhile to do so, growth and invested assets are liquidated whenever the continuation value is lower than the value of debt payments. Thus, after endogenizing the default decision, growth options would only have different recovery rates than invested assets if the shareholders of potential industry buyers of growth assets were more cash constrained in recession than the shareholders of potential industry buyers of value assets. It is not obvious why this should be the case. By contrast, our model's explanation for lower recovery rates of growth assets holds up in a dynamic setting. The intuition for the result derives from the observation that default thresholds of firms with a higher asset composition ratio are more countercyclical than the ones of value firms. As a consequence, shareholders' propensity to default in recession, when recovery rates are low, is more pronounced for growth firms.

The derivation of endogenously determined optimal option exercise boundaries also helps to relate our work to the empirical investment literature. The model is in line with a strong procyclical pattern of aggregate investment as reported in Barro (1990). When the regime switches from recession to boom, all firms in the region between the two option exercise boundaries in Figure 2 immediately exercise their expansion option by investing K. Moreover, the high asset value drift positively affects the probability of firms reaching the exercise boundary during boom. At the other end, investment dries out when the economy changes from boom to recession, because the optimal exercise boundary jumps up. The lower drift additionally decreases the probability of firms reaching the option exercise boundary during recession. Our model also predicts that observed investment clustering should be mainly driven by firms with high expansion options.

Hackbarth, Miao, and Morellec (2006) generate countercyclical leverage ratios in their macroeconomic model. As in our framework, their optimal coupon rate, which determines the value of debt, in boom exceeds the coupon rate in recession. At the same time, the value of assets is greater in boom. The second effect dominates the first, generating the countercyclicity in leverage. We additionally incorporate the empirical fact that asset volatility is regime-dependent. Because the latter decreases in boom and increases in recession, our optimal coupon rate varies more than in Hackbarth, Miao, and Morellec (2006) when the regime changes. The change in the value of optimal debt then always dominates the change in the value of assets, generating procyclical leverage ratios. An average firm in our baseline scenario, for example, exhibits an optimal leverage ratio of 45.5% in boom, and 38.8% in recession. These predictions for the cyclicality of leverage ratios, obtained with more realistic parameters than used in existing work, are consistent with the empirical findings in Choi and Richardson (2008). The authors document a very strong negative relation between asset volatility, which is usually higher in recession, and leverage. Our results, however, only partially conform to Korajczyk and Levy (2003) who find that financially constrained firms indeed exhibit pro-cyclical leverage, but unconstrained firms' leverage ratios vary counter-cyclically.

6. Conclusion

It is now well-accepted that macroeconomic risk is central for understanding capital structure choices. Specifically, defaults are more likely in recession, when they are particularly costly. This increases the costs of debt for all firms. But to explain the cross-sectional variation in apparent underleverage and apparently excessive costs of debt, we need variation inside the firm. This paper formalizes the role of one particularly important aspect of this heterogeneity, the asset composition of firms. It is not surprising that the asset composition can be important for optimal capital structure. After all, economists have devoted much effort to understanding the difference between value and growth firms in terms of their financial structure, starting with Myers (1977) and Jensen (1986). In this paper, we show that, in fact, incorporating this factor goes a long way towards quantitatively explaining both average levels and the cross-sectional variation in costs of debt and leverage, without the need to appeal to factors such as agency costs.

Our model implies that companies with a high portion of expansion opportunities tend to be riskier in general, and, at the same time, particularly sensitive to macroeconomic risk. The reason is that they are not only more volatile (because they represent a levered claim), but also have a higher propensity to default in bad times than firms with a low portion of expansion opportunities. Thus, the countercyclicity of the default probability is higher the greater the ratio of expansion options to total assets. This relation (exacerbated by costly liquidation in recession) implies higher costs of debt and more important endogenous shadow costs of leverage for firms with growth opportunities than for those with only invested assets. Thus, our findings explain why the credit spread and under-leverage puzzles are empirically more pronounced for growth firms. Moreover, because the economy is made up of a mix of firms, the model accounts, in quantitatively fairly accurate ways, for the average credit spread and under-leverage puzzles.

We have studied one type of (arguably important) real options of firms, namely, growth opportunities. However, firms have a wide and varying range of options, including abandonment and shut-down options. A structural model incorporating these options could, therefore, yield further cross-sectional predictions.

While recent research has made important progress in enhancing our understanding of average credit risk, the cross-section of credit risk has not received sufficient attention. Analyzing it empirically is, fortunately, quite feasible. Liquid credit default swap quotes are now widely available on a firm-by-firm basis, which allows researchers to investigate specific relationships between firm-specific characteristics such as growth opportunities and credit spreads. Our paper also provides a theoretical basis that can guide empirical research in this direction.

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7. Figures

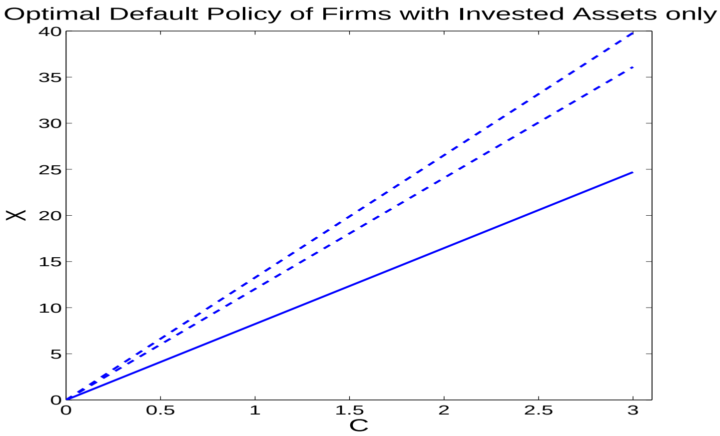

Figure 1. The solid line represents the default threshold in boom, the two dashed lines the default thresholds in recession for various coupon levels. The asset volatility is set to $\sigma_B = \sigma_R = 0.23$ to generate the upper dashed line, and to $\sigma_B = 0.23 < \sigma_R = 0.28$ for the lower dashed line.

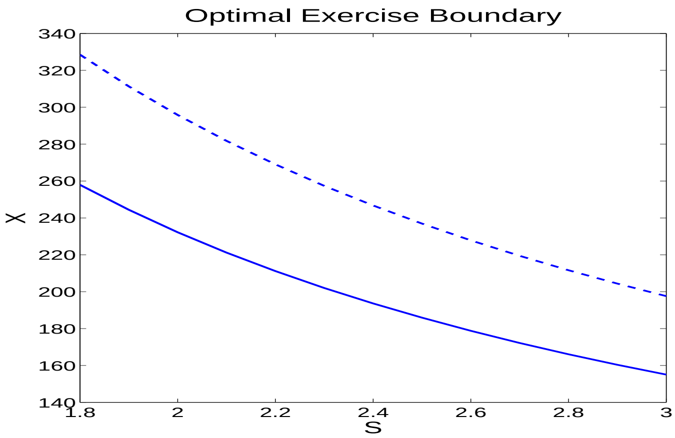

Figure 2. The dashed line shows the optimal exercise boundary for the idiosyncratic asset value X in recession for a range of scale parameters s. The solid line represents the corresponding exercise boundary in boom. The graph is drawn for optimal leverage. The baseline parameter specification from Table I is used.

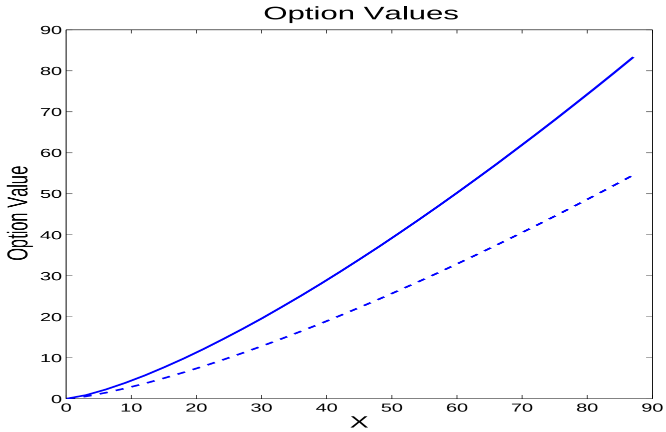

Figure 3. The solid line represents the value of the expansion option in boom for a range of scale parameters s between 0 and 2.5. The dashed line shows the corresponding values of the same option in recession. The graph is drawn for optimal leverage. The baseline parameter specification from Table I is used.

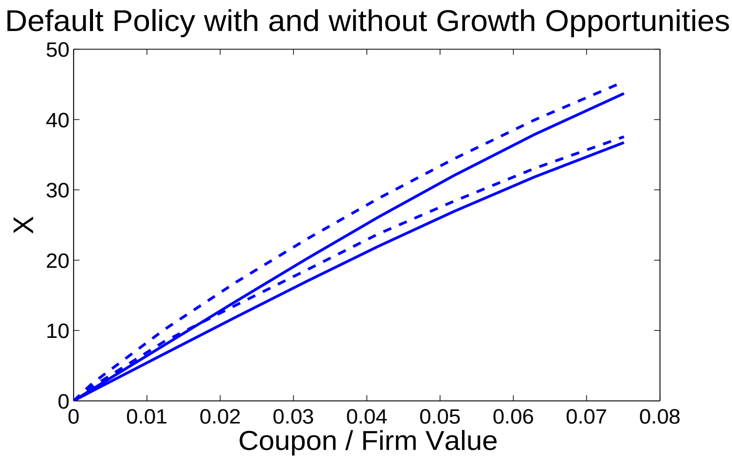

Figure 4. The solid lines represent the default thresholds of a firm consisting of only invested assets, the dashed lines the default thresholds of a firm with an asset composition ratio of 2.4. The baseline parameter specification from Table I is used, with s being varied to generate the desired asset composition ratio.

8. Tables

Table I
Baseline Parameter Calibration

This table depicts our baseline scenario. Panel A contains the calibrated parameters of a typical, Baa-rated S&P 500 firm. Panel B and C show the parameter choice for the expansion option and our workhorse macro economy, respectively. Later sections present results for alternative parameter choices.

Panel A. Firm Characteristics	
Initial value of idiosyncratic asset value	$X = 100$
Tax advantage of debt	τ=0.15
Payout ratio	δ=0.03
Asset volatility in recession	$\sigma_R = 0.28$
Asset volatility in boom	$\sigma_B = 0.23$
Recovery rate in recession	$\alpha_R = 0.5$
Recovery rate in boom	$\alpha_B = 0.7$
Panel B. Expansion Option Parameters	
Exercise price	$K = 140$
Scale parameter	$s = 1.2$
Panel C. Macroeconomic Variables	
Regime persistency in recession	$\lambda_R = 0.15$
Regime persistency in boom	$\lambda_B = 0.1$
Interest rate	$r = 0.06$
Aggregate shock variable in recession	$y_R = 0.85$
Aggregate shock variable in boom	$y_B = 1.15$

Table II
Explaining the Aggregate Credit Spread Puzzle

This table shows average credit spreads in basis points (bps) generated by various models. Credit spreads are defined as the coupon divided by the debt value, minus the riskless interest rate. Debt maturity is assumed to be infinite. Parameters are calibrated to a typical *Baa*-rated firm with a leverage ratio equal to 43.3%. In the one regime model, parameters are set to match their unconditional mean, calculated by weighting their values in boom and recession by the average time spent in each regime. The standard macroeconomic model is a two regime model with constant recovery rate and asset volatility. In our model, we allow for regime-dependent recovery rates and asset volatility as outlined in Table I. The average asset composition ratio is 1.7, where the ratio is defined as the sum of the value of the growth option and invested assets divided by invested assets. Average credit spreads in the two regime models are obtained by weighting credit spreads in boom and recession by the average expected times spent in each regime, respectively.

(1) **Observed Average Credit Spread (bps)**	**111-145.5**
(2) One Regime Model	71
(3) Standard Macroeconomic Model	94.2
(4) Our Model for Invested Assets	105
(5) Our Model for Average Asset Composition Ratio	**126.8**

Table III
Expansion Opportunities and Credit Spreads

This table demonstrates the relationship between growth opportunities and credit spreads expressed in basis points (bps) for various models. Credit spreads are defined as the coupon divided by the debt value, minus riskless interest rate. Debt maturity is assumed to be infinite. Parameters are calibrated to a typical *Baa*-rated firm with a leverage ratio equal to 43.3%. In the one regime model, parameters are set to match their unconditional mean. The standard macroeconomic model is a two regime model with constant recovery rate and asset volatility. In our model, we allow for regime-dependent recovery rates and asset volatility as outlined in Table I. The asset composition ratio is defined as the sum of the value of the growth option and invested assets divided by invested assets. Credit spreads in the two regime models are obtained by weighting credit spreads in boom and recession by the average expected times spent in each regime, respectively.

