Estimating Market-implied Recovery Rates from Credit Default Swap Premia

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Abstract

In this paper, we explore the stochastic nature of implied recovery rates. We exploit the fact that differently-ranking debt instruments of the same issuer face identical default risk but different default-conditional recovery rates. Specifically, we extract information from Credit Default Swaps (CDSs) referencing different types of debt and, in particular, make use of Loan-only Credit Default Swaps, an altogether new asset class. This enables us to overcome a well-known separation problem: In most CDS pricing equations, loss and default rates are essentially multiplicatively linked, making a dissection intrinsically difficult. Our approach permits estimating a firm's entire implied probability distribution of recovery given default at a particular point in time. We allow the mean and the standard deviation of this distribution to vary stochastically and do not impose any parametric relationship to implied default rates. Our estimation results are reliable, robust, as well as economically meaningful and can serve as a basis for deducing firms' implied probabilities of default.

JEL Codes: G0, G1, G33.

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1. INTRODUCTION

Research on the determinants of historical recovery rates shows that there is a systematic component in recovery risk and that the market practice of assuming constant recovery rates results in a significant underestimation of economic capital (cf. Renault and Scaillet (2003), Altman, Brady, Resti, and Sironi (2005), Bruche and Gonzáles-Aguado (2009), and others). The Basel Committee on Banking Supervision (2006) accordingly demands that recovery estimates "reflect economic downturn conditions where necessary to capture the relevant risks". Understanding the dynamics of implied recovery rates should thus be of interest to a variety of market participants, be it traders, risk managers or developers of forward-looking credit risk models.

In this paper, we explore the stochastic nature of implied recovery rates. We exploit the fact that differently-ranking debt instruments of the same issuer face identical default risk but different default-conditional recovery rates. Specifically, we extract information from Credit Default Swaps (CDSs) referencing different types of debt and, in particular, make use of Loan-only Credit Default Swaps (LCDSs), an altogether new asset class. This enables us to overcome a well-known separation problem: In most CDS pricing equations, loss and default rates are essentially multiplicatively linked, making a dissection intrinsically difficult. Our approach permits estimating a firm's entire implied probability distribution of recovery given default at a particular point in time. We allow the mean and the standard deviation of this distribution to vary stochastically and do not impose any parametric relationship to implied default rates. Our estimation results are reliable, robust, as well as economically meaningful and can serve as a basis for deducing firms' implied probabilities of default.

We thus eschew some of the shortcomings of earlier research concerned with the estimation of implied recovery rates: Bakshi, Madan, and Zhang (2006) and Gaspar and Slinko (2008) specify an explicit link between implied default and recovery rates. Zhang (2003), Pan and Singleton (2008), and Schneider, Sögner, and Veza (2009) exploit the fact that premia of long-lived CDSs are particularly sensitive to changes in implied recovery rates and therefore require CDSs of various maturities. Further, they assume implied recovery rates to be constant and only Zhang arrives at estimates that are close to market practice. Other approaches require information from multiple asset classes: Jarrow (2001) proposes a model for estimating recovery rates and probabilities of default from debt and equity prices. Janosi, Jarrow, and Yildirim (2003) show that the implementation of this model is feasible but find that equity price bubbles can impair the reliability of estimation results. Das and Hanouna (2009) use a binomial tree to estimate the entire term structure of recovery rates and probabilities of

default. Their approach is original in that it requires as input only the current term structure of CDS premia, equity prices and equity volatility and thus evites use of time series data. However, they, too, explicitly specify default and recovery rates as functions of a common state variable. Madan and Unal (1998) use prices of certificates of deposit on senior and junior debt of the same issuer. In a similar approach, Güntay, Madan, and Unal (2003) estimate recovery rates from debt prices and balance sheet information. They show that, given various types of debt of the same borrower, it is feasible to construct a metric that reflects recovery risk but is void of default risk. Using capital structure data and approximate prices of senior and junior zero coupon bonds, they infer implied firm-wide recovery rates. Song (2008) imposes no-arbitrage restrictions between spot and forward CDSs. The limited availability of the latter, however, effectively limits the applicability of this method to sovereign CDSs.

Our approach to estimating market-implied recovery rates is tailored to the particularities of default swaps. First, we screen the universe of outstanding CDSs for pairs that reference the same issuer but different types of debt of that issuer. We find that the overwhelming majority of CDSs references either senior unsecured bonds or senior subordinated bonds and identify a number of firms for which both instruments are outstanding. Alternative combinations are unworkable as seniorities other than the mentioned are referenced only in negligibly few cases.[1] To broaden the basis of our investigation nonetheless, we take advantage of LCDSs, a new type of credit derivative that has emerged in recent years. LCDSs share the purpose of "traditional" CDSs in that they allow trading the credit risk associated with some debt obligation but are intended for use with leveraged loans as opposed to bonds. Leveraged loans are senior secured loans of sub-investment grade issuers and usually rank senior to all other debt of a borrower.[2] As LCDS data has become available for a relatively large number of issuers, we are able to compose a second data set, this time containing firms on which LCDSs as well as CDSs on senior unsecured bonds are outstanding.

We show that the ratio of (L)CDS premia referencing the same firm is a function of implied recovery rates but not of the implied probability of default. We then derive equations for implied expected instrument-specific recovery rates based on issuers' capital structure. Using a parametric approach, we are able to estimate the entire implied probability distribution of recovery given default by calibrating model-implied ratios of premia to actual ratios.

[1] Of 4,408 USD-, EUR-, GBP-, and JPY-denominated CDSs on obligations of financial and corporate issuers quoted by Markit as of December 2007, reference obligations are senior unsecured bonds in 3,603 cases (82%), senior subordinated bonds in 409 cases (9%), senior secured bonds in 62 cases (1%), and other in 334 cases (8%).

[2] Conversations with practitioners suggest that this generally also holds true when compared to senior secured bonds but inter-creditor agreements may stipulate aberrant provisions.

Our analysis suggests that the implied probability distribution of recovery is related to proxies for firm- and industry-specific financial health. In particular, implied expected recovery rates tend to be higher for issuers with low leverage, a high share of tangible assets, strong liquidity, and more so if an issuer's industry is in a robust condition. This extends earlier findings on the determinants of physical recovery rates (Cantor and Varma (2005), Acharya, Bharath, and Srinivasan (2007), and others) to the risk-neutral world.

Further, we find that the shape of the implied probability density of recovery differs significantly from that of its physical counterpart: While the physical density is known to be approximately bell-shaped, the implied density is U-shaped due to the high standard deviation of implied recovery rates. This suggests that ex ante there is substantial uncertainty as to where recovery rates will come out in the event of a default, a proposition practitioners would probably endorse.[3]

Our observation period includes two periods of major economic shakeups, namely the time from 2001 to 2003 which was characterized by the repercussions from the burst of the internet bubble and the time from summer 2007 to July 2008 when the current economic crisis started to unfold. We observe that implied expected recovery rates are strongly affected by the economic environment, falling substantially in times of distress and reverting to a "non-crisis-level" in between. This is consistent with earlier research showing that historically, recovery rates were higher when the economy fared well. The standard deviation of implied recovery rates, though, behaves contrarily, generally rising in times of distress.

Instrument-specific recovery rates are first and foremost driven by the reference obligation's seniority and the issuer's capital structure. We find that implied expected recovery rates are on average more than twice as high for senior secured loans as for senior unsecured bonds and more than four times as high for senior unsecured bonds as for senior subordinated bonds. Further, within a particular type of debt, we observe significant inter-company differences. Quantifying the sensitivity of recovery rates to the size of debt cushion below and above, we find that capital structure characteristics explain most of these differences. In addition, we examine whether implied expected firm-wide recovery rates are related to corporate family ratings but find no evidence in this regard. This is consistent with Emery (2007) who does the same for historical recovery rates and likewise detects no such link. The picture is, however, quite different when instrument-specific ratings are considered as these explicitly account for the recovery prospects of a particular debt issue.

[3] "It is extremely difficult to predict firm-wide loss given default rates well in advance of default.", cf. Cantor, Emery, and Stumpp (2006), p. 10.

To validate our results, we implement two alternative parameterizations of the implied probability distribution of recovery. We find that estimation results do not differ materially and that, in particular, the U-shape of implied distribution persists. Further, we relate our estimates of implied expected recovery rates to actual recovery rates, finding that the first are significantly lower than the second. Using an exponential utility function, we calculate coefficients of risk aversion for firm-wide and instrument-specific recovery rates and find these to be comparable in size, quite stable over time and that investors seem to be slightly more risk averse for lower-ranking debt instruments.

Finally, we use our estimates of implied expected recovery rates to deduce implied probabilities of default. This is feasible since our approach to estimating recovery rates does not impose any relationship upon default and recovery rates. We find that implied probabilities of default are strongly affected by changes in the economic environment, too, rising substantially in times of distress. Consequently, they are inversely related to implied expected recovery rates, a result consistent with Bakshi, Madan, and Zhang (2006) and Das and Hanouna (2009). Further, we show that the coefficient of risk aversion for the implied probability of default is somewhat lower than that of implied recovery rates, possibly due to investors being more comfortable with taking default risk than with taking (less understood) recovery risk.

The remainder of this paper is organized as follows: Section 2 discusses our approach to estimating implied recovery rates. Section 3 gives an overview of the data and shows some descriptive statistics. Section 4 details the empirical specification of the model. Section 5 presents estimation results for implied recovery rates and discusses robustness. Section 6 uses these results to deduce implied probabilities of default. Section 7 concludes.

2. METHODOLOGY

2.1 The Ratio of CDS Premia

CDSs allow trading the credit risk associated with a certain debt instrument (reference obligation), issued by some firm or sovereign (reference entity). If the reference entity defaults, the CDS seller compensates the CDS buyer for the loss in value of the reference obligation. In return, the CDS buyer pays a periodic premium to the CDS seller until a default occurs or the life of the CDS ends, whichever is earlier. The payments made by the CDS seller and buyer constitute the "protection leg" and the "premium leg", respectively. At inception of the CDS, the premium is commonly chosen such that the value of both legs is identical.

Schläfer and Uhrig-Homburg (2009) show that U.S. LCDS standard terms are for the most part comparable to CDS standard terms but differ in certain respects, two of which are of particular relevance. First, leveraged loans usually have no or only limited call protection and are thus likely to be pre-paid if a borrower's re-financing costs decline substantially. This means that the reference obligation may cease to exist prior to the end of the LCDS in which case the latter can terminate early. This usually comes at the expense of protection sellers: They retain limited upside if a borrower's credit situation improves but bear all the downside if it deteriorates. Ceteris paribus, this should result in higher premia. However, U.S. LCDS standard terms stipulate various covenants that restrict cancellation considerably and we disregard the topic for the purpose of this paper.[4] Second, restructuring does not constitute a credit event under U.S. LCDS standard terms but is eligible under CDS standard terms. This implies that default risk can be different for LCDSs and CDSs referencing the same issuer. We solve this issue by applying an adjustment factor that makes different credit event definitions comparable, as discussed in *Section 3.1*.

Assuming that the (L)CDS premium s is paid continuously, the value of the premium leg $P(s,\tau,T)$ is equal to the product of s and the price of an annuity $A(\tau,T)$ paying one until the reference entity defaults or the (L)CDS expires, whichever happens first:

$$P(s,\tau,T) = sA(\tau,T) \qquad (1)$$

where τ is the time of default and T is the term of the (L)CDS.

The value of the (L)CDS protection leg $PR(\rho,\tau,T)$ can be expressed as the present value of the expected compensation payment under the T-forward measure Q:

$$PR(\rho,\tau,T) = E^{Q}\left[(1-\rho)1_{\{\tau<T\}}\right]B(T) \qquad (2)$$

where ρ is the reference obligation's default-conditional recovery rate at time T,[5] $1_{\{\tau<T\}}$ is an indicator function that equals one if a default occurs prior to T, and $B(T)$ is the current price of the default-free zero coupon bond with maturity T.

[4] Under U.S. standard terms, LCDSs cancel only if no substitute reference obligation exists. This is likely to be the case only if the reference entity prepays all its leveraged loans at the same time, for instance due to a merger or move to investment grade status (cf. Schläfer and Uhrig-Homburg (2009)).

[5] This implicitly assumes that recovery of treasury applies. Bakshi, Madan, and Zhang (2006) test several recovery assumptions and find that recovery of treasury indeed matches market prices best. However, Guha (2003) observes that defaulted bonds of the same issuer and the same seniority mostly trade at very similar prices regardless of maturities and finds that only recovery of face value can explain this pattern satisfactorily.

Rearranging *Eq. 2* shows that the value of the protection leg is equal to the product of loss given default, the probability of default, and the price of the risk-free zero coupon bond:

$$\begin{aligned} PR(\rho,\tau,T) &= E^Q\left[(1-\rho)1_{\{\tau<T\}}|1_{\{\tau<T\}}=1\right]Q\left(1_{\{\tau<T\}}=1\right)B(T) \\ &\quad + E^Q\left[(1-\rho)1_{\{\tau<T\}}|1_{\{\tau<T\}}=0\right]Q\left(1_{\{\tau<T\}}=0\right)B(T) \\ &= \left(1-E^Q\left[\rho\,|1_{\{\tau<T\}}=1\right]\right)Q\left(1_{\{\tau<T\}}=1\right)B(T). \end{aligned} \tag{3}$$

At inception, s is thus given by

$$s = \frac{PR(\rho,\tau,T)}{A(\tau,T)} = \frac{\left(1-E^Q\left[\rho\,|1_{\{\tau<T\}}=1\right]\right)Q\left(1_{\{\tau<T\}}=1\right)B(T)}{A(\tau,T)}. \tag{4}$$

For our analysis we form pairs of i) LCDSs vs. CDSs on senior unsecured bonds and ii) CDSs on senior unsecured bonds vs. CDSs on senior subordinated bonds. In either case, both instruments have identical terms, identical credit event definitions (after adjusting for restructuring), reference the same firm but refer to different types of debt of that firm. Hence, both have different default-conditional recovery rates but identical probabilities of default. Let s_{loan}, s_{uns} and s_{sub} denote observed LCDS premia, senior unsecured CDS premia, and senior subordinated CDS premia, respectively. From *Eq. 4* it then follows that ratios s_{loan}/s_{uns}, denoted by R_1 and s_{uns}/s_{sub}, denoted by R_2, are solely functions of the recovery rates of respective reference obligations, denoted by ρ_{loan}, ρ_{uns} and ρ_{sub}:

$$R_1 = \frac{s_{loan}}{s_{uns}} = \frac{1-E^Q\left[\rho_{loan}\,|1_{\{\tau<T\}}=1\right]}{1-E^Q\left[\rho_{uns}\,|1_{\{\tau<T\}}=1\right]}, \tag{5a}$$

$$R_2 = \frac{s_{uns}}{s_{sub}} = \frac{1-E^Q\left[\rho_{uns}\,|1_{\{\tau<T\}}=1\right]}{1-E^Q\left[\rho_{sub}\,|1_{\{\tau<T\}}=1\right]}. \tag{5b}$$

2.2 The Link to Capital Structure

According to the absolute priority rule (APR), claims under a certain liability are satisfied only if all claims that are relatively senior have been satisfied *in full*. If the APR holds, recovery rates are a function only of the ratio of firm value to total liabilities, denoted by x, and capital structure, both at the time of default.[6] We proxy capital structure by the share that

[6] Deviations from absolute priority of debt claims over equity claims have become very rare for publicly traded firms (c.f. Baird, Bris, and Zhu (2007) and Bharath and Panchapegesan (2007)). APR violations within different classes of debt are however less well-researched.

senior secured loans, senior secured bonds, senior unsecured bonds[7], and senior subordinated bonds constitute of total liabilities and denote the respective percentages by $loan$, sec, uns, and sub. As no debt other than the mentioned is included in total liabilities, it always holds that $loan + sec + uns + sub = 1$.

