Sovereign Default Risk and the U.S. Equity Market

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Abstract

I develop a two-country general equilibrium model with firms, governments, and endogenous default decisions. This paper shows that the risk of sovereign default abroad is important in the explanation of the level and the volatility of U.S. equity returns. The intuition is that negative economic shocks deteriorate the fiscal situation of foreign governments, thereby increasing the risk of a sovereign default that would trigger a local contraction in economic growth. The rise in the risk of an economic slowdown abroad amplifies the direct effect of these shocks on the level and the volatility of equity returns in the U.S. through i) a decrease in the present value of future firm earnings due to the unfavorable adjustment of the real exchange rate for U.S. exporters and ii) a fall in U.S. equity prices that originates from the investors' incentive to rebalance their portfolio towards risk-free securities. The amplification effect is strongest during periods of financial distress when the risk of corporate default is high in the U.S. A structural estimation of the model provides strong support for this prediction using monthly data for Brazil and the U.S. over the period 1994-2008.

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1 Introduction

The risk of sovereign default is no longer limited to emerging economies. Today, the attention is shifting to the risk of a sovereign debt crisis in Southern Europe, which is widely considered to be the main threat for financial markets in 2010.[1] In particular, the last few months have provided a clear illustration of the interactions between the U.S. equity market and the risk of sovereign default in Greece.[2] While sovereign debt clearly plays an increasing role in today's financial environment, the expected adverse consequences of sovereign default crises on the U.S. financial markets have been thus far largely ignored.

In this paper, I develop a theoretical framework that explains how negative economic shocks increase the risk of sovereign default abroad, thus producing adverse consequences for the U.S. equity market through a decrease in returns and an increase in volatility. The intuition is that a negative economic shock, which can originate either in the U.S. or abroad, worsens the fiscal situation of foreign governments, thereby increasing the risk of a sovereign default that would trigger a contraction in domestic economic growth. The rise in such a risk amplifies the initial effect of this shock on the level and the volatility of equity returns in the U.S., especially during periods of financial distress when the risk of corporate default in the U.S. is high. A structural estimation of the model provides strong support for the prediction that the risk of sovereign default abroad generates a strong leverage effect during economic downturns that helps explain the level and the volatility of U.S. equity returns.

This article endogenizes the default decisions of firms and governments within a general equilibrium model with international trade. The building block is a two-country, two-good consumption-based asset-pricing model with a representative risk-averse agent for each country. The world consists of two countries, namely, Home and Foreign. Home is a large country with a default-free government and Foreign is a small country with a defaultable government. Embedded in each country is a representative firm that produces a specific good and is financed by equity and debt. Both countries are subject to production

[1] The Financial Times (December 27, 2009), for example, writes "after two years of worrying about mortgage and corporate risk, attention is now shifting to managing the risk of country defaults, say bankers." Similarly, Moody's (2009) reports that "sovereign debt would be sharply sold off next year, leading to a wider downturn in financial markets, if central banks failed to implement perfect exit strategies." The report states that "the end of exceptionally low financing conditions will expose the true cost of the crisis on government debt affordability across the world." Finally, the report adds that the crisis of public finances is "the final – and disturbingly long-lasting – stage of the global crisis after the financial and subsequent economic crises."

[2] See, for example, the front page article of the Financial Times (February 5, 2010) titled "Europe fears rock markets", which suggests that "investor fears which had initially been confined to Greece have now spread to Portugal and Spain and spilled over into equity markets in the U.S." The article titled "U.S. Stocks drop as sovereign debt concerns overshadows data" (Bloomberg, March 30, 2010) mentions that "U.S. stocks retreated as concern that deteriorating government finances will derail the global economic recovery". A similar conclusion can be found in the article "Greek fears hit global stocks, bond spreads" (Reuters, April 8, 2010).

shocks that are transmitted internationally through the real exchange rate, which is defined as the price of the Home good per unit price of the Foreign good. A shock in a country is perfectly shared with the other country. The effect of negative economic shocks in either country is then to depress the level of firm earnings in both countries. In turn, a decrease in firm earnings increases volatility of equity returns through financial leverage, which arises from the presence corporate debt in a firm's balance sheet, and through operational leverage, which is generated from the presence of firm operating costs. Equity return volatility is thus countercyclical.[3] The same economic shocks also deteriorate the fiscal situation of the Foreign government, which raises fiscal revenues by taxing the value of its domestic firm's earnings. Sovereign credit risk, which captures the probability that this government will be unable to service its debt, is then high during adverse economic conditions.[4]

I assume that defaulting on sovereign debt causes a contraction in economic growth in addition to the initial negative economic shocks that triggered the default event.[5] The risk of a contraction in economic growth abroad exacerbates the effect of these negative shocks on the evaluation of the Home firm's assets through two complementary channels. First, a rise in the risk of economic slowdown abroad reduces the expected value of future export revenues for the Home firm through a depreciation of the terms of trade.[6] Second, the increase in the same risk triggers an incentive for portfolio rebalancing towards the risk-free bond. However, no rebalancing takes place at the equilibrium level because all households have identical portfolios and they must jointly hold the entire supply of each market. Therefore, equity prices in both countries move downwards to counteract the incentive to rebalance.[7] The risk of a contraction

[3]This prediction is line with the countercyclical nature of equity return volatility documented in Schwert (1989), Forbes and Rigobon (2002), Bae, Karolyi, and Stulz (2003), Engle and Rangel (2008), and Engle, Ghysels, and Sohn (2009), among others.

[4]This prediction is line with the countercyclical nature of sovereign credit risk documented in Cantor and Packer (1996), Hu, Kiesel, and Perraudin (2002), Catao and Sutton (2002), Hilscher and Nosbusch (2009), Jeanneret (2009), and Longstaff et al. (2009), among others.

[5]This assumption is consistent with the empirical evidence of Reinhart, Rogoff, and Savastano (2003), De Paoli, Hoggarth, and Saporta (2006), Sturzenegger and Zettelmeyer (2006), Borensztein and Panizza (2008), and Bordo, Meissner, and Stuckler (2009). However, the direction of causality in the empirical relationship between sovereign default and GDP growth documented in these studies raises some questions: debt default is a direct consequence of economic shocks that also hurt growth in a direct fashion. In addition, the anticipation of the default costs can affect output growth before the event.

[6]In line with the model's assumption, Forbes (2002), and Kaminsky and Reinhart (2000) find evidence that international trade linkages allow country-specific crises to spread to stock markets elsewhere in the world. In particular, Forbes (2002) finds that trade linkages explain 25% of the variation in stock market returns during crises. In another study (Forbes, 2000), this author uses firm-level data to analyze the Asian and the Russian crises and shows that firms which had sales exposure to the crisis country had significantly lower stock returns during the crisis event. Similarly, Bae, Karolyi, and Stulz (2003) and Ehrmann, Fratzscher, and Rigobon (2005) conclude that movements in exchange rates explain a large fraction of the contagion across international equity markets.

[7]The mechanism of financial contagion is closely related to Kyle and Xiong (2001) and Cochrane, Longstaff, and Santa-Clara (2008). When investors suffer a large loss in investment in the crisis country, they may have to liquidate their positions in other countries and thus cause equity prices to depreciate in these other countries. Van Rijckeghem and Weder (2001) and Boyer, Kumagai, and Yuan (2006) find strong evidence that crises spread internationally through the asset holdings of international investors.

in economic growth abroad amplifies, through these two channels, the initial fall in firm revenues and equity value, and thus the rise in equity return volatility in the Home country. This paper suggests a new amplification mechanism of volatility that is, in essence, a "macro leverage effect."

A structural estimation of the model provides strong support for this new prediction. That is, the presence of sovereign default risk affects the level and volatility of equity returns in the U.S. The structural test of the model consists of estimating the expected loss in economic growth upon sovereign default using the generalized method of moments (GMM) developed by Hansen (1982). The moments under consideration are the first two moments of equity returns in Brazil and in the U.S. over the past 15 years. I use information on monthly industrial production data for Brazil and the U.S. to generate the asset prices predicted by the model, thus producing the moment conditions, which are matched to those of the data as closely as possible.[8] Brazil is good candidate for a representative foreign country as it is the largest debt issuer in emerging markets, with a current level of debt of more than 1 trillion U.S. dollars (Moody's, 2009), it has sizable sovereign credit risk, and it is a large trading partner of the U.S. Furthermore, the data on sovereign credit spreads, stock market prices, and industrial production for Brazil cover a longer period than for any other comparable country. A goodness-of-fit test suggests that the model, and thus the four moment conditions, cannot be rejected at 90% confidence level. More importantly, the estimate of the expected perpetual loss in economic growth upon sovereign default, which is shared in both countries, is statistically significant at 99% confidence level and equals 0.2 percent, which is economically important compared to the average annual growth rates of industrial production in Brazil and in the U.S. (1.5 percent and 2.1 percent, respectively).

The core result of the paper is thus that the risk of sovereign default contributes to the explanation of the level and volatility of equity returns in the U.S., as well as in Brazil. While the effect of the risk of economic contraction upon sovereign default on the level of equity return volatility is marginal in periods of high economic growth, this effect appears to be particularly strong during adverse economic conditions. The potential adverse consequences of a sovereign default crisis amplify the effect of shocks on equity return volatility in periods of economic downturns, precisely when corporate credit risk is high. The model developed in this paper is thus successful in explaining the high peaks in equity return volatility observed in periods of financial distress, in addition to generating time-varying, countercyclical, and high level of unconditional equity return volatility.

[8]The model is calibrated to match the dynamics of industrial production in both countries, the corporate leverage ratios, and the government debt-to-GDP ratio in Brazil.

An additional empirical analysis provides evidence for the positive relationship between equity return volatility in the U.S. and sovereign credit risk, as predicted by the model, using daily data over the period of 1998-2008. The relationship is found to be particularly strong during the recent period of financial distress (2007-2008). The measure of sovereign credit risk is computed as the daily average of JPMorgan EMBI+ sovereign spreads for Brazil, Bulgaria, Ecuador, Mexico, Panama, Peru, Philippines, Russia, and Venezuela. Equity return volatility is estimated with a GARCH(1,1) model on S&P500 returns. The positive relationship between these two variables has induced the empirical literature to conclude on a single direction of causality, which is that equity return volatility in the U.S. explains sovereign credit spreads in foreign markets.[9] The model in this paper offers a complementary explanation: the risk of sovereign default affects the level of equity return volatility in the U.S.

This paper builds on a number of models belonging to separate strands of literature. The two-country, two-good consumption-based asset-pricing model used in this paper is essentially that of Pavlova and Rigobon (2007, 2008), which is based on the works of Helpman and Razin (1978), Cole and Obstfeld (1991), Dumas (1992), and Zapatero (1995). The theoretical contribution of the present paper is to introduce levered firms, governments, and endogenous default decisions into this framework. The modeling of the Foreign government, which issues some debt and decides the timing of the default, follows Gibson and Sundaresan (2001), François (2006), Arellano (2008), Jeanneret (2009), and Yue (2010), among others. While sovereign default is opportunistic in these studies, the present paper assumes that sovereign default occurs when the fiscal revenues become insufficient to cover the debt service. Hence, a sovereign default is triggered by the inability rather than the unwillingness to pay. By assumption, defaulting causes local contraction in economic growth. This output cost of sovereign debt default is also present in the works of Cohen and Sachs (1986), Bulow and Rogoff (1989), Arellano (2008), Andrade (2009), Guimaraes (2009), Hatchondo and Martinez (2009), and Yue (2010), among others.[10]

[9] Various empirical studies document a positive relationship between sovereign credit spreads in emerging markets and equity market volatility in the U.S. as measured by the option-implied volatility index on the S&P500 (VIX). See, for example, McGuire and Schrijvers (2003), González-Rozada and Yeyati (2008), Pan and Singleton (2008), Remolona, Scatigna, and Wu (2008), Hilscher and Nosbusch (2009), and Longstaff et al. (2009). To date, this literature has focused on how sovereign credit spreads relate to the VIX, which is a forward-looking measure, rather than to equity return volatility, which is a realized volatility measure. However, I provide evidence that the positive relationship between sovereign credit risk and the VIX arises from the positive relationship between sovereign credit risk and equity return volatility. Equity return volatility explains 80% of the variation of the VIX. The remaining part is attributed to a volatility risk premium, which has an unclear relationship with sovereign credit spreads.

[10] It is not clear, thus far, what the exact costs of sovereign default are; there is weak empirical support for the default costs due to reputation effect on future borrowing opportunities (Eichengreen, 1987; Gelos, Sahay, and Sandleris, 2004), trade sanctions (Rose, 2005; Martinez and Sandleris, 2008), and armed interventions since World War II (Sturzenegger and Zettelmeyer, 2006). However, sovereign default seems to weaken the domestic financial system and thereby increase the probability of banking crisis (De Paoli, Hoggarth, and Saporta, 2006; Sturzenegger and Zettelmeyer, 2006; Borensztein and Panizza, 2008). As major creditors of the government, domestic banks may thus be prevented from competing their intermediary duties of providing liquidity and credit to the economy.

The evaluation of firm assets builds upon the corporate finance literature (e.g., Mello and Parsons (1992), Leland (1994, 1998), and Morellec (2004)). That is, shareholders select the default policy that maximizes the value of equity by trading off the tax benefits of debt and bankruptcy costs in default. This paper also relates to Hackbarth, Miao, and Morellec (2006), Bhamra, Kuehn, and Strebulaev (2010a,b), and Chen (2010), who analyzed how macroeconomic conditions affect corporate capital structure decisions and the evaluation of assets; in this paper, the sovereign default triggers the change of macroeconomic regime, which reduces the valuation of future firm earnings through a contraction in output growth. A new outcome of the present paper is that the probability of sovereign default affects a firm's probability of defaulting, which thereby reduces the value of its assets. The value of a firm's assets then depends on this firm's decision to default before or after the sovereign defaults, which is determined *ex ante* by shareholders to maximize the value of equity. The Foreign government's default decision and the evaluation of Home asset prices are then closely related.

Finally, the paper accounts for the role of the real exchange rate in the evaluation of asset prices through the effect on sovereign and corporate credit risk. The reason is that the balance sheets are skewed towards debt denominated in a world basket and revenues denominated in the local good. That is, depreciation of the real exchange rate has a negative balance sheet effect because it decreases the worth of both government fiscal revenues and corporate earnings relative to their level of debt, thereby reducing their capacity to honor their debt.[11] To my knowledge, this paper is the first attempt to account for the interactions between the foreign exchange market, international corporate asset prices, and sovereign default risk.

The remainder of the paper is organized as follows. Section 2 outlines a two-country consumption-based asset-pricing model with endogenous default decisions. Section 3 offers theoretical predictions on equity return volatility and discuss the calibration of the parameters. Section 4 consists of structural estimation of the model. Section 5 offers an empirical analysis of the relationship between equity return volatility and sovereign credit risk. I conclude my analysis in Section 6.