Asset Composition Ratio	1 (Invested Assets)	1.7 (Average Firm)	2.4 (Growth Firm)	Difference due to Asset Composition Ratio (2.4 instead of 1.0)
(1) **Observed Average Credit Spread**		**111-145.5**		
(2) One Regime Model Calibrated to Unconditional Parameters	71	89	98	+27
(3) Extra Credit Spread due to Business Cycle Risk	+23.2	+23.5	+24.8	
(4) Standard Macroeconomic Model	94.2	112.5	122.8	+28.6
(5) Extra Credit Spread due to Regime-Dependent Volatility and Recovery Rate	+10.8	+14.3	+15.8	
(6) **Predicted Credit Spreads from our Model**	105	**126.8**	137.6	+32.6

Table IV
Explaining the Aggregate Under-Leverage Puzzle

This table shows optimal leverage levels generated by various models. Leverage is defined as the market value of debt divided by firm value. Debt maturity is assumed to be infinite. Parameters are calibrated to a typical *Baa*-rated firm. In the one regime model, parameters are set to match their unconditional mean. The standard macroeconomic model is a two regime model with constant recovery rate and asset volatility. In our model, we allow for regime-dependent recovery rates and asset volatility as outlined in Table I. The average asset composition ratio is 1.7, where the ratio is defined as the sum of the value of the growth option and invested assets divided by invested assets. Average leverage ratios in the two regime models are obtained by weighting optimal leverages in boom and recession by the average expected times spent in each regime, respectively.

(1) **Observed Average Leverage Ratio**	**43.3%**
(2) One Regime Model	49.7%
(3) Standard Macroeconomic Model	45.9%
(4) Our Model for Invested Assets	44.8%
(5) **Our Model for Average Asset Composition Ratio**	**42.8%**

Table V
Expansion Opportunities and Leverage Ratios

This table analyzes the relationship between expansion opportunities, leverage ratios, and credit spreads. Leverage is defined as the market value of debt divided by firm value, and credit spreads as the coupon divided by the debt value, minus riskless interest rate. Debt maturity is assumed to be infinite. Parameters are calibrated to a typical *Baa*-rated firm. Our macroeconomic model represents the baseline parameter specification with regime-dependent recovery rates and asset volatility from Table I. Average leverage ratios (credit spreads) are obtained by weighting optimal leverages (credit spreads) in boom and recession by the average expected times spent in each regime. The asset composition ratio is defined as the sum of the value of the growth option and invested assets, divided by invested assets.

Asset Composition Ratio	1 (Invested Assets)	1.7 (Average Firm)	2.4 (Growth Firm)
(1) **Observed Average Leverage Ratio**		**43.3%**	
(2) Our Macroeconomic Model	44.8%	42.8%	42.6%
(3) Credit Spreads at Optimal Leverage	110.2 bps	124.6 bps	129.2 bps

Table VI

Alternative Parameter Specification and Credit Spreads

This table analyzes the sensitivity of our model to alternative parameter specifications. Credit spreads are defined as the coupon divided by the debt value, minus riskless interest rate. Debt maturity is assumed to be infinite. Parameters are calibrated to a typical *Baa*-rated firm as shown in Table I with a leverage ratio equal to 43.3%. The alternative parameter specifications are indicated in parentheses in column 1. The asset composition ratio is defined as the sum of the value of the growth option and invested assets divided by invested assets. Credit spreads are obtained by weighting credit spreads in boom and recession by the average expected times spent in each regime, respectively.

Asset Composition Ratio	1 (Invested Assets)	1.7 (Average Firm)	2.4 (Growth Firm)	Difference due to Asset Composition Ratio (2.4 instead of 1.0)
(1) **Observed Average Credit Spread**		**111-145.5**		
(2) Low Macroeconomic Variation ($y_B = 1.1$, $y_R = 0.9$)	89.6	110	119.2	+29.6
(3) High Macroeconomic Variation ($y_B = 1.2$, $y_R = 0.8$)	127.2	151	161.9	+34.7
(4) Short Recession Relative to Boom ($\lambda_B = 0.1$,$\lambda_R = 0.5$)	103.8	125.9	135.6	+ 31.8
(5) Long Recession Relative to Boom ($\lambda_B = 0.1$, $\lambda_R = 0.1$)	109	131.1	141.6	+32.6
(6) High Volatility Difference ($\sigma_B = 0.21$, $\sigma_R = 0.303$)	108	131.7	141.2	+33.2

Table VII
Alternative Parameter Specification and Leverage Ratios

This table analyzes the relationship between expansion opportunities and leverage ratios for alternative parameter choices. Leverage is defined as the market value of debt divided by firm value. Debt maturity is assumed to be infinite. Parameters are calibrated to a typical *Baa*-rated firm. Our macroeconomic model represents the baseline parameter specification with regime-dependent recovery rates and asset volatility from Table I. Average leverage ratios in the two regime models are obtained by weighting optimal leverages in boom and recession by the average expected times spent in each regime, respectively. The asset composition ratio is defined as the sum of the value of the growth option and invested assets, divided by invested assets.

Asset Composition Ratio	1 (Invested Assets)	1.7 (Average Firm)	2.4 (Growth Firm)
(1) **Observed Average Leverage Ratio**		**43.3%**	
(2) Our Macroeconomic Model	44.8%	42.8%	42.6%
(3) High Macroeconomic Variation ($y_B = 1.2$, $y_R = 0.8$)	43.1%	41.2%	40.7%
(4) High Volatility Difference ($\sigma_B = 0.21$, $\sigma_R = 0.303$)	45.7%	44.1%	43.6%

Table VIII
Alternative Settings and Credit Spreads

This table shows the results of our model for alternative model settings. Credit spreads are defined as the coupon divided by the debt value, minus riskless interest rate. Debt maturity is assumed to be infinite. Parameters are calibrated to a typical *Baa*-rated firm as shown in Table I with a leverage ratio equal to 43.3%. The setting is indicated in parentheses in column 1. The asset composition ratio is defined as the sum of the value of the growth option and invested assets divided by invested assets. Credit spreads are obtained by weighting credit spreads in boom and recession by the average expected times spent in each regime, respectively.

Asset Composition Ratio	1 (Invested Assets)	1.7 (Average Firm)	2.4 (Growth Firm)	Difference due to Asset Composition Ratio (2.4 instead of 1.0)
(1) **Observed Average Credit Spread**		**111-145.5**		
(2) Main Analysis	105	126.8	137.6	+32.6
(3) Equity-financed Exercise Price K	105	125	135.5	+30.5
(4) Firm-specific Expansion Option	105	142.4	162.8	+57.8

Table IX
Alternative Settings and Leverage Ratios

This table analyzes the relationship between expansion opportunities and leverage ratios in alternative model settings. Leverage is defined as the market value of debt divided by firm value. Debt maturity is assumed to be infinite. Parameters are calibrated to a typical *Baa*-rated firm. Our macroeconomic model represents the baseline parameter specification with regime-dependent recovery rates and asset volatility from Table I. Average leverage ratios in the two regime models are obtained by weighting optimal leverages in boom and recession by the average expected times spent in each regime, respectively. The asset composition ratio is defined as the sum of the value of the growth option and invested assets, divided by invested assets.

Asset Composition Ratio	1 (Invested Assets)	1.7 (Average Firm)	2.4 (Growth Firm)
(1) **Observed Average Leverage Ratio**		**43.3%**	
(2) Main Analysis	44.8%	42.8%	42.6%
(3) Equity-financed Exercise Price K	44.8%	43.2%	43.4%
(4) Firm-specific Expansion Option	44.8%	36.4%	33.5%

Table X
Summary of Main Predictions

This table summarizes the main predictions of our model for firms with low and high asset composition ratios, respectively.

Asset Composition Ratio	Low	High
Credit Spreads	Low	High
Leverage	High	Low
Cyclicality of Credit Spreads	Countercyclical	Countercyclical
Default Clustering	Moderate	High
Recovery Rates	High	Low
Investment Clustering	Moderate	High
Cyclicality of Leverage	Procyclical	Procyclical

A. Appendix

A.1. Firms with only invested assets

A.1.1. The valuation of corporate debt

Case V1: $\hat{D}_B < \hat{D}_R$.[20] We use the notation $\hat{}$ to indicate that a parameter or function refers to a firm with only invested assets (e.g. the default boundaries $\hat{D}_i$). An investor investing in corporate debt requires an instantaneous return equal to the risk-free rate r. Once the firm defaults, debtholders receive a fraction α_i of the asset value $V_i(X)$. The required rate of return on debt must be equal to the realized rate of return plus the proceeds of debt. Therefore, an application of Ito's lemma with regime switches shows that debt satisfies the following system of ODEs:

For $0 \leq X \leq \hat{D}_B$:

$$\begin{cases} \hat{d}_B(X) &= \alpha_B V_B(X) \\ \hat{d}_R(X) &= \alpha_R V_R(X). \end{cases} \tag{A-1}$$

For $\hat{D}_B < X \leq \hat{D}_R$:

$$\begin{cases} r\hat{d}_B(X) &= c + \mu_B X \hat{d}'_B(X) + \frac{\sigma 2_B}{2} X 2 \hat{d}''_B(X) + \lambda_B \left(\alpha_R V_R(X) - \hat{d}_B(X)\right) \\ \hat{d}_R(X) &= \alpha_R V_R(X). \end{cases} \tag{A-2}$$

For $X > \hat{D}_R$:

$$\begin{cases} r\hat{d}_B(X) &= c + \mu_B X \hat{d}'_B(X) + \frac{\sigma 2_B}{2} X 2 \hat{d}''_B(X) + \lambda_B \left(\hat{d}_R(X) - \hat{d}_B(X)\right) \\ r\hat{d}_R(X) &= c + \mu_R X \hat{d}'_R(X) + \frac{\sigma 2_R}{2} X 2 \hat{d}''_R(X) + \lambda_R \left(\hat{d}_B(X) - \hat{d}_R(X)\right). \end{cases} \tag{A-3}$$

The functional form of the solution is

$$\hat{d}_i(X) = \begin{cases} \alpha_i V_i(X) & X \leq \hat{D}_i & i = B, R \\ \hat{C}_1 X^{\beta_1^B} + \hat{C}_2 X^{\beta_2^B} + C_3 X + C_4 & \hat{D}_B < X \leq \hat{D}_R, & i = B \\ \hat{A}_{i1} X^{\gamma_1} + \hat{A}_{i2} X^{\gamma_2} + A_5 & X > \hat{D}_R, & i = B, R, \end{cases} \tag{A-4}$$

where $\hat{A}_{B1}, \hat{A}_{B2}, \hat{A}_{R1}, \hat{A}_{R2}, A_5, \hat{C}_1, \hat{C}_2, C_3, C_4, \gamma_1, \gamma_2, \beta_1^B$, and β_2^B are real-valued parameters to be determined. We first consider the region $X > \hat{D}_R$, and use the standard approach of plugging in the functional form $\hat{d}_i(X) = \hat{A}_{i1} X^{\gamma_1} + \hat{A}_{i2} X^{\gamma_2} + A_5$ into both equations of (A-3). Comparing coefficients, we find first that $A_5 = \frac{c}{r}$, and then that $\hat{A}_{Rk}$ is always a multiple of $\hat{A}_{Bk}$, $k = 1, 2$, with the factor $l_k := \frac{1}{\lambda_B}(r + \lambda_B - \mu_B \gamma_k - \frac{1}{2}\sigma 2_B \gamma_k (\gamma_k - 1))$, i.e., $A_{Bk} = l_k A_{Rk}$.

[20] The solution of the case $\hat{D}_B > \hat{D}_R$ can be found by the according change in notation.