Assuming that the ratio of firm value to liabilities at default is between zero and e percent of total liabilities, i.e. $x \in]0, e[$ and that debt holders cannot recover more than 100%, the firm-wide recovery rate ρ_{firm} and instrument-specific recovery rates ρ_{loan}, ρ_{uns}, and ρ_{sub} are given by

$$\rho_{firm}(x) = \begin{cases} x & x \in]0,1] \\ 1 & x \in]1, e[\end{cases} \tag{6a}$$

$$\rho_{loan}(x) = \begin{cases} \dfrac{x}{loan} & x \in]0, a] \\ 1 & x \in]a, e[\end{cases} \tag{6b}$$

$$\rho_{uns}(x) = \begin{cases} 0 & x \in]0, b] \\ \dfrac{(x-b)}{uns} & x \in]b, c] \\ 1 & x \in]c, e[\end{cases} \tag{6c}$$

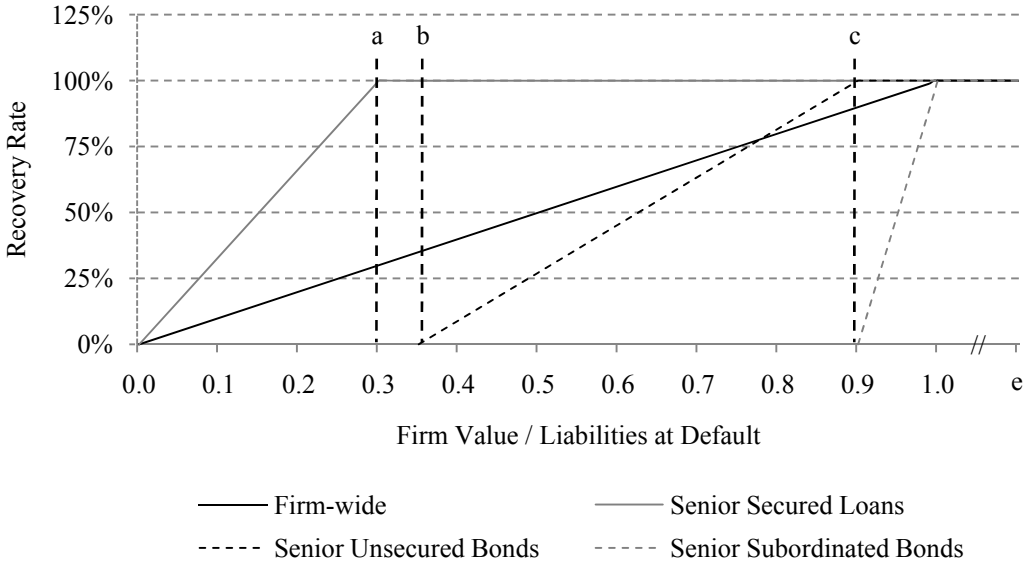
$$\rho_{sub}(x) = \begin{cases} 0 & x \in]0, c] \\ \dfrac{(x-c)}{sub} & x \in]c, 1] \\ 1 & x \in]1, e[\end{cases} \tag{6d}$$

for $e \geq 1$, where $a = loan$, $b = a + sec$, and $c = b + uns$.

Figure 1 illustrates this for a borrower with $loan = 30\%$, $sec = 5\%$, $uns = 55\%$, and $sub = 10\%$. If $0 < x < a = 30\%$, senior secured loan-holders recover only a fraction of their claims and bond-holders receive nothing. If $x \geq a$, senior secured loan-holders recover 100%. Holders of senior unsecured bonds receive proceeds if $x > b = 35\%$ and recover 100% if $x \geq c = 90\%$. For holders of senior subordinated bonds, the relevant barriers are 90% and 100%. The firm-wide recovery rate is equal to x but does not exceed 100%.

[7] We use "senior unsecured bond" as a general term for all senior unsecured debt. This also includes senior unsecured loans and notes as well as certain other non-debt items that are generally treated as pari passu to senior unsecured debt, as discussed in *Section 3.2*.

Figure 1: Recovery Rates Given Default



This figure illustrates the relationships between the ratio of firm value to liabilities at default and recovery rates for the entire firm, senior secured loans, senior unsecured bonds, and senior subordinated bonds. It is assumed that total liabilities consist of 30% senior secured loans, 5% senior secured bonds, 55% senior unsecured bonds, and 10 % senior subordinated bonds and that the APR holds. Results are obtained using *Eqs. 6*.

For any given implied probability density function of recovery given default $h(x)$, implied expected recovery rates and the variance of implied recovery rates are given by:

$$E^Q\left[\rho \,|1_{\{\tau<T\}} = 1\right] = \int_0^e \rho(x)h(x)dx, \tag{7}$$

$$Var^Q\left[\rho \,|1_{\{\tau<T\}} = 1\right] = \int_0^e \left(\rho(x) - \int_0^e \rho(x)h(x)dx\right)^2 h(x)dx\,. \tag{8}$$

Substituting *Eqs. 6b – 6d* and *7* in *Eqs. 5* permits expressing model-implied ratios s_{loan}/s_{uns}, denoted by $\bar{R}_1$ and s_{uns}/s_{sub}, denoted by $\bar{R}_2$, as functions of the borrower's capital structure and $h(x)$:

$$\bar{R}_1 = \frac{1 - \int_0^a \frac{x}{loan} h(x)dx + \int_a^e h(x)dx}{1 - \int_b^c \frac{(x-b)}{uns} h(x)dx + \int_c^e h(x)dx}, \tag{9a}$$

$$\bar{R}_2 = \frac{1 - \int_b^c \frac{(x-b)}{uns} h(x)dx + \int_c^e h(x)dx}{1 - \int_c^1 \frac{(x-c)}{sub} h(x)dx + \int_1^e h(x)dx}. \tag{9b}$$

We can now estimate $h(x)$ by calibrating model-implied ratios $\overline{R}$ to actual ratios R. We pursue a parametric approach and thus need to specify the functional form of $h(x)$. Therefore, it is examined next what requirements such specification should fulfill.

2.3 The Implied Probability Distribution of Recovery

By definition, recovery rates are strictly non-negative and cannot assume arbitrarily high values. Based on Moody's Ultimate Recovery Database, Cantor, Emery, and Stumpp (2006) and Emery (2007) find that historical realizations of the ratio of firm value to liabilities at default lie mostly between zero and 100% with very few observations reaching values as high as 120%. Ratios above 100% can occur if a firm chooses to strategically default, for instance to obtain relief from lenders and regulators. In such case, debt holders recover 100% with equity holders receiving the remainder. As these instances are, however, quite rare, we assume x to have support in the unit interval and require that $\int_0^1 h(x)dx = 1$. At default, the firm-wide recovery rate is then equal to the ratio of firm value to liabilities at default, i.e. $\rho_{firm} = x, x \in]0,1[$, and it follows that the expected firm-wide recovery rate is equal to the mean of $h(x)$, i.e. $E^Q[\rho_{firm}|1_{\{\tau<T\}} = 1] = \int_0^1 xh(x)dx$. The same is true with respect to the variance, i.e. $Var^Q[\rho_{firm}|1_{\{\tau<T\}} = 1] = \int_0^1 \left(x - \int_0^1 xh(x)dx\right)\text{^}2\, h(x)dx$.

We follow the approach of Madan and Unal (1998), Gaspar and Slinko (2008), and rating agencies such as Moody's (cf. Gupton and Stein (2002) and Cantor, Emery, and Stumpp (2006)) and model recovery rates using a beta distribution. Beta distributions are bounded on both sides, can assume a variety of shapes and are fully specified by their first two moments. Assuming the lower and upper bound to be zero and unity, respectively, the density function of the beta distribution is given by

$$bet(x) = \frac{x^{p-1}(1-x)^{q-1}}{\int_0^1 y^{p-1}(1-y)^{q-1}dy} \quad p, q \in]0, \infty[\tag{10}$$

where p and q are shape parameters.

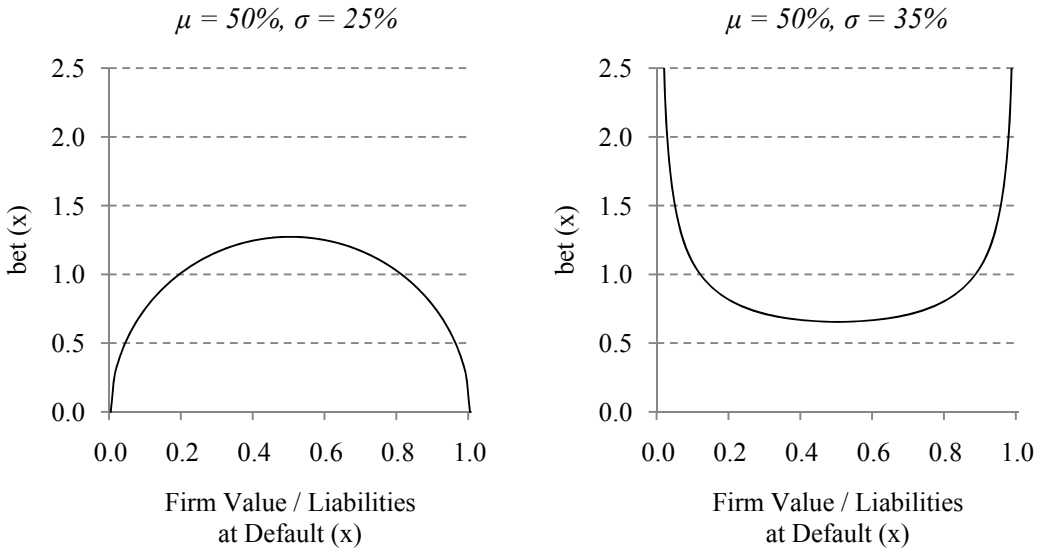
As shown in *Appendix A*, the supremum and infimum of the standard deviation of the beta distribution for a given mean are

$$\sigma_{sup} = \sqrt{\mu - \mu^2} \quad \mu \in]0,1[, \tag{11a}$$

$$\sigma_{inf} = 0. \tag{11b}$$

The first two moments of the beta distribution are thus related: As the mean approaches zero or unity, the standard deviation approaches zero. The highest possible standard deviation is 50% and requires that $\mu = 50\%$. For relatively small standard deviations, distributions are approximately bell-shaped. As the standard deviation increases, probability masses concentrate on either side and distributions eventually assume a U-shape. In this case, realizations of x close to zero or unity are more likely than realizations in between and probability densities approach infinity at endpoints. *Figure 2* illustrates this for $\mu = 50\%$.

Figure 2: Exemplary Densities of the Beta Distribution



This figure illustrates exemplary probability densities of a beta distribution with support in the unit interval, assuming $\mu = 50\%$, $\sigma = 25\%$ (left chart) and $\mu = 50\%$, $\sigma = 35\%$ (right chart). Results are obtained using *Eq. 10*.

3. Data and Descriptive Statistics

3.1 CDS and LCDS Premia

We use weekly, mid-market premia of USD-denominated (L)CDSs with a maturity of 5 years. LCDS premia are obtained from a leading investment bank, and CDS premia are obtained from Markit. We search for firms on which either i) LCDSs and senior unsecured CDSs or ii) senior unsecured CDSs and senior subordinated CDSs are outstanding at the same time. Further, we require these instruments to be USD-denominated as European LCDS standard terms differ substantially from U.S. LCDS standard terms and are less com-

parable to CDS standard terms.[8] To assure comparability of our capital structure analyses, we exclude financial institutions and require that firms report according to US-GAAP.

Table 1 shows an overview of resulting samples: Sample 1 (LCDSs and senior unsecured CDSs) comprises 20 U.S. firms, all with a sub-investment grade rating.[9] Data lie between May-2006 and Jul-2008, and we observe on average 64 pairs of premia for each firm. Due to the relative newness of the LCDS market, earlier information is not available or of limited quality. Sample 2 (senior unsecured CDSs and senior subordinated CDSs) comprises 17 U.S. firms, two of which have an investment-grade rating. Data lie between Jan-2001 and Dec-2007, and we observe on average 162 pairs of premia for each firm. Constituents of both samples belong to a variety of industries including consumer goods & services, technology, industrial, media, automotive, and healthcare. For a breakdown of key statistics by firm, refer to *Appendix B*.

Table 1: Overview of Samples

	Sample 1	Sample 2
Instruments	LCDSs, senior unsecured CDSs	Senior unsecured CDSs, senior subordinated CDSs
Constituents	20 U.S. companies	17 U.S. companies
Rating (IG/sub-IG)	0/20	2/12
First - Last Observation	May-2006 - Jul-2008	Jan-2001 - Dec-2007
Average Count	64	162

This table shows an overview of the two samples on which all analyses in this paper are based. Ratings are Moody's corporate family ratings.

As mentioned earlier, restructuring does not constitute a credit event under U.S. LCDS standard terms but is eligible under CDS standard terms. We find that approximately half the CDSs in our samples stipulate "modified restructuring", the other half exclude restructuring. Markit uses adjustment factors to make CDS quotes that are based on different restructuring definitions comparable.[10] We follow this approach and divide by 1.0565 to adjust "modified restructuring" to "no restructuring". This is in close agreement with Berndt, Jarrow, and Kang (2007) who investigate the price of including restructuring as a default event and find that CDS premia stipulating "modified restructuring" are on average 5.69% higher than those excluding restructuring.

[8] Cf. Schläfer and Uhrig-Homburg (2009).

[9] Remember that, by definition, leveraged loans are issued by sub-investment grade borrowers.

[10] Factors to adjust back to "old restructuring" are 1.0417 for "modified-modified restructuring", 1.0695 for "modified restructuring", and 1.1299 for "no restructuring".

Table 2 gives an overview of premia by sample, averaged over all firms and the entire observation period. As one would expect, seniority is reversely related to premia: For Sample 1 constituents, the average LCDS premium is 309 BPs while the average senior unsecured CDS premium is 536 BPs. LCDS premia are thus on average just 58.1% of senior unsecured CDS premia or 227 BPs lower. For Sample 2 constituents, the average senior unsecured CDS premium is 183 BPs while the average senior subordinated CDS premium is 243 BPs. Senior unsecured CDS premia are thus just 75.8% of senior subordinated CDS premia or 60 BPs lower. For a breakdown of observations by firm, refer to *Appendix B*.

Table 2: Overview of Average Premia

	Sample 1 Constituents	Sample 2 Constituents
LCDSs	309 BPs	NA
Senior Unsecured CDSs	536 BPs	183 BPs
Senior Subordinated CDSs	NA	243 BPs
Ratio	58.1%	75.8%
Difference	227 BPs	60 BPs

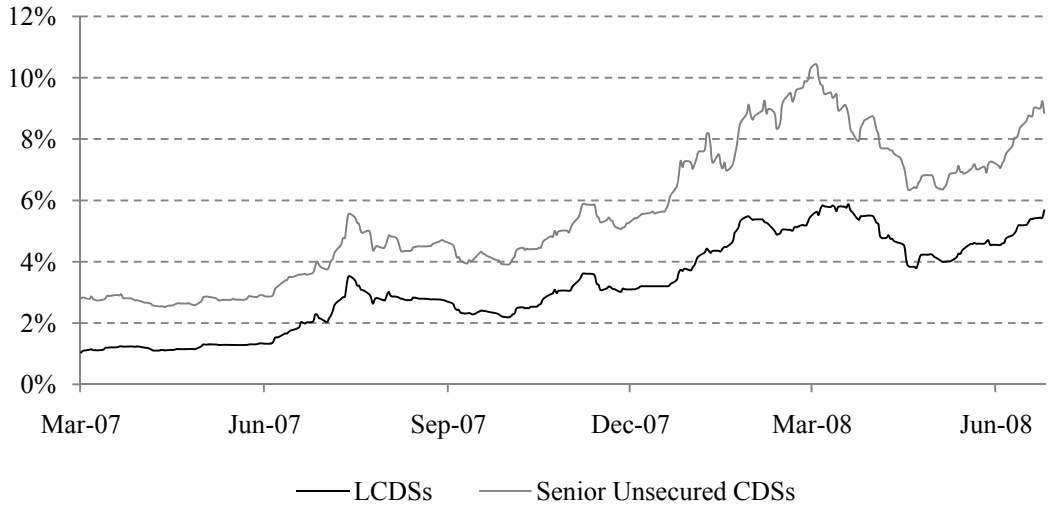
This table shows LCDS premia, senior unsecured CDS premia, senior subordinated CDS premia, ratios of premia (as defined by *Eqs. 5*), and differences of premia (defined as $s_{uns} - s_{loan}$ for Sample 1 constituents and as $s_{sub} - s_{uns}$ for Sample 2 constituents). Figures are averages over all firms and the entire observation period.