[11] Empirical evidence on the relationship between exchange rate depreciation and default include, for example, Reinhart (2002), De Paoli, Hoggarth, and Saporta (2006), and Bordo, Meissner, and Stuckler (2009). In addition, Longstaff et al. (2009) provide recent empirical evidence that, after controlling for large set of global and local macroeconomic factors, sovereign credit risk increases as the sovereign's currency depreciates relative to the U.S. dollar.

2 The Model

The world I model consists of two types of countries, namely, Home and Foreign. Home is a large country with a default-free government and a firm. Foreign is a small market economy with a defaultable government and a firm. Financial markets are complete before and after default. The tax environment consists of a constant tax rate for corporate income and a zero tax rate for individual income. All parameters in the model are assumed to be common knowledge.

2.1 Structure of the Economy

Each country consists of a representative firm that raises revenues by producing a country-specific perishable good. There is a large number of infinitely-living households with logarithmic preferences in both countries. They are the owners and the lenders of the firms and the lenders of the governments. These households receive the produced goods, which are then traded across countries and consumed. In equilibrium, households do not save. The real exchange rate, which is equal to the terms of trade, is defined in terms of the price of the Home good per unit of the price of the Foreign good. Both countries are subject to production shocks, which are propagated internationally through the real exchange rate. A shock in a country is then perfectly shared with the other country.

Each government raises fiscal revenues by taxing the value of its domestic firm's earnings. While the debt issued by the Home government is risk-free, the Foreign government can default on its debt obligation. It does so when the fiscal revenues cannot meet the required coupon payment. Therefore, the Foreign country's creditworthiness essentially depends on the level of the Foreign firm's earnings. The Foreign government also plays an important role in the path of the Foreign country through its decision to issue and default on its debt. On one hand, the issuance of greater sovereign debt allows for the fostering of production growth, which is beneficial for the Foreign firm's earnings. On the other hand, the increase in indebtedness raises the risk of a sovereign default.

In the event of default, the Foreign country enters a recession, which is characterized by a fall in the production growth rate. Thus, sovereign default occurs after negative economic shocks and induces a significant cost for subsequent economic activity. Avoidance of this default cost in terms of economic performance, in particular for future fiscal revenues, is the sovereign country's motivation not to default. The fall in the Foreign country's growth rate also has adverse consequences for the Home firm revenues through unfavorable real exchange rate adjustments. Sovereign credit risk thus affects the evaluation of a firm's assets in both countries. The absence of regime in the Home country arises from the assumption that the government debt in that country is risk-free.

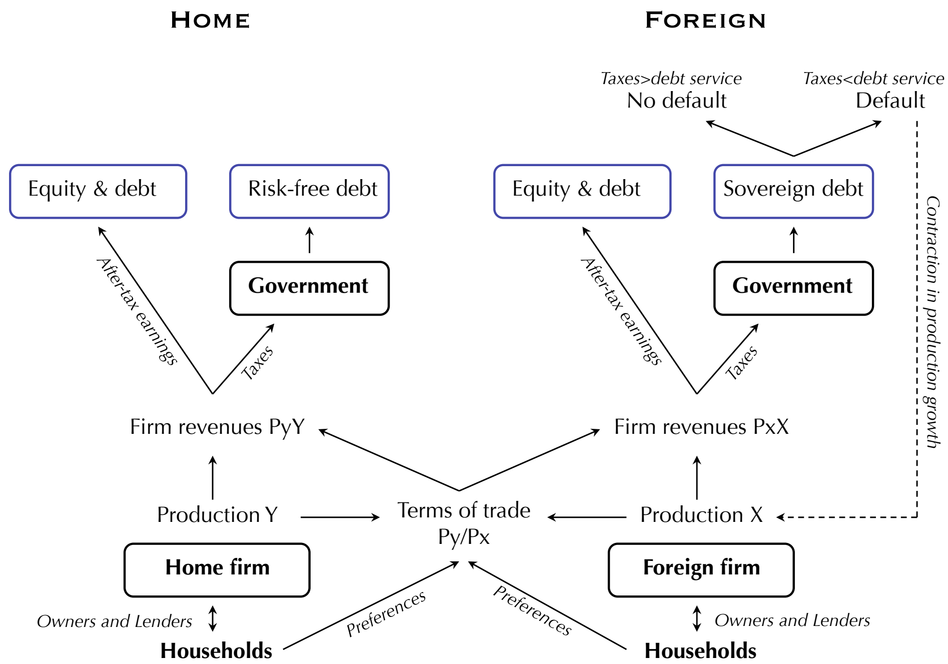

Figure 1: **Structure of the Model.**

Because firms pay taxes on their earnings, they have an incentive to issue debt. Firms are then financed by equity and debt. A firm is liquidated when it defaults on its debt obligations. Shareholders decide whether the firm defaults before or after the government defaults. Default is triggered by the shareholder decision to optimally cease injecting funds into the firm. At that time, a new representative firm with identical value and level of debt emerges. The bankruptcy costs upon default consist of liquidation fees paid to a third party (e.g., lawyers) that are subject to taxes. The government raises taxes from the new firm's earnings and from the third party's gain after the firm's default. Therefore, there is continuity in production, consumption, and fiscal revenues.

2.2 Dynamics of Production and Macroeconomic Regimes

Let Y_t denote the perpetual stream of output produced by the representative firm located in the Home country at time t, which evolves according to the process

$$\frac{dY_t}{Y_t} = \theta_y dt + \sigma_y dW_t^y \tag{1}$$

where W_t^y is a Brownian motion defined on the probability space $(\Omega, \mathscr{F}, \mathbb{P})$. The standard filtration of W_t^y is $F_y = \{\mathscr{F}_t : t \geq 0\}$. The conditional moments θ_y and σ_y represent the expected growth rate and the volatility of Home production.

The Foreign country is characterized by two different states of growth, namely, a normal regime H until the Foreign government defaults on its debt and a low, or recession, regime L after the default event. The dynamics of the perpetual stream of output generated by the Foreign firm is governed by the process

$$\frac{dX_t}{X_t} = \theta_{x,i} dt + \sigma_x dW_t^x, \quad i = \{L, H\} \tag{2}$$

where W_t^x is a Brownian motion independent of W_t^y, which generates idiosyncratic shocks specific to the Foreign country, defined on the probability space $(\Omega, \mathscr{F}, \mathbb{P})$. The standard filtration of W_t^x is $F_x = \{\mathscr{F}_t : t \geq 0\}$. The firm's idiosyncratic volatility is denoted by σ_x. Finally, the growth rate $\theta_{x,i}$ is defined by

$$\theta_{x,i} = \bar{\theta}_{x,i} + \theta_{x,c} C, \quad i = \{L, H\} \tag{3}$$

where $\bar{\theta}_{x,i}$ is the growth rate of output that prevails in the country in the absence of sovereign debt. The growth rate is lower in recession than in normal times, such that $\theta_{x,H} - \theta_{x,L} = \bar{\theta}_{x,H} - \bar{\theta}_{x,L} = \Delta\theta > 0$.[12] I model the change of the regime as an endogenous decision of the government. I also assume that sovereign borrowing enhances economic growth through higher productivity growth, with $\theta_{x,c} > 0$.[13] The fostering of economic growth is thus the government's motivation to issue debt of coupon payment C.

2.3 Investor Preferences and Consumption

The representative household has logarithmic preferences, which allow for closed-form solutions for consumption allocations and the real exchange rate, as well as ensure a constant marginal rate of substitution between goods. There is heterogeneity in consumer tastes to capture the possible home bias in the consumption baskets. The weights of the Foreign good in the utility function of the Foreign and Home households are expressed by a_x and a_y, respectively.

I determine the equilibrium allocation by solving the world social planner's problem to ensure Pareto optimality, which is similar to Pavlova and Rigobon (2007). The initial wealth of the representative

[12] Periods of recession are typically associated with a rise in macroeconomic volatility. However, I explicitly do not consider a regime switch in macroeconomic volatility to avoid generating time-variation in the level of equity return volatility in a rather adhoc fashion.

[13] Pattillo, Poirson, and Ricci (2004) analyze 61 developing countries over the period 1969-1998 and find strong empirical support for the impact of debt on economic growth, in particular through total factor productivity growth.

household of each country is such that the central-planning welfare function allocates weights of λ_x and $\lambda_y \equiv 1-\lambda_x$ to the utility levels of the Foreign and Home households, respectively. Accordingly, the planner chooses country consumption so as to maximize the weighted sum of the utilities of the representative agents:

$$U \equiv Max \ \mathbb{E}_t \int_0^\infty e^{-\rho t} \lambda_x \left\{ a_x log(C_{xx,t}) + (1-a_x) log(C_{xy,t}) \right\} dt \tag{4}$$

$$+\mathbb{E}_t \int_0^\infty e^{-\rho t} \lambda_y \left\{ a_y log(C_{yx,t}) + (1-a_y) log(C_{yy,t}) \right\} dt \tag{5}$$

subject to the resource contraints

$$C_{xx,t} + C_{yx,t} = X_t \tag{6}$$

$$C_{yy,t} + C_{xy,t} = Y_t \tag{7}$$

where ρ is the rate of time preference, and C_{kl} denotes consumption of good l by the representative agent of country k. The optimal allocation of consumption is determined by

$$C_{xx,t} = \frac{\lambda_x a_x}{\lambda_y a_y + \lambda_x a_x} X_t, \ C_{yx,t} = \frac{\lambda_y a_y}{\lambda_y a_y + \lambda_x a_x} X_t, \tag{8}$$

$$C_{xy,t} = \frac{\lambda_x (1-a_x)}{\lambda_y (1-a_y) + \lambda_x (1-a_x)} Y_t, \ C_{yy,t} = \frac{\lambda_y (1-a_y)}{\lambda_y (1-a_y) + \lambda_x (1-a_x)} Y_t \tag{9}$$

The prices per unit of the Foreign good X and the Home good Y are denoted by P_x and P_y, respectively. I fix the world numéraire basket to be the Home consumption basket; it is determined by a Cobb-Douglas function of quantities of good Y and X with weights $\alpha = a_x$ and $1-\alpha = 1-a_x$, respectively. I normalize the price of this basket $P_x^{1-\alpha} P_y^{\alpha}$ as equal to unity.[14] Everything is denominated in units of that basket.

2.4 The Exchange Rate

Following Dumas (1992), the real exchange rate S is expressed by the ratio of either country's marginal

[14] An alternative world numéraire basket would be $\alpha Y + (1-\alpha) X$ with $\alpha \in (0,1)$. However, such a basket is much less tractable than the basket suggested in this paper when computing asset prices; it does not allow for analytical solutions of the first two moments of equity returns.

utilities of the Foreign and Home goods (see *Appendix 7.1*):[15]

$$S_t = \frac{P_{y,t}}{P_{x,t}} = \frac{\frac{\lambda_y \partial u_y(C_{yy,t}, C_{yx,t})}{\partial C_{yy,t}}}{\frac{\lambda_x \partial u_x(C_{xx,t}, C_{xy,t})}{\partial C_{xx,t}}} = \overline{S}\frac{X_t}{Y_t} \tag{10}$$

with

$$\overline{S} = \frac{\lambda_y (1 - a_y) + \lambda_x (1 - a_x)}{\lambda_x a_x + \lambda_y a_y} \tag{11}$$

From Itô's lemma, the exchange rate S follows the process

$$\frac{dS_t}{S_t} = \theta_{s,i} dt + \sigma_x dW_t^x - \sigma_y dW_t^y, \quad i = \{L, H\} \tag{12}$$

with

$$\theta_{s,i} = r_{x,i} - r_y + \sigma_x^2 \tag{13}$$

The mean appreciation rate θ_s is the difference between the Foreign risk-free interest rate and the Home risk-free interest rate $r_x = \rho + \theta_x - \sigma_x^2$ and $r_y = \rho + \theta_y - \sigma_y^2$, respectively, augmented by some compensation for bearing aggregate output risk.[16] When a country experiences an output shock, the exchange rate adjusts exactly to offset any net payoff. This exchange rate satisfies the no-arbitrage conditions, which prove the redundancy of having a risk-free bond in each country. Interestingly, while the key drivers of the level of the exchange rate are the relative preferences for goods and the central planner's welfare weights, the dynamics (i.e., time-variation) of the exchange rate solely depend on the dynamics of macroeconomic fundamentals. The exchange rate plays an important role in linking asset prices in the two countries.

2.4.1 International Transmission of Economic Shocks

Within the model, the propagation of shocks from one country to another arises from a Ricardian response to economic shocks. To see this, consider a negative shock in the Home country. This shock is accompanied by an increase in the real exchange rate $S = \frac{P_y}{P_x}$ (i.e., an improvement of the terms of trade due to an increase in the price P_y) because the Home good becomes relatively rare. However, the

[15] In competitive equilibrium, the price of one unit of the Foreign good to be delivered at time t in state w is $\xi^x = P_x \xi$ and the price of one unit of the Home good to be delivered at time t in state w is $\xi^x = P_x \xi$, where ξ_t is the state-price density in unit of the world numéraire (see *Appendix 7.1*). Therefore, consistent with Backus et al. (2001), Brandt et al. (2006), and Bakshi et al. (2008), the exchange rate can also be expressed as the ratio of ξ^y and ξ^x. Given the preferences of agents, prices are unique, as is the ratio of the two.

[16] There exists only one risk-free asset, namely, the Home government bond denominated in the Home good. As such, the risk-free rate r_x represents the rate of return on this risk-free bond when measured in units of the Foreign good.

improvement in the terms of trade implies a decrease in the relative price of the Foreign good P_x in unit of the world numéraire, leading to a fall in the value of Foreign output $P_x X$, although the quantity of output X remains unchanged. Firm revenues in both countries thus move in the same direction in response to an economic shock in one of the countries despite the independence of the output innovations of a country.[17]

2.5 State-Price Density

The state-price density ξ can be used to compute prices of any contingent asset, irrespective of the good in which the asset is denominated. It follows the process defined by (see *Appendix 7.3*)

$$\frac{d\xi_t}{\xi_t} = -r_{z,i}dt - \sigma_{z,y}dW_t^y - \sigma_{z,x}dW_t^x, \quad i = \{L, H\} \tag{14}$$

where r_z is the risk-free rate prevailing under the basket numéraire, given by

$$r_{z,i} = \rho + \theta_{z,i} - \left(\sigma_{z,x}^2 + \sigma_{z,y}^2\right) \tag{15}$$

and

$$\theta_{z,i} = \theta_{x,i} - \alpha\theta_{s,i} + \frac{\alpha(1+\alpha)}{2}\left(\sigma_x^2 + \sigma_y^2\right) - \alpha\sigma_x^2 \tag{16}$$

$$\sigma_{z,x} = (1-\alpha)\sigma_x \tag{17}$$

$$\sigma_{z,y} = \alpha\sigma_y \tag{18}$$

The state-price density is driven by the same set of shocks that drive aggregate output in the Home and the Foreign countries. As systematic shocks affect the marginal utility of investors through today's consumption levels, the risk price of these shocks rises with economic volatility. Either a higher level of uncertainty or a lower economic growth rate induce greater demand for the risk-free government bond. This flight-to-quality response lowers the risk-free interest rate in recession.

