A Reduced Form Model of Default Spreads with Markov Switching Macroeconomic Factors

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Abstract

An important research area of the corporate yield spread literature seeks to measure the proportion of the spread explained by factors such as the possibility of default, liquidity or tax differentials. We contribute to this literature by assessing the ability of observed macroeconomic factors and the possibility of changes in regime to explain the proportion in yield spreads caused by the risk of default in the context of a reduced form model. For this purpose, we extend the Markov Switching risk-free term structure model of Bansal and Zhou (2002) to the corporate bond setting and develop recursive formulas for default probabilities, risk-free and risky zero-coupon bond yields. The model is calibrated with consumption, inflation, risk-free yields and default data for Aa, A and Baa bonds over the 1987-2008 period. We find that our macroeconomic factors are linked with two out of three sharp increases in the spreads during this sample period, indicating that these variations can be related to macroeconomic undiversifiable risk. We also find that the estimated default spreads can explain close to half of the 10 years to maturity industrial Baa zero-coupon yields in some regime with different sensitivities to consumption and inflation through time while this proportion is found to be much smaller for Aa and A bonds with numbers around 10%.

Keywords: credit spread, default spread, Markov switching, macroeconomic factors, reduced form model of default, random subjective discount factor.

JEL classifications: G12, G13

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1 Introduction

Several empirical studies have been recently performed on corporate yield spreads, measured as the difference between the corporate and treasury yield to maturity. These studies attempt to explain some of the observed features of corporate spreads through time. In this article we blend two research directions recently explored in this literature. We investigate if a reduced form model with observed macroeconomic risk factors following a Markov Switching process, can help in explaining the spread behavior through time. The model and the empirical study proposed here can also be seen as an extension of the Elton et al. (2001) model to a risk averse setting. In Elton et al. (2001), an assumption of risk neutrality was needed to justify the use of observed default probabilities in a bond pricing model specified under the risk neutral measure. Although risk premia were estimated empirically with Fama-French factors, risk aversion was not explicitly incorporated in their theoretical modeling approach. The model developed here is entirely specified under the objective measure in a risk averse setting, avoiding the need for a risk neutrality assumption.

The motivation for examining macroeconomic fundamentals as drivers of the spread behavior comes from the link between interest rates and output from firms and the macroeconomy. These variables, which should influence yield spreads, fluctuate over the business cycle. It should thus be anticipated that macroeconomic fundamentals play a role in explaining the spread behavior through time. Recently, some attempts have been made to tie macroeconomic activity with the spreads in the context of structural models. For example, Pesaran et al. (2006) examine an econometric model linking credit risk and macroeconomic variables in a Merton-type structural model. Chen et al. (2009) examine how a structural model using pricing kernels that are successful in solving the equity premium puzzle performs to explain the spread. David (2008) looks at how investors learning from inflation helps in generating realistic credit spread levels. To our knowledge, in the context of reduced form models, few attempts have been made apart from Amato and Luisi (2006) where a model of credit spread with both latent and observed macro variables is examined. Further work on the reduced form type models and the macroeconomy is thus an interesting addition to the literature as these models often require fewer inputs in the calibration stage.

Another distinctive feature of the model examined here is the Markov switching environment.

The motivation for examining the influence of macroeconomic variables in such a framework comes from empirical evidence suggesting that switching regimes are better descriptions of these variables and risk-free interest behavior than single regime models. See for example Evans (2003), Ang, Bekeart and Wei (2008), Bansal and Zhou (2002) and Dai, Singleton and Yang (2007). Because the possibility of changes in regime might influence macroeconomic factors and risk-free interest rates, it is only natural to assume that this might also affect the corporate yield spreads. Davies (2008) finds that this is indeed the case and that a markov switching model summarizes well some properties of spread times series. Cenesizoglu and Essid (2010) also find some evidences of switching behavior while examining the effect of monetary policy on credit spreads.

To introduce macroeconomic factors and the possibility of changes in regime in a reduced form spread model, we extend the switching regime risk-free term structure model of Bansal and Zhou (2002) to the risky corporate setting. Starting from the first order condition of the intertemporal consumption problem with a power utility function and a random subjective discount factor, we assume that consumption and inflation dynamics are governed by two independent Markov chains. Using a log linear approximation we derive closed form recursive formulas for risk-free and risky bond yields as well as for default probabilities which are all functions of the growth rates of our two observed factors. We also consider recovery rates varying with the states of consumption growth. We then measure the default spread generated by this approach by calibrating the model with financial data.

Our calibration approach proceeds without yield spread data. Corporate yield spread levels might be influenced by several factors such as the possibility of default, liquidity or taxes. Since our goal is to measure the proportion of the spread brought by the possibility of default, our model only accounts for this dimension, avoiding the potential misspecifications of liquidity and taxes factors. In such a context, fitting the model developed here with spread data could produce biased results because of these omitted factors. We therefore rely on an indirect strategy which uses aggregate consumption growth, inflation, risk-free yield curves and default data to obtain the parameter values required to measure the proportion of the yield spreads which can be explained by the possibility of default alone.

This calibration approach proceeds in three steps. First, we estimate the Markov switching parameters with aggregate consumption and inflation data. Using the parameters obtained in the first step, we then extract utility parameters from the term structure of risk-free rates. In a third step, with the parameter values obtained in the first two steps, we calibrate the parameters linking our theoretical default probabilities with the macroeconomic risk factors to match the observed default probabilities obtained from default data. The default yield spreads implied by our model can then be computed and analyzed with an assumed structure for the recovery rates.

Our results show that the default spread exhibit different sensitivities to consumption and inflation depending on the different possible regimes. We find that the model can reproduce some key properties of observed spreads, such as for example, the sharp increases observed in two out of three recessions present in our sample period. These two sharp increases are associated with a low-consumption growth and high inflation uncertainty regime identified by our Markov Switching process. This result is interesting because it indicates that, in some regime, the spread level is sensitive to a macroeconomic market wide undiversifiable risk. Such a result is supported by recent studies such as Farnsworth and Li (2007) who provide evidences about the presence of systematic factors associated with default risk. Our results also indicate that sharp increases in spreads are not necessarily linked to macroeconomic variables. For example, the sharp increase in 2001 is not captured by our model as this period is found to be in a high consumption growth and low inflation risk regime. We also find that risk aversion does not influence much the proportion of the spread caused by the risk of default. This result can be, in part, attributed to the low volatility of consumption growth and inflation during the studied period which are used as the sole factors in the model. Finally, we obtain estimates of default spread proportions varying through the different regimes. For example, these proportions, for 10 years to maturity Baa yields range from 28% to 43%, while they are around 10% for Aa and A yields. (Table 7).

Section 2 presents our theoretical models and formulas for the risk-free zero-coupon bonds, the risky zero-coupon bonds and the default probabilities. Section 3 presents our estimation results and calibration procedures. Section 3.7 analyses the estimated default spreads for industrial Aa, A and Baa bonds. Section 4 concludes.

2 Models

The model developed here starts from the well known first order condition of the intertemporal consumption problem as described in, for example, Cochrane (2005). Because we attempt to model nominal bond prices, we account for the future growth rates of the price level and real consumption. We assume that the future evolution of these variables is well described by a Markov Switching process.

Let C_t denote the real personal consumption expenditures per capita at time t, and Π_t the ratio of nominal over real consumptions (consumption price index) at time t with $t \in \mathcal{N}$. Here, the time variable is expressed in quarters and the continuously compounded quarterly growth rates are defined as $c_t = \ln C_t - \ln C_{t-1}$ and $\pi_t = \ln \Pi_t - \ln \Pi_{t-1}$. We assume that c_t and π_t follow an autoregressive model with switching regimes

$$c_t = a^c_{s^c_t} + b^c_{s^c_t} c_{t-1} + e^c_t \tag{1a}$$

$$\pi_t = a^\pi_{s^\pi_t} + b^\pi_{s^\pi_t} \pi_{t-1} + e^\pi_t \tag{1b}$$

where $s^c_t \in \{1, 2\}$ is the state of consumption at time t and $s^\pi_t \in \{1, 2\}$ is the state of inflation at time t. The error terms e^c_t and e^π_t are i.i.d. Gaussian noises with zero mean, standard deviations $\sigma^c_{s^c_t}$ and $\sigma^\pi_{s^\pi_t}$ and covariance $\rho_{s_t} \sigma^c_{s^c_t} \sigma^\pi_{s^\pi_t}$ with $s_t = \{s^c_t, s^\pi_t\}$ i.e. $s_t \in \{(1, 1), (1, 2), (2, 1), (2, 2)\}$. The states of consumption growth and inflation are assumed to follow two independent Markov chains with transition matrices

$$\phi^c = \begin{pmatrix} \phi^c_{11} & 1 - \phi^c_{11} \\ 1 - \phi^c_{22} & \phi^c_{22} \end{pmatrix}, \qquad \phi^\pi = \begin{pmatrix} \phi^\pi_{11} & 1 - \phi^\pi_{11} \\ 1 - \phi^\pi_{22} & \phi^\pi_{22} \end{pmatrix}.$$

These Markov chains are also assumed to be independent of past values of c and π.

Define the $\sigma-$field $\mathcal{G}_t = \sigma \left(C_u, \Pi_u, s_u : u \in \{0, 1, ..., t\} \right)$. It may be interpreted as the information available at time t if one observes the evolution of consumption growth, inflation, and the state of consumption and inflation up to time t.

To obtain our pricing equations, we assume a standard time-separable power utility function for the investor with a subjective discount factor coefficient which depends on the future states of the Markov chain. We are thus allowing the subjective discount factor parameter to be different in the different possible regimes[1]. As discussed in Cochrane (2005), the standard power utility framework with lognormal consumption has some difficulties in reproducing key features of observed risk-free term structures such as, for example, a positive average slope. This parameterization for the time-preference coefficient provides an additional flexibility that will help generating average yield curves with a positive slope. Appendix A shows that, from the first order condition of the intertemporal consumption problem, the time t value of a security without default risk worth X_{t+1} at time $t+1$ is given by

$$\mathrm{E}_t\left[\beta_{s_{t+1}}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\frac{\Pi_t}{\Pi_{t+1}}X_{t+1}\right]=\mathrm{E}_t\left[M_{t,t+1}X_{t+1}\right] \tag{2}$$

where

$$M_{t,t+1}=\exp\left(\ln\beta_{s_{t+1}}-\gamma c_{t+1}-\pi_{t+1}\right) \tag{3}$$

is the nominal discount factor or the default-free pricing kernel for the time period $]t,t+1]$, $\beta_{s_{t+1}}$ is the subjective discount factor in state s_{t+1} and γ the risk aversion coefficient. $\mathrm{E}_t[\bullet]$ is a short hand notation for $\mathrm{E}[\bullet|\mathcal{G}_t]$, the conditional expectation with respect to available information at time t. Equation (2) thus proposes a pricing kernel for default-free securities which is a function of the consumption, inflation and Markov chain processes. This pricing kernel accounts for regime shifts uncertainty as parameters of the consumption and inflation at $t+1$ are function of the state for the Markov chain in $t+1$. As mentioned above, we also assume that this uncertainty affects the subjective discount factor β that also depends on the state of the Markov chain at $t+1$.

2.1 Risk-free zero-coupon bond

An exact formula for the time t value of a default-risk-free zero-coupon bond paying one dollar at time T can be obtained using the framework described above. However, such a solution is not practical. For example, with quarterly time steps, the value of a zero-coupon bond maturing in 40 quarters would roughly contain 4^{40} terms to compute. This would make the numerical implementation of the exact solution unmanageable. For this reason, we instead rely on an analytical

[1]Such preferences are coherent with the general framework proposed in Higashi et al. (2009). In their model, a decision maker believes that his discount factors change randomly over time according to i.i.d. shocks. Our formulation assumes a Markov chain. Other than that, our model is consistent with their formulation. See also Salanié and Treich (2006).

approximation developed in Bansal and Zhou (2002) for the price of a risk-free zero-coupon bond with n periods to maturity:

$$P(t,n,s_t) = \exp\left(A^p_{n,s_t} - B^{p,c}_{n,s_t} c_t - B^{p,\pi}_{n,s_t} \pi_t\right) \tag{4}$$

where $s_t = \{s^c_t, s^\pi_t\}$ and expressions for $A^p_{n,s_t}, B^{p,c}_{n,s_t}$, and $B^{p,\pi}_{n,s_t}$ are given in Appendix B. The pricing formula is a function of our observed factors and the states of the Markov chains. The sensitivities to the factors are given by the B functions which are determined recursively using backward induction and the terminal condition $A^p_{0,s_T} = B^{p,c}_{0,s_T} = B^{p,\pi}_{0,s_T} = 0$. These expressions are functions of the Markov switching parameters and the actual states of consumption and of inflation s^c_t and s^π_t. At each point in time, four different bond prices can thus be computed since four different states are possible. Because the state of the economy is unknown at a particular point in time, we will define the theoretical zero-coupon bond price as the expected bond price, with the expectation computed over the possible states of the Markov chain whose probability can be conveniently estimated. Section 3.4 provides further details about this procedure. The factor sensitivities are also functions of the time to maturity, $n = T - t$, and the utility function parameters.

Although the formula in Appendix B is complex, it is possible to get some intuition by looking at the one period case, rewritten in terms of an annualized yield to maturity:

$$y_p(t,1,s_t) = 4\sum_{i=1}^{2}\sum_{j=1}^{2} \phi^c_{s^c_t,i}\phi^\pi_{s^\pi_t,j} \left[\begin{array}{c} -\ln \beta_{i,j} + \gamma\left(a^c_i + b^c_i c_t\right) + \left(a^\pi_j + b^\pi_j \pi_t\right) \\ -\frac{1}{2}\left(\sigma^\pi_j\right)^2 - \frac{1}{2}\gamma^2\left(\sigma^c_i\right)^2 - \gamma\rho_{i,j}\sigma^c_i\sigma^\pi_j \end{array} \right]$$

where the term inside brackets is the expression for the yield to maturity, in state i, j, of a one period risk-free bond within the power utility lognormal framework. The one period bond yield in state s_t is the conditional expected value of the bond yields in the different possible states where the ϕ's are the transition probabilities. The various terms forming the bond yield in state i, j are then interpreted the usual way.

