A WhIMS for Financial Crises*

(Work in progress)

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- August 2010 -

Abstract

This paper introduces a quantitative measure of financial disturbances, which captures the heterogeneity of investor horizons - from day traders to pension funds. This risk measure, called "Wavelet-heterogeneous Index of Market Shocks" (WhIMS), is based on the combination of two methods: the Wavelet Packet Sub-band Decomposition and the constrained Independent Component Analysis. We apply this measure on the French Stock Market to date and gauge the severity of financial crises. A state separation of financial disturbances is finally performed using a nonlinear classification.

JEL Classification: C16, G11, G14.

Keywords: financial crisis, wavelets, nonlinear classification, highfrequency, regime switching.

*We thank Thierry Chauveau, Benjamin Hamidi, Patrick Kouontchou, Paul Merlin and Jean-Philippe Médecin for encouragements and a kind support when preparing this work. We here acknowledge Thierry Michel, Madalina Olteanu, Patrick Rousset and Joseph Rynkiewicz for active participations in previous collaborations on the subject. We should finally mention the excellent preliminary research assistance by Caroline Barrault, Rachid Bokreta, Louride Sekhri and Alexander Subbotin on related topics. The third author thanks the Europlace Institute of Finance for financial support. Preliminary version (work in progress): do not quote or circulate without an explicit permission. The usual disclaimers apply.

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1 Introduction

Extreme price movements in financial markets are of primary concern not only for practitioners but also for policy makers and monetary authorities by their potential consequences on macroeconomic and financial stability. However, a clear understanding of financial stability requires an accurate measure of financial turbulences. It is important to know when such pressure occurs and what its intensity is to detect, identify and compare the severity of different crises. Scales to measure the severity of extreme risks are frequently used in geology, meteorology, astronomy and other sciences: the Guttenberg-Richter, Saffir-Simpson, TORRO and Torino scales. By contrast, there is still no consensus about a scale measure of market shocks. Few attempts have been proposed in the literature (e.g., Mishkin and White, 2003) aiming to provide comparisons of financial crises across markets, or over time in a single market. But financial crises are still most often measured by simple binary variables based on extreme values (1, 2 or 3 standard deviations above the mean) of one or few underlying financial variables.

The objective of this paper is to propose a new quantitative measure of financial turbulences, which we call the WhIMS (for Wavelet-heterogeneous Index of Market Shocks) and to identify regimes in financial turbulences, *i.e.* normal and crisis states. A nonlinear classification using Self Organizing Maps (SOM) based on the WhIMS and conditional probabilities of a Markow switching model is performed to characterize and identify these regimes.

The WhIMS quantifies the intensity of market turbulences. In short, the WhIMS gauges how much are extreme the volatilities at different time scales by assessing the scarcity of factors that underlie the time scale components of the volatility. This measure of market disturbances is a refinement of the so-called Scale of Market Shocks (SMS) by Zumbach *et al.* (2000) and the Index of Market Shocks (IMS) by Maillet and Michel (2003). Its construction is based on an analogy with the so-called Richter scale used for measuring earthquake intensity. The main steps consist in applying a Robust Wavelet Packet Subband Decomposition constrained Independent Component Analysis (RWPSD-cICA) to first decompose the return volatility at different time-scales (see Lu and Rajapakse, 2005; Kopriva and Seršić, 2007) and, extract secondly independent factors on the volatility components. Then, we fit volatility factor extreme values from a Generalized Pareto Distribution (GPD) based on the L-moment method. Finally, the WhIMS is compute based on the scarcity of the volatility underlying factors.

The motivation behind the wavelet time scale decomposition is that financial markets are populated by investors with various time horizons. The heterogeneous market hypothesis was first introduced by Dacorogna *et al.* (1993) which observed a scaling law relating time horizon and size of price movements (Müller *et al.*, 1990). While short-term traders, such as day-traders, are constantly watching the market and re-evaluate the risk over high-frequency horizons, long-term inverstors, such as pension funds, may look at the market less frequently. Intuitively, an asset price drop of 0.5% followed by a rise of the same size during the same trading day is a major event for the day trader but a non-event for mutual funds and pension funds. In contrast, some asset price movements have a significant impact on the timing of both day-trader and long-term investor transactions and investment decisions. One of the objective of the WhIMS is to gauge the severity of these market fluctuations that impact both day-traders, financial and banking institutions, mutual funds, hedge funds and pension funds.

The wavelet analysis consists in a decomposition of a signal into its set of basis functions (wavelets), analogous to the use of sines and cosines in the Fourier analysis. These basis functions are obtained from dilations or contractions (scaling), and translations of the mother wavelet. The main advantages of wavelet analysis are (1) the ability to decompose data into several time scales and to deal with both stationary and non-stationary data, and (2) their localization in time. Wavelet analysis, popular in disciplines such as signal processing and medical sciences, is an increasing used tool in economics and finance. Wavelets have been used in a variety of financial applications, such as outlier testing (Greenblatt, 1995), the modelling of non-stationary processes (Ramsey and Zhang, 1997) and long-memory processes (Jensen, 1999), the contagion measure (Cipollini and LoCascio, 2009), forecasting (Stevenson, 2001), scaling analysis (Gençay *et al.*, 2002), CAPM and ICAPM testing (Gençay *et al.*, 2003 and 2005; Fernandez, 2006) and analysis of higher-order systematic co-moments (Galagedera and Maharaj, 2008). A thorough discussion of the use of wavelets in economics and finance can be found in the survey articles written by Ramsey (1999 and 2002).

Conventional time-series analyses focuse exclusively on a time-series at a given scale but some recent researches explore the behavior of financial time-series, especially volatilities, at different time scales. The HARCH models of Müller *et al.* (1997) and Dacorogna *et al.* (2001) belong to the ARCH family but differ from ARCH-type processes by considering the volatilities of returns measured over different interval sizes. The HARCH model captures the asymmetry in the interaction between volatilities measured at different frequencies such that a coarsely defined volatility, which captures the view and actions of long-term traders, predicts a fine volatility (short-term traders) better than most of the other models. Aït-Sahalia *et al.* (2005) also propose an estimator of the volatility based on overlapping sub-sampling schemes and an appropriate combination of two realized volatilities computed at two different time-scales. More recently, Aït-Sahalia *et al.* (2006) have generalized the Two Scales estimator to a multiple time-scales estimator that combines realized volatilities computed at more than two return frequencies and reaches the same asymptotic efficiency of the kernel-based estimator. Our empirical study is also related to the increasing literature that emphasizes the structural breaks in financial volatility and more generally in financial time-series (see Lamoureux and Lastrapes, 1990; Andreou and

Ghysels, 2002).

The outline of this paper is as follows. Section 2 introduces the time scale decomposition of the volatility and the technique for revealing hidden factors that underlie these time scale components. Section 3 presents the Generalized Pareto Distribution estimated based on L-moments method to assess the scarcity of financial disturbances. Section 4 describes the WhIMS computation. Section 5 presents the preliminary empirical results. Section 6 identifies states in financial turbulences based on a Markow switching model coupled with a SOM nonlinear classification. Section 7 concludes.

2 Identifying Factors of Time Scale Components of the Volatility

We decompose the volatility into different time-scales by applying a new wavelet and factor analysis method: the Wavelet Packet Sub-band Decomposition constrained Independent Component Analysis (WPSD-cICA). The Independent Component Analysis (ICA) is well-known in Signal Processing and applied in various fields such as biomedicine, speech and telecommunication signals. The ICA is a factor analysis method that extracts independent components from a set of multi-dimensional observations. This method corresponds to an extension of the Principal Component Analysis which provides orthogonal directions. However, the ICA presents several limits. Indeed, Chang and Zhang (2005) show that the independence property of the extracting independent components is not always verified. Some extensions have been proposed to outcome this principal limit of the traditional ICA. Throughout this paper, we will introduce a new method which combines two decompositions and factor analysis methods: the Wavelet Packet Sub-band Independent Composition Analysis (WPSD-ICA - Kopriva and Seršić, 2007) and the constrained Independent Component Analysis (cICA – see Lu and Rajapakse, 2005). The result of this mix gives the WPSD-cICA.

2.1 An Introduction to Wavelet Analysis

Our approach to multi-scale analysis is based on the wavelet transform of the original time-series of stock returns. The definition of WhIMS resides on the estimation of the vector of volatilities at different time-scales $\left\{\sigma^{(1)}, ..., \sigma^{(m)}\right\}$. A natural tool for this purpose is wavelet analysis. It is widely used in signal processing in order to decompose a signal into a hierarchical set of approximations and details (multi-resolution analysis), and decompose the energy of a signal on a scale-by-scale basis.

Multiresolution wavelet analysis is a useful tool for studying the time and frequency properties of an economic or financial time series. Using a wavelet filter, returns $r(t)$ can be decomposed as:

$$r(t) = A_J(t) + \sum_{j=1}^{J} D_j(t), \tag{1}$$

where at each decomposition level $j = [1, \cdots, J]$, approximations A_j and details D_j are built.

The suitable decomposition level, J, can be chosen either endogenously, *i.e.* based on the properties of the signal $r(t)$ or using *a priori* hypotheses.