Interestingly, the average senior unsecured CDS premium is significantly higher for Sample 1 firms than for Sample 2 firms, the difference being 353 BPs. We identify three reasons for this observation: First, ratings are slightly better for Sample 2 firms, as mentioned earlier. Second, more than 70% of observations in Sample 1 lie after the outbreak of the crisis in summer 2007, while this figure is less than 10% for Sample 2. Third, we find a systematic difference in the capital structure across samples. As we shall see below, this difference results in higher implied expected recovery rates for senior unsecured bonds of Sample 2 firms than for those of Sample 1 firms, ceteris paribus.

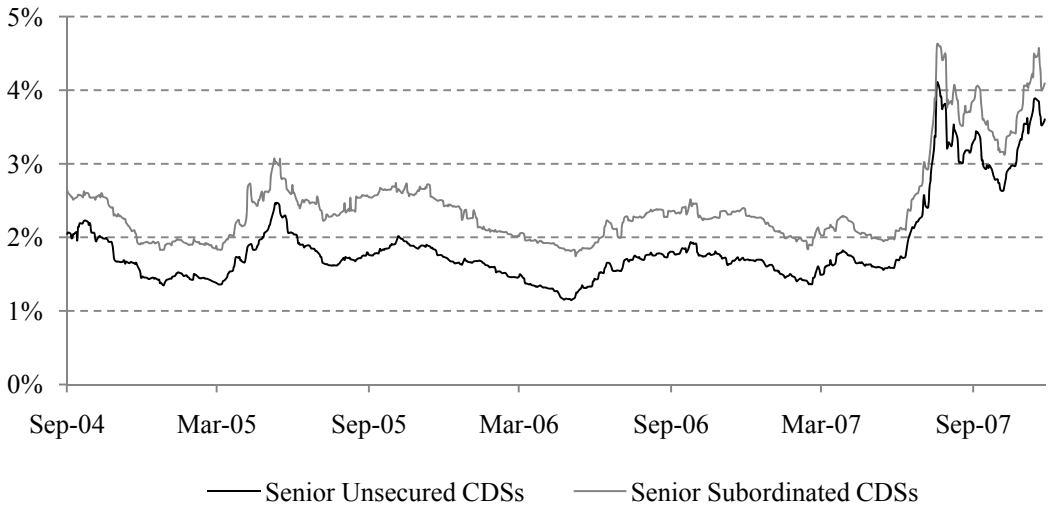
Figure 3 illustrates the evolution of average premia by type of debt. Shown are only the periods for which data for at least 10 firms is available. We observe a strong positive correlation of respective pairs and a sharp increase in premia since the outbreak of the crisis in summer 2007.

Figure 3: Evolution of Average Premia

Sample 1 Constituents



Sample 2 Constituents



This figure illustrates the evolution of LCDS premia and senior unsecured CDS premia for Sample 1 constituents and of senior unsecured CDS premia and senior subordinated CDS premia for Sample 2 constituents. Figures are averages over all firms. Shown are only the respective periods for which data for at least 10 firms are available.

3.2 Capital Structure Data

Based on the 10-K reports of the firms in our samples, we identify the percentage of senior secured loans, senior secured bonds, senior unsecured bonds, and senior subordinated bonds for each firm at various points in time. In senior secured loans we include secured term loans

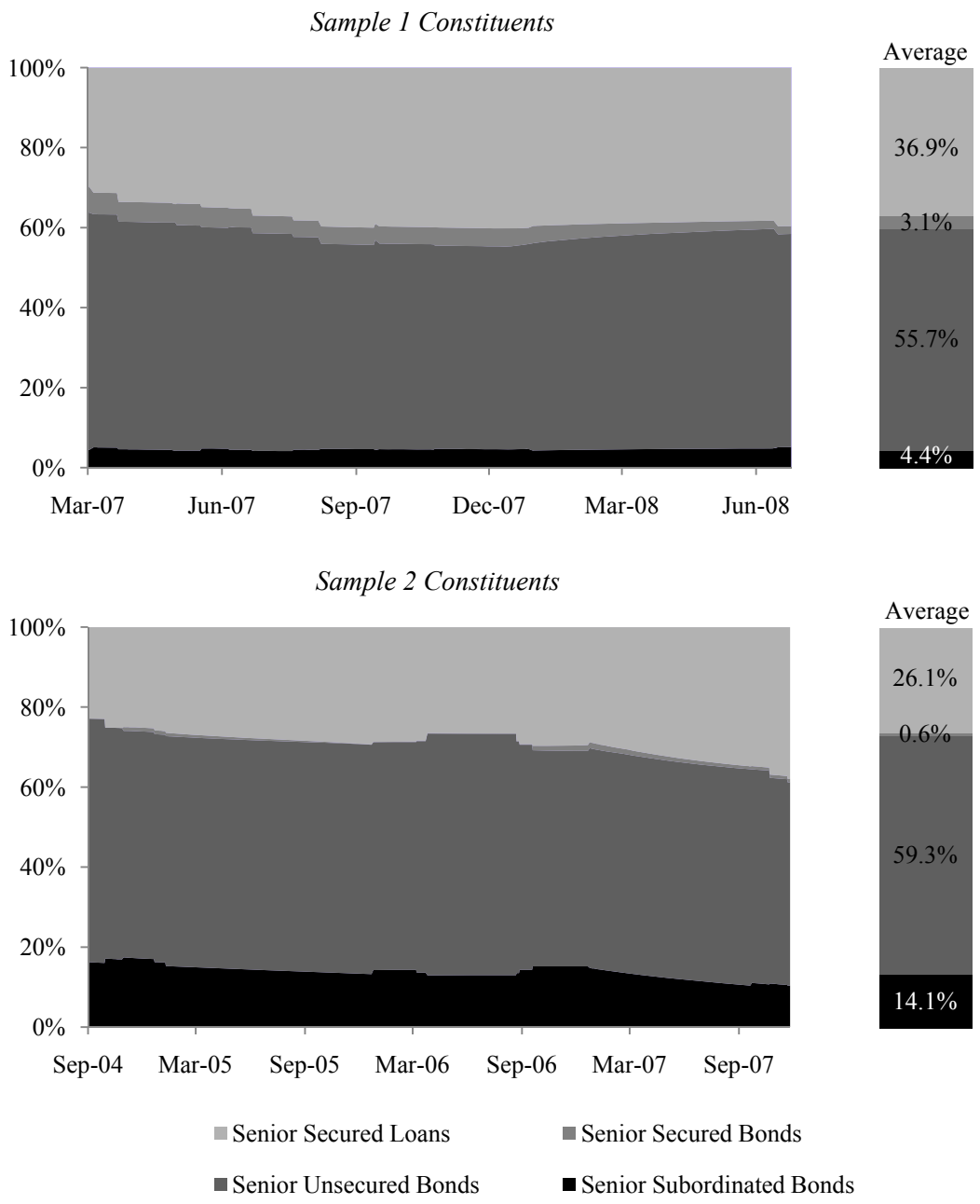
and secured revolving credit facilities. To provide a realistic estimate of the capital structure at default, we also take in undrawn revolving credit facilities as these are frequently triggered once a borrower faces financial distress. In senior unsecured bonds we include senior unsecured loans, bonds, and notes, as well as certain non-debt items that are generally treated as pari passu to senior unsecured debt. In particular, these are accounts payables, pension deficits in defined benefit schemes (i.e. projected benefit obligations less fair amount of plan assets) as well as operating and capital lease obligations.[11] For each firm, we conduct this analysis at each fiscal year-end, starting with the fiscal year that precedes the year in which the first (L)CDS premium is observed and ending with the fiscal year in which the last premium is observed. We then linearly interpolate to receive weekly figures.

Figure 4 illustrates the evolution of average capital structures by sample. In both samples, senior unsecured bonds are the most prevalent liability type, accounting on average for 55.7% and 59.3% of total liabilities in Sample 1 and Sample 2, respectively. Senior secured loans constitute a substantial lot as well (36.9% and 26.1%). In Sample 2, their share has increased significantly over time, mostly at the expense of senior unsecured bonds. This may be due to the fact that ratings of Sample 2 firms have generally declined throughout the observation period, making senior secured loans more important a source of funding. The use of senior subordinated bonds is more prevalent in Sample 2 than in Sample 1 (14.1% and 4.4%). The share of senior secured bonds is negligible in both samples (3.1% and 0.6%).

Note that there is a systematic bias in capital structures: As all Sample 1 firms have liquidly-traded LCDSs outstanding, it is likely that leveraged loans play a major role in the financing efforts of these firms. The same argument applies to Sample 2 firms with respect to senior subordinated bonds. This results in systematically different cushions for senior unsecured bonds: In Sample 1, on average 40.0% of liabilities are senior to senior unsecured bonds and only 4.4% are junior. For Sample 2, these figures are 26.7% and 14.1%, respectively.

[11] Cf. Solomon (2006).

Figure 4: Evolution of Average Capital Structures



This figure illustrates the evolution of average capital structures, measured by the share that senior secured loans, senior secured bonds, senior unsecured bonds, and senior subordinated bonds constitute of total liabilities. Shown are only the respective periods for which data for at least 10 firms is available but averages comprise all data.

4. EMPIRICAL SPECIFICATION

Past studies on defaulted corporate debt suggest that historically observed recovery rates can be explained to some extend by firm-specific, industry-specific, and macroeconomic factors: Altman, Brady, Resti, and Sironi (2005) find that recovery rates are positively related to GDP growth and S&P 500 stock returns. Rösch and Scheule (2005) document a positive relation to indicators that measure the health of the economy. Cantor and Varma (2005) obtain similar results and, in addition, examine the impact of a number of firm- and industry-specific factors. Inter alia, they find that recovery rates are negatively related to firm-specific financial leverage and the level of speculative-grade credit spreads, and positively related to proxies for firm and industry growth prospects, firm-specific asset tangibility, firm and industry stock returns, and industry capacity utilization. Acharya, Bharath, and Srinivasan (2007) concentrate on the impact of industry factors, in particular in conjunction with asset specificity. They show that recovery rates are lower if the issuer's industry is in distress, illiquid or highly leveraged, particularly so if that industry's assets are specific, i.e. of limited use to other industries.

It is likely that variables that explain historical recovery rates to some extent also drive implied recovery rates. We therefore allow the mean of the implied probability distribution of recovery to depend on factors that have proved relevant for explaining historical recovery rates. In particular, we consider the following four firm-specific factors: i) *Financial leverage* (F_Lev), the ratio of long-term debt to total assets. High financial leverage implies that assets need to be shared among more debt-holders in the event of default. Further, Acharya et al. argue that high leverage may be associated with a greater dispersion of ownership, resulting in a more complex and lengthy resolution of bankruptcy proceedings.[12] Both effects should result in lower recovery rates in default; ii) *Asset tangibility* (F_Tan), the ratio of hard assets (proxied by property, plant, and equipment) to total assets. Cantor and Varma argue that firms with a high percentage of hard (i.e. likely to be revenue-producing and therefore more easily sellable) assets should achieve higher recovery rates in default; iii) *Interest coverage ratio* (F_IntCov), the ratio of EBITDA to interest expenses. Firms with high interest coverage dispose of assets that generate high earnings relative to interest expenses. In default, a liquidation of such assets should result in higher recovery rates; and iv) *Quick ratio* (F_Quick), the ratio of current assets minus inventories to current liabilities. In default, firms with a high quick ratio should be able to repay a higher share of their current liabilities out of their liquid current assets.

[12] However, Acharya, Bharath, and Srinivasan, (2007) also note that in several occasions, highly leveraged transactions were particularly easily restructured.

Further, we consider three industry-specific factors suggested by Acharya et al.: i) *Industry distress* (I_Diss), a dummy variable that takes the value 1 if the median 12-months stock return for the issuer's industry is less than -30%. Acharya et al. argue that industry distress is indicative of a downturn in the economic prospects of an industry and therefore associated with a reduction in the value of firms' assets and hence with lower recovery rates in default. They also test continuous, un-truncated industry equity returns but find that these do not possess explanatory power, suggesting that the effect of industry equity returns on recovery rates is essentially non-linear and restricted to situations where the industry is in distress; ii) *Industry illiquidity* (I_Illiq), the median inverse quick ratio of the issuer's industry;[13] and iii) *Industry financial leverage* (I_Lev), the median financial leverage of the issuer's industry. The latter two metrics are indicative of the financial condition of an issuer's peer firms. If this condition is delicate, the demand for the issuer's assets in the event of a default should be impaired, and recovery rates would suffer accordingly.

We determine the industry of each firm using 3 digit SIC codes, as reported in *Appendix B*. For each distinct code, the respective industry is then proxied by selecting the ten firms with the largest market capitalization that have the same SIC code. Accounting data for firm- and industry-specific metrics are obtained from COMPUSTAT and companies' 10-K reports. Metrics are calculated based on fiscal year-end data and then interpolated to receive weekly figures.

Mentioned accounting metrics potentially capture differences that might exist between implied recovery rates of individual firms or industries. However, they are less qualified to reflect the evolution of implied recovery rates over time. For that end, macroeconomic variables are more apt. We therefore consider as final explanatory variable the level of the *CDX High-Yield* (CDX), an index of CDS premia published by Markit that includes 100 equally-weighted, non-investment grade U.S. borrowers. If higher levels of the CDX imply higher recovery risk, they should be associated with lower implied recovery rates. We do not consider GDP growth and S&P 500 returns, two other macroeconomic indicators that have proved useful for explaining historical recovery rates, as discussed earlier. GDP data is available on a quarterly basis only which conflicts with the relative brevity of our observation period. Equity returns are indirectly factored in through our proxy for industry distress.

Table 3 shows summary statistics of all explanatory variables.

[13] Acharya, Bharath, and Srinivasan, (2007) also implement an alternative definition of industry illiquidity, the median inverse interest coverage ratio, and obtain similar results.

Table 3: Overview of Explanatory Variables

	Count	Average	Stdev.	Min.	25th Percentile	Median	75th Percentile	Max.
F_Lev	5,229	0.531	0.712	0.046	0.280	0.403	0.583	6.408
F_Tan	5,229	0.355	0.216	0.009	0.217	0.358	0.522	0.803
F_IntCov	5,229	5.238	3.027	0.000	2.920	4.400	8.089	10.000
F_Quick	5,229	0.926	0.431	0.055	0.695	0.823	1.067	2.608
I_Diss	6,873	0.045	0.207	0.000	0.000	0.000	0.000	1.000
I_Illiq	7,585	1.025	0.519	0.186	0.722	0.970	1.235	5.403
I_Lev	7,861	0.247	0.141	0.000	0.145	0.220	0.327	0.716
CDX	394	1.498	0.636	0.686	1.048	1.237	1.777	3.963

This table shows summary statistics of the firm-specific, industry-specific, and macroeconomic factors that serve as explanatory variables for modelling the mean and standard deviation of the implied probability distribution of recovery. Firm-specific factors are i) *Financial leverage* (F_Lev), the ratio of long-term debt to total assets, ii) *Asset tangibility* (F_Tan), the ratio of hard assets (proxied by property, plant, and equipment) to total assets, iii) *Interest coverage ratio* (F_IntCov), the ratio of EBITDA to interest expenses (for negative EBITDA (negative interest expenses) the interest coverage ratio is set to 0x (10x), otherwise, the ratio is capped at 10x), and iv) *Quick ratio* (F_Quick), the ratio of current assets minus inventories to current liabilities. Industry-specific factors are i) *Industry distress* (I_Diss), a dummy variable that takes the value 1 if the median 12 months stock return for the issuer's industry is less than -30%, ii) *Industry illiquidity* (I_Illiq), the median inverse quick ratio of the issuer's industry, and iii) *Industry financial leverage* (I_Lev), the median financial leverage of the issuer's industry. The *CDX High-Yield* (CDX), an index of CDS premia published by Markit that includes 100 equally-weighted, non-investment grade U.S. borrowers is chosen as macroeconomic factor. As the CDX is not available for the earlier years of the observation period, we extrapolate using its initial set of constituents. The time series is scaled to 100% of its initial value. Accounting data for firm- and industry-specific metrics are obtained from COMPUSTAT and companies' 10-K reports. Metrics are calculated based on fiscal year-end results and then interpolated to receive weekly figures.