Using these results and comparing coefficients again, we find that γ_1 and γ_2 correspond to the negative roots of the quartic equation

$$\left(\mu_R\gamma + \frac{1}{2}\sigma_R 2\gamma(\gamma-1) - \lambda_B - r\right)\left(\mu_B\gamma + \frac{1}{2}\sigma_B 2\gamma(\gamma-1) - \lambda_R - r\right) = \lambda_R\lambda_B, \tag{A-5}$$

with the reason for taking the negative roots being the no-bubbles condition for debt stated below. Note that as pointed out by Guo (2001), this quartic equation always has four distinct real roots, two of them being negative, and two of them positive.

Next, we consider the region $\hat{D}_B \leq X \leq \hat{D}_R$ i.e., the realized state of the Markov chain is boom (if not, the solution is already known by the second equation of system (A-2)). Again, plugging in the functional form $d_B(X) = \hat{C}_1 X^{\beta_1^B} + \hat{C}_2 X^{\beta_2^B} + C_3 X + C_4$ into the first equation of (A-2), we find by comparison of coefficients that

$$\begin{aligned}
\beta_{1,2}^B &= \frac{1}{2} - \frac{\mu_B}{\sigma_B 2} \pm \sqrt{\left(\frac{1}{2} - \frac{\mu_B}{\sigma_B 2}\right)2 + \frac{2(r+\lambda_B)}{\sigma_B 2}} \\
C_3 &= \frac{\lambda_B \alpha_R y_R}{r + \lambda_B - \mu_B} \\
C_4 &= \frac{c}{r+\lambda_B},
\end{aligned} \tag{A-6}$$

where we used that $r + \lambda_B - \mu_B > 0$ by (5). The unknown parameters are now $\hat{A}_{B1}, \hat{A}_{B2}, \hat{C}_1$ and $\hat{C}_2$. The boundary conditions are

$$\lim_{X\to\infty} \frac{\hat{d}_i(X)}{X} < \infty, \quad i = B, R \tag{A-7}$$

$$\lim_{X\searrow\hat{D}_R} \hat{d}_B(X) = \lim_{X\nearrow\hat{D}_R} \hat{d}_B(X) \tag{A-8}$$

$$\lim_{X\searrow\hat{D}_R} \hat{d}'_B(X) = \lim_{X\nearrow\hat{D}_R} \hat{d}'_B(X) \tag{A-9}$$

$$\lim_{X\searrow\hat{D}_B} \hat{d}_B(X) = \alpha_B D_B y_B \tag{A-10}$$

$$\lim_{X\searrow\hat{D}_B} \hat{d}_R(X) = \alpha_R D_R y_R. \tag{A-11}$$

Condition (A-7) is the no-bubbles condition used above in determining the appropriate roots of equation (A-5). The default thresholds $\hat{D}_R$ and $\hat{D}_B$ are chosen by the equityholders, and are taken as given by the debtholders. The boundary conditions are, hence, the value-matching conditions (A-8), (A-10), and (A-11), and the smooth-pasting condition at the higher default threshold $\hat{D}_B$ for the debt function in recession $\hat{d}_R(\cdot)$, equation (A-9). As the default thresholds are not related to an optimality concept from the point of view of the debtholders, there are no smooth-pasting conditions at default to consider.

We plug in the functional form (A-4) into conditions (A-8)-(A-11), and obtain a four-dimensional linear system in the four unknowns $\hat{A}_{B1}, \hat{A}_{B2}, \hat{C}_1$ and $\hat{C}_2$:

$$\begin{aligned}
\hat{A}_{B1}\hat{D}_R^{\gamma_1} + \hat{A}_{B2}\hat{D}_R^{\gamma_2} + A_5 &= \hat{C}_1\hat{D}_R^{\beta_1^B} + \hat{C}_2\hat{D}_R^{\beta_2^B} + C_3\hat{D}_R + C_4 \\
\hat{A}_{B1}\gamma_1\hat{D}_R^{\gamma_1} + \hat{A}_{B2}\gamma_2\hat{D}_R^{\gamma_2} &= \hat{C}_1\beta_1^B\hat{D}_R^{\beta_1^B} + \hat{C}_2\beta_2^B\hat{D}_R^{\beta_2^B} + C_3\hat{D}_R \\
\alpha_B\hat{D}_B y_B &= \hat{C}_1\hat{D}_R^{\beta_1^B} + \hat{C}_2\hat{D}_R^{\beta_2^B} + C_3\hat{D}_R + C_4 \\
l_1\hat{A}_{B1}\hat{D}_R^{\gamma_1} + l_2\hat{A}_{B2}\hat{D}_R^{\gamma_2} + A_5 &= \alpha_R\hat{D}_R y_R.
\end{aligned} \tag{A-12}$$

We define the matrices

$$\begin{aligned}
\hat{M} &:= \begin{bmatrix} \hat{D}_R^{\gamma_1} & \hat{D}_R^{\gamma_2} & -\hat{D}_R^{\beta_1^B} & -\hat{D}_R^{\beta_2^B} \\ \gamma_1\hat{D}_R^{\gamma_1} & \gamma_2\hat{D}_R^{\gamma_2} & -\beta_1^B\hat{D}_R^{\beta_1^B} & -\beta_2^B\hat{D}_R^{\beta_2^B} \\ 0 & 0 & \hat{D}_R^{\beta_1^B} & \hat{D}_R^{\beta_2^B} \\ l_1\hat{D}_R^{\gamma_1} & l_2\hat{D}_R^{\gamma_2} & 0 & 0 \end{bmatrix} \\
\hat{b} &:= \begin{bmatrix} C_3\hat{D}_R + C_4 - A_5 \\ C_3\hat{D}_R \\ \alpha_B\hat{D}_B y_B - C_3\hat{D}_R - C_4 \\ \alpha_R\hat{D}_R y_R - A_5 \end{bmatrix},
\end{aligned}$$

such that $\hat{M}\begin{bmatrix}\hat{A}_{B1} & \hat{A}_{B2} & \hat{C}_1 & \hat{C}_2\end{bmatrix}^T = \hat{b}$. Hence the solution of the unknowns left is given by

$$\begin{bmatrix}\hat{A}_{B1} & \hat{A}_{B2} & \hat{C}_1 & \hat{C}_2\end{bmatrix}^T = \hat{M}^{-1}\hat{b}. \tag{A-13}$$

Note that none of the parameters depends on X, as we already used in the calculation. Hence we can confirm the derivative to be

$$\hat{d}_i'(X) = \begin{cases} \alpha_i y_i & X \le \hat{D}_i \quad\quad i = B, R \\ \hat{C}_1\beta_1^B X^{\beta_1^B - 1} + \hat{C}_2\beta_2^B X^{\beta_2^B - 1} + C_3 & \hat{D}_B < X \le \hat{D}_R, \quad i = B \\ \hat{A}_{i1}\gamma_1 X^{\gamma_1 - 1} + \hat{A}_{i2}\gamma_2 X^{\gamma_2 - 1} & X > \hat{D}_R, \quad\quad i = B, R. \end{cases} \tag{A-14}$$

This result will be used to determine the first derivative of the equity value.

Case V2: $\hat{D}_B = \hat{D}_R$**.** Define $\hat{D}_1 =: \hat{D}_R$, and consider the case $y_R = y_B =: y$[21]. It follows that $\hat{d}_R(X) = \hat{d}_B(X) =: \hat{d}(X)\ \forall\ X \ge 0$. Consequently, $\sigma_R = \sigma_B =: \sigma, \lambda_R = \lambda_B =: \lambda$, and thus $\mu_R = \mu_B =: \mu$. In words, this situation describes the case that the system is not subject to regime switches. Note that for $y = 1$, this case corresponds to the model of Leland (1994). Using that the required return must be equal to

[21] The case $\hat{D}_R = \hat{D}_B$, and $y_R \neq y_B$ can be solved without further difficulties.

the expected realized one plus the proceeds from debt, we find the system to solve:

$$\begin{aligned} r\hat{d}(X) &= c + \mu X \hat{d}'(X) + \tfrac{\sigma 2}{2} X 2 \hat{d}''(X) \quad X > \hat{D} \\ \hat{d}(X) &= \alpha X y \qquad\qquad\qquad\qquad\quad X \leq \hat{D}. \end{aligned} \tag{A-15}$$

The boundary conditions are the no bubbles condition, as well as value-matching at default:

$$\begin{aligned} \lim_{X\to\infty} \frac{\hat{d}(X)}{X} &< \infty \\ \lim_{X\searrow\hat{D}} \hat{d}\,(X) &= \alpha y \hat{D}. \end{aligned} \tag{A-16}$$

The functional form of the solution is

$$\hat{d}(X) = \begin{cases} \alpha y X & X < \hat{D} \\ \hat{B} X^{\beta_2} + \frac{c}{r} & X \geq \hat{D}, \end{cases} \tag{A-17}$$

where $\hat{B}$ and β_2 are real-valued parameters. It is straightforward to show that

$$\beta_2 = \frac{1}{2} - \frac{\mu}{\sigma 2} - \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma 2}\right) 2 + \frac{2r}{\sigma 2}} \tag{A-18}$$

$$\hat{B} = \left(\alpha y \hat{D} - \frac{c}{r}\right) \hat{D}^{-\beta_2}. \tag{A-19}$$

We repeat that the derivative is given by

$$\hat{d}'(X) = \begin{cases} \alpha y & X \leq \hat{D} \\ \hat{B}\beta_2 X^{\beta_2 - 1} & X > \hat{D}. \end{cases} \tag{A-20}$$

A.1.2. The valuation of tax benefits

The value of tax benefits $\hat{t}_i(X)$ corresponds to the value of debt with recovery rates equal to zero, and a coupon of $c\tau$ (and analogously for Case V2). Further details can be found in the main text for the more general case of a firm consisting of both assets in place and a growth option.

A.1.3. The valuation of default costs

As there are no continuous cash-flows associated with default costs, its value function $\hat{b}_i(X)$ can be calculated as the value of a debt contract with recovery rates $1 - \alpha_B$ and $1 - \alpha_R$, respectively, and a coupon of zero. Again, for details see the main text for the more general case. Case V2 can be treated analogously.

A.1.4. Firm value

Total firm value $\hat{f}_i$ in regime $i = B, R$ corresponds to the value of assets $y_i X$, plus the value of tax benefits from debt $\hat{t}_i(X)$ less the value of potential default costs $\hat{b}_i(X)$, i.e.,

$$\hat{f}_i(X) = Xy_i + \hat{t}_i(X) - \hat{b}_i(X).$$

Analogously, for Case V2, we have

$$\hat{f}(X) = Xy + \hat{t}(X) - \hat{b}(X).$$

A.1.5. The valuation of equity

The levered firm value equals the sum of debt and equity values. Hence, equity value $\hat{e}_i(X)$, $i = B, R$, may be written as

$$\hat{e}_i(X) = \hat{f}_i(X) - \hat{d}_i(X) = Xy_i + \hat{t}_i(X) - \hat{b}_i(X) - \hat{d}_i(X), \tag{A-21}$$

or, for the Case V2,

$$\hat{e}(X) = \hat{f}(X) - \hat{d}(X) = Xy + \hat{t}(X) - \hat{b}(X) - \hat{d}(X). \tag{A-22}$$

This is the closed-form expression for equity.

A.1.6. Default policy

Once debt has been issued, managers select the ex-post default policy that maximizes the value of equity. As in Leland (1998), default is triggered by shareholders' decision to cease injecting funds in the firm. Formally, the default policy is determined by postulating that the derivative of the equity value has to be zero at the according default boundary. It is straightforward to calculate the first derivative of equity in closed form, using (A-21) (or (A-22)), and the derivatives of debt, taxes, and bankruptcy cost (A-14) (or (A-20)). The system to solve is hence

for **Case V1: $\hat{D}_B < \hat{D}_R$:**

$$\begin{cases} \hat{e}'_B(\hat{D}^*_B) &= 0 \\ \hat{e}'_R(\hat{D}^*_R) &= 0. \end{cases} \tag{A-23}$$

For **Case V2: $\hat{D}_B = \hat{D}_R$,** the system is

$$\hat{e}'(D^*) = 0. \tag{A-24}$$

Then we solve this problem numerically.

Note that for a given coupon, all value functions can be calculated by following the calculations up to system (A-23) or system (A-24), depending on the case. This will be used later for the calculation of the value of corporate securities of a firm consisting of both assets in place and an expansion option.