The first term of the expression within brackets is a function of the subjective discount factor. A smaller subjective discount factor (more impatient investor) is associated with higher yields since the impatient investor prefers consumption to saving. The second term, $\gamma\left(a^c_i + b^c_i c_t\right)$, is the risk aversion parameter multiplied by the conditional expected growth of consumption in state i, j. Given positive a_i^c and b_i^c, higher values of these coefficients will lead to higher expected consumption growth and higher yields. The risk aversion parameter $\gamma > 0$ makes the yield more or less sensitive to the expected consumption growth rate. The sum of the third and fourth term, $(a_j^\pi + b_j^\pi \pi_t) - \frac{1}{2}\left(\sigma_j^\pi\right)^2$, is the portion of yield rewarding the investor for the expected loss in real purchasing power on the nominal one dollar bond payoff at maturity and where the variance of inflation appears because of the convexity of the bond pricing function. The fifth term, $\frac{1}{2}\gamma^2\left(\sigma_j^c\right)^2$, is the precautionary savings effect brought by the volatility of consumption. An increase in the volatility of consumption brings more extreme low and high paths of future consumption. Because investors worry more about the low consumption states than they are pleased by the high ones, a demand for savings is created which drives down the yield on the bond. Finally, the last term, $\gamma\rho_{i,j}\sigma_i^c\sigma_j^\pi$, is the inflation risk premium. A negative correlation will obtain a positive risk premia because inflation decreases the real nominal bond payoff in states where the investor needs it the most. For example, a future low consumption state would likely be associated with a high inflation path and low real value for the nominal payoff.

2.2 Risky zero-coupon bond and default spread

We consider a risky zero-coupon bond paying one dollar at T if it has not defaulted before. In case of default, the bondholder receives at the default time τ, a fraction of its market value if it had not defaulted. In this well studied context (see Duffie and Singleton 1999), the time t value of the survived risky zero-coupon bond is

$$\widetilde{V}(t, n, s_t) = \mathrm{E}_t\left[M_{t,t+1}\left(1 - L_{s_{t+1}} h_{t+1}\right)\widetilde{V}(t+1, n-1, s_{t+1})\right] \tag{5}$$

where $L_{s_{t+1}}$is the loss given default (LGD) defined as one minus the recovery rate and assumed to depend on the states of the Markov chain. Here, $h_{t+1} = \mathrm{Pr}_{\mathcal{G}_{t+1}}[\tau = t+1 \mid \tau > t]$ represents the conditional probability that the default arises within the next period of time knowing that the firm as survived at time t and having the information available at time $t+1$. Since default probabilities are usually small, it is reasonable to use a first order Taylor expansion to approximate $1 - L_{s_{t+1}} h_{t+1}$ by $\exp\left(-L_{s_{t+1}} h_{t+1}\right)$. Hence

$$\widetilde{V}(t, n, s_t) \cong \mathrm{E}_t\left[M_{t,t+1}\exp\left(-L_{s_{t+1}} h_{t+1}\right)\widetilde{V}(t+1, n-1, s_{t+1})\right]. \tag{6}$$

where $M_{t,t+1} \exp\left(-L_{s_{t+1}} h_{t+1}\right)$ is our pricing kernel for the risky zero-coupon bond. We also assume that the conditional default probability h_{t+1} is approximated by an affine function of c_{t+1} and π_{t+1}, that is,

$$h_{t+1} \cong \alpha_{s_{t+1}} + \alpha^c_{s_{t+1}} c_{t+1} + \alpha^\pi_{s_{t+1}} \pi_{t+1} \tag{7}$$

where $\alpha_{s_{t+1}}, \alpha^c_{s_{t+1}}$ and $\alpha^\pi_{s_{t+1}}$ are parameters. Note that the specification (7) can produce negative probabilities as well as probabilities larger than one. Using these assumptions and those required by the approach of Bansal and Zhou (2002), Appendix C develops the following analytical approximation for the prices of risky zero-coupon bonds :

$$V(t, n, s_t) = \exp\left(A^v_{n,s_t} - B^{v,c}_{n,s_t} c_t - B^{v,\pi}_{n,s_t} \pi_t\right). \tag{8}$$

As shown in Appendix C, the coefficients A^v_{n,s_t}, $B^{v,c}_{n,s_t}$ and $B^{v,\pi}_{n,s_t}$ are obtained recursively starting with $A^v_{0,s_T} = B^{v,c}_{0,s_T} = B^{v,\pi}_{0,s_T} = 0$. The resulting pricing formula is very similar to the risk-free case developed earlier. It is a function of the current values of our two observed factors with the loadings given by the B functions. These quantities are functions of the Markov switching model parameters, the actual states of consumption and of inflation s^c_t and s^π_t, the utility function parameter values and the time to maturity. Unlike the risk-free case, however, we find the additional $L_{i,j}\alpha_{i,j}, L_{i,j}\alpha^c_{i,j}$ and $L_{i,j}\alpha^\pi_{i,j}$ terms appearing because of the possibility of default.

Using the above analytical approximation and the earlier approximation for the risk-free yield, an expression for the annualized default spread, defined as the difference between the risky yield to maturity and the risk-free yield to maturity, is given by:

$$DS(t, n, s_t) = \frac{A^p_{n,s_t} - A^v_{n,s_t} + (B^{v,c}_{n,s_t} - B^{p,c}_{n,s_t})\, c_t + (B^{v,\pi}_{n,s_t} - B^{p,\pi}_{n,s_t})\, \pi_t}{n/4}. \tag{9}$$

Again, to get some intuition about the role of the different parameters on the spread, it is interesting to look at the one period case :

$$DS(t, 1, s_t) = 4 \sum_{i=1}^{2} \sum_{j=1}^{2} \phi^c_{s^c_t, i} \phi^\pi_{s^\pi_t, j} \tag{10}$$

$$\times L_{i,j} \begin{bmatrix} \alpha_{i,j} + \alpha^c_{i,j}\left(a^c_i + b^c_i c_t\right) + \alpha^\pi_{i,j}(a^\pi_j + b^\pi_j \pi_t) \\ -\frac{1}{2} L_{i,j} \left(\left(\alpha^c_{i,j} \sigma^c_i\right)^2 + \left(\alpha^\pi_{i,j} \sigma^\pi_j\right)^2 + 2\alpha^\pi_{i,j}\alpha^c_{i,j}\rho_{i,j}\sigma^c_i\sigma^\pi_j \right) \\ -\alpha^\pi_{i,j}\left(\sigma^\pi_j\right)^2 - \gamma\left(\alpha^c_{i,j}\left(\sigma^c_i\right)^2 + \left(\alpha^\pi_{i,j} + \alpha^c_{i,j}\right)\rho_{i,j}\sigma^c_i\sigma^\pi_j\right) \end{bmatrix}$$

where the term inside brackets is the expression for the default spread, in state i, j, for a one period risky bond. The default spread in state s_t is the conditional expected value of the bond yield spreads in the different possible states next period.

The first line of the term within brackets can be interpreted as one of the portions forming the expected loss next period in state i, j. In the context of a one period bond, $L_{i,j}$ represents the loss given default. This quantity is multiplied by the conditional expected default probability in state i, j that is $\alpha_{i,j} + \alpha^c_{i,j}\left(a^c_i + b^c_i c_t\right) + \alpha^\pi_{i,j}(a^\pi_j + b^\pi_j \pi_t)$. The signs of the $\alpha_{i,j}, \alpha^c_{i,j}$ and $\alpha^\pi_{i,j}$ will determine the influence of consumption and inflation on this portion of the spread. The second and third lines are the additional impacts of potential losses brought by the convexity of our recovery factor model. Again, the sign of these terms will depend on the signs of the α's. For example, the effect of a change in consumption volatility is not clear as it depends on the magnitude and signs of the α's and the correlation. Hence, given $\rho_{i,j} > 0$ with a negative and large $\alpha^c_{i,j}$ (relatively to the $\alpha^\pi_{i,j}$), an increase in consumption volatility will increase the spread. Finally, it is interesting to note that the risk aversion parameter interacts only with the squared volatilities and covariance of the factors. Hence, risk aversion is here a second order effect whose magnitude will be determined by the relative importance of the volatilities, covariance term and the magnitudes and signs of the $\alpha^c_{i,j}$ and $\alpha^\pi_{i,j}$ parameters. In the context of this model, this proportion caused by risk aversion can be conveniently assessed. One can first compute the portion of the spread which is caused by the actuarial loss. This is the default spread a risk neutral investor would be satisfied with. This quantity, that we label the *default risk spread*, can be computed by setting $\gamma = 0$ in the default spread equation i.e.

$$DR(t, n, s_t) = DS\left(t, n, s_t \mid \gamma = 0\right). \tag{11}$$

The portion of the spread caused by risk aversion, which we label the *default premium spread*, can then be computed by difference with

$$DP(t, n, s_t) = DS\left(t, n, s_t\right) - DR(t, n, s_t). \tag{12}$$

This is the spread a risk averse investor would ask for in addition of the default risk spread. In the context of a one period bond, this quantity becomes

$$DP(t, 1, s_t) = 4 \times \sum_{i=1}^{2}\sum_{j=1}^{2} \phi^c_{s^c_t, i}\phi^\pi_{s^\pi_t, j}\left[-\gamma L_{i,j}\left(\alpha^c_{i,j}\left(\sigma^c_i\right)^2 + \left(\alpha^\pi_{i,j} + \alpha^c_{i,j}\right)\rho_{i,j}\sigma^c_i\sigma^\pi_j\right)\right].$$

2.3 Survival probability

A final theoretical quantity obtained within the framework of this model is the term structure of survival probabilities. This quantity will be used to calibrate our model to match the default probabilities that are observed for the sample period examined in this study.

The survival probability at t, $\Pr_{\mathcal{G}_t}[\tau > t+n \mid \tau > t]$, is the probability that the default occur in more than n periods from t knowing that the firm has not defaulted at time t in a given state of our macro factors at t. Because this probability is usually small, we use the approximation $e^{-h_u} \cong 1 - h_u$ to write

$$\Pr_{\mathcal{G}_t}[\tau > t+n \mid \tau > t] = \mathrm{E}_t\left[\prod_{u=t+1}^{t+n}(1-h_u)\right]$$

as

$$\Pr_{\mathcal{G}_t}[\tau > t+n \mid \tau > t] \cong \mathrm{E}_t\left[\exp\left(-\sum_{u=t+1}^{t+n}\left(\alpha_{s_u} + \alpha^c_{s_u}c_u + \alpha^\pi_{s_u}\pi_u\right)\right)\right]$$

from our assumption given in equation (7). As shown in Appendix D, an analytical approximation for this expected value is given by:

$$q(t,n,s_t) = \exp\left(-A^q_{n,s_t} - B^{q,c}_{n,s_t}c_t - B^{q,\pi}_{n,s_t}\pi_t\right). \tag{13}$$

As in the previous cases, the coefficients A^q_{n,s_t}, $B^{q,c}_{n,s_t}$ and $B^{q,\pi}_{n,s_t}$ are obtained recursively starting with $A^q_{0,s_T} = B^{q,c}_{0,s_T} = B^{q,\pi}_{0,s_T} = 0$. These coefficients are function of the maturity n, the Markov switching parameters, the unobserved state s_t and the unknown parameters linking the one period default probability h_t with the real consumption growth and inflation.

3 Calibration and estimation

3.1 Empirical yield curves

To measure the capacity of our model to generate realistic default spread levels through time, estimates of the credit yield spread curves of Aa, A and Baa zero-coupon bonds are required. The spreads curves are computed as the difference between the zero-coupon corporate yields minus the zero-coupon risk-free yields. We explain here how these risk-free and corporate zero yields are obtained.

The risk free zero-coupon yield data comes from Gurkaynack et al. (1997) which is available from a Federal Reserve web site[2]. This data contains, at a daily frequency, the parameter estimates of the Svensson (1994) model from cross sections of risk-free coupon bonds. To build our quarterly risk-free curves, we first extract the parameters at the dates that are the nearest to the quarterly dates of the National Income and Product Accounts data. We then use these parameters in the Svensson (1994) model with maturities of 0.25, 0.5, 0.75, ..., 10 years to get a risk-free curve for each date of our sample.

For dates ranging from 1987-I to 1996-IV, the corporate zero-coupon yield curves are extracted from the Warga (1998) fixed income database which contains information on monthly prices, accrued interest, coupons, ratings, callability and returns on investment-grade corporate bonds quoted at Lehman Brothers. All bonds with matrix prices and options were removed; bonds not in Lehman Brothers' bond indexes and bonds with an odd frequency of coupon payments were also dropped. We proceed as in Elton et al. (2001) to get the month-end estimates of the yield curves on zero-coupon bonds for each rating class from Moody's (Aa, A and Baa). These yield curves are obtained by first estimating the parameters associated with the Nelson and Siegel (1987) model with a non-linear least-square procedure on bonds with maturities of 10 years or less[3]. All bonds with a pricing error higher than \$5 are dropped. We then repeat this estimation and data removal procedure until all bonds with a pricing error larger than \$5 were removed. Using this procedure, 776 bonds were removed (one Aa, 90 A and 695 Baa) out of a total of 33,401 bonds found in the industrial sector. We then use these parameters in the Nelson Siegel (1987) model with maturities of 0.25, 0.5, 0.75, ..., 10 years to get a risk-free curve for each date of our sample.

For dates between 1997-I to 2008-IV, we use the zero-coupon yield data available from Bloomberg. These yields are extracted by Bloomberg on samples of coupon bond prices with a spline approach. Bloomberg makes available maturities of 0.25, 0.5, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 15, 20 and 30 years on bonds from the industrial sector rated (Standard and Poor's) AA, A+, A, A-, BBB+, BBB, BBB- on a daily frequency. We first extract the data for the dates that are the nearest to the quarterly dates of the National Income and Product Account data. For this sample, for each date

[2]http://www.federalreserve.gov/pubs/feds/2006

[3]To minimize the chances of converging to a local rather than a global minimum, a grid search of $20^4 = 160000$ points is performed to find a suitable starting point for the numerical minimization.

and maturity, we first aggregate the yields of A rated bonds with an average i.e. the yield of an A rated bond is the average yield from A+, A, and A- rated bonds. The same is done for BBB bonds. Because our study uses maturities 0.25, 0.5, 0.75, ..., 10 years, we interpolate for these maturities using Nelson-Siegel. We thus estimate for each date, the parameters of the Nelson-Siegel model on the Bloomberg zero-coupon yields with maturities of 0.5, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 and use these parameters to compute the yields of 0.25, 0.5, 0.75, ..., 10 year bonds. Table 1 presents the average zero-coupon yield spreads for industrial Aa, A and Baa for maturities of 1 to 10 years for the 1987-2008 period.