In summary, the mean μ of the implied probability distribution of recovery at time t and for firm n is given by

$$\mu_{t,n} = \beta_0 + \beta_1 F_Lev_{t,n} + \beta_2 F_Tan_{t,n} + \beta_3 F_IntCov_{t,n} + \beta_4 F_Quick_{t,n} \quad (12)$$
$$+\beta_5 I_Diss_{t,n} + \beta_6 I_Illiq_{t,n} + \beta_7 I_Lev_{t,n} + \beta_8 CDX_t$$

where $\beta_0, \dots, \beta_8$ are constant parameters.

There is no theoretical reason why the standard deviation of implied recovery rates should be constant over time. Rather, it might be higher in times of high uncertainty and vice versa. To account for this, we model the standard deviation as a function of the CDX. If higher levels of the CDX are indicative of higher uncertainty, they should be associated with a higher standard deviation. Such approach is, however, complicated by the fact that the first two moments of the beta distribution are related, as mentioned earlier. For instance, a particular standard deviation can be relatively high or even unattainable if the mean of the distribution

is, say, very low or it can be relatively low if the mean is close to 50%. Therefore, instead of modeling the absolute standard deviation directly, we model the excess standard deviation over the minimum standard deviation, denoted by $\sigma_{prem,t}$, and have the nice property that $\sigma_{prem,t}$ always lies in the unit interval and, being a function only of the CDX, is identical across all firms. Using *Eqs. 11*, the absolute standard deviation σ at time t and for firm n is given by is then given by

$$\begin{aligned}\sigma_{t,n} &= \left(\sigma_{sup,t,n} - \sigma_{inf,t,n}\right)\sigma_{prem,t} + \sigma_{inf,t,n} \\ &= \sigma_{sup,t,n}\,\sigma_{prem,t} \quad \sigma_{prem,t} \in \,]0,1[\end{aligned} \tag{13}$$

where

$$\sigma_{prem,t} = \gamma_0 + \gamma_1 CDX_t \tag{14}$$

and γ_0, γ_1 are constant parameters.

With the mean and the standard deviation of the implied distribution of recovery being specified by *Eqs. 12* and *14*, the model-implied ratios $\bar{R}_{1,t,n}$ and $\bar{R}_{2,t,n}$ are a function of time-dependent firm-specific capital structure variables, time-dependent firm- and industry-specific accounting metrics, the level of the CDX, and unknown, constant parameters $\gamma_0, \gamma_1, \beta_0, \ldots, \beta_8$. The latter are estimated by least squares, i.e. we minimize the sum of squared differences between actual and model-implied ratios over the entire observation period and over all N firms:

$$RMSE = \min_{\gamma_0,\gamma_1,\beta_0,\ldots,\beta_8} \sqrt{\frac{\sum_{n=1}^{N}\sum_{t=1}^{T_n}\left(\left(R_{1,t,n} - \bar{R}_{1,t,n}\right)^2 + \left(R_{2,t,n} - \bar{R}_{2,t,n}\right)^2\right)}{\sum_{n=1}^{N} T_n}} \tag{15}$$

where T_n denotes the number of observations for firm n.

Table 4 shows estimation results for parameters $\gamma_0, \gamma_1, \beta_0, \ldots, \beta_8$ and associated test statistics. Without exception, signs of coefficients are as expected: Financial leverage (both, firm- and industry-specific), industry illiquidity, industry distress, and the CDX are negatively related to the mean of the implied probability distribution. For instance, if an industry is in distress or the level of the CDX doubles, implied expected firm-wide recovery rates decrease 2.6% and 3.1% on average, ceteris paribus. Asset tangibility, interest coverage, and the quick ratio are positively related to the mean. Further, the CDX is positively related to the excess standard deviation $\left(\sigma_{prem}\right)$. Due to the interrelation of μ and σ discussed earlier, this does, however, not imply that the absolute standard deviation must always increase if the CDX de-

creases. We examine the evolution of the absolute standard deviation over time in the *Section 5.3*.

With the exception of asset tangibility and industry-specific financial leverage, all coefficients are significant at the 5% confidence level or higher. The RMSE is 0.137 and the adjusted R^2 is 0.330. In absolute terms this is quite low, indicating that the explanatory variables fail to capture much of the week-to-week variation in observed ratios.

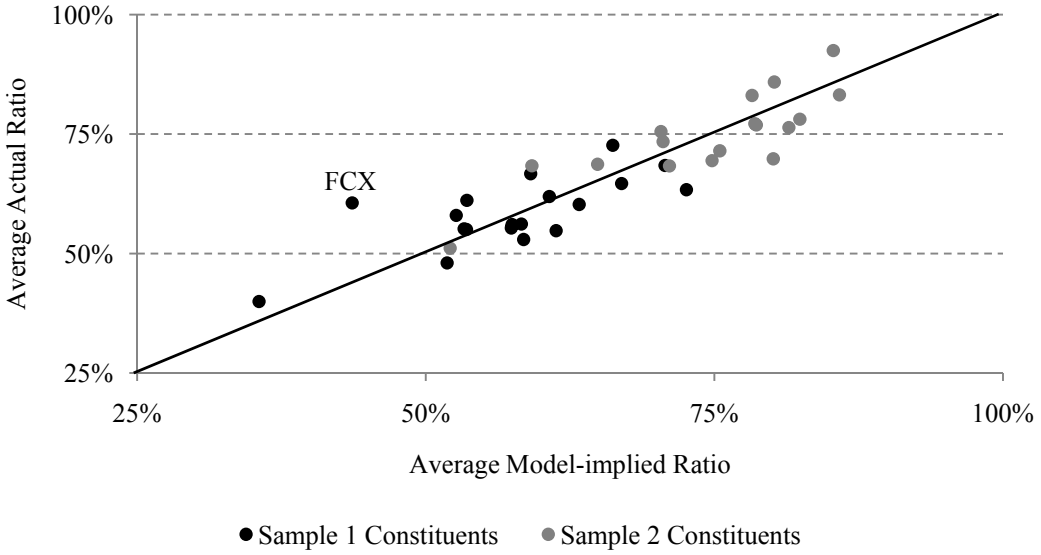
Table 4: Estimation Results for the Implied Distribution of Recovery

	β_0	β_1 (F_Lev)	β_2 (F_Tan)	β_3 (F_IntCov)	β_4 (F_Quick)	β_5 (I_Diss)	β_6 (I_Illiq)	β_7 (I_Lev)	β_8 (CDX)
Coeff.	0.3557***	-0.0101**	0.0031	0.0052***	0.0298***	-0.0257**	-0.0214***	-0.0217	-0.0312***
Std. Err.	0.0162	0.0043	0.0152	0.0011	0.0066	0.0119	0.0069	0.0176	0.0057
	γ_0	γ_1 (CDX)							
Coeff.	0.6308***	0.0431***							
Std. Err.	0.0106	0.0073				RMSE	0.137	Adj. R^2	0.330

This table shows estimation results for the implied probability distribution of recovery. It is assumed that implied recovery rates follow a beta distribution. The mean of this distribution is modelled as a function of financial leverage (F_Lev), asset tangibility (F_Tan), interest coverage ratio (F_IntCov), quick ratio (F_Quick), industry distress (I_Diss), industry illiquidity (I_Illiq), industry financial leverage (I_Lev), and the CDX (*Eq. 12)*. The excess standard deviation of the distribution is modelled as a function of the CDX (*Eq. 14)*. Standard errors for $\beta_0, \dots, \beta_8$ are conditional on estimates of γ_0, γ_1 and vice versa. *** and ** indicate significance at the 1% and 5% confidence level, respectively. RMSE is obtained using *Eq. 15*.

The model does, however, fare very well when it comes to explaining the general level of actual ratios as opposed to their week-to-week variation. This is true for constituents of either sample. To see this, we compare firms' average model-implied ratios (independent variable) to average actual ratios (dependent variable). As *Figure 5* illustrates, pairs of ratios lie close to the 45-degree line, one exception being the pair for Freeport McMoran (FCX). If FCX is disregarded, the hypothesis that the ordinary least squares line has an intercept equal to zero and a slope equal to one cannot be rejected at the 10% confidence level. The adjusted R^2 is 0.816.

Figure 5: Average Model-implied vs. Average Actual Ratios



This figure illustrates the relation of average model-implied ratios and average observed ratios. Figures are averages over the respective observation period.

5. ESTIMATES OF MARKET-IMPLIED RECOVERY RATES

5.1 Implied Firm-wide and Industry-specific Recovery Rates

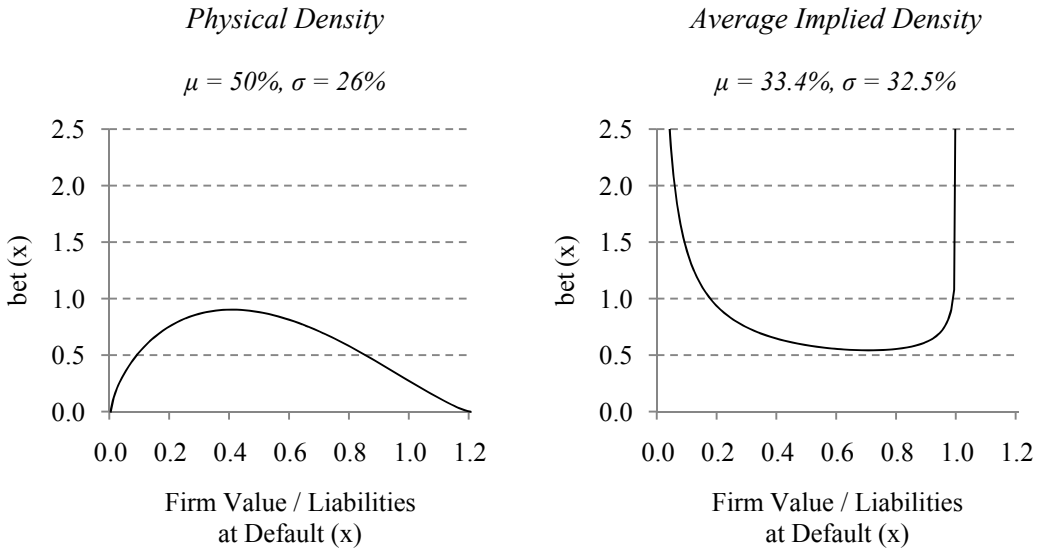
Based on estimated model parameters $\gamma_0, \gamma_1, \beta_0, \dots, \beta_8$ and issuers' capital structure data, we can now calculate implied expected firm-wide and instrument-specific recovery rates and the standard deviation of implied recovery rates for each firm and for each week. *Appendix C* lists results for each firm, averaged over the respective observation period and *Table 5* further aggregates figures over all firms. The average implied expected firm-wide recovery rate is 33.4%. Maximum and minimum results vary notably, mostly due to observation periods differing from firm to firm[14], but firm-and industry-specific factors are of relevance, as well.

Cantor, Emery, and Stumpp (2006) find that the dispersion of historically realized ratios of firm value to liabilities at default is well-described by a beta distribution that is restricted to $[0,1.2]$ with an average firm-wide recovery rate of 50% and a standard deviation of 26%. It is insightful to visualize the density of this distribution and to compare it to the implied density of recovery. The latter is constructed from the average implied expected firm-wide re-

[14] For instance, observations for Solectron (average firm-wide recovery rate: 41.0%) all lie prior to the outbreak of the credit crisis in July 2007 whereas observations for MichaelStores (average firm-wide recovery rate: 22.7%) all lie thereafter. We examine the evolution of implied expected recovery rates over time in *Section 5.3*.

covery rate (33.4%) and the average standard deviation of implied firm-wide recovery rates (32.5%). *Figure 6* illustrates that while the physical density is bell-shaped, the implied density is approximately U-shaped (due to its higher standard deviation) with much of the probability mass concentrating at the lower bound.

Figure 6: Physical vs. Average Implied Densities of Recovery



This figure illustrates the shape of the probability density of historically observed ratios of firm value to liabilities at default (Cantor, Emery, and Stumpp (2006)), taken as a proxy for the physical density of firm-wide recovery rates. Also shown is the implied probability density of recovery rates, obtained by averaging over all firms and the entire observation period. In both cases, it is assumed that recovery rates follow a beta distribution.

This indicates that ex ante there is high uncertainty as to how much borrowers will recover, should a default occur. This finding does not come at much of a surprise: Studies on the determinants of historically observed recovery rates show that a substantial share of their variation remains unexplained, even if a broad range of issue-, firm-, and industry-specific factors as well as macroeconomic indicators are employed as explanatory variables.[15] Further, note that the mean of the implied distribution is much lower than that of its physical counterpart. This suggests that investors require a premium for taking recovery risk, a topic we examine more closely in *Section* 5.*5.2*.

[15] Depending on regression model specifications, R^2s obtained by some of these studies are: Keisman, Van de Castle, and Yang (2000): 37 – 48%, Covitz and Han (2004): 33 – 44%, Cantor and Varma (2005): 53 – 59%, Chava, Stefanescu, and Turnbull (2006): 12 – 27%, Acharya, Bharath, and Srinivasan (2007): 51 – 68%.

There is ample empirical evidence for *physical* instrument-specific recovery rates varying greatly across seniorities (c.f. Altman and Kishore (1996), Altman, Resti, and Sironi (2004), Kelhoffer, Schwartz, and Zennario (2005), Altman and Pasternack (2006), Emery and Ou (2009), and others). For instance, Emery and Ou (2009) analyze Moody's recovery data on corporate issuers that defaulted between 1982 and 2008 and find that average issuer-weighted recovery rates, as measured by trading prices 30 days after default, are 69.9% for senior secured loans, 36.4% for senior unsecured bonds, and 31.7% for senior subordinated bonds. Studies investigating S&P data obtain similar results (c.f. Kelhoffer, Schwartz, and Zennario (2005)).

Table 5 shows that seniority is similarly important a characteristic for *implied* instrument-specific recovery rates, average estimates being 53.7% for senior secured loans, 24.0% for senior unsecured bonds, and 5.8% for senior subordinated bonds. Again, implied figures are significantly below historical averages, further substantiating the hypothesis that there is a risk premium in implied recovery rates. It is, however, striking that the gap is particularly large for senior subordinated bonds (physical: 31.7%, implied: 5.8%), both in absolute as well as in relative terms. We investigate the cause for this in *Section 5.5.2* and find the extraordinarily high standard deviation of physical senior subordinated recovery rates to be responsible.

Table 5: Implied Firm-wide and Instrument-specific Recovery Rates

	Firm-wide	Senior Secured Loans	Senior Unsecured Bonds	Senior Subordinated Bonds
Implied Expected RR				
Average	33.4%	53.7%	24.0%	5.8%
Median	33.3%	53.4%	22.8%	4.7%
Max.	41.0%	75.7%	42.9%	11.2%
Min.	22.7%	36.3%	10.0%	1.4%
Avg. Stdev. of Implied RRs	32.5%	41.4%	34.3%	18.4%

This table shows average, median, maximum, and minimum implied expected recovery rates for the entire firm, senior secured loans, senior unsecured bonds, and senior subordinated bonds as well as average standard deviations of respective recovery rates. Estimates of implied expected recovery rates are obtained using *Eqs. 6* and *7*. Estimates of the standard deviation of implied recovery rates are obtained using *Eqs. 6* and *8*.