The Fourier transform breaks down a time series into constituent sinusoids of different frequencies. Since a sine wave function has a specific frequency but infinite duration in time, it is perfectly localized in frequency but not localized in time. The wavelet transform, on the other hand, breaks down a time series into shifted and scaled versions of a mother wavelet function that has limited spectral band and limited duration in time. One major advantage afforded by wavelet transform is thus the ability to perform natural local analysis of a time series in the sense that the length of wavelets (windows) varies endogenously in an optimal way (see, for instance, Percival and Walden, 2000). In wavelet analysis, a signal is represented as a linear combination of wavelet functions. In particular, a wavelet allows for decomposing a signal into multi-resolution components: fine and coarse resolution components. There are father and mother wavelets. Father wavelets, $\Psi(.)$, represent the smooth and low-frequency parts of a signal, whereas mother wavelets, $\Phi(.)$, represents the detailed and high-frequency parts of a signal. In particular, the orthogonal wavelet series approximation to continuous retuns $r(t)$ is given by:

$$r(t) \simeq \sum_{k=1}^{K} s_{J,k} \Psi_{J,k}(t) + \sum_{j=1}^{J} \sum_{k=1}^{K} d_{j,k} \Phi_{j,k}(t), \tag{2}$$

where J is the number of multi-resolution components or scales, and $k = [1, \cdots, K]$ corresponds to positions of wavelet on the time axis.

The coefficients $s_{J,k}$ and $d_{j,k}$ are the wavelet transform coefficients, whereas the functions $\Psi_{J,k}(t)$ and $\Phi_{j,k}$ are the approximating wavelet functions.

These functions are generated as follows:

$$\begin{cases} \Psi_{J,k}(t) = 2^{\frac{-J}{2}} \Psi\left(\frac{t-2^J k}{2^J}\right) \\ \Phi_{j,k}(t) = 2^{\frac{-j}{2}} \Phi\left(\frac{t-2^j k}{2^j}\right) \end{cases}. \tag{3}$$

The wavelet coefficients are approximately given by the integrals:

$$\begin{cases} s_{J,k} = \int_{-\infty}^{+\infty} \Psi_{J,k}(t) f(t)\, dt \\ d_{j,k} = \int_{-\infty}^{+\infty} \Phi_{j,k}(t) f(t)\, dt \end{cases} \tag{4}$$

These coefficients are a measure of the contribution of the corresponding wavelet function to the total signal.

In general, there is no close-form solution for father and mother wavelets, and they have to be computed by the so-called dilation equations:

$$\begin{cases} \Psi(t) = \sqrt{2}\sum\limits_{k=1}^{K} l_k \Psi(2t-k) \\ \Phi(t) = \sqrt{2}\sum\limits_{k=1}^{K} h_k \Phi(2t-k) \end{cases} \tag{5}$$

The l_k and h_k coefficients are called the scaling (low-pass) and wavelet (high-pass) filter coefficients, respectively, which are defined by:

$$\begin{cases} l_k = \frac{1}{\sqrt{2}} \int\limits_{-\infty}^{+\infty} \Psi\left(t\right) \Psi\left(2t-k\right) dt \\ h_k = \frac{1}{\sqrt{2}} \int\limits_{-\infty}^{+\infty} \Phi\left(t\right) \Psi\left(2t-k\right) dt \end{cases} \tag{6}$$

The continuous version of the wavelet transform (known as the CWT) assumes an underlying continuous signal, whereas a discrete wavelet transform (DWT) assumes a variable or signal consisting of observations sampled at evenly-spaced points in time.

The Wavelet Packet Sub-band Decomposition presented in this paper is a generalization of the DWT that offers a large set of decomposition structures. Wavelet Packets were introduced for a better treatment of non-stationnarity data. We mobilize this tool to decompose and reconstruct return trajectories of the CAC40 returns.

There are several types of wavelet functions available, such as Morlet, Mexican hat, Haar, Shannon, Daubechies wavelet function, etc. The choice of the wavelet function depends on the application. With respect to time and frequency localization, the Haar and Shannon wavelets take opposite extremes. Having compact support in time, the Haar wavelet has poor decay in frequency, whereas the Shannon wavelet has compact support in frequency with poor decay in time. Other wavelets typically fall in the middle of these two extremes. In fact, having exponential decay in both the time and frequency domain, the Morlet wavelet has optimal joint time-frequency concentration (Teolis, 1964). The wavelet that is used for analysis of economics fluctuations in this paper is Morlet wavelet, which is a modulated Gaussian function with exponential decay property. We will show in the last section that the main results are qualitatively the same whatever the wavelet function used. Concerning the choice of the decomposition level, no such easy rule is defined. We select in our application a number of levels allowing us to determine very long-term fluctuations (*i.e.* fluctuations that may concern pension funds).

2.2 A Wavelet Packet Sub-band Decomposition constrained Independent Component Analysis

We propose a combination of two Blind Source Separation methods (WPSD and cICA) in order to form the Wavelet Packet Sub-band Decomposition constrained Independent Component Analysis (WPSD-cICA). The cICA allows us to overcome two of the main limits of the traditional ICA: the independence property that is not verified and without dimension data reduction.

A standard ICA algorithm is a statistical technique allowing to extract non-Gaussian and statistically independent source signals given only the observed or measured data. The task is to find $\mathbf{W}$ such that the distribution of $\mathbf{Z}$ is as far from Gaussian as possible and statistically independent.

$$\mathbf{Z} = \mathbf{W}\mathbf{R}, \tag{7}$$

where $\mathbf{Z}$ represents the vector of measured signals, $\mathbf{W}$ represents the demixing matrix and $\mathbf{R}$ represents the estimated vector of the unknown vector of the source signals $\mathbf{S}$. Kopriva and Seršić (2007) show that source signals can be dependent, although some of their sub-components are independent. They generalize this basic ICA model by a Sub-band Decomposition ICA (SD ICA). Thus, source signals can be represented as:

$$\mathbf{S} = \mathbf{s}_1 + \mathbf{s}_2 + \cdots + \mathbf{s}_L, \tag{8}$$

where $\mathbf{s}_k$, with $k = [1, \cdots, L]$, are narrow-band sub-components. The standard ICA algorithm can be applied to a selected (reduced) set of k sub-components in order to learn demixing matrix $\mathbf{W}$ so that $\mathbf{y}_k = \mathbf{W}\mathbf{r}_k$. The sub-components are extracted by the Wavelet Packet algorithm (WP - see Kopriva and Seršić, 2007; Kotnik and Kacic, 2007). The result of the decomposition is often represented as a tree of J levels and of N nodes as follows:

$$r^j_{k,n}(t) = \sum_{m=1}^{M} c^j_{k,n,m} \phi_{j,m}(t), \tag{9}$$

where j represents scale level, k represents sub-band index, n represents the sensor index, $m = [1, \ldots, M]$ represents shift index, $\phi_j(t)$ is the chosen wavelet function (or atom) at time t and $c^j_{k,n,m}$ are decomposition coefficients.

A first method for selecting sub-bands (or at least dependent components) is to use the Kullback-Leibler Information *Criterion* (KLIC - see Gouriéroux and Jasiak, 2008), which is a special case of the α-divergence measure (see Csiszar, 2005), and defined by $I(\mathbf{r}^j_{k,n})$ such as:

$$I(\mathbf{r}^j_{k,n}) = -\int_{-\infty}^{+\infty} f_{\mathbf{r}^j_{k,n}}(\mathbf{u}) \ln[f_{\mathbf{r}^j_{k,n}}(\mathbf{u})\ f_0(\mathbf{u})^{-1}]\ d\mathbf{u}, \tag{10}$$

where $f_{\mathbf{r}^j_{k,n}}(.)$ is the "true" density function of the observations and $f_0(.)$, the approximant density, is the standardized Gaussian distribution.

Since the estimation of $I(.)$ requires the knowledge of the true probability distribution function, this measure is difficult to estimate. It is usually approximated in practice with an Edgeworth expansion of the Gaussian approximant density, based on the computation of (conventional) order cumulants, such as:

$$I(\mathbf{r}^j_{k,n}) \simeq \frac{1}{12}\kappa_3(\mathbf{r}^j_{k,n})^2 + \frac{1}{48}\kappa_4(\mathbf{r}^j_{k,n})^2, \tag{11}$$

where $\kappa_i(\mathbf{r}^j_{k,n})$ is the i-th order conventional cumulant of $\mathbf{r}^j_{k,n}$.

3 Scarcity Assessment of Turbulences

Recent attempts for modelling distributions in a multivariate framework are built on the order-statistics, for calibrating a Bernstein Copula in Baker (2008) or for defining extreme comovements using L-moments in Serfling and Xiao (2007). The latter, which are linear functions of the expectations of order statistics, were introduced under this name by Sillitto (1951) and comprehensively reviewed by Hosking (1989). As so-called U-statistics, one of their main advantages over the conventional (C-)moments is that their empirical counterparts are less sensitive to the effects of sampling variability, since they are linear functions of the ordered data. They are shown to provide more robust estimators of higher moments than traditional sample moments and they have then found wide applications in fields where extreme events matter, such as meteorology, hydrology and also earthquake analysis with the Richter Scale (see Thompson *et al.*, 2007). More precisely, L-moments are defined as certain linear functions of Probability Weighted Moments and can characterize a wider range of distributions compared to usual moments. Indeed, they exist whenever the mean of the distribution does, even though some conventional moments do not. Moreover, they are easy to compute and reliable estimators for extreme distributions.

