

Interest-rate spread and public-debt dynamics in a two-country monetary-union portfolio model*

Roberto Tamborini

Department of Economics

University of Trento

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Abstract

Tracking, monitoring and steering the evolution of public debt over time will be a major policy challenge for almost all governments in the advanced countries in the years to come, in particular for those belonging to the European Monetary Union. The most common technique used to these purposes, the well-known dynamic equation of public debt, is based on some (over)simplifying assumptions, one of which is that the interest rate paid on the outstanding debt is exogenous to the evolution of debt itself. This contrasts with financial theory as well as statistical analyses of the ongoing explosion of intra-EMU interest-rate spreads since 2009. In this paper I study public debt dynamics in a two-country monetary union where a representative, risk-averse wealth-owner optimizes his/her portfolio of sovereign bonds issued in the common currency. Two are the main results with respect to the standard one-country equation. First, the higher-debt country pays a risk premium which is proportional to the level of its own debt with respect to the debt of the other. Second, its debt dynamic path becomes non-linear and dependent on the evolution of the other country's debt. Some relevant policy implications follow.

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1. Introduction

Tracking, monitoring and steering the evolution of public debt over time will be a major policy challenge for almost all governments in the advanced countries in the years to come. The members of the European Monetary Union (EMU) will also be committed to the new "corrective part" of the Stability and Growth Pact (SGP) stating that each year member states should submit a Stability and Convergence Plan (SCP), a key component of which is debt convergence towards a defined target; member states in excess of the 60% debt/GDP ratio in the previous three years should reduce it at a pace defined in 1/20th of the excess per year (European Commission, IP/10/1199).

The most common technique used to these purposes, the well-known dynamic equation of public debt (EC (2011), pp. 10-11), is based on some (over)simplifying assumptions, one of which is that the interest rate paid on the outstanding debt is exogenous to the evolution of debt itself. For instance, basic notions of portfolio analysis indicate that the interest rate should be commensurate to the underlying debt stock, and also that the risk premium demanded for holding each debt stock depends on its relative level with respect to alternative sovereign debts. Observational evidence as well as statistical analyses of the ongoing explosion of intra-EMU interest-rate spreads since 2009 indicate that both the level of each country's debt and its distance from that of Germany appear among the determinants of the spreads, albeit in conjunction with other factors (see e.g. Manganelli and Wolswijk (2009), Sgherri and Zoli (2009), Attinasi et al. (2009), Caceres et al. (2010), Favero and Missale (2011)). As a consequence, two main complications may arise, one is the *endogeneity* between each country's interest rate and its own outstanding debt, the other is the *interdependence* of each country's interest rate with the evolution of all other countries' debts.

In this paper I present an extension of the standard dynamic equation of public debt that seeks to take into account both phenomena. To this effect, I consider a two-country monetary union where a representative, risk-averse wealth-owner optimizes his/her portfolio of the two-countries' sovereign bonds issued in the common currency. Hence, the only portfolio risk component comes from sovereign risk (i.e. the variance of the returns to the two bonds). Two are the main results with respect to the standard one-

country equation. First, the higher-debt country pays a risk premium which is proportional to the level of its own debt with respect to the debt of the other. Second, its debt dynamic path becomes non-linear and dependent on the evolution of the other country's debt.

Admittedly, this model rests on quite a streamlined treatment of portfolio choices, since we know that country-risk assessment has been an intermittent activity in the EMU (and elsewhere), and that risk premia are driven by complex, time-varying combinations of different factors, some of which may be consistent with financial fundamentals while others may be not (Favero and Missale (2011)). If the model is still too simple for empirical purposes, it nonetheless may shed light on critical factors for better design and future implementation of SCPs. Two policy implications are worth stressing. One is that non-linearity of debt paths generates acceleration of divergence from debt targets, and consequently a higher "fiscal effort" (the ratio of the government's primary balance to GDP) in order to keep debt on track. Another is that it may be harmful that SCPs are engineered as single-country "homeworks" without taking into account the interdependencies created by endogenous risk premia.

The paper begins with a brief treatment of the basic single-country model of debt dynamics in section 2. Next, section 3 opens with an overview of empirical evidence on the relationships between debt stocks and interest-rate spreads in the ongoing crisis of euro sovereign debts, and then the two-country model of portfolio selection and debt dynamics is presented and discussed. Section 4 concludes.