2.6 The Foreign Government

The government of the Foreign country raises fiscal revenues by taxing the value of the Foreign firm's earnings at the tax rate τ net of the tax-deductible debt service of the firm. The capacity to service

[17] This results follows Helpman and Razin (1978), Cole and Obstfeld (1991), Zapatero (1995), and Pavlova and Rigobon (2007, 2008). A natural implication of this prediction is the co-movement in international equity markets, which is documented by Karolyi and Stulz (1996), Forbes and Rigobon (2002), Bae, Karolyi, and Stulz (2003), Hartmann, Straetmans, and de Vries (2004), and Andersen et al. (2007), among others.

this debt depends on the dynamics of the government revenues $R = \tau\left(Z - K - C_f\right)_{t\geq 0}$, where $Z \equiv P_x X$ denotes the firm's revenues; K is the firm's operating costs per unit of time (e.g., constant wages paid to workers); and τC_f is the firm's tax-shield. All variables are measured in units of the Home consumption basket. From Itô's formula, Foreign firm revenues Z satisfy (see *Appendix 7.2*)

$$\frac{dZ_t}{Z_t} = \theta_{z,i}dt + \sigma_{z,x}dW_t^x + \sigma_{z,y}dW_t^y, \quad i = \{L, H\} \tag{19}$$

The sovereign defaults on its debt obligation when the fiscal revenues cannot meet the required coupon payment C, such that $R = \tau\left(Z - K - C_f\right) \leq C$. In contrast to corporations, sovereigns are unable to issue additional financial claims to cover a revenue shortage. In addition, households are unwilling to freely give up part of consumption to finance the government budget deficit. Therefore, the sovereign defaults when the firm's revenues fall below the default threshold

$$Z^D = \frac{C}{\tau} + K + C_f \tag{20}$$

at time $T(Z^D) = \inf\{t \geq 0 \mid Z_t \leq Z^D\}$.[18] The default boundary Z^D characterizes the sovereign's default policy, which is Pareto optimal from market completeness. The likelihood of defaulting increases when Foreign output decreases and/or when the terms of trade depreciate.[19] The sovereign is also more likely to default, and thus trigger an contraction in economic growth, when the level of sovereign debt C, the firm's operating costs K, and the firm's level of debt C_f are high, and when the level of tax rate τ is low.[20]

2.6.1 The Price of Sovereign Debt

The lenders anticipate the behavior of the sovereign and reflect the associated wealth loss in the pricing of the sovereign debt. They require a risk-free rate of return r_z per unit of time. The sovereign pays a constant total coupon C at each moment in time. Upon defaulting, the sovereign and its lenders restructure the terms of the debt contract and agree on a reduction $0 \leq \phi \leq 1$ of the debt service.[21] I

[18] Sovereigns do not tend to default once but several times (Reinhart et al., 2003). Generalizing the framework to account for multiple defaults is left for future research.

[19] Hilscher and Nosbusch (2009) and the references therein show empirical evidence that terms of trade fluctuations are a significant predictor of sovereign credit spread and, thus, of the probability of defaulting. Recent examples are found in Russia and Ecuador, where falling export prices (e.g., oil prices) led to a deterioration of the macroeconomic and fiscal conditions and a sovereign default in 1998 and 1999, respectively.

[20] Pattillo, Poirson, and Ricci (2004) find the relationship between sovereign debt and growth to be nonlinear (the "debt Laffer curve"): for low levels of debt, greater debt fosters growth; but for high levels of debt, additional debt has negative impact on growth through the increased likelihood that in the future debt will be larger than the country's repayment ability (the "debt overhang" effect).

[21] For simplicity, the recovery rate is assumed to be exogenous. Alternatively, Jeanneret (2009) and Yue (2010) develop a model that accounts for endogenous renegotiation upon default. Once default occurs, the sovereign country and its lenders renegotiate the terms of their debt contracts, which determine the recovery rate. In both studies, the outcome of the

assume, for simplicity, that the sovereign cannot scale up its debt after default. The value of sovereign debt is (see *Appendix 7.4*)

$$D(Z) = \mathbb{E}_0^{\mathbb{Q}}\left[\int_0^{T(Z^D)} Ce^{-r_{z,H}t}dt\right] + \mathbb{E}_0^{\mathbb{Q}}\left[\int_{T(Z^D)}^{\infty} (1-\phi)Ce^{-r_{z,L}t}dt\right] \tag{21}$$

The first term is equal to the present value of the promised cash flows C to debtholders until default. The second term corresponds to the present value of the recovered value of the debt after the government has defaulted. The market credit spread is determined by $CS(Z) \equiv \frac{C}{D(Z)} - r_z$.

2.6.2 Economic Conditions and Sovereign Credit Risk

A negative output shock in the Home country is transmitted internationally through a terms of trade response and, thus, decreases Foreign firm revenues Z. The reduction in taxable corporate income in the Foreign country reduces the level of fiscal revenues $R = \tau\left(Z - K - C_f\right)$ necessary to service the sovereign debt C, which raises the likelihood of defaulting and thus the sovereign credit spread CS. Sovereign credit risk is thus high during adverse economic conditions in either the Home or the Foreign country.

2.7 The Firms

In this section, I determine the value of the firms' assets, which depend on whether firms default before or after the government defaults. The evaluation of Home and Foreign firms are obtained under identical assumptions. However, the value of the assets differs across countries because of heterogeneous levels of leverage C_f and operating costs K.

Within the model, markets are frictionless. I assume that the management acts in the best interests of the shareholders. I consider an exogenous infinite-maturity debt structure in a stationary environment. On the one hand, the perpetuity feature is shared with numerous other models, including those presented in Fischer, Heinkel, and Zechner (1989), Leland (1994), and Strebulaev (2007). On the other hand, the level of debt is assumed to be exogonous because, most of the time, firm leverage deviates from "optimal leverage".[22] I first discuss the firm value upon default, then derive the values of corporate debt and equity, and finally determine the default thresholds selected by shareholders.

restructuring process involves a Nash bargaining solution. However, the consideration of an endogenous recovery rate in the present analysis would not substantially alter the core result of this paper.

[22]For reference, see Strebulaev (2007) and Bhamra et al. (2010b). Because of issuance costs, most firms optimally refinance only periodically. Hence, as shown by Strebulaev (2007), if leverage deviates from its target substantially, then the response of firms to changes in economic conditions will not be in line with the predictions of comparative statics at refinancing points.

2.7.1 Firm Value in Default

The shareholders strategically declare default on their debt obligation when firm revenues Z fall below the default boundary Z_f^D at time $T(Z_f^D) = \inf\{t \geq 0 \mid Z_t \leq Z_f^D\}$. For the Home firm, the revenues $\overline{S}Z$ replace the Foreign firm revenues Z.[23] I follow Mello and Parsons (1992) and Leland (1994) and presume that the value of the firm upon default is $(1-\eta)V_u\left(Z_f^D\right)$, where $\eta \in (0,1)$ is the fraction of asset value lost in default, and $V_u\left(Z_f^D\right)$ is the value of the unlevered firm's assets.

2.7.2 Valuation of Firm Debt

I start by determining the value of corporate debt for a given default boundary. The debt has value equal to the sum of the present value of the earnings that accrue to debtholders until the default time and the change in this present value that arises in default. The expected value of the firm's cash flows is discounted with the risk-free rate r_z under the risk-neutral probability measure. The risk-neutral measure $\mathbb{Q}$ adjusts for risks by changing the distributions of shocks. Cash flows are risky for an investor when they are positively correlated with its marginal utility, which is accounted for by lowering the expected growth rate under $\mathbb{Q}$ (see *Appendix 7.2.1*).

The value of firm debt is (see *Appendix 7.5.1*)

$$D_f(Z)\mid_{T^- \leq T^+} \quad = \quad \mathbb{E}_0^{\mathbb{Q}}\left[\int_0^{T^-} C_f e^{-r_{z,H}t}dt + e^{-r_{z,H}T^-} D_f(Z)\mid_{t=T^-}\right] \tag{22}$$

with

$$D_f(Z)\mid_{t=T^-} \quad = \quad \mathbb{E}_{T^-}^{\mathbb{Q}}\left[\int_{T^-}^{T^+} C_f \mathbf{1}_{\left[T\left(Z_f^D\right) > T\left(Z^D\right)\right]} e^{-r_{z,L}t}dt\right] \tag{23}$$

$$+\mathbb{E}_{T^-}^{\mathbb{Q}}\left[\int_{T^-}^{T^+} (1-\eta)(1-\tau)\left(Z_t - K\right)\mathbf{1}_{\left[T\left(Z_f^D\right) \leq T\left(Z^D\right)\right]} e^{-r_{z,H}t}dt\right] \tag{24}$$

$$+\mathbb{E}_{T^-}^{\mathbb{Q}}\left[\int_{T^+}^{\infty} (1-\eta)(1-\tau)\left(Z_t - K\right) e^{-r_{z,L}t}dt\right] \tag{25}$$

where $T^+ = T\left(Z_f^D\right) \vee T\left(Z^D\right)$, $T^- = T\left(Z_f^D\right) \wedge T\left(Z^D\right)$, and $\mathbf{1}_{[a]}$ is an indicator function equals to one if the function a is true and zero, otherwise. Consider, for example, that the firm defaults after the Foreign government defaults, such that $T\left(Z_f^D\right) > T\left(Z^D\right)$, $T^+ = T\left(Z_f^D\right)$, and $T^- = T\left(Z^D\right)$. The value of debt is determined by the present value of the promised coupon payment C_f discounted at the risk-free rate

[23] Under the basket numéraire, Home firm revenues are $P_yY = \overline{S}Z$, while Foreign firm revenues are $P_xX = Z$.

$r_{z,h}$ until sovereign default at time $T\left(Z^D\right)$, $\mathbb{E}_0^{\mathbb{Q}}\left[\int_0^{T\left(Z^D\right)} C_f e^{-r_{z,H}t}dt\right]$, plus the present value of debt at the time of sovereign default, $\mathbb{E}_0^{\mathbb{Q}}\left[e^{-r_{z,H}T\left(Z^D\right)}D_f(Z)\mid_{t=T(Z^D)}\right]$. The value of debt at the time of sovereign default, $D_f(Z)\mid_{t=T(Z^D)}$, is equal to the present value of the promised coupon payment C_f discounted at the risk-free rate $r_{z,l}$ until the firm defaults at time $T\left(Z_f^D\right)$, $\mathbb{E}_{T(Z^D)}^{\mathbb{Q}}\left[\int_{T(Z^D)}^{T\left(Z_f^D\right)} C_f e^{-r_{z,L}t}dt\right]$, plus the value of the firm upon liquidation, which is determined by the unlevered firm value net of liquidation costs, $\mathbb{E}_{T(Z^D)}^{\mathbb{Q}}\left[\int_{T\left(Z_f^D\right)}^{\infty}(1-\eta)(1-\tau)\left(Z_t-K\right)e^{-r_{z,L}t}dt\right]$.

2.7.3 Total Firm Value

The total value of the levered firm equals the unlimited liability value of a perpetual claim to the current flow of after-tax earnings $(1-\tau)\left(Z_t-K\right)$, plus the present value of a perpetual claim to the current flow of tax benefits of debt τC_f, minus the change in those present values arising in default due to the liquidation costs η. Thus, the levered firm value $V(Z)$ satisfies (see *Appendix 7.5.2*)

$$V(Z)\mid_{T^-\leq T^+} = \mathbb{E}_0^{\mathbb{Q}}\left[\int_0^{T^-}\left((1-\tau)\left(Z_t-K\right)+\tau C_f\right)e^{-r_{z,H}t}dt\right] \quad (26)$$

$$+\mathbb{E}_0^{\mathbb{Q}}\left[e^{-r_{z,H}T^-}V(Z)\mid_{t=T^-}\right] \quad (27)$$

with

$$V(Z)\mid_{t=T^-} = \mathbb{E}_{T^-}^{\mathbb{Q}}\left[\int_{T^-}^{T^\infty}\left((1-\tau)\left(Z_t-K\right)+\tau C_f\right)\mathbf{1}_{\left[T\left(Z_f^D\right)>T(Z^D)\right]}e^{-r_{z,L}t}dt\right] \quad (28)$$

$$-\mathbb{E}_{T^-}^{\mathbb{Q}}\left[\int_{T^+}^{T^\infty}\left(\eta(1-\tau)\left(Z_t-K\right)+\tau C_f\right)\mathbf{1}_{\left[T\left(Z_f^D\right)>T(Z^D)\right]}e^{-r_{z,L}t}dt\right] \quad (29)$$

$$+\mathbb{E}_{T^-}^{\mathbb{Q}}\left[\int_{T^-}^{T^+}(1-\eta)(1-\tau)\left(Z_t-K\right)\mathbf{1}_{\left[T\left(Z_f^D\right)\leq T(Z^D)\right]}e^{-r_{z,H}t}dt\right] \quad (30)$$

$$+\mathbb{E}_{T^-}^{\mathbb{Q}}\left[\int_{T^+}^{\infty}(1-\eta)(1-\tau)\left(Z_t-K\right)e^{-r_{z,L}t}dt\right] \quad (31)$$

As an example, consider, as before, that the firm defaults after the Foreign government defaults, such that $T\left(Z_f^D\right)>T\left(Z^D\right)$, $T^+=T\left(Z_f^D\right)$, and $T^-=T\left(Z^D\right)$. The value of the firm is determined by the present value of the sum of after-tax earnings $(1-\tau)\left(Z_t-K\right)$ and tax benefits of debt τC_f discounted at the risk-free rate $r_{z,h}$ until sovereign default at time $T\left(Z^D\right)$, $\mathbb{E}_0^{\mathbb{Q}}\left[\int_0^{T\left(Z^D\right)}\left((1-\tau)\left(Z_t-K\right)+\tau C_f\right)e^{-r_{z,H}t}dt\right]$. The firm value at the time of sovereign default, $V(Z)\mid_{t=T(Z^D)}$, is equal to the present value of the sum of perpetual after-tax earnings $(1-\tau)\left(Z_t-K\right)$ and tax benefits of debt τC_f discounted at the risk-free rate $r_{z,l}$, $\mathbb{E}^{\mathbb{Q}}_{T(Z^D)}\left[\int_{T(Z^D)}^{\infty}\left(\left(1-\tau\right)\left(Z_t-K\right)+\tau C_f\right)e^{-r_{z,L}t}dt\right]$, net of the present value of the sum of the unlevered firm value lost upon liquidation $\eta(1-\tau)\left(Z_t-K\right)$ and the tax benefits of debt τC_f, $\mathbb{E}^{\mathbb{Q}}_{T(Z^D)}\left[\int_{T\left(Z_f^D\right)}^{\infty}\left(\eta(1-\tau)\left(Z_t-K\right)+\tau C_f\right)e^{-r_{z,L}t}dt\right]$.