3.2 Markov Switching parameters

This section describes how the parameters of the Markov Switching model are estimated. Let θ denote the set of parameters associated with the growth rate equations that is $\theta = (a_1^c,\ a_2^c,\ b_1^c,\ b_2^c,\ a_1^\pi,\ a_2^\pi,\ b_1^\pi,\ b_2^\pi,\ \sigma_1^c,\ \sigma_2^c,\ \sigma_1^\pi,\ \sigma_2^\pi,\ \rho_{1,1},\ \rho_{1,2},\ \rho_{2,1},\ \rho_{2,2})$ and the transition probabilities parameters $\phi = (\phi_{11}^c,\ \phi_{22}^c,\ \phi_{11}^\pi,\ \phi_{22}^\pi)$. From the time series of consumption levels $C_0, ..., C_T$ and price index levels $\Pi_0, ..., \Pi_T$ from which we create the sample $c_1, ...c_T, \pi_1, ..., \pi_T$, we define $v_t = (x_1, ..., x_t)$ as the set of observed data point up to time t and $x_t = (c_t, \pi_t)$ as the set of observed consumption growth and inflation at t. From Hamilton (1994), the log-likelihood function based on the observed sample v_T up to time T is then computed with

$$\mathcal{L}\left(\theta,\phi; v_T\right) = \sum_{t=2}^{T} \ln f\left(x_t \mid v_{t-1}; \theta,\phi\right) \tag{14}$$

where

$$f\left(x_t \mid v_{t-1}; \theta,\phi\right) = \eta_t' \times \xi_{t|t-1}$$

represents the conditional likelihood function of x_t given the observed sample v_{t-1}. The 4×1 vector η_t contains the likelihood value of x_t conditional on states i, j and the observed sample v_{t-1}. The 4×1 vector $\xi_{t|t-1}$ contains the probability of being in state i, j at time t conditional on the observed sample v_{t-1}. Appendix E describes how these quantities can be computed. The maximization of the log-likelihood function $\mathcal{L}\left(\theta, \phi; v_T\right)$ is done numerically using a hill climbing algorithm.

The data series used here are the growth rate of non-durable and services personal consumption expenditures per capita (real) from the first quarter of 1957 to the last quarter of 2008 and the growth rate of the consumption price index for the same period[4]. The data comes from the U.S. Department of Commerce, Bureau of Economic Analysis. The data period contains nine recessions according to the NBER and many of them should be important enough to generate regime shifts. Figure 1 illustrates the temporal evolution of the two growth rates.

The results of the estimation procedure are presented in Table 2. For consumption, regime switching appears in the mean and the volatility that show different levels in both. For inflation, only the volatility parameter is found to differ with the regime shifts. We also observe that ρ_{12} and ρ_{22} are statistically different from zero.

Within the context of our regime switching model, two different conditional probabilities are of interest. The ex-ante probability, $\xi_{t|t}$, is useful in forecasting future inflation and consumption rates based on an evolving information set. The smoothed probability, $\xi_{t|T}$, estimated using the entire information set available, is of interest for the determination of the prevailing regime at each time point within the sample period. To estimate $\xi_{t|T} = f(s_t \mid v_T; \theta, \phi)$ for $s_t \in \{(1,1), (1,2), (2,1), (2,2)\}$, we use the following algorithm developed in Kim (1994):

$$\xi_{t|T} = \xi_{t|t} (\times) \left[\Phi \left(\xi_{t+1|T} (\div) \xi_{t+1|t}\right)\right]$$

where $(\times)$ and $(\div)$ means element-by-element multiplication and division respectively and where the transition matrix Φ is described in appendix E.

Using these probabilities, Figures 2 and 3 examine the fit of the Markov switching model. Each figure shows three graphs. The first graph plots the actual and fitted values; the second graph shows a quantile-to-quantile plot (qqplot) of the standardized residuals; the third graph looks at the sample autocorrelation coefficients of the residuals. Because there is uncertainty about which state prevails, the fitted values are computed as the expected fitted values over the two possible states. These expected values are computed with the smoothed probabilities. The same procedure is adopted to form the residual series, which are standardized by dividing by the estimated standard deviation in each state. For the consumption growth series, the actual values are often far from the fitted ones. Despite these large residuals, the qqplot and sample autocorrelation coefficients

[4]Non-durable goods and services expenditures are aggregated naively with a simple sum of both components. We have verified that this approach gives results almost identical those from the chain aggregation approach, described in, for example, Whelan (2000).

show that the model produces close to white noise residuals that are well described by a normal distribution. The sample autocorrelations are in most cases within the two standard deviation confidence interval around a value of zero for all lags, except for lags 3 and 8 that are slightly out. For the inflation series, the actual values are closer to the fitted one. Again, from the qqplot and sample autocorrelation coefficients, the model is shown to produce well behave Normal residuals with some significant autocorrelations in lags 1, 3 and 6.

3.3 States of consumption and inflation

We examine here more closely the estimated probabilities for the state of the Markov chain for the period 1987-I- to 2008-IV which corresponds to the data period we have, regarding our risky bond information. Note that we used the estimates of θ and ϕ from Table 2.

The results of the estimation procedure are presented in Figure 4. In most quarters, one of the four values of the mass function clearly dominates the other. On a total of 88 observations, 80 are larger than 0.6 and 73 are larger than 0.8. The estimated states $\widehat{s}_t$ at time t is the one for which the estimated probability in vector $\xi_{t|T}$ is the highest among all the possible states. The results are reported in Figure 5.

The interpretation of the estimated states are as follows: $s_t = (1, 1)$ corresponds to a state of high level and low consumption growth volatility combined with low volatility of inflation; $s_t = (1, 2)$ is the state of high level and low volatility of consumption growth with high volatility of inflation; $s_t = (2, 1)$ corresponds to the low level and high volatility of consumption growth combined with low volatility of inflation; finally $s_t = (2, 2)$ is for low level and high volatility of consumption growth with more volatile inflation.

A detailed examination of the results reveals that the estimated state of consumption is 1 for two distinct time periods: 1987-I to 1990-I and for 1991-IV to 2007-I. Otherwise, the consumption's estimated state is 2. For inflation, we note two changes of regime. Indeed, the state of inflation is estimated to 2 for the time period 1987-I to 1990-IV and becomes 1 for the time period 1991-I to 2005-I and then comes back to two until the end of the sample. If we consider the system globally, the estimated state is $s_t = (2, 2)$ near the 1991 and 2008 NBER recessions. In between these NBER recessions, the system stays in state $s_t = (1, 1)$, even with the presence of an official NBER recession in 2001. Prior to the 1991 NBER recession, the estimated state is $s_t = (1, 2)$. Figure 6 illustrates the changes of regime behavior for the growth rate of our two factors. The consumption growth rate and inflation clearly exhibit different behavior in each regime.

It is interesting to note that the observed average consumption growth rate and volatility are 0.51% and 0.27% during the periods corresponding to $\widehat{s}_t^c = 1$ while they are -0.15% and 0.37% during the periods corresponding to $\widehat{s}_t^c = 2$. The observed average inflation growth rate and volatility are 0.64% and 0.21% during periods for which $\widehat{s}_t^\pi = 1$ and 0.93% and 0.59% when $\widehat{s}_t^\pi = 2$. These numbers correspond roughly to the unconditional mean and standard deviations which can be computed from the parameter estimates. For consumption, these unconditional mean and standard deviations are 0.61% and 0.38% for $\widehat{s}_t^c = 1$ and 0.075% and 0.55% for $\widehat{s}_t^c = 2$ while they are 0.68% and 0.30% for inflation when $\widehat{s}_t^\pi = 1$ and 1.2% and 0.75% when $\widehat{s}_t^\pi = 2$.

As mentioned, our model did not capture the 2001 recession for consumption growth. This conclusion seems more related to the consumption variable than the model. It is well documented in the literature on forecasting models that standard leading indicators did not perform well to forecast the 2001 recession. While the 1991 recession is explained by a sharp reduction in consumption, the 2001 recession was more associated to a reduction in investment and particularly in the information technology sector. Evans et al. (2002) show that their personal consumption and housing indicator of economic activity did not fall below zero in 2000 and hardly registered negative values in 2001, contrarily to other indicators. The same conclusion was obtained by Stock and Watson (2003) who verified that consumer spending remained strong during the first quarters of the 2001 recession. They also report that the consumer expectation series of 2000 reflected positive consumer expectations.

3.4 Preference parameters

In this section, we explain how the subjective discount factors $\beta = (\beta_{11}, \beta_{12}, \beta_{21}, \beta_{22})$ and the risk aversion coefficient γ are estimated. We assume that the parameters θ and ϕ of the Markov Switching processes are known and are set to their estimated value.

Because the state of the economy is unknown at a particular point in time, we define the theoretical zero-coupon bond price as the expected bond price, with the expectation computed over the possible states of the Markov chain[5]. Using $\widehat{\theta}$ and $\widehat{\phi}$, the estimated parameters for the Markov chain, the zero-coupon risk-free bond price at time t is defined as

$$\overline{P}\left(t,n,\beta,\gamma\right)=\sum_{k=1}^{4}\hat{\xi}_{t|t}(k)\times P\left(t,n,s_t(k)\right)$$

where $\hat{\xi}_{t|t}(k)$ is the estimated ex-ante probability of being in one of the four possible states at time t, $s_t(k)$ denotes the kth possible value of s_t and $P(\bullet)$ is the risk-free zero-coupon bond price computed with equation (4). Notice that this price is a function of the estimated Markov switching model parameters $\widehat{\theta}$ and $\widehat{\phi}$ and the preference parameters. The estimates of the preference parameters are obtained by minimizing the following objective function with respect to β and γ :

$$Q\left(t,\beta,\gamma\right)=\sum_{t}\sum_{n=1}^{40}\left(-\frac{\ln\overline{P}\left(t,n,\beta,\gamma\right)}{n/4}-y_g\left(t,n\right)\right)^2 \tag{15}$$

with the constraints that $0<\beta_{i,j}<1$ and where $y_g\left(t,n\right)$ is the yield to maturity of a zero-coupon government bond estimated with the Nelson and Siegel (1987). We are thus using our quarterly time series of estimated risk-free spot rates term structures, covering the 1987 to 2008 period, to estimate the preference parameters. Each term structure in this sample covers maturities up to ten years (40 quarters).

This calibration procedure obtains estimates of $\widehat{\gamma}=0.7919$ and $\widehat{\beta}=\{0.9999,\ 0.9984,\ 0.9925,\ 0.9999\}$. As in other studies, our estimates of the time preference parameters are close to one. See for example Hansen and Singleton (1982, 1984). As discussed in Kocherlakota (1990), such values for this coefficient are not unrealistic and coherent with well-defined equilibria in growth economy. We note that the restriction that these parameters be smaller or equal to one is imposed during the estimation. This avoids the potential problem of having negative yields in some states. To study how good the model fits the data, we report the root mean squared errors (*rmse*), the average absolute errors (*aae*), the average errors (*ae*) and the average fitted yields (*avg fitted*) in Table 3. The average fitted yields have a positive slope, just like the average observed yield, but the average errors are large. The top graph in Figure 7 illustrates the evolution of the observed and fitted 10

[5]Another approach could use the price prevailing in the state with the highest filtered probability. However, we have verified that such an approach has little impact on the results. The highest filtered probability is often in the neighbourhood of 0.9 or 0.95. Because of this, the average of the bond prices in the four states using the filtered probabilities is almost identical to the price in the state with the highest probability.

years to maturity yield from which we can visualize the large errors. A detailed examination of the fitted and observed risk-free yield curves shows that in many cases, the slope and curvature do not agree.

Because of these large errors in the fitted risk-free yields, we look at an additional calibration approach that will provide a robustness check of our final results regarding the estimation of default spreads. This alternative calibration approach allows a tighter fit to the risk-free curves, at the cost of some time inconsistencies, by fitting different values of β and γ through time. This cross-sectional estimation procedure is similar to the one adopted in Brown and Dybvig (1986) for the case of the Cox et al. (1985) model. It produces, at each time-point, implied estimates with the available cross section of bond yields at that time. More precisely, at each quarter of our sample, we estimate a set of preference parameters by minimizing the following objective function with respect to β and γ :

$$\widetilde{Q}\left(t,\beta,\gamma\right)=\sum_{n=1}^{40}\left(-\frac{\ln\overline{P}\left(t,n,\beta,\gamma\right)}{n/4}-y_{g}\left(t,n\right)\right)^{2}. \tag{16}$$

This procedure gives a calibrated model which can accurately replicate the level, slope and curvature of the risk-free term structures at each time point of our sample. These time varying parameter values will affect the spreads estimates only through the risk-aversion parameters since the $\beta_{i,j}$ are not functions of the theoretical spread expression (see equation (9) and (10)). The preference parameters estimated with this calibration procedure are presented in Figure 8. The average of the γ's is 0.1782 while they are 0.9964, 0.9956, 0.9880, and 0.9859 for the β's. Table 4 reports the fit of this calibration procedure which is, as expected, closer to the actual yields when compared with the results from the earlier procedure. The bottom graph in Figure 7 illustrates the evolution of the observed and fitted 10 years to maturity yield which are much closer than with the constant preference parameter.

3.5 Conditional default probability parameters

We describe here the calibration procedure for the conditional default probability parameters $\alpha = (\alpha_{1,1},\ \alpha_{1,2},\ \alpha_{2,1},\ \alpha_{2,2},\ \alpha^{c}_{1,1},\ \alpha^{c}_{1,2},\ \alpha^{c}_{2,1},\ \alpha^{c}_{2,2},\ \alpha^{\pi}_{1,1},\ \alpha^{\pi}_{1,2},\ \alpha^{\pi}_{2,1},\ \alpha^{\pi}_{2,2})$ required by our corporate bond pricing and credit spread model.

Because we want to capture the time-varying nature of default probabilities, we calibrate these parameters with a yearly sample of default probabilities term structures. This sample is estimated from transition matrices obtained from Moody's Corporate Bond Default Database with the cohort method of Carty and Fons (1993) and Carty (1997). One matrix is estimated for each year of our sample period, for a total of 22 different matrices. Each matrix is estimated with one year of default data. For example, the transition matrix for 1987 is estimated using the cohort method with default data from the beginning of January 1987 to the end of December 1987. The default probability matrix is then obtained from those matrices by first converting these into a generator (see Christensen et al. (2004)) with the approach suggested in Israel et al. (2001). With this generator, the transition matrix for a horizon of n periods and the corresponding default probability can be computed with

$$\exp\left(\frac{n}{4}\mathbf{G}\right) = \sum_{i=0}^{\infty} \frac{\left(\frac{n}{4}\mathbf{G}\right)^i}{i!}$$

where $\mathbf{G}$ is the generator matrix. The term structure of empirical survival probabilities for the appropriate credit class is then extracted from these computed matrices generated with $n = 1$ to 40.