2. The basic single-country model of public debt dynamics

The basic model tracks the evolution of public debt of a single country in isolation by means of two equations

$$(1) \quad D_t = D_{t-1} - B_t$$

$$(2) \quad B_t = B'_t - i_t D_{t-1}$$

where D_{t-1} is the outstanding public debt, B_t is the current budget balance, B'_t is the current primary balance, and i_t is the current interest rate paid on the outstanding debt. All variables are expressed in nominal terms.

These expressions are easily converted into GDP ratios, obtaining

$$(3) \quad d_t = \frac{1+i_t}{1+n_t} d_{t-1} - b'_t$$

where small-case letters for fiscal variables denote GDP ratios, and n_t is the current nominal growth rate of GDP¹ (unless otherwise stated, I will refer to the fiscal variables as GDP ratios). This equation says that the debt d_t grows over time owing to two factors: (i) an interest rate greater than the nominal growth rate, $i_t > n_t$, and/or (ii) primary deficits, $b'_t < 0$. Here, i_t and n_t are taken as exogenous variables. To focus on the key issues, let us first assume that i and n are constant.

The first issue is whether the debt is on an increasing or decreasing path. To this effect, let us take the first difference of debt, $\Delta d_t \equiv d_t - d_{t-1}$. Hence,

$$(4) \quad \Delta d_t = \frac{i-n}{1+n} d_{t-1} - b'_t$$

Therefore, the primary budget has to be regarded as the policy control variable of debt dynamics. If Δd^*_t is the debt-*change* target, then

$$(5) \quad b'(\Delta d^*_t) = \frac{i-n}{1+n} d_{t-1} - \Delta d^*_t$$

As can be seen, the choice of the path of the primary budget depends on the interplay of three variables: i , n , Δd^*_t . For instance, a debt reduction plan ($\Delta d^*_t < 0$) calls for a primary surplus, also known as "fiscal effort"; its extent is magnified if $i > n$ (or it is mitigated, or even reversed, otherwise).

A second relevant issue is the primary budget associated with a given debt-*level* target d^* . According to equation (5), this occurs when

$$(6) \quad b'(d^*) = \frac{i-n}{1+n} d^*$$

Hence, the budget target should be in surplus if $i > n$ (it can be in deficit otherwise), and its absolute value should be proportional to the debt target.

Finally, a less discussed issue is whether the debt target is stable or not. This depends on whether the slope of the debt dynamic path at the initial level d_0 ,

$$(7) \quad \partial d_t / \partial d_{t-1} |_{d_0} = \frac{1+i}{1+n}$$

is greater or smaller than unity. If the debt is on target and $i > n$, d^* is unstable, otherwise it is stable. An unstable target entails that any shock will make the debt diverge from the target; this, however, may occur upwards or downwards depending on the shock. A policy implication of an

¹ An alternative formulation decomposes the nominal growth rate into the real growth rate and the inflation rate. Then, the coefficient of equation (3) results reformulated as the ratio between the (ex-post) real interest rate and the real growth rate. In principle, the two formulations are equivalent.

unstable target is that the government is expected to be able to control the primary budget with great precision and rapidity. Figure 1 exemplifies debt dynamics with $i > n$. The line corresponding to the initial primary budget $b'_0 = 0$ entails that the initial debt d_0 is on a divergent path from the target value d^* . The lower line is identified by the primary surplus $b'(d^*)$ consistent with the debt target d^* . Note that the debt target is unstable.

[Figure 1]

3. The role of interest-rate spreads

In spite of its widespread use as a policy tool even by the EMU institutions (EC (2011)), the basic debt dynamic equation expounded above suffers from serious limitations, especially for countries belonging to a highly integrated area like the EMU. In this section I will focus on one of the most patent problems that the ongoing euro sovereign debt crisis has brought to the forefront, namely the fact that the interest rate paid on debt is not exogenous with respect to the dynamic process, as suggested by the explosion of intra-EMU spreads since 2009. For instance, standard portfolio theory explains why each sovereign debt should in fact command a different interest rate according to its stock dimension. I will begin with a cursory view of some empirical evidence, and then I will examine how debt dynamics evolve in a two-country setup with endogenous interest-rate spread.