A.1.7. Capital structure

For each coupon level $\hat{c}$, debtholders evaluate debt at issuance anticipating the ex-post optimal default decision of shareholders. As the proceeds of the issue accrue to shareholders, the latter do not only care about the value of equity, but also about the value of debt. Hence, the optimal capital structure is determined ex-ante by the coupon level $\hat{c}^*$ which maximizes the value of equity and debt, i.e., the value of the firm. Denote by $\hat{f}_i^*(X)$ the firm value of a firm with only invested assets, given optimal ex-post default thresholds. The ex-ante optimal coupon of this firm then solves
in Case V1

$$\hat{c}^* := \text{argmax}_{\hat{c}} \hat{f}_i^*(X), \tag{A-25}$$

and in Case V2

$$\hat{c}^* := \text{argmax}_{\hat{c}} \hat{f}^*(X). \tag{A-26}$$

A.2. The value of the growth option

Case G1: $X_R > X_B$**.** Recall that the system to solve is:
For $0 \leq X < X_B$:

$$\begin{cases} rG_B(X) &=& \mu_B X G_B'(X) + \frac{\sigma 2_B}{2} X2G_B''(X) + \lambda_B \left(G_R(X) - G_B(X)\right) \\ rG_R(X) &=& \mu_R X G_R'(X) + \frac{\sigma 2_R}{2} X2G_R''(X) + \lambda_R \left(G_B(X) - G_R(X)\right) \end{cases} \tag{A-27}$$

For $X_B \leq X < X_R$:

$$\begin{cases} G_B(X) &=& sXy_B - K \\ rG_R(X) &=& \mu_R X G_R'(X) + \frac{\sigma 2_R}{2} X2G_R''(X) + \lambda_R \left(sXy_B - K - G_R(X)\right) \end{cases} \tag{A-28}$$

For $X \geq X_R$:

$$\begin{cases} G_B(X) &=& sXy_B - K \\ G_R(X) &=& sXy_R - K, \end{cases} \tag{A-29}$$

subject to the boundary conditions:

$$\lim_{X \searrow 0} G_i(X) = 0, \quad i = B, R \tag{A-30}$$

$$\lim_{X \searrow X_B} G_R(X) = \lim_{X \nearrow X_B} G_R(X) \tag{A-31}$$

$$\lim_{X \searrow X_B} G_R'(X) = \lim_{X \nearrow X_B} G_R'(X) \tag{A-32}$$

$$\lim_{X \nearrow X_R} G_R(X) = sX_R y_R - K \tag{A-33}$$

$$\lim_{X \nearrow X_B} G_B(X) = sX_B y_B - K \tag{A-34}$$

Then the solution can be written as

$$G_i(X) = \begin{cases} \tilde{A}_{i1}X^{\gamma_3} + \tilde{A}_{i2}X^{\gamma_4} & X < X_B, \quad i = B, R \\ \tilde{C}_1 X^{\beta_1^R} + \tilde{C}_2 X^{\beta_2^R} + \tilde{C}_3 X + \tilde{C}_4 & X_B \leq X < X_R, \quad i = R \\ sXy_i - K & X \geq X_i \quad i = B, R, \end{cases} \tag{A-35}$$

where $\tilde{A}_{B1}, \tilde{A}_{B2}, \tilde{A}_{R1}, \tilde{A}_{R2}, \tilde{C}_1, \tilde{C}_2, \tilde{C}_3, \tilde{C}_4, \gamma_3, \gamma_4, \beta_1^R$, and β_2^R are real-valued parameters to be determined. The notation $\tilde{}$ indicates that a parameter refers to the value of the growth option (and only to this). We first consider the region $X < X_B$, and use the standard approach of plugging in the functional form $G_i(X) = \tilde{A}_{i1}X^{\gamma_3} + \tilde{A}_{i2}X^{\gamma_4}$ into both equations of (A-27). Comparison of coefficients yields that $\tilde{A}_{Rk}$ is always a multiple of $\tilde{A}_{Bk}$, $k = 1, 2$, with the factor $\tilde{l}_k := \frac{1}{\lambda_B}(r + \lambda_B - \mu_B\gamma_{k+2} - \frac{1}{2}\sigma 2_B \gamma_{k+2}(\gamma_{k+2} - 1))$, i.e., $\tilde{A}_{Bk} = \tilde{l}_k \tilde{A}_{Rk}$. Note that even though the factor $\tilde{l}_k$ is of similar structure as the one found in the calculation of the value of debt of a firm with only invested assets, their values differ due to the different roots γ_i in the formulae. Using this relationship and comparing coefficients again, we find that γ_3 and γ_4 correspond to the positive roots of the quartic equation

$$\left(\mu_R\gamma + \frac{1}{2}\sigma_R 2\gamma(\gamma - 1) - \lambda_B - r\right)\left(\mu_B\gamma + \frac{1}{2}\sigma_B 2\gamma(\gamma - 1) - \lambda_R - r\right) = \lambda_R\lambda_B, \tag{A-36}$$

with the reason for taking the positive roots being that the option value has to approach zero as the asset value approaches zero.

Next, we consider the region $X_B \leq X < X_R$. Note that in the case of interest the Markov chain is in recession (otherwise, the solution is already known). Again, plugging in the functional form $G_R(X) = \tilde{C}_1 X^{\beta_1} + \tilde{C}_2 X^{\beta_2} + \tilde{C}_3 X + \tilde{C}_4$ into the second equation of (A-28), we find by comparison of coefficients that

$$\begin{aligned} \beta_{1,2}^R &= \frac{1}{2} - \frac{\mu_R}{\sigma_R 2} \pm \sqrt{\left(\frac{1}{2} - \frac{\mu_R}{\sigma_R 2}\right) 2 + \frac{2\,(r + \lambda_R)}{\sigma_R 2}} \\ \tilde{C}_3 &= \frac{s\lambda_R y_B}{r - \mu_R + \lambda_R} \\ \tilde{C}_4 &= -\frac{K\lambda_R}{r + \lambda_R}. \end{aligned} \tag{A-37}$$

It is left to solve for the unknown parameters $\tilde{A}_{B1}, \tilde{A}_{B2}, \tilde{C}_1$ and $\tilde{C}_2$. Plugging in the functional form (A-35) into conditions (A-31)-(A-34) yields

$$\tilde{C}_1 X_B^{\beta_1^R} + \tilde{C}_2 X_B^{\beta_2^R} + \tilde{C}_3 X_B + \tilde{C}_4 = \tilde{l}_1 \tilde{A}_{B1} X_B^{\gamma_3} + \tilde{l}_2 \tilde{A}_{B2} X_B^{\gamma_4} \tag{A-38}$$

$$\tilde{C}_1 \beta_1^R X_B^{\beta_1^R} + \tilde{C}_2 \beta_2^R X_B^{\beta_2^R} + \tilde{C}_3 X_B = \tilde{l}_1 \tilde{A}_{B1} \gamma_3 X_B^{\gamma_3} + \tilde{l}_2 \gamma_4 \tilde{A}_{B2} X_B^{\gamma_4} \tag{A-39}$$

$$\tilde{C}_1 X_R^{\beta_1^R} + \tilde{C}_2 X_R^{\beta_2^R} + \tilde{C}_3 X_R + \tilde{C}_4 = sy_R X_R - K \tag{A-40}$$

$$\tilde{A}_{B1} X_B^{\gamma_3} + \tilde{A}_{B2} X_B^{\gamma_4} = sy_B X_B - K \tag{A-41}$$

This is a four-dimensional system in four unknowns, which is linear in $\tilde{A}_{B1}, \tilde{A}_{B2}, \tilde{C}_1$ and $\tilde{C}_2$. Hence we define the matrices

$$\tilde{M} := \begin{bmatrix} \tilde{l}_1 X_B^{\gamma_3} & \tilde{l}_2 X_B^{\gamma_4} & -X_B^{\beta_1^R} & -X_B^{\beta_2^R} \\ \tilde{l}_1 \gamma_3 X_B^{\gamma_3} & \tilde{l}_2 \gamma_4 X_B^{\gamma_4} & -\beta_1^R X_B^{\beta_1^R} & -\beta_2^R X_B^{\beta_2^R} \\ 0 & 0 & X_R^{\beta_1^R} & X_R^{\beta_2^R} \\ X_B^{\gamma_3} & X_B^{\gamma_4} & 0 & 0 \end{bmatrix}$$

$$\tilde{b} := \begin{bmatrix} \tilde{C}_3 X_B + \tilde{C}_4 \\ \tilde{C}_3 X_B \\ -\tilde{C}_3 X_R - \tilde{C}_4 + s y_R X_R - K \\ s y_B X_B - K \end{bmatrix},$$

such that $\tilde{M} \begin{bmatrix} \tilde{A}_{B1} & \tilde{A}_{B2} & \tilde{C}_1 & \tilde{C}_2 \end{bmatrix}^T = \tilde{b}$. Hence the solution to the unknowns left is given by

$$\begin{bmatrix} \tilde{A}_{B1} & \tilde{A}_{B2} & \tilde{C}_1 & \tilde{C}_2 \end{bmatrix}^T = \tilde{M}^{-1} \tilde{b}. \tag{A-42}$$

Note that indeed none of the parameters of the solution depends on the value of X. Hence we confirm that the derivative of the option value, its Delta, is given by

$$G_i'(X) = \begin{cases} \gamma_3 \tilde{A}_{i1} X^{\gamma_3 - 1} + \tilde{A}_{i2} \gamma_4 X^{\gamma_4 - 1} & X < X_B, \quad i = B, R \\ \tilde{C}_1 \beta_1 X^{\beta_1 - 1} + \tilde{C}_2 \beta_2 X^{\beta_2 - 1} + \tilde{C}_3 & X_B \leq X < X_R, \quad i = R \\ s y_i & X \geq X_i \quad i = B, R, \end{cases} \tag{A-43}$$

with the parameters as given above. Consequently, the relative price change sensitivity is

$$\frac{G_i'(X)}{G_i(X)} = \begin{cases} \frac{\gamma_3 \tilde{A}_{i1} X^{\gamma_3 - 1} + \tilde{A}_{i2} \gamma_4 X^{\gamma_4 - 1}}{\tilde{A}_{i1} X^{\gamma_3} + \tilde{A}_{i2} X^{\gamma_4}} & X < X_B, \quad i = B, R \\ \frac{\tilde{C}_1 \beta_1 X^{\beta_1 - 1} + \tilde{C}_2 \beta_2 X^{\beta_2 - 1} + \tilde{C}_3}{\tilde{C}_1 X^{\beta_1} + \tilde{C}_2 X^{\beta_2} + \tilde{C}_3 X + \tilde{C}_4} & X_B \leq X < X_R, \quad i = R \\ \frac{s y_i}{s y_i X - K} & X \geq X_i \quad i = B, R. \end{cases} \tag{A-44}$$

Case G2: $X_R = X_B$. Define $X_1 := X_R$, and consider the case $y_R = y_B =: y$.[22] It follows that $G_R(X) = G_B(X) := G(X)\ \forall\ X \geq 0$, considering the dynamics of X. Consequently, $\sigma_R = \sigma_B =: \sigma, \lambda_R = \lambda_B =: \lambda$, which implies $\mu_R = \mu_B =: \mu$. In words, Case G2 treats the situation of being subject to only one regime. Using again the approach that the required return and expected realized return must by equal, we find that

[22] Technically, the case $X_R = X_B$, and $y_R \neq y_B$ can be solved without further difficulties. The calculations are straightforward, but not presented here, as they are not of interest for our applications.

the system to solve is given by:

$$\begin{aligned} rG(X) &= \mu X G'(X) + \tfrac{\sigma 2}{2} X 2 G''(X) & X < X_1 \\ G(X) &= sXy - K & X \geq X_1 \end{aligned} \tag{A-45}$$