To compute the WhIMS provided, we use a GPD estimated with Generalized Method of TL-moments (Bali, 2003a and 2003b and Hosking, 2007). The GPD distribution of the Independent Factors of semi-volatility is denoted $\widehat{\sigma}$. The GPD is defined also by three parameters: $\upsilon \in I\!R$, the location parameter, $\alpha \in I\!R_+$, the scale parameter and $\xi \in I\!R$ the tail index. It is given by:

$$G_\xi(\widehat{\sigma}) = \begin{cases} 1 - \left[1 + \xi\frac{(\widehat{\sigma}-\upsilon)}{\alpha}\right]^{-\xi^{-1}} & \text{if } \xi \neq 0 \\ 1 - exp\left[-\frac{(\widehat{\sigma}-\upsilon)}{\alpha}\right] & \text{otherwise,} \end{cases}$$

for every $\widehat{\sigma} \in \mathcal{D}$, defined by:

$$\mathcal{D} = \begin{cases}]-\infty; \upsilon - \frac{\alpha}{\xi}[& \text{if } \xi < 0 \\ I\!R & \text{if } \xi = 0 \\ ]\upsilon - \frac{\alpha}{\xi}; +\infty[& \text{if } \xi > 0. \end{cases}$$

This allows us to deduce the cumulants of order 3 and 4 for a GPD normalized distribution:

$$\left\{ \begin{array}{l} \kappa_3 = \frac{2\alpha^3(1+\xi)}{(1-\xi)^3(1-2\xi)(1-3\xi)} \\ \kappa_4 = \frac{3\alpha^4(3+\xi+2\xi^2)}{(1-\xi)^4(1-2\xi)(1-3\xi)(1-4\xi)} - 3. \end{array} \right. \tag{12}$$

Moreover, the first three TL-moments, as a function of the three characteristic parameters of a GPD distribution, are given, for every $(s,t) \in I\!N^2$, by:

$$\left\{ \begin{array}{lcl} \lambda_1^{(s,t)} & = & \upsilon - \frac{\xi}{\alpha} + \frac{(1+s+t)!}{t!} \frac{\Gamma(t-\xi+1)}{\Gamma(2+s+t-\xi)} \frac{\xi}{\alpha} \\ \lambda_2^{(s,t)} & = & \frac{(2+s+t)!}{2(t+1)!} \frac{\Gamma(t-\xi+1)}{\Gamma(3+s+t-\xi)} \alpha, \\ \lambda_3^{(s,t)} & = & \frac{(3+s+t)!}{3(t+2)!} \frac{\Gamma(t-\xi+1)}{\Gamma(4+s+t-\xi)} (1+\xi)\alpha, \end{array} \right. \tag{13}$$

where $\lambda_r^{(s,t)}$ is the r-th TL-moment of truncation order (s,t), $\upsilon \in I\!R$ the location parameter, $\alpha \in I\!R_+$ the scale parameter, $\xi \in I\!R$ the tail index and $\Gamma(a) = \int_0^{+\infty} t^{a-1} e^{-t} dt$ is the Gamma function.

4 The Wavelet-heterogeneous Index of Market Shocks

The measure of market disturbances we propose is a refinement of the so-called Scale of Market Shocks (SMS) by Zumbach *et al.* (2000) and the Index of Market Shocks (IMS) by Maillet and Michel (2003, 2005). The SMS is an indicator of market volatility, analogous to the Richter scale in geophysics (Richter, 1958). The Richter scale is a measure of the logarithm of the total energy liberated during an earthquake. According to recent evidences, the probability of occurrence of large earthquakes grouped within temporal clusters of high seismic activity obeys the inverse power law (see Bak *et al.*, 2002 and Mega *et al.*, 2003). The distribution of the time intervals between one earthquake and the next is wall represented by an inverse power law. By definition, a one point increase in the corresponding $\log_2$ scale is associated with an event that is twice as unlikely (or twice as intense). It has been widely documented that the probability of large price changes decays as a power law, which results in fat-tailed return distributions (Gabaix *et al.*, 2003). The analogy with the Richter scale for financial markets is a complex matter mainly because the notion of "total energy" is hard to specify: many indicators can be potential candidates for this. Zumbach *et al.* (2000) characterize the size of shocks on the financial markets by a continuous operator, mapping price volatility to a logarithmic scale. The latter stands as a counterpart of mechanical work, *i.e.* the rate of change of energy in time. The corresponding formula is:

$$SMS_t = \gamma \log_{10} \left[P\left(v_t\right) \right]^{-1}, \tag{14}$$

with $P\left(v_t\right)$ the distribution function associated with the indicator of agregated realized volatility v_t at time t and γ a scaling parameter.

This indicator is designed to be used as a basis for the definition of a crisis, which is a period when the indicator is higher than some pre-specified threshold. Note that the SMS by Zumbach *et al.* (2000) accounts not only for the size but also for the scale of fluctuations. The basic idea is that the price dynamics is driven by the actions of heterogeneous market participants, operating at different frequencies (Müller *et al.*, 1997). To make this idea clear, compare an institutional investor who operates over medium and long-term targets and a small private investor exploiting short-term market fluctuations. Hardly do they share the same opinion on what is trend and fluctuation on the market. Namely, what is seen as a conjectural fluctuation by the first may often be regarded as a trend by the second. Sometime, however, they would agree to characterize the situation on the market as a crisis or a crash. The underlying intuition is that volatility is characterized not only relatively to particular time-series, but also relatively to particular scales of observations. Economically, these scales can be seen as the decision and portfolio adjustment horizons of investors. The existence of multiple scales in volatility, whatever the underlying model could be, motivates by designing an indicator, which characterizes the overall vector of volatilities at different scales. The Scale of Market Shocks (SMS), proposed in Zumbach *et al.* (2000) for the currency markets, takes the form of an average realized volatility across different scales:

$$SMS_t = \sum_{k=1}^{K} w_k f\left(\sigma_t^{(k)}\right), \tag{15}$$

where $\sigma_t^{(k)}$ is the volatility of returns at scale $k = [1, \ldots, K]$, w_k is the convolution kernel measuring the contribution of scale τ_k to the overall volatility and $f(.)$ is some properly chosen mapping function.

Let M_k be the number of observations of price available for any period of time of length τ_k. An estimate of (realized) price volatility for a given scale is then given by:

$$\sigma_t^{(k)} = \frac{1}{\sqrt{I}} \sum_{i=1}^{I} r_{t,\delta}^2, \tag{16}$$

with $r_{t,\delta}^2$ represents squared log-returns computed for time intervals $\delta = k\,(I)^{-1}$ where $I = M_k - 1$. As the SMS was constructed for inhomogeneous tick-by-tick data, formula (2) could not be applied directly and Zumbach *et al.* (2000) used smoothed volatilities, computed over moving windows, of the form:

$$\sigma_t^{(k)} = \int_{t-2\tau^{(k)}}^{t} \Gamma(\phi)\, w_\phi^{(k)} \sigma_\phi^{(k)} d\phi, \tag{17}$$

with $\Gamma(.)$ an appropriately chosen kernel function and $w_\phi^{(k)}$ a scale dependent weight. A potential problem is that the scaling method suggested in Zumbach and Müller (2001) gives no idea about the range of scales to be considered and their relative importance.

They assert that the choice of the convolution kernel, denoted in formula (17), is not a crucial issue and it suffices to take a function which satisfies some regularity conditions and tends to zero at the edge of the range of frequencies, which is 15 minutes - 64 days in their case. To our knowledge, this choice is made *ab nihilo* and it is not based on any economic notion or statistical properties of the time-series. The assumption that the mass point, or the most important frequency is located in the center of the range, is also doubtful. Another important drawback is that (15) does not account for the possible interdependence between scales.

Maillet and Michel (2003, 2005) adapted the multi-scale approach to the stock market with regards to the complexities due to the discontinuity of the time of trades. They use a different approach for computation and aggregation of volatilities at multiple frequencies, applying the Principal Components Analysis (PCA). The new indicator, called Index of Market Shocks (IMS), was used for the detection and comparison of severity of different crises. The IMS is designed for stock market data and uses a different method of multi-resolution analysis.

Instead of computing volatilities at different scales by successively moving a window of fixed length, the IMS uses different sampling frequencies within each interval to obtain the scale components of volatility. The aggregation scheme is more delicate: it is based on new information contained in each scale relatively to other scales. Instead of scaled volatilities themselves, principal components are used in the definition of IMS:

$$IMS_t = -\alpha \sum_{k=1}^{K} \{w_k \log_2 [1 - F(c_k)]\}, \tag{18}$$

where $F(.)$ is the cumulative density function, $\{c_k, ..., c_K\}$ are normalized principal components (or factors underlying multi-scale volatility), w_k is the weight of each component determined as the portion of variance of the data explained by the corresponding component, and α a scaling constant. The use of a logarithm in base 2 gives an obvious economic sense to the indicator: a unique point increase of IMS corresponds to the energy vector approximately twice as unlikely.

Several shortcomings of the IMS should however be mentioned. Firstly, the method of estimating volatilities at different scales by varying sampling frequency has one important drawback which becomes very important if applied to low-frequency data. If the length of computation window is fixed and volatilities are calculated from the samples with different numbers of points, estimation results will be different in terms of statistical error and may become incomparable. Secondly, the definition of the vector $\{c_k, ..., c_K\}$ in (5) is too restrictive: it relies on linear decorrelation and may not have the power to identify factors if they are not symmetrically distributed (Loretan, 1997) or if the underlying model is not a simultaneous linear mix. Thirdly, the weightening scheme is simply based on the eigenvalues of the PCA mixing matrix, so it does not differentiate between factors influencing short or long scales. So volatilities at different horizons are treated as if they were all of equal importance to investors.

These drawbacks lead to the development of the WhIMS defined by:

$$WhIMS_t = -\widetilde{\alpha}\sum_{k=1}^{K}\left\{\widetilde{w}_k \log_2\left[1 - \widetilde{F}\left(\widetilde{c}_k\right)\right]\right\}, \tag{19}$$

where $\widetilde{F}(.)$ is the cumulative density function, $\widetilde{c}_k$ represents each Independent Factors of volatilities at different time-scales, $\widetilde{w}_k$ is the weight of each Independent Component obtained from a Wavelet Packet Sub-band Decomposition constrained Independent Component Analysis, and $\widetilde{\alpha}$ a scaling constant.