3.1. Some empirical evidence

At the launch of the EMU there was widespread confidence in the role of financial integration and freedom from exchange-rate risks as means to equalize interest rates across member countries. The first decade of the EMU by and large delivered this outcome, whereas the more recent financial turmoil has shown that the so-called "country risks" may still boost substantial interest-rate differentials. Table 1 and Table 2 present some summary statistics about interest-rate spreads and debt stocks. For the sake of comparison over time, only early accession countries (Greece included) are considered (EMU12). A glance at the data suggests the following stylized facts:

- there is clear evidence of a break between the early years of financial tranquillity (2000-2008) and those of the sovereign debt crisis (2009-2011)
- from 2000 to 2008, the average monthly spreads with the 10-year German *Bund* were small and concentrated in small range (the EMU12 average was 0.19% with standard deviation of 0.09)
- as a matter of fact, a (slow) convergence process of fiscal convergence was in place; in 2008 the 62% debt/GDP ratio of the EMU as a whole was close to the Maastricht Treaty target and 0.6 points lower than in 2000, with more marked reductions in the two largest debtors (Belgium -17.8 , Italy -3.4) *vis-à-vis* increases in major low-debt countries (Austria $+3.8$, France $+10.1$, Germany $+6.2$, Ireland $+6.4$, Netherlands $+4.4$); the standard deviation was reduced from 30.2 to 27.1
- these tendencies have substantially been reversed since 2009: in each and all countries the mean value of the spread and its volatility have increased abruptly; for the EMU12 as a whole, the mean value of the spread has jumped to 1.8% with standard deviation of 1.75
- concomitantly, all countries have increased their debt/GDP ratio (the EMU12 average has reached 85.9%), but with larger disparities across countries (the standard deviation has risen from 27.1 to 36.7).

These same facts are confirmed by more formal statistical analyses, such as those by Manganelli and Wolswijk (2009), Sgherri and Zoli (2009), Attinasi et al. (2009), Caceres et al. (2010), Favero and Missale (2011). The reasons of these phenomena are not yet clear, though. In particular, it is not clear whether the surge of spreads is significantly explained by the increase in debt stocks with respect to the previous era, or also by other factors. However, the different levels of debt/GDP ratios consistently appear as one of the determinants, albeit in conjunction with other factors². According to the data in Table 1 each country's change in the average spread between the

² In the pre-crisis years, Wyplosz (2006), drawing on Bayoumi et al. (1995) for US states, shows that up to 2005 quadratic equations of debt/GDP ratios were good predictors, albeit quantitatively modest, of interest-rate spreads. These equations, however, do not include foreign substitutes. Schiavo (2008) offers an extended analysis of the theoretical and empirical issues involved, and further original estimates with various techniques also including foreign substitutes as well as flow variables (i.e. budget deficits and new issuances of bonds). Stocks alone are seldom significant, but in the cases where they are significant, the signs of coefficients are consistent with portfolio theory.

two periods seems roughly consistent with the respective change in the average debt stock (excluded the abnormal case of Greece): see Figure 2..

[Figure 2]

Portfolio theory also indicates that what should matter for the determination of risk premia is the *relative* weight of the different stocks in the market (or "relative supply effect"). Attinasi et al. (2009) and Missale and Favero (2011) find evidence, with some qualifications, of this effect with reference to the debt stock of Germany, regarded as the "safe haven" by investors. Figure 3 highlights this relationship for each country by plotting the change in the average spread from 2000-08 to 2009-2011 against the change in the average difference with Germany's debt stock. Consider that in 2000 only 3 countries had their debt larger than Germany's (Belgium, Greece, Italy); in 2008 these were 4 (France joined the group), but all other countries had reduced their distance or kept it almost unchanged. In 2011, 6 countries recorded debts higher than Germany, but the list does not coincide with the so-called 'PIIGS', for France and Belgium are in the list whereas Spain is not (and Italy's distance with Germany was less in 2011 than in 2008).

[Figure 3]

According to Favero and Missale (2011), however, "the evidence reported shows that there is a relationship between yield spreads and fiscal fundamentals, but this is non-linear. Fiscal fundamentals do not matter *per se* [... they] are important in the determination of the domestic spread as they define the distance between countries and therefore select the reference group relevant to determine the global spread variable" (p. 14).

A balanced judgement may be that each country's own or relative debt stock is not the single determinant of its spread, and that this relative supply effect may be time-varying according to news, "frames", and "conventions". Though the historical experience of the EMU is open to different interpretations, psychology, if not fundamental analysis, suggests that the current sovereign debt turmoil may represent an important change in investors' "frames" and "conventions" with a long lasting shift of focus on country risk assessment. Hence the theoretical analysis that follows may be valuable in order to highlight the role of relative supply effects in the evolution of sovereign debts in the EMU.

