Volatility, Correlation and Tails for Systemic Risk Measurement

Christian T. Brownlees* Robert Engle*

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Abstract

The Great Recession of 2007/2009 has motivated market participants, academics and regulators to better understand systemic risk. Regulation is now designed to reduce systemic risk. However it is not yet clear how to measure systemic risk and in particular to determine which firms are the major contributors to the overall risk of the economy. This paper focuses on constructing measures of systemic risk based on public market data and consequently provides a quick and inexpensive approach to determining which firms deserve more careful scrutiny and regulation. The measure examined in this paper is the Marginal Expected Shortfall or MES. This is the expected loss an equity investor in a financial firm would experience if the overall market declined substantially. This measure can then be extrapolated to estimate equity losses for this firm in a future crisis and consequently the capital shortage that would be experienced as a consequence of the initial leverage. The contribution to systemic risk is then estimated as the percentage of capital shortfall that can be expected in a future crisis. MES depends upon the volatility of a firm equity price, its correlation with the market return and the comovement of the tails of the distributions. These in turn are estimated by asymmetric versions of GARCH, DCC and non-parametric tail estimators. Empirical results with 102 US financial firms find predictability in both time series and cross section and useful ranking of firms at various stages of the financial crisis.

Keywords: Systemic Risk, Volatility, Correlations, Tails, Forecasting.

JEL classification: C22, C23, C53.

*Department of Finance, Stern School of Business, New York University, e-mail: ctb@stern.nyu.edu, rengle@stern.nyu.edu. An appendix with detailed results is available on request to the authors. Weekly systemic risk analysis results are published on the Risk section of the Vlab (http://www.systemicriskranking.stern.nyu.edu/).

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1 Introduction

The Great Recession of 2007/2009 has motivated market participants, academics and regulators to better understand systemic risk. A useful definition of systemic risk by Federal Reserve Governor Daniel Tarullo, is

> *"Financial institutions are systemically important if the failure of the firm to meet its obligations to creditors and customers would have significant adverse consequences for the financial system and the broader economy."*

In this definition, it is the failure of a firm to meet obligations that is the cause of systemic distress. Thus measures of systemic risk are associated with firm bankruptcies or near bankruptcies which are inevitable consequences of a decline in equity valuations for highly levered firms.

In order to estimate the capital weakness of firms in a potential crisis, we seek to first estimate the equity losses that would result from such a crisis. Following Acharya *et al.* (2010) we focus on the Marginal Expected Shortfall of an individual financial company. This is defined as the expected equity loss per dollar invested in a particular company if the overall market declines by a certain amount. The companies with the highest MES are the companies that contribute the most to the market decline and are therefore the most important candidates to be systemically risky. Equity holders in a company that is systemically risky will suffer major losses in a financial crisis and consequently will reduce positions if a crisis becomes more likely. MES measures this effect. It clearly relies on investors recognizing which companies will do badly in a crisis.

In this work we engage in a modeling and forecasting exercise of systemic risk using MES. We begin by reviewing the MES risk measure and showing how it can be expressed as a function of volatility, correlation and tail expectations. We then propose a multi step modeling approach based on GARCH, DCC (Engle (2002b), Engle (2009)) and nonparametric tail expectation estimators (Scaillet (2005)) to estimate it. We apply the proposed methodology to analyse the systemic risk contribution of a sample of 102 top U.S. Financial firms between 1990 and 2008, both in the time series and cross sectional dimension. Financial firms are grouped in 4 categories: Depositories, Insurance, Broker-Dealers and a residual category labelled Others which also contains non depositary institutions and real estate related firms. Our GARCH/DCC/Nonparametric approach allows us to extract the time series of MES and to investigate its dynamics. Cross sectional regressions in the spirit of Engle and Rangel (2008) highlight the linkages of systemic risk with individual firm characteristics: financial industry group, market capitalization and leverage. Finally, we engage in a multi horizon forecasting application of MES and propose MES evaluation criteria for the assessment of aggregated losses, loss rankings and loss concentration predictions. There are no closed form solutions available for multi horizon MES but simulation based methods allow us to compute such predictions in a straightforward way.

Empirical results show that firm volatility and correlation with the market are the most important determinants of MES: the more a firm is volatile and the less diversified with respect to the market, the higher the systemic risk contribution. The time series analysis highlights that the level of MES during the financial crisis is extreme relative to the last 20 years. However, the MES industry group rankings have been stable in time, with Broker-Dealers and the Other sectors being the most systemically risky ones and, indeed, these sectors turned out to be the most problematic in the financial crisis. The cross sectional analysis shows how MES is an increasing function of market size and financial leverage. Moreover, leverage has an asymmetric role and its impact is higher on those quarters in which the market falls. The forecasting application shows that our MES predictions are able to perform well relative to the benchmark proposed in Acharya *et al.* (2010).

Different strands of the literature relate to this work. MES is not the only systemic risk measure currently proposed. Among others, the CoVaR (Adrian and Brunnermeier (2009)) has received some notable attention. Our work also relates the burgeoning literature on volatility and correlation modeling using, respectively, GARCH and DCC models. A detailed glossary of the ARCH universe can be found Bollerslev (2008). The DCC approach for correlations has been introduced by Engle (2002a) and recently surveyed in Engle (2009). Contributions in this area include Engle and Sheppard (2001), Aielli (2006) and Engle *et al.* (2009). Dynamic models for Value–at–Risk and Expected Shortfall have been developed in Engle and Manganelli (2004) and Taylor (2008).

The rest of the paper is organized as follows. Section 2 presents the MES systemic risk measure and introduces the econometric approach used to estimate it. Section 3 describes the sample of companies used in this analysis and basic descriptive statistics. Section 4 reports the results of the MES analysis while Section 5 investigates its cross section. Section 6 engages in a forecasting exercise with a special focus on the financial crisis. Concluding remarks follow in Section 7.

2 Econometric Methodology

2.1 Systemic Risk Measurement

In this section we introduce the Marginal Expected Shortfall measure of systemic risk proposed by Acharya *et al.* (2010). Let I denote the set of firms in the economy. The return generated in the system at time t can be measured by the value weighted average return of all firms, which we name the market return, that is

$$r_{m\,t} = \sum_{i \in I} w_i r_{i\,t},$$

where $r_{i\,t}$ and w_i denote respectively the return and market size weight of firm i. We adopt Expected Shortfall as the risk measure for the whole system, which is defined as the expected downside loss in case a negative shock beyond a given threshold is observed

$$\mathsf{ES}_{t-1}(C) \equiv -\mathsf{E}_{t-1}(r_{m\,t} | r_{m\,t} < C),$$

with C some negative constant. A realisation of the condition $r_{m\,t} < C$ is called a systemic event. Several authors have strongly advocated the use of ES as opposed to VaR, in that the latter does not take into account the entity of the damage while the former does and is also coherent (cf. Artzner *et al.* (1999)). Note that in our formulation of the problem we are considering the *conditional* expectation of observing an *unconditional* systemic event. We this choice because we think of systemic risk as of something regarded as harmful regardless of current market conditions. The implications of this assumption will be further discussed later. We define the marginal contribution of firm i to the system risk, named *Marginal Expected Shortfall* (MES), as the partial derivative of the system's ES with respect to the weight of firm i in the economy. It follows from the definition of market return and linearity of expectations that

$$\mathsf{ES}_{t-1}(C) = -\sum_{i \in I_t} w_i \mathsf{E}_{t-1}(r_{i\,t} | r_{m\,t} < C),$$

hence, it is trivial to see that

$$\mathsf{MES}_{i\,t-1}(C) \equiv \frac{\partial \mathsf{ES}_{t-1}(C)}{\partial w_i} = -\mathsf{E}_{t-1}(r_{i\,t} | r_{m\,t} < C).$$

2.2 Econometric Approach

We apply time series methods to obtain estimates of MES using the bivariate series of market and firm returns. The base MES estimator proposed in Acharya *et al.* (2010), which we label as "Historical" is

$$\mathsf{MES}^{\mathsf{his}}_{i\,t-1}(C) \equiv \frac{\sum_{\tau=t-W}^{t-1} r_{i\,\tau} I(r_{m\,\tau} < C)}{\sum_{\tau=t-W}^{t-1} I(r_{m\,\tau} < C)}, \tag{1}$$

which is an average of firm returns on event days over a given window of most recent observations (say, W = 4 years), and is inspired by basic risk management tools where rolling averages are often used to obtain estimates of ES or VaR. In the forecasting application this method will be used as a reference benchmark.

Our approach starts from a description of the bivariate process of firm and market returns:

$$\begin{aligned} r_{m\,t} &= \sigma_{m\,t}\, \epsilon_{m\,t} \\ r_{i\,t} &= \sigma_{i\,t}\, \rho_{i\,t} \epsilon_{m\,t} + \sqrt{1 - \rho_{i\,t}^2} \xi_{i\,t} \\ (\epsilon_{m\,t}, \xi_{i\,t}) &\sim F \end{aligned}$$

the disturbances are independent and identically distributed over time and have zero mean, unit variance and zero covariance. However they are not assumed to be independent. Indeed, there are important reasons to believe that extreme values of these disturbances could occur at the same time for systemically risky firms. When the market is in its tail, the firm disturbances may be even further in the tail if there is serious risk of default. The stochastic specification is completed by a description of the two conditional standard deviations and the conditional correlation. These will be discussed in the next section but are familiar models of asymmetric GARCH and asymmetric DCC. There is substantial scope for examining these assumtions.

Straightforward algebra allows us to decompose MES in a function of volatility, correlation and tail expectations of the standardised innovations distribution

$$\begin{aligned}
\mathsf{MES}_{i\,t-1} &= \mathsf{E}_{t-1}(r_{i\,t}|r_{m\,t} < C) \\
&= \sigma_{i\,t}\mathsf{E}_{t-1}(\epsilon_{i\,t}|\epsilon_{m\,t} < C/\sigma_{m\,t}) \\
&= \sigma_{i\,t}\mathsf{E}_{t-1}(\rho_t\epsilon_{m\,t} + \sqrt{1-\rho_t^2}\xi_{i\,t}|\epsilon_{m\,t} < C/\sigma_{m\,t}) \\
&= \sigma_{i\,t}\rho_t\mathsf{E}_{t-1}(\epsilon_{m\,t}|\epsilon_{m\,t} < C/\sigma_{m\,t}) + \\
&\quad \sigma_{i\,t}\sqrt{1-\rho_t^2}\mathsf{E}_{t-1}(\xi_{i\,t}|\epsilon_{m\,t} < C/\sigma_{m\,t}),
\end{aligned} \tag{2}$$

and the conditional probability of a systemic event is

$$P_{t-1}(r_{m\,t} < C) = P(\epsilon_{m\,t} < C/\sigma_{m\,t}). \tag{3}$$

Some comments on the formulas in Equations (2) and (3) are in order under the assumption that the conditional correlation of the firm with the market is positive. Firstly MES is an increasing function of a firm's own volatility. Depending on whether correlation is high or low, the MES formula gives more weight to, respectively, either the tail expectation of the standardised market residual or the tail expectation of standardised idiosyncratic firm residual. Also note that the second term in Equation (2) arises because of the nonlinear dependence assumption between $\epsilon_{m\,t}$ and $\xi_{i\,t}$ and it would otherwise be zero if dependence was captured entirely by correlation. It is also important to stress the implication of the conditioning systemic event C. Typically, VaR and ES are expressed in conditional terms, that is the conditioning event is a quantile from the conditional return distribution. On the other hand, in this work the conditioning event is unconditional. Thus, while in the conventional approach the probability of observing the conditioning event is constant, in our framework such probability is time varying: the higher the volatility the higher the probability of observing a loss above a fixed threshold.