Estimates of instrument-specific recovery rates vary widely across firms, too (36.3% to 75.7% for senior secured loans, 10.0% to 42.9% for senior unsecured bonds, 1.4% to 11.2% for senior subordinated bonds). Part of this variation is due to the same factors that drive variation in firm-specific recovery rates (i.e. firm- and industry specific explanatory va-

riables, different observation periods). For instrument-specific recovery rates, differences in issuers' capital structure are, however, a major driver as well, as the next section shows.

5.2 The Impact of Debt Cushion

Studies on the determinants of historically observed recovery rates mostly rely on dummy variables for seniority to indirectly proxy debt cushion effects.[16] Such approach, however, comes at the disadvantage of not being able to capture the variation in recovery rates within a particular type of debt, and as we saw earlier, this variation can be substantial. A more thorough exploration of this matter is therefore in order.

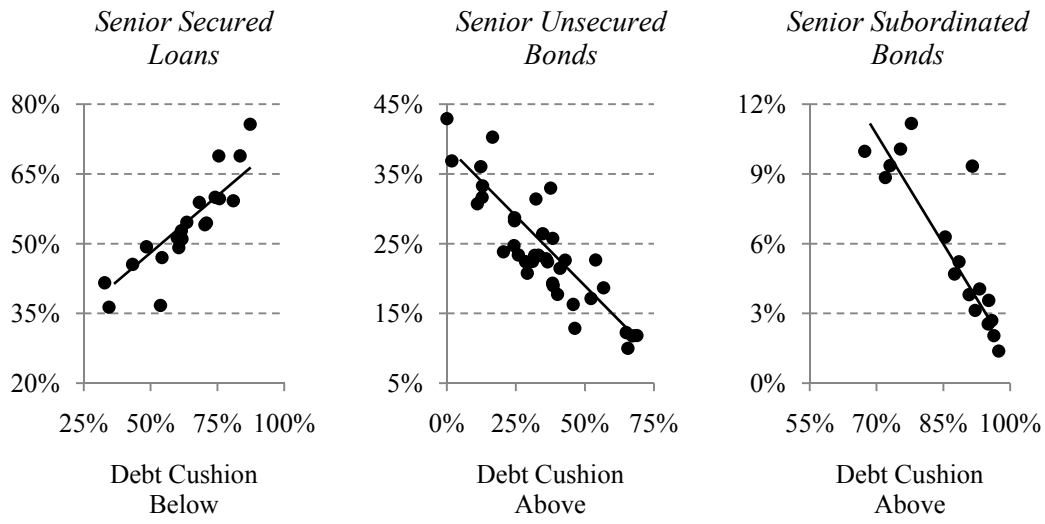
For a given implied distribution of firm-wide recovery rates, the theoretical impact of capital structure (as measured by debt cushion above and below) on instrument-specific recovery rates can be easily deduced using *Eqs. 6* and *7*. Considering, though, that implied distributions are not only different for each firm but also change over time, it is expedient to examine the *average* relation of instrument-specific recovery rates and issuers' capital structure, as implied by estimation results.

Figure 7 illustrates the sensitivity of implied expected instrument-specific recovery rates to debt cushions below and above. For senior secured loans, a one percentage point increase in debt cushion below is on average associated with a 0.60 percentage point increase in recovery rates, the adjusted R^2 being 0.758. For senior unsecured bonds (senior subordinated bonds), an increase in debt cushion above of the same magnitude is on average associated with a 0.40 (0.29) percentage point decrease, the adjusted R^2s being 0.749 and 0.736.[17] This implies that issuers' capital structure is responsible for most of the tremendous variation in recovery rates of a particular type of debt. Of course, these figures are only approximate because the true relationships are not linear. For instance, as debt cushion below approaches zero (one), recovery rates of senior secured loans approach the firm-wide recovery rate (100%).

[16] One exception is Cantor and Varma (2005) who use debt cushion below to explain historically observed recovery rates. They find that issues with more than 60% debt cushion below recover about 76% more than issues with less than 15% debt cushion below.

[17] The relation of debt cushion below and recovery rates of senior unsecured bonds (not shown) is positive but less clear cut. This is due to debt cushion above being the dominant driver for this type of debt.

Figure 7: The Impact of Debt Cushion



This figure illustrates the impact of capital structure (as measured by debt cushion below and debt cushion above) on implied expected recovery rates of senior secured loans, senior unsecured bonds, and senior subordinated bonds (y-axes). Figures are averages over the respective observation period.

5.3 The Impact of the Economic Environment

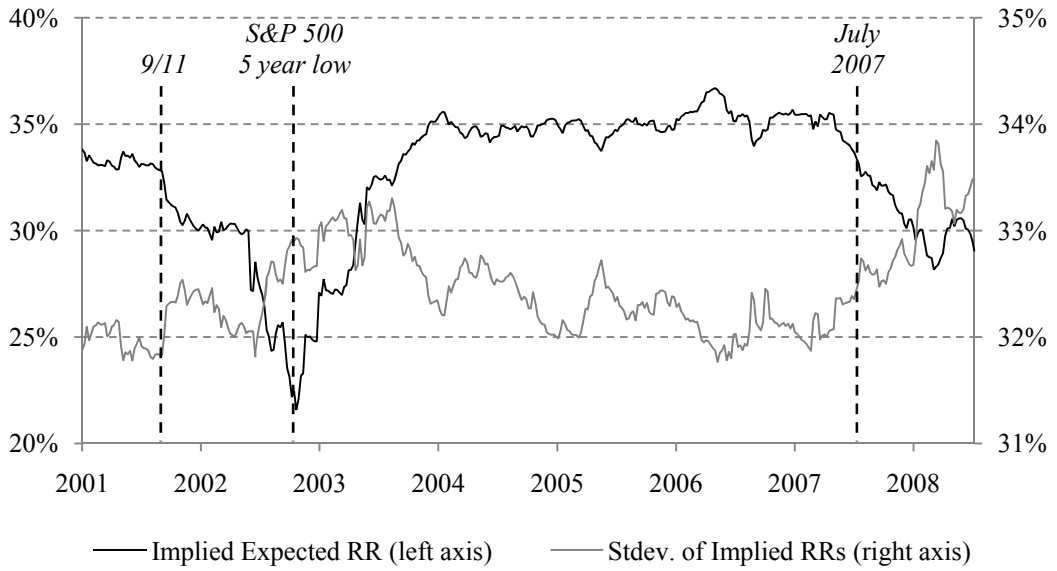
Historically, recovery rates showed substantial co-movement with the business cycle. For instance, Rösch and Scheule (2005) find that senior unsecured bonds recovered only around 40% in 1990 and 30% in 2000 – 2002, years in which many economies fared badly. By contrast, this figure was approximately 50% for the time in between. Covitz and Han (2004), Düllmann and Trapp (2004), Harpaintner, Rachev, and Trück (2005), and Bruche and Gonzáles-Aguado (2009) obtain comparable results, and there is little reason why a similar behavior should not be observable under the pricing measure.

Figure 8 illustrates the evolution of implied expected firm-wide recovery rates and the standard deviation of implied firm-wide recovery rates over the entire observation period. Between early 2001 and autumn 2002, a time that saw the burst of the internet bubble, 9/11, and various accounting scandals (Enron, WorldCom, and Tyco, to name but a few), recovery rates plunged by more than 12 percentage points from 33.8% to 21.6%. Until year-end 2004, however, they recovered to what could be called their "non-crisis-level" (approximately 35%) and stayed there for the three years to come. Around July 2007, when the distortions in the U.S. sub-prime mortgage market unveiled for the first time, recovery rates started to deteriorate again and fell to below 30% at the end of the observation period.

The standard deviation of implied firm-wide recovery rates fluctuates only moderately and does not leave the band of 31.8% to 33.8%. We observe a jump subsequent to 9/11, general-

ly higher values between autumn 2002 and autumn 2003, and an increase since July 2007. Note that this is despite the simultaneous drop of expected recovery rates, meaning that the excess standard deviation over the minimum standard deviation (not shown) is extremely sensitive to distress. This suggests that in uncertain times, investors not only reduce their estimates of recovery given default, but are also aware that their predictions may have become less reliable.

Figure 8: Evolution of Implied Firm-wide Recovery Rates



This figure illustrates the evolution of implied expected firm-wide recovery rates and the standard deviation of implied firm-wide recovery rates. Figures are averages over all firms.

5.4 The Impact of Ratings

Historical evidence as to whether credit ratings are indicative of recovery given default is mixed: Emery (2007) studies the relation of Moody's corporate family ratings and historical firm-wide recovery rates. Corporate family ratings mainly convey information as to an issuer's general default risk, rather than specifically accounting for the risk associated with a particular debt instrument. His results suggest that there is no systematic link between the two, from which it follows that higher default risk is not necessarily associated with lower default-conditional recovery. Altman, Resti, and Sironi (2004), on the other hand, examine the link between Moody's instrument-specific credit ratings and historical instrument-specific recovery rates. In addition to default risk, instrument-specific recovery rates take capital structure and collateral quality into consideration and thus account specifically for the recovery prospects of a particular debt issue. Consistent with intuition, Altman et al. find that

bonds with an investment grade rating recover a substantially higher fraction than those with a sub-investment grade rating.

To see whether these relations persist for our implied estimates, we calculate average implied expected firm-wide recovery rates for firms with the same Moody's corporate family rating as well as average implied expected instrument-specific recovery rates for debt instruments with the same Moody's instrument-specific credit rating. *Table 6* shows that the average firm-wide recovery rate is 35.7% for Baa-rated firms and only slightly lower for Ba- and B-rated firm (33.4 and 32.7%). Using standard statistical tests, we are not able to reject the null hypothesis of no relation between corporate family ratings and firm-wide recovery rates at the 10% confidence level, a result analogous to that of Emery.[18] Instrument-specific ratings, on the other hand, seem to qualify as indicators for recovery prospects: Average instrument-specific recovery rates range from as high as 45.0% for Baa-rated debt to as low as 16.1% for Caa-rated debt. We are able to reject the null hypothesis of no relation at the 1% confidence level and thus find that results of Altman et al. persist, too.

Table 6: Implied Expected Recovery Rates by Moody's Rating

	Baa	Ba	B	Caa
Firm-wide	35.7%	33.4%	32.7%	NA
Count	2	17	15	
Instrument-specific	45.0%	36.4%	21.1%	16.1%
Count	9	27	21	8

This table shows average implied expected firm-wide recovery rates for firms with the same corporate family rating and average implied expected instrument-specific recovery rates for debt with the same instrument-specific credit rating. Ratings are as published by Moody's and obtained from Bloomberg. In each case, the average rating during the observation period is used. "Count" indicates the number of firms/issues with a particular rating.

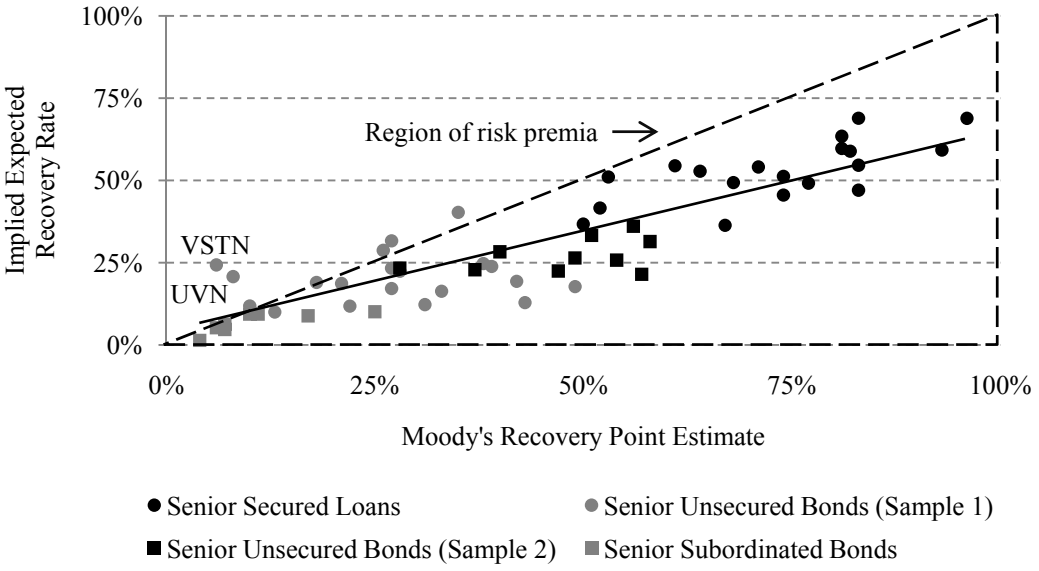
As mentioned, corporate family as well as instrument-specific ratings reflect issuers' default risk, albeit not necessarily solely. However, rating agencies also derive pure estimates of default-conditional recovery rates for debt instruments of a particular seniority. These recovery estimates are based on an analysis of priority of claims and the distribution of *historical* recovery rates and, thus, should not comprise a premium for recovery risk.[19] *Figure 9* illustrates that there is a strong relation between our estimates of implied-expected instrument-specific recovery rates and Moody's recovery point estimates of the same instruments, the adjusted R^2 being 80.2%. With the exception of Univision Communications (UVN) and Visteon (VSTN), observations lie within or close to the "region of risk premia" (i.e. below

[18] Statistical tests are performed using all rating notches and not just full letter designations.

[19] Solomon (2006) describes Moody's approach to deriving recovery estimates. S&P and Fitch apply similar methodologies.

the 45-degree line), meaning that Moody's estimates of (physical) recovery are greater than our implied estimates. The slope of the ordinary least squares line is 0.632 and the intercept is not significantly different from zero. These results show that our approach to prioritizing claims produces results similar to those of Moody's and that investors require a substantial premium for taking recovery risk.

Figure 9: Implied Expected Recovery Rates vs. Moody's Recovery Estimates



This figure illustrates the relation of Moody's recovery point estimates (as of Jun-2008 and where available) and implied expected instrument-specific recovery rates, averaged over the respective observation period.

5.5 Robustness

5.5.1 Alternative Parameterization

To assure that our approach to estimating implied recovery rates is robust with regard to the chosen functional form of the implied probability distribution of recovery $h(x)$, we re-estimate the model based on an alternative parameterization. Following Güntay, Madan, and Unal (2003), Düllmann and Trapp (2004), and Rösch and Scheule (2005), we assume that the ratio of firm value to total liabilities at default x is related to a normally distributed variable y with mean $\bar{\mu}$ and variance $\bar{\sigma}^2$ by the logit transformation $x = e^y/(1+e^y)$.[20] x then follows the transformed normal density

[20] Güntay et al. use this assumption for their estimation of implied recovery rates, Düllmann and Trapp and Rösch and Scheule analyze the determinants of historical recovery rates.

$$t(x) = \frac{1}{\sqrt{2\pi}\bar{\sigma}x(1-x)} exp\left(-\frac{\left(log\left(\frac{x}{1-x}\right)-\bar{\mu}\right)^2}{2\bar{\sigma}^2}\right) \quad x \in]0,1[. \tag{16}$$

Similar to the beta distribution, the transformed normal distribution is fully specified by its first two moments, can assume U- as well as bell shapes, and is capable of reproducing all combinations of mean and standard deviation that are theoretically conceivable for a probability density with support in the unit interval.

Table 7: Estimation Results for the Transformed Normal Distribution

	β_0	β_1 (F_Lev)	β_2 (F_Tan)	β_3 (F_IntCov)	β_4 (F_Quick)	β_5 (I_Diss)	β_6 (I_Illiq)	β_7 (I_Lev)	β_8 (CDX)
Coeff.	0.3551***	-0.0081*	0.0059	0.0054***	0.0329***	-0.0243**	-0.0174**	-0.0214	-0.0251***
Std. Err.	0.0162	0.0042	0.0151	0.0011	0.0066	0.0119	0.0069	0.0175	0.0057
	γ_0	γ_1 (CDX)							
Coeff.	0.6404***	0.0404***							
Std. Err.	0.0107	0.0074				RMSE	0.138	Adj. R^2	0.327

This table shows estimation results for the implied probability distribution of recovery. It is assumed that implied recovery rates follow a transformed normal distribution. The mean of this distribution is modelled as a function of financial leverage (F_Lev), asset tangibility (F_Tan), interest coverage ratio (F_IntCov), quick ratio (F_Quick), industry distress (I_Diss), industry illiquidity (I_Illiq), industry financial leverage (I_Lev), and the CDX (*Eq. 12)*. The excess standard deviation of the distribution is modelled as a function of the CDX (*Eq. 14)*. Standard errors for $\beta_0, \dots, \beta_8$ are conditional on estimates of γ_0, γ_1 and vice versa. ***, **, and * indicate significance at the 1%, 5%, and 10% confidence level, respectively. RMSE is obtained using *Eq. 15*.

