Credit ratings: a combined approach

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Abstract In this contribution we propose to estimate the probability of financial default of companies and the correlated rating classes, using efficiently the information contained in different databases. In this respect, we propose a novel approach, based on the recursive usage of Bayes theorem, that can be very helpful in integrating default estimates obtained from different sets of covariates. Our approach is ordinal: on one hand, the default response variable is binary; on the other hand, covariates that induce partitioning of companies are measured on an ordinal scale. We use our approach not only in a Bayesian variable averaging perspective but also to binarize ordinal variables in the most predictive way. The method is based on a mixture of Binomial and Beta random variables since we model the proportions of default companies in each level of the covariate as independent Binomials with a Beta prior distribution. The application of our proposal to an Italian credit risk database shows that it performs quite efficiently, allowing to predict for each company the probability of default by averaging the covariates contribute.

Key words: credit risk, Bayesian variable averaging, rating classes

1 Background

The financial meltdown of 2008-2009 questioned the validity of risk models and their practical implications. Within the framework of existing regulatory models, Basel II and III, banks have a tendency to uniform their models of risk evaluation

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and enterprise funding generating a pro-cyclical approach that in the current economic and financial environment highlights and exacerbates the difficult conditions in which the firms operate. Over the last 25 years, technological advances and information sharing have increased the use of credit scoring in almost all forms of loan origination (Altman and Saunders, 1997). However, the use of credit scoring is not without limitations (Mester 1997). For example, origination credit scores cannot account for institutional factors such as local economic conditions or business cycle peaks and troughs (Avery et al. 2004). Theoretical studies have demonstrated the importance of information sharing in mitigating the problems of adverse selection (Jappelli and Pagano, 1993) and moral hazard (Padilla and Pagano, 2000). However, the high dimensional data available from public financial statements make credit analysis difficult, and the problems are exacerbated by the necessity to account jointly for qualitative and quantitative data. In order to improve empirical results, and obtain credit scores that are more predictive and less procyclical, research is needed in the area of ”scoritisation” of ordinal variables and in variable selection, preliminary to the inclusion in a full Bayesian model averaging perspective.

2 Proposal

In this contribution we propose to estimate the probability of financial default of companies and the correlated rating classes, using efficiently the provided information typically contained in several and not homogeneous databases.

We want to classify companies into groups in a supervised way. Such groups, in order to comply with BASEL II requirements, have to be: homogeneous with regard to target variable (default, not default), order preserving and stable with regard to horizon time. In this context we are typically provided with databases of various origin, often not transparent and made of both qualitative and quantitative variables. Our proposal is to build, effective but easy to explain, ordinal rating models integrated by means of Bayes theorem.

Given the available data $\underline{X}$, the model we propose can be essentially described as follows:

$$E(\theta_i \mid \underline{X}, Y) = \sum_{k=1}^{K} E(\theta_j \mid g_k, Y) \cdot p(g_k \mid Y) \tag{1}$$

where g_k is a partition induced by a covariate X_k that classifies each unit i into one and only one level j and Y is the observed response variable that assumes two levels: default or not default. Equation (1) is obtained using a Bayesian model based on a mixture of Binomial and Beta random variables, corresponding respectively to the sample and the a priori distributions of the default frequency.

More precisely, our aim is to predict the target event, default or not default of a company, by averaging the most relevant available variables with the employment of Bayes theorem. We follow a factorial approach: each covariate is dichotomized into two classes that respect the binary nature of the target variable. The best dichotomization and consequently the most relevant variables are chosen by maximizing the marginal likelihoods. We assume the probability of default of each company to be constant within the same level j of the covariate. Assuming that θ_j are independent Beta random variables with parameters α β which implies that $E(\theta_j) = \frac{\alpha}{\alpha+\beta}$ a priori, the posterior distribution of each θ_j is easily shown to be a Beta distribution with parameters $\alpha + d_j$ and $\beta + nd_j$, where d_j and nd_j are respectively the number of defaults and non defaults in each level of the covariate. As a consequence the Bayesian estimate of θ_j is:

$$E(\theta_j \mid Y, g_k) = \frac{\alpha + d_j}{\alpha + \beta + n_j} = \left[\frac{\alpha + \beta}{\alpha + \beta + n_j}\right] \frac{\alpha}{\alpha + \beta} + \left[\frac{n_j}{\alpha + \beta + n_j}\right] \frac{d_j}{n_j} \quad (2)$$

Equation (2) implies that each covariate induces through the associated partition g_k, j estimates of the probability of default that are weighted averages between the prior guess and the sample mean default. In a typical credit scoring application, many partitions could arise from available databases. Equation (1) shows how to combine the corresponding estimates of default through the theorem of total probability.

In Equation (1) the posterior probability of the partition g_k, $p(g_k|\underline{X})$ is proportional to $p(\underline{x}|g_k) * p(g_k)$ where $p(\underline{x}|g_k)$ is the marginal likelihood and $p(g_k)$ can be set a priori, for example according to the uniform distribution: $p(g_k) \propto \frac{1}{S}$ where S is a constant. On the other hand can be shown that the marginal likelihood is equal to:

$$p(y \mid g_k) = \left[\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\right]^J \prod_{j=1}^{J} \frac{\Gamma(\alpha + d_j)\Gamma(\beta + nd_j)}{\Gamma(\alpha + \beta + nd_j)} \quad (3)$$