Different strategies can be employed to obtain estimates of MES starting from Equation (2). In this work rely on a multi stage modeling approach which is inspired by the DCC (Engle (2002a), Engle (2009)). In the first step we model volatilities using GARCH models to obtain conditional volatility and standardised residuals. We then resort to a DCC specification to obtain conditional correlation and the standardised idiosyncratic firm residual. Finally, we resort to a nonparametric estimator to compute the tail expectations in Equation (2). The appealing feature of such modeling paradigm are simplicity and flexibility. Estimation of fully bivariate conditionally heteroskedastic model with nonlinear residual dependence for a large panel of assets (some of which not too long) can be quite challenging. On the other hand our approach is much easier to implement and it allows for considerable flexibly in the specifications, by changing the different types of volatility (splines, high frequency based, etc.) and correlation (standard DCC, factor DCC, asymmetric, breaks in correlation, etc.).

2.3 Volatility, Correlation and Tails

Volatility. In the wide universe of GARCH specifications, we pick the TARCH specification to model volatility (Rabemananjara and Zakoïan (1993), Glosten *et al.* (1993)). The evolution of the conditional variance dynamics in this model class is given by

$$\begin{aligned}\sigma^2_{m\,t} &= \omega_{m\,G} + \alpha_{m\,G}\, r^2_{m\,t-1} + \gamma_{m\,G}\, r^2_{m\,t-1} I^-_{m\,t-1} + \beta_{m\,G}\, \sigma^2_{m\,t-1} \\ \sigma^2_{i\,t} &= \omega_{i\,G} + \alpha_{i\,G}\, r^2_{i\,t-1} + \gamma_{i\,G}\, r^2_{i\,t-1} I^-_{i\,t-1} + \beta_{i\,G}\, \sigma^2_{i\,t-1}\end{aligned}$$

with $I^-_{i\,t} = r_{i\,t} < 0$ and $I^-_{m\,t} = r_{m\,t} < 0$. The main highlight of this specification is its ability to capture the so called leverage effect, that is the tendency of volatility to increase more with negative news rather than positive ones. This model is also successful from a forecasting standpoint and it turns out to be quite difficult to beat. We estimate the model using QML which guaranties consistent estimates of the model parameters as long as the conditional variance equation is correctly specified.

Correlations. We model time varying correlations using the DCC approach (Engle (2002a), Engle (2009)). Let P_t denote the time varying correlation matrix of the market and firm return, that is, using matrix notation,

$$\begin{aligned}\mathsf{Var}_{t-1}\begin{pmatrix} r_{i\,t} \\ r_{m\,t}\end{pmatrix} &= D_t\, P_t\, D_t \\ &= \begin{bmatrix}\sigma_{i\,t} & 0 \\ 0 & \sigma_{m\,t}\end{bmatrix}\begin{bmatrix}1 & \rho_t \\ \rho_t & 1\end{bmatrix}\begin{bmatrix}\sigma_{i\,t} & 0 \\ 0 & \sigma_{m\,t}\end{bmatrix}.\end{aligned}$$

Rather than directly modeling the P_t matrix, the DCC framework models the, so called, pseudo correlation matrix Q_t, a positive definite matrix which is then mapped in a correlation matrix through the transformation

$$P_t = \mathsf{diag}(Q_t)^{-1/2}\, Q_t\, \mathsf{diag}(Q_t)^{-1/2},$$

where the $\mathsf{diag}(A)$ matrix operator denotes a matrix with the same elements of the A matrix on the diagonal and zero otherwise. We formulate the pseudo correlation matrix Q_t dynamics using the DCC formulation proposed in Aielli (2006) and Aielli (2009).

The basic (scalar) symmetric DCC specification is defined as

$$Q_t = (1 - \alpha_C - \beta_C)S + \alpha_C\, \epsilon^*_{t-1}\epsilon^{*\,\prime}_{t-1} + \beta_C\, Q_{t-1}, \tag{4}$$

where S is an intercept matrix and ϵ^*_t contains the rescaled standardised (or degarched) returns, that is $\epsilon^*_t = Q^*_{t-1}\epsilon_t$ with $Q^*_t = \mathsf{diag}(Q_t)^{1/2}$. The pseudo conditional correlation matrix Q_t is thus an exponentially weighted moving average of past outer products of the rescaled standardised returns. Necessary and sufficient conditions for Q_t to be positive definite are $\alpha_C > 0$, $\beta_C > 0$, $\alpha_C + \beta_C < 1$ and the positive definiteness of the S matrix. The rescaling device ensures that $\{\epsilon^*_t, Q_t\}$ is a MGARCH process (Ding and Engle (2001)) and, under the assumption of stationarity of the model ($\alpha_C + \beta_C < 1$), this implies that S is the unconditional covariance matrix of ϵ^*_t

$$S = \mathsf{E}(\epsilon^*_t \epsilon^{*\,\prime}_t).$$

This properties is useful for highly dimensional DCC estimation in that it justifies the use of the unconditional covariance matrix of the ϵ_t^* as a correlation targeting (Mezrich and Engle (1996)) estimator for S, that is

$$\hat{S} = \frac{1}{n} \sum \epsilon_t^* \epsilon_t^{*\prime},$$

which drastically reduces the number of parameter that need be optimized to estimate the model.

A limitation of the DCC model of Equation (4) is that products of jointly positive or negative standardised returns have the same impact on the evolution of the future correlation matrix. Cappiello *et al.* (2006) relax such an assumption introducing different types of asymmetric specifications. Their models are also motivated by an analogy to the leverage effect in volatility, which is found to be relevant in the vast majority of applications on equity data. Moreover, it is of particular interest to assess if strings of joint common negative shocks are able to increase correlation significantly more than positive ones, implying, for instance, that market drops can significantly reduce the ability to diversify investments. We define an Asymmetric DCC model using the Aielli (2006) device as

$$Q_t = ((1 - \alpha_C - \beta_C)S) - \gamma_C N + \alpha_C \; \epsilon_{t-1}^* \epsilon_{t-1}^{*\prime} + \gamma_C \; n_{t-1}^* n_{t-1}^{*\prime} + \beta_C \; Q_{t-1}$$

where S and N are intercept matrices and $n_t^* = \epsilon_t^* \odot I[\epsilon_t^* < 0]$ is used to capture the asymmetric impact of jointly negative news. Necessary and sufficient conditions for Q_t to be positive definite are $\alpha_C > 0$, $\gamma_C > 0$ and $\beta_C > 0$ together with the positive definiteness of the intercept $(1 - \alpha_C - \beta_C)S - \gamma_C N$, which is ensured by the condition

$$\alpha_C + \beta_C + \delta_C \gamma < 1$$

where δ is the maximum eigenvalue of $S^{-1/2} N S^{-1/2}$ (cf Cappiello *et al.* (2006)). Again, to reduce the dimensionality of the model the S and N intercept matrices can be replaced by sample moments of the ϵ_t^* and n_t^* process, that is

$$\hat{S} = \frac{1}{n} \sum_t \epsilon_t^* \epsilon_t^{*\prime}, \qquad \hat{N} = \frac{1}{n} \sum_t n_t^* n_t^{*\prime}.$$

A slight complication of the $\hat{S}$ and $\hat{N}$ estimators is that they are based on the diagonal of the Q_t matrix which in turns depends on the parameter values of the model. Hence, in the evaluation of the likelihood, for instance, the estimators have to be computed each time. Also note that the diagonal elements of S and N are either known or can be computed without knowledge of the model parameters. The computation of the likelihood proceeds by first computing the diagonal of Q_t, evaluating the $\hat{S}$ and $\hat{N}$ matrices and finally computing the off diagonal elements of Q_t and the likelihood of the model.

Tails. The last ingredients needed to measure MES are estimates of the tail expectations

$$\mathsf{E}(\epsilon_{mt} | \epsilon_{mt} < \kappa) \qquad \text{and} \qquad \mathsf{E}(\xi_{it} | \epsilon_{mt} < \kappa).$$

These expectation can be simply estimated for a particular value of the variances $(\sigma^2_{m\,t}, \sigma^2_{i\,t})$ and conditional correlation ρ_t by simply looking at the average of the two residuals in all cases which satisfy the condition. However, when κ is large, this estimator will be unstable as there are only a small number of observations. A nonparametric kernel estimation approach can be used to improved the efficiency of these simple estimators. Let

$$K_h(t) = \int_{\infty}^{t/h} k(u)du,$$

where $k(u)$ is a kernel function and h is a positive bandwidth. Then

$$\widehat{\mathsf{E}}_h(\epsilon_{m\,t}|\epsilon_{m\,t} < \kappa) = \frac{\sum_{i=1}^{n} \epsilon_{m\,t} K_h(\epsilon_{m\,t} - \kappa)}{(n\hat{p}_h)}, \tag{5}$$

and

$$\widehat{\mathsf{E}}_h(\xi_{i\,t}|\epsilon_{m\,t} < \kappa) = \frac{\sum_{i=1}^{n} \xi_{i\,t} K_h(\epsilon_{m\,t} - \kappa)}{(n\hat{p}_h)}, \tag{6}$$

where

$$\hat{p}_h = \frac{\sum_{i=1}^{n} K_h(\epsilon_{m\,t} - \kappa)}{n}.$$

An advantage of the nonparametric estimators defined in Equations (5) and (6) is that they are smooth functions of the cutoff point κ which, in turns, deliver smooth estimates of MES as a function of $C/\sigma_{m\,t}$. This is an appealing property for ranking purposes in that the MES ranks will not be too sensitive to small changes in the threshold level of the systemic loss.