3.2. The model

In this paragraph I draw on the basic portfolio diversification model to provide some insights on the issue of the endogenous evolution of interest rate spreads and their consequences on debt dynamics.

The spread

Let us consider a two-country monetary union, with two sovereign debt stocks, D_A , D_B , issued by the national governments A and B , and the union's representative wealth owner who, for any period t , can hold the two stocks under the wealth constraint $D_{At} + D_{Bt} = W_t$. To normalize the value of wealth, we can re-express this constraint in terms of the wealth/income ratio, where the income of the representative wealth-owner is the average nominal GDP of the two countries. These are assumed to be equal, $YN_{At} = YN_{Bt} = YN_t$, so that YN_t is also the income of the representative wealth-owner. Dividing the whole wealth constraint by YN , and using our previous notational convention, $w_t \equiv W_t/YN_t$, $d_{mt} \equiv D_{mt}/YN_t$, $m = A, B$, we obtain

$$(8) \quad d_{At} + d_{Bt} = w_t$$

The expected return rates to the two bonds, conditional on information available at t , are normally and independently distributed with mean, respectively, i_{At+1} , i_{Bt+1} , variance σ_A^2 , σ_B^2 , and covariance σ_{AB} . The demand for bonds of the representative wealth owner is the result of his/her maximization of an exponential utility function of wealth, with constant absolute risk aversion $\rho \in [0,1]$, under the constraint (8). As is well known, under these conditions the Lagrange maximand function results to be

$$(9) \quad i_{At+1} d_{At} + i_{Bt+1} d_{Bt} - (\rho/2)(\sigma_A^2 d_{At}^2 + \sigma_B^2 d_{Bt}^2 + 2\sigma_{AB} d_{At} d_{Bt}) + \lambda(d_{At} + d_{Bt} - w_t)$$

The demand functions for the two stocks are:

$$(10) \quad d_{At} = \frac{1}{\alpha + \beta} (i_{At+1} - i_{Bt+1}) + \frac{\beta}{\alpha + \beta} w_t$$

$$(11) \quad d_{Bt} = -\frac{1}{\alpha + \beta} (i_{At+1} - i_{Bt+1}) + \frac{\alpha}{\alpha + \beta} w_t$$

with $\alpha = \rho(\sigma_A^2 - \sigma_{AB})$, $\beta = \rho(\sigma_B^2 - \sigma_{AB})$.

As is well known, the basic portfolio model for N risky assets determines $N-1$ independent interest rates. For simplicity, and with transparent analogy with the EMU, I assume that one country's debt, namely B , provides of the N -th interest rate, or "the anchor" of the interest rate array. Again with transparent analogy to the EMU, one may think of B as the

"virtuous" country with low i_B and β ; the representative investor considers B 's debt an almost risk-free asset, and i_B is kept close to the interest rate directly controlled by the central bank. Consequently, the interest rate spread observed in period t must be consistent with the demand functions of the outstanding debt stocks held from $t-1$, that is

$$(12) \quad i_{At} - i_{Bt} = \alpha d_{At-1} - \beta d_{Bt-1}$$

Hence a positive spread on sovereign debt A may develop according to combinations of: (i) the sign and size of *the risk factors* α and β , (ii) the stocks of outstanding debt d_{At-1} , d_{Bt-1} , i.e. *the relative supply effect*³. Note that it is not necessarily true that a positive spread reflects a larger debt stock. If $\alpha > 0$, $\beta < 0$ ($\sigma_A^2 > \sigma_{AB}$, $\sigma_B^2 < \sigma_{AB}$) the spread is positive independently of the relative supply effect, which only determines the size of the spread. In this case it is the higher riskiness of A 's debt ($\sigma_A^2 > \sigma_B^2$) that dominates. The relative supply effect is necessary to obtain a positive spread when $\alpha > 0$, $\beta > 0$, and $\beta/\alpha \geq 1$, so that it must be $d_{At-1}/d_{Bt-1} > \beta/\alpha$. Clearly, $\sigma_A^2 > \sigma_B^2$ is still a possible case, but also the reverse is (β/α is greater than 1 but smaller than the debt ratio). In the latter case, the relative supply effect of A 's debt dominates the relative riskiness of B 's debt.