In the absence of arbitrage, the levered firm value equals the sum of debt and equity values. Hence, the value of the firm's equity $E(Z)$ is determined by

$$E(Z)\mid_{T^-\leq T^+}=V(Z)\mid_{T^-\leq T^+}-D_f(Z)\mid_{T^-\leq T^+} \tag{32}$$

2.7.4 The Firm's Decision to Default

Default is triggered by the shareholder decision to optimally cease injecting funds into the firm, following Leland (1998) and Morellec (2004), among others. The firm's default policy is characterized by the default boundary $Z_f^D\mid_{T^-\leq T^+}$, which maximizes the shareholder value such that the smooth-pasting condition $\frac{\partial\left[E(Z)\mid_{T^-\leq T^+}\right]}{\partial Z}\mid_{Z=Z_f^D\mid_{T^-\leq T^+}}=0$ is satisfied (see *Appendix 7.5.3* for the value of Z_f^D). The decision to default before or after the government is determined *ex ante* to maximize the shareholder value. The optimal default boundary thus satisfies

$$Z_f^D = \begin{cases} Z_f^D\mid_{T\left(Z_f^D\right)\leq T(Z^D)} \text{ if } E(Z)\mid_{T\left(Z_f^D\right)\leq T(Z^D)}\geq E(Z)\mid_{T\left(Z_f^D\right)>T(Z^D)} \\ Z_f^D\mid_{T(Z^D)<T\left(Z_f^D\right)} \text{ if } E(Z)\mid_{T\left(Z_f^D\right)\leq T(Z^D)}< E(Z)\mid_{T\left(Z_f^D\right)>T(Z^D)} \end{cases} \tag{33}$$

The above rule determines the conditions under which the firm defaults before or after the government. The model predicts that the firm tends to default first when i) the firm is relatively more leveraged than the government (i.e., high C_f and low C); ii) the firm has large operating costs (i.e., high K); iii) the loss of economic growth rate upon the change of regime is important (i.e., high $\Delta\theta$); iv) volatility in either country's economic fundamentals is low (i.e., low σ_x and σ_y); v) either economy grows rapidly (i.e., high θ_x and θ_y); and finally, vi) when the corporate tax burden is severe (i.e., high τ).

3 Theoretical Predictions on Equity Return Volatility

This section uses the model developed in this paper to analyze the drivers of equity return volatility in the Home country, which is given by (see *Appendix 7.5.4*)[24]

$$\sigma_{E(Z)} \quad = \frac{\frac{\partial E(Z)}{\partial Z} Z}{E(Z)} \sqrt{\sigma_{z,x}^2 + \sigma_{z,y}^2} > \sigma_z \tag{34}$$

Equity return volatility is predicted to depend negatively on the growth rates of output in both countries θ_x and θ_y, and on the corporate tax rate τ. However, equity return volatility is predicted to rise with increasing macroeconomic volatilities of both countries σ_x and σ_y, financial leverage C_f, operational costs K, and sovereign indebtedness C. A negative economic shock increases the volatility of equity returns through a reduction in Home firm revenues. Equity return volatility is thus countercyclical, which is in line with the empirical findings of Schwert (1989), Forbes and Rigobon (2002), Bae et al. (2003), Engle and Rangel (2008), and Engle, Ghysels, and Sohn (2009), among others. Finally, the level of equity return volatility is predicted to be greater than the volatility of the firm's revenues, $\sigma_z = \sqrt{\sigma_{z,x}^2 + \sigma_{z,y}^2}$.

Both the countercyclical nature and the high level of equity return volatility arise from three effects.[25] First, the presence of corporate debt generates the financial leverage introduced by Black (1976) and Christie (1982). When afflicted by a negative output shock, the value of a firm declines, which raises the probability of defaulting and lowers the value of equity (i.e., the junior claim) relative to the value of debt (i.e., the senior claim). The increase in the firm's financial leverage raises the volatility of equity returns. Second, the presence of constant production costs K borne by the firm generates an operational leverage effect. Lev (1974) demonstrated early on that the presence of operating leverage raises the volatility of a firm's earnings, thereby increasing a firm's equity return volatility.

The model suggests a third effect: the presence of sovereign default risk additionally raises the sensitivity of equity returns to economic shocks, which is due to the risk of a drop in the growth rate of firm revenues upon the change of macroeconomic regime. The risk that the Foreign country enters a recession, which is triggered in the model through a sovereign default in that country, decreases the present value of firm revenues in both countries, thereby depressing the value of equity and increasing the volatility of

[24] The analysis focuses on the volatility of equity return when the Home firm defaults after the Foreign government defaults, such that $T\left(Z_f^D\right) > T\left(Z^D\right)$, $T^+ = T\left(Z_f^D\right)$, and $T^- = T\left(Z^D\right)$. Should the firm default before the Foreign government, the value of the firm's equity would be independent of sovereign credit risk. This case is of limited interest.

[25] Alternative explanations of the countercyclicality of equity return volatility include Bansal and Yaron (2004) and Tauchen (2005). These authors argue that investors with a preference for early resolution of uncertainty require compensation, thereby inducing negative co-movements between ex-post returns and volatility. Some models on limited equity market participation such as Basak and Cuoco (1998) are also able to generate asymmetric equity return volatility movements.

equity returns. The model accounts for all three effects.[26] Let us examine now how these effects help explain the level of equity return volatility in the U.S.

3.1 Calibration

In the empirical analysis of this paper, the U.S. represents the Home country, while Brazil represents the Foreign country. Brazil is a natural candidate because it is a large trading partner of the U.S. with sizable sovereign credit risk. In addition, the data on sovereign credit spreads, stock market prices, and industrial production for Brazil cover a longer period than for any other emerging country. I calibrate the model for the means and the standard deviations of Home and Foreign output growths to be equal the U.S. and Brazilian annual growth rates of industrial production, respectively, over the period from June 1994 through December 2008. Industrial production data are take from *Datastream*. The parameter values related to firms are chosen to match the characteristics of representative firms in the U.S. and Brazil, and those related to sovereign debt match the indebtedness level of the government in Brazil. Finally, the central planner weights are chosen to match the relative size of Brazil's economy measured with GDP data with respect to the U.S. The parameter values are presented in Table 1.

3.2 Equity Return Volatility and Economic Shocks

Figure 2 provides an illustration of the sensitivity of equity return volatility to economic shocks, using the parameter values in Table 3.1. It displays and decomposes the level of equity return volatility in the U.S., as predicted by the model.

I consider, as a benchmark case, the model of Pavlova and Rigobon (2007) in the absence of demand shocks, which is essentially the model developed within this paper in the absence of defaultable debt in the firm's balance sheet, of operating costs, and of the risk of a sovereign default. In this case, volatility of Home equity return is constant and equal to the volatility of the revenues. The prediction for the U.S. would be a level of annual volatility of 2.3%, which is much lower than the average level of S&P 500 return volatility over the 1994-2008 period (equal to 15.3%.)

In contrast, the model of the present paper can generate time-varying, countercyclical, and high level of equity return volatility. As illustrated in Figure 2, the level of equity return volatility severely increases i) when the firm is levered, ii) when there are operating costs, and finally iii) when there is a risk of sovereign default abroad. While the effect of the risk of economic contraction upon sovereign

[26] Alternatively, the introduction of portfolio constraints can also increase equity return volatility. For example, Pavlova and Rigobon (2008) show that the presence of a constraint that limits the fraction of wealth at a country's agents may invest in the assets of the other country amplifies the asset price reaction to economic shocks.

Table 1: **Parameter Choices.** This table presents the parameter values adopted for the estimation and simulation. All variables are annualized when applicable.

Variable	Symbol	Value	Source
Preferences			
Time preference	ρ	0.02	Author's assumption
Preference of Foreign households for the Foreign good	a_x	0.75	Author's assumption
Preference of Home Households for the Foreign good	a_y	0.25	Author's assumption
Central planner's weight for the Foreign households	λ_x	0.1	$\frac{\text{GDP}_{\text{Brazil}}}{\text{GDP}_{\text{Brazil}}+\text{GDP}_{\text{US}}}$, average over 1994-2008
Home country			
Growth rate	θ_y	0.01	Average growth rate of industrial production in the U.S. (1994-2008)
Volatility	σ_y	0.02	Growth rate volatility of industrial production in the U.S. (1994-2008)
Initial level of production	Y	100	[Normalization]
Foreign country			
Fixed growth rate	$\bar{\theta_x}$	0.02	Match average growth rate of industrial production in Brazil (1994-2008)
Variable growth rate	$\theta_{x,c}$	0.001	Match average growth rate of industrial production in Brazil (1994-2008)
Volatility	σ_x	0.07	Growth rate volatility of industrial production in Brazil (1994-2008)
Initial level of production	X	7.7	$100\frac{\text{GDP}_{\text{Brazil}}}{\text{GDP}_{\text{US}}}$ in 1994
Government debt			
Debt service	C	1	Match Debt/GDP for Brazil (Sturzenegger and Zettelmeyer, 2006)
Haircut	ϕ	0.66	Moody's (2006)
Firms			
Debt service in Foreign country	C_f	10	Match leverage ratio in Brazil (Lins, 2003)
Debt service in Home country	C_{fy}	20	Match leverage ratio in U.S. (Morellec et al., 2009)
Fixed costs in Foreign country	K	15	Match leverage ratio in Brazil (Lins, 2003)
Fixed costs in Home country	K_y	40	Match leverage ratio in U.S. (Morellec et al., 2009)
Bankruptcy costs	η	0.5	Morellec et al. (2009)
Tax rate	τ	0.3	Morellec et al. (2009)

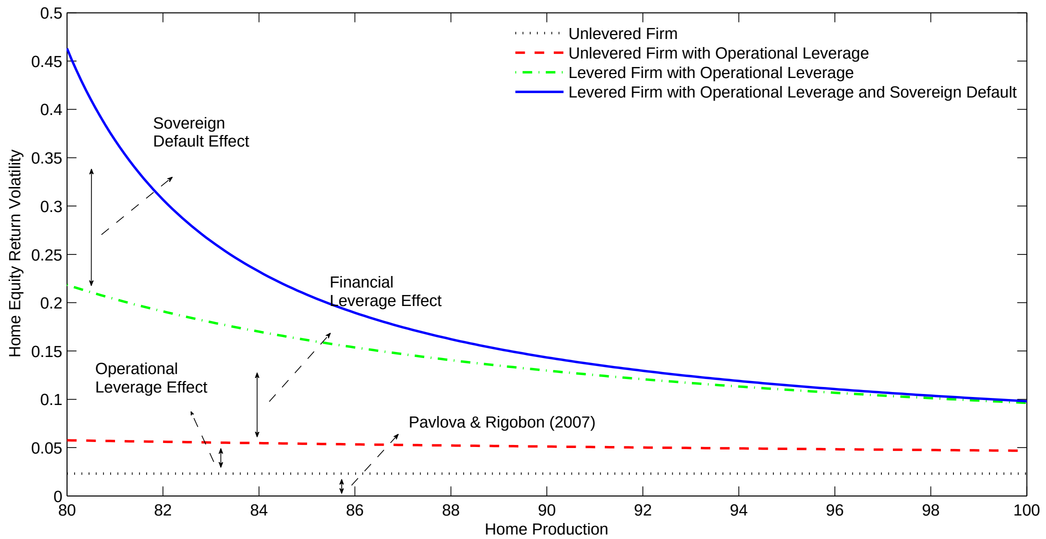

Figure 2: **Equity Return Volatility, Economic Shocks, and Leverage Effects.** This figure shows the effect of economic conditions on the level of equity return volatility, which depends on the presence of financial leverage, of operational leverage, and of sovereign default risk. Equity return volatility is determined by $\sigma_{E(Z)} = \frac{\frac{\partial E(Z)}{\partial Z} Z}{E(Z)} \sqrt{\sigma_{z,x}^2 + \sigma_{z,y}^2}$. The parameters of the models are those presented in Table 3.1 with $\Delta\theta = 0.005$.

default on the level of equity return volatility is marginal in period of high economic growth, this effect appears to be particularly strong during adverse economic conditions (see Figure 3). For example, the proportion of the level of equity return volatility that is explained by the risk of sovereign default abroad rises over 50 percent in Fall 2008. The potential adverse consequences of a sovereign default crisis amplify the effect of shocks on equity return volatility in periods of economic downturns, precisely when the risk of corporate default is high.

The main prediction of the paper is that a higher level of sovereign credit risk increases the level of Home equity return volatility through a higher risk of a contraction in economic growth abroad. This effect arises through two complementary channels. First, a contraction in economic growth abroad affects Home fundamentals through the trade linkages between these two countries. A rise in the risk of an economic slowdown abroad reduces the expected value of future firm exports and thus the value of the firm's assets in the Home country through a depreciation of the real exchange rate. Second, the risk of a contraction in economic growth affects both countries' financial asset prices through a financial contagion channel, due to the common pricing kernel for all financial assets. An increase in the risk of an economic slowdown abroad triggers a portfolio rebalancing towards the risk-free bond. However, at the

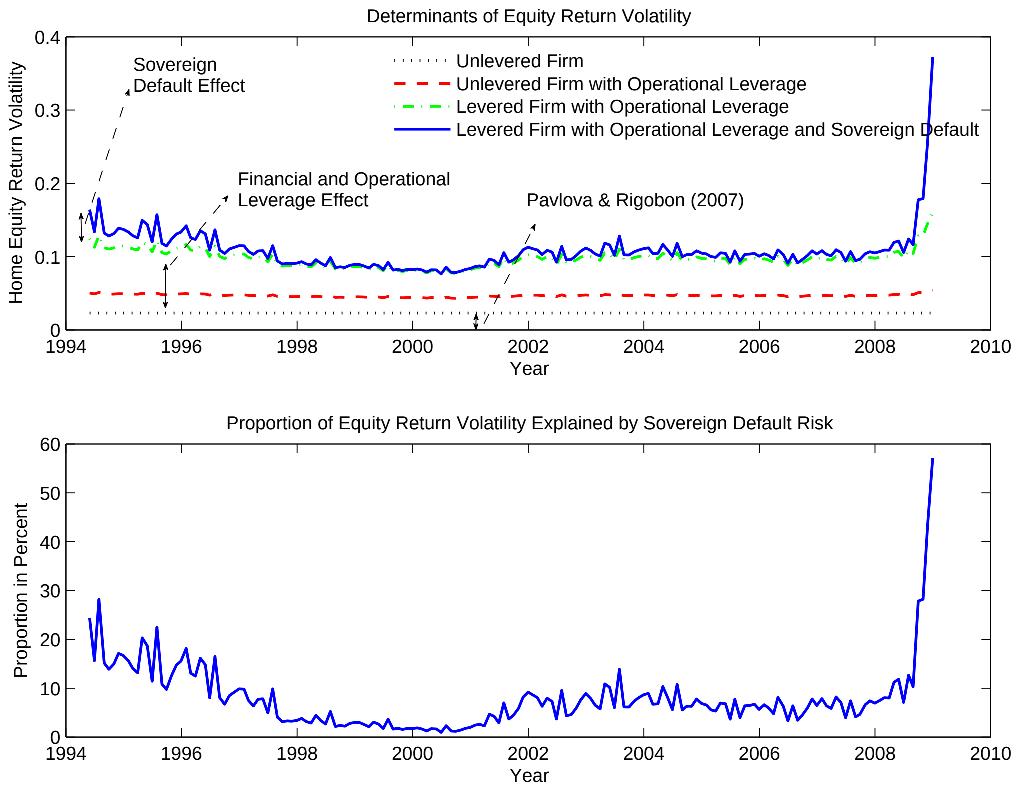

Figure 3: **Decomposition of the U.S. Equity Return Volatility Predicted by the Model, 1994-2008.** The upper panel illustrates the contribution of operational leverage, financial leverage, and the risk of sovereign default in the level of equity return volatility in the U.S., as predicted by the model. The lower panel shows the proportion (in percent) of the level of equity return volatility in the U.S., as predicted by the model, that is explained by the risk of sovereign default in Brazil. The input series are monthly industrial production data over the period 1994-2008.

equilibrium level, no rebalancing takes place because all agents have identical portfolios and they must jointly hold the entire supply of each market. Therefore, as illustrated in Figure 4 (upper panels), the value of all financial assets move downwards to counteract the incentive to rebalance. Eventually, the fall in equity prices in the Home country is accompanied by a rise in the level of equity return volatility when the probability of sovereign default increases, beyond the operational and financial leverage effects (Figure 4, lower panels).