3 U.S. Financials in 1990–2008

The empirical analysis focuses of the systemic risk contribution of the top U.S. financial firms between January 2, 1990 and December 31, 2008. We study the same panel of institutions studied in Acharya *et al.* (2010). The panel contains all U.S. financial firms with a market capitalization greater than 5 bln USD as of the end of June 2007 and it is unbalanced in that not all companies have continuously been trading during the sample period. We extract daily returns and market capitalization from CRSP and the quarterly book value of equity (ceqq) from COMPUSTAT. SIC codes are used to divide firms into 4 groups: Depositories (such as Bank of America or JP Morgan Chase), Broker-Dealers (Goldman Sachs or Lehman Brothers), Insurance (AIG) and Others (non depositary institutions, real estate) (Freddie and Fannie). We make one exception to this rule, Goldman Sachs (GS) should have been classified within the Others group, but instead we put it with Brokers-Dealers. We also use the daily CRSP market value weighted index return as the market index return. In what follows we will refer to the period January 1990 to December 2008 as the "full sample" and July 2007 to December 2008 as the "financial crisis". The full list of tickers and company names divided by industry groups is reported in Table 1.

Depositories		Others		Insurance		Broker-Dealers	
BAC	Bank of America	ACAS	American Capital	ABK	AMBAC Financial Group	AGE	Edwards A G
BBT	BB&T	AMP	Ameriprise Financial	AET	AETNA	BSC	Bear Stearns
BK	bank of new york mellon	AMTD	t d ameritrade holding	AFL	A F L A C INC	ETFC	E-Trade Financial
C	citigroup inc	AT	alltel corp	AIG	American International Group	GS	Goldman Sachs
CBH	commerce bancorp inc nj	AXP	American Express	AIZ	ASSURANT INC	LEH	Lehman Brothers
CMA	comerica inc	BEN	Franklin Resources	ALL	ALLSTATE CORP	MER	Merrill Lynch
HBAN	huntington bancshares inc	BLK	Blackrock	AOC	aon corp	MS	Morgan Stanley
HCBK	hudson city bancorp inc	BOT	c b o t holdings inc	BER	berkley w r corp	NMX	Nymex Holdings
JPM	JP Morgan Chase	CBG	c b richard ellis group	BRKA	berkshire hathaway inc del	SCHW	Schwab Charles
KEY	keycorp new	CBSS	compass bancshares inc	BRKB	berkshire hathaway inc del	TROW	T. Rowe Price
MI	marshall & ilsley	CIT	CIT group	CB	chubb corp		
MTB	m & t bank corp	CME	c m e group	CFC	countrywide financial corp		
NCC	national city corp	COF	capital one financial	CG	loews corp		
NTRS	northern trust corp	EV	eaton vance corp	CI	c i g n a corp		
NYB	new york community bancorp	FIS	fidelity national info svcs	CINF	cincinnati financial corp		
PBCT	peoples united financial	FITB	fifth third bancorp	CNA	c n a financial corp		
PNC	p n c financial services	FNM	federal national mortgage assn	CVH	coventry health care inc		
RF	regions financial	FRE	federal home loan mortgage	FNF	fidelity national finl inc new		
SNV	synovus financial	ICE	intercontinentalexchange	GNW	genworth financial inc		
SOV	sovereign bancorp	JNS	janus cap group inc	HIG	hartford financial svcs group		
STI	suntrust banks inc	LM	legg mason inc	HNT	health net inc		
STT	state street corp	LUK	leucadia national	HUM	humana inc		
UB	unionbancal corp	MA	Mastercard	LNC	lincoln national corp in		
USB	u s bancorp del	NYX	NYSE Euronext	LTR	loews corp		
WB	Wachovia	SEIC	s e i investments company	MBI	m b i a inc		
WFC	Wells Fargo & co	SLM	s l m corp	MET	metlife inc		
WM	Washington Mutual	UNP	union pacific	MMC	marsh & mclennan cos inc		
WU	Western Union			PFG	principal financial group inc		
ZION	Zion			PGR	progressive corp oh		
				PRU	prudential financial inc		
				SAF	safeco corp		
				TMK	torchmark corp		
				TRV	travelers companies inc		
				UNH	unitedhealth group inc		
				UNM	unum group		
				WLP	Wellpoint i		

Table 1: Tickers, company names, industry groups.

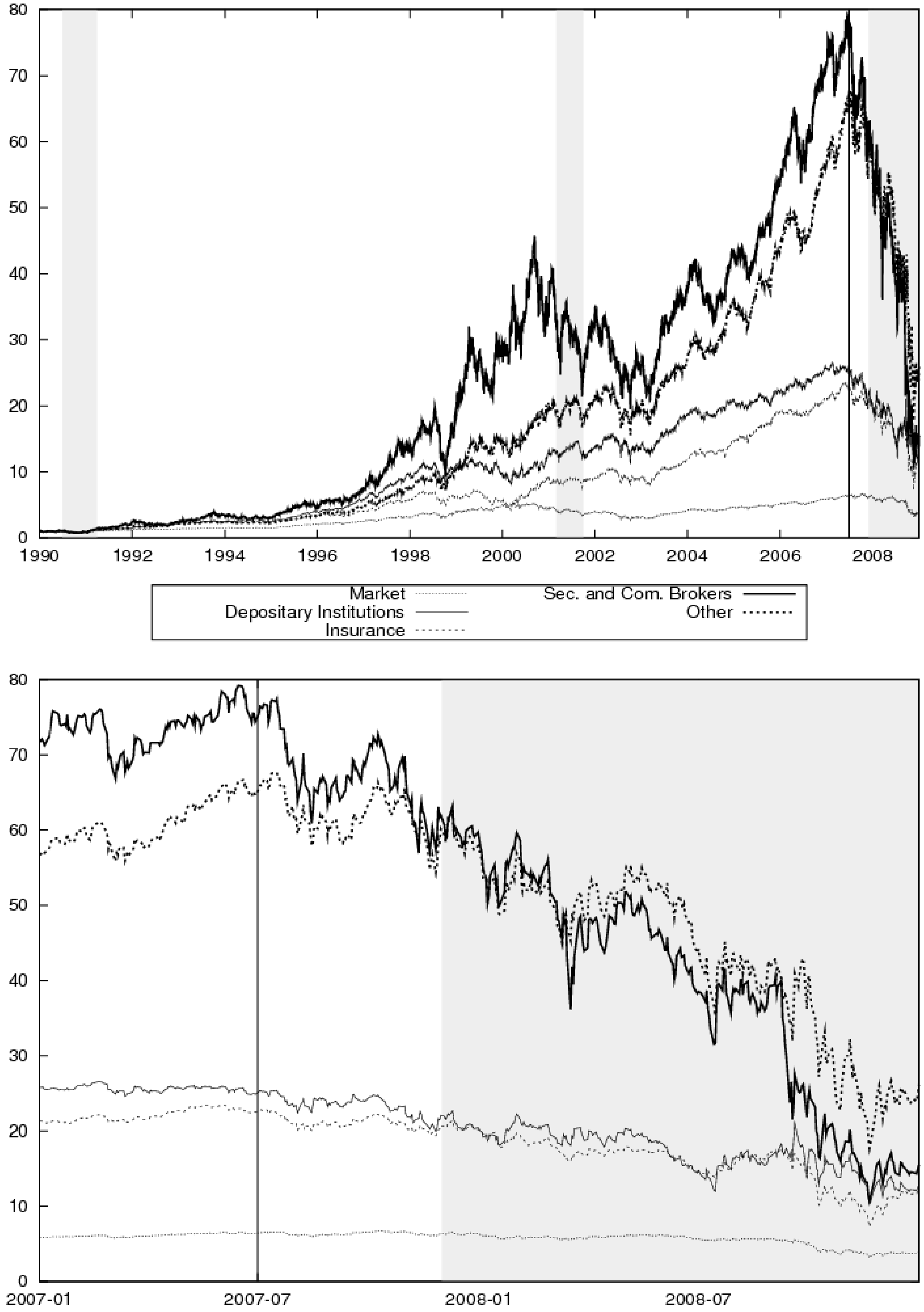

Figure 1: Cumulative average return by financial industry group.

	Ret.	Vol.	Beta	Cor.	Kurt.	Skew.	Ret.	Vol.	Beta	Cor.	Kurt.	Skew.
	Full Sample						Crisis Period					
	Depositary Institutions											
$Q_{0.25}$	5.01	32.38	0.95	0.46	14.02	0.2	-75.56	57.84	1.11	0.59	6.8	0.29
Median	9.75	36.76	1.06	0.53	19.27	0.41	-35.05	73.19	1.43	0.67	9.72	0.44
$Q_{0.75}$	12.98	40.46	1.22	0.59	64.32	0.78	-17.42	97.62	1.69	0.73	12.83	0.63
	Others											
$Q_{0.25}$	-8.23	37.96	1.12	0.49	9.03	0	-98.14	61.93	1.37	0.62	5.87	-0.17
Median	8.43	45.65	1.31	0.59	12.68	0.4	-57.31	73.15	1.64	0.71	7.81	0.29
$Q_{0.75}$	19.3	57.16	1.64	0.65	22.97	0.88	-34.86	86.94	1.75	0.78	10.34	0.68
	Insurance											
$Q_{0.25}$	2.27	30.44	0.85	0.42	18.45	-0.34	-79.64	49.92	0.9	0.51	10.09	0.08
Median	7.07	38.3	0.95	0.48	27.22	0.43	-38.49	59.62	1.25	0.64	12.85	0.43
$Q_{0.75}$	10.2	46.62	1.21	0.57	42.7	1.27	-12.65	84.54	1.62	0.74	17.22	1.07
	Security Dealers and Commodity Brokers											
$Q_{0.25}$	-4.77	43.04	1.43	0.54	8.09	0.21	-178	64.43	1.45	0.57	7.07	-0.11
Median	6.2	46.42	1.66	0.6	15.82	0.49	-78.36	85.58	1.75	0.74	11.16	0.24
$Q_{0.75}$	14.54	50.34	1.9	0.68	110.9	0.82	-26.82	122.6	2.4	0.74	32.85	0.86

Table 2: U.S. financials descriptive statistics.

It is important to give some further details about the industry grouping. The Broker-Dealers group contains the top U.S. investment banks. Many of these companies were in severe distress in the crisis: Lehman Brothers (LEH) was liquidated, Bear Stearns (BNC) and Merrill Lynch (MER) sold, Goldman Sachs (GS) and Morgan Stanley (MS) became commercial banks switching to a more stringent regulatory regime. The residual group Others contains real estate hardly hit by the crisis, like Freddie (FRE) and Fannie (FAN) which were placed into conservatorship. As we will highlight in the cross sectional analysis these institutions as of June 2007 were also large and highly leveraged. Moreover, they were also subject to looser regulations than the standard commercial banks which make up the Depositories group.