The WhIMS is computed in two main steps. The first one is a combination of two methods which are applied on volatility signals: a Wavelet Packet and a Sub-band Decomposition Independent Component Analysis (See Lu and Rajapakse, 2005 and Kopriva and Seršić, 2007). This method enables to determine quantitatively the range of frequencies computing the various volatilities instead of fixing it arbitrarily (Maillet and Michel, 2003 and 2005). The difference with the traditional ICA is that this method allows us to get components truly independent and introduce a constraint in the signals to reduce data dimension. Moreover, this tool is more appropriate than the Principal Component Analysis used in the original IMS computation when data are non-linear and non-Gaussian. The second step consists in fitting cumulative density functions of Independent Components with a Generalized Pareto Distribution based on L-moments estimation method (which are order-statistic cumulants). Indeed, this step guarantees the WhIMS not to depend anymore on the hypothesis of Log-normality of volatilities.

Finally, the algorithm for computing the WhIMS can be stated as follows. First, perform the Wavelet Packet Sub-band Decomposition of returns; second, reconstruct the trajectories of returns for each scale; third, calculate from each scale return, the log-squared returns; fourth, compute constrained Independent Components and their weights; fifth, fit the Generalized Pareto Distribution for the tails of each Independent Component; and sixth, compute the WhIMS using formula (19). This algorithm is briefly summarized in table 1.

5 Empirical Analysis

In this section, we apply the WhIMS to intraday data on the CAC40 index. We first study the statistical properties of our time-series. Next, we apply the WPSD-cICA tool to the returns of the CAC40 index in order to decompose the source signal into several ones at different time-scales and reduce the dimension of these components in Independent Components. Finally, we compute the WhIMS using formula (19) and discuss the time evolution of our measure of extreme volatilities.

5.1 Data

We estimate the WhIMS by using CAC40 high-frequency data from January 2^{nd}, 1997 to the June 2^{nd}, 2009. The CAC40 index, computed by Euronext France, is available over a 30-second frequency. However, as widely documented in recent financial literature (see Corsi *et al.*, 2001, and Oomen, 2005, for a detailed discussion), significant microstructure effects can appear at highest frequencies. So, we restrict our analysis to long enough 15 minutes intervals. This period covers two major financial events: the Asian crisis of 1998, the dot-com crisis from 2000 to 2003 as well asthe recent subprime turmoil. The Figure 1 represents the time evolution of the CAC40 index and intra-day returns. This figure shows that turbulent periods alternate with calm periods and that CAC40 returns are characterized by volatility clusters. Table 2 presents some statistical properties of CAC40 returns. These statistics suggest that CAC40 returns are characterized by an asymmetric distribution with fat tails. The maximum positive and negative variation of prices are respectively equals to 7.14% and -7.00%. Moreover the *kurtosis* is equal to 32. These features testify the use of statistical methods that deal with extreme events.

5.2 An Illustration of the Heterogeneity Concept

We illustrate the heterogeneity phenomenon in the Figure 2, which represents the CAC40 index for the aribitrarychosen period 10/08/2008 to 11/13/2008 at different frequencies (30 minutes, 1 day and 1 week). The Figure 2 shows that one month after the Lehman Brother's bankruptcy, shocks at few days horizon had probably no consequence for a long-term investor such as pension funds. On the contrary, in crisis periods, all investors are concerned by falling stock prices. Figure 3 gives an intuitive idea of our multi-scale definition of financial crisis. The state of volatility is represented for each frequency considered: if the volatility is an extreme event defined as a value above the 80th percentile, the value 1 is set, otherwise the value 0 indicates a calm period. We can see that during the 2000-2003 and 2008 periods, all investors whatever their investment horizon are touched by the crisis. In other words, a crisis is a multi-scale *phenomenon*.

5.3 The WhIMS applied to Intraday CAC40 Data

The basic idea of the wavelet analysis is to decompose a signal into a certain number of components at different time-scales. We apply this method to the 30 minute intraday returns of the CAC40. After decomposing the source signal, we build new return trajectories at different time-scales. Each decomposition level j corresponds to a frequency of 2^j periods. Figure 4 represents the different rebuilt trajectories (approximations) of CAC40 returns at different time-scales going from one hour to six months.

We first identify regimes of financial crises that are defined when our risk measure exceeds the arbitrary threshold corresponding to a - for the moment - arbitary 80% confidence level. We applied the WhIMS algorithm to our data in order to detect and identify market turbulences during the studied period using formula (19).

Figure 5 represents the evolution of the CAC40 index and the WhIMS over the 1997-2009 period. The high level of the WhIMS during the 2000-2003 and 2008 periods suggests that the French market is characterized by a strong instability over these years, whereas it is rather quite calm between 2003 and 2007. Crises - defined at this step by the 80% percentile threshold - are shaded in grey. We observe that extreme volatilities can also occur in calm periods even if they are less frequent. This feature testifies to the need of a state separation of the WhIMS for identifying more precisely turbulent and calm periods.

6 A State Separation of Financial Turbulences

We finally propose to identify regimes in financial turbulences (*i.e.* normal and crisis states) using a Markow switching model coupled to a Robust Self-Organizing Map. We use the Perez-Quiros and Timmermann (2000) Markov switching framework with time-varying transition probabilities based on prior work of Hamilton (1989) and Gray (1996). We first present the Markow switching framework and estimate the model based on a French Stock Market index (CAC40 Index). Next, a state separation of financial disturbances based on the WhIMS and conditional probabilities of the Markow model is finally performed by using a Robust Self-Organizing Map.

6.1 The Markow Switching Framework

Markov-Switching Autoregressive (MS-AR) models, first introduced by Hamilton (1989), have emerged to describe the behavior of macroeconomic variables as well as exchange rates, stock returns and the volatility (see *e.g.*, Ang and Bekaert, 2003; Garcia, Luger, and Renault, 2003; Dai, Singleton, and Wei, 2003). The success of regime switching models to capture discrete regime shifts in time series lies in allowing the intercept term, slope coefficients, and volatility to depend on a single, latent (unobservable) state variable S_t.

Assume then that the WhIMS, $WhIMS_t$, is generated by the following MS-AR(1) model[1]:

$$WhIMS_t = \mu_{s_t} + \phi_{s_t} WhIMS_{t-1} + \varepsilon_t, \tag{20}$$

where $\varepsilon_t | s_t \sim N(0, \sigma^2_{st})$. The unobserved random state variable s_t is independent of past $WhIMS_t$ conditional on s_{t-1} and is assumed to follow an ergodic Markov chain with a finite number of states, M, which is defined by the transition probabilities:

$$\Pr\{s_t = j | s_{t-1} = i\} = p_{ij}, \tag{21}$$

with

[1] This lag length was chosen in accordance with findings from Akaike and Schwartz tests.

$$\sum_{j=1}^{M} p_{ij} = 1 \tag{22}$$

for i, $j = 1, ..., M$.

We assume that s_t follows an ergodic M-state Markov process with an irreducible transition matrix:

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1M} \\ p_{21} & p_{22} & \cdots & p_{2M} \\ \ldots & \ldots & \ddots & \ldots \\ p_{M1} & \cdots & \cdots & p_{MM} \end{bmatrix}, \tag{23}$$

where $p_{i1} + p_{i2} + \ldots + p_{iM} = 1$ for $i = 1, \ldots, M$. In a two-regime case (*i.e.* M=2), this specification assumes that, if the economy was in crisisin the last period, the probability of a regime switching is constant and independent of the persistence of the crisis.

We focused on daily values of the WhIMS, computed from the CAC40 French Stock Market index, from July 9th, 1987 until June 11th, 2009 (5,567 observations). We first investigate whether the series is "regime switching" and then the number of states required. We employ the test developed by Ang and Bekaert (1998), which, under the null hypothesis that there is no regime shift. The test is approximately distributed as as a chi-squared with q degrees of freedom, where q represents the number of restrictions.

The second relevant issue is how to determine the number of states required by each model to be an adequate characterization of the observed data. Unfortunately, simple and direct statistical criteria cannot be used. Our empirical procedure follows Psaradakis and Spagnolo (2003), who suggest to select the number of regimes using the Akaike Information Criterion (AIC). Monte Carlo experiments show that selection procedures based on the AIC (and on the so-called "three-pattern method") are generally successful in determining the correct number of regimes. Garcia (1998) has tabulated critical values for some Markov switch. Indeed, the usual tests (LR, LM and Wald) do not have the standard asymptotic distribution (e.g., Carrasco, 2004).

Preliminary comparisons of the mean squared error[2] suggest there is no special interest in considering a Markow-Switching model for forecasting the exact value of the WhIMS as there is no sensible improvement when compared to the linear model. Intuitively, this negative result is not surprising when recalling that sudden and violent movements in the market are clearly difficult to predict, and that crises are often due to exogenous and unforecastable events (*e.g.* for the September 2001 terrorist attack, bank failures). Nevertheless, a state representation of the turbulence is of interest since a qualitative assessment (crisis *versus* non-crisis period) is worthy and decisive for financial applications.

[2]Available upon request.

We will focus hereafter on studying the state separation of the possible series considering a three-state Hidden Markov Chain model.