In *Section 4*, I provide a structural estimation of the model and test whether the presence of the expected loss in economic growth upon sovereign default helps explain the level and the volatility of equity returns in the U.S. beyond the financial and operational leverage effects. An empirical analysis in *Section 5* provides strong support for the prediction of the model that sovereign credit risk and Home

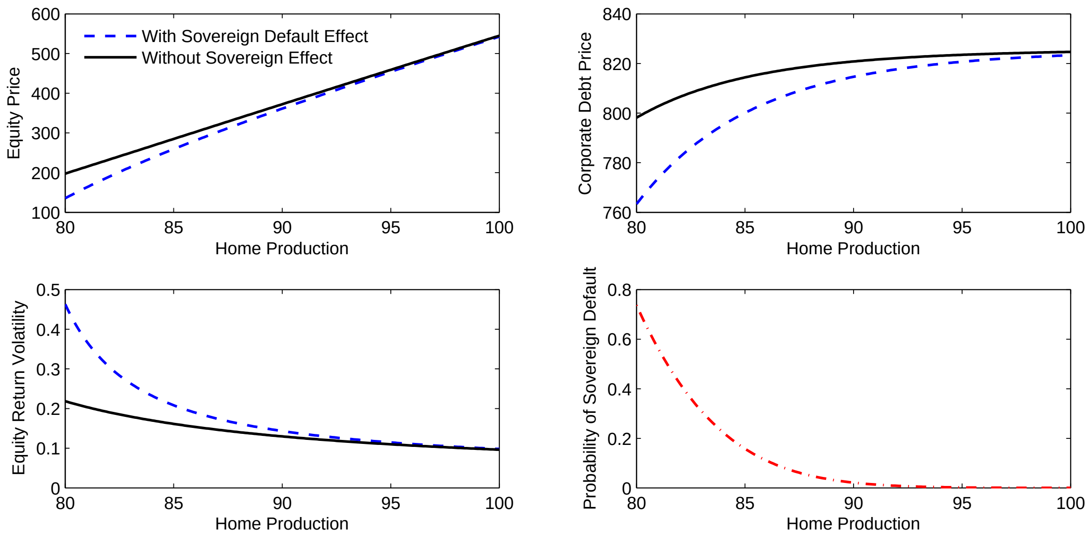

Figure 4: **Corporate Assets, Equity Return Volatility, and Probability of Sovereign Default.** This figure shows the effect of economic conditions on the value of corporate assets (equity and debt), on the level of equity return volatility, as well as on the probability of sovereign default. The figure illustrates these effects with and without the presence of a contraction in economic growth upon sovereign default, where $\Delta\theta$ equals 0.005 and 0, respectively. The other parameters of the models are those presented in Table 3.1.

equity return volatility are countercyclical, as they are determined endogenously by the same economic shocks. Sovereign credit spreads abroad and equity return volatility in the U.S. are then indeed positively related.

4 A Structural Estimation of the Model

The model developed in this paper is based on the assumption that a sovereign default event triggers a local contraction in economic growth, which is then transmitted to the U.S. both through the foreign exchange market and through the financial contagion channel. Should the expected loss in economic growth upon default $\Delta\theta$ be positive, the model predicts that the risk of sovereign default would negatively affect firm asset valuation and thus increase the volatility of equity returns. In this section, I provide a structural test of this hypothesis, which consists of estimating the expected loss in economic growth $\Delta\theta$ upon sovereign default using the generalized method of moments (GMM). I use monthly industrial production data for Brazil and the U.S. to generate the equity prices predicted by the model and the moment conditions. I show that this expected loss in economic growth is both significant and large and thus helps explain the level and the volatility of equity returns in the U.S. and Brazil over the past 15 years. I first describe the data, then discuss the estimation approach, and finally present the results.

4.1 Data

Financial data for this section consist of the S&P500 for the U.S. equity price index, MSCI Brazil for the Brazilian equity price index (measured in U.S. dollars), the JPMorgan EMBI+ spreads Brazil for sovereign credit spreads in Brazil, and finally the CBOE option-implied volatility index on the S&P500. Data are taken from two sources, namely, i) *Datastream* for equity and bond market indices and for the U.S. dollars/Brazilian Reals exchange rate, and ii) *Bloomberg* for the EMBI+ spreads. All series consist of daily or monthly observations from June 1, 1994, to December 31, 2008.

4.2 GMM Estimation and the Choice of Moments

This section describes the econometric approach that I use to estimate the parameter of interest, $\Delta\theta$. The econometric approach consists of testing a set of overidentifying restrictions on a system of moment equations using the generalized method of moments (GMM) developed by Hansen (1982). The moments under consideration are the mean and variance of the equity returns in the U.S. and Brazil. In comparison to the Maximum Likelihood estimation, the GMM technique is particularly attractive for an estimation of this type of asset pricing model. First, the GMM approach does not require that the distribution of equity returns or equity return volatility be normal;[27] second, the GMM estimators and their standard errors are consistent even if the assumed disturbances are conditionally heteroskedastic.

The GMM estimation procedure chooses the parameter estimates that minimize the quadratic form

$$J(\Delta\theta) \quad = \quad m'(\Delta\theta)W(\Delta\theta)m(\Delta\theta) \tag{35}$$

with

$$m(\Delta\theta) = \begin{cases} \frac{1}{N-1}\sum_{t=2}^{N}\left(r_{us,t} - r_{E_{fy},t}(\Delta\theta)\right) \\ \frac{1}{N-1}\sum_{t=2}^{N}\left(r_{br,t} - r_{E_f,t}(\Delta\theta)\right) \\ \frac{1}{N-1}\sum_{t=2}^{N}\left[\left(r_{us,t} - \bar{r}_{us}\right)^2 - \left(r_{E_{fy},t}(\Delta\theta) - \bar{r}_{E_{fy}}(\Delta\theta)\right)^2\right] \\ \frac{1}{N-1}\sum_{t=2}^{N}\left[\left(r_{br,t} - \bar{r}_{br}\right)^2 - \left(r_{E_f,t}(\Delta\theta) - \bar{r}_{E_f}(\Delta\theta)\right)^2\right] \end{cases} \tag{36}$$

where $W(\Delta\theta)$ is a positive-definite symmetric weighting matrix and $m(\Delta\theta)$ is a vector of orthogonality

[27] The asymptotic justification for the GMM procedure requires only that the distribution of equity return and equity return volatility be stationary and ergodic and that the relevant expectations exist.

conditions, which correspond to the model's pricing errors. The historical monthly returns on U.S. and Brazilian equity indices between time $t-1$ and t are denoted by $r_{us,t}$ and $r_{br,t}$, respectively, while the monthly equity returns as predicted by the model for the U.S. and Brazil between time $t-1$ and t are denoted by $r_{E_{fy},t}$ and $r_{E_f,t}$, respectively.

Because I consider more moment conditions than parameters, not all of the moment restrictions are satisfied. The weighting matrix $W(\Delta\theta)$ determines the relative importance of the various moment conditions so as to give more weight to the moment conditions with less uncertainty. Following Hansen (1982), when equal to the inverse of the asymptotic covariance matrix, the weighting matrix $W(\Delta\theta) = S^{-1}(\Delta\theta)$ is optimal because $\widehat{\Delta\theta}$ is determined with the smallest asymptotic variance. I estimate the covariance matrix using the Newey and West (1987) approach to account for heteroskedasticity and serial correlation with a correction for small samples. This covariance matrix is used to test the significance of the parameter.

The optimal weighting matrix $W(\Delta\theta)$ requires an estimate of the parameter $\Delta\theta$; at the same time, estimating the parameter $\Delta\theta$ requires the weighting matrix. To solve this dependency, I account for a two-stage estimation method. I first set the initial weighting matrix to be equal to the identity matrix $W_0 = I$ and then calculate the parameter estimate. I then compute a new weighting matrix with the parameter estimate obtained at the first stage. The parameter $\Delta\theta$ is obtained by matching the moments of the model to those of the data as closely as possible.

4.3 Empirical Results

In this section, I present the empirical results and examine the explanatory power of the asset-pricing model developed in this paper. Table 2 reports the parameter estimate, the asymptotic standard deviations and their associated p-values, and the GMM minimized criterion (χ^2) values.

First, it is worth analyzing how well the model fits the data. As the model is over-identified, it is not possible to set every moment to zero. Therefore, the key concern is the distance from zero. The minimized value of the quadratic form $J(\Delta\theta)$ is χ^2-distributed under the null hypothesis that the model is true with the number of degrees of freedom equal to the number of orthogonality conditions net of the number of parameters to be estimated. This χ^2 measure thus provides a goodness-of-fit test for the model.

The χ^2 tests for goodness-of-fit suggest that the model cannot be rejected at the 90% confidence level (see Table 2). The table uses the covariance matrix of the moments to test the significance of

Table 2: **Results of the Model Estimation.** This table provides the results of the model estimation using the general method of moments. The moments under consideration are the mean and the variance of equity returns in the U.S. and in Brazil. Equity returns in the U.S., r_{us}, are computed with the S&P500 and equity returns in Brazil, r_{br}, are computed with the MSCI Brazil Index. I use monthly industrial production data for Brazil and the U.S. from June 1994 through December 2008 to generate the equity prices predicted by the model and the moment conditions. I estimate the expected economic costs $\Delta\theta$ of a sovereign default to match the moments as closely as possible. The remaining parameter values are presented in Table 1. The heteroskedasticity consistent standard errors, presented in parenthesis, are corrected for serial correlation using the Newey and West's non-parametric variance covariance estimator.

Moment conditions (pricing errors)	Value	p-value	GMM parameter estimates and J-test	Value	p-value
Home country: U.S.			**Parameter estimate**		
Average equity return	0.0055	0.113	$\Delta\theta$	0.2132	0.000
$\frac{1}{N-1}\sum_{t=2}^{N}\left(r_{us,t} - r_{E_{fy},t}\right)$	(0.0034)			(0.0045)	
Equity return volatility	0.0127	0.165			
$\frac{1}{N-1}\sum_{t=2}^{N}\left[\left(r_{us,t} - \bar{r}_{us}\right)^2 - \left(r_{E_{fy},t} - \bar{r}_{E_{fy}}\right)^2\right]$	(0.0092)				
			Test of over-identifying restrictions		
Foreign country: Brazil			$J(\Phi_1)$	8.383	0.136
Average equity return	-0.0061	0.454			
$\frac{1}{N-1}\sum_{t=2}^{N}\left(r_{br,t} - r_{E_f,t}\right)$	(0.0082)				
Equity return volatility	0.0168	0.734	**Observations** $N = 176$		
$\frac{1}{N-1}\sum_{t=2}^{N}\left[\left(r_{br,t} - \bar{r}_{br}\right)^2 - \left(r_{E_f,t} - \bar{r}_{E_f}\right)^2\right]$	(0.0549)				

individual moments (i.e., the model's pricing errors) and provides the corresponding p-values. Table 2 suggests that the estimate of $\Delta\theta$ is statistically different from zero. Therefore, the risk of the adverse economic consequences of sovereign default helps explain the level and the volatility of international equity returns beyond the financial leverage effect studied by Black (1976) and Christie (1982) and the operational leverage effect documented by Lev (1974). Moreover, we cannot reject the fact that the moment conditions are not satisfied at the 90% confidence level. Thus, the model can simultaneously satisfy the first two moments of equity returns in the U.S and in Brazil.

To date, the existing international asset pricing literature has largely ignored the presence of defaultable debt in a firm's balance sheet, operating costs, or the risk of sovereign default. The corresponding prediction would be that the volatility of an unlevered firm's equity return is lower than or equal to the volatility of this firm's output, depending on whether or not there is an offsetting terms-of-trade effect. However, the data suggest that the volatility of equity returns is far greater than the volatility of output (see Table 3). To date, it has been difficult to offer an adequate response to Shiller's (1981) critique:

stock prices are too volatile to be explained by a simple asset pricing model with dividends or earnings.[28] In contrast, the results of Table 2 show that the model resolves this issue when accounting for financial leverage, operational leverage, and in particular for sovereign default risk.

Table 3: **Statistics on Industrial Production and Equity Markets, 1994-2008.** This table compares the mean and the standard deviation (volatility) of industrial production's growth with the mean and the volatility of returns on equity market indices for Brazil and the U.S. All values are annualized over the period 1994-2008.

	Industrial Production Growth		Equity Market Return	
	Mean (%)	Volatility (%)	Mean (%)	Volatility (%)
Brazil	1.50	7.99	4.68	41.95
United States	2.11	7.49	8.17	15.32

Finally, I analyze the magnitude of the loss in output growth due to sovereign default. The results suggest an estimate of 0.21% for Brazil. As Brazil grows at 1.5% per annum (see Table 3), the economic loss upon default corresponds to 13% of the average growth rate. Because this estimate captures the loss in output growth due to the default event in excess of the average growth and not in excess of the relatively weak economic growth at the time of this event, this estimate should be viewed as a lower bound. The magnitude of this estimate is close to that measured by De Paoli, Hoggarth, and Saporta (2006), who looked at the annual difference between potential and actual output, where potential output is based on the country's pre-crisis (HP filter) trend. Analyzing 45 sovereign default crises over the period of 1970-2000, these authors found a loss in GDP growth of 15.1% per annum.

One may easily take objection that the impact of default on growth is assumed permanent and not short-lived. Yet what matters in the explanation of the first two moments of equity returns is the loss in equity value that the risk of such a contraction induces and not the level of contraction per se. Should the assumed perpetual contraction be statistically significant in the GMM estimation, so will be a short-term contraction; for a given loss in present value, a short-term contraction is necessarily of a greater magnitude than a perpetual contraction. Given the aim of the paper, the assumption of a perpetual output cost upon default is then appropriate, thus following the works of Eaton and Gersovitz (1981), Cohen and Sachs (1986), Bulow and Rogoff (1989), Arellano (2008), Andrade (2009), Guimaraes (2009), Hatchondo and Martinez (2009), among others.

[28] Shiller's (1981) initial findings apply to stock market in the U.S. However, this "excess-volatility" puzzle is confirmed by Campbell (1996) for a number of other countries.