Figure 1 gives some visual insights on the boom and burst of the financial sector. The figures show the cumulative average return by industry group over the full sample, together with the cumulative return of the market. The shaded gray areas denote NBER recession periods and the vertical black line marks the beginning of the financial crisis. The top panel shows the overall series while the bottom panel zooms on the last two years of the sample. Between January 1990 and June 2007 all financial groups have had a steep growth with respect to the market. Moreover, Broker-Dealers and the Other groups had roughly grown four times more than Depositories and Insurance. Starting from July 2007, the fall of financials has been dramatic, with the biggest winners transforming into the biggest losers. Interestingly, Broker-Dealers had the biggest correction among groups non only in the financial crisis but also in the recession of the early 2000's.

Table 2 reports descriptive statistics of the set of tickers divided by industry group over the full sample and the financial crisis. For each statistic the table reports the median, 1st and 3rd quartiles across each group. The annualised return on the full sample is mildly positive, this is a consequence of boom and burst effect of financials cancelling each other out. In the financial crisis, the median annualised return is strongly negative. Broker-Dealers undertook the most extreme losses, followed by the Other group, Depositories and Insurance compa-

	vol	α_G	γ_G	β_G	cor	α_C	γ_C	β_C
	Depositary Institutions							
$Q_{0.25}$	32.0	0.034	0.063	0.878	0.44	0.021	0.003	0.958
Median	36.2	0.046	0.079	0.903	0.56	0.051	0.004	0.964
$Q_{0.75}$	39.9	0.069	0.094	0.921	0.59	0.056	0.005	0.905
	Others							
$Q_{0.25}$	37.3	0.020	0.063	0.894	0.50	0.046	0.001	0.943
Median	45.4	0.031	0.082	0.913	0.53	0.051	0.003	0.963
$Q_{0.75}$	54.4	0.048	0.105	0.940	0.61	0.058	0.005	0.974
	Insurance							
$Q_{0.25}$	30.3	0.020	0.069	0.875	0.41	0.039	0.002	0.951
Median	37.4	0.033	0.091	0.907	0.46	0.042	0.006	0.972
$Q_{0.75}$	45.8	0.056	0.125	0.928	0.53	0.052	0.008	0.974
	Security Dealers and Commodity Brokers							
$Q_{0.25}$	42.7	0.026	0.066	0.893	0.56	0.024	0.002	0.950
Median	45.9	0.031	0.082	0.925	0.62	0.027	0.003	0.961
$Q_{0.75}$	49.7	0.040	0.109	0.939	0.66	0.028	0.004	0.969

Table 3: TARCH and ADCC estimation results.

nies. Median annualised volatility in the crisis has doubled and also median correlations with the market increased by approximately by 0.10. Interestingly, unconditional kurtosis and skewness in the crisis period do not appear to be greater than the full sample estimates.

4 The Dynamics of MES

4.1 Estimation Results

We estimate the volatility, correlations and tail expectations using the methodology outlined in the Section 2 using for the full list of financial firms over the full sample.

Table 3 reports a summary of the estimation results. For each asset class, the table shows selected quantiles of the parameter estimates of the TARCH (left side) and ADCC (right side) models. The TARCH parameters do not appear to fluctuate significantly within groups, with the only exception of intercept term which is on average higher for Broker-Dealers, followed by Others, Insurance and Depositories. The point estimates are in line with the typical GARCH estimates, with slightly higher α's and γ's and lower β's implying higher level of unconditional kurtosis. Turning to the ADCC, parameters appear, again, to be close to the typical set of ADCC estimates. Intercept aside, parameters are similar across groups, with the only exception of the α for Broker-Dealers which is smaller than the one of other groups. The γ parameter capturing asymmetric effects has a small magnitude from an economic standpoint. Broker-Dealers have the highest level of unconditional correlation, followed by the Others and Depository institution which are close and, finally, Insurance.

4.2 The time series of volatility, correlation and MES

The time series of volatility, correlation and MES provide some insights on the dynamics of systemic risk in the financial sector over the last 20 years.

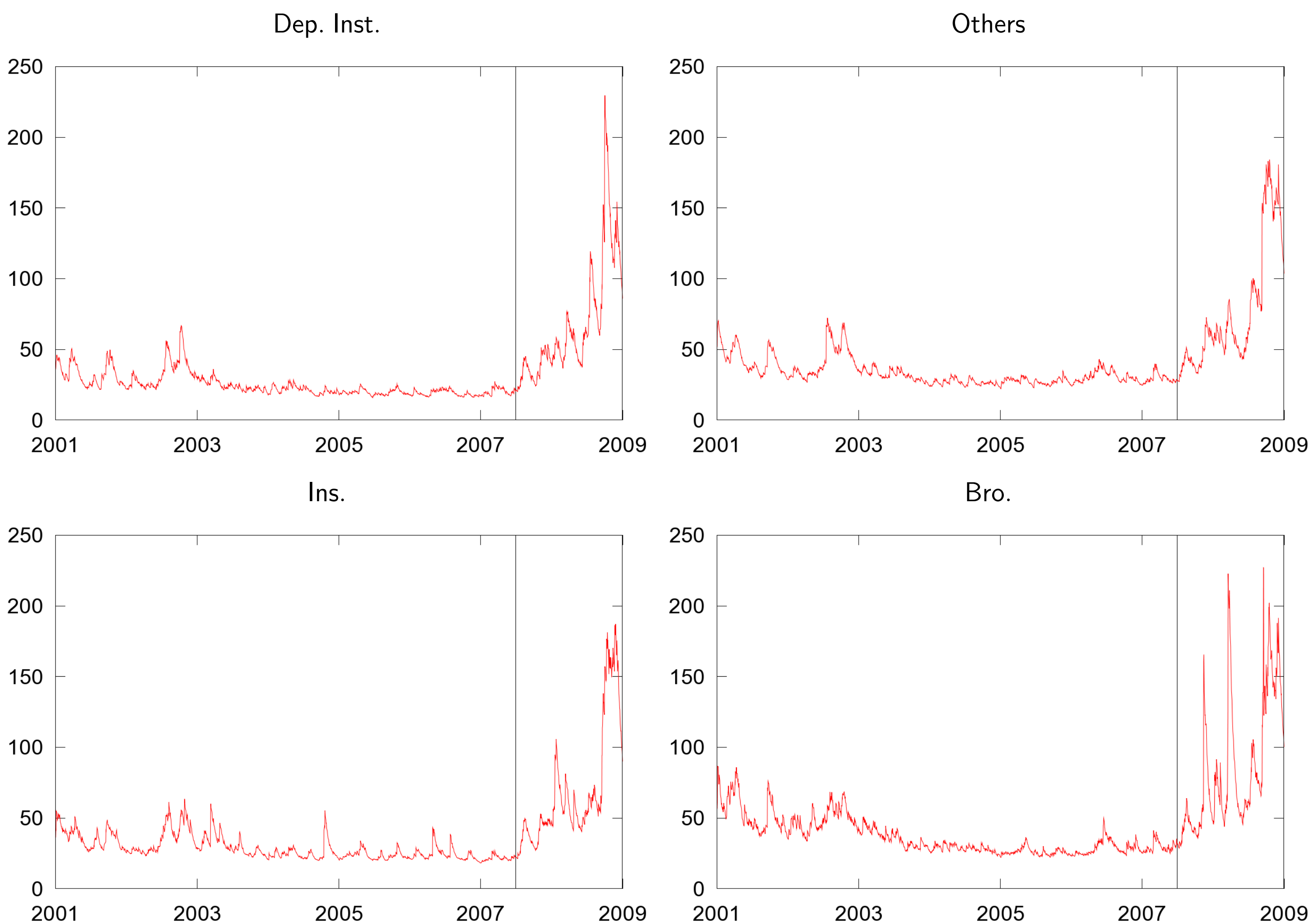

Figure 2: Average volatility by financial industry group.

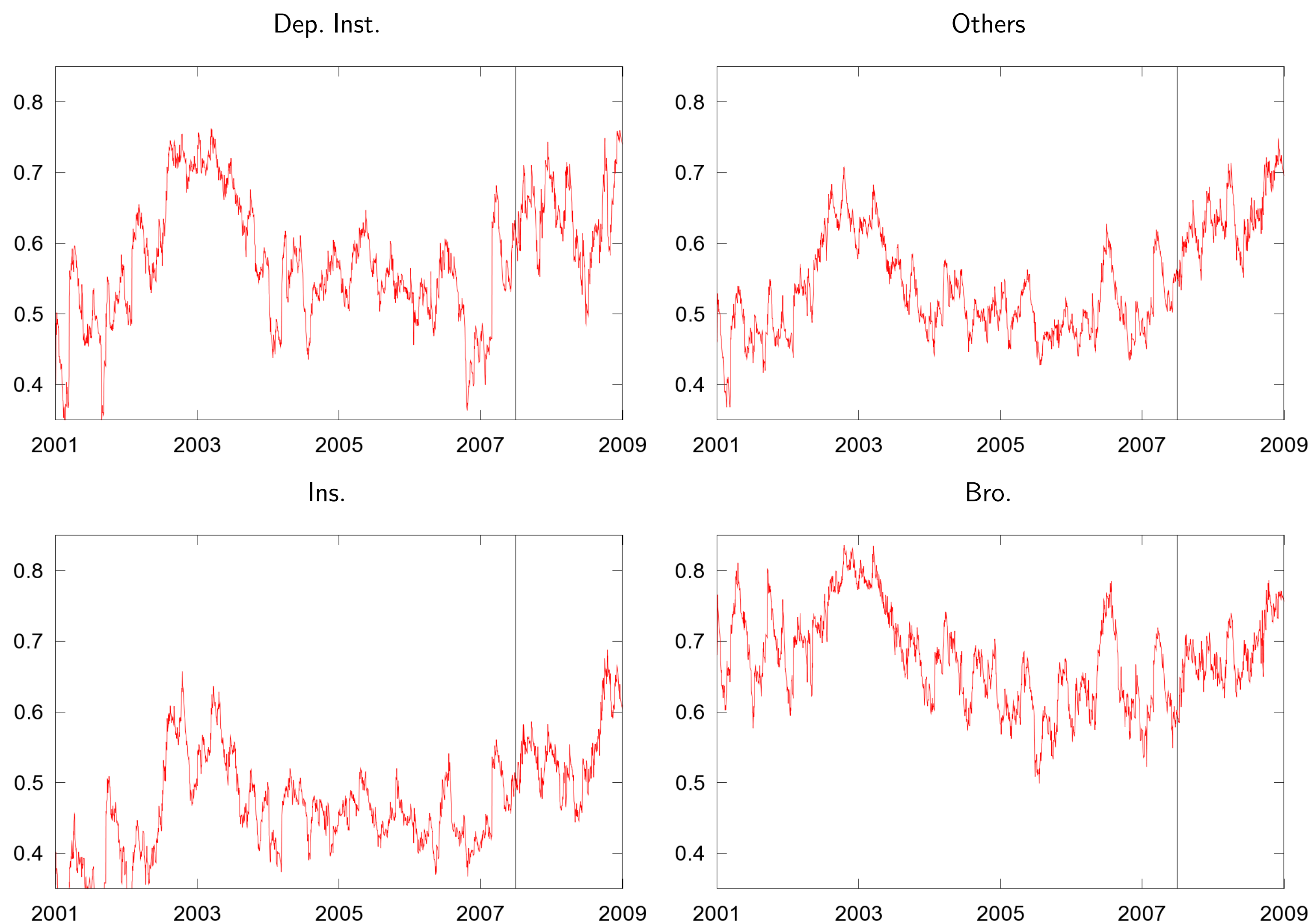

Figure 3: Average correlation by financial industry group.