Table 7 shows estimation results obtained for the transformed normal distribution. Without exception, signs of coefficients are identical to those presented earlier for the beta distribution (*Table 4*). Confidence levels, the size of coefficients, the RMSE, and the adjusted R^2 are very similar, too and for practical purposes can be considered identical. The resulting implied expected firm-wide recovery rate is 35.2% and the standard deviation of implied recovery rates is 33.2% (not shown). Again, these figures are not markedly different from results for the beta distribution (33.4% and 32.5%, respectively). Further, the central finding that the implied density of recovery is U-shaped persists for the transformed normal distribu-

tion.[21] This suggests that estimation results are robust for different parameterizations of $h(x)$.

5.5.2 Risk Aversion in Implied Recovery Rates

To further assess the validity of our results, it is expedient to examine whether the gap between implied and physical recovery rates is reasonable. For this purpose, we resort to the work of Bakshi, Madan, and Zhang (2006) who show that the implied expected recovery rate is related to the probability density of recovery under the physical measure $h^P(x)$ by

$$E^Q\left[\rho|1_{\{\tau<T\}}=1\right]=\int_0^1 \frac{x\,U'(w_0-(1-x)F)h^P(x)}{\int_0^1 U'(w_0-(1-x)F)h^P(x)\,dx}\,dx \qquad (17)$$

where $U'(\cdot)=\partial U(\cdot)/\partial x$, $U(\cdot)$ is the utility-of-wealth-function of the representative agent, w_0 is initial wealth, and F is the notional principal at stake. From *Eq. 17* it follows that in the absence of risk aversion, i.e. constant $U'(\cdot)$, expected recovery rates are identical under the T-forward and the physical measure, i.e. $E^Q\left[\rho|1_{\{\tau<T\}}=1\right]=E^P[\rho]$.[22] However, for concave utility functions, Bakshi et al. show that $\partial E^Q\left[\rho|1_{\{\tau<T\}}=1\right]/\partial\eta<0$ where η is the coefficient of risk aversion. It follows that for $\eta>0$, we have $E^Q\left[\rho|1_{\{\tau<T\}}=1\right]<E^P[\rho]$. This relation is intuitive: If investors are risk averse, they require a premium for taking recovery risk which results in implied expected recovery rates being lower than physical expected recovery rates. We saw earlier that this is the case for the estimates produced by our model.

In the following, we assume that the representative agent has an exponential utility function of the form $U(w)=-e^{-\eta w}$ where w denotes wealth. Exponential utility functions are appealing in that they imply constant absolute risk aversion, i.e. $-(\partial^2U(w)/\partial w^2)/(\partial U(w)/\partial w)=\eta$. Assuming further that $F=100\%$ and that recovery rates follow a beta distribution, *Eq. 17* simplifies to

[21] To confirm that the observed U-shape is not an artifact of the mathematical properties of the beta and the transformed normal distribution, we re-estimate the model based on a third parameterization of $h(x)$, given by the quadratic density function

$$U(x)=a_0+a_1\left(x-\frac{1}{2}\right)+a_2\left(x-\frac{1}{2}\right)^2$$

with $\int_0^1 u(x)\,dx=1$ and $u(x)\geq 0$ for all $x\in[0,1]$ where α_0, α_1 and α_2 are shape parameters. Again, we find the resulting implied density to be U-shaped.

[22] For constant $U'(\cdot)$, *Eq. 17* simplifies to $E^Q\left[\rho|1_{\{\tau<T\}}=1\right]=\int_0^1\frac{x\,h^P(x)}{\int_0^1 h^P(x)dx}dx=E^P[\rho]$.

$$E^Q\left[\rho|1_{\{\tau<T\}}=1\right]=\frac{C\left(\frac{(1-\mu)(\mu^2+\sigma^2)}{\sigma^2},\frac{(1-\mu)\mu}{\sigma^2},-\eta\right)\mu}{C\left(\frac{\left((1-\mu)\mu-\sigma^2\right)\mu}{\sigma^2},\frac{(1-\mu)\mu}{\sigma^2}-1,-\eta\right)} \quad (18)$$

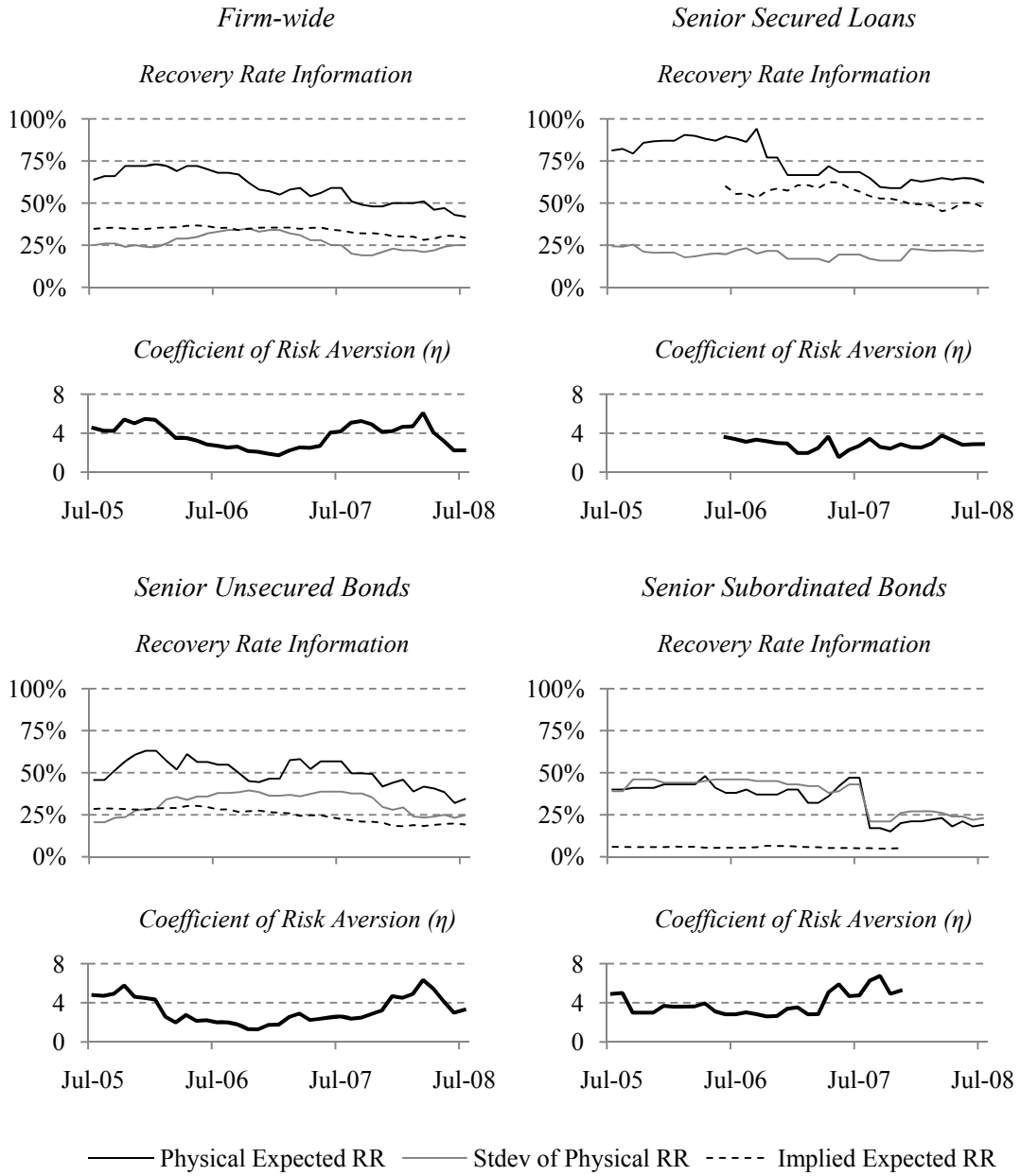
where $C(\cdot)$ is the confluent hypergeometric function and μ and σ are the mean and standard deviation of the distribution of recovery rates under the physical measure. For a given implied expected recovery rate and the physical distribution of recovery, the resulting η can thus be determined.

We proxy the physical distribution of recovery at a particular point in time by the mean and standard deviation of actual recovery rates during the time period starting 6 months before and ending 6 months ahead. We find that using a narrower interval results in very erratic figures, in particular for senior secured loans and senior subordinated bonds for which observations occur less frequently. Using forward-looking data seems to be justified since the market shows a tendency to anticipate changes in the economic environment by several months.

Figure 10 illustrates the evolution of the thusly calculated mean and standard deviation of physical recovery rates between July 2005 and July 2008. Also shown is the evolution of implied expected recovery rates during that time and the resulting coefficient of risk aversion. We observe that physical recovery rates are at all times higher than implied recovery rates. For firm-wide recovery rates, η fluctuates between 2 and 6, with few exceptions. The average η over the entire observation period is 3.7. For recovery rates of senior secured loans, senior unsecured bonds, and senior subordinated bonds, η is mostly confined to values between 2 and 6, too, the averages being 2.6, 3.2, and 3.9, respectively. This suggests that the degree of aversion to recovery risk is quite similar for different types of debt but that investors are slightly more risk averse for lower-ranking debt instruments.

A further point to mention is that the tremendous difference between historical and implied expected recovery rates of senior subordinated bonds observed earlier seems to be justified indeed. For example, throughout 2005, the physical expected recovery rate is above 40% while its implied counterpart is slightly below 6%, yet η does not exceed 5. This is due to the extraordinarily high standard deviation of the physical expected recovery rate which reaches values up to 46% during that time. This indicates that our estimates of implied recovery rates are economically sensible, both, on a firm-wide and on an instrument-specific level.

Figure 10: Risk Aversion in Implied Expected Recovery Rates



This figure illustrates the evolution of the expected physical recovery rate and its standard deviation. For a particular point in time, these are calculated using actual recovery rates during the time period starting 6 months before and ending 6 months ahead. Recovery rates are trading prices 30 days after default of U.S. corporate issuers as reported by Moody's (Hamilton and Varma (2006), Hamilton (2007), Emery, Ou, and Tennant (2008), and Emery and Ou (2009)). The figure also shows the evolution of implied expected recovery rates (averages over all firms) and resulting coefficients of risk aversion. Results are obtained using *Eq. 18*.

6 Application: Deducing the Implied Probability of Default

Our approach to extracting implied recovery rates did not require a specification of the relation between implied default and recovery rates and, in particular, did not presume independence. Based on our estimates of implied expected recovery rates, we can thus deduce implied probabilities of default without risking that a possible misspecification of this relation contaminates results. We employ the reduced-form framework developed by Jarrow and Turnbull (1995), Lando (1998), Duffie and Singleton (1999), and others and for ease of expedition assume the implied arrival rate λ^Q to be constant. The (L)CDS premium is then approximately equal to the product of loss given default and λ^Q:

$$s \approx \left(1 - E^Q\left[\rho | 1_{\{\tau<T\}} = 1\right]\right)\lambda^Q.^{23} \qquad (19)$$

It follows that the one-year implied probability of default PD^Q is given by

$$PD^Q = 1 - exp\left(-\frac{s}{\left(1 - E^Q\left[\rho | 1_{\{\tau<T\}} = 1\right]\right)}\right), \qquad (20)$$

and we can use observed premia and implied expected instrument-specific recovery rates to infer implied probabilities of default for each firm.[24]

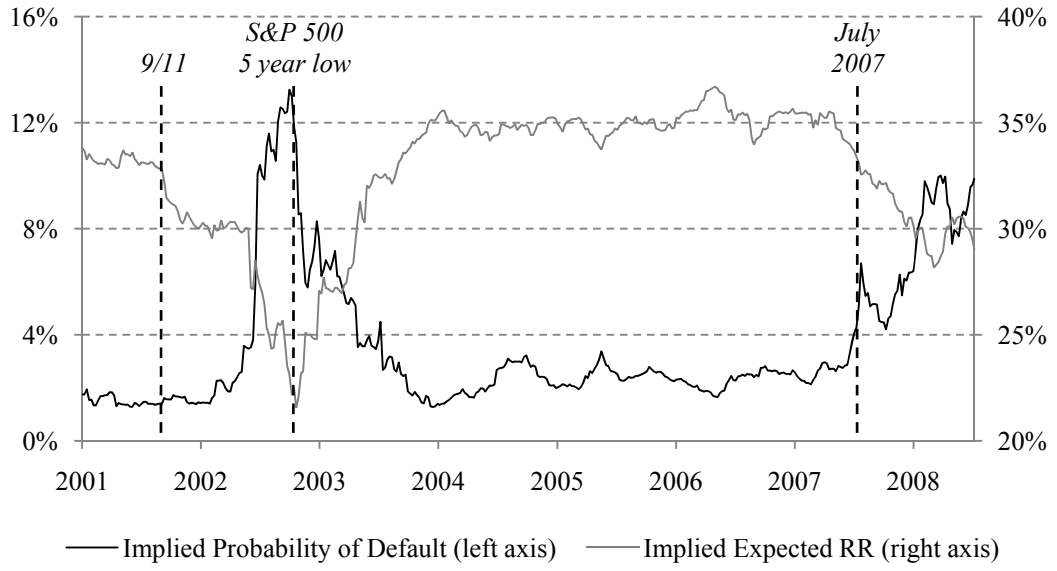
Figure 11 illustrates the evolution of the average one-year implied probability of default and, for comparison, once more shows the average implied expected firm-wide recovery rate. Throughout 2001, the probability of default sojourned below 2% but increased swiftly in course of the 2002/2003 calamities to levels beyond 12%. 2004 to 2006 was a period of relative calmness, followed by yet another jump around July 2007 and a subsequent increase to about 9% at the end of the observation period. Also, we observe that the relation to recovery rates is negative: A one percentage point increase in the probability of default is on average associated with a 0.91 percentage point decrease in recovery rates. The intercept is 35.9%, a figure that we identified earlier as the approximate "non-crisis-level" of recovery rates. Both figures are significant at the 1% confidence level. With 0.5965, the adjusted R^2 is quite high.

[23] Cf. Longstaff, Mithal, and Neis (2005). This further assumes that (L)CDS premia are fully determined by default and recovery risk. There is abundant literature indicating that other factors such as liquidity risk and counterparty risk are of relevance as well, and more so for higher ratings (cf. Janosi, Jarrow, and Yildirim (2002), Longstaff, Mithal, and Neis (2005), Ericsson and Renault (2006), and others). For the sake of simplicity, we however ignore these (the fact that most of our sample firms have a sub-investment grade rating should extenuate the issue) and leave a more thorough implementation to future research.

[24] To calculate implied probabilities of default, we use senior unsecured CDS premia and implied expected recovery rates of senior unsecured bonds. We find that using the respective senior (Sample 1) or junior (Sample 2) instrument leads to virtually identical results (in the absence of estimation error, both would be identical).

This extends earlier research showing that historical default and recovery rates are negatively related (cf. Frye (2000), Altman, Resti, and Sironi (2004), Altman, Brady, Resti, and Sironi (2005), Cantor and Varma (2005), Emery and Ou (2009), and others). It is well-understood that assuming constant recovery rates thus results in an underestimation of value-at-risk and economic capital. The Basel Committee on Banking Supervision (2005) accordingly requires that "if recovery rates are negatively related to default rates, loss given default parameters must embed forecasts of future recovery rates that are lower" where appropriate.