The option must become worthless as the asset value approaches zero; furthermore, value-matching requires:

$$\lim_{X \searrow 0} G(X) = 0 \tag{A-46}$$

$$\lim_{X \nearrow X_1} G(X) = syX_1 - K \tag{A-47}$$

The functional form of the solution is

$$G(X) = \begin{cases} \tilde{A} X^{\beta_1} & X < X_1 \\ sXy - K & X \geq X_1, \end{cases} \tag{A-48}$$

where $\tilde{A}$ and β_1 are real-valued parameters to be determined. It is then straightforward to show that

$$\beta_1 = \frac{1}{2} - \frac{\mu}{\sigma 2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma 2}\right) 2 + \frac{2r}{\sigma 2}} \tag{A-49}$$

$$\tilde{A} = (syX_1 - K) X_1^{-\beta_1}, \tag{A-50}$$

which is the solution for the option for Case G2. The Delta of the option in this case is hence

$$G'(X) = \begin{cases} \tilde{A} \beta_1 X^{\beta_1 - 1} & X < X_1 \\ sy & X \geq X_1, \end{cases} \tag{A-51}$$

and its relative sensitivity

$$\frac{G'(X)}{G(X)} = \begin{cases} \frac{\beta_1}{X} & X < X_1 \\ \frac{sy}{syX - K} & X \geq X_1. \end{cases} \tag{A-52}$$

A.3. Firms with invested assets and expansion opportunities

A.3.1. The valuation of corporate debt

Case 1: $D_B < D_R$, $\hat{D}_B < \hat{D}_R$, and $X_R > X_B$. This case corresponds to the one presented in the main text. Recall that the system to solve is:

For $0 \leq X \leq D_B$:

$$\begin{cases} d_B(X) = \alpha_B (V_B(X) + G_B(X)) \\ d_R(X) = \alpha_R (V_R(X) + G_R(X)) \end{cases} \tag{A-53}$$

For $D_B < X \leq D_R$:

$$\begin{cases} rd_B(X) &= c + \mu_B X d'_B(X) + \frac{\sigma 2_B}{2} X 2 d''_B(X) + \lambda_B \left(\alpha_R \left(V_R(X) + G_R(X)\right) - d_B(X)\right) \\ d_R(X) &= \alpha_R \left(V_R(X) + G_R(X)\right) \end{cases} \tag{A-54}$$

For $D_R < X < X_B$:

$$\begin{cases} rd_B(X) &= c + \mu_B X d'_B(X) + \frac{\sigma 2_B}{2} X 2 d''_B(X) + \lambda_B \left(d_R(X) - d_B(X)\right) \\ rd_R(X) &= c + \mu_R X d'_R(X) + \frac{\sigma 2_R}{2} X 2 d''_R(X) + \lambda_R \left(d_B(X) - d_R(X)\right) \end{cases} \tag{A-55}$$

For $X_B \leq X < X_R$:

$$\begin{cases} d_B(X) &= \hat{d}_B \left(\bar{s}X - \frac{K}{y_B}\right) \\ rd_R(X) &= c + \mu_R X d'_R(X) + \frac{\sigma 2_R}{2} X 2 d''_R(X) + \lambda_R \left(\hat{d}_B \left(\bar{s}X - \frac{K}{y_B}\right) - d_R(X)\right) \end{cases} \tag{A-56}$$

For $X \geq X_R$:

$$\begin{cases} d_B(X) &= \hat{d}_B \left(\bar{s}X - \frac{K}{y_B}\right) \\ d_R(X) &= \hat{d}_R \left(\bar{s}X - \frac{K}{y_R}\right). \end{cases} \tag{A-57}$$

Recall that $\hat{d}_i(\cdot)$ denotes the value of debt for a firm with only invested assets. These functions are calculated in Appendix A.1, and hence taken as known at this point.
The system is subject to the boundary conditions:

$$\begin{aligned} \lim_{X \searrow D_R} d_B(X) &= \lim_{X \nearrow D_R} d_B(X) \\ \lim_{X \searrow D_R} d'_B(X) &= \lim_{X \nearrow D_R} d'_B(X) \\ \lim_{X \searrow D_B} d_B(X) &= \alpha_B \left(D_B y_B + G_B(D_B)\right) \\ \lim_{X \searrow D_R} d_R(X) &= \alpha_R \left(D_R y_R + G_R(D_R)\right) \\ \lim_{X \searrow X_B} d_R(X) &= \lim_{X \nearrow X_B} d_R(X) \\ \lim_{X \searrow X_B} d'_R(X) &= \lim_{X \nearrow X_B} d'_R(X) \\ \lim_{X \nearrow X_B} d_B(X) &= \hat{d}_B \left(\bar{s}X_B - \frac{K}{y_B}\right) \\ \lim_{X \nearrow X_R} d_R(X_R) &= \hat{d}_R \left(\bar{s}X_R - \frac{K}{y_R}\right). \end{aligned} \tag{A-58}$$

In order to solve this system, we start with the functional form of the solution:

$$d_i(X) = \begin{cases} \alpha_i\left(V_i(X) + G_i(X)\right) & X \le D_i & i = B, R, \\ C_1 X^{\beta_1^B} + C_2 X^{\beta_2^B} + C_3 X + C_4 & & \\ \quad + C_5 X^{\gamma_3} + C_6 X^{\gamma_4} & D_B < X \le D_R, & i = B \\ A_{i1} X^{\gamma_1} + A_{i2} X^{\gamma_2} & & \\ \quad + A_{i3} X^{\gamma_3} + A_{i4} X^{\gamma_4} + A_5 & D_R < X \le X_B, & i = B, R \\ B_1 X^{\beta_1^R} + B_2 X^{\beta_2^R} + Z(X) & X_B \le X \le X_R, & i = R \\ \hat{d}_i\left(\bar{s}X - \frac{K}{y_i}\right) & X > X_i, & i = B, R, \end{cases} \tag{A-59}$$

where $A_{B1}, A_{B2}, A_{R1}, A_{R2}, C_1, C_2, C_3, C_4, C_5, C_6, B_1, B_2, \beta_1^B, \beta_2^B, \beta_1^R, \beta_2^R, \gamma_3$, and γ_4 are real-valued parameters to be determined (or to be confirmed). The function $Z(X)$ as stated in the sixth line of the functional form is of closed form. It will be given explicitly in the following calculations.

We first consider the region $X_B \ge X > D_R$. Using the standard approach of plugging in the functional form $d_i(X) = A_{i1}X^{\gamma_1} + A_{i2}X^{\gamma_2} + A_{i3}X^{\gamma_3} + A_{i4}X^{\gamma_4} + A_5$ into both equations of (A-55) and comparing coefficients, we confirm that $A_5 = \frac{c}{r}$, and we find again that A_{Rk} is always a multiple of A_{Bk}, $k = 1, 2, 3, 4$, with the factor $l_k := \frac{1}{\lambda_B}(r + \lambda_B - \mu_B\gamma_k - \frac{1}{2}\sigma 2_B \gamma_k(\gamma_k - 1))$ as before, i.e., $A_{Bk} = l_k A_{Rk}$. Using this relationship and comparing coefficients again, we find that $\gamma_1, \gamma_2, \gamma_3$, and γ_4 correspond to the roots of the quartic equation (A-5), which is:

$$\left(\mu_R\gamma + \frac{1}{2}\sigma_R 2\gamma(\gamma - 1) - \lambda_B - r\right)\left(\mu_B\gamma + \frac{1}{2}\sigma_B 2\gamma(\gamma - 1) - \lambda_R - r\right) = \lambda_R\lambda_B. \tag{A-60}$$

Recall that by Guo (2001), we know that this quartic equation always has four distinct real roots, two of them being negative, and two positive. Technically, the reason for taking all four exponents γ_k now is that the value of debt in both regimes will be subject to boundary conditions from both below (default) and above (exercise of expansion option). Therefore, in order to meet all boundary conditions, we need four terms with the according factors A_{ik}. The no-bubbles condition is already implemented in the value function of a firm with only invested assets $\hat{d}_i$, and hence does not need to be imposed one more time. The unknown parameters left for this region are now $A_{Bk},\ k = 1, \ldots, 4$.

Next, we consider the region $D_B \le X \le D_R$ i.e. the realized state of the Markov chain is boom (if not the solution is already known by the second equation of system (15)). Plugging in the functional form $d_B(X) = C_1X^{\beta_1^B} + C_2X^{\beta_2^B} + C_3X + C_4 + C_5X^{\gamma_3} + C_6X^{\gamma_4}$ into the second equation of (15), we find by comparison of coefficients that

$$\beta_{1,2}^{B} = \frac{1}{2} - \frac{\mu_B}{\sigma_B 2} \pm \sqrt{\left(\frac{1}{2} - \frac{\mu_B}{\sigma_B 2}\right) 2 + \frac{2\,(r + \lambda_B)}{\sigma_B 2}} \tag{A-61}$$

$$C_3 = \frac{\lambda_B \alpha_R y_R}{r + \lambda_B - \mu_B} \tag{A-62}$$

$$C_4 = \frac{c}{r + \lambda_B} \tag{A-63}$$

$$C_5 = \frac{\lambda_B \alpha_R \tilde{l}_1 \tilde{A}_{B1}}{r - \mu_B \gamma_3 - \frac{1}{2}\sigma_B 2 \gamma_3\,(\gamma_3 - 1) + \lambda_B} \tag{A-64}$$

$$C_6 = \frac{\lambda_B \alpha_R \tilde{l}_2 \tilde{A}_{B2}}{r - \mu_B \gamma_4 - \frac{1}{2}\sigma_B 2 \gamma_4\,(\gamma_4 - 1) + \lambda_B}. \tag{A-65}$$

We used again that $r + \lambda_B - \mu_B > 0$ by (5), and that by equation (A-60), the denominators of C_5 and C_6 are different from zero as long as the Markov chain I is recurrent, i.e., if $\lambda_i > 0$, $i = B, R$. The parameters $\beta_{1,2}^{B}, C_3$, and C_4 are as before. C_5 and C_6 are influenced by the parameters in the solution of the growth option, $\tilde{l}_1, \tilde{l}_2, \tilde{A}_{B1}$, and $\tilde{A}_{B2}$, see Appendix A.2. The two additional terms of the solution for this region, $C_5 X^{\gamma_3}$, and $C_6 X^{\gamma_4}$, reflect the fact that the firm does not only consist of assets in place, but also of the growth option. As debtholders get also a fraction of the growth option value at regime-switching induced default, the growth option value directly influences the solution in this region. This impact of the option value explains the occurrence of the growth option parameters in C_5 and C_6, as well as the use of the same exponents as in the calculation of the value of the option, γ_3 and γ_4. As the latter were already calculated in the value of the growth option, we take them as given at this point, and must not determine them. Note that the approach and the intuition regarding the exponents γ_3 and γ_4 for this region is completely different than for the previously discussed region $X_B \geq X > D_R$, where these exponents occur only due to the valuation of debt itself, independent of the growth option, and must be calculated as a part of the solution. The unknown parameters left for this region are hence C_1 and C_2.

Finally, consider the region $X_B \leq X \leq X_R$, i.e., we are interested in the case that $i = R$. We solve now the according differential equation in (A-56):

$$r d_R(X) = c + \mu_R X d_R'(X) + \frac{\sigma 2_R}{2} X 2 d_R''(X) + \lambda_R \hat{d}_B(\bar{s}X - \frac{K}{y_B}). \tag{A-66}$$

In order to solve this inhomogeneous differential equation, we will use a standard approach and first find a fundamental system of solutions of the homogenous differential equation, and then calculate the solution to the inhomogeneous equation as the sum of the solutions of the homogenous equation and a particular solution of the nonhomogeneous equation. A reference for this approach is Polyanin and Zaitsev (2003), pages 21-23. The reason that this system is not straightforward to solve is that the function $\hat{d}$ is not evaluated at X, but at $\bar{s}X - \frac{K}{y_B}$.[23]

(A-66) is equivalent to

$$X2d_R''(X) + \frac{2\mu_R}{\sigma_R 2} X d_R'(X) - \frac{2\,(r+\lambda_R)}{\sigma_R 2} d_R(X) = -\frac{2c}{\sigma_R 2} - \frac{2\lambda_R}{\sigma_R 2}\hat{d}_B(\bar{s}X - \frac{K}{y_B}). \tag{A-67}$$

Therefore the according homogenous differential equation is

$$X2d_R''(X) + \frac{2\mu_R}{\sigma_R 2} X d_R'(X) - \frac{2\,(r+\lambda_R)}{\sigma_R 2} d_R(X) = 0. \tag{A-68}$$

A fundamental system of solutions is given by $\{z_1, z_2\}$, with

$$\begin{aligned} z_1 &:= X^{\beta_1^R}, \\ z_2 &:= X^{\beta_2^R}, \end{aligned}$$

and

$$\beta_{1,2}^R = \frac{1}{2} - \frac{\mu_R}{\sigma_R 2} \pm \sqrt{\left(\frac{1}{2} - \frac{\mu_R}{\sigma_R 2}\right)2 + \frac{2\,(r+\lambda_R)}{\sigma_R 2}}. \tag{A-69}$$

These solutions can be calculated by using the functional form, plugging it into the homogenous ODE (A-68), and solving for $\beta_{1,2}^R$.