The estimates for α are obtained by minimizing the squared errors between our theoretical and empirical survival probabilities. Again, as in the case of the theoretical risk-free bond prices, we define the survival probability as the expected survival probability, with the expectation taken over the regime of the Markov chain. More formally, the expected survival probability is defined as :

$$\overline{q}\left(t, n, \alpha\right) = \sum_{k=1}^{4} \hat{\xi}_{t|t}(k) \times q\left(t, n, s_t(k)\right)$$

where $\hat{\xi}_{t|t}(k)$ is the estimated ex-ante probability of being in one of the four possible states at time t, $s_t(k)$ denotes the kth possible value of s_t and $q\left(\bullet\right)$ is the survival probability computed with equation (13). The sum of squared error function is then defined as:

$$R\left(\alpha\right) = \frac{1}{40} \sum_{t=1}^{88} \sum_{n=1}^{40} \left(q^{\text{emp}}\left(t, n\right) - \overline{q}\left(t, n, \alpha\right)\right)^2$$

where $q^{\text{emp}}\left(t, n\right)$ is the empirical survival probability obtained at quarter t from a transition matrix. Notice that since we have one term structure of default probability per year, $q^{\text{emp}}\left(t, n\right)$ is identical for the four quarters of a given year.

The minimization of the above function is done numerically under the constraint that the one-period conditional default probability is non-negative. The estimated parameters of the conditional default probability are shown in Table 5. It is interesting to notice that for each credit classes, the consumption and inflation parameters in state $s_t = (2,2)$ are large negative numbers, indicating that large negative values for these variables will increase the one period conditional default probability specified as $h_t = \alpha_{s_t} + \alpha_{s_t}^c c_t + \alpha_{s_t}^\pi \pi_t$.

Figure 9 reproduces the estimated one period conditional default probability computed as $1 - \overline{q}(t, 1, \widehat{\alpha})$ along with the consumption growth and inflation. It is interesting to note that the conditional default probability jumps during state $s_t = (2,2)$ i.e. the low level high volatility of consumption growth and high volatility of inflation. These periods are within two out of the three NBER economic recessions found in our sample. Table 6 reports the correlation between consumption and inflation as well as their correlations with the estimated default probabilities.

Without any conditioning on the regimes, the estimated probabilities for all credit classes are negatively correlated with the real consumption growth rate with estimated correlations around -0.5 over the 1987-2008 period. These sign are however changing when conditioning on the regime. For state $s_t = (2,2)$, the correlation is around -0.4 for all classes but switches in sign for state $s_t = (1,2)$. For the other states, the correlations can be positive or negative, depending on the credit class. For inflation, without conditioning on the regime, the estimated probabilities are also negatively linked with the probabilities. Again, as for consumption, the correlation is strong and negative in state $s_t = (2,2)$. For the other states, the signs are changing across the different credit classes.

3.6 Recovery rates

The loss given default parameters $L_{i,j}$ (1-the recovery rate) are estimated with an annual time series of recovery rates from Moody's (2009) for the years 1987 to 2008. Moody's rates are defined as the ratio of the defaulted bond's market price to its face value, as observed 30 days after the default date, for all bonds irrespective of their rating. Since these recovery rates are for all bond ratings, they can be interpreted as the recovery rates of bonds with an average risk.

The limited length of the annual 1987-2008 time series makes it difficult to obtain meaningful estimates for all four possible states. Because of this, we assume a recovery rate varying with the states of consumption only. We obtain an average recovery rate of 0.41 for the state of low volatility of consumption growth (years 1987 to 1989 and 1992 to 2006) prevailing in our 1987-2008 sample period and an average of 0.38 for the high volatility state (years 1990 to 1991 and 2007 to 2008).

3.7 Default spreads in corporate yield spreads

With the parameters of our default spread model obtained from consumption, inflation, risk-free yields and default data, we examine in this section the theoretical default spreads generated by the model. As mentioned in the introduction, the default spread estimates are computed without using the actual corporate yield spreads to avoid the potential bias associated with missing factors.

Figure 10 plots a two scale graph showing the evolution of the estimated default spread for ten years to maturity Aa, A and Baa zero-coupon bonds in conjunction with the observed yield spread for the case of the constant preference parameter estimates i.e. with $\widehat{\gamma} = 0.7919$ and $\widehat{\beta} = \{0.9999, 0.9984, 0.9925, 0.9999\}$. As shown in these graphs, the estimated default spreads show some similarities with the observed yields spreads. For example, the sharp increases at the end of 1990 and in 2007-08 is well captured by the model, without the use of yield spread data. The 2001 sharp increase is however not captured by our model. Looking at figure 6, we see that our observed factors do not show large variations for this 2001 NBER recession. Hence, our model which specifies default probabilities that are functions of these state variables is not able to capture the spread increase for this period.

Table 7 presents some statistics about these estimated default spreads. This table also reports the statistics across the different states of consumption and inflation. The estimated default spread represents, on average, 8% and 14% and 37% of the 10 years yield spreads for A, Aa and Baa bonds. For Baa bonds, this is higher than numbers found in Elton et al. (2001) which reported a maximum of 25% from an analysis partially using the same data. For Aa and A bonds, the proportions are roughly stable across the different states of the Markov switching process. For Baa bonds, this proportion varies in the different states. For example, in state $s_t = (1, 2)$, the high consumption growth and volatile inflation state, the proportion is 43% while it goes down to 28% in state $s_t = (2, 2)$, the low consumption and high volatility of inflation state. For this state, the default spreads did show sharp increases, but not as large as the increases observed for the credit spreads. It can also be noticed that the volatility of our theoretical default spreads are small when compared to the yield spread volatility for the whole sample and in all subperiods. In general, our estimated default spreads are positively related to the yields spreads with correlations around 50% for all states. Across the different regimes, these correlations are typically positive and strong in state $s_t = (2, 2)$. For the other states, this correlation is changing in sign across the different ratings. This indicates that an increase in default spread is not typically linked to an increase in the overall spread.

The correlation of the default spreads with consumption and inflation is negative. When conditioning on the possible states, we observe that this link with consumption is not constant across the different regimes and ratings except for state $s_t = (2, 2)$ which is found negative for all ratings. Figure 11 shows the links between our estimated spreads and our observed factors for Baa bonds. Much of the variations is found for state $s_t = (2, 2)$. For the other states, the positive or negative relation are weak as the estimated default spread do not vary much when the factors are varied. Similar pictures are obtained for the Aa and A rating classes.

The part of the default spread associated with the default premium is estimated to be negligible. Hence, using the preference parameters from the fit of the theoretical risk-free yield curves with the observed yield curves, we compute default risk premia that are small relatively to our default spread. The expression for the one period yield spread shows that the risk aversion parameter impacts on the spreads through the squared volatilities and covariance; it is a second order effect brought by the convexity of our pricing kernel and approximate recovery factor. A possible explanation for the low default premia spread obtained here is thus the low volatility levels of consumption growth and inflation. For reasonable values of the risk aversion parameter, these low volatilities have difficulties to produce high default premia. In other words, given the estimated volatility levels, a much higher value for the risk-aversion coefficient would be required to generate higher default premia.

We examine here the robustness of our results to the preference parameter estimates. Table 8 presents the credit spreads and proportions computed with the time-varying set of preference parameters estimated in section 3.4. The results are similar to those from the constant preference parameter presented in Table 7. This can be explained by looking at equations (9) and (10) which shows that the theoretical expressions for the default spread is a function of the risk-aversion parameter only i.e. the time preference parameters in the A^p_{n,s_t} and A^v_{n,s_t} functions are canceling out. Given that, at the current level of volatilities, the theoretical default spreads are not sensitive to the risk aversion parameter, the constant and time varying parameter estimates for γ produce similar results, even if the two estimates are different on average.

4 Conclusion

We proposed here an approach for estimating the default spread component of corporate yield spreads. Our model uses observed macroeconomic factors in a reduced form framework and is built on the objective measure. We use a pricing model with discrete regime shifts in consumption growth and inflation. The parameters of consumption, inflation and conditional default probability variations over time are also functions of the discrete regime shifts. Using consumption, inflation, risk-free yield and default data, the model is calibrated over the 1987-2008 period.

Our results indicate that our factors are linked to sharp increases in default spreads in two out of three NBER economic recessions. During these recessions, both inflation and consumption growth are negatively linked with default spreads. This result might indicate that, in some regime, the spread level is sensitive to a macroeconomic market wide undiversifiable risk. Our results also indicate that sharp increases in spreads are not necessarily linked to macroeconomic variables. The sharp increase in 2001 is not captured by our model as this period is found to be in a high consumption growth and low inflation risk regime. This result is consistent with the literature on forecasting models indicating that consumption growth hardly registered negative values in 2001. Our results can explain about half of the yield spread for Baa bond that can be considered as the average bond in the market. This is coherent with the recent study of Giesecke et al (2010) who show that, over the last 150 years, default risk represented about fifty percent of credit spreads. These authors also document that illiquidity of the bond market is probably the main factor that explains the difference between default spread and credit spread. Our results indicate that this illiquidity effect is perhaps stronger in recession periods. Finally, we find that almost of the estimated default spread is explained by the default risk while a negligible fraction is due to the default risk premium. This finding is explained by the low volatility of the consumption growth and inflation which are the main drivers of the default risk premium in this model.

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Appendices

A Asset prices with a stochastic subjective discount factor

Let p_t be the price of an asset at time t, X_{t+1} denotes the asset value at time $t+1$, ω_t represents the number of assets bought at time t, C_t is the consumption and ϵ_t stands for other earnings. With a time-separable utility function, a consumer determines at time t its consumptions, present and future, such to maximize

$$g_t\left(C_t,...,C_T\right)=\mathrm{E}_t\left[\sum_{s=t}^{T}\left(\prod_{s'=t+1}^{s}\beta_{s'}\right)u\left(C_s\right)\right] \tag{17}$$

under the budget constraint $C_t=\epsilon_t+\omega_{t-1}X_t-\omega_t p_t$. Here, $\beta_{s'}$ represent the random subjective discount factors with the convention $\prod_{s'=t+1}^{t}\beta_{s'}=1$. The first-order condition requires that $\frac{\partial g_t}{\partial \omega_{s^*}}=0$ if and only if

$$\mathrm{E}_t\left[\left(\prod_{s'=t+1}^{s^*}\beta_{s'}\right)u'\left(C_{s^*}\right)p_{s^*}\right]=\mathrm{E}_t\left[\left(\prod_{s'=t+1}^{s^*+1}\beta_{s'}\right)u'\left(C_{s^*+1}\right)X_{s^*+1}\right], \tag{18}$$

where $s^*\in\{t,t+1,...,T-1\}$. As a special case, using $s^*=t$, the time t price must satisfy

$$p_t=\mathrm{E}_t\left[\beta_{t+1}\frac{u'\left(C_{t+1}\right)}{u'\left(C_t\right)}X_{t+1}\right],\ t\in\{0,1,...,T-1\}. \tag{19}$$

The time consistency of the solution is shown by replacing back Equation (19) in the left hand side of Equation (18). One can show that Equation (18) is therefore satisfied for any $s^*\in\{t,t+1,...,T-1\}$. This result can be adapted to the case with inflation and a power utility function to obtain equation (2).

B Risk-free zero-coupon bond price analytical approximation

In this section, we derive functions A^{p}_{n,s_t}, $B^{p,c}_{n,s_t}$, and $B^{p,\pi}_{n,s_t}$ appearing in the analytical approximation formula of the zero-coupon risk-free bond price $P\left(t,n,s_t\right)$. Note that for the derivation in this appendix, for notational convenience, we drop the p superscript from the A and B functions. The derivation is based on two approximations: (i) the true time t value of the zero-coupon bond given the actual states of consumption and inflation, $\widetilde{P}\left(t,n\right)$, is well approximated by an exponential function (instead of a sum of exponential functions) and (ii) the function $\exp\left(x\right)$ may be replaced by its Taylor expansion around zero truncated after the second term, that is, $\exp\left(x\right)\cong 1+x$. Starting from equation (2), we have

$$1=\mathrm{E}_t\left[M_{t,t+1}\frac{\widetilde{P}\left(t+1,n-1,s_{t+1}\right)}{\widetilde{P}\left(t,n,s_t\right)}\right]\cong\mathrm{E}_t\left[M_{t,t+1}\frac{P\left(t+1,n-1,s_{t+1}\right)}{P\left(t,n,s_t\right)}\right].$$

Substituting $M_{t,t+1}$, $P\left(t,n,s_t\right)$ and $P\left(t+1,n-1,s_{t+1}\right)$ using equations (3) and (4), rewriting s_{t+1} as $\left\{s^c_{t+1},s^\pi_{t+1}\right\}$, applying embedded conditional expectation rule $\mathrm{E}_t\left[\bullet\right]=\mathrm{E}_t\left[\mathrm{E}_t\left[\bullet|\,s_{t+1}\right]\right]$, and using the fact that for a Gaussian random variable z, $\mathrm{E}[\exp z]=\exp\left(\mathrm{E}\left[z\right]+\frac{1}{2}\mathrm{Var}\left[z\right]\right)$, we get

$$\sum_{i=1}^{2}\sum_{j=1}^{2}\phi^c_{s^c_t,i}\phi^\pi_{s^\pi_t,j}\exp\left(\begin{array}{c}\ln\beta_{i,j}+A_{n-1,i,j}-A_{n,s^c_t,s^\pi_t}+B^c_{n,s^c_t,s^\pi_t}c_t+B^\pi_{n,s^c_t,s^\pi_t}\pi_t\\ -\left(B^c_{n-1,i,j}+\gamma\right)\left(a^c_i+b^c_i c_t\right)-\left(B^\pi_{n-1,i,j}+1\right)\left(a^\pi_j+b^\pi_j\pi_t\right)\\ +\frac{1}{2}\left(B^c_{n-1,i,j}+\gamma\right)^2\left(\sigma^c_i\right)^2+\frac{1}{2}\left(B^\pi_{n-1,i,j}+1\right)^2\left(\sigma^\pi_j\right)^2\\ +\left(B^c_{n-1,i,j}+\gamma\right)\left(B^\pi_{n-1,i,j}+1\right)\rho_{i,j}\sigma^c_i\sigma^\pi_j\end{array}\right)\cong 1.$$