6.2 Identifying Financial Crises

The research of a three-state model having the most significant segmentation of the series leads to select an architecture with the following estimated transition matrix:

$$\hat{A} = \begin{pmatrix} .91 & .06 & .03 \\ .02 & .83 & .15 \\ .02 & .13 & .85 \end{pmatrix}$$

The interpretation of a three-state model is however not straightforward than a simple two-state one (crisis *versus* non-crisis period), but preliminary results suggest that a three-state model allows a better discrimination of crisis and non-crisis periods than a two-state model. We present hereafter a robust Kohonen classification for describing the behavior of the Wavelet-heterogeneous Index of Market Shocks subject to the three experts. The basic idea of Kohonen map is to display high dimensional information on a plane by putting together observations with related characteristics (see Kohonen, 2000, for the description of Self-organizing Maps algorithm). But, since during the learning process outliers influence the model by moving neurons towards the outliers, we use a robust version of the SOM (see Maillet *et al.*, 2005).

We consider the WhIMS and the conditional probabilities related to the MS-AR model as input variables. Thus each cell in the map will incorporate information about the current state of the market but also some information about the dynamics of WhIMS trajectories through the conditional probabilities of belonging to each of the three states. The three probabilities come from the Markow switching model and thus correspond to a denoized estimate of market conditions.

We first conduct a hierarchical classification performed on the map. If we aggregate the information contained in this Kohonen map, by cutting the classification tree and keeping only three clusters, we note that WhIMS values migrate through clusters, from small to high values; the first and the last cluster are homogeneous and correspond, respectively, to periods of calm and crisis respectively (see Figure 6). The intermediate cluster - associated to medium WhIMS values and the third expert, but also to mixtures of experts mentioned above - is less homogeneous. This is even more obvious if we cut the classification tree at five clusters: no significant change can be noted regarding the first and the last clusters, but the middle ones are splitted according to different expert combinations (results available upon request).

The separation between high values of the WhIMS - associated with the first expert - and low values of the Index - linked to the predictions of the second expert - are quite insensitive to the number of clusters. Performing a Kohonen map analysis has here the major advantage that the cut between regimes is less arbitrary, and shows that a clear separation can be made based on WhIMS values and on the value of conditional probabilities to be in some states. Note that state separation is more effectively achieved with the observed WhIMS instead of the predicted one.

Finally, we investigate the behavior of returns in the three identified clusters. Table 3 presents some performance measures of portfolios corresponding to each identified regime: the annualized mean return, the volatitily, the Sharpe ratio (a risk-adjusted return measure) and the frequencies of up and large down returns. We compare the characteristics of the constructed portfolios between each identified cluster and between different Markow switching models of financial disturbances. Indeed, we consider four different measures of financial turbulences: the WhIMS, the Index of Market Shocks proposed by Maillet and Michel (2003), the VIX index and the volatility, which correspond respectively to an aggregate measure of the implied volatility of a wide range of S&P 500 options and to the one-year daily volatility of returns.

Differences between the three clusters clearly reflect the various disturbance regimes from high return-low volatility (cluster 1) to low return-high volatility (cluster 3), with an intermediate state which is undetermined and corresponds to a wider range of WhIMS. The mean conditional expected daily returns are not significantly different, but the cumulated differences on the whole sample amount to large discrepancies between conditional performances. Moreover, the state separation based on the WhIMS leads to a better discrimination of market conditions than those based on the IMS, the VIX or the classical volatility.

The three portfolios derived from the WhIMS are shown on the Figure 7: the series "State 1" ("State 2", "State 3") is built considering benchmark returns when the period is classified in the first (second, third) cluster of the three-category classification and a zero return when classified elsewhere, either in the second (first, first) clusters or third (third, second). While the first state (first cluster) is clearly associated with low WhIMS and low returns, the second cluster is more in line with volatile markets, whilst the third cluster is generally associated with large drops in the market. These results suggest that jumps or regime switching in volatility could potentially strongly affect portfolio allocation.

7 Conclusion

From the definition of a financial crisis as a multi-scale phenomenon, we proposed a quantitative measure of financial crisis, the WhIMS, based on a multi-resolution analysis of market volatility. This indicator aims to characterize volatility as perceived by different investors who have various investment horizons. It can be computed both for large samples of low-frequency data and high-frequency data. The algorithm of computing WhIMS is based on a wavelet analysis to decompose the volatility at different time-scales combined with a factor analysis to identify volatility factors at each scale. The WhIMS does not rely on the log-normality of these factors, making it more robust to distributional properties of the data.

We established a quantitative definition of crises based on the distribution of the WhIMS to date events on financial markets, as well as to compare the relative severity of these events. Finally, we estimate a Markow switching model for the WhIMS to account for potential regime switching in financial turbulences. A non-linear classification, such a robust Kohonen map analysis, based on the WhIMS and conditional probabilities of the Markow switching model allows to identify and characterize stock market conditions without arbitrary threshold. From an asset allocation and risk-management perspective, a promising direction of future research would be to investigate how the identification of market condition regimes based on expert conditional probabilities altogether with WhIMS values affects the investor's portfolio optimization problem.

8 References

Ang, A., and Bekaert, G. (2001), "Regime Switches in Interest Rates", *Journal of Business and Economic Statistics* 20(2), 163–182.

Aït-Sahalia Y., P. Mykland and L. Zhang, (2005), "How Often to Sample a Continuous-time Process in the Presence of Market Microstructure Noise", *Review of Financial Studies 18(2)*, 351–416.

Aït-Sahalia Y., P. Mykland and L. Zhang, (2006), "Ultra High-frequency Volatility Estimation with Dependent Microstructure Noise", National Bureau of Economic Research, NBER Working Papers: 11380, 43 pages.

Aguiar-Conraria L., N. Azevedo and M. Soares, (2008), "Using Wavelets to Decompose the Time–frequency Effects of Monetary Policy", *Physica A 387(12)*, 2863–2878.

Andreou E. and E. Ghysels, (2002), "Detecting Multiple Breaks in Financial Market Volatility Dynamics", *Journal of Applied Econometrics 17(5)*, 579–600.

Arneodo A., J. Muzy and D. Sornette, (1998), "Casual Cascade in Stock Market from the 'Infrared' to the 'Ultraviolet'", *European Physical Journal B 2*, 277–282.

Bali T., (2003-a), "An Extreme Value Approach to Estimating Volatility and Value-at-Risk", *Journal of Business 76(1)*, 83–108.

Bali T., (2003-b), "The Generalized Extreme Value Distribution", *Economics Letters 79(3)*, 423–427.

Bak P., K. Christensen, L. Danon and T. Scanlon, (2002), "Unified Scaling Law for Earthquakes", *Physical Review Letters 88(17)*, 178501, 1–4.

Baker R., (2008), "An Order-statistics based Method for Constructing Multivariate Distributions with Fixed Marginals", *Journal of Multivariate Analysis 99(10)*, 2312–2327.

Baum L. and T. Petrie, (1966), "Statistical Inference for Probabilistic Functions of Finite State Markov Chains", *Annals of Mathematical Statistics 37(6)*, 1554–1563.

Baum L., T. Petrie, G. Soules and N. Weiss, (1970), "A Maximization Technique occurring in the Statistical Analysis of Probabilistic Functions of Markov Chains", *Annals of Mathematical Statistics 41(1)*, 164–171.

Brockwell P. and R. Davis, (2002). *Introduction to Time-series and Forecasting*, Second Edition, Springer-Verlag, New York, 434 pages.

Capobianco E., (2004), "Multiscale Analysis of Stock Index Return Volatility", *Computational Economics 23(3)*, 219–237.

Carrasco M. (2002), "Misspecified Structural Change, Threshold, and Markov-switching Models", *Journal of Econometrics* 109, 239–237.

Garcia, R. (1998), "Asymptotic Null Distribution of the Likelihood Ratio Test in Markov Switching Models", *International Economic Review* 39, 763–788.

Cheng G., Y. Choi and Y. Zhou, (2008), "Detections of Changes in Return by a Wavelet Smoother with Conditional Heteroskedastic Volatility", *Journal of Econometrics 143(2)*, 227-262.

Cipollini A. and I. LoCascio, (2009), "Contagion between Asset Markets: a Wavelet based Testing Methodology", *mimeo*, 6 pages.

Corsi F., G. Zumbach, U. Müller and M. Dacorogna, (2001), "Consistent High-precision Volatility from High-frequency Data", *Economic Notes 30(2)*, 183–204.

Csiszar I., (1975), "I-Divergence Geometry of Probability Distributions and Minimization Problems", *Annals of Probability 3(1)*, 146–158.

Dacorogna M., R. Gencay, U. Müller, R. Olsen and O. Pictet, (2001). *An Introduction to High-frequency Finance*, Academic Press, San Diego, 383 pages.

Dacorogna M., U. Müller, R. Nagler, R. Olsen and O. Pictet, (1993), "A Geographical Model for the Daily and Weekly Seasonal Volatility in the FX Market", *Journal of International Money and Finance 12(4)*, 413–438.

Elamir E. and A. Seheult, (2003), "Trimmed L-moments", *Computational Statistics and Data Analysis 43(3)*, 299–314.

Fernandez V., (2004), "Time-scale Decomposition of Price Transmission in International Markets", Center for Applied Economics, University of Chile, Working paper 189, 33 pages.

Fernandez V., (2006), "The International CAPM and a Wavelet-based Decomposition of Value-at-Risk", National Bureau of Economic Research, NBER Working Papers: 12233, 26 pages.

Forbes K. and R. Rigobon, (2002), "No Contagion, only Interdependence: Measuring Stock Market Co-movements", *Journal of Finance 57(5)*, 2223–2261.

Gabaix X., P. Gopikrishnan, V. Plerou and H. Stanley (2003), "A Theory of Power Law Distributions in Financial Market Fluctuations", *Nature 423*, 267–270.

Galagedera U. and E. Maharaj, (2008), "Wavelet Time-scales and Conditional Relationship between Higher-order Systematic Co-moments and Portfolio Returns", *Quantitative Finance 8(2)*, 201–215.