5 Equity Return Volatility and Sovereign Credit Risk

The structural estimation of the model presented in Section 4 provides strong support for the effect of sovereign credit risk on the level of equity return volatility in the U.S. As predicted by the model, this effect arises from the positive relationship between sovereign credit risk and Home equity return volatility, in particular in periods of economic downturns. In this section, I provide additional empirical evidence that this prediction is consistent with the data. I compute the time series of equity return volatility in the U.S. using a GARCH(1,1) model on S&P500 daily returns over the period from January 1, 1998, to December 31, 2008 and compare it with an aggregate measure of sovereign credit risk, which is computed with the daily average JPMorgan EMBI+ sovereign spreads for Brazil, Bulgaria, Ecuador, Mexico, Panama, Peru, Philippines, Russia, and Venezuela.

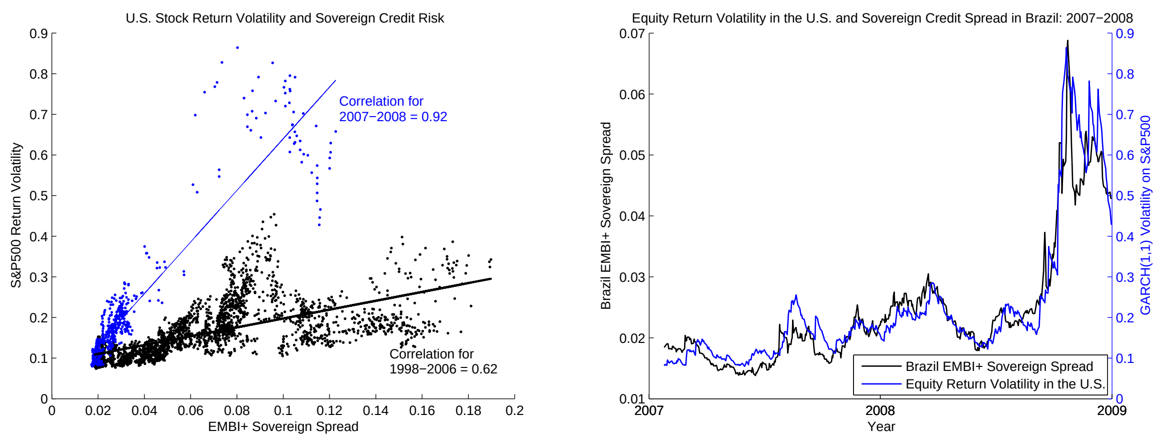

Figure 5: **Equity Return Volatility and Sovereign Credit Risk, 1998-2008.** This figure (left panel) compares the volatility on S&P500 returns computed with the GARCH(1,1) model and sovereign credit risk, which is computed with the daily average JPMorgan EMBI+ sovereign spreads for Brazil, Bulgaria, Ecuador, Mexico, Panama, Peru, Philippines, Russia, and Venezuela. The figure breaks down the relationship between these series into two subsamples: from June 1, 1994 through December 31, 2006 and from January 1, 2007 through December 31, 2008. The right panel displays the evolution of the volatility on S&P500 returns and the EMBI+ sovereign credit spread in Brazil for the period from January 1, 2007 through December 31, 2008.

Figure 5 (left panel) shows a positive relationship between sovereign credit spreads in emerging markets and equity return volatility in the U.S. The relationship holds during both the pre-crisis period (1998-2006) and the recent crisis period (2007-2008).[29] The right panel compares the evolution of sovereign credit spread in Brazil, as an example, and equity return volatility in the U.S. during the 2007-2008

[29]The 2007-2008 crisis period is characterized by sharp increase in the correlation between equity return volatility and sovereign credit spreads in Brazil. This feature is typical in periods of high volatility (i.e., volatility of sovereign credit spread or of equity return volatility), as suggested by Forbes and Rigobon (2002).

financial crisis. The correlation is extremely high (0.95) in this sample period. Noteworthy, the failure of Lehman Brothers in September 2009 has triggered a sharp increase in equity return volatility and sovereign credit risk.

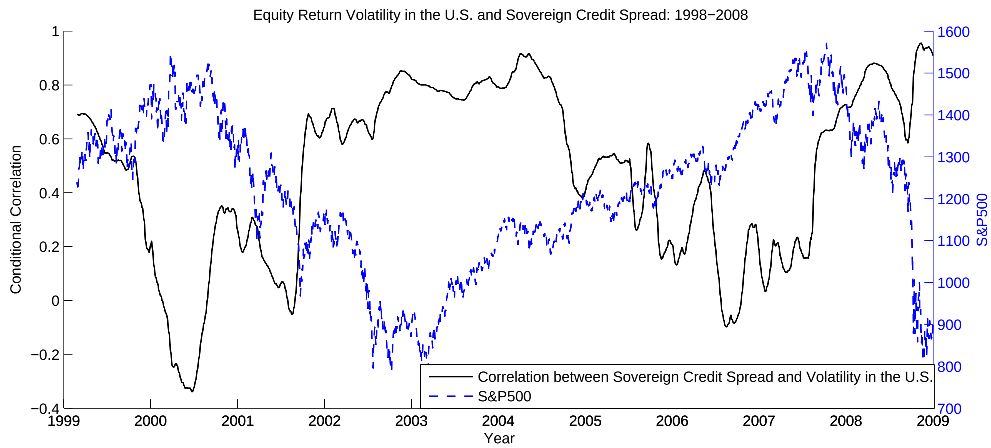

Figure 6: **Conditional Correlation between Equity Return Volatility in the U.S. and Sovereign Credit Risk, 1998-2008.** This figure plots the conditional correlation between equity return volatility in the U.S., as computed with a GARCH(1,1) model on daily S&P500 returns, and sovereign credit risk, which is computed with the daily average JPMorgan EMBI+ sovereign spreads for Brazil, Bulgaria, Ecuador, Mexico, Panama, Peru, Philippines, Russia, and Venezuela. The correlation is computed using a 500-day rolling window over the period from January 1, 1998 through December 31, 2008. The figure also displays the S&P500 for comparison.

The high correlation between equity return volatility in the U.S. and sovereign credit risk during the recent financial crisis is not specific to that time period. The Figure 6 illustrates the long-term conditional correlation between these two series, which is computed with 500-day rolling windows. The correlation between equity return volatility in the U.S. and sovereign credit risk in those countries is very high (almost one) in periods of financial distress (when the S&P500 is low), while it is relatively low (around zero) in good times (when the S&P500 is high).

The countercyclical nature of the correlation between these two measures offers interesting insights on the importance of sovereign default risk in the explanation of the level of equity return volatility. In periods of economic downturns, a large fraction of equity return volatility is attributed to the macro leverage effect because the risk of an economic slowdown triggered by a sovereign default abroad is particularly important. Hence, the correlation between sovereign credit risk and equity return volatility in the U.S. is high. In contrast, the macro leverage effect is negligible in periods of high economic growth, as theoretically predicted by the model (see Figure 2). In such periods, there is a low correlation between sovereign credit risk and equity return volatility in the U.S. because equity return volatility is mostly explained by factors specific to the U.S. economy.

5.1 Equity Return Volatility and Option-implied Volatility

Over recent years, the relationship between equity market volatility in the U.S. measured with the option-implied volatility index on the S&P500 (VIX), and sovereign credit spread movements abroad has attracted a great deal of interest. Recent studies include McGuire and Schrijvers (2003), González-Rozada and Yeyati (2008), Pan and Singleton (2008), Remolona et al. (2008), Hilscher and Nosbusch (2009), and Longstaff et al. (2009). To date, this literature has only focused on the VIX, which is a forward-looking measure, rather than on equity return volatility, which is a realized or historical volatility measure. In this section, I suggest that the positive correlation between sovereign credit risk and the VIX documented in thoses studies arises from the positive relationship between sovereign credit risk and equity return volatility highlighted in the previous section.

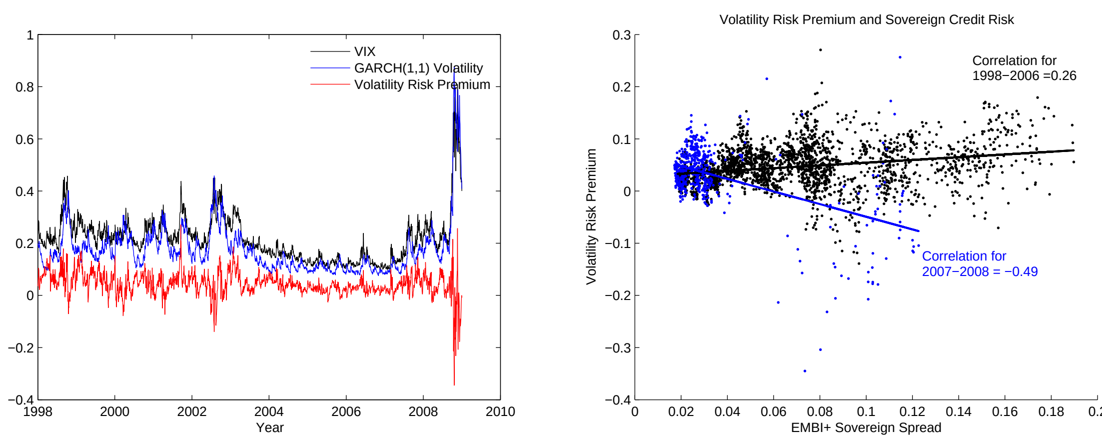

Figure 7: **Equity Return Volatility, Volatility Risk Premium, and Sovereign Credit Risk, 1998-2008.** The left panel plots the dynamics of the option-implied volatility (VIX), the historical volatility computed with the GARCH(1,1) model, and the difference between the two series. The right panel shows the relationship between sovereign credit risk, which is computed as the average EMBI+ sovereign credit spread for Brazil, Bulgaria, Ecuador, Mexico, Panama, Peru, Philippines, Russia, and Venezuela, and the volatility risk premium on S&P500 returns, which is determined as the difference between the option-implied volatility (VIX) and the historical volatility computed with a GARCH(1,1) model. The figure breaks down the relationship between these series into two subsamples: from January 1, 1998 through December 31, 2006 and from January 1, 2007 through December 31, 2008.

There are several reasons. First, equity return volatility in the U.S. computed with a GARCH(1,1) model on S&P500 returns is the major component of the VIX; it explains 80% of its time variation over the sample period (see Figure 7, left panel). Second, the relationship between sovereign credit risk and the volatility risk premium, which is the difference between the VIX and return volatility on S&P500, is unclear (see Figure 7, right panel); the correlation between these two measures is positive but weak during the 1998-2006 period; however, it is negative during the subprime crisis period (2007-2008). This result arises because the volatility risk premium was negative and large in the fall of 2008 (see Figure 7, left panel) when sovereign credit spreads considerably widened (see Figure 5).[30]

We can thus conclude that the positive relation between the VIX and sovereign credit spreads is mostly driven by the positive relation between historical equity return volatility and sovereign credit spreads rather than by the relation between the volatility risk premium and sovereign credit spreads. Therefore, the model developed in this paper offers an intuitive and economic explanation of the relationship between sovereign credit risk abroad and equity market volatility in the U.S. that has been recently documented in various studies.

6 Conclusion

This paper shows that the consequences of economic shocks in the U.S. on the U.S. financial market are greater than those predicted by the current literature. The analysis is based on a simple concept: a negative economic shock to the U.S. economy also affects its trading partners. In particular, this shock increases the risk of sovereign default abroad, which triggers a "boomerang effect" that amplifies the negative effect of this initial shock on the U.S. financial markets. In line with the prediction of this paper, sovereign defaults have generally followed a negative economic shock in the U.S. and have thus exacerbated the level of equity return volatility in the U.S. Examples include the 1998 default of Russia after the failure of Long-Term Capital Management, the 2001 default of Argentina after the attack of September 11, the 2002 default in Brazil after the stock market sell-off in July, and the 2008 defaults of Ecuador and Iceland after the collapse of Lehman Brothers.

The framework developed in this paper lends itself to several international finance implications and extensions for further research. For example, the recent financial crisis has brought into question the diversification benefits of investing across asset classes. This paper offers insights on how governments and firms are closely linked in the economy. In addition, the model predicts strong co-movement among corporate debt, sovereign debt, and the equity markets. The capacity to service sovereign debt and thus

[30] Longstaff et al. (2009) also obtain a negative effect of changes in the volatility risk premium on changes in sovereign credit spreads, when controlling for other country-specific and global factors over the period 2000-2009.

to avoid defaulting depends on the level of fiscal revenues determined by the level of the domestic firm earnings. At the same time, the present value of firm earnings depends on the likelihood of entering a recession, which itself depends on the risk of a sovereign default. Therefore, a thorough understanding of the interactions between firm and government default decisions can yield important new asset pricing predictions.

This paper is also useful in improving our understanding of the drivers of equity return volatility and of its variation over time. For example, the combination of the transmission of shocks through the foreign exchange market and the leverage effects can explain the stylized fact that equity return volatility moves across countries in a coordinated fashion (e.g., Hamao, Masulis, and Ng (1990), Lin, Engle, and Ito (1994), Edwards and Susmel (2001), Forbes and Rigobon (2002)). Understanding the factors that lead some countries to have higher levels of equity return volatility than those of others is also of crucial importance for international portfolio allocation. The model predicts, for example, that heterogeneity in financial leverage, operational leverage, tax rate, bankruptcy costs, and the dynamics of the economy could help explain the cross-sectional variation in equity return volatility.