An interesting role is played by the covariance σ_{AB} . In general, a positive covariance may not be unusual for assets belonging to the same class such as government bonds which, in a monetary union, reflect a number of common shocks. If $\sigma_{AB} > 0$ the two assets are (imperfect) substitutes in the portfolio, but the signs of α and β are undefined and so are the effects on the spread. An interesting case may be one where B 's debt has lower risk, $\sigma_A^2 > \sigma_B^2$, and also $\sigma_B^2 < \sigma_{AB}$. Hence, $\alpha > 0$, $\beta < 0$, and we have seen that the spread is the *weighted sum* of the two stocks. A 's spread is magnified by its own relative supply but a low-debt union fellow is beneficial (positive externality). On the other hand, a negative covariance may develop in the presence of asymmetric shocks to public finances or to portfolio dislocations such as "flight to quality". If $\sigma_{AB} < 0$ the two assets become strong complements in the portfolio, and α and β are positive. As a consequence of complementarity, A 's spread is the *weighted difference* of the two stocks, and

³ Favero and Missale (2011), insert the difference of debt/GDP ratios into their estimation equations of interest-rate spreads with respect to Germany, but they apply a single coefficient to it. This model indicates that each debt stock should have a specific coefficient.

hence it is smaller than in the previous case⁴, but on the other hand now it is larger the larger is *A*'s debt and the smaller is *B*'s debt, or the low-debt union fellow exerts a negative externality.

Before proceeding it may be instructive to look at some data about the risk indicators discussed above. These, except the risk-aversion coefficient ρ , are reported in Table 3. As in previous analysis, the 2000-08 period is compared with the 2009-2011 period. The following list of noteworthy facts can be considered.

1) The 'PIIGS' group usually identified as high-debt/high-risk countries, actually display larger variances in the crisis with respect to the tranquillity period.

2) In the tranquillity period all covariances with Germany were positive and sizeable, denoting a regime of (close) substitutability; given $\rho > 0$, all α 's were positive and all β 's negative, corresponding to the previously discussed case of unconditional positive spreads but limited by the small scale of the risk parameters and by the positive externality of the lowest debt country. Overall, these data are consistent with the well-known story of the "euro dividend" enjoyed by high-debt countries in terms of lower interest rates.

3) The crisis period presents itself as a sudden regime shift. The covariances with Germany of all the 'PIIGS' have turned to negative, probably as a result of a "flight to quality" effect. All the α 's and β 's have become larger and positive leading to the previously discussed case where larger spreads are driven by relative supply effects and the negative externality of the lowest debt country.

As already noted above, this pure mean-variance approach to portfolio choices alone is not sufficient to explain in detail and quantitative precision the EMU sovereign debt crises. In particular, it misses the shift from ordinary financial risk as measured by historical variances to default risk, which is a unique prospective event, or other events that may impinge upon the risk attitude of investors. Nonetheless, the baseline spread equation (12) to some extent seems nested in the data, so that it may be used to provide guidance to better understanding debt dynamics when interest-rate spreads are endogenous⁵.

⁴ In fact complementarity means that the demand for the riskier asset is higher because its negative covariance reduces the overall portfolio risk.

⁵ I do not attempt at quantifying risk-aversion, first because it is not strictly necessary in the present context, and second because it is notoriously unreliable

Debt dynamics

We can now complete our two-country model. Since the two countries have the same nominal GDP, and face the same inflation rate determined by the union's central bank, we also let them grow at the same nominal rate n . This is a simplifying assumption that insulates the single role of the endogenous interest-rate spread in the debt dynamics with respect to other factors⁶.

We can now substitute equation (12) into the debt equation (3) for country A obtaining the following system of dynamic equations:

$$(13) \quad \begin{aligned} \text{A) } d_{At} &= \frac{\alpha}{1+n} d_{At-1}^2 + \frac{1+i_{Bt} - \beta d_{Bt-1}}{1+n} d_{At-1} - b'_{At} \\ \text{B) } d_{Bt} &= \frac{1+i_{Bt}}{1+n} d_{Bt-1} - b'_{Bt} \end{aligned}$$

In the discussion of this system, I will assume that $\alpha > 0$, $\beta > 0$ since this seems to characterize the present regime in the EMU (see previous paragraph). Accordingly, this system has the following noteworthy features, in particular for A .

and questionable. However, it may be interesting to gauge the dimension of this parameter as implied by the model equation (12) given the observed spreads of each member country m . Let us define

$$\rho'_t = (i_{mt} - i_{GERt}) / [(\sigma_m^2 - \sigma_{m,GER})d_{mt-1} - (\sigma_{GER}^2 - \sigma_{m,GER})d_{GERt-1}].$$

Note that this can also be seen as an indirect test of the extent to which equation (12) fits the observed spreads. Theoretically, the true ρ should be small (typically a few centesimal points or less) and time and space invariant. Substantial deviations from this benchmark would indicate that the actual spreads "contain" other factors unaccounted for by the model. The actual ρ' for each country and sub-period is deployed in the last column of Table 3.