Because $\exp(x) \cong 1 + x$ for x in the neighborhood of zero and since $\sum_{i=1}^{2}\sum_{j=1}^{2}\phi^c_{s^c_t,i}\phi^\pi_{s^\pi_t,j} = 1$,

$$\begin{aligned}
0 \cong & -A_{n,s^c_t,s^\pi_t} + \sum_{i=1}^{2}\sum_{j=1}^{2}\phi^c_{s^c_t,i}\phi^\pi_{s^\pi_t,j}G^A_{n-1,i,j} \\
& + \left(B^c_{n,s^c_t,s^\pi_t} - \sum_{i=1}^{2}\sum_{j=1}^{2}\phi^c_{s^c_t,i}\phi^\pi_{s^\pi_t,j}b^c_i\left(B^c_{n-1,i,j}+\gamma\right)\right)c_t \\
& + \left(B^\pi_{n,s^c_t,s^\pi_t} - \sum_{i=1}^{2}\sum_{j=1}^{2}\phi^c_{s^c_t,i}\phi^\pi_{s^\pi_t,j}b^\pi_j\left(B^\pi_{n-1,i,j}+1\right)\right)\pi_t
\end{aligned}$$

where

$$\begin{aligned}
G^A_{n-1,i,j} = & A_{n-1,i,j} + \ln\beta_{i,j} - a^c_i\left(B^c_{n-1,i,j}+\gamma\right) - a^\pi_j\left(B^\pi_{n-1,i,j}+1\right) \\
& + \frac{1}{2}\left(B^c_{n-1,i,j}+\gamma\right)^2\left(\sigma^c_i\right)^2 + \frac{1}{2}\left(B^\pi_{n-1,i,j}+1\right)^2\left(\sigma^\pi_j\right)^2 \\
& + \left(B^c_{n-1,i,j}+\gamma\right)\left(B^\pi_{n-1,i,j}+1\right)\rho_{i,j}\sigma^c_i\sigma^\pi_j.
\end{aligned}$$

Since this relation must be true for any c_t and π_t, we set the coefficients in front of c_t and π_t and the remaining term equal to zero to obtain:

$$\begin{aligned}
A_{n,s^c_t,s^\pi_t} &= \sum_{i=1}^{2}\sum_{j=1}^{2}\phi^c_{s^c_t,i}\phi^\pi_{s^\pi_t,j}G^A_{n-1,i,j}, \\
B^c_{n,s^c_t,s^\pi_t} &= \sum_{i=1}^{2}\sum_{j=1}^{2}\phi^c_{s^c_t,i}\phi^\pi_{s^\pi_t,j}b^c_i\left(B^c_{n-1,i,j}+\gamma\right), \\
B^\pi_{n,s^c_t,s^\pi_t} &= \sum_{i=1}^{2}\sum_{j=1}^{2}\phi^c_{s^c_t,i}\phi^\pi_{s^\pi_t,j}b^\pi_j\left(B^\pi_{n-1,i,j}+1\right).
\end{aligned}$$

C Risky zero-coupon bond price analytical approximation

In this section, we derive the functions A^v_{n,s_t}, $B^{v,c}_{n,s_t}$, and $B^{v,\pi}_{n,s_t}$ appearing in the analytical approximation formula of the zero-coupon risky bond price $V(t,T,s_t)$. Note that for the derivation in this appendix, for notational convenience, we drop the v superscript from the A and B functions. The derivations are based on our assumption of affine default probability in consumption growth and inflation and three approximations: (i) the time t value of the zero-coupon bond given the actual states of consumption and inflation, $\widetilde{V}(t,T,s_t)$, is well approximated by an exponential function and (ii) the function $\exp(x)$ may be replaced by its Taylor expansion around zero truncated after the second term, that is, $\exp(x) \cong 1 + x$; (iii) $\exp(-L_{s_{t+1}}h_{t+1}) \simeq 1 - L_{s_{t+1}}h_{t+1}$. Starting from equations (6) and (7) we obtain:

$$1 \cong \mathrm{E}_t\left[\exp\begin{pmatrix} \left(\ln\beta_{s_{t+1}} - L_{s_{t+1}}\alpha_{s_{t+1}}\right) \\ -\left(\gamma + L_{s_{t+1}}\alpha^c_{s_{t+1}}\right)c_{t+1} \\ -\left(1 + L_{s_{t+1}}\alpha^\pi_{s_{t+1}}\right)\pi_{t+1} \end{pmatrix}\frac{V(t+1,n-1,s_{t+1})}{V(t,n,s_t)}\right].$$

Substituting $V(t,n,s_t)$ and $V(t+1,n-1,s_{t+1})$ using equation (8), we use the same solution techniques as in Appendix B to obtain

$$\begin{aligned}
A_{n,s_t^c,s_t^\pi} &= \sum_{i=1}^{2}\sum_{j=1}^{2}\phi^c_{s_t^c,i}\phi^\pi_{s_t^\pi,j}G^A_{n-1,i,j},\\
B^c_{n,s_t^c,s_t^\pi} &= \sum_{i=1}^{2}\sum_{j=1}^{2}\phi^c_{s_t^c,i}\phi^\pi_{s_t^\pi,j}b^c_i\left(L_{i,j}\alpha^c_{i,j}+B^c_{n-1,i,j}+\gamma\right),\\
B^\pi_{n,s_t^c,s_t^\pi} &= \sum_{i=1}^{2}\sum_{j=1}^{2}\phi^c_{s_t^c,i}\phi^\pi_{s_t^\pi,j}b^\pi_j\left(L_{i,j}\alpha^\pi_{i,j}+B^\pi_{n-1,i,j}+1\right),
\end{aligned}$$

with

$$\begin{aligned}
G^A_{n-1,i,j} &= A_{n-1,i,j}+\ln\beta_{i,j}-L_{i,j}\alpha_{i,j}-a^c_i\left(L_{i,j}\alpha^c_{i,j}+B^c_{n-1,i,j}+\gamma\right)-a^\pi_j\left(L_{i,j}\alpha^\pi_{i,j}+B^\pi_{n-1,i,j}+1\right)\\
&\quad+\frac{1}{2}\left(L_{i,j}\alpha^c_{i,j}+B^c_{n-1,i,j}+\gamma\right)^2\left(\sigma^c_i\right)^2+\frac{1}{2}\left(L_{i,j}\alpha^\pi_{i,j}+B^\pi_{n-1,i,j}+1\right)^2\left(\sigma^\pi_j\right)^2\\
&\quad+\left(L_{i,j}\alpha^c_{i,j}+B^c_{n-1,i,j}+\gamma\right)\left(L_{i,j}\alpha^\pi_{i,j}+B^\pi_{n-1,i,j}+1\right)\rho_{i,j}\sigma^c_i\sigma^\pi_j.
\end{aligned}$$

D Survival probability analytical approximation

We assume that $\Pr_{\mathcal{G}_t}[\tau>t+n\mid\tau>t]\cong q(t,n,s_t)$ where

$$q(t,n,s_t)=\exp\left(-A^q_{n,s_t}-B^{q,c}_{n,s_t}c_t-B^{q,\pi}_{n,s_t}\pi_t\right).$$

The coefficients A^q_{n,s_t}, $B^{q,c}_{n,s_t}$ and $B^{q,\pi}_{n,s_t}$ are obtained recursively starting with $A^q_{0,s_T}=B^{q,c}_{0,s_T}=B^{q,\pi}_{0,s_T}=0$. Indeed, since

$$\begin{aligned}
\Pr_{\mathcal{G}_t}[\tau>t+n\mid\tau>t] &\cong \mathrm{E}_t\left[\exp\left(-h_{t+1}\right)\mathrm{E}_t\left[\exp\left(-\sum_{u=t+2}^{t+n}h_u\right)\right]\right]\\
&\cong \mathrm{E}_t\left[\exp\left(-h_{t+1}\right)\Pr_{\mathcal{G}_{t+1}}[\tau>t+n\mid\tau>t+1]\right]
\end{aligned}$$

where $\sum_{u=t+2}^{t+1}h_u$ is set to zero whenever it happens, then

$$\begin{aligned}
1 &\cong \mathrm{E}_t\left[\exp\left(-h_{t+1}\right)\frac{\Pr_{\mathcal{G}_{t+1}}[\tau>t+n\mid\tau>t+1]}{\Pr_{\mathcal{G}_t}[\tau>t+n\mid\tau>t]}\right]\\
&\cong \mathrm{E}_t\left[\exp\left(-h_{t+1}\right)\frac{q(t+1,n-1,s_{t+1})}{q(t,n,s_t)}\right]\\
&\cong \mathrm{E}_t\left[\exp\begin{pmatrix}-\alpha_{s_{t+1}}-A^q_{n-1,s_{t+1}}+A^q_{n,s_t}\\ +B^{q,c}_{n,s_t}c_t-\left(\alpha^c_{s_{t+1}}+B^{q,c}_{n-1,s_{t+1}}\right)c_{t+1}\\ +B^{q,\pi}_{n,s_t}\pi_t-\left(\alpha^\pi_{s_{t+1}}+B^{q,\pi}_{n-1,s_{t+1}}\right)\pi_{t+1}\end{pmatrix}\right]
\end{aligned}$$

where the last line is obtained by replacing h_{t+1} by the approximation (7). Using the same solution technique as in Appendix B we get

$$
\begin{aligned}
A^q_{n,s^c_t,s^\pi_t} &= \sum_{i=1}^{2}\sum_{j=1}^{2}\phi^c_{s^c_t,i}\phi^\pi_{s^\pi_t,j}G^A_{n-1,i,j},\\
B^{q,c}_{n,s^c_t,s^\pi_t} &= \sum_{i=1}^{2}\sum_{j=1}^{2}\phi^c_{s^c_t,i}\phi^\pi_{s^\pi_t,j}b^c_i\left(\alpha^c_{i,j}+B^{q,c}_{n-1,i,j}\right),\\
B^{q,\pi}_{n,s^c_t,s^\pi_t} &= \sum_{i=1}^{2}\sum_{j=1}^{2}\phi^c_{s^c_t,i}\phi^\pi_{s^\pi_t,j}b^\pi_j\left(\alpha^\pi_{i,j}+B^{q,\pi}_{n-1,i,j}\right),
\end{aligned}
$$

with

$$
\begin{aligned}
G^A_{n-1,i,j} = {} & A^q_{n-1,i,j}+\alpha_{i,j}+a^c_i\left(\alpha^c_{i,j}+B^{q,c}_{n-1,i,j}\right)+a^\pi_j\left(\alpha^\pi_{i,j}+B^{q,\pi}_{n-1,i,j}\right)\\
&-\frac{1}{2}\left(\alpha^c_{i,j}+B^{q,c}_{n-1,i,j}\right)^2\left(\sigma^c_i\right)^2-\frac{1}{2}\left(\alpha^\pi_{i,j}+B^{q,\pi}_{n-1,i,j}\right)^2\left(\sigma^\pi_j\right)^2\\
&-\left(\alpha^c_{i,j}+B^{q,c}_{n-1,i,j}\right)\left(\alpha^\pi_{i,j}+B^{q,\pi}_{n-1,i,j}\right)\rho_{i,j}\sigma^c_i\sigma^\pi_j.
\end{aligned}
$$

E Log-likelihood function of the Markov Switching model

The terms of the log likelihood function are computed as follows. Let the conditional likelihood be rewritten using Bayes rule as:

$$
\begin{aligned}
f\left(x_t \mid v_{t-1};\theta,\phi\right) &= \frac{f\left(x_t, v_{t-1};\theta,\phi\right)}{f\left(v_{t-1};\theta,\phi\right)}\\
&= \frac{\sum_{s_t} f\left(x_t, s_t, v_{t-1};\theta,\phi\right)}{f\left(v_{t-1};\theta,\phi\right)}\\
&= \sum_{s_t} f\left(x_t \mid s_t, v_{t-1};\theta\right)\times f\left(s_t \mid v_{t-1};\theta,\phi\right)\\
&= \eta_t' \times \xi_{t|t-1}.
\end{aligned}
$$

where the sum over s_t is performed for values of $s_t \in \{(1,1),(1,2),(2,1),(2,2)\}$ and

$$
\eta_t = \begin{pmatrix} f\left(x_t \mid (1,1), v_{t-1};\theta\right)\\ f\left(x_t \mid (1,2), v_{t-1};\theta\right)\\ f\left(x_t \mid (2,1), v_{t-1};\theta\right)\\ f\left(x_t \mid (2,2), v_{t-1};\theta\right)\end{pmatrix} \quad \text{and} \quad \xi_{t|t-1} = \begin{pmatrix} f\left((1,1) \mid v_{t-1};\theta,\phi\right)\\ f\left((1,2) \mid v_{t-1};\theta,\phi\right)\\ f\left((2,1) \mid v_{t-1};\theta,\phi\right)\\ f\left((1,2) \mid v_{t-1};\theta,\phi\right)\end{pmatrix}.
$$

The components of η_t are computed analytically using the bivariate Gaussian density function. Indeed, from the Markov Switching model, the likelihood function of $x_t = (c_t, \pi_t)$ given $x_{t-1} = (c_{t-1}, \pi_{t-1})$ and the actual states of consumption and inflation $s_t = (s^c_t, s^\pi_t)$ is

$$
f\left(x_t \mid s_t, v_{t-1};\theta\right) = \frac{1}{2\pi}\frac{\exp\left(-\frac{1}{2}\left(z^c_t\right)^2-\frac{1}{2}\left(z^\pi_t\right)^2+\rho_{s^c_t,s^\pi_t}z^c_t z^\pi_t\right)}{\sigma^c_{s^c_t}\sigma^\pi_{s^\pi_t}\sqrt{1-\rho^2_{s^c_t,s^\pi_t}}}
$$

with

$$
z^c_t = \frac{c_t - a^c_{s^c_t} - b^c_{s^c_t}c_{t-1}}{\sigma^c_{s^c_t}\sqrt{1-\rho^2_{s^c_t,s^\pi_t}}}
$$

and

$$z_t^\pi = \frac{\pi_t - a^\pi_{s_t^\pi} - b^\pi_{s_t^\pi}\pi_{t-1}}{\sigma^\pi_{s_t^\pi}\sqrt{1-\rho^2_{s_t^c,s_t^\pi}}}.$$