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7 Appendix

7.1 The Exchange Rate

The price of one unit of Foreign good to be delivered at time t in state w is equal to

$$\xi_{x,t} = \lambda_x e^{-\rho t} \frac{\partial u_x \left(C_{xx,t}, C_{xy,t}\right)}{\partial C_{xx,t}} \tag{37}$$

$$= \lambda_x e^{-\rho t} \frac{\partial \left[a_x log \left(C_{xx,t}\right) + (1-a_x) log \left(C_{xy,t}\right)\right]}{\partial C_{xx,t}} \tag{38}$$

$$= \frac{\lambda_x e^{-\rho t} a_x}{C_{xx,t}} = \frac{e^{-\rho t} \left(\lambda_y a_y + \lambda_x a_x\right)}{X_t} = f\left(t, X_t\right) \tag{39}$$

Dropping the time and the regime subscript and applying Itô's formula to $\xi_{x,t}$ yields,

$$df\left(t, X\right) = f_t dt + f_x dX + \frac{1}{2} f_{xx} dX dX \tag{40}$$

$$= -\rho f dt - \frac{f}{X}\left(\theta_x X dt + \sigma_x X dW^x\right) + \frac{f}{X^2}\left[\left(\sigma_x X\right)^2 dt\right] \tag{41}$$

$$= f\left[(-\rho - \theta_x + \sigma_x^2) dt + \sigma_x X dW^x\right] \tag{42}$$

The price of one unit of Foreign good to be delivered at time t in state w thus follows the process defined by

$$\frac{d\xi_{x,t}}{\xi_{x,t}} = -r_{x,i} dt - \sigma_x dW_t^x, \quad i = \{L, H\} \tag{43}$$

where r_x is the risk-free rate prevailing in the Foreign country, given by

$$r_{x,i} = \rho + \theta_{x,i} - \sigma_x^2 \tag{44}$$

Using a similar approach to obtain the price of one unit of Home good to be delivered at time t in state w, defined by $\xi_{y,t} = \frac{\lambda_y e^{-\rho t}(1-a_y)}{C_{yy,t}}$, we obtain

$$\frac{d\xi_{y,t}}{\xi_{y,t}} = -r_y dt - \sigma_y dW_t^y \tag{45}$$

where r_y is the risk-free rate prevailing in the Home country, given by

$$r_y = \rho + \theta_y - \sigma_y^2 \tag{46}$$

Finally, the exchange rate is defined by $S_t = f(t, \xi_{y,t}, \xi_{x,t}) = \frac{\xi_{y,t}}{\xi_{x,t}}$, which is the same as the ratio of either country's marginal utilities of the Home good and the Foreign good.[31] From Itô's formula,

[31] For reference, see Dumas (1992), Backus et al. (2001), Brandt et al. (2006), Pavlova and Rigobon (2007, 2008), and Bakshi et al. (2008).

dropping the time and the regime subscript,

$$
\begin{aligned}
df(t,\xi_y,\xi_x) &= f_t dt + f_{\xi_y} d\xi_y + f_{\xi_x} d\xi_x + \frac{1}{2}\left(f_{\xi_y\xi_y} d\xi_y d\xi_y + f_{\xi_x\xi_x} d\xi_x d\xi_x + 2 f_{\xi_y\xi_x} d\xi_y d\xi_x\right) & (47)\\
&= 0 + \frac{1}{\xi_x}\left(-r_y\xi_y dt - \sigma_y \xi_y dW^y\right) & (48)\\
&\quad -\frac{\xi_y}{(\xi_x)^2}\left(-r_x\xi_x dt - \xi_x\sigma_x dW^x\right) + \frac{\xi_y}{(\xi_x)^3}\left(\xi_x\sigma_x\right)^2 dt & (49)\\
&= \frac{\xi_y}{\xi_x}\left[\ \left(r_x - r_y + (\sigma_x)^2\right) dt + \sigma_x dW^x - \sigma_y dW^y\ \right] & (50)
\end{aligned}
$$

The exchange rate S thus follows the process defined by

$$\frac{dS_t}{S_t} = \theta_{s,i} dt + \sigma_x dW_t^x - \sigma_y dW_t^y, \quad i = \{L,H\} \tag{51}$$

with

$$\theta_{s,i} = r_{x,i} - r_y + \sigma_x^2 \tag{52}$$

7.2 The Dynamics of the Firm Revenues

Let's define the Foreign firm's revenues as $f = Z_t \equiv P_{x,t}X_t = (S_t)^{-\alpha} X_t$ (from the world basket numeraire) with

$$\frac{dX_t}{X_t} = \theta_{x,i} dt + \sigma_x dW_t^x, \quad i = \{L,H\} \tag{53}$$

and

$$\frac{dS_t}{S_t} = \theta_{s,i} dt + \sigma_x dW_t^x - \sigma_y dW_t^y, \quad i = \{L,H\} \tag{54}$$

From Itô's formula, dropping the time and regime substricts,

$$
\begin{aligned}
df(t,X_t,S_t) &= f_t dt + f_x dX + f_s dS + \frac{1}{2}\left(f_{xx} dX dX + f_{ss} dS dS + 2 f_{xs} dX dS\right) & (55)\\
&= 0 + \tau S^{-\alpha} X\left(\theta_x dt + \sigma_x dW^x\right) & (56)\\
&\quad -\alpha\tau S^{-\alpha} X\left(\theta_s dt + \sigma_x dW^x - \sigma_y dW^y\right) & \\
&\quad +0 + \frac{\alpha(1+\alpha)}{2}\tau S^{-\alpha} X\left(\sigma_x^2 + \sigma_y^2\right) dt - \alpha\tau S^{-\alpha} X \sigma_x^2 dt & (57)\\
& & (58)\\
&= \tau S^{-\alpha} X \left\{ \begin{array}{c} \left[\theta_x - \alpha\theta_s + \frac{\alpha(1+\alpha)}{2}\left(\sigma_x^2+\sigma_y^2\right) - \alpha\sigma_x^2\right] dt \\ +(1-\alpha)\sigma_x dW^x + \alpha\sigma_y dW^y \end{array} \right\} & (59)
\end{aligned}
$$

The dynamics of Z_t is thus characterized by the process defined by

$$\frac{dZ_t}{Z_t} = \theta_{z,i} dt + \sigma_{z,x} dW_t^x + \sigma_{z,y} dW_t^y, \quad i = \{L,H\} \tag{60}$$

with

$$\theta_{z,i} = \theta_{x,i} - \alpha\theta_{s,i} + \frac{\alpha(1+\alpha)}{2}\left(\sigma_x^2 + \sigma_y^2\right) - \alpha\sigma_x^2 \tag{61}$$

$$\sigma_{z,x} = (1-\alpha)\sigma_x \tag{62}$$

$$\sigma_{z,y} = \alpha\sigma_y \tag{63}$$

7.2.1 The Risk-Neutral Measure

Let be $(\Omega, \mathscr{F}, \mathbb{P})$ the probability space on which the Brownian motions are defined. The corresponding information filtration is $F = \{\mathscr{F}_t : t \geq 0\}$.

First, we define the risk-neutral measure associated with the pricing kernel under the world basket numeraire ξ_t by specifying the density process φ_t,

$$\varphi_t = \mathbb{E}_t \left[\frac{d\mathbb{Q}}{d\mathbb{P}} \right] \tag{64}$$

which evolves according to the process

$$\frac{d\varphi_t}{\varphi_t} = -\sigma_{z,y} dW_t^y - \sigma_{z,x} dW_t^x \tag{65}$$

Applying the Girsanov theoreom, we obtain new Brownian motions under $\mathbb{Q}$, $\tilde{W}_t^y$ and $\tilde{W}_t^x$, which solve

$$dW_t^y = d\tilde{W}_t^y - \sigma_{z,y} dt \tag{66}$$

$$dW_t^x = d\tilde{W}_t^x - \sigma_{z,x} dt \tag{67}$$

Under the risk-neutral probability measure $\mathbb{Q}$, Home and Foreign firm revenues then follow the process

$$\frac{dZ_t}{Z_t} = \tilde{\theta}_{z,i} dt + \sigma_{z,x} d\tilde{W}_t^x + \sigma_{z,y} d\tilde{W}_t^y, \quad i = \{L, H\} \tag{68}$$

with

$$\tilde{\theta}_{z,i} = \theta_{z,i} - \left(\sigma_{z,x}^2 + \sigma_{z,y}^2 \right) \tag{69}$$

7.3 The State-Price Density

The state-price density ξ_t that prevails in a competitive equilibrium is equal to

$$\xi_t = (\xi_t P_x)^{1-\alpha} (\xi_t P_y)^{\alpha} = (\xi_{x,t})^{1-\alpha} (\xi_{y,t})^{\alpha} \tag{70}$$

$$= e^{-\rho t} \left(\frac{\lambda_y a_y + \lambda_x a_x}{X_t} \right)^{1-\alpha} \left(\frac{\lambda_y (1 - a_y) + \lambda_x (1 - a_x)}{Y_t} \right)^{\alpha} \tag{71}$$

$$= \frac{e^{-\rho t}}{Z_t} (\lambda_y a_y + \lambda_x a_x) \tag{72}$$

where the first equality follows from the price normalization $P_x^{1-\alpha} P_y^{\alpha} = 1$ and the last equality from $Z = XP_x = XS^{-\alpha} = X(\overline{S}\frac{X}{Y})^{-\alpha} = (\overline{S})^{-\alpha} X^{1-\alpha} Y_t^{\alpha}$, with $\overline{S} = \frac{\lambda_y(1-a_y)+\lambda_x(1-a_x)}{\lambda_x a_x + \lambda_y a_y}$.

Dropping the time and the regime subscript and applying Itô's formula to ξ_t yields,

$$\begin{aligned} df(t,Z) &= f_t dt + f_z dZ + \frac{1}{2} f_{zz} dZ dZ && (73) \\ &= -\rho f dt - \frac{f}{Z}\left(\theta_z Z dt + \sigma_{z,x} Z dW^x + \sigma_{z,y} Z dW^y\right) && (74) \\ &\quad + \frac{f}{Z^2}\left[\left(\sigma_{z,x} Z\right)^2 dt + \left(\sigma_{z,y} Z\right)^2 dt\right] \\ &= f\left[(-\rho - \theta_z + \sigma_{z,x}^2 + \sigma_{z,y}^2)dt + \sigma_{z,x} Z dW^x + \sigma_{z,y} Z dW^y\right] && (75) \end{aligned}$$

The state-price density thus follows the process defined by

$$\frac{d\xi_t}{\xi_t} = -r_{z,i} dt - \sigma_{z,y} dW_t^y - \sigma_{z,x} dW_t^x, \quad i = \{L, H\} \tag{76}$$

where r_z is the risk-free rate prevailing under the world basket numeraire, given by

$$r_{z,i} = \rho + \theta_{z,i} - \left(\sigma_{z,x}^2 + \sigma_{z,y}^2\right) \tag{77}$$

7.4 Evaluation of Sovereign Debt

The price of the debt is determined subject to a number of conditions. First, when firm revenues Z tend to infinity (and so do the revenues R), the value of the sovereign debt D tends to the value of the risk-free debt

$$Lim_{Z\to\infty} D(Z) = \mathbb{E}_0^{\mathbb{Q}}\left[\int_0^\infty C e^{-r_{z,H} t} dt\right] = \frac{C}{r_{z,H}} \tag{78}$$

Second, lenders value this debt upon default, which depends on the recovery rate. Upon default, the sovereign and its lenders restructure the terms of the debt contract and agree on a reduction of the debt service. I determine the value matching conditions that impose equality between the value of the sovereign debt and the value of the restructured debt in default. At default time $T(Z^D)$, the value of the sovereign debt is

$$Lim_{Z\to Z^D} D(Z) = \frac{C(1-\phi)}{r_{z,L}} \tag{79}$$

where $0 \leq 1-\phi \leq 1$ denotes the recovery rate on the debt service C. The stochastic discount factor is defined as the Arrow-Debreu price of default $\mathbb{E}_0^{\mathbb{Q}}\left[e^{-r_{z,H} T\left(Z^D\right)}\right] = \left(\frac{Z}{Z^D}\right)^{\beta_H}$, where β_i is the negative root of the quadratic equation $\frac{1}{2}\sigma_z^2 \beta_i(\beta_i - 1) + \tilde{\theta}_{z,i}\beta - r_{z,i} = 0$ in regime i, defined by

$$\beta_i = \frac{1}{2} - \frac{\tilde{\theta}_{z,i}}{\sigma_z^2} - \sqrt{\left(\frac{1}{2} - \frac{\tilde{\theta}_{z,i}}{\sigma_z^2}\right)^2 + \frac{2 r_{z,i}}{\sigma_z^2}} < 0, \quad i = \{L, H\} \tag{80}$$

with $\sigma_z = \sqrt{\sigma_{z,x}^2 + \sigma_{z,y}^2}$. The value of the sovereign debt can then be rewritten as

$$\begin{aligned} D(Z) &= \mathbb{E}_0^{\mathbb{Q}}\left[\int_0^{T\left(Z^D\right)} C e^{-r_{z,H} t} dt\right] + \mathbb{E}_0^{\mathbb{Q}}\left[\int_{T\left(Z^D\right)}^\infty (1-\phi) C e^{-r_{z,L} t} dt\right] && (81) \\ &= \frac{C}{r_{z,H}}\left[1 - \left(\frac{Z}{Z^D}\right)^{\beta_H}\right] + \frac{(1-\phi)C}{r_{z,L}}\left(\frac{Z}{Z^D}\right)^{\beta_H} && (82) \end{aligned}$$

where the default time on sovereign debt can be written as

$$\begin{aligned} T(R^D) &= \inf\{t \geq 0 \mid R_t \leq R^D\} & (83)\\ &= \inf\{t \geq 0 \mid Z_t \leq Z^D\} & (84)\\ &= T(Z^D) & (85) \end{aligned}$$

7.5 Evaluation of Firm Assets and Default Policy

I here provide the valuation of the Home firm assets and default policy when this firm defaults either before or after the Foreign government defaults. The evaluation of Foreign firm assets is obtained by the same formulae when Foreign firm revenues Z_t replace Home firm revenues $\overline{S}Z_t$.

7.5.1 Debt Evaluation

Case I: the Firm Defaults after the Foreign Government Defaults, $T\left(Z_f^D\right) > T\left(Z^D\right)$

The corporate debt value is

$$\begin{aligned} D_f(Z) \mid_{T\left(Z_f^D\right)>T(Z^D)} &= \mathbb{E}_0^{\mathbb{Q}}\left[\int_0^{T\left(Z^D\right)} C_f e^{-r_{z,H}t}dt + e^{-r_{z,H}T\left(Z^D\right)} D_f(Z) \mid_{t=T(Z^D)}\right] & (86)\\ &= \frac{C_f}{r_{z,H}}\left[1-\left(\frac{Z}{Z^D}\right)^{\beta_H}\right] + D_f(Z) \mid_{t=T(Z^D)} \left(\frac{Z}{Z^D}\right)^{\beta_H} & (87) \end{aligned}$$

with

$$\begin{aligned} D_f(Z) \mid_{t=T(Z^D)} &= \mathbb{E}_{T(Z^D)}^{\mathbb{Q}}\left[\int_{T(Z^D)}^{T\left(Z_f^D\right)} C_f e^{-r_{z,L}t}dt\right] & (88)\\ &\quad +\mathbb{E}_{T(Z^D)}^{\mathbb{Q}}\left[\int_{T\left(Z_f^D\right)}^{\infty} (1-\eta)(1-\tau)\left(\overline{S}Z_t - K\right)e^{-r_{z,L}t}dt\right] & (89)\\ &= \frac{C_f}{r_{z,L}}\left[1-\left(\frac{Z^D}{Z_f^D}\right)^{\beta_L}\right] & (90)\\ &\quad +(1-\eta)(1-\tau)\left(\frac{\overline{S}Z_f^D}{r_{z,L}-\tilde{\theta}_{z,L}} - \frac{K}{r_{z,L}}\right)\left(\frac{Z^D}{Z_f^D}\right)^{\beta_L} & (91) \end{aligned}$$

Case II: the Firm Defaults before the Government Defaults, $T\left(Z_f^D\right) < T\left(Z^D\right)$