In the tranquillity period, the ρ' were in fact in the order of magnitude of centesimal points (except Italy and Luxembourg), and also surprisingly similar across countries. In the crisis period the picture is quite different. Three countries (Austria, Belgium and France) have $\rho' < 0$, which indicates that their actual spread has the wrong sign. Two other low-debt countries (Finland and Netherlands) have a larger ρ' than before, which indicates that their spreads have grown abnormally. By contrast the 'PIIGS', except Spain, have a much lower ρ' than before indicating the opposite. This finding suggests that a change in risk aversion, or other "market sentiment" factors, may have occurred in the crisis period.

⁶ For a model of debt dynamics with different growth rates see Tamborini (2011).

1) Because of the dependence of the interest rate on the evolution of debt, the debt equation is no longer linear. Specifically, it is a quadratic function. Taking i_B , n , d_B as constants, equation (13)-A draws the positive arm of a concave quadratic map. There is a whole family of curves corresponding to the primary balance chosen by the government, which is measured as the intercept of the curve along the vertical axis. Figure 4 exemplifies the case when a constant $b'_A = 0$ is given. In this case, $b'_A = 0$ implies that any $d_t > d^*$ is diverging from d^* . Note that the curve is drawn keeping d_B constant; this condition will be removed below.

[Figure 4]

2) Debt dynamics now depends not only on the domestic policy instrument b'_{At} , but also on the other country's debt and interest rate. Note that with $\beta > 0$, this interdependence with a low-debt union fellow works in two opposite directions. On the one hand, a low debt stock of B has a negative externality in that A 's *interest-rate spread is higher*, which accelerates its debt dynamics. On the other hand, a low interest rate i_B has a positive externality in that the *absolute level of all interest rates is lower*, which decelerates A 's debt dynamics.

3) The debt dynamic equation of the basic model (3) also applies to individual countries in the two-country context only if the two countries are financially identical. In fact it is easily verified that equations (13) boils down to (3) if $d_{At-1} = d_{Bt-1}$, $\sigma^2_A = \sigma^2_B = \sigma^2$, $\sigma_{AB} = 1$, $\alpha = \beta$, $i_{Bt} = i_{At} = i_t$.

Before turning to some policy implications, it is worth stressing that the non-linear debt dynamics discussed above may shed light on self-sustained phenomena of fast acceleration of debt/GDP ratios and yawning of spreads that are common in sovereign debt crises and have also been observed in the recent EMU events concerning Ireland, Spain or Italy. This model suggests that this phenomena may not only be driven by abnormal changes in risk aversion or in market sentiments, but also by particular combinations of normal market forces.

4. Policy implications

Now we can examine how the present reformulation of the debt motion law modifies the policy indications drawn from the basic model. Various aspects can be investigated. We may begin with the provision of the new SGP that all countries exceeding the 60% debt/GDP ratio should commit to

a SCP to be accomplished in a given number of years. This implies that both countries aim at the same debt target given by the SGP rule, $d^*_A = d^*_B = d^* = 60\%$.

As discussed previously, the SCP is a meaningful requirement if the initial ratio exceeds 60%, $d_{A0}, d_{B0} > d^*$, and it is on a divergent path. To check this condition we should examine the slope of the debt path at the initial level and for the trend values of i_B and n , that is,

$$(14) \quad \begin{aligned} \text{A) } \partial d_{At} / \partial d_{At-1} | d_{A0} &= \frac{1+i_B}{1+n} + \frac{2\alpha}{1+n} d_{A0} - \frac{\beta}{1+n} d_{B0} \\ \text{B) } \partial d_{Bt} / \partial d_{Bt-1} | d_{B0} &= \frac{1+i_B}{1+n} \end{aligned}$$

Recall that divergence occurs if these expressions exceed unity. Clearly, the result is the same as in the basic model for country B ($i_B > n$), whereas country A now faces two additional conditions. One depends on its own initial debt value d_{A0} and its effect on its interest-rate spread: this initial condition favours divergence. The other consists of the opposite-sign effect of the other country's initial debt value d_{B0} . Consequently, we may say that a country's debt will more likely be set on a divergent path from the 60% target (see Figure 4) if the initial conditions include a high-debt/high risk history and a lower-debt/lower-risk counterparty. It seems that the so-called 'PIIGS' find themselves in this position *vis-à-vis* Germany.