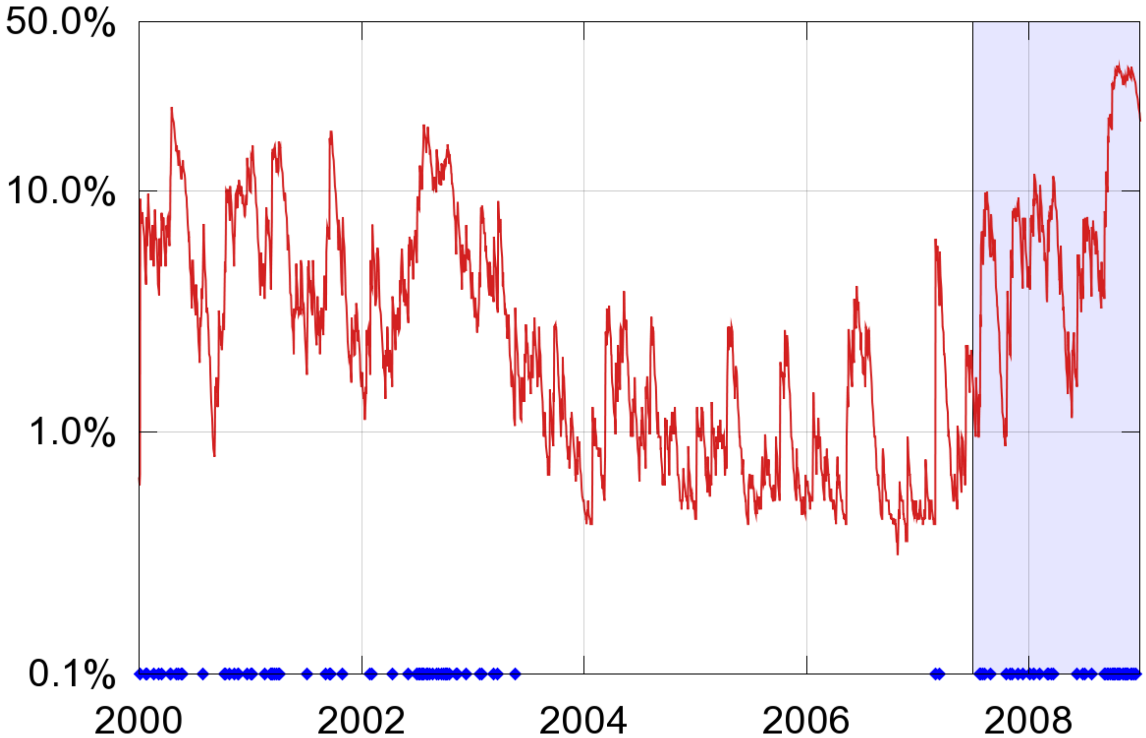

Figure 4: Probability of a 1-step-ahead daily 2% loss in the market.

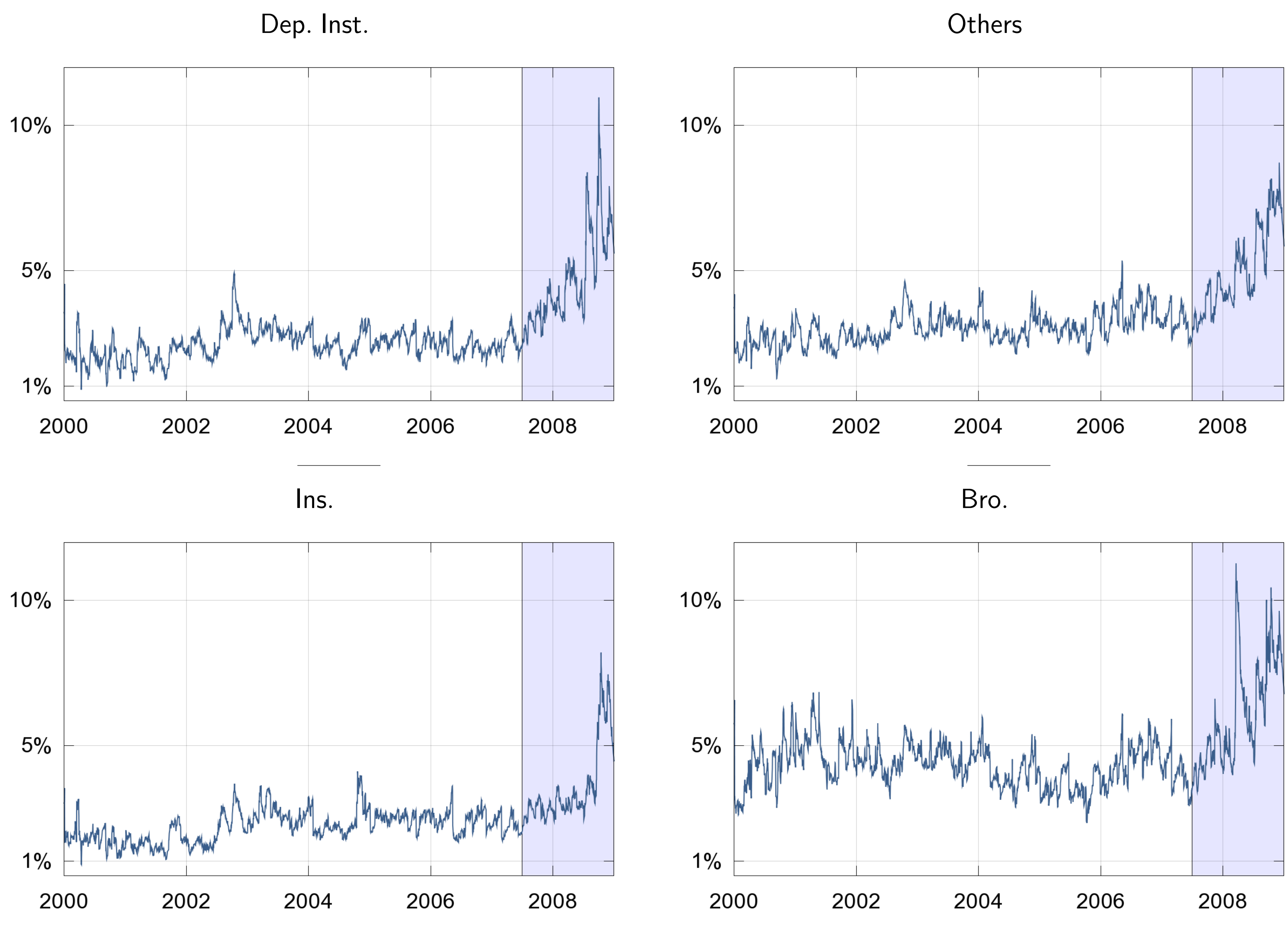

Figure 5: Average MES by financial industry group.

Figure 2 displays the annualised volatility series averages by industry group. Over ally, all groups exhibit a similar time series pattern which can be associated with the general level of risk in the economy. The early 2000s are characterised by high levels of volatility which correspond with the dot com bubble burst and the recession of the beginning of the 2000s. This is followed by a protracted period of low volatility that spans, roughly, 2003 to mid 2007. Starting from July of the same year, volatility gradually surges as the financial crisis begins and in December 2008 it peaks to the highest levels ever measured over the last 20 years of data. While the overall volatility trend is similar across groups the average differs, with Broker-Dealers and the Others being be the most volatile groups. Many of the extreme spikes in the series can be associated with well known distress days in the financial industry. For instance, Depositories have a spike around 200% in correspondence of the bankruptcy of Washington Mutual (September 2008) and Broker-Dealers go beyond 200% with the acquisition of Bears Sterns (March 2008) and the bankruptcy of Lehman (September 2008).

Figure 3 shows correlation averages by industry group. Again, over ally the time series pattern is similar across sectors although there are some differences between Broker-Dealers and the other three groups. Correlations at the beginning of the 2000s are moderate (approximately 0.40) for all sector but Broker-Dealers, which enter the decade with an average correlation level around 0.70. As commonly observed in periods of financial distress, correlation steadily increase in correspondence of the first recession of the 2000s. Subsequently, average correlations do not decrease much in the low volatility period (2003 to mid 2007). The, so called, Chinese correction of February 29, sharply shifts average correlations upwards for all sectors, with the biggest increase in linear dependence for commercial banks. As the financial crisis unwinds correlation further levitate, going beyond 0.75 for Depositories and investment banks in December 2008. Once more, the average levels of correlation differ across groups. Broker-Dealers are always the most correlated sector, Insurance the least one and Depositories together with the Other group lie in the middle.

We finally turn to the analysis of MES. Figure 4 shows the time varying probability of observing a systemic event $\{r_{mt} < C\}$, reported on a log-scale. The rug plot displays the actual days on which a systemic event is observed. The probability of systemic event is a function of the market volatility and the series exhibits the same time series pattern, with high probabilities in correspondence to the early recession of the 2000s and the financial crisis.

Figure 5 reports the average level of MES by industry group. At the beginning of the 2000s investment banks have the higher level of MES, while all other groups cluster together. The levels of MES appear to be roughly stable in time in the subsequent part of the sample. Interestingly, starting from 2006 the Others group has a mild shift in the level of the series, which brings it closer to the MES level of Broker-Dealers. The series start to increase in July 2007, reaching their peaks in December 2008. Depositories reach a MES of roughly 7% and go beyond 10% at the end of September 2008, in conjunction with the bankruptcy of Washington Mutual. Broker-Dealers on the other hand have the biggest increase in mid March 2008 and mid September 2008 with the acquisition Bears Sterns and liquidation of Lehman. MES ranking of the industry groups has been highly consistent in time: Broker-

	Avg Hit	Binomial Test	Crisis Test
	Depositary Institutions		
$q_{0.25}$	0.010	0.103	0.019
Median	0.011	0.383	0.037
$q_{0.75}$	0.013	0.717	0.181
	Others		
$q_{0.25}$	0.009	0.557	0.024
Median	0.010	0.683	0.257
$q_{0.75}$	0.011	0.925	0.466
	Insurance		
$q_{0.25}$	0.011	0.081	0.021
Median	0.012	0.269	0.114
$q_{0.75}$	0.013	0.592	0.258
	Broker-Dealers		
$q_{0.25}$	0.010	0.317	0.030
Median	0.010	0.644	0.083
$q_{0.75}$	0.011	0.906	0.222

Table 4: Residuals Diagnostics.