The corporate debt value is

$$D_f(Z)\mid_{T\left(Z_f^D\right)<T(Z^D)} = \mathbb{E}_0^{\mathbb{Q}}\left[\int_0^{T\left(Z_f^D\right)} C_f e^{-r_{z,H}t}dt + e^{-r_{z,H}T\left(Z_f^D\right)} D_f(Z)\mid_{t=T\left(Z_f^D\right)}\right] \tag{92}$$

$$= \frac{C_f}{r_{z,H}}\left[1-\left(\frac{Z}{Z_f^D}\right)^{\beta_H}\right] + D_f(Z)\mid_{t=T\left(Z_f^D\right)}\left(\frac{Z}{Z_f^D}\right)^{\beta_H} \tag{93}$$

with

$$D_f(Z)\mid_{t=T\left(Z_f^D\right)} = \mathbb{E}_{T\left(Z_f^D\right)}^{\mathbb{Q}}\left[\int_{T\left(Z_f^D\right)}^{T(Z^D)} (1-\eta)(1-\tau)\left(\overline{S}Z_t - K\right)e^{-r_{z,H}t}dt\right] \tag{94}$$

$$+\mathbb{E}_{T\left(Z_f^D\right)}^{\mathbb{Q}}\left[\int_{T(Z^D)}^{\infty} (1-\eta)(1-\tau)\left(\overline{S}Z_t - K\right)e^{-r_{z,L}t}dt\right] \tag{95}$$

$$= (1-\eta)(1-\tau)\left(\frac{\overline{S}Z_f^D}{r_{z,H}-\tilde{\theta}_{z,H}} - \frac{K}{r_{z,H}}\right)\left[1-\left(\frac{Z_f^D}{Z^D}\right)^{\beta_H}\right] \tag{96}$$

$$+(1-\eta)(1-\tau)\left(\frac{\overline{S}Z_f^D}{r_{z,L}-\tilde{\theta}_{z,L}} - \frac{K}{r_{z,L}}\right)\left(\frac{Z_f^D}{Z^D}\right)^{\beta_H} \tag{97}$$

7.5.2 Firm Value

Case I: the Firm Defaults after the Foreign Government Defaults, $T\left(Z_f^D\right) > T\left(Z^D\right)$

The levered firm value $V(Z)$ satisfies

$$V(Z)\mid_{T\left(Z_f^D\right)>T(Z^D)} = \mathbb{E}_0^{\mathbb{Q}}\left[\int_0^{T\left(Z^D\right)} \left((1-\tau)\left(\overline{S}Z_t - K\right) + \tau C_f\right)e^{-r_{z,H}t}dt\right] \tag{98}$$

$$+\mathbb{E}_0^{\mathbb{Q}}\left[e^{-r_{z,H}T\left(Z^D\right)} V(Z)\mid_{t=T(Z^D)}\right] \tag{99}$$

$$= \left(\frac{(1-\tau)\overline{S}Z}{r_{z,H}-\tilde{\theta}_{z,H}} + \frac{\tau C_f - (1-\tau)K}{r_{z,H}}\right)\left[1-\left(\frac{Z}{Z^D}\right)^{\beta_H}\right] \tag{100}$$

$$+V(Z)\mid_{t=T(Z^D)}\left(\frac{Z}{Z^D}\right)^{\beta_H} \tag{101}$$

with

$$
\begin{aligned}
V(Z)\mid_{t=T(Z^D)} &= \mathbb{E}^{\mathbb{Q}}_{T(Z^D)}\left[\int_{T(Z^D)}^{\infty}\left((1-\tau)\left(\overline{S}Z_t-K\right)+\tau C_f\right)e^{-r_{z,L}t}dt\right] && (102)\\
&\quad -\mathbb{E}^{\mathbb{Q}}_{T(Z^D)}\left[\int_{T\left(Z_f^D\right)}^{\infty}\left(\eta(1-\tau)\left(\overline{S}Z_t-K\right)+\tau C_f\right)e^{-r_{z,L}t}dt\right] && (103)\\
&= (1-\tau)\left(\frac{\overline{S}Z}{r_{z,L}-\tilde{\theta}_{z,L}}-\frac{K}{r_{z,L}}\right)+\frac{\tau C_f}{r_{z,L}}\left[1-\left(\frac{Z^D}{Z_f^D}\right)^{\beta_L}\right] && (104)\\
&\quad -\eta(1-\tau)\left(\frac{\overline{S}Z_f^D}{r_{z,L}-\tilde{\theta}_{z,L}}-\frac{K}{r_{z,L}}\right)\left(\frac{Z^D}{Z_f^D}\right)^{\beta_L} && (105)
\end{aligned}
$$

Case II: the Firm Defaults before the Foreign Government Defaults, $T\left(Z_f^D\right) < T\left(Z^D\right)$

The levered firm value $V(Z)$ satisfies

$$
\begin{aligned}
V(Z)\mid_{T\left(Z_f^D\right)<T(Z^D)} &= \mathbb{E}^{\mathbb{Q}}_{0}\left[\int_{0}^{T\left(Z_f^D\right)}\left((1-\tau)\left(\overline{S}Z_t-K\right)+\tau C_f\right)e^{-r_{z,H}t}dt\right] && (106)\\
&\quad +\mathbb{E}^{\mathbb{Q}}_{0}\left[e^{-r_{z,H}T\left(Z_f^D\right)}V(Z)\mid_{t=T\left(Z_f^D\right)}\right] && (107)\\
&= \left(\frac{(1-\tau)\overline{S}Z}{r_{z,H}-\tilde{\theta}_{z,H}}+\frac{\tau C_f-(1-\tau)K}{r_{z,H}}\right)\left[1-\left(\frac{Z}{Z_f^D}\right)^{\beta_H}\right] && (108)\\
&\quad +V(Z)\mid_{t=T\left(Z_f^D\right)}\left(\frac{Z}{Z_f^D}\right)^{\beta_H} && (109)
\end{aligned}
$$

with

$$
\begin{aligned}
V(Z)\mid_{t=T\left(Z_f^D\right)} &= \mathbb{E}^{\mathbb{Q}}_{T\left(Z_f^D\right)}\left[\int_{T\left(Z_f^D\right)}^{T(Z^D)}(1-\eta)(1-\tau)\left(\overline{S}Z_t-K\right)e^{-r_{z,H}t}dt\right] && (110)\\
&\quad +\mathbb{E}^{\mathbb{Q}}_{T\left(Z_f^D\right)}\left[\int_{T(Z^D)}^{\infty}(1-\eta)(1-\tau)\left(\overline{S}Z_t-K\right)e^{-r_{z,L}t}dt\right] && (111)\\
&= (1-\eta)(1-\tau)\left(\frac{\overline{S}Z_f^D}{r_{z,H}-\tilde{\theta}_{z,H}}-\frac{K}{r_{z,H}}\right)\left[1-\left(\frac{Z_f^D}{Z^D}\right)^{\beta_H}\right] && (112)\\
&\quad +(1-\eta)(1-\tau)\left(\frac{\overline{S}Z_f^D}{r_{z,L}-\tilde{\theta}_{z,L}}-\frac{K}{r_{z,L}}\right)\left(\frac{Z_f^D}{Z^D}\right)^{\beta_H} && (113)
\end{aligned}
$$

7.5.3 Default Policy

Case I: the Firm Defaults after the Foreign Government Defaults, $T\left(Z_f^D\right) > T\left(Z^D\right)$

As the value of the firm until sovereign default is, by assumption, independent from the default policy, the optimal default policy in this case is the one that maximizes equity value at time $T\left(Z^D\right)$

$$
\begin{aligned}
E(Z\mid_{t=T(Z^D)})\mid_{T\left(Z_f^D\right)>T(Z^D)} &= V(Z\mid_{t=T(Z^D)})\mid_{T\left(Z_f^D\right)>T(Z^D)} && (114)\\
&\quad -D_f(Z\mid_{t=T(Z^D)})\mid_{T\left(Z_f^D\right)>T(Z^D)} && (115)\\
&= (1-\tau)\left(\frac{\overline{S}Z}{r_{z,L}-\tilde{\theta}_{z,L}}-\frac{K}{r_{z,L}}\right)-(1-\tau)\frac{C_f}{r_{z,L}}\left[1-\left(\frac{Z}{Z_f^D}\right)^{\beta_L}\right] && (116)\\
&\quad -(1-\tau)\left(\frac{\overline{S}Z_f^D}{r_{z,L}-\tilde{\theta}_{z,L}}-\frac{K}{r_{z,L}}\right)\left(\frac{Z}{Z_f^D}\right)^{\beta_L} && (117)
\end{aligned}
$$

The first-order maximization yields

$$
\frac{\partial E\left(Z\mid_{t=T(Z^D)}\right)}{\partial Z} = \frac{(1-\tau)\overline{S}}{r_{z,L}-\tilde{\theta}_{z,L}}-\frac{\beta_L}{Z_f^D}(1-\tau)\left(\frac{\overline{S}Z_f^D}{r_{z,L}-\tilde{\theta}_{z,L}}-\frac{K+C_f}{r_{z,L}}\right)\left(\frac{Z}{Z_f^D}\right)^{\beta_L-1} \qquad (118)
$$

Using the smooth-pasting condition $\frac{\partial\left[E(Z)\mid_{t=T(Z^D)}\right]}{\partial Z}\mid_{Z=Z_f^D}=0$, we have

$$
Z_f^D\mid_{T\left(Z_f^D\right)>T(Z^D)} = \frac{\beta_L\,(K+C_f)\left(r_{z,L}-\tilde{\theta}_{z,L}\right)}{(\beta_L-1)\,\overline{S}r_{z,L}} \qquad (119)
$$

Case II: the Firm Defaults before the Foreign Government Defaults, $T\left(Z_f^D\right) < T\left(Z^D\right)$

The value of equity is given by

$$
\begin{aligned}
E(Z\mid_{t=0})\mid_{T\left(Z_f^D\right)<T(Z^D)} &= V(Z\mid_{t=0})\mid_{T\left(Z_f^D\right)<T(Z^D)} -D_f(Z\mid_{t=0})\mid_{T\left(Z_f^D\right)<T(Z^D)} && (120)\\
&= (1-\tau)\left(\frac{\overline{S}Z}{r_{z,H}-\tilde{\theta}_{z,H}}-\frac{K}{r_{z,H}}\right) && (121)\\
&\quad -(1-\tau)\frac{C_f}{r_{z,H}}\left[1-\left(\frac{Z}{Z_f^D}\right)^{\beta_H}\right] && (122)\\
&\quad -(1-\tau)\left(\frac{\overline{S}Z_f^D}{r_{z,H}-\tilde{\theta}_{z,H}}-\frac{K}{r_{z,H}}\right)\left(\frac{Z}{Z_f^D}\right)^{\beta_H} && (123)
\end{aligned}
$$

The first-order maximization yields

$$
\frac{\partial E\left(Z\mid_{t=0}\right)}{\partial Z} = \frac{(1-\tau)\overline{S}}{r_{z,H}-\tilde{\theta}_{z,H}}-\frac{\beta_H}{Z_f^D}(1-\tau)\left(\frac{\overline{S}Z_f^D}{r_{z,H}-\tilde{\theta}_{z,H}}-\frac{K+C_f}{r_{z,H}}\right)\left(\frac{Z}{Z_f^D}\right)^{\beta_H-1} \qquad (124)
$$

Using the smooth-pasting condition $\frac{\partial[E(Z)|_{t=0}]}{\partial Z}\mid_{Z=Z_f^D}=0$, we have

$$Z_f^D\mid_{T(Z_f^D)<T(Z^D)} = \frac{\beta_H\left(K+C_f\right)\left(r_{z,H}-\tilde{\theta}_{z,H}\right)}{\left(\beta_H-1\right)\overline{S}r_{z,H}} \tag{125}$$

7.5.4 Equity Return Volatility

To determine the volatility of a firm's equity return, we first compute the dynamics of equity return denoted by $\frac{dE}{E}$. Dropping the time and the regime subscript and applying Itô's formula to E_t yields

$$\begin{aligned} dE\left(t,Z\right) &= E_t dt + E_z dZ + \frac{1}{2}E_{y,zz}dZdZ & (126)\\ &= E_z\left(\theta_z Z dt + \sigma_{z,x}ZdW^x + \sigma_{z,y}ZdW^y\right) & (127)\\ &\quad +E_{zz}\left[\left(\sigma_{z,x}Z\right)^2 dt + \left(\sigma_{z,y}Z\right)^2 dt\right] \\ &= \left[\theta_z Z + \left(\sigma_{z,x}^2+\sigma_{z,y}^2\right)E_{y,zz}Z^2\right]dt + E_z Z\left(\sigma_{z,x}dW^x + \sigma_{z,y}dW^y\right) & (128)\end{aligned}$$

Hence, the dynamics of the equity return is given by

$$\frac{dE}{E} = \frac{1}{E}\left[\theta_z Z + \left(\sigma_{z,x}^2+\sigma_{z,y}^2\right)E_{zz}Z^2\right]dt + \frac{E_z Z}{E}\left(\sigma_{z,x}dW^x + \sigma_{z,y}dW^y\right) \tag{129}$$

where E_z and E_{zz} denote the first and second derivatives of E with respect to the state variable Z, respectively. Finally, equity return volatility is given by

$$\sigma_E = \frac{E_z Z}{E}\sqrt{\sigma_{z,x}^2+\sigma_{z,y}^2} \tag{130}$$

Should the Home firm defaults after the government defaults, the level of equity return volatility is determined by the above expression where the value of equity E equals

$$\begin{aligned} E(Z)\mid_{T(Z_f^D)>T(Z^D)} &= (1-\tau)\left(\frac{\overline{S}Z}{r_{z,H}-\tilde{\theta}_{z,H}} - \frac{C_f+K}{r_{z,H}}\right)\left[1-\left(\frac{Z}{Z^D}\right)^{\beta_H}\right] & (131)\\ &\quad +E(Z)\mid_{t=T(Z^D)}\left(\frac{Z}{Z^D}\right)^{\beta_H} & (132)\end{aligned}$$

with

$$\begin{aligned} E(Z)\mid_{t=T(Z^D)} &= (1-\tau)\left(\frac{\overline{S}Z}{r_{z,L}-\tilde{\theta}_{z,L}} - \frac{K}{r_{z,L}}\right) - (1-\tau)\frac{C_f}{r_{z,L}}\left[1-\left(\frac{Z^D}{Z_f^D}\right)^{\beta_L}\right] & (133)\\ &\quad -(1-\tau)\left(\frac{\overline{S}Z_f^D}{r_{z,L}-\tilde{\theta}_{z,L}} - \frac{K}{r_{z,L}}\right)\left(\frac{Z^D}{Z_f^D}\right)^{\beta_L} & (134)\end{aligned}$$

and the first derivative E_z is given by

$$E_z(Z) \quad = \quad = \frac{(1-\tau)\overline{S}}{r_{z,H}-\tilde{\theta}_{z,H}}\left[1-\left(\frac{Z}{Z^D}\right)^{\beta_H}\right]+\frac{(1-\tau)\overline{S}}{r_{z,L}-\tilde{\theta}_{z,L}}\left(\frac{Z}{Z^D}\right)^{\beta_H} \tag{135}$$

$$+\frac{\beta_H}{Z^D}\left(\frac{Z}{Z^D}\right)^{\beta_H-1}\left[E(Z)\mid_{t=T(Z^D)}-(1-\tau)\left(\frac{\overline{S}Z}{r_{z,H}-\tilde{\theta}_{z,H}}-\frac{C_f+K}{r_{z,H}}\right)\right] \tag{136}$$