A typical SCP will consist of a year-by-year change in the debt ratio $\Delta d^*_{mt} < 0$, namely $\Delta d^*_{mt} = (d^* - d_{mt})/T$. The policy instrument is the primary balance $b'(\Delta d^*_{mt})$ obtained from equations (13):

$$(15) \quad \begin{aligned} \text{A) } b'(\Delta d^*_{At}) &= \frac{\alpha}{1+n} d_{At-1}^2 + \frac{i_{Bt} - n - \beta d_{Bt-1}}{1+n} d_{At-1} - \Delta d^*_{At} \\ \text{B) } b'(\Delta d^*_{Bt}) &= \frac{i_{Bt} - n}{1+n} d_{Bt-1} - \Delta d^*_{Bt} \end{aligned}$$

These adjustment equations provide the basis for an assessment of the fiscal effort of the convergence plan. As is clear, the two countries are in different positions. Country A faces additional factors: the square of its own outstanding debt, and its dependence on the foreign country's debt at each point in time.

Consider the following numerical example, with $\beta = 0.01$, $\alpha = 0.05$,⁷ and where $i_B = 3\%$, $n = 5\%$ are assumed to remain constant. In the first place we

⁷ These are orders of magnitude consistent with the 2009-11 data in Table 3.

can examine the case where country A starts with debt/GDP at 100% and zero primary balance, whereas B already fulfils the mandatory 60%. The initial interest rate on A 's debt results 7.4%, i.e. a spread of 4.4%. Note that the values for i_B and n design a favourable environment for debt reduction; nonetheless, according to equation (14), A 's debt is on a divergent path. Consequently, A adopts, say, a ten-year SCP. The SCP path of country A is reproduced in Table 4, Case 1. In the first year of the plan, the primary surplus is $b'(\Delta d^*_{A1}) = 6.3\%$. Then both the debt and the primary surplus ratios decrease; as soon as the debt target is reached, the primary balance can be levelled at $b'(d^*_A) = 0.2\%$ of GDP. That is to say, to keep its debt on target, A should sustain a small primary surplus. The spread is reduced, but it remains substantially high at 2.4%. An important message of portfolio theory is contained in this result, namely that history may matter, in the sense that a country with a history of higher risk reflected in $\alpha > \beta$, should be compelled to pay for a sovereign risk premium even when the Promised Land of the SGP is reached.

A second important issue, one which is almost ignored in official documents, is whether different countries moving towards the target simultaneously have a better or worse chance to accomplish the task. The problem is relevant because we have seen that portfolio choices may generate spillovers across the debt dynamic paths of different countries. Hence we now examine the case where both countries start with 100% debt/GDP, zero primary balance, and adopt a ten-year SCP: see Table 4, Case 2. The first observation is that the fact that B has the same initial debt ratio as A entails a smaller spread all along the SCP path. As a consequence, A also faces less fiscal effort as measured by its primary balance. On the other hand, despite the equal initial conditions, B enjoys much less fiscal effort; once on target, it can even afford a small primary deficit. These differences between A and B are entirely due to the risk factors α and β .

This numerical example seems to suggest that simultaneous convergence is not jeopardized. However, this may well be due to the particular choice of figures. A rigorous analysis of the problem would require the investigation of the dynamic properties of the two-equation system (15), but since one equation is non-linear there may be no analytical solution. On the other hand, the present setup is simplified in that there is only a one-way spillover from country B to A , therefore the key problem is how fast B 's debt

falls with respect to A 's. In principle, if B 's debt falls faster than A 's, the interest rate spread for A may increase instead of decreasing. If this happens, A 's primary surplus should also increase, instead of decreasing, along the convergence plan in a sort of Sisyphus Fatigue. Let us see Case 3 in Table 4, where B adopts a faster five-year SCP. As soon as it reaches the 60% debt/GDP target, A 's fiscal effort worsens with respect to Case 2. This is also known as the "laggard problem" that may hurt the risk assessment of countries that lag behind the adjustment pace of others.

A third issue to be examined is whether the assigned SGP target is also stable. This is another neglected problem, perhaps because it may appear too abstract with respect to the practical problems of managing large debt stocks. However, the SGP prescriptions, in particular those relating to the 'S', are ranked high in the institutional design of the EMU, and are pinpointed by authoritative theoretical justifications. But an unstable target is not a particularly appealing normative concept. Probably, discovering that once the 60% debt target has been reached it is unstable is, for any member country, not only a mathematical *curiosum*.