The vectors $\xi_{2|1}, ..., \xi_{T|T-1}$ are obtained recursively using

$$\xi_{t|t-1} = \Phi' \times \frac{\eta'_{t-1}\,(\times)\;\xi_{t-1|t-2}}{\eta'_{t-1}\;\xi_{t-1|t-2}}$$

where $(\times)$ denotes element-by-element multiplication and Φ is the transition matrix

$$\Phi = \begin{pmatrix} \phi^c_{1,1}\phi^\pi_{1,1} & \phi^c_{1,1}\phi^\pi_{1,2} & \phi^c_{1,2}\phi^\pi_{1,1} & \phi^c_{1,2}\phi^\pi_{1,2} \\ \phi^c_{1,1}\phi^\pi_{2,1} & \phi^c_{1,1}\phi^\pi_{2,2} & \phi^c_{1,2}\phi^\pi_{2,1} & \phi^c_{1,2}\phi^\pi_{2,2} \\ \phi^c_{2,1}\phi^\pi_{1,1} & \phi^c_{2,1}\phi^\pi_{1,2} & \phi^c_{2,2}\phi^\pi_{1,1} & \phi^c_{2,2}\phi^\pi_{1,2} \\ \phi^c_{2,1}\phi^\pi_{2,1} & \phi^c_{2,1}\phi^\pi_{2,2} & \phi^c_{2,2}\phi^\pi_{2,1} & \phi^c_{2,2}\phi^\pi_{2,2} \end{pmatrix}.$$

To initialize the recursion, we let $\eta_1 = (1,1,1,1)'$ and $\xi_{1|0}$ is set to the stationary distribution of the Markov chain associated with Φ. Using the independence between the evolution of the state of consumption and the state of inflation, the stationary distribution is obtained as the product of the stationary distribution of s^c and s^π:

$$\xi_{1|0} = \begin{pmatrix} \frac{1-\phi^c_{2,2}}{1-\phi^c_{1,1}+1-\phi^c_{2,2}} \times \frac{1-\phi^\pi_{2,2}}{1-\phi^\pi_{1,1}+1-\phi^\pi_{2,2}} \\ \frac{1-\phi^c_{2,2}}{1-\phi^c_{1,1}+1-\phi^c_{2,2}} \times \frac{1-\phi^\pi_{1,1}}{1-\phi^\pi_{1,1}+1-\phi^\pi_{2,2}} \\ \frac{1-\phi^c_{1,1}}{1-\phi^c_{1,1}+1-\phi^c_{2,2}} \times \frac{1-\phi^\pi_{2,2}}{1-\phi^\pi_{1,1}+1-\phi^\pi_{2,2}} \\ \frac{1-\phi^c_{1,1}}{1-\phi^c_{1,1}+1-\phi^c_{2,2}} \times \frac{1-\phi^\pi_{1,1}}{1-\phi^\pi_{1,1}+1-\phi^\pi_{2,2}} \end{pmatrix}.$$

Table 1: Average treasury spot rates and corporate spreads 1987-2008

Maturity (years)	Treasuries (%)	Aa spreads (%)	A spreads (%)	Baa spreads (%)
1	4.832	0.627	0.777	1.305
2	5.088	0.527	0.755	1.241
3	5.291	0.583	0.842	1.321
4	5.468	0.640	0.911	1.385
5	5.626	0.674	0.949	1.420
6	5.769	0.688	0.963	1.432
7	5.897	0.690	0.961	1.429
8	6.011	0.684	0.949	1.416
9	6.113	0.674	0.933	1.397
10	6.201	0.663	0.914	1.376

This table reports the average treasury yields and corporate yield spreads of industrial Aa, A and Baa zero-coupon bonds for maturities from one to ten years. The treasury spot rates are taken from Gurkaynak et al. (2007). For the 1987-1996 period, the corporate spot rates are extracted from samples of coupon bond prices taken from Warga (1998) and computed with the Nelson-Siegel (1987) approach. The corporate spot rates for the 1997-2008 period are taken directly from Bloomberg. Corporate yield spreads are calculated as the difference between the corporate spot rates and treasury spot rates for a given maturity.

Table 2: Parameter estimates for the Markov Switching model

	a_1^c	a_2^c	b_1^c	b_2^c	σ_1^c	σ_2^c
Point estimate	**0.00377**	0.00068	**0.38479**	0.09514	**0.00349**	**0.00549**
Standard deviation	0.00057	0.00123	0.07820	0.18919	0.00022	0.00071
p-value	0.00 %	58.02 %	0.00 %	61.51 %	0.00 %	0.00 %

	a_1^π	a_2^π	b_1^π	b_2^π	σ_1^π	σ_2^π
Point estimate	**0.00246**	**0.00366**	**0.63932**	**0.71340**	**0.00235**	**0.00524**
Standard deviation	0.00065	0.00121	0.08783	0.07608	0.00025	0.00045
p-value	0.01 %	0.26 %	0.00 %	0.00 %	0.00 %	0.00 %

	$\rho_{1,1}$	$\rho_{1,2}$	$\rho_{2,1}$	$\rho_{2,2}$
Point estimate	0.15218	**-0.50253**	-0.44210	**0.02355**
Standard deviation	0.10896	0.12014	0.28546	0.00000
p-value	16.25 %	0.00 %	12.14 %	84.69 %

	$\phi_{1,1}^c$	$\phi_{2,2}^c$	$\phi_{1,1}^\pi$	$\phi_{2,2}^\pi$
Point estimate	**0.96464**	**0.86163**	**0.96691**	**0.96201**
Standard deviation	0.01698	0.07908	0.01914	0.02845
p-value	0.00 %	0.00 %	0.00 %	0.00 %

This table reports the point estimates and estimated standard deviations for the parameters of the Markov Switching model given by $c_t = a^c_{s^c_t} + b^c_{s^c_t} c_{t-1} + e^c_t$ and $\pi_t = a^\pi_{s^\pi_t} + b^\pi_{s^\pi_t} \pi_{t-1} + e^\pi_t$ with $\phi^c_{ij} = \Pr(s^c_t = i \mid s^c_{t-1} = j)$ and $\phi^\pi_{ij} = \Pr(s^\pi_t = i \mid s^\pi_{t-1} = j)$ and ρ_{ij} the correlation between e^c_t and e^π_t in state i, j. The last line of each panel reports the p-value associated with the test of a zero parameter value. These estimates have been obtained with the growth rate of non-durable and services personal consumption expenditures per capita (real) from the first quarter of 1957 to the last quarter of 2008 and the growth rate of non-durable and services consumption price index for the same period. Parameters in bold are significant at the 1% level.

Table 3: Fit of the risk-free zero-coupon bond pricing model: constant preference parameters

Maturity ($n/4$)	1	3	5	8	10
rmse (%)	1.938	1.838	1.734	1.641	1.618
aae (%)	1.533	1.520	1.455	1.397	1.368
ae (%)	0.450	0.242	0.015	-0.284	-0.439
avg fitted (%)	5.281	5.534	5.641	5.727	5.763
avg obs. (%)	4.832	5.291	5.626	6.011	6.201

rmse is the root-mean-squared error for a given maturity computed for 1987:I to 2008:IV with $\varepsilon_{t,n} = -\frac{\ln \overline{P}(t,n,\beta,\gamma)}{n/4} - y_g(t,n)$, the difference between our fitted theoretical risk-free yield and the estimated risk-free yield to maturity taken from Gurkaynac et al. (2007). *aae* is the absolute average error while *ae* is the average error. *avg obs.* and *avg fitted* are, respectively, the average yield from Gurkaynac et al. (2007) and average fitted theoretical yield for a given maturity.

Table 4: Fit of the risk-free zero-coupon bond pricing model: time varying preference parameters

Maturity ($n/4$)	1	3	5	8	10
rmse (%)	0.833	0.610	0.395	0.224	0.284
aae (%)	0.441	0.300	0.182	0.109	0.176
ae (%)	0.317	0.286	0.172	-0.023	-0.133
avg fitted (%)	5.148	5.577	5.798	5.988	6.069
avg obs. (%)	4.832	5.291	5.626	6.011	6.201

rmse is the root-mean-squared error for a given maturity computed for 1987:I to 2008:IV with $\varepsilon_{t,n} = -\frac{\ln \overline{P}(t,n,\beta,\gamma)}{n/4} - y_g(t,n)$, the difference between our fitted theoretical risk-free yield and the estimated risk-free yield to maturity taken from Gurkaynac et al. (2007). *aae* is the absolute average error while *ae* is the average error. *avg obs.* and *avg fitted* are, respectively, the average yield from Gurkaynac et al. (2007) and average fitted theoretical yield for a given maturity.

Table 5: Parameter estimates for the conditional default probabilities

	$s_t = (1,1)$	$s_t = (1,2)$	$s_t = (2,1)$	$s_t = (2,2)$
		Aa		
$\alpha_{i,j}$	0.000055	0.000171	0.000420	0.001126
$\alpha^c_{i,j}$	-0.001906	0.009432	-0.009109	-0.028082
$\alpha^\pi_{i,j}$	0.004704	-0.005518	0.024932	-0.067682
		A		
$\alpha_{i,j}$	0.001038	0.000991	0.000181	0.002705
$\alpha^c_{i,j}$	0.001018	0.019711	0.005935	-0.067490
$\alpha^\pi_{i,j}$	-0.063422	-0.056948	-0.006430	-0.162663
		Baa		
$\alpha_{i,j}$	0.001195	0.001150	0.000044	0.011359
$\alpha^c_{i,j}$	0.137513	0.004280	-0.006709	-0.513363
$\alpha^\pi_{i,j}$	-0.001813	0.079485	0.005585	-0.522254

The table reports the parameter estimates for the conditional default probability function $h_t = \alpha_{s_t} + \alpha^c_{s_t} c_t + \alpha^\pi_{s_t} \pi_t$ obtained by minimizing the distance between our model generated theoretical term structures of survival probabilities and the empirical survival probabilities obtained from Moody's rating transition matrices over 1987:I to 2008:IV.

Table 6: Correlations of the estimated default probability with consumption growth and inflation.

	All	$s_t = (1,1)$	$s_t = (1,2)$	$s_t = (2,1)$	$s_t = (2,2)$
nobs	88	54	20	3	11
Corr c_t and π_t	-0.0189	0.0838	-0.3933	-0.9989	0.0758
			Aa		
Corr h_t and c_t	-0.5154	-0.2697	0.3432	-0.9952	-0.3951
Corr h_t and π_t	-0.5087	0.1654	-0.7313	0.9896	-0.9276
			A		
Corr h_t and c_t	-0.3227	0.0348	0.4338	0.4989	-0.2754
Corr h_t and π_t	-0.7788	-0.9457	-0.9854	-0.5384	-0.9668
			Baa		
Corr h_t and c_t	-0.4706	0.7877	0.1469	-0.2985	-0.4474
Corr h_t and π_t	-0.4508	0.0390	0.7392	0.2540	-0.8905

The table reports the correlation between the estimated conditional default probability h_t and consumption growth and inflation. Column All report results for the full sample i.e. 1987:I to 2008:IV. Columns $s_t = (1,1)$, $s_t = (1,2)$, $s_t = (2,1)$ and $s_t = (2,2)$ report the statistics computed over the sample periods for which the given state is prevailing. *nobs* is the number of observations.

Table 7: Statistics on estimated default spread: constant preference parameters

	Aa					A					Baa				
	All	(1,1)	(1,2)	(2,1)	(2,2)	All	(1,1)	(1,2)	(2,1)	(2,2)	All	(1,1)	(1,2)	(2,1)	(2,2)
Nobs.	88	54	20	3	11	88	54	20	3	11	88	54	20	3	11
Average YS_t (%)	0.66	0.57	0.69	0.55	1.10	0.91	0.83	0.82	0.88	1.51	1.38	1.28	1.19	1.38	2.18
Average DS_t (%)	0.04	0.04	0.04	0.05	0.05	0.11	0.11	0.11	0.11	0.12	0.46	0.44	0.49	0.44	0.55
Average DS_t prop. (%)	8	9	6	9	5	14	16	13	12	8	37	38	43	32	28
Stdv. YS_t (%)	0.31	0.28	0.17	0.07	0.32	0.41	0.34	0.13	0.02	0.61	0.54	0.43	0.23	0.05	0.80
Stdv. DS_t (%)	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.02	0.04	0.00	0.01	0.02	0.05
corr. DS_t and YS_t	0.49	-0.04	0.49	-0.49	0.74	0.41	-0.06	-0.12	0.78	0.75	0.53	0.00	0.51	-0.02	0.64
corr. DS_t and c_t	-0.55	0.03	0.23	0.05	-0.28	-0.11	-0.07	0.54	1.00	-0.12	-0.48	0.28	-0.28	0.98	-0.14
corr. DS_t and π_t	-0.30	0.19	-0.63	-0.10	-0.97	-0.92	-0.90	-0.96	-0.99	-0.99	-0.04	0.11	-0.05	-0.99	-0.93
corr. ΔYS_t and $\Delta y_{g,t}$	-0.11	-0.17	0.11	-0.71	-0.08	-0.18	-0.24	0.61	-0.41	-0.63	-0.26	-0.43	0.49	-0.13	-0.83
corr. ΔDS_t and $\Delta y_{g,t}$	0.13	0.10	-0.16	0.98	0.45	0.04	-0.11	-0.08	-0.73	0.32	0.08	-0.09	-0.36	-0.56	0.44
Average DR_t (%)	0.04	0.04	0.04	0.05	0.05	0.11	0.11	0.11	0.11	0.12	0.46	0.44	0.49	0.44	0.55
Average DR_t prop. (%)	8	9	6	9	5	14	16	14	12	8	37	38	43	32	28
Average DP_t (%)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Average DP_t prop. (%)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

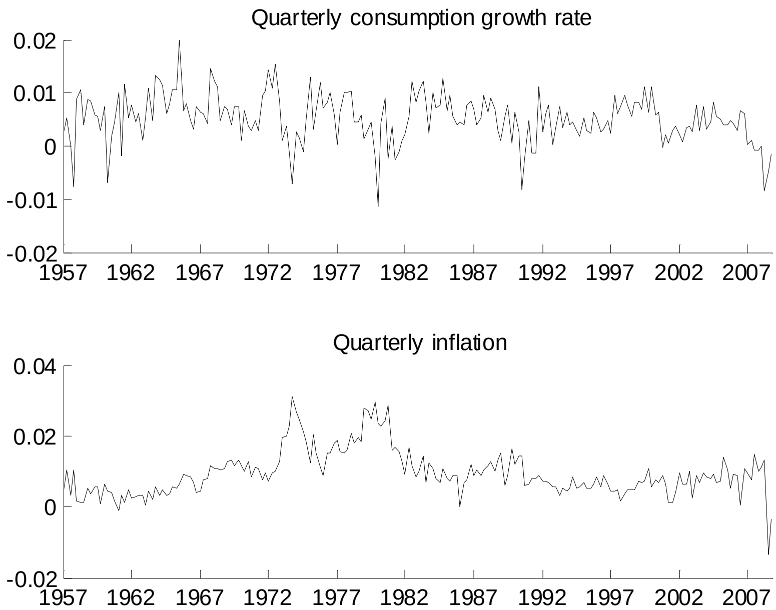
Nobs is the number of observations; YS_t is the yield spread computed as the difference between the industrial corporate and treasury zero-coupon yields. DS_t, DR_t and DP_t represent, respectively, the default spread, the default risk and the default premia computed with equations (9), (11) and (12) and the calibrated parameters; DR_t *prop* and DP_t *prop* are, respectively, DR_t/YS_t and DP_t/YS_t; *Stdv* are standard deviations and *corr* are correlations; ΔYS_t, ΔDS_t and $\Delta y_{g,t}$ are the first differences in the yield spread, default spread and risk-free zero-coupon yield. Columns All report results for the full sample i.e. 1987:I to 2008:IV. Columns $(1,1)$, $(1,2)$, $(2,1)$ and $(2,2)$ report the statistics computed over the sample periods for which the given state (i,j) is prevailing.