Garcia R., R. Luger and E. Renault (2003), "Empirical Assessment of an Intertemporal Option Pricing Model with Latent Variables", *Journal of Econometrics* 116, 49-83.

Gassiat E., (2002), "Likelihood Ratio Inequalities with Applications to Various Mixtures", *Annales de l'institut Henri Poincaré (B) Probabilités et Statistiques 38(6)*, 897–906.

Gençay R., F. Selçuk and B. Whitcher, (2001). *An Introduction to Wavelets and Other Filtering Methods in Finance and Economics*, Academic Press, San Diego, 359 pages.

Gençay R., F. Selçuk and B. Whitcher, (2003), "Systematic Risk and Time-scales", *Quantitative Finance 3(2)*, 108–116.

Gençay R., F. Selçuk and B. Whitcher, (2005), "Multiscale Systematic Risk", *Journal of International Money and Finance 24(1)*, 55–70.

Gouriéroux Ch. and J. Jasiak, (2008), "Dynamic Quantile Models", *Journal of Econometrics 147(1)*, 198–205.

Gray, S.F., (1996), Modeling the Conditional Distribution of Interest Rates as a Regime Switching Process, *Journal of Financial Economics* 42, 27–62.

Greenblatt S., (1995), "Wavelets in Econometrics: An Application to Outlier Testing", in *Computational Economic Systems: Models, Methods, and Econometrics,* M. Gilli (eds), vol. 5 of Advances in Computational Economics, Kluwer Academic, 139–160.

Hamilton, J.D., (1989), A New Approach to the Economic Analysis of Non-stationary Time Series and the Business Cycle, *Econometrica* 57, 357–384.

Hosking J., (1989), "Some Theoretical Results concerning L-moments", *Research Report RC14492*, IBM Research Division, 13 pages.

Hosking J., (2007), "Some Theory and Practical Uses of Trimmed L-moments", *Journal of Statistical Planning and Inference 137(9)*, 3024–3039.

Illing M. and Y. Liu, (2003), "An Index of Financial Stress for Canada", Bank of Canada Working Paper 2003-14, 63 pages.

Jensen M., (1999), "Using Wavelets to Obtain a Consistent Ordinary Least Squares Estimator of the Long-Memory Parameter", *Journal of Forecasting 18*, 17–32.

Kohonen T., (2000). *Self-organizing Maps*, Springer Series in Information Sciences, Vol. 30, Springer, New-York, 501 pages.

Kopriva I. and D. Seršić, (2007), "Wavelet Packet Approach to Blind Separation of Statistically Dependant Sources", *Neurocomputing 71(7-9)*, 1642–1655.

Kotnik B. and Z. Kacic, (2007), "A Noise Robust Feature Extraction Algorithm using Joint Wavelet Packet Sub-band Decomposition and AR Modeling of Speech Signals", *Signal Processing 87*, 1202–1223.

Lamoureux C. and W. Lastrapes, (1990), "Persistence in Variance, Structural Change and the GARCH Model", *Journal of Business and Economic Statistics 8(2)*, 225–234.

Lee H., (2004), "International Transmission of Stock Market Movements: A Wavelet Analysis", *Applied Economics Letters 11(3)*, 197–201.

Longin F. and B. Solnik, (2001), "Extreme Correlation of International Equity Markets", *Journal of Finance 56(2)*, 649–676.

Loretan M., (1997), "Generating Market Risk Scenario using Principal Component Analysis: Methodological and Practical Considerations", *manuscript*, Federal Reserve Board, 60 pages.

Lu W. and C. Rajapakse, (2005), "Approach and Applications of Constrained ICA", *Transactions on Neural Networks 16(1)*, 203–212.

Maillet B., P. Merlin and P. Rousset, (2005), "Robust SOM for Realistic Data Completion", Proceedings of the 5th Workshop On Self-organizing Maps, 371–378.

Maillet B. and T. Michel, (2003), "An Index of Market Shocks based on Multiscale Analysis", *Quantitative Finance 3(2)*, 88–97.

Maillet B. and T. Michel, (2005), "The Impact of the 9/11 Events on the American and French Stock Markets", *Review of International Economics 13(3)*, 597–611.

Maillet B., M. Olteanu and J. Rynkiewicz, (2004), "Non-linear Analysis of Shocks when Financial Markets are Subject to Changes in Regime", Proceedings of ESANN 2004 - European Symposium on Artificial Neural Networks, Bruges, Belgium.

Mega M., P. Allegrini, P. Grigolini, V. Latora, L. Palatella, A. Rapisarda and S. Vinciguerra, (2003), "Power Law Time Distributions of Large Earthquakes", *Physical Review Letters 90(18)*, 188501.1-188501.4.

Mishkin F. and E. White (2002), "U.S. Stock Market Crashes and Their Aftermath:

Implications for Monetary Policy", NBER Working Paper No. 8992, National Bureau of Economic Research, 55 pages.

Müller U., M. Dacorogna, R. Dave, O. Pictet, R. Olsen and J. von Weizsäcker, (1997), “Volatilities of Different Time Resolutions – Analysing the Dynamics of Market Components”, *Journal of Empirical Finance 4(2-3)*, 213–239.

Müller U., M. Dacorogna, R. Olsen, O. Pictet, M. Schwarz and C. Morgenegg, (1990), “Statistical Study of Foreign Exchange Rates, Empirical Evidence of a Price Scaling Law, and Intraday Analysis”, *Journal of Banking and Finance 14(6)*, 1189–1208.

Olteanu M., (2006), “A Descriptive Method to Evaluate the Number of Regimes in a Switching AutoRegressive Model”, *Neural Networks 19(6-7)*, 963–972.

Oomen R., (2005), “Properties of Bias-corrected Realized Variance under Alternative Sampling Schemes”, *Journal of Financial Econometrics 3(4)*, 555–577.

Perez-Quiros, G. and A. Timmermann, (2000), Firm Size and Cyclical Variations in Stock Returns, *Journal of Finance* 55, 1229–1262.

Percival D. and A. Walden, (2000). *Wavelet Methods for Time-series Analysis*, Cambridge University Press, 594 pages.

Psaradakis Z., Spagnolo N. (2003), “On the Determination of the Number of Regimes in Markov–switching Autoregressive Models”, *Journal of Time Series Analysis* 24, 237–252.

Ramsey J., (1999), “Regression over Time-scale Decompositions: A Sampling Analysis of Distributional Properties”, *Economic Systems Research 11(2)*, 163–183.

Ramsey J., (2002), “Wavelets in Economics and Finance: Past and Future”, *Studies in Nonlinear Dynamics & Econometrics 6(3)*, 1–29.

Ramsey J. and Z. Zhang, (1997), “The Analysis of Foreign Exchange Rate Data using Waveform Dictionaries”, *Journal of Empirical Finance 4(4)*, 341–372.

Richter C., (1958). *Elementary Seismology*, W. Freeman and Company, 768 pages.

Rigobon R., (2003), “On the Measurement of the International Propagation of Shocks: Are they Stable?”, *Journal of International Economics 61(2)*, 261–283.

Serfling R. and P. Xiao, (2007), “A Contribution to Multivariate L-moments: L-comoment Matrices”, *Journal of Multivariate Analysis 98(9)*, 1765–1781.

Sillitto G., (1951), “Interrelations between certain Linear Systematic Statistics of Sample from any Continuous Population”, *Biometrika 38*, 377–382.

Stevenson M., (2001), “Filtering and Forecasting Spot Electricity Prices in the Increasingly Deregulated Australian Electricity Market”, Quantitative Finance Research Center, Working Papers 63, University of Technology, Sydney, 31 pages.

Teolis, A., 1964. Computational signal processing with wavelets, Birkhauser.

Thompson E., L. Baise and R. Vogel, (2007), “A Global Index Earthquake Approach to Probabilistic Assessment of Extremes”, *Journal of Geophysical Research 112(B6)*, 1–12.

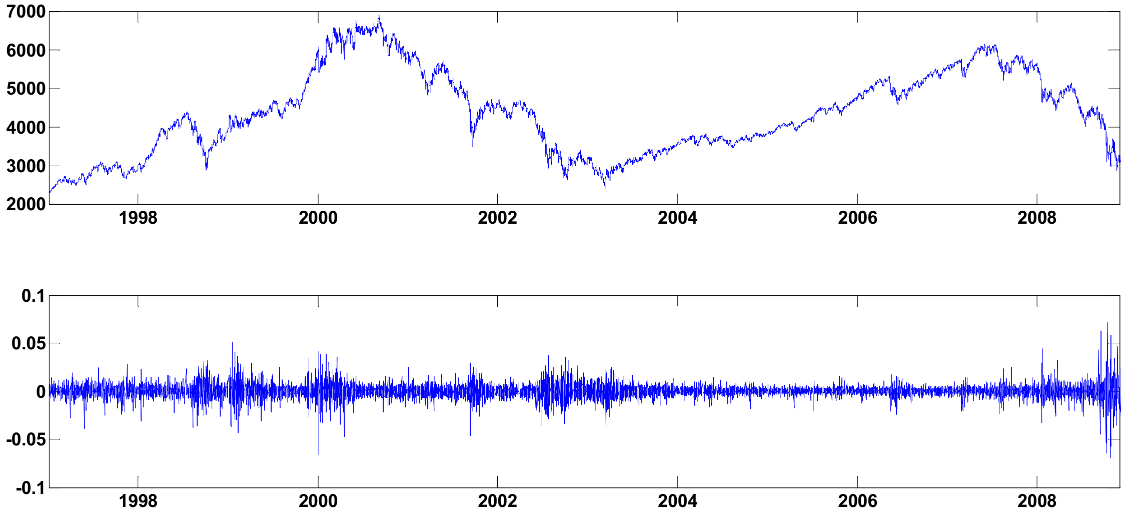
Viterbi A., (1967),“Error Bounds for Convolutional Codes and an Asymptotically Optimum Decoding Algorithm”, *IEEE Transactions on Information Theory 13*, 260-269.