Dealers are the more systemically risky group followed by the Others group, Depositories and Insurance.

4.3 Residual Diagnostics

We report some residual diagnostics on the GARCH-DCC residuals series $(\hat{\epsilon}_{m\,t}, \hat{\xi}_{i\,t})$. We focus on assessing the degree of dependence in the lower tail between the two innovations. In order to do this, we analysis the Hit_t variable constructed as follows: we first transform the residual series $(\hat{\epsilon}_{m\,t}, \hat{\xi}_{m\,t})$ into their ranks and then construct the Hit_t variable as a dummy indicator which is one when both residuals on period t are simultaneously below their 10% rank. Under the null of correct specification and absence of tail dependence the sequence of hits behaves as an iid Bernoulli sequence with $p = 1\%$, and we can then use this to construct tests in the same spirit of Christoffersen (1998), Engle and Manganelli (2004) and Patton (2002). We perform a binomial test and a "crisis" test. The null of the binomial test is that the hit sequence is generated by a Bernoulli with $p = 1\%$. The crisis test assess whether the proportion of hits is higher during the financial crisis period. The test is performed by running logit regression on the hit variable using a financial crisis dummy as explanatory variable. Table 4 reports the average of the Hit variable together with the p-values of the tests. Again, results are broken down by industry group and we report Median, 1st and 3rd quantile of each statistic within each group. The average hit is almost always systematically higher than the null of 1%, however the deviation appears to be mild and in fact the binomial test is almost always non significant. However, the crisis test p-value signals that for a relevant percentage of cases during the financial crisis the average of the Hit variable has increased. Over ally speaking, results point that although the performance of the GARCH-DCC approach is satisfactory, there is some evidence of neglected lower tail dependence which might turn to be quite important for MES forecasting.

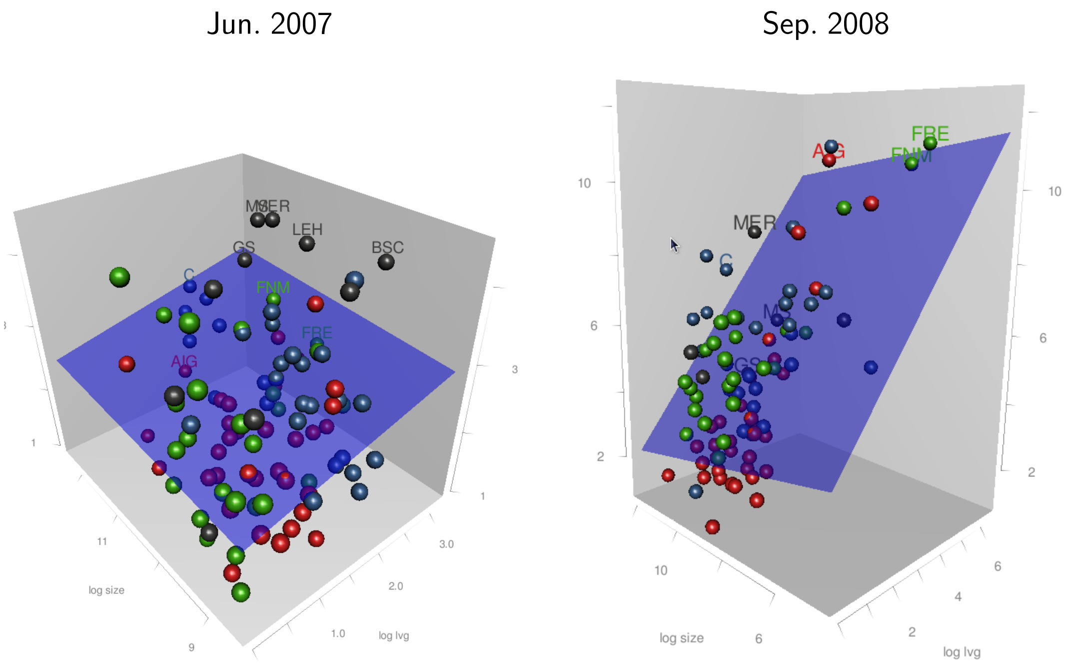

Figure 6: MES scatter plots. The graphs show the value of MES against market capitalization and leverage.

5 The Cross Section of MES

The time series methodology described in the previous sections allows us measure the systemic risk contribution of a firm across time. What the time series analysis does not fully answer however, is what firm characteristics make one institution riskier than another one. Following Engle and Rangel (2008), in this section we resort to cross sectional regression to shed light on this question by regressing low frequency aggregated MES measurements on a set of explanatory variables of interest. As MES is a function of volatility and correlation, it is also of interest to use the same methodology on volatility and correlation to disentangle the role of the explanatory variables.

We define the quarterly average of the daily MES estimates as

$$\text{mes}_{i\,j} = \frac{1}{|Q_j|} \sum_{t \in Q_j} \text{MES}_{i\,d}(C),$$

where Q_j denotes the set of t in quarter j; and we analogously define quarterly (annualised) volatility and correlation as

$$\text{vol}_{i\,j} = \frac{1}{|Q_j|} \sum_{t \in Q_j} \sqrt{252} \cdot \sigma_{i\,t} \quad \text{and} \quad \text{cor}_{i\,j} = \frac{1}{|Q_j|} \sum_{t \in Q_j} \rho_{i\,t},$$

respectively. The explanatory variables we us in the cross sectional analysis are a yearly dummy (Y_j), industry group dummy (G_i), (log) market capitalisation ($\log(\text{size})$) and (log)

	MES	Vol	Cor
Insurance	-0.203^{***} 0.039	0.046^{***} 0.005	-0.064^{***} 0.004
S. & C. Brokers	1.617^{***} 0.054	0.121^{***} 0.008	0.102^{***} 0.005
Other	0.367^{***} 0.046	0.086^{***} 0.007	0.003 0.004
$\log(\mathsf{siz})$	0.105^{***} 0.013	-0.036^{***} 0.002	0.051^{***} 0.001
$\log(\mathsf{lvg})$	0.139^{***} 0.022	0.030^{***} 0.003	0.006^{***} 0.002
$\log(\mathsf{lvg}) \times I^-$	0.095^{***} 0.021	0.041^{***} 0.003	0.005 0.002
R^2	32.3%	45.1%	50.2%

Table 5: Cross sectional determinants of systemic risk, volatility and correlation.

leverage ($\log(\mathsf{lvg})$). The yearly dummy is used to capture common long range dependence which might be present in series (cf. Barigozzi *et al.* (2010)). The plots displayed in the previous section suggest that different industry groups often exhibit different average values of MES, volatility and correlation, hence, industry dummies allow us to control for these type of effects. Market capitalisation is defined as the product of the share price multiplied by the number of share outstanding as of the last day of the previous quarter. The (ab)use of leverage has been identified as one of the factors contributing to the financial crisis. The period of low volatility and positive growth of the market after the recession of the early 2000s has lead several financial companies to significantly increase their leverage in the attempt to obtain a higher rate of return, and this has subsequently lead to more severe losses in the market downturn of July 2007. We allow leverage to have a nonlinear impact in our cross sectional regressions by considering the interaction of leverage with a dummy indicator of negative quarterly return on the market. Figure 6 gives some graphical evidence as of the asymmetric role of leverage. The two scatter show MES as a function of log size and log leverage in June 2007 and September 2008. These two quarters had, respectively, a positive and negative market return over the quarter. It is evident from the scatters that the slope of MES with respect to leverage is much higher in quarters with negative market returns.

We are going to use the same regression specification for the three variables of interest. We specify a mixed linear regression, which, in the MES case is defined as

$$\begin{aligned}
\mathsf{mes}_{i\,j} &= \tilde{c}_i + c_0 + c_1' Y_j + c_2' G_i \\
&\quad + c_3 \log(\mathsf{size})_{i\,j} \\
&\quad + c_4 \log(\mathsf{lvg})_{i\,j} + c_5 \log(\mathsf{lvg})_{i\,j} \times I_j^- \\
&\quad + u_{it}
\end{aligned}$$

where I_j^- is a dummy variable which is one when the quarterly return of the market is negative, $\tilde{c}_i \sim N(0, \sigma_c^2)$ is a Gaussian random effect, c_k, $k = 1, ..., 5$ are fixed effetcs and u_{it} is a Gaussian error term.

Table 5 reports the estimation regression of the systemic risk, volatility and correlation

	GARCH/DCC/NP		HIS	
	1995-2007	2007-2009	1995-2007	2007-2009
RMSE $\overline{L}$	1.02	2.80	1.08	4.06
Rel. Bias $\overline{L}$	6%	9%	4%	82%
Rank Cor.	0.36	0.44	0.34	0.36

Table 6: 1 day ahead forecasting performance comparison.

panels. The industry group dummy variable is significant for all groups, in line with the visual evidence of the dynamic plots of the previous section. The impact of size on MES is positive implying that bigger firm are more systemically risk. Leverage has a strongly significant positive impact on MES and its marginal effect increases by 2/3 when the market return is negative. The second column of the table shows the estimation results on volatility. Again, the differences in volatility levels between groups are strongly significant, with Broker-Dealers and the Other groups being, respectively, 12% and 9% more volatile than the baseline Depositary institutions. The role of size on volatility is significantly negative, in line with the intuition that, *ceteris paribus*, bigger financial institutions are less risky. Finally, leverage is strongly significantly and asymmetric, its impact more than doubles in negative quarters. Lastly, correlation results in the third column highlight that only Insurance and Broker-Dealers have significantly different level of correlation with the market, with the latter group being being more correlated by approximately 10%. The impact of size on correlation is positive, the bigger the institution the large its weight in the economy hence the higher the correlation. Leverage has a significant positive impact on correlation too, but there is negligible evidence of asymmetric effects.

6 MES Prediction and Evaluation

Finally, we engage in a MES forecasting exercise. Our goal is to forecast (compound) MES from a day to a month ahead and to develop MES prediction evaluation metrics. We perform two out–of–sample of exercises: 1-step ahead and multi–step ahead. In the 1–step ahead case we compare the forecasting ability of our model against the historical MES estimator of Equation (1), and use novel MES evaluation metrics to compare forecasting performance. In the multi–step ahead case we do not have an obvious benchmark to compare our predictions with and we will only describe how much the multi–step predictions differ from the 1–step ahead ones.