For notational convenience, we now define $f_2 := X2$, $f_1 := \frac{2\mu_R}{\sigma_R 2}X$, $f_0 := -\frac{2(r+\lambda_R)}{\sigma_R 2}$, and

$$g\,(X) := -\frac{2c}{\sigma_R 2} - \frac{2\lambda_R}{\sigma_R 2}\hat{d}_B(\bar{s}X - \frac{K}{y_B}). \tag{A-70}$$

These notations allow to write the ODE (A-67) as:

$$f_2 d_R''(X) + f_1 d_R'(X) + f_0 d_R(X) = g(X). \tag{A-71}$$

The general solution of this inhomogeneous ODE is given by

$$d_R\,(X) = B_1 z_1 + B_2 z_2 + z_2 \underbrace{\int z_1 \frac{g}{f_2}\frac{\mathrm{d}X}{W}}_{=:I_1(X)} - z_1 \underbrace{\int z_2 \frac{g}{f_2}\frac{\mathrm{d}X}{W}}_{=:I_2(X)}, \tag{A-72}$$

where $W = z_1 z_2' - z_2 z_1'$ is the Wronskian determinant, and B_1 and B_2 are coefficients (see e.g. Polyanin and Zaitsev (2003), page 22, (7)). The first two terms are a linear combination of the solutions of the homogenous ODE, and the last two terms are a particular solution of the inhomogeneous ODE.

[23] This is due to the assumption that the exercise of the option is financed by selling a part of the assets in place. Under the alternative assumption that the exercise of the option is equity-financed, the function $\hat{d}$ is evaluated at a multiple of X instead. Then we can exploit the additive nature of the ODE, and calculate the solution as a weighted sum of solutions, including the value function of debt of a firm with only invested assets. This would result in a functional form for this region comparable to the one for the region $D_B \leq X \leq D_R$.

We start by calculating the Wronskian determinant

$$\begin{aligned} W &= z_1 z_2' - z_2 z_1' && \text{(A-73)} \\ &= \beta_2^R X^{\beta_1^R} X^{\beta_2^R - 1} - \beta_1^R X^{\beta_1^R - 1} X^{\beta_2^R} && \text{(A-74)} \\ &= \left(\beta_2^R - \beta_1^R\right) X^{\beta_1^R + \beta_2^R - 1}. && \text{(A-75)} \end{aligned}$$

The integral $I_1(X)$ is, hence:

$$\begin{aligned} I_1(X) &= \int z_1 \frac{g}{f_2} \frac{\mathrm{d}X}{W} \\ &= \int X^{\beta_1^R} X^{-2} \frac{1}{\beta_2^R - \beta_1^R} X^{1-\beta_1^R - \beta_2^R} g(X) \mathrm{d}X \\ &= \frac{1}{\beta_2^R - \beta_1^R} \int x^{-1-\beta_2^R} g(X) \mathrm{d}X && \text{(A-76)} \\ &= \frac{1}{\beta_2^R - \beta_1^R} \int x^{-1-\beta_2^R} \left(-\frac{2c}{\sigma_R 2} - \frac{2\lambda_R}{\sigma_R 2} \hat{d}_B \left(\bar{s}X - \frac{K}{y_B} \right) \right) \mathrm{d}X \\ &= \frac{1}{\beta_2^R - \beta_1^R} \int x^{-1-\beta_2^R} \left(-\frac{2c}{\sigma_R 2} \right. \\ &\quad \left. -\frac{2\lambda_R}{\sigma_R 2} \left\{ \hat{A}_{B1} \left(\bar{s}X - \frac{K}{y_B} \right)^{\gamma_1} + \hat{A}_{B2} \left(\bar{s}X - \frac{K}{y_B} \right)^{\gamma_2} + \frac{c}{r} \right\} \right) \mathrm{d}X \\ &= -\frac{2\lambda_R \hat{A}_{B1}}{\left(\beta_2^R - \beta_1^R\right) \sigma_R 2} \underbrace{\int X^{-1-\beta_2^R} \left(\bar{s}X - \frac{K}{y_B} \right)^{\gamma_1} \mathrm{d}X}_{=:I_{11}(X)} \\ &\quad -\frac{2\lambda_R \hat{A}_{B2}}{\left(\beta_2^R - \beta_1^R\right) \sigma_R 2} \underbrace{\int X^{-1-\beta_2^R} \left(\bar{s}X - \frac{K}{y_B} \right)^{\gamma_2} \mathrm{d}X}_{=:I_{12}(X)} && \text{(A-77)} \\ &\quad + \frac{2c\left(\lambda_R + r\right)}{\left(\beta_2^R - \beta_1^R\right) r \beta_2^R \sigma_R 2} X^{-\beta_2^R}. \end{aligned}$$

We used the definition of the function $g(X)$, see (A-70), and the solution of the debt value of a firm with only invested assets $\hat{d}_R(\cdot)$, see Appendix A.1, (A-4).

In order to calculate for the integrals $I_{11}(X)$ and $I_{12}(X)$,[24] we will use the following expansion based on the binomial theorem

$$(x+y)^r = \sum_{l=0}^{\infty} \binom{r}{l} x^{r-l} y^l, \qquad \text{(A-78)}$$

with

$$\binom{r}{l} = \frac{(-r)_l}{l!}.$$

[24] We are grateful to Simon Broda, who completed this calculation by providing the closed-form solution of these integrals, which we solved numerically in previous versions of the paper.

Here, the notation $(\cdot)_l$ denotes the Pochhammer symbol, representing rising factorials:

$$(a)_l := a\,(a+1)\cdot\ldots\cdot(a+k-1)\,. \tag{A-79}$$

Using these definitions, the binomial theorem, and Fubini's theorem, the integral $I_{11}(X)$ can be calculated as follows:

$$\begin{aligned}
I_{11}(X) &= \int X^{-1-\beta_2^R}\left(\bar{s}X-\frac{K}{y_B}\right)^{\gamma_1}\mathrm{d}X\\
&= \bar{s}^{\gamma_1}\int X^{-1-\beta_2^R}\left(X-\frac{K}{\bar{s}y_B}\right)^{\gamma_1}\mathrm{d}X\\
&= \bar{s}^{\gamma_1}\int X^{-1-\beta_2^R}\left(\sum_{l=0}^{\infty}\frac{(-\gamma_1)_l}{l!}X^{\gamma_1-l}\left(-\frac{K}{\bar{s}y_B}\right)^l\right)\mathrm{d}X\\
&= \bar{s}^{\gamma_1}\sum_{l=0}^{\infty}\frac{(-\gamma_1)_l}{l!}\left(-\frac{K}{\bar{s}y_B}\right)^l\int X^{-1-\beta_2^R+\gamma_1-l}\mathrm{d}X\\
&= \bar{s}^{\gamma_1}\sum_{l=0}^{\infty}\frac{(-\gamma_1)_l}{l!}\left(-\frac{K}{\bar{s}y_B}\right)^l\frac{1}{\gamma_1-\beta_2^R-l}X^{\gamma_1-\beta_2^R-l}\mathrm{d}X\\
&= \bar{s}^{\gamma_1}X^{\gamma_1-\beta_2^R}\sum_{l=0}^{\infty}\frac{(-\gamma_1)_l}{l!}\left(-\frac{K}{\bar{s}Xy_B}\right)^l\frac{1}{\gamma_1-\beta_2^R-l}\mathrm{d}X\\
&= \bar{s}^{\gamma_1}X^{\gamma_1-\beta_2^R}\sum_{l=0}^{\infty}\frac{(-\gamma_1)_l}{l!}\left(-\frac{K}{\bar{s}Xy_B}\right)^l\frac{\left(\beta_2^R-\gamma_1\right)_l}{\left(1+\beta_2^R-\gamma_1\right)_l}\frac{1}{\gamma_1-\beta_2^R}\mathrm{d}X\\
&= \frac{1}{\gamma_1-\beta_2^R}\bar{s}^{\gamma_1}X^{\gamma_1-\beta_2^R}\,{}_2F_1\left(-\gamma_1,\beta_2^R-\gamma_1,\beta_2^R-\gamma_1+1;-\frac{K}{\bar{s}Xy_B}.\right)
\end{aligned} \tag{A-80}$$

Here, ${}_2F_1(\cdot,\cdot,\cdot;\cdot)$ is Gauss' hypergeometric function:

$${}_2F_1(a,b,c;z) := \sum_{l=0}^{\infty}\frac{(a)_l\,(b)_l\,z^l}{(c)_l\,l!}, \quad (|z|<1)\vee(|z|=1\wedge Re\,(c-a-b)>0)\,. \tag{A-81}$$

If $X_B > \frac{K}{\bar{s}y_B}$ (i.e., the option is never exercised if its payoff is negative), the fourth argument in Gauss' hypergeometric function fulfills the condition $\left|-\frac{K}{\bar{s}Xy_B}\right| < 1$, and hence the function is well-defined.

Analogously, we obtain

$$I_{12}(X) = \frac{1}{\gamma_2-\beta_2^R}\bar{s}^{\gamma_2}X^{\gamma_2-\beta_2^R}\,{}_2F_1\left(-\gamma_2,\beta_2^R-\gamma_2,\beta_2^R-\gamma_2+1;-\frac{K}{\bar{s}Xy_B}.\right) \tag{A-82}$$

Plugging the solutions (A-80) and (A-82) back into the expression for the integral I_1, (A-77) yields

$$\begin{aligned} I_1(X) &= -\frac{2\lambda_R \hat{A}_{B1}}{\left(\beta_2^R-\beta_1^R\right)\sigma_R 2}\frac{1}{\gamma_1-\beta_2^R}\bar{s}^{\gamma_1}X^{\gamma_1-\beta_2^R}{}_2F_1\left(-\gamma_1,\beta_2^R-\gamma_1,\beta_2^R-\gamma_1+1;-\frac{K}{\bar{s}Xy_B}\right) \\ &\quad -\frac{2\lambda_R \hat{A}_{B2}}{\left(\beta_2^R-\beta_1^R\right)\sigma_R 2}\frac{1}{\gamma_2-\beta_2^R}\bar{s}^{\gamma_2}X^{\gamma_2-\beta_2^R}{}_2F_1\left(-\gamma_2,\beta_2^R-\gamma_2,\beta_2^R-\gamma_2+1;-\frac{K}{\bar{s}Xy_B}\right) \\ &\quad +\frac{2c\left(\lambda_R+r\right)}{\left(\beta_2^R-\beta_1^R\right)r\beta_2^R\sigma_R 2}X^{-\beta_2^R}. \end{aligned} \tag{A-83}$$

Similarly, we find for the second integral $I_2(X)$:

$$\begin{aligned} I_2(X) &= -\frac{2\lambda_R \hat{A}_{B1}}{\left(\beta_2^R-\beta_1^R\right)\sigma_R 2}\frac{1}{\gamma_1-\beta_1^R}\bar{s}^{\gamma_1}X^{\gamma_1-\beta_1^R}{}_2F_1\left(-\gamma_1,\beta_1^R-\gamma_1,\beta_2^R-\gamma_1+1;-\frac{K}{\bar{s}Xy_B}\right) \\ &\quad -\frac{2\lambda_R \hat{A}_{B2}}{\left(\beta_2^R-\beta_1^R\right)\sigma_R 2}\frac{1}{\gamma_2-\beta_1^R}\bar{s}^{\gamma_2}X^{\gamma_2-\beta_1^R}{}_2F_1\left(-\gamma_2,\beta_1^R-\gamma_2,\beta_2^R-\gamma_2+1;-\frac{K}{\bar{s}Xy_B}\right) \\ &\quad +\frac{2c\left(\lambda_R+r\right)}{\left(\beta_2^R-\beta_1^R\right)r\beta_1^R\sigma_R 2}X^{-\beta_1^R}. \end{aligned} \tag{A-84}$$

Plugging (A-83) and (A-84) into (A-72), we finally obtain the solution

$$d_R(X) = B_1X^{\beta_1^R}+B_2X^{\beta_2^R}+Z(X), \tag{A-85}$$

with

$$\begin{aligned} Z(X) &= \frac{2}{\beta_1\beta_2\sigma_R 2}\frac{c}{r}\left(\lambda_R+r\right) \\ &\quad +\sum_{i,k=1,2}\frac{2(-1)^{i+1}\bar{s}^{\gamma_k}\hat{A}_{Bk}}{\sigma_R 2\left(\beta_2^R-\beta_1^R\right)\left(\gamma_k-\beta_i^R\right)}X^{\gamma_k}{}_2F_1\left(-\gamma_k,\beta_i^R,\beta_i^R-\gamma_k+1;-\frac{K}{\bar{s}Xy_B}\right), \end{aligned} \tag{A-86}$$

for some parameters B_1 and B_2 determined by the boundary conditions.