Figure 11: Evolution of the Implied Probability of Default



This figure illustrates the evolution of the one-year implied probability of default *(*obtained using *Eq. 20)* and for comparison also the evolution of the implied expected firm-wide recovery rate. Figures are averages over all firms.

To determine the risk premium in the implied probability of default, we once again resort to Bakshi, Madan, and Zhang (2006). They show that the implied probability of default is related to the physical probability of default PD^P and the physical probability density of recovery by

$$PD^Q = \frac{PD^P \int_0^1 U'(w_0 - (1-x)F)h^P(x)\,dx}{(1-PD^P)U'(w_0) + PD^P \int_0^1 U'(w_0 - (1-x)F)h^P(x)\,dx}. \qquad (21)$$

In the hypothetical case of full expected recovery, i.e. $E^P[\rho] = 1$, we have $\int_0^1 U'(w_0 - (1-x)F)h^P(x)\,dx = U'(w_0)$ which implies that probabilities of default are identical under the T-forward and the physical measure, i.e. $PD^Q = PD^P$. This is also true in the absence of

risk aversion.[25] For $E^P[\rho] < 1$ and concave utility functions, we, however, have $\int_0^1 U'(w_0 - (1-x)F)h^P(x)\,dx > U'(w_0)$ which implies $PD^Q > PD^P$. This relation is intuitive, too: If investors are risk averse, they require a premium for taking default risk which results in the probability of default being higher under the pricing than under the physical measure.

Table 8 shows average implied and physical probabilities of default for firms with the same Moody's corporate family rating. Again, observations are as expected: Physical probabilities are lower than their implied counterparts for all ratings. This indicates that investors require a compensation for taking default risk, a finding consistent with earlier research (c.f. Fons (1987), Berndt et al. (2005), Driessen (2005), Saita (2006), and Pan and Singleton (2008)).

Table 8: Historical vs. Implied One-Year Probabilities of Default by Rating

	Baa3	Ba1	Ba2	Ba3	B1	B2	B3
Implied	1.0%	2.4%	2.6%	4.8%	6.2%	6.6%	8.6%
Count	2	1	10	6	6	5	4
Historical (Moody's)	0.3%	0.7%	0.7%	1.8%	2.5%	3.8%	7.7%

This table shows average implied one-year probabilities of default for firms with the same corporate family rating. Ratings are as published by Moody's and obtained from Bloomberg. For each firm, the average rating during the observation period is used. Estimates are obtained using *Eq. 20*. "Count" indicates the number of firms/issues with a particular rating. Also shown are respective historical one-year default rates, as reported by Emery and Ou (2009) (data from 1983 to 2008).

Under the same assumptions as employed in *Section 5.5.2*, *Eq. 21* simplifies to

$$E^Q\left[\rho|1_{\{\tau<T\}}=1\right] = 1 + \left(\frac{C\left(\frac{(\mu-1)^2\mu}{\sigma^2}+\mu-1, \frac{(1-\mu)\mu}{\sigma^2}-1, \eta\right)PD^P}{PD^P-1} - 1\right)^{-1}, \qquad (22)$$

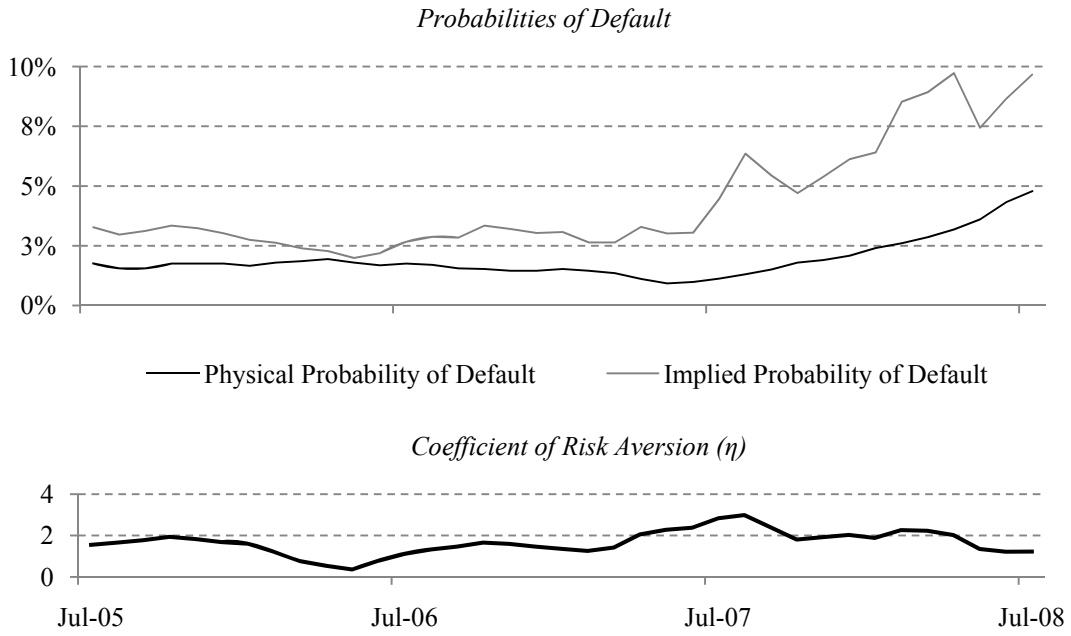
and for given (implied and physical) probabilities of default and the physical distribution of recovery rates, the resulting η can be determined. *Figure 12* illustrates the evolution of one-year physical and implied probabilities of default.[26] Also shown is the resulting coefficient of risk aversion for the calculation of which we additionally use the mean and standard deviation of the physical distribution of firm-wide recovery rates. We observe that the physical probability of default is at all times lower than its implied counterpart and that both start off to new heights around July 2007. η fluctuates between 1 and 3, with one exception in the

[25] For constant $U'(\cdot)$, *Eq. 21* simplifies to $PD^Q = \frac{PD^P \int_0^1 h^P(x)dx}{(1-PD^P)+PD^P \int_0^1 h^P(x)dx} = PD^P$.

[26] We approximate the physical probability of default at a particular point in time using the average default rate of U.S. sub-investment grade corporate issuers during the time period starting 6 months before and ending 6 months ahead. To assure comparability, the implied probability of default is calculated using only those firms with a sub-investment grade rating.

first half of 2006, where both probabilities almost cross. The average η over the entire observation period is 1.7. This is somewhat lower than the ηs obtained for recovery risk. One explanation might be that default risk is better researched and understood than recovery risk, making investors more comfortable with taking the first than the second.

Figure 12: Risk Aversion in the Implied Probability of Default



This figure illustrates the evolution of the physical and the implied probability of default. The physical probability for a particular point in time is calculated using actual default rates of sub-investment grade corporate issuers during the time period starting 6 months before and ending 6 months ahead. Data are taken from Moody's (Emery and Ou (2009)). The implied probability is obtained by averaging over all firms with a sub-investment grade Moody's corporate family rating. The figure also shows the resulting coefficient of risk aversion for the calculation of which we additionally use the mean and standard deviation of the distribution of firm-wide recovery rates under the physical measure. Results are obtained using *Eq. 22*.

7. CONCLUSION

Our approach to estimating market-implied recovery rates exploits the fact that differently-ranking debt instruments of the same issuer face identical arrival risk but different default-conditional recovery rates. We show that given (L)CDSs on two such instruments, it is feasible to derive a metric that is a function of implied recovery rates only. Based on this metric and additionally employing capital structure data, we can infer the entire implied distribution of recovery without imposing any parametric relationship upon implied recovery and default rates.

We use various firm- and industry-specific factors for the empirical specification of our model and find that most of these possess explanatory power as to implied expected recovery rates. Observed relations are intuitive and add to findings of earlier research regarding the determinants of historical recovery rates. Robust to the choice of parameterization, the implied probability density of recovery is approximately U-shaped, indicating that the implied likelihood of extreme outcomes is particularly high. We attribute this to actual recovery rates being very difficult to predict.

The reference obligation's seniority and the issuer's capital structure are the dominant drivers of instrument-specific recovery rates. Implied expected recovery rates are on average more than twice as high for senior secured loans as for senior unsecured bonds. Senior subordinated bonds, being the most junior debt instrument we consider, benefit from no debt cushion below and recovery rates are accordingly meager. Recovery rates for a particular type of debt differ significantly across companies, and most of this difference can be explained by capital structure characteristics.

Our results suggest that implied recovery rates depend on the general state of the economy and vacillate over time. In particular, they seem to be quite sensitive to crises and accordingly have plummeted since summer 2007. We illustrate how our estimates of recovery rates can be used to infer the implied probability of default and here, too, observe such dependency, albeit with opposite sign. It follows that the implied probability of default is negatively related to implied expected recovery rates, as it is known to be under the physical measure.

Recovery rates are considerably lower under the pricing measure than under the physical measure. This is unsurprising considering that investors should require a premium for taking recovery risk. Assuming an exponential utility function, we calculate the coefficient of risk aversion for firm- and instrument-specific recovery rates. Results indicate that investors are more risk averse for lower-ranking debt but not much so. We perform the same exercise for implied recovery rates and find that investors require a somewhat lower compensation for taking default risk than for taking recovery risk.

This paper adds to the growing literature on implied recovery. Importantly, we eschew some of the shortcomings of earlier approaches to separating default and recovery rates and arrive at economically meaningful and robust results. Our findings shed further light on the stochastic nature of implied recovery rates and should be of particular interest to modelers of credit-risky assets. Our deduction of implied probabilities of default is, however, based on greatly simplified assumptions, and a more rigorous estimation approach could be a valuable extension.

APPENDIX A: *Supremum and Infimum Standard Deviation of the Beta Distribution*

The mean and the variance of the beta distribution are

$$\mu = \int_0^1 xbet(x)dx = \frac{p}{p+q}, \tag{A.i}$$

$$\sigma^2 = \int_0^1 (x-\mu)^2 bet(x)dx = \frac{pq}{(p+q)^2(p+q+1)} \quad p,q \in]0,\infty[. \tag{A.ii}$$

Solving *Eq. A.i* for q and substituting into *Eq. A.ii*, we obtain

$$\sigma = \sqrt{\frac{p\frac{(1-\mu)p}{\mu}}{\left(p+\frac{(1-\mu)p}{\mu}\right)^2\left(p+\frac{(1-\mu)p}{\mu}+1\right)}} \tag{A.iii}$$

$$= \sqrt{\frac{(1-\mu)}{\left(1+\frac{(1-\mu)}{\mu}\right)^2\left(p+\frac{(1-\mu)p}{\mu}+1\right)\mu}}$$

$$= \sqrt{\frac{(1-\mu)}{\left(\frac{\mu+p}{\mu^3}\right)\mu}}$$

$$= \sqrt{\frac{(1-\mu)\mu^2}{\mu+p}}.$$

Subject to the constraint $p > 0$, the supremum and infimum of the standard deviation for a given μ thus are

$$\sigma_{sup} = \lim_{p\to 0}\sqrt{\frac{(1-\mu)\mu^2}{\mu+p}} = \sqrt{\mu-\mu^2} \quad \mu \in]0,1[, \tag{A.iv}$$

$$\sigma_{inf} = \lim_{p\to\infty}\sqrt{\frac{(1-\mu)\mu^2}{\mu+p}} = 0. \tag{A.v}$$

APPENDIX B: *Descriptive Statistics by Firm*

Sample 1 Constituents

	Advanced Micro Devices Inc.	ArvinMeritor Inc.	Allied Waste North America Inc.	Cablevision Systems Corp.	Dean Foods Co.	Directv Holdings LLC.	El Paso Corp.	Ford Motor Co.	Freeport McMoran Copper & Gold Inc.	Freescale Semiconductor Inc.	Hospital Corporation of America Inc.	Idearc Inc.	Lear Corp.	Michaels Stores Inc.	Realogy Corp.	Sungard Data Systems Inc.	Tribune Co.	TRW Automotive Inc.	Univision Communications Inc.	Visteon Corp.	Average
Ticker Symbol	AMD	ARM*	AW	CVC	DF*	DTV	EP	F*	FCX	FSL	HCA*	IDARQ*	LEA*	MIKE	REAL*	SGDS	TRB*	TRW	UVN*	VSTN*	
SIC Code	367	371	495	484	202	484	492	371	102	367	806	274	371	594	653	737	271	371	483	371	
Observations																					
First	Mar-07	Sep-07	Feb-07	Jun-06	Apr-07	Dec-06	May-06	Dec-06	Mar-07	Mar-07	Nov-06	Jul-07	Jun-07	Jun-07	May-07	Aug-07	May-07	Mar-07	Mar-07	Sep-06	
Last	Aug-07	Jun-08	Jul-08	Jul-08	Jul-08	Jul-08	Jul-08	Jul-08	Sep-07	Jul-08	Jul-08	Jul-08	Jul-08	Jan-08	Jul-08	Jul-08	Nov-07	Jul-08	Jul-08	Jul-08	
Count	24	40	74	110	67	81	114	81	29	71	88	54	56	33	61	50	25	69	69	95	65
Moody's Corporate Family Rating	B1	Ba3, B1	NA	NA	Ba3, B1	Ba2	B3, Ba3	B3	Ba2	Ba3, B1	Ba2	Ba3, B1	B2	B2	B3	B2	Ba3	Ba2	B1	B2, B3	
LCDS Premia (BPs)																					
Average	181	350	193	192	245	139	126	421	62	334	226	407	324	299	854	290	411	181	457	478	309
Median	137	340	160	144	260	150	120	415	60	350	248	295	300	300	615	275	460	175	375	375	
Max.	380	495	370	445	450	270	275	915	90	650	435	800	560	480	2,017	505	545	360	885	1,372	
Min.	100	200	85	75	75	52	45	169	50	94	80	140	200	130	240	150	215	80	145	140	
Premia of Senior Unsecured CDS (BPs)																					
Average	438	642	320	309	334	224	181	731	104	674	384	654	522	465	1,523	442	726	339	838	865	536
Median	383	650	328	264	345	196	181	674	98	632	388	436	482	447	1,625	431	834	329	587	739	
Max.	710	868	499	575	606	370	335	1,643	181	1,231	599	1,556	851	724	2,716	620	902	632	2,148	2,136	
Min.	293	320	158	148	122	121	91	419	87	290	241	190	310	273	477	293	419	174	335	373	
Avg. Ratio	40.0%	55.0%	58.0%	60.3%	72.7%	61.2%	66.7%	55.2%	60.6%	48.0%	56.1%	68.4%	61.9%	63.4%	54.8%	64.7%	56.2%	52.9%	55.3%	51.1%	58.1%
Avg. Difference	257	292	127	117	89	85	55	310	42	340	159	247	198	166	668	152	315	159	380	387	227

Sample 2 Constituents

	Amkor Technology Inc.	Best Buy Co. Inc.	Boyd Gaming Corp.	Cox Communications Inc.	D.R. Horton Inc.	Harrah's Entertainment Inc.	Health Management Associates Inc.	KB Home	MGM Mirage	Solectron Corp.	Station Casinos Inc.	TJX Cos. Inc.	Toll Brothers Inc.	Triad Hospitals Inc.	TRW Automotive Inc.	Tesoro Corp.	United Rentals Inc.	Average
Ticker Symbol	AMKR	BBY*	BYD	COX*	DHI	HET*	HMA*	KBH	MGM	SLR	STN	TJX*	TOL*	TRI	TRW	TSO*	URI	
SIC Code	367	573	799	484	153	799	806	153	799	367	799	565	153	806	371	131	735	
Observations																		
First	Jun-02	May-03	Oct-04	Jan-01	Mar-06	Dec-04	Sep-06	Apr-04	Jun-04	Apr-06	Sep-04	3-16-3	Mar-04	Jul-04	Jan-05	Nov-04	Jul-04	
Last	Dec-07	Dec-07	Dec-07	Dec-05	Sep-07	Dec-07	Dec-07	Nov-07	Dec-07	Aug-06	Dec-07	Dec-07	Oct-07	Dec-06	Dec-07	Sep-06	Dec-07	
Count	286	241	166	261	81	154	63	189	183	21	170	219	188	129	151	95	180	163
Moody's Corporate Family Rating	B1, B2	Baa3	Ba2	NA	Ba2	B3	Ba3	Ba1	Ba1, Ba2	B1	Ba2, B2	NA	Baa3	NA	Ba2	Ba3, Ba1	B2	
Premia of Senior Unsecured CDS (BPs)																		
Average	626	44	200	119	132	157	199	178	181	187	180	27	93	181	219	100	286	183
Median	542	39	183	95	97	75	135	151	171	187	157	26	81	163	208	95	258	
Max.	1,777	129	485	500	411	609	411	517	514	215	501	56	297	278	420	158	744	
Min.	214	24	108	35	56	47	89	66	113	149	95	14	45	134	127	78	150	
Premia of Senior Subordinated CDS (BPs)																		
Average	863	70	214	206	165	185	231	228	230	276	233	41	122	253	262	151	397	243
Median	800	59	197	163	132	81	166	197	222	255	203	45	110	242	258	151	371	
Max.	2,179	296	473	852	458	671	445	583	538	332	634	65	330	361	465	259	814	
Min.	326	31	140	40	76	51	121	96	163	210	141	17	63	188	181	99	260	
Avg. Ratio	73.5%	68.9%	92.5%	69.8%	76.9%	85.9%	83.3%	77.2%	78.2%	68.4%	76.3%	68.4%	75.5%	71.5%	83.1%	69.4%	70.4%	75.8%
Avg. Difference	237	27	15	87	32	28	32	50	49	88	53	15	28	72	42	51	111	60

For each of the firms in Sample 1 and Sample 2, this table shows ticker symbols, three digit SIC codes, observation periods, the first and last Moody's corporate family rating in the respective observation period, LCDS premia, senior unsecured CDS premia, senior subordinated CDS premia, ratios of premia (as defined by *Eqs. 5*), and differences of premia (defined as $s_{uns} - s_{loan}$ for Sample 1 constituents and as $s_{sub} - s_{uns}$ for Sample 2 constituents). * indicates that CDS terms of the respective firm stipulate "modified restructuring" as a credit event. Premia of respective CDSs are adjusted to "no restructuring" using an adjustment factor of 1/1.0565.