In the one–day ahead case, forecasts can readily be obtained from Equation (2) using 1–step ahead predictions of volatility and correlation together the conditional tail estimators based past data. The systemic event threshold value is C =-2%, as in the analysis of the previous sections. The multi–step forecasts cannot be obtained in closed form and are computed using simulation based methods. The procedure is designed as follows: on day $t-1$

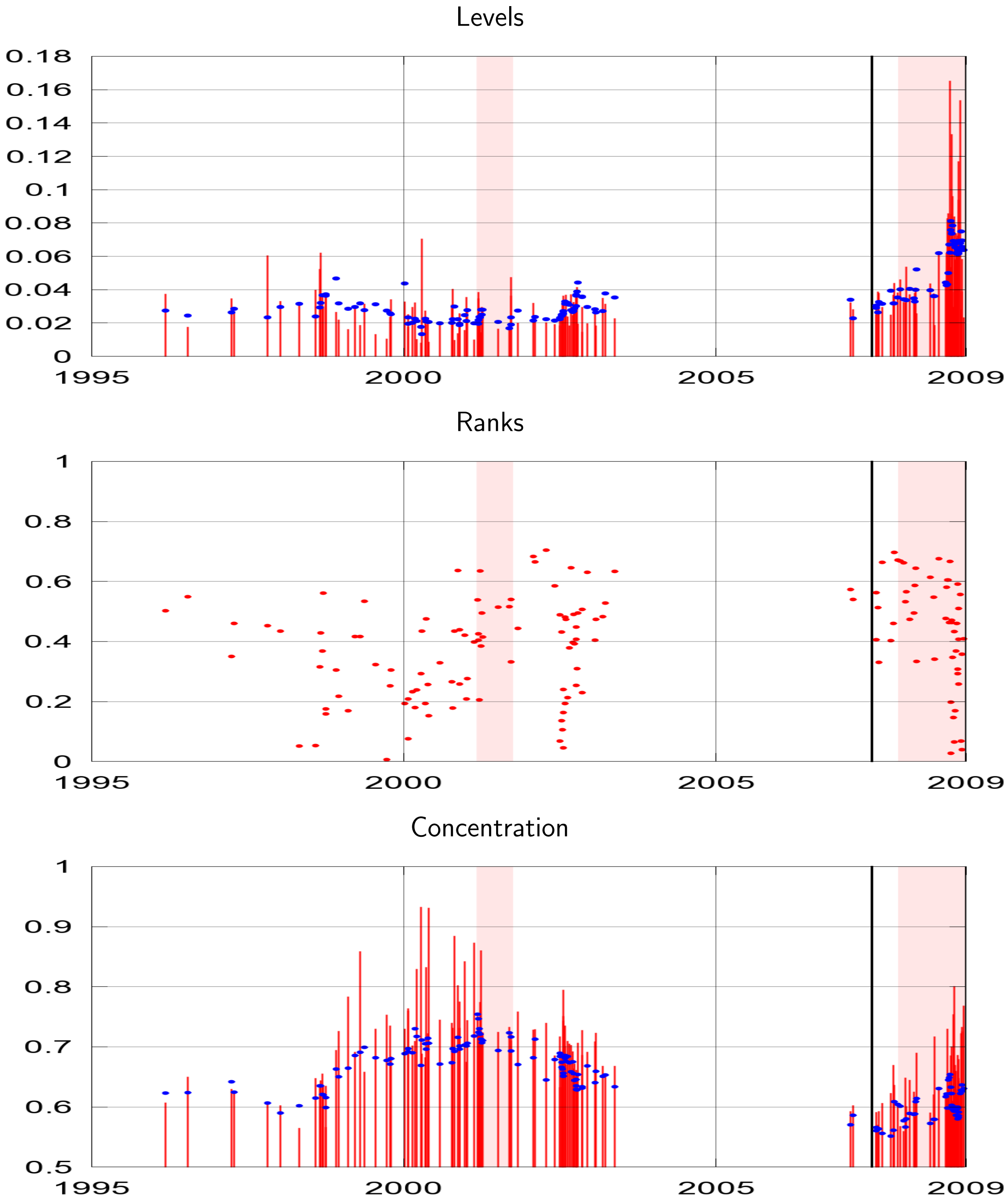

Figure 7: 1 day ahead forecasting performance 1995-2008.

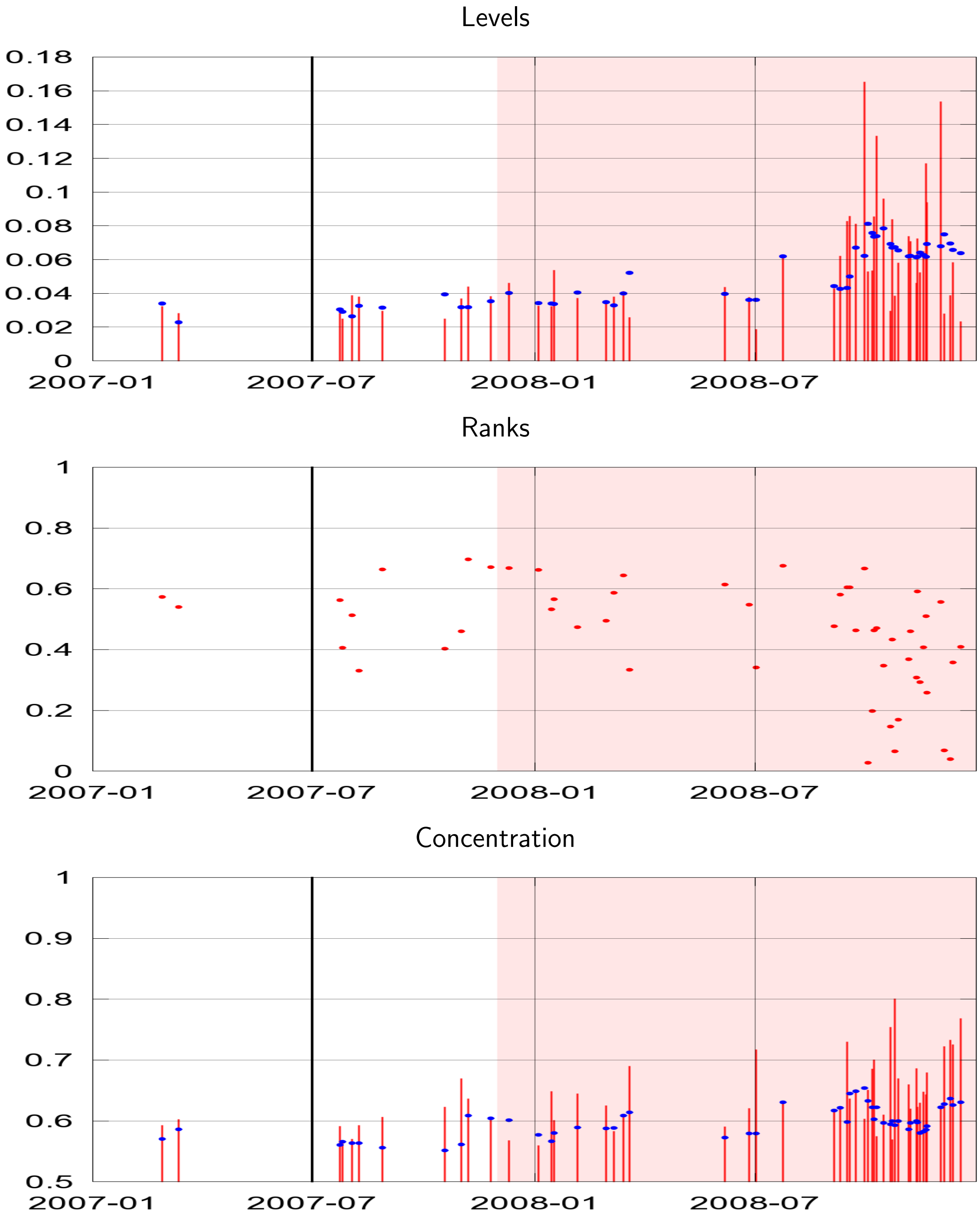

Figure 8: 1 day ahead forecasting performance 2007-2008.

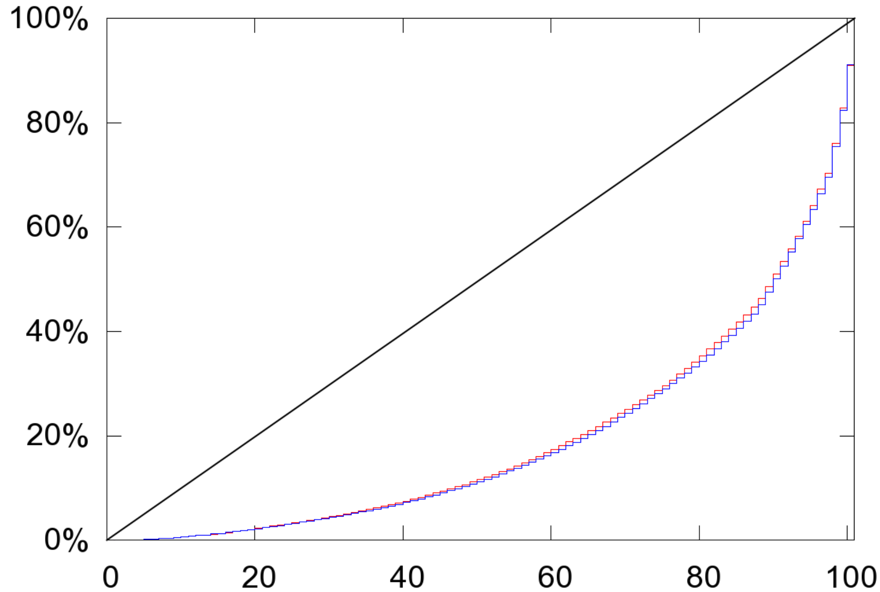

Figure 9: Predicted and actual 1 day ahead loss concentration in the financial crisis.

we simulate a path of length h for $s = 1, ..., S$

$$\left\{ \begin{array}{c} r^s_{m\,t+\tau-1} \\ r^s_{i\,t+\tau-1} \end{array} \right\}_{\tau=1}^{h}$$

drawing values from the estimated GARCH/DCC model with current levels of volatility and correlations as starting conditions. The innovations process used in the simulations is the empirical distribution of standardised system innovations up to time $t-1$:

$$(\hat{\epsilon}_{m\,j}, \hat{\xi}_{m\,j})_{j=1}^{t-1}.$$

This procedure is replicated for each t in the sample to obtain approximate multiperiod MES as

$$\mathsf{MES}^h_{i\,t-1}(C_h) \approx \frac{\sum_{s=1}^{S} r^s_{i\,t:t+h-1} I\{r^s_{m\,t:t+h-1} < C_h\}}{\sum_{s=1}^{S} I\{r^s_{m\,t:t+h-1} < C_h\}},$$

where $r^s_{i\,t:t+h-1}$ denotes the simulated cumulative return of firm i from period t to period $t+h-1$. The multi–step ahead horizon we consider are 1 week (5 days) and 1 month (22 days). The systemic loss level C_h are set to -4%, -7% respectively, which roughly correspond to the 5% quantile of the unconditional market return distribution at the same horizon.