Table 8: Statistics on estimated default spread: time varying preference parameters

	Aa					A					Baa				
	All	(1,1)	(1,2)	(2,1)	(2,2)	All	(1,1)	(1,2)	(2,1)	(2,2)	All	(1,1)	(1,2)	(2,1)	(2,2)
Nobs.	88	54	20	3	11	88	54	20	3	11	88	54	20	3	11
Average YS_t (%)	0.66	0.57	0.69	0.55	1.10	0.91	0.83	0.82	0.88	1.51	1.38	1.28	1.19	1.38	2.18
Average DS_t (%)	0.04	0.04	0.04	0.05	0.05	0.11	0.12	0.11	0.11	0.12	0.47	0.44	0.50	0.45	0.56
Average DS_t prop. (%)	8	9	6	9	5	14	16	14	13	9	38	38	44	32	28
Stdv. YS_t (%)	0.31	0.28	0.17	0.07	0.32	0.41	0.34	0.13	0.02	0.61	0.54	0.43	0.23	0.05	0.80
Stdv. DS_t (%)	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.01	0.05	0.00	0.01	0.02	0.04
corr. DS_t and YS_t	0.54	0.07	0.44	-0.24	0.74	0.43	-0.09	-0.04	0.79	0.76	0.51	0.04	0.48	-0.00	0.65
corr. DS_t and c_t	-0.57	-0.04	0.28	0.35	-0.27	-0.13	-0.05	0.52	0.99	-0.12	-0.47	0.20	-0.28	0.98	-0.14
corr. DS_t and π_t	-0.31	0.26	-0.70	-0.35	-0.96	-0.92	-0.96	-0.98	-0.99	-0.99	-0.00	0.19	-0.04	-0.98	-0.90
corr. ΔYS_t and $\Delta y_{g,t}$	-0.11	-0.17	0.11	-0.71	-0.08	-0.18	-0.24	0.61	-0.41	-0.63	-0.26	-0.43	0.49	-0.13	-0.83
corr. ΔDS_t and $\Delta y_{g,t}$	0.13	0.13	-0.18	1.00	0.45	0.03	-0.12	-0.10	-0.73	0.33	0.08	-0.07	-0.33	-0.54	0.45
Average DR_t (%)	0.04	0.04	0.04	0.05	0.05	0.11	0.12	0.11	0.11	0.12	0.47	0.44	0.50	0.45	0.56
Average DR_t prop. (%)	8	9	6	9	5	14	16	14	13	9	38	38	44	32	28
Average DP_t (%)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Average DP_t prop. (%)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

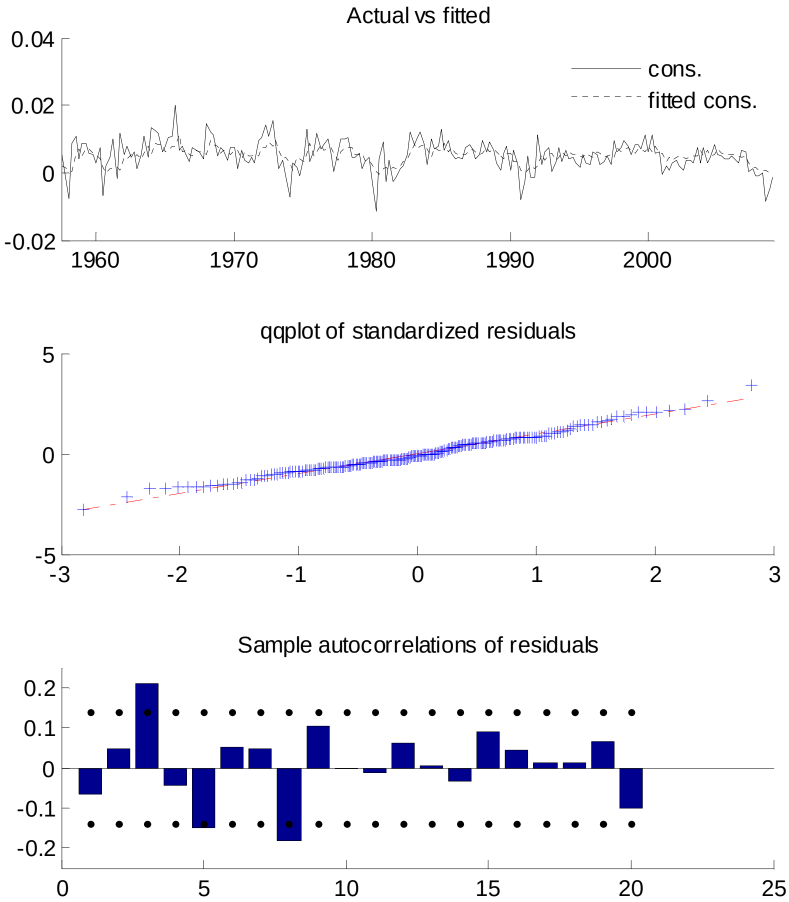
Nobs is the number of observations; YS_t is the yield spread computed as the difference between the industrial corporate and treasury zero-coupon yields. DS_t, DR_t and DP_t represent, respectively, the default spread, the default risk and the default premia computed with equations (9), (11) and (12) and the calibrated parameters; DR_t *prop* and DP_t *prop* are, respectively, DR_t/YS_t and DP_t/YS_t; *Stdv* are standard deviations and *corr* are correlations; ΔYS_t, ΔDS_t and $\Delta y_{g,t}$ are the first differences in the yield spread, default spread and risk-free zero-coupon yield. Columns All report results for the full sample i.e. 1987:I to 2008:IV. Columns $(1,1)$, $(1,2)$, $(2,1)$ and $(2,2)$ report the statistics computed over the sample periods for which the given state (i,j) is prevailing.

Figure 1: Non-durable and services consumption growth and inflation



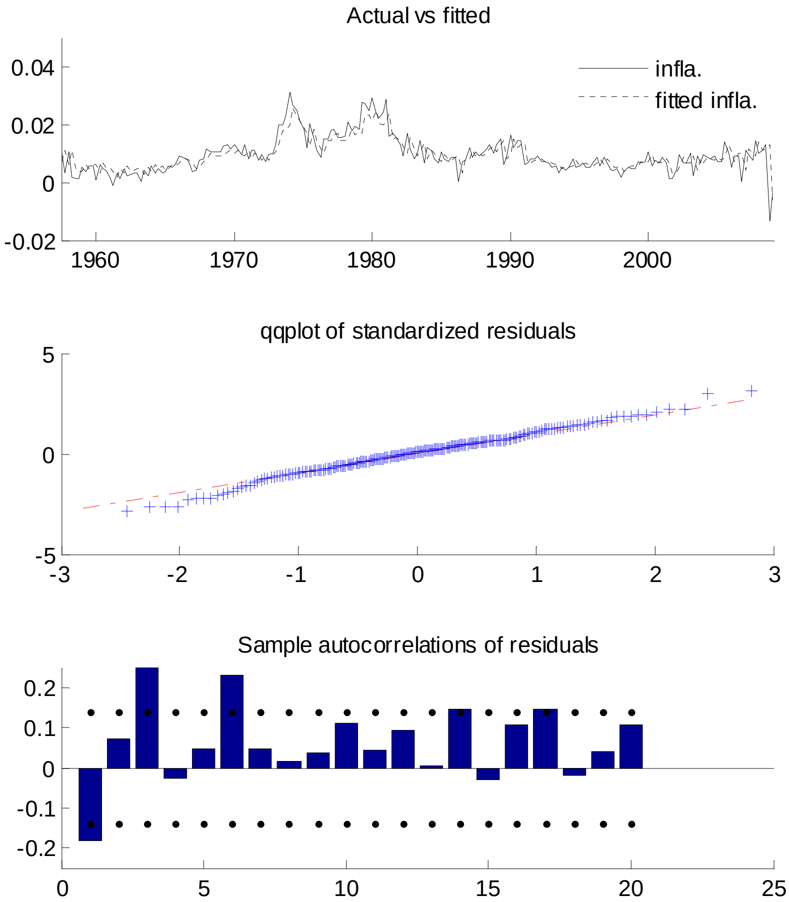
This figure plots the data series of the observed bond pricing factors used for the estimation of the Markov Switching parameters. The data series used here are the growth rate of non-durable and services real consumption expenditures per capita (c_t) from 1957-I to the last quarter of 2008-IV and the growth rate of the consumption price index (π_t) for the same period. Source: U.S. Department of Commerce, Bureau of Economic Analysis.

Figure 2: Fitted values and residuals analysis for consumption growth



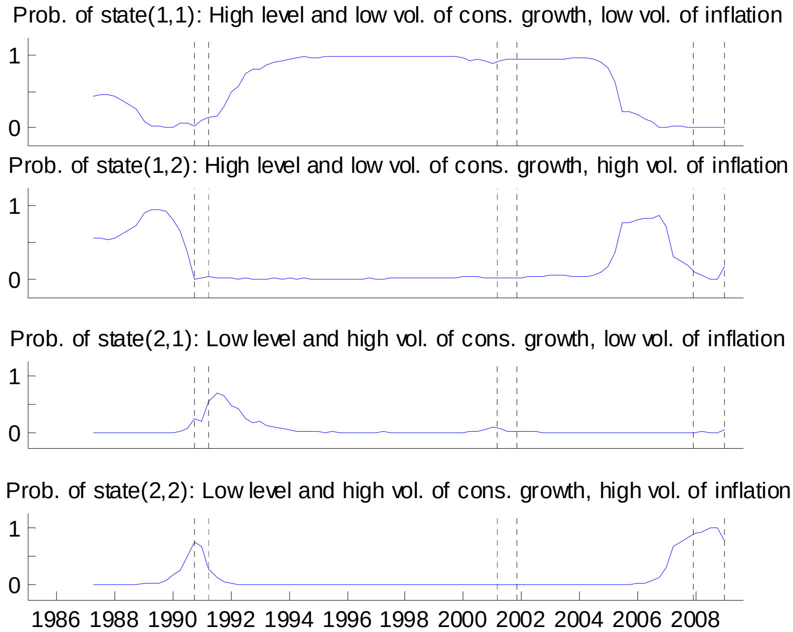
The top graph plots the actual and expected consumption growth obtained from the estimated Markov Switching model. The second graph shows the qqplot of the expected standardized residuals. The third graph shows the sample autocorrelation coefficients computed with the expected standardized residuals. All expected values are computed with the smoothed probabilities estimates.

Figure 3: Fitted values and residuals analysis for inflation



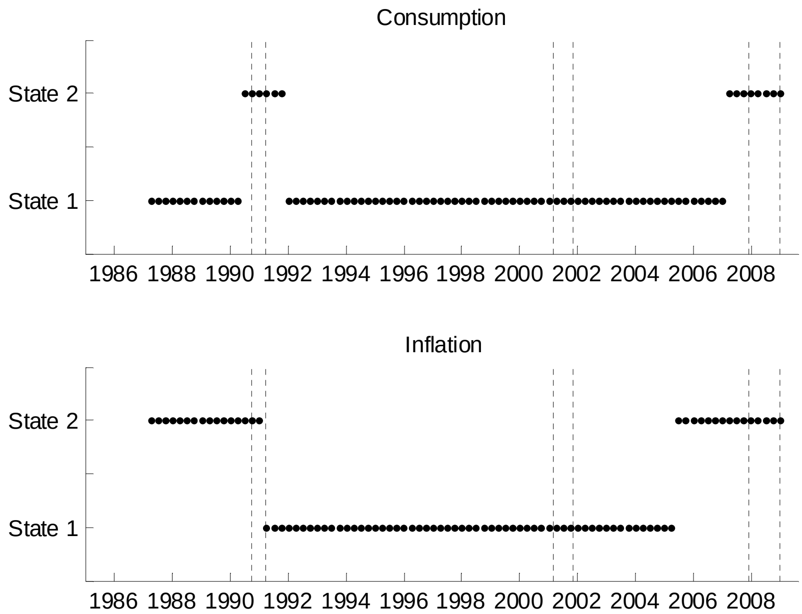
The top graph plots the actual and expected inflation obtained from the estimated Markov Switching model. The second graph shows the qqplot of the expected standardized residuals. The third graph shows the sample autocorrelation coefficients computed the expected standardized residuals. All expected values are computed with the smoothed probabilities estimates.