In order to treat the boundary conditions, we also need the first derivative of Z:

$$\begin{aligned} Z'(X) &= \frac{\mathrm{d}}{\mathrm{d}X}Z(X) \\ &= \frac{\mathrm{d}}{\mathrm{d}X}\left(X^{\beta_2^R}I_1(X)-X^{\beta_2^R}I_2(X)\right) \\ &= \beta_2^RX^{\beta_2^R}I_1(X)+\frac{1}{\beta_2^R-\beta_1^R}X^{\beta_2^R}X^{-1-\beta_1^R}g(X) \\ &\quad -\beta_1^RX^{\beta_1^R}I_2(X)-\frac{1}{\beta_2^R-\beta_1^R}X^{\beta_1^R}X^{-1-\beta_1^R}g(X) \\ &= \beta_2^RX^{\beta_2^R}I_1(X)-\beta_1^RX^{\beta_1^R}I_2(X) \\ &= \sum_{i,k=1,2}\frac{2(-1)^{i+1}\bar{s}^{\gamma_k}\hat{A}_{Bk}\beta_i^R}{\sigma_R 2\left(\beta_2^R-\beta_1^R\right)\left(\gamma_k-\beta_i^R\right)}X^{\gamma_k}{}_2F_1\left(-\gamma_k,\beta_i^R,\beta_i^R-\gamma_k+1;-\frac{K}{\bar{s}Xy_B}\right). \end{aligned} \tag{A-87}$$

In total, we are now left with the unknown parameters $A_{B1}, A_{B2}, A_{B3}, A_{B4}, C_1, C_2, B_1$ and B_2.

Finally, we plug the functional form (A-59) into the system of boundary conditions (A-58):

$$
\begin{aligned}
\sum_{k=1}^{4} A_{Bk} D_R^{\gamma_k} + A_5 &= C_1 D_R^{\beta_1^B} + C_2 D_R^{\beta_2^B} + C_3 X + C_4 + C_5 X^{\gamma_3} + C_6 X^{\gamma_4} \\
\sum_{k=1}^{4} A_{Bk} \gamma_k D_R^{\gamma_k} &= C_1 \beta_1^B D_R^{\beta_1^B} + C_2 \beta_2^B D_R^{\beta_2^B} + C_3 X + C_5 \gamma_3 X^{\gamma_3} + C_6 \gamma_4 X^{\gamma_4} \\
\alpha_B \left(D_B y_B + G_B(D_B)\right) &= C_1 D_B^{\beta_1^B} + C_2 D_B^{\beta_2^B} + C_3 D_B + C_4 + C_5 D_B^{\gamma_3} + C_6 D_B^{\gamma_4} \\
\sum_{k=1}^{4} l_k A_{Bk} D_R^{\gamma_k} + A_5 &= \alpha_R \left(D_R y_R + G_R(D_R)\right) \\
\sum_{k=1}^{4} l_k A_{Bk} X_B^{\gamma_k} + A_5 &= B_1 X_B^{\beta_1^R} + B_2 X_B^{\beta_2^R} + Z(X_B) \qquad \text{(A-88)} \\
\sum_{k=1}^{4} l_k A_{Bk} \gamma_k X_B^{\gamma_k} &= B_1 \beta_1^R X_B^{\beta_1^R} + B_2 \beta_2^R X_B^{\beta_2^R} + X_B Z'(X_B) \\
\sum_{k=1}^{4} A_{Bk} X_B^{\gamma_k} + A_5 &= \hat{d}_B \left(\bar{s} X_B - \frac{K}{y_B}\right) \\
B_1 X_R^{\beta_1^R} + B_2 X_R^{\beta_2^R} + Z(X_R) &= \hat{d}_R \left(\bar{s} X_R - \frac{K}{y_R}\right).
\end{aligned}
$$

Using matrix notation, we can write

$$
M := \begin{bmatrix}
D_R^{\gamma_1} & D_R^{\gamma_2} & D_R^{\gamma_3} & D_R^{\gamma_4} & -D_R^{\beta_1^B} & -D_R^{\beta_2^B} & 0 & 0 \\
\gamma_1 D_R^{\gamma_1} & \gamma_2 D_R^{\gamma_2} & \gamma_3 D_R^{\gamma_3} & \gamma_4 D_R^{\gamma_4} & -\beta_1^B D_R^{\beta_1^B} & -\beta_2^B D_R^{\beta_2^B} & 0 & 0 \\
0 & 0 & 0 & 0 & D_B^{\beta_1^B} & D_B^{\beta_2^B} & 0 & 0 \\
l_1 D_R^{\gamma_1} & l_2 D_R^{\gamma_2} & l_3 D_R^{\gamma_3} & l_4 D_R^{\gamma_4} & 0 & 0 & 0 & 0 \\
l_1 X_B^{\gamma_1} & l_2 X_B^{\gamma_2} & l_3 X_B^{\gamma_3} & l_4 X_B^{\gamma_4} & 0 & 0 & -X_B^{\beta_1^R} & -X_B^{\beta_2^R} \\
l_1 \gamma_1 X_B^{\gamma_1} & l_2 \gamma_2 X_B^{\gamma_2} & l_3 \gamma_3 X_B^{\gamma_3} & l_4 \gamma_4 X_B^{\gamma_4} & 0 & 0 & -\beta_1^R X_B^{\beta_1^R} & -\beta_2^R X_B^{\beta_2^R} \\
X_B^{\gamma_1} & X_B^{\gamma_2} & X_B^{\gamma_3} & X_B^{\gamma_4} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & X_R^{\beta_1^R} & X_R^{\beta_2^R}
\end{bmatrix}
$$

$$
b := \begin{bmatrix}
-A_5 + C_3 D_R + C_4 + C_5 D_R^{\gamma_1} + C_6 D_R^{\gamma_2} \\
C_3 D_R + \gamma_1 C_5 D_R^{\gamma_1} + C_6 \gamma_2 D_R^{\gamma_2} \\
-C_3 D_B - C_4 - C_5 D_B^{\gamma_3} - C_6 D_B^{\gamma_4} + \alpha_B \left(D_B y_B + G_B(D_B)\right) \\
-A_5 + \alpha_R \left(D_R y_R + G_R(D_R)\right) \\
-A_5 + Z\left(X_B\right) \\
X_B Z'\left(X_B\right) \\
-A_5 + \hat{d}_B \left(\bar{s} X_B - \frac{K}{y_B}\right) \\
-Z\left(X_R\right) + \hat{d}_R \left(\bar{s} X_R - \frac{K}{y_R}\right)
\end{bmatrix}.
$$

Thus the solution to the unknowns left is given by

$$\begin{bmatrix} A_{B1} & A_{B2} & A_{B3} & A_{B4} & C_1 & C_2 & B_1 & B_2 \end{bmatrix}^T = M^{-1}b. \tag{A-89}$$

Due to the fact that none of the parameters depend on X, we confirm the derivative to be

$$d_i'(X) = \begin{cases} \alpha_i \left(y_i + G_i'(X)\right) & X \leq D_i \qquad i = B, R, \\ C_1\beta_1^B X^{\beta_1^B - 1} + C_2\beta_2^B X^{\beta_2^B - 1} + C_3 & \\ \quad + C_5\gamma_3 X^{\gamma_3 - 1} + C_6\gamma_4 X^{\gamma_4 - 1} & D_B < X \leq D_R, \quad i = B \\ A_{i1}\gamma_1 X^{\gamma_1 - 1} + A_{i2}\gamma_2 X^{\gamma_2 - 1} & \\ \quad + A_{i3}\gamma_3 X^{\gamma_3 - 1} + A_{i4}\gamma_4 X^{\gamma_4 - 1} & D_R < X \leq X_B, \quad i = B, R \\ B_1\beta_1^R X^{\beta_1^R - 1} + B_2\beta_2^R X^{\beta_2^R - 1} + Z'(X) & X_B \leq X \leq X_R, \quad i = R \\ \hat{d}_i'\left(\bar{s}X - \frac{K}{y_i}\right) & X > X_i, \qquad i = B, R. \end{cases} \tag{A-90}$$

Case 2: $D_B = D_R$, $\hat{D}_B = \hat{D}_R$, and $X_R = X_B$. For brevity of notation, define $D_1 = D_B$, and recall that $\hat{D}_1 := \hat{D}_B$ and $X_1 := X_B$. Again, we consider the case $y_R = y_B =: y$. Then, we know that $\hat{d}_R(X) = \hat{d}_B(X) =: \hat{d}(X)\ \forall X \geq 0$ for the value of debt of a firm with only assets in place (see Appendix A.1), as well as $G_R(X) = G_B(X) := G(X)\ \forall X \geq 0$ for the value of the expansion option, since $G_R(X) = G_B(X)\ \forall\ X \geq 0$. Hence, $\sigma_R = \sigma_B =: \sigma, \lambda_R = \lambda_B =: \lambda$, and thus $\mu_R = \mu_B =: \mu$. Consequently, it must be that $d_R(X) = d_B(X) =: d(X)\ \forall X \geq 0$. In words, Case 2 treats the situation that the system is not exposed to regime switches, i.e., that there is only one regime.

Postulating that in the continuation region the required return must be equal to the expected realized return plus the proceeds from debt, we find that the system to solve can be written as:

$$\begin{aligned} d(X) &= \alpha\left(yX + G(X)\right) & X \leq D_1 \\ rd(X) &= c + \mu X d'(X) + \tfrac{\sigma 2}{2} X 2 d''(X) & D_1 < X < X_1 \\ d(X) &= \hat{d}\left(\bar{s}X - \tfrac{K}{y}\right) & X \geq X_1 \end{aligned} \tag{A-91}$$

$\hat{d}(\cdot)$ denotes the value of debt of a firm with only invested assets in the case of only one regime, see Appendix A.1 Case V2. $G(X)$ is the value of the expansion option in the case of only one regime, see Appendix A.2, Case G2. The first and second equations are analogous to the two regime case. In the third equation, we postulate that above the exercise boundary X the value of debt of the firm must be equal to the value of debt of a firm with only invested assets. As in the two regime case, the conversion of the growth option into assets in place is arranged such that the total value of the firm's assets remains unchanged at the exercise of the option. The boundary conditions are the value-matching conditions at default and optimal exercise:

$$\lim_{X \searrow D_1} d(X) = \alpha y D_1 \tag{A-92}$$

$$\lim_{X \nearrow X_1} d(X) = \hat{d}\left(\bar{s}X_1 - \frac{K}{y}\right). \tag{A-93}$$

Note that for $X > X_1$ the value of debt is equal to the one of a firm with only invested assets. As the latter is calculated using a no-bubbles condition, we do not have to postulate this condition for the function $d(X)$ again.