Zhang K. and L. Chan, (2006), “An Adaptive Method for Subband Decomposition ICA”, *Neural Computation 18(1)*, 191–223.

Zumbach G. and U. Müller, (2001), “Operators on Inhomogeneous Time-series”, *International Journal of Theoretical and Applied Finance 4(1)*, 147–178.

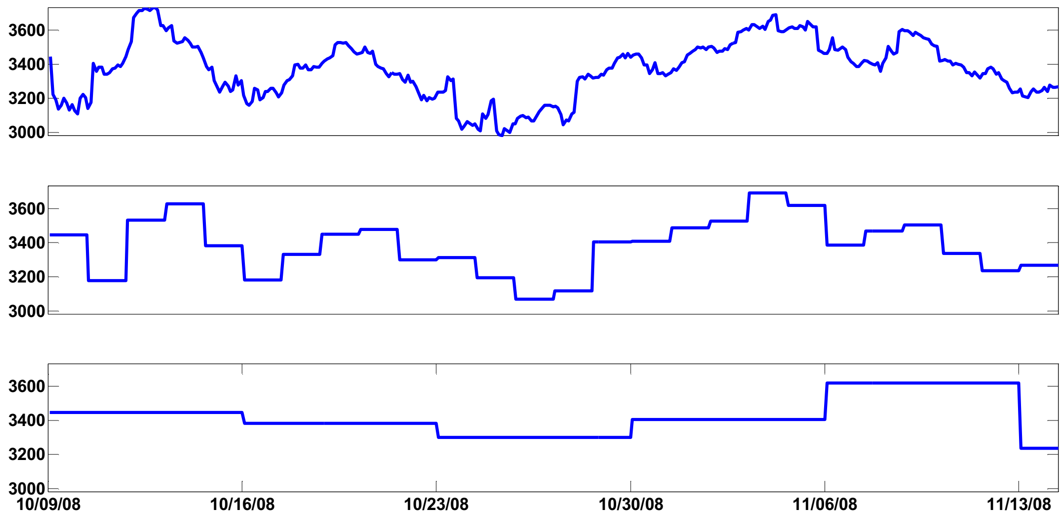
Zumbach G., M. Dacorogna, J. Olsen and R. Olsen, (2000), “Measuring Shocks in Financial Markets”, *International Journal of Theoretical and Applied Finance 3(3)*, 347–355.

Figure 1: Time Evolution of the CAC40 and the CAC40 returns



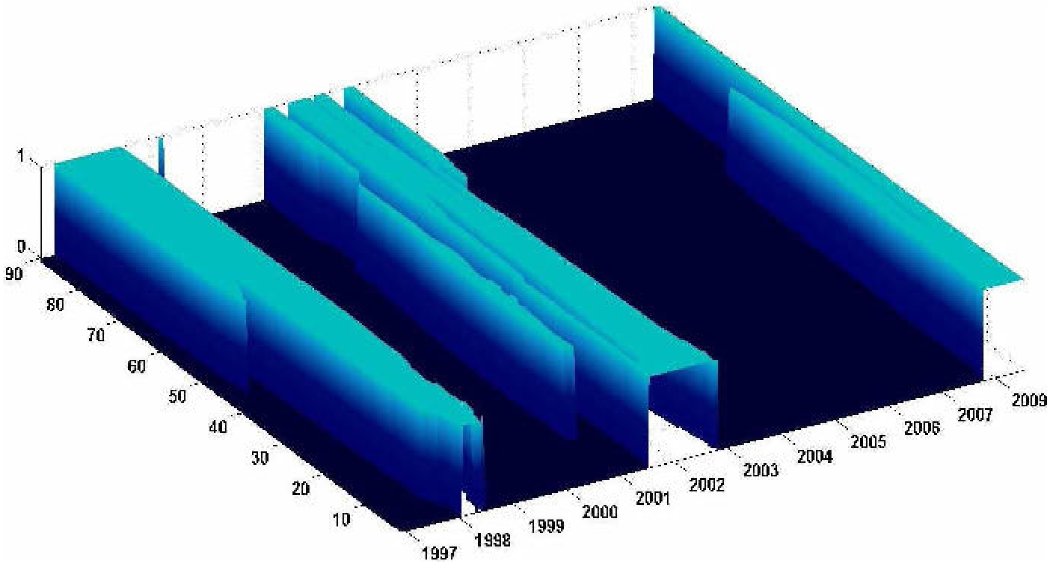
Source: *Euronext*. CAC40 30 minutes intraday data from the January 2nd, 1997 to the June 2nd, 2009. The chart on the top represents the evolution of the CAC40 whereas the chart on the bottom is an illustration of the CAC40 intraday returns. Computations by the authors.

Figure 2: Time-Series Evolution of the CAC40 at Different Calendar Frequencies on the "Post-Lehman" Sample



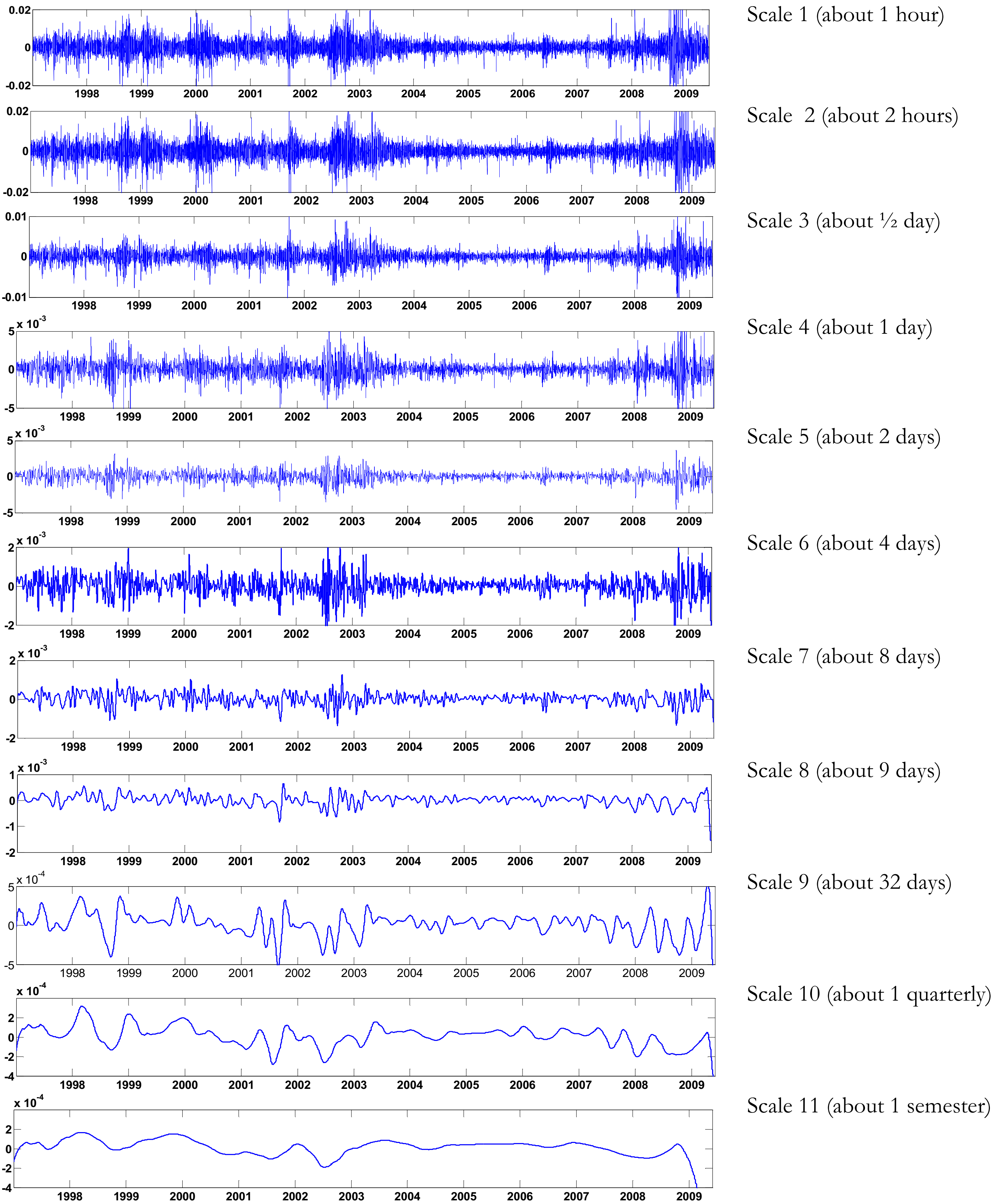
Source: *Euronext*. CAC40 30 minutes intraday data during the period 10/09/2008 to 11/13/2008. The first chart represents the 30 minutes CAC40 index, the second one is the closed daily CAC40 index and the third and last chart illustrates the (closed) weekly CAC40 index. Computations by the authors.