We propose to evaluate forecasts by comparing the predicted MES with the realised losses on event days (or periods). That is, in the one–period case we compare

$$\mathsf{MES}_{i\,t-1}(C_1) \quad \text{and} \quad L^C_{i\,t} = -r_{i\,t} I\{r_{i\,t} < C_1\},$$

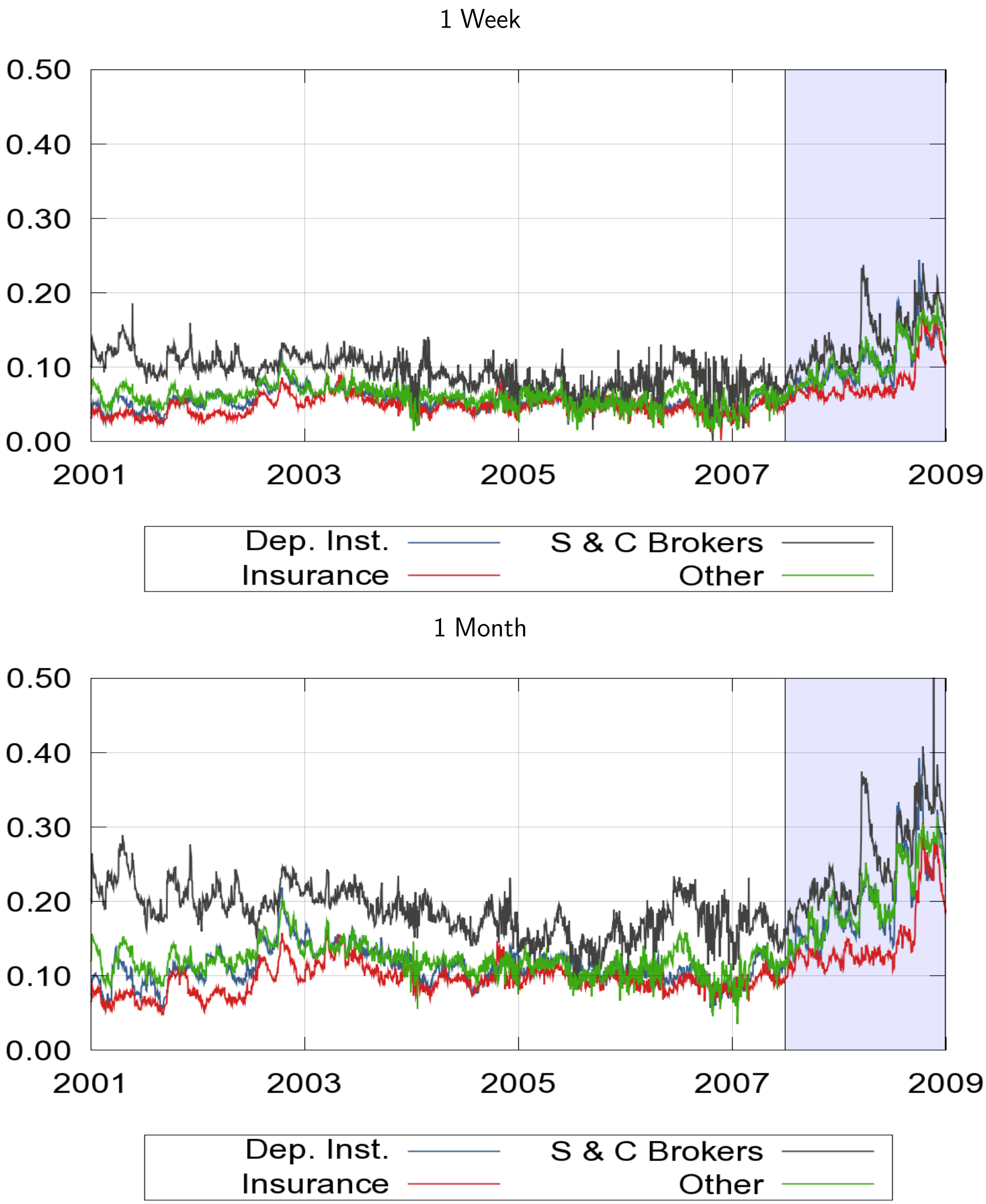

Figure 10: 1 day ahead forecasting performance 1995-2008.

and in the h–period case ahead we have

$$\mathsf{MES}^h_{i\,t-1}(C_h) \quad \text{and} \quad L^C_{i\,t:t+h-1} = -r_{i\,t:t+h-1} I\{r_{i\,t:t+h-1} < 0\}.$$

We summarise forecasting ability along three dimensions: aggregate level, ranking and concentration of losses. The first prediction metric measures the dispersion of aggregate loss level forecasts. Let

$$\hat{\bar{L}}^C_{t:t+h-1} = \sum_i \mathsf{MES}^h_{i\,t-1}(C_h) \quad \text{and} \quad \bar{L}^C_{t:t+h-1} = \sum_i L^C_{i\,t:t+h-1}$$

denote the predicted and actual aggregate loss levels. We compute the Root Mean Square Error (RMSE) and Relative Bias (RB) of the predictions as

$$RMSE = \sqrt{\sum \left(\hat{\bar{L}}^C_{t:t+h-1} - \bar{L}^C_{t:t+h-1} \right)^2},$$

and

$$RB = \sum \left(\hat{\bar{L}}^C_{t:t+h-1} / \bar{L}^C_{t:t+h-1} \right).$$

In the context of systemic risk analysis it is of interest to also compute the relative Bias of predictions to measure how much aggregate losses are being over or under predicted. The rank correlation between MES prediction and the actual losses is the second evaluation metric we propose. Despite the ability of producing accurate forecasts of the system average losses, MES predictions could be of great interest if able to successfully rank firms according to the severity of the individual loss contributions. The Rank Correlation (RC) is computed as the Pearson correlation coefficient between ranks, that is

$$RC_t = \frac{\sum (y^L_i - \bar{y}^L)(y^{\mathsf{MES}}_i - \bar{y}^{\mathsf{MES}})}{\sqrt{\sum (y^L_i - \bar{y}^L) \sum (y^{\mathsf{MES}}_i - \bar{y}^{\mathsf{MES}})}}$$

where y^L_{it} and y^{MES}_{it} are the loss and MES ranks for firm i. Lastly, we want to compare the predicted and actual level of dispersion of losses in the financial system. We are interested in predicting what is the share of the total losses generated by, say, the top 10% distressed firms, and compare it with the actual one. This can readily be done using the Gini dispersion coefficient

$$G_t = 1 - \frac{2}{I-1} \left(I - \frac{\sum_{i \in I} i L^C_{(i)\,t}}{\sum_{i \in I} L^C_{(i)\,t}} \right) \quad \text{and} \quad \hat{G}_t = 1 - \frac{2}{I-1} \left(I - \frac{\sum_{i \in I} i \mathsf{MES}(C)_{(i)\,t-1}}{\sum_{i \in I} \mathsf{MES}(C)_{(i)\,t-1}} \right)$$

where $L^C_{(i)\,t}$ and $\mathsf{MES}(C)_{(i)\,t}$ denote the series of losses and MES on day t sorted in ascending order.

In the 1–step ahead exercise we consider two different out–of–sample windows: January 1995 to June 2007 and July 2007 to December 2008. Figures 7 and 8 report summary plots of the 1-day ahead MES forecasts. There are 96 event days in between January 1995 and

July 2007 and 50 failure between July 2007 and December 2008. The first picture shows the period January 1995 to December 2008 while the second ones zooms on the last 2 years of the sample. The aggregate losses cluster between 2% and 4%. At the peak of the financial crises after the Lehman's failure the average level of losses shifts up as volatility surges. Rank correlation of losses and MES is dispersed between 0 and 0.80 and appears to be substantially stable in time. The concentration of losses (bottom panel of the figures) has been quite high (always above 55%), and it has been well tracked by the MES predictions. Interestingly, it has been much higher around the recession of the early 2000s rather than during the financial crisis. Table 6 reports summary statistics of the 1-day ahead MES forecasts against the historical ones. From a RMSE perspective the dynamic approach always performs better. And even if predictive power deteriorates sensibly in the crisis, the deterioration is greater for the latter approach. The relative bias of prediction is positive, indicating that realized losses have always been larger than the predicted ones and the bias is larger in the financial crisis. However, while the bias of the dynamic approach is moderate, the historical approach severely undershoots the level of losses, with a relative bias of 82%. Also in terms of loss rankings, the dynamic approach does systematically better than the historical one and average rank correlation actually increases in the crisis.

Figure 10 reports the time series plots of multi–step ahead simulation based predictions. The top panel displays the group averages of the 1 week MES predictions while the bottom one the 1 month one. The time series profile of the forecast is essentialy the same one of the 1-step ahead ones; however, as the horizon increases, the distance between the average predictions across groups becomes wider. As the plots suggest, the correlation with 1-step ahead forecast is quite high, 0.67 and 0.59 for the 1 week and 1 month predictions respectively. Also note that the simulation based estimator becomes more noisy in the low volatility periods. When volatility is low the systemic event $r^s_{m\,t:t+h-1} < C_h$ becomes more rare, and keeping the number of simulation fixed for each t, the set of simulated paths that satisfy the condition becomes smaller.

7 Conclusion

The 2007/2009 financial crisis calls for a better understanding of systemic risk. The systemic risk measure we focus on in this work is the recently proposed MES, which is a function of volatility, correlation and tail expectations. We develop ways to estimate and predict MES using some of the modeling tools of the financial econometrics toolbox (GARCH and DCC) together with nonparametric tail expectation estimators. We engage in a systemic risk analysis of top U.S. financial firms between 1990 and 2008. Results shows that despite the fact that the levels of MES reached in the Fall of 2008 are rather extreme by historical standards, the ranking of systemically risky industry groups has been substantially stable over time. Among other findings, cross sectional regressions highlight the role of leverage: firms with high leverage have a higher level of MES. Moreover the impact of leverage differs depending on whether the market grows or declines, and is more severe in the latter rather than the former state. The analysis shows that the most systematically risky sectors are the Broker-

Dealers and the Other groups, which indeed do contain many of the companies more severely hit by the crisis. MES analysis provides useful tools for monitoring systemic risk and, in retrospective, it captures several of the early signs of the crisis.

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