The functional form of the solution is

$$d(X) = \begin{cases} \alpha\left(yX + G(X)\right) & X \leq D_1 \\ \bar{B}_1 X^{\beta_1} + \bar{B}_2 X^{\beta_2} + A_5 & D_1 < X < X \\ \hat{d}\left(\bar{s}X - \frac{K}{y}\right) & X \geq X, \end{cases} \tag{A-94}$$

where $\bar{B}_1, \bar{B}_2, A_5, \beta_1$, and β_2 are real-valued parameters to be determined (or to be confirmed). The only region left to solve for is $D_1 < X < X_1$. Plugging in the functional form (A-94) into the differential equation (A-91) and comparing coefficients, we find that

$$\beta_{1,2} = \frac{1}{2} - \frac{\mu}{\sigma 2} - \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma 2}\right) 2 + \frac{2r}{\sigma 2}} \tag{A-95}$$

$$A_5 = \frac{c}{r}. \tag{A-96}$$

Finally, $\bar{B}_1$ and $\bar{B}_2$ are determined by the two-dimensional linear system defined by the above boundary conditions:

$$\bar{B}_1 D_1^{\beta_1} + \bar{B}_2 D_1^{\beta_2} + \frac{c}{r} = \alpha y D_1 \tag{A-97}$$

$$\bar{B}_1 X_1^{\beta_1} + \bar{B}_2 X_1^{\beta_2} + \frac{c}{r} = \hat{d}\left(\bar{s}X_1 - \frac{K}{y}\right). \tag{A-98}$$

Using matrix notation, we have with

$$\bar{M} := \begin{bmatrix} D_1^{\beta_1} & D_1^{\beta_2} \\ X_1^{\beta_1} & X_1^{\beta_2} \end{bmatrix},$$

$$\bar{b} := \begin{bmatrix} \alpha y D_1 - \frac{c}{r} \\ \hat{d}\left(\bar{s}X_1 - \frac{K}{y}\right) - \frac{c}{r} \end{bmatrix},$$

that

$$\begin{bmatrix}\bar{B}_1 & \bar{B}_2\end{bmatrix}^T = \bar{M}^{-1}\bar{b} \tag{A-99}$$

$$= \frac{1}{D_1^{\beta_1}X_1^{\beta_2} - D_1^{\beta_2}X_1^{\beta_1}}\begin{bmatrix} X_1^{\beta_2} & -D_1^{\beta_2} \\ -X_1^{\beta_1} & D_1^{\beta_1}\end{bmatrix}\begin{bmatrix}\alpha y D_1 - \frac{c}{r} \\ \hat{d}\left(\bar{s}X_1 - \frac{K}{y}\right) - \frac{c}{r}\end{bmatrix}, \tag{A-100}$$

which completes the calculation of the solution.

The derivative is given by

$$d'(X) = \begin{cases} \alpha\left(y + G'(X)\right) & X \leq D \\ \bar{B}_1\beta_1 X^{\beta_1 - 1} + \bar{B}_2\beta_2 X^{\beta_2 - 1} & D \leq X \leq X_1 \\ \hat{d}'\left(\bar{s}X - \frac{K}{y}\right) & X \geq X_1. \end{cases} \tag{A-101}$$

A.3.2. The valuation of tax benefits

For Case 1, see the main text. Analogously, the value of tax benefits $t(X)$ in Case 2 can be calculated as the value of debt with a recovery rate of zero and a coupon $c\tau$.

A.3.3. The valuation of default costs

Case 1 can be found in the main text. Analogously, the value of default costs $b(X)$ in Case 2 corresponds to the value of debt with a recovery rate of $1 - \alpha$ and a coupon of zero.

A.3.4. Firm value

The main text states the firm value in Case 1. For Case 2, analogously, the firm value $f(X)$ is

$$f(X) = Xy + G(X) + t(X) - b(X). \tag{A-102}$$

A.3.5. The valuation of equity

Case 1 is given in the main text. For Case 2, the value of equity $e(X)$ is given by

$$e(X) = f(X) - d(X) = Xy + G(X) + t(X) - b(X) - d(X). \tag{A-103}$$

A.3.6. Default policy

With similar arguments as in Case 1 (main text), the optimal default and investment policies in Case 2, D^* and X^*, are determined by the conditions

$$\begin{cases} e'\left(D^*\right) & = & 0 \\ e'\left(X^*\right) & = & \hat{e}'\left(sX^* - \frac{K}{y}\right) \end{cases} \tag{A-104}$$

A.3.7. Capital structure

Analogously to Case 1 in the main text, denote for Case 2 by $f^*(X)$ the firm value given ex-post optimal default and expansion thresholds as determined by the system (A-104). The optimal coupon of this firm then solves

$$c^* := \text{argmax}_c f^*(X). \tag{A-105}$$

A.4. Numerical Feasibility

We comment briefly on the numerical feasibility of our solutions, as not all of them are straightforward. We discuss only the case that the exercise and default boundaries are different (Cases G1, V1 and 1). The case that the boundaries coincide does not constitute any numerical difficulties.

First, note that for the value of the growth option, as well as for all corporate securities, the calculation includes solving a quartic equation (see equations (A-36), (A-5), and (A-60)). Using Ferrari's method, all solutions are known in closed-form, and we, therefore, do not need to apply an algorithm to find the roots.

The main step in the calculations of the value of corporate securities of a firm with only invested assets and of the growth option, as given in Appendix A.1 and A.2, is to invert a four-dimensional matrix.

The solution for the value of corporate securities of a firm with invested assets and expansion opportunities is more involved (Appendix A.3). The solution is found by inverting an eight-dimensional matrix. The calculations of the entries of the matrix include twelve evaluations of Gauss' hypergeometric function ${}_2F_1$. This can be seen from system (A-88), as it involves two evaluations of the function $Z\left(\cdot\right)$ and one evaluation of the function $Z'\left(\cdot\right)$, which in turn each needs four evaluations of Gauss' hypergeometric function, see definitions (A-86) and (A-87). Efficient algorithms exist for evaluating the ${}_2F_1$ function; see, e.g., Zhang and Jin (1996). The calculation of the optimal default and exercise boundaries corresponds to solving a four-dimensional non-linear equation.

Finally, solving for the optimal coupon can be done by using a simple one-dimensional optimization. Here, due to the structure of the solution, it is important not to rely on a gradient-based algorithm, but to use a simplex-based algorithm instead.

A.5. Robustness Tests

A.5.1. Financing the exercise of the growth option by issuing additional equity

We consider the case that the exercise of the growth option is financed by issuing additional equity. The resulting system of ODEs for corporate debt is then:

For $0 \leq X \leq D_B$:

$$\begin{cases} d_B(X) & = & \alpha_B \left(V_B(X) + G_B(X)\right) \\ d_R(X) & = & \alpha_R \left(V_R(X) + G_R(X)\right) \end{cases} \tag{A-106}$$

For $D_B < X \leq D_R$:

$$\begin{cases} rd_B(X) & = & c + \mu_B X d_B'(X) + \frac{\sigma 2_B}{2} X 2 d_B''(X) + \lambda_B \left(\alpha_R \left(V_R(X) + G_R(X)\right) - d_B(X)\right) \\ d_R(X) & = & \alpha_R \left(V_R(X) + G_R(X)\right) \end{cases} \tag{A-107}$$

For $D_R < X < X_B$:

$$\begin{cases} rd_B(X) & = & c + \mu_B X d_B'(X) + \frac{\sigma 2_B}{2} X 2 d_B''(X) + \lambda_B \left(d_R(X) - d_B(X)\right) \\ rd_R(X) & = & c + \mu_R X d_R'(X) + \frac{\sigma 2_R}{2} X 2 d_R''(X) + \lambda_R \left(d_B(X) - d_R(X)\right) \end{cases} \tag{A-108}$$

For $X_B \leq X < X_R$:

$$\begin{cases} d_B(X) & = & \hat{d}_B(\bar{s}X) \\ rd_R(X) & = & c + \mu_R X d_R'(X) + \frac{\sigma 2_R}{2} X 2 d_R''(X) + \lambda_R \left(\hat{d}_B(\bar{s}X) - d_R(X)\right) \end{cases} \tag{A-109}$$

For $X \geq X_R$:

$$\begin{cases} d_B(X) & = & \hat{d}_B(\bar{s}X) \\ d_R(X) & = & \hat{d}_R(\bar{s}X). \end{cases} \tag{A-110}$$

The boundary conditions now read:

$$\begin{aligned} \lim_{X \searrow D_R} d_B(X) & = \lim_{X \nearrow D_R} d_B(X) \\ \lim_{X \searrow D_R} d_B'(X) & = \lim_{X \nearrow D_R} d_B'(X) \\ \lim_{X \searrow D_B} d_B(X) & = \alpha_B \left(D_B y_B + G_B(D_B)\right) \\ \lim_{X \searrow D_R} d_R(X) & = \alpha_R \left(D_R y_R + G_R(D_R)\right) \\ \lim_{X \searrow X_B} d_R(X) & = \lim_{X \nearrow X_B} d_R(X) \\ \lim_{X \searrow X_B} d_R'(X) & = \lim_{X \nearrow X_B} d_R'(X) \\ \lim_{X \nearrow X_B} d_B(X) & = \hat{d}_B(\bar{s}X_B) \\ \lim_{X \nearrow X_R} d_R(X_R) & = \hat{d}_R(\bar{s}X_R). \end{aligned} \tag{A-111}$$

The solution to this system follows by standard arguments from the theory of differential equations. Technically, this modification constitutes a simplification of the presented main case: The functional form is straightforward and does not need to be determined as the solution of an inhomogeneous ODE using the fundamental system of solutions of the homogenous ODE (cf. footnote 23). Therefore, we do not present the solution here.

A.5.2. Firm-specific growth options

Assuming that the recovery rates for the assets in place, α_i and for the growth option, defined as α_i^{GO}, might be different, the resulting system of ODEs for the value of corporate debt is:

For $0 \leq X \leq D_B$:

$$\begin{cases} d_B(X) &= \alpha_B V_B(X) + \alpha_B^{GO} G_B(X) \\ d_R(X) &= \alpha_R V_R(X) + \alpha_R^{GO} G_R(X) \end{cases} \tag{A-112}$$

For $D_B < X \leq D_R$:

$$\begin{cases} r d_B(X) &= c + \mu_B X d_B'(X) + \frac{\sigma 2_B}{2} X 2 d_B''(X) + \lambda_B \left(\alpha_R V_R(X) + \alpha_R^{GO} G_R(X) - d_B(X)\right) \\ d_R(X) &= \alpha_R V_R(X) + \alpha_R^{GO} G_R(X) \end{cases} \tag{A-113}$$

For $D_R < X < X_B$:

$$\begin{cases} r d_B(X) &= c + \mu_B X d_B'(X) + \frac{\sigma 2_B}{2} X 2 d_B''(X) + \lambda_B \left(d_R(X) - d_B(X)\right) \\ r d_R(X) &= c + \mu_R X d_R'(X) + \frac{\sigma 2_R}{2} X 2 d_R''(X) + \lambda_R \left(d_B(X) - d_R(X)\right) \end{cases} \tag{A-114}$$

For $X_B \leq X < X_R$:

$$\begin{cases} d_B(X) &= \hat{d}_B \left(\bar{s} X - \frac{K}{y_B}\right) \\ r d_R(X) &= c + \mu_R X d_R'(X) + \frac{\sigma 2_R}{2} X 2 d_R''(X) + \lambda_R \left(\hat{d}_B \left(\bar{s} X - \frac{K}{y_B}\right) - d_R(X)\right) \end{cases} \tag{A-115}$$

For $X \geq X_R$:

$$\begin{cases} d_B(X) &= \hat{d}_B \left(\bar{s} X - \frac{K}{y_B}\right) \\ d_R(X) &= \hat{d}_R \left(\bar{s} X - \frac{K}{y_R}\right). \end{cases} \tag{A-116}$$

The boundary conditions are:

$$
\begin{aligned}
\lim_{X \searrow D_R} d_B(X) &= \lim_{X \nearrow D_R} d_B(X) \\
\lim_{X \searrow D_R} d'_B(X) &= \lim_{X \nearrow D_R} d'_B(X) \\
\lim_{X \searrow D_B} d_B(X) &= \alpha_B D_B y_B + \alpha_B^{GO} G_B(D_B) \\
\lim_{X \searrow D_R} d_R(X) &= \alpha_R D_R y_R + \alpha_B^{GO} G_R(D_R) \\
\lim_{X \searrow X_B} d_R(X) &= \lim_{X \nearrow X_B} d_R(X) \\
\lim_{X \searrow X_B} d'_R(X) &= \lim_{X \nearrow X_B} d'_R(X) \\
\lim_{X \nearrow X_B} d_B(X) &= \hat{d}_B\left(\bar{s}X_B - \frac{K}{y_B}\right) \\
\lim_{X \nearrow X_R} d_R(X_R) &= \hat{d}_R\left(\bar{s}X_R - \frac{K}{y_R}\right).
\end{aligned}
\tag{A-117}
$$

The solution can be obtained analogously to the presented base case.