Figure 4: Smoothed probabilities estimates



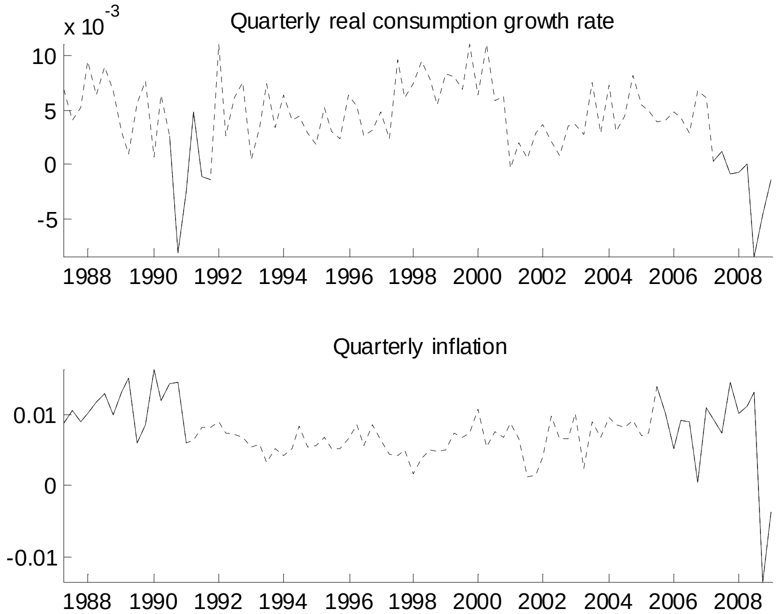
This figure plots the estimated smoothed probabilities $\hat{\xi}_{t|T}$ of being in state $s_t = (i, j)$ for the 1987-I- to 2008-IV period corresponding to the sample period of our corporate bond prices. Vertical lines indicate the official NBER recessions within our sample period. State (1,1): high level and low volatility of consumption growth with low volatility of inflation; State (1,2): high level and low volatility of consumption growth with high volatility of inflation; State (2,1): low level and high volatility of consumption growth with low volatility of inflation; State (2,2): low level and high volatility of consumption growth with high volatility of inflation.

Figure 5: Estimated states for consumption growth and inflation



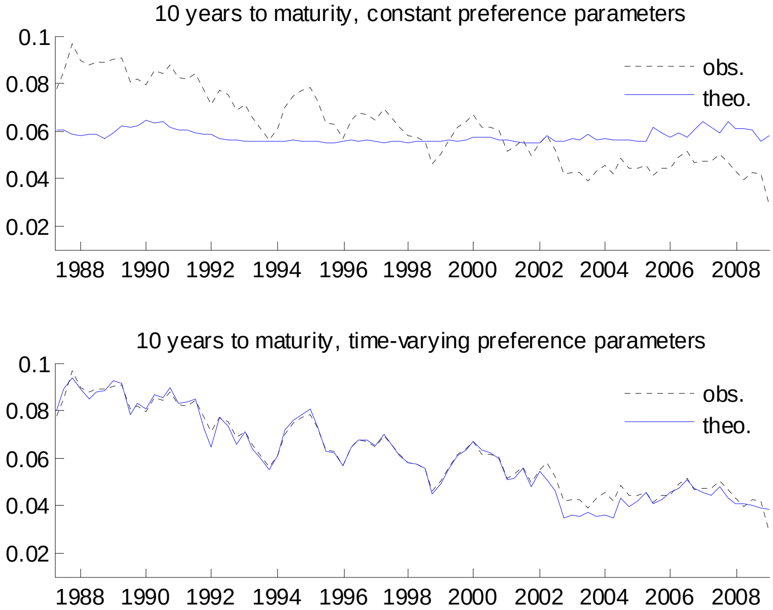
This figure plots the estimated state $\hat{s}_t = (i, j)$ for the 1987-I- to 2008-IV period corresponding to the sample period of our corporate bond prices. Vertical lines indicate the official recessions within our sample period. State (1,1): high level and low volatility of consumption growth with low volatility of inflation; State (1,2): high level and low volatility of consumption growth with high volatility of inflation; State (2,1): low level and high volatility of consumption growth with low volatility of inflation; State (2,2): low level and high volatility of consumption growth with high volatility of inflation.

Figure 6: Consumption growth and inflation - 1987:I to 2008:IV



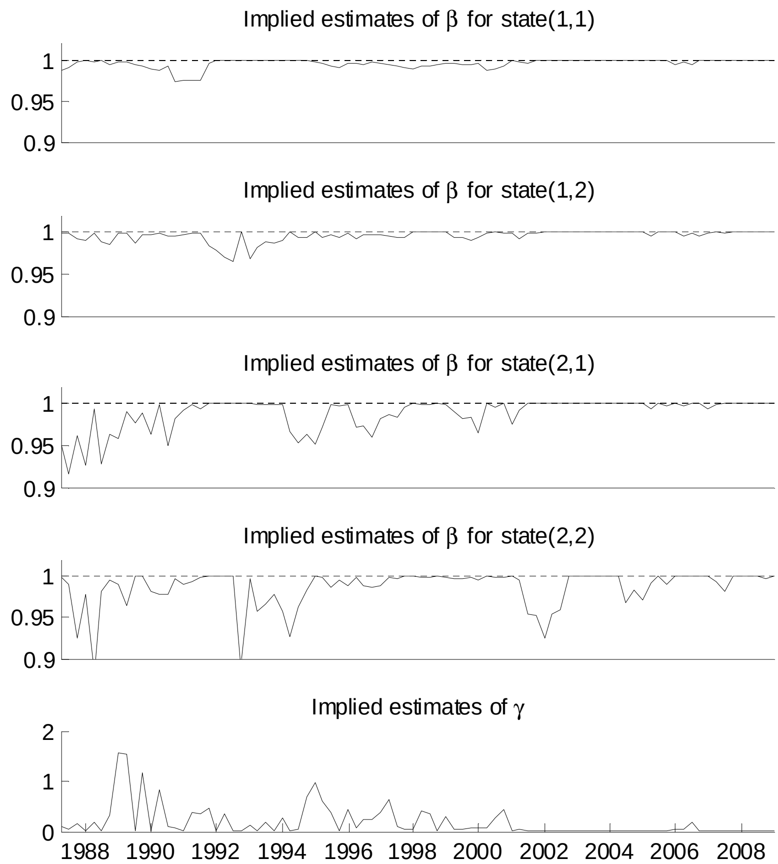
This figure plots the data series of our observed bond pricing factors for the 1987-I- to 2008-IV period corresponding to the sample period of our corporate bond prices. For consumption, the dashed line indicates the periods for which state 1 prevails (high level, low volatility). For inflation, the dashed line indicates state 1 (low volatility.)

Figure 7: Fitted and observed risk-free yields to maturity



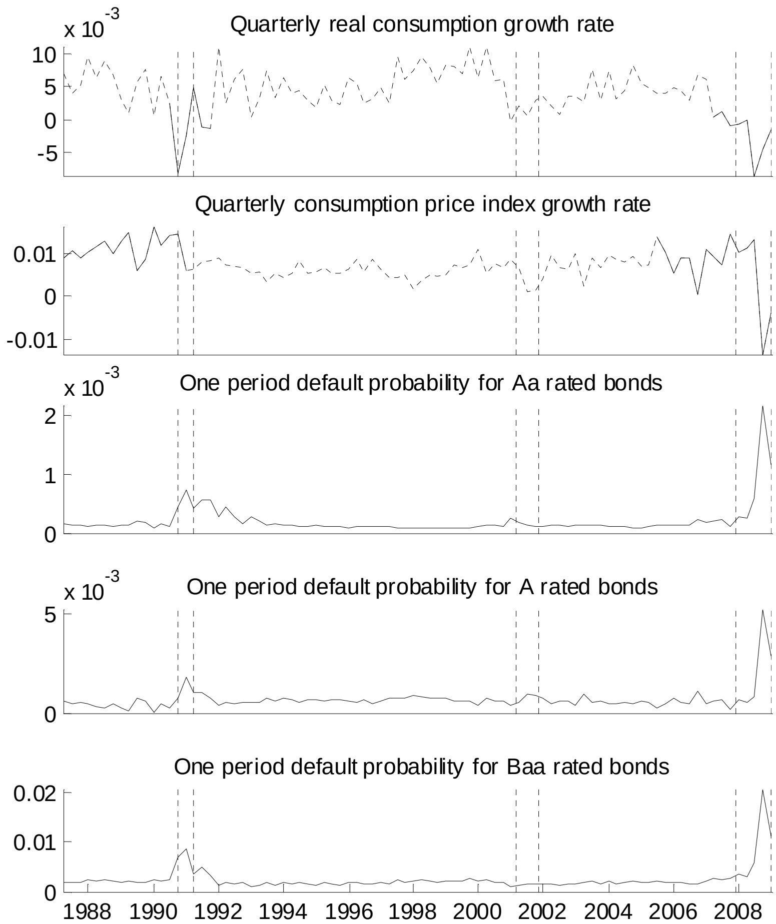
This figure plots the fitted and observed risk-free yield to maturity for the 10 year case. The top graph is obtained by fitting a common set of utility parameters (risk aversion and time preference) for all quarters while the bottom graph is obtained by fitting a different set of utility parameters each quarter.

Figure 8: Implied estimates of preference parameters



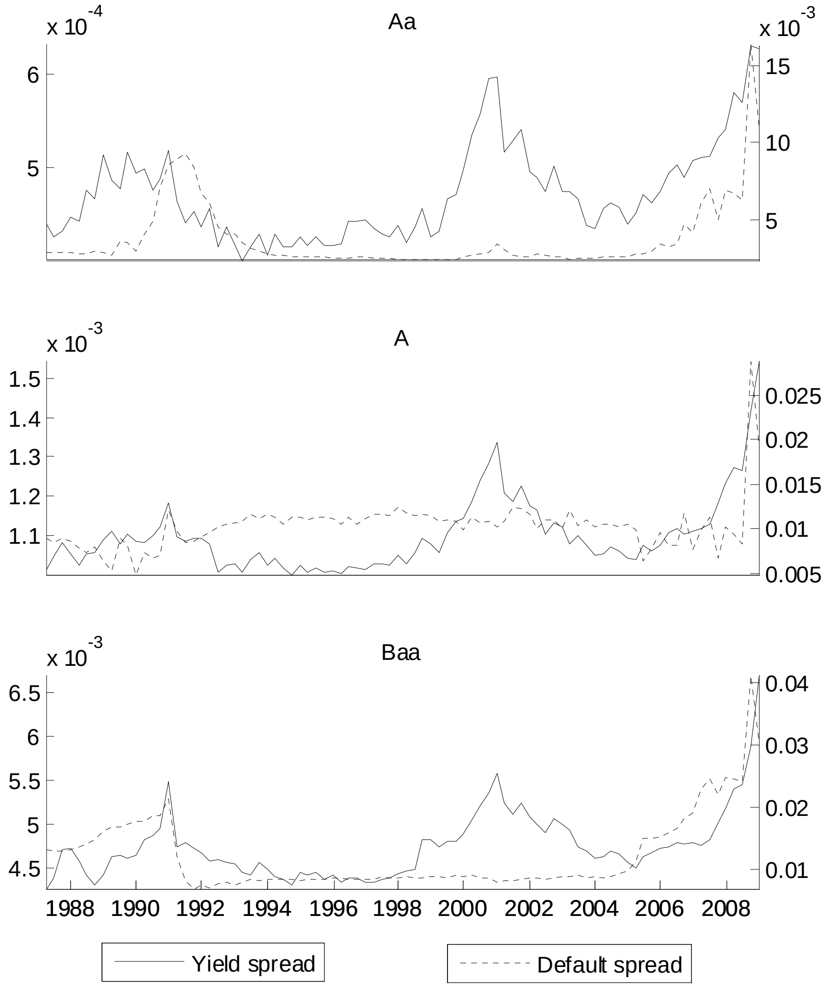
This figure plots the implied estimates for the preference parameters obtained by fitting at each quarter the Nelson-Siegel observed risk-free yield curve of the period with the risk-free bond yield model. State (1,1): high level and low volatility of consumption growth with low volatility of inflation; State (1,2): high level and low volatility of consumption growth with high volatility of inflation; State (2,1): low level and high volatility of consumption growth with low volatility of inflation; State (2,2): low level and high volatility of consumption growth with high volatility of inflation.

Figure 9: One period default probability with consumption growth and inflation



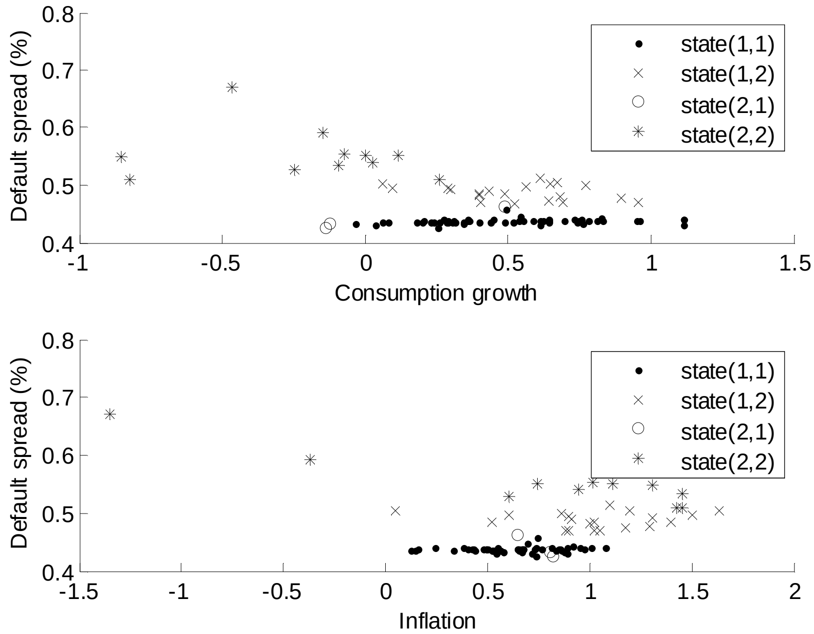
This figure plots the estimated one period conditional default probability function $h_t = \alpha_{s_t} + \alpha^c_{s_t} c_t + \alpha^\pi_{s_t} \pi_t$ along with consumption growth and inflation. For consumption, the dashed line indicates the periods for which state 1 prevails (high level, low volatility). For inflation, the dashed line indicates state 1 (low volatility.) The conditional default probability jumps occur during states of low level and high volatility of consumption and high volatility of inflation. These periods are within 2 of the 3 economic recessions identified by the NBER during the sample period.

Figure 10: Corporate yield spreads and estimated default spreads



This figure shows on two scale graphs the evolution of the yield spread (right scale) with the estimated default spread (left scale) for ten years to maturity zero-coupon bonds.

Figure 11: Baa default spreads with consumption growth and inflation



This figure shows for the different estimated states of the Markov Switching model, the links between the computed Baa default spreads with consumption growth and inflation.