3 Empirical Evidence

In order to evaluate the performance of the proposed model, we were provided from an Italian bank a database containing a list of variables on a set of corporate companies. The dataset is made up of 1000 companies and 13 variables: a target one describing the occurrence of the default or not default event; the remaining 12 on an ordinal measurement. These 12 variables can be divided in two subsets: the first containing 3 independent external rating evaluations and the second containing 9 items from the internal rating questionnaire. The external ratings, measured on a ordinal scale of 9 levels, are based on three different databases : 'Ai' describing the banking transactions of the involved companies,'Dir' containing macro-economic scenarios and 'Cebi' including balance sheets information. On the internal rating side we have 9 ordinal variables characterized by a 4 levels measurement, Those variables give information either on the historical relation between the company and the bank (if there exists) or on the management and structure of the company itself. In particular:

Q1 is ’Competitive position of the company’; Q2 is ’Ability to change the management board without financial consequences’; Q3 is ’Payment of suppliers’; Q4 is ’Number of clients which the 50% of sales refers to’; Q5 is ’Payment from clients’; Q6 ’Trend of the demand for the goods produced by the company’; Q7 ’Historical relation with the bank’; Q8 ’Financial ability of the board to face adverse economic conditions’; Q9 ’Professional experience of the management’.

As mentioned before, we compute the marginal likelihood for each covariate by varying the dichotomization threshold. Such procedure allows us to choose the covariate configuration with the highest power of prediction for the default event.

In Table 1 we report the marginal likelihoods of the 9 internal rating items (columns) crossed with the dichotomization thresholds (rows). We underline that the covariates Q3, Q4, Q5 and Q7 have been treated with particular attention since on of the modalities of the measurement scale represents the ’not available’ state, thus we removed it from the computation, reducing consequently the possible thresholds.

Table 1 Marginal Likelihoods varying the dichotomization of item covariates (in bold best configuration)

Dichotomization	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9
1 vs 2:4	6.47e-24	5.47e-23				3.96e-23		4.72e-23	5.44e-23
1:2 vs 3:4	**1.05e-23**	**7.30e-23**				**4.84e-23**		**7.31e-23**	**5.91e-23**
1:3 vs 4	6.25e-24	4.02e-23				4.19e-23		3.91e-23	4.92e-23
1 vs 2:3			**8.65e-23**	**9.84e-24**	**7.03e-23**		**2.92e-23**		
1:2 vs 3			5.17e-23	9.83e-24	5.54e-23		1.08e-23		

It clearly appears that for Q1, Q2, Q6, Q8 and Q9 the best configuration is attained putting together modalities 1 and 2, on one side, 3 and 4 on the other one. With regard to Q3, Q4, Q5 and Q7 the best configuration is modality 1 against 2 and 3. The important result from table 1 is that all item variables have comparable likelihoods, in the order of e-23 and the most important are: Q3, Q8, Q2 and Q5.

Similarly in Table 2 we report the marginal likelihoods of the 3 external rating variables crossed with the dichotomization thresholds.

In this case Cebi, Ai and Dir select the configuration in which the modalities from 1 to 6 are put together against modalities from 7 to 9. Those three variables share also the level of importance in the order of e-23. The 12 best marginal likelihoods from table 1 and table 2 (one for each covariate) can now be employed to obtain the posterior probability $p(g_k|\underline{X})$ that will be the covariate weight in equation 1. Table 3 shows such results:

Table 2 Marginal Likelihoods varying the dichotomization of macro-economic covariates

Dichotomization	Cebi	Ai	Dir
1 vs 2:9	8.18e-24	8.85e-24	3.12e-23
1:2 vs 3:9	8.68e-24	1.22e-23	3.13e-23
1:3 vs 4:9	9.02e-24	2.07e-23	3.17e-23
1:4 vs 5:9	1.22e-23	2.64e-23	3.11e-23
1:5 vs 6:9	1.59e-23	3.03e-23	3.11e-23
1:6 vs 7:9	**1.69e-23**	**3.76e-23**	**3.20e-23**
1:7 vs 8:9	8.14e-24	3.56e-23	3.14e-23
1:8 vs 9	7.85e-24	1.57e-23	3.10e-23

Table 3 Marginal likelihoods of the best configuration for the 12 covariates and relative weights

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Cebi	Ai	Dir
1.05e-23	7.30e-23	8.65e-23	9.84e-24	7.03e-23	4.84e-23	2.92e-23	7.31e-23	5.91e-23	1.69e-23	3.76e-23	3.20e-23
2%	13%	16%	2%	13%	9%	5%	13%	11%	3%	7%	6%

4 Conclusions

Our objective was to develop an integration method within credit risk assessment, which typically uses multiple sources of information: balance sheets, assessment questionnaires, bank account flows, rating from credit agencies. We therefore need a structured approach, that fully employs the potential of Bayesian modelling, a natural way to merge different information. We address the problem of integrating several credit scores, by homogenizing the variables level measurement and importance. In particular we binarize the ordinal variables, typically arising from assessment questionnaires and ratings, to be matched with the target variable (default or not default). We have also proposed a novel Bayesian variable averaging procedure that leads to produce efficient estimation of probabilities of default and relative rating classes when dealing with several variables of different nature on a set of companies. Such variables averaging approach will also be able to select the most important covariates and the relative best binarization in terms of predicting the target event: default or non default. Future research development include a thorough predictive performance tests of the model and the extension to longitudinal data. In view of such extensions we may need a methodology able to summarize internal rating variables into summary indicators. One possibility is to follow what recently proposed by Cerchiello et al (2010) that suggests to employ stochastic dominance and quantile-based indicators. A further need in modelling of microeconomic credit risk data is to take into account interdependencies between risk variables, and their causal factors; one possibility is to employ bayesian network modelling as suggested in Cornalba and Giudici (2004) or Bonafede et al.(2007) or, alternatively, Bayesian graphical models as suggested in Giudici (2001).

Finally we remark that what proposed here can be applied to other assessment context such as quality and reputational risk as introduced in Cerchiello (2012).

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