APPENDIX C*: Implied Recovery Rates by Firm*

Sample 1

	AMD	ARM	AW	CVC	DF	DTV	EP	F	FCX	FSL	HCA	IDARQ	LEA	MIKE	REAL	SGDS	TRB	TRW	UVN	VSTN	Avg.
Firm-wide Recovery Rate																					
Implied Expected	37.3%	30.9%	30.3%	29.8%	31.7%	34.6%	30.9%	32.3%	38.5%	36.1%	33.5%	27.3%	31.4%	22.7%	30.2%	32.6%	31.8%	33.3%	30.7%	32.8%	31.9%
Implied Stdev.	33.0%	33.4%	32.4%	31.7%	33.0%	33.4%	32.0%	32.8%	33.4%	34.0%	33.0%	31.9%	33.2%	29.2%	32.7%	33.7%	32.3%	33.4%	32.5%	32.8%	32.7%
Recovery Rate of Senior Secured Loans																					
Implied Expected	75.7%	59.2%	54.1%	47.0%	41.6%	58.9%	51.2%	59.7%	68.9%	68.9%	49.3%	36.3%	51.0%	36.7%	49.1%	45.5%	52.8%	54.6%	54.4%	60.0%	53.7%
Implied Stdev.	37.5%	43.5%	43.0%	41.5%	39.6%	42.0%	41.8%	42.3%	39.6%	40.5%	41.2%	39.1%	42.8%	40.3%	42.4%	41.3%	42.2%	42.5%	43.0%	42.1%	41.4%
Recovery Rate of Senior Unsecured Bonds																					
Implied Expected	31.6%	23.8%	12.3%	16.3%	11.8%	23.3%	17.7%	24.7%	28.7%	40.3%	11.8%	10.0%	19.3%	12.9%	17.1%	18.7%	19.0%	22.4%	20.8%	23.4%	20.3%
Implied Stdev.	34.3%	33.6%	27.3%	30.8%	26.9%	33.5%	30.2%	34.5%	34.7%	43.4%	26.9%	25.1%	32.0%	29.2%	33.2%	35.2%	31.5%	34.6%	32.1%	32.9%	32.1%

Sample 2

	AMKR	BBY	BYD	COX	DHI	HET	HMA	KBH	MGM	SLR	STN	TJX	TOL	TRI	TRW	TSO	URI	Avg.
Firm-wide Recovery Rate																		
Implied Expected	34.4%	36.2%	33.4%	30.5%	33.4%	33.2%	38.8%	35.1%	32.5%	41.0%	31.8%	34.0%	35.2%	40.0%	34.8%	35.3%	36.1%	35.0%
Implied Stdev.	32.8%	32.8%	32.0%	32.0%	31.8%	31.9%	33.3%	32.4%	31.8%	32.8%	31.6%	32.2%	32.4%	33.1%	32.3%	32.2%	32.6%	32.3%
Recovery Rate of Senior Unsecured Bonds																		
Implied Expected	36.0%	36.9%	22.7%	22.4%	22.5%	22.8%	22.6%	26.4%	21.5%	42.9%	25.8%	30.7%	33.3%	33.0%	23.3%	28.3%	31.4%	28.4%
Implied Stdev.	39.8%	34.2%	39.4%	34.1%	32.7%	34.7%	39.1%	37.7%	35.3%	34.2%	39.8%	34.7%	36.9%	41.9%	33.9%	36.3%	41.2%	36.8%
Recovery Rate of Senior Subordintd Bonds																		
Implied Expected	9.3%	2.7%	9.4%	3.1%	1.4%	3.8%	10.0%	6.3%	4.7%	3.6%	8.8%	2.6%	5.2%	11.2%	2.0%	4.1%	10.1%	5.8%
Implied Stdev.	24.3%	13.7%	23.9%	12.1%	9.8%	15.9%	24.9%	20.2%	17.6%	15.7%	23.3%	13.3%	18.5%	26.3%	11.9%	15.8%	24.9%	18.4%

This table shows expected recovery rates for the entire firm, senior secured loans, senior unsecured bonds, and senior subordinated bonds as well as standard deviations of respective recovery rates. Figures are averages over the respective observation period. Estimates of implied expected recovery rates are obtained using *Eqs. 6* and *7*. Estimates of the standard deviation of implied recovery rates are obtained using *Eqs. 6* and *8*.

REFERENCES

Acharya, Viral, Sreedhar Bharath, and Anand Srinivasan, 2007, Does Industry-wide Distress Affect Defaulted Firms? Evidence from Creditor Recoveries, Journal of Financial Economics, Vol. 85, Issue 3, p787-821.

Altman, Edward, Brooks Brady, Andrea Resti, and Andrea Sironi, 2005, The Link between Default and Recovery Rates: Theory, Empirical Evidence and Implications, Journal of Business, Vol. 78, Issue 6, p2203-2227.

Altman, Edward and Vellore Kishore, 1996, Almost Everything You Wanted to Know About Recoveries on Defaulted Bonds, Financial Analysts Journal, Vol. 52, Issue 6, p57-64.

Altman, Edward and Brent Pasternack, 2006, Defaults and Returns in the High Yield Bond Market: The Year 2005 in Review and Market Outlook, Journal of Applied Research in Accounting and Finance, Vol. 1, Issue 1, p3-29.

Altman, Edward, Andrea Resti, and Andrea Sironi, 2004, Default Recovery Rates in Credit Risk Modelling: A Review of the Literature and Empirical Evidence, Economic Notes, Vol. 33, Issue 2, p183-208.

Baird, Douglas, Arturo Bris, and Ning Zhu, 2007, The Dynamics of Large and Small Chapter 11 Cases: An Empirical Study, Working Paper.

Bakshi, Gurdip, Dilip Madan, and Frank Zhang, 2006, Understanding the Role of Recovery in Default Risk Models: Empirical Comparison and Implied Recovery Rates, Working Paper.

Basel Committee on Banking Supervision, 2005, Guidance on Paragraph 468 of the Framework Document, Bank for International Settlements, http://www.bis.org.

_____, 2006, International Convergence of Capital Measurement and Capital Standards – A Revised Framework, Bank for International Settlements, http://www.bis.org.

Berndt, Antje, Rohan Douglas, Darrel Duffie, Mark Ferguson, and David Schranz, 2005, Measuring Default Risk Premia from Default Swap Rates and EDFs, Working Paper.

Berndt, Antje, Robert Jarrow, and ChoongOh Kang, 2007, Restructuring Risk in Credit Default Swaps: An Empirical Analysis, Stochastic Processes and their Applications, Vol. 177, Issue 11, p1724-1749.

Bharath, Sreedhar and Venky Panchapegesan, 2007, The Changing Nature of Chapter 11, Working Paper.

Bruche, Max and Carlos González-Aguado, 2009, Recovery Rates, Default Probabilities and the Credit Cycle, Journal of Banking and Finance, Vol. 34, Issue 4, p754-764..

Cantor, Richard, Kenneth Emery, and Pamela Stumpp, 2006, Probability of Default Ratings and Loss Given Default Assessments for Non-Financial Speculative-Grade Corporate Obligors in the United States and Canada, Moody's, http://www.moodys.com.

Cantor, Richard and Praveen Varma, 2005, Determinants of Recovery Rates on Defaulted Bonds and Loans for North American Corporate Issuers: 1983-2003, Journal of Fixed Income, Vol. 14, Issue 4, p29-44.

Chava, Sudheer, Catalina Stefanescu, and Stuart Turnbull, 2008, Modeling the Loss Distribution, Working Paper.

Covitz, Daniel and Song Han, 2004, An Empirical Analysis of Bond Recovery Rates: Exploring a Structural View of Default, Working Paper.

Das, Sanjiv and Paul Hanouna, 2009, Implied Recovery, Journal of Economic Dynamics and Control, Vol. 33, Issue 11, p1837-1857.

Driessen, Joost, 2005, Is Default Event Risk Priced in Corporate Bonds?, Review of Financial Studies, Vol. 18, Issue 1, p165-195.

Duffie, Darrell and Kenneth Singleton, 1999, Modeling Term Structures of Defaultable Bonds, Review of Financial Studies, Vol. 12, Issue 4, p687-720.

Düllmann, Klaus and Monika Trapp, 2004, Systematic Risk in Recovery Rates – An Empirical Analysis of US Corporate Credit Exposures, Working Paper.

Emery, Kenneth, 2007, Moody's Ultimate Recovery Database, Moody's, http://www.moodys.com.

Emery, Kenneth and Sharon Ou, 2009, Corporate Default and Recovery Rates 1920-2008, Moody's, http://www.moodys.com.

Emery, Kenneth, Sharon Ou, and Jennifer Tennant, 2008, Corporate Default and Recovery Rates 1920-2007, Moody's, http://www.moodys.com.

Ericsson, Jan and Olivier Renault, 2006, Liquidity and Credit Risk, Journal of Finance, Vol. 61, Issue 5, p2219-2250.

Fons, Jerome, 1987, The Default Premium and Corporate Bond Experience, Journal of Finance, Vol. 42, Issue 1, p81-97.

_____, 1994, Using Default Rates to Model the Term Structure of Credit Risk, Financial Analysts Journal, Vol. 50, Issue 5, p25-33.

Friedman, Craig and Sven Sandow, 2003, Ultimate Recoveries, Risk Magazine, Vol. 16, Issue 8, p69-73.

Frye, Jon, 2000, Depressing Recoveries, Risk Magazine, Vol. 13, Issue 11, p108-111.

_____, 2003, A False Sense of Security, Risk Magazine, Vol. 16, Issue 8, p63-67.

Gaspar, Raquel and Irina Slinko, 2008, On Recovery and Intensity's Correlation – A New Class of Credit Risk Models, Journal of Credit Risk, Vol. 4, Issue 2, p3-35.

Guha, Rajiv, 2003, Recovery of Face Value at Default: Empirical Evidence and Implications for Credit Risk Pricing, Working Paper.

Gupton, Greg and Roger Stein, 2002, Losscalc^{TM}: Model for Predicting Loss Given Default, Moody's, http://www.moodys.com.

Güntay, Levent, Dilip Madan, and Haluk Unal, 2003, Pricing the Risk of Recovery in Default with Absolute Priority Rule Violation, Journal of Banking and Finance, Vol. 27, Issue 6, p1001-1025.

Hamilton, David, 2007, Corporate Default and Recovery Rates 1920-2006, Moody's, http://www.moodys.com.

Hamilton, David and Praveen Varma, 2006, Corporate Default and Recovery Rates 1920-2005, Moody's, http://www.moodys.com.

Harpaintner, Stefan, Svetzlozar Rachev, and Stefan Trück, 2005, A Note on Forecasting Aggregate Recovery Rates with Macroeconomic Variables, Working Paper.

Janosi, Tibor, Robert Jarrow, and Yildiray Yildirim, 2002, Estimating Expected Losses and Liquidity Discounts Implicit in Debt Prices, Journal of Risk, Vol. 5, Issue 1, p1-38.

_____, 2003, Estimating Default Probabilities Implicit in Equity Prices, Journal of Investment Management, Vol. 1, Issue 1, p1-30.

Jarrow, Robert, 2001, Default Parameter Estimation Using Market Prices, Financial Analysts Journal, Vol. 57, Issue 5, p75-92.

Jarrow, Robert and Stuart Turnbull, 1995, Pricing Derivatives on Financial Securities Subject to Credit Risk, Journal of Finance, Vol. 50, Issue 1, p53-85.

Keisman, David, Karen Van de Castle, and Ruth Yang, 2000, Suddenly Structure Mattered: Insights into Recoveries from Defaulted Debt, Standard & Poor's Credit Week, May Issue, p61-68.

Kelhoffer, Kevin, David Schwartz, and Jane Zennario, 2005, Standard & Poor's Recovery Highlights of 2004, S&P, www.standardandpoors.com.

Lando, David, 1998, On Cox Processes and Credit Risky Securities, Review of Derivatives Research, Vol. 2, Issue 2-3, p99-120.

Longstaff, Francis, Sanjay Mithal, and Eric Neis, 2005, Corporate Yield Spreads: Default Risk or Liquidity? New Evidence from the Credit Default Swap Market, Journal of Finance, Vol. 60, Issue 5, p2213-2253.

Madan, Dilip and Haluk Unal, 1998, Pricing the Risks of Default, Review of Derivatives Research, Vol. 2, Issue 2-3, p121-160.

Pan, Jun and Kenneth Singleton, 2008, Default and Recovery Implicit in the Term Structure of Sovereign CDS Spreads, Journal of Finance, Vol. 63, Issue 5, p2345-2384.

Renault, Olivier and Olivier Scaillet, 2003, On the Way to Recovery: A Nonparametric Bias Free Estimation of Recovery Rate Densities, Journal of Banking and Finance, Vol. 28, Issue 12, p2915-2931.

Rösch, Daniel and Harald Scheule, 2005, A Multifactor Approach for Systematic Default and Recovery Risk, Journal of Fixed Income, Vol. 15, Issue 2, p63-75.

Saita, Leandro, 2006, The Puzzling Price of Corporate Default Risk, Working Paper.

Schläfer, Timo and Marliese Uhrig-Homburg, 2010, Loan-only Credit Default Swaps, forthcoming in: Structured Credit Products, 2nd edition, editor: Moorad Choudhry, John Wiley & Sons, Chichester.

Schneider, Paul, Leopold Sögner, and Tanja Veza, 2009, The Economic Role of Jumps and Recovery Rates in the Market for Corporate Default Risk, forthcoming in: Journal of Financial and Quantitative Analysis.

Solomon, Russel, 2006, Loss Given Default Analytics: User's Guide to Prioritizing Claims and Applying the LGD Model, Moody's, http://www.moodys.com.

Song, Jeong, 2008, Loss Given Default Implied by Cross-sectional No Arbitrage, Working Paper.

Zhang, Frank, 2003, What Did the Credit Market Expect of Argentina Default? Evidence from Default Swap Data, Working Paper.