Figure 3: Time-series Evolution of the CAC40 Return Volatility States
(0 if calm period, 1 if in the 90% extreme)



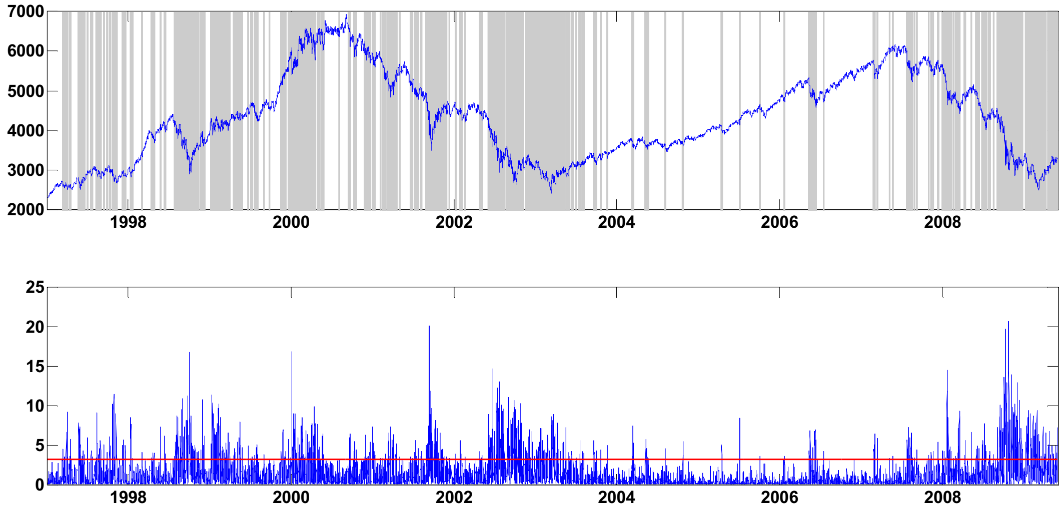
Source: *Bloomberg*. Daily CAC40 Index data from 01/01/1996 to 06/11/2009. This chart represents the volatility state in the y-axis, the frequency in the z-axis (1 day to three months) and time in the x-axis. Computations by the authors.

Figure 4: Evolution of Some Levels of Decomposition of CAC40 Returns



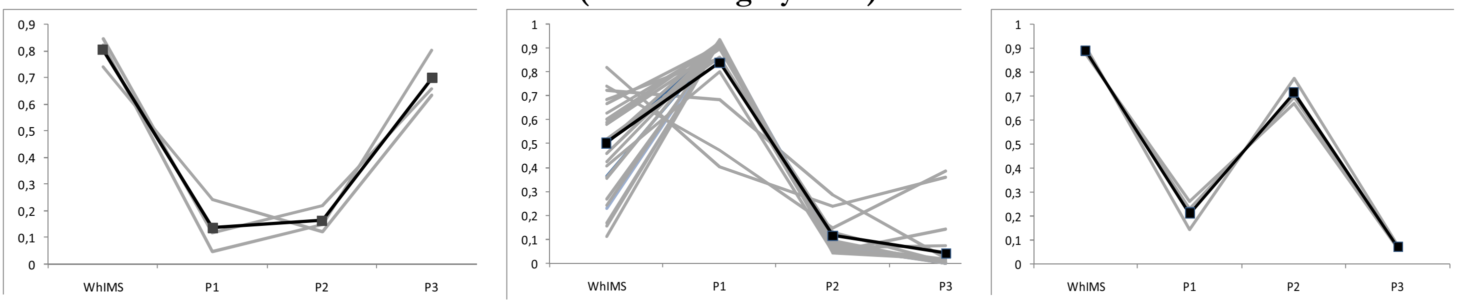
Sources: *Euronext*, CAC40 30 minutes intraday data from the 01/02/1997 to 06/02/2009. These charts represent eleven decomposition levels of the CAC40 intraday returns from a Wavelet Packets constrained Independent Component Analysis. Computations by the authors.

Figure 5: Evolution of the CAC40 Index and the Wavelet-heterogeneous Index of Market Shocks



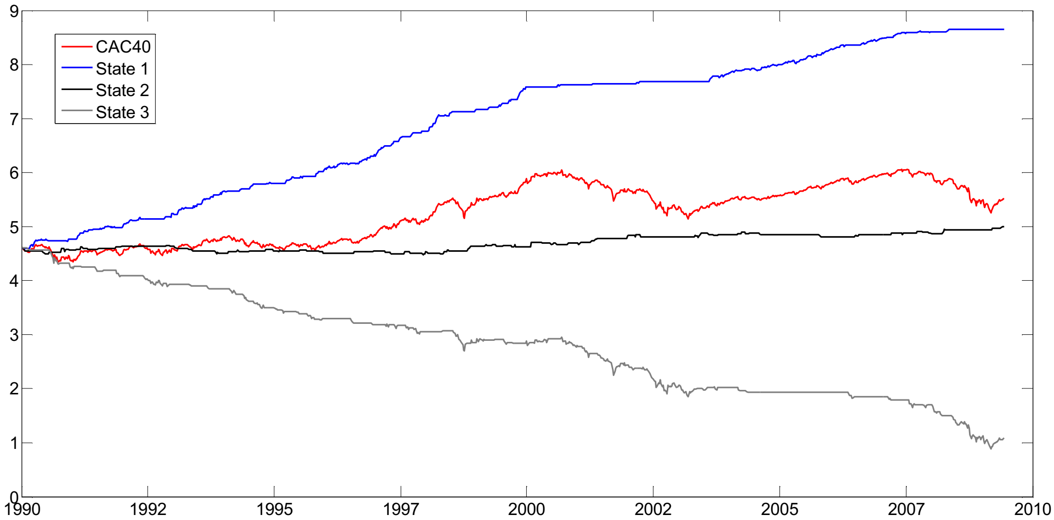
Sources: *Euronext.* CAC40 30 minutes intraday data from the January 2nd, 1997 to the June 2nd, 2009. The chart on the top represents the evolution of the CAC40 in brown and crises in gray. The chart on the bottom represents the WhIMS applied to CAC40 in black line and the 80% threshold in red line. Computations by the authors.

Figure 6: Clustering of the Kohonen Map of States of Market Turbulences (Three-category Tree)



Source: *Bloomberg*. Weekly CAC40 Index data between 01/01/1987 and 06/11/2009. Clusters resulting from a Three-category Hierarchical classification of the four normalized inputs of the Kohonen Map: the WhIMS (first (x,y) coordinate), and the three-state conditional probabilities (the three following (x,y) coordinates: P_1, P_2 and P_3). The bold line represents the mean code vector of each cell. Computations by the authors.

Figure 7: Cumulated Return Series Related to States of Turbulence of the CAC 40



Source: *Bloomberg*, Weekly CAC40 Index data between 01/01/1990 and 06/11/2009. “State 1” (“State 2”. “State 3”) corresponds to a series of cumulated returns (base 100 in 01/01/1990) when the WhIMS and the conditional expert probabilities are classified in the first (second, third) cluster on a three-category string (semi-logarithmic scale). Computations by the authors.

Table 1: Sketch of algorithm

Main steps of the WhIMS algorithm
Step 1. Perform the Wavelet Packet Sub-band Decomposition of returns
Step 2. Reconstruct the trajectories of returns for each scale
Step 3. Transform negative returns for each scale into log-squared returns
Step 4. Compute constrained Independent Components and their weights
Step 5. Fit the Generalized Pareto Distribution for the tails of each Independent Component
Step 6. Compute the WhIMS using formula (19)

Table 2: Statistics of Intraday Returns on the CAC40

Min.	Max.	Mean	Standard Deviation	Mediane	Skewness	***Kurtosis***	Negative Frequency
-7.00%	7.14%	.00%	.38%	.00%	-.18	32.16	48.99%

Source: *Euronext*. CAC40 30 minutes intraday data during January 2nd, 1997 to the June 2nd, 2009. Computations by the authors.

Table 3: Comparisons between Market Characterizations based on a HMM-MLP Modelling of the WhIMS, IMS, VIX and Volatility

		Frequency	Return	Volatility	Sharpe ratio	Up	Large Down
All States	**CAC40**	100.00	4.80	20.41	0.23	52.56	9.96
State 1	**WHIMS**	43.20	61.30	12.57	4.87	68.72	0.46
	IMS	63.31	11.92	15.92	0.74	54.21	6.39
	VIX	35.21	19.03	14.24	1.33	58.82	5.04
	Volatility	33.33	3.06	12.55	0.23	51.78	3.25
State 2	**WHIMS**	16.37	13.13	15.92	0.82	50.60	3.01
	IMS	18.93	-15.96	20.72	-0.77	46.88	13.02
	VIX	20.81	5.10	15.24	0.33	52.61	5.21
	Volatility	16.67	31.18	17.48	1.78	59.76	7.69
State 3	**WHIMS**	40.43	-36.17	25.97	-1.40	36.10	22.93
	IMS	17.75	4.87	31.28	0.15	52.78	19.44
	VIX	43.98	-5.49	25.92	-0.21	47.53	16.14
	Volatility	50.00	-1.68	24.97	-0.07	50.69	15.19

Source: *Bloomberg*. Weekly CAC40 Index data between January 1st. 1990 and June 11th. 2009. Computations by the authors. The IMS corresponds to the Index of Market Shocks (Maillet and Michel, 2003), the WhIMS to the Wavelet-heterogeneous Index of Market Shocks, and the Volatility corresponds to the one-year daily annualized volatility of returns on the CAC40. The VIX is an aggregate measure of the implied volatility of a wide range of S&P 500 options. All figures – except Sharpe ratios – are expressed in percentages. The column "State" indicates the regime issued from the classification. Frequency represents the percentage of periods in each corresponding state. Mean and volatility represent annualized first and second central conventional moments of the conditional return in the various states. The Sharpe ratio is calculated by subtracting the risk free rate (Eonia) from the rate of return on the portfolio and dividing by the standard deviation of the portfolio returns. Up (Large down) indicates the frequency of positive (large negative) returns in each state conditional samples.