The problem can be split into two parts. First, each government should choose its primary balance such that the 60% debt/GDP ratio is a s-s. This is the solution of equations (15) for $d_{At-1} = d_{Bt-1} = d^*$, $\Delta d^*_{At} = \Delta d^*_{Bt} = 0$, that is,

$$(16) \quad \begin{aligned} \text{A) } b'(d_A^*) &= \frac{\alpha - \beta}{1+n} d^{*2} + \frac{i_B - n}{1+n} d^* \\ \text{B) } b'(d_B^*) &= \frac{i_B - n}{1+n} d^* \end{aligned}$$

In the previous numerical example we have seen that these values are, respectively, 0.2% and -1.1%. Note that a coordination issue is involved here, because government A should safely assume that government B is in fact aiming at d^* . It can now be seen algebraically what was said in the numerical example: the difference in the two countries' s-s primary balance is entirely due to the first addendum in the equation for A , which reflects the determinants of the domestic interest-rate spread. Given n , the low-interest-rate union fellow B grants a relaxing effect on the primary surplus. But also note that $\alpha - \beta = \rho(\sigma_A^2 - \sigma_B^2)$; hence the low-interest-rate fellow is also likely to have $\sigma_B^2 < \sigma_A^2$ exerting an upward pressure on A 's spread and a restrictive effect on its primary balance.

Second, the stability conditions at the debt levels $d^*_A = d^*_B = d^*$, along the respective paths given by $b'(d^*_A)$, $b'(d^*_B)$ are

$$(17) \quad \text{A) } \partial d_{At} / \partial d_{At-1} |_{d^*} = \frac{1+i_B}{1+n} + \frac{2\alpha-\beta}{1+n} d^* < 1$$

$$\text{B) } \partial d_{Bt} / \partial d_{Bt-1} |_{d^*} = \frac{1+i_B}{1+n} < 1$$

The stability condition for B is the same as in the basic model (7) ($i_B < n$), whereas it is more restrictive for A . In fact, it may no longer be sufficient that the nominal growth rate exceeds the interest rate (set by the foreign country). The second term should also be taken into account. Given α , β , stability for A depends on the magnitude of d^* itself, namely

$$(18) \quad \frac{n-i_B}{2\alpha-\beta} > d^*$$

Unlike B , A cannot choose its debt target arbitrarily if it is also to be stable; or otherwise, the 60% debt target may not be stable for all countries. Once more, "one size does not fit all". In the previous numerical example condition (18) does not hold for country A (the left-hand side is $0.22 < 0.6$). To make the 60% debt target stable, country A would need stronger nominal growth or lower interest rate. The consequence is another form of asymmetry between the two countries. Table 5 exemplifies the effects of various types of on-off shocks on A 's debt path, and the changes in its primary surplus that are necessary to keep debt on target. Note that in the first two cases, a procyclical fiscal manoeuvre is necessary.

It might be argued that instability may also be beneficial, since a small *favourable* shock to the debt/GDP ratio sets in motion a progressive fall of the ratio itself with no fiscal effort. Yet this argument is just the other side of the coin that the government has aimed at (or has been directed to) the wrong target for its own country. It is therefore of some interest to examine what our model indicates as the correct s-s debt \bar{d}_m . To this end we should generalize equations (17) as follows:

$$(19) \quad \text{A) } \partial d_{At} / \partial d_{At-1} |_{\bar{d}_A} = \frac{1+i_B-\beta\bar{d}_B}{1+n} + \frac{2\alpha}{1+n} \bar{d}_A < 1$$

$$\text{B) } \partial d_{Bt} / \partial d_{Bt-1} |_{\bar{d}_B} = \frac{1+i_B}{1+n} < 1$$

and therefore,

$$(20) \quad \bar{d}_A < \frac{n-i_B+\beta\bar{d}_B}{2\alpha}$$

The important innovation of the two-country analysis is that, ideally, a specific s-s solution should exist for each country, and it should hold for both simultaneously. Again, A cannot choose its s-s debt arbitrarily: it should be

smaller, the larger is α , and the smaller is \bar{d}_B . Hence, interdependence matters: a country confronted with a low-debt union fellow should aim at a low debt too. If, with the previously given figures, B sticks to the 60% debt target, then A should search for its s-s debt level below 26%. If B cuts its target to 50%, A 's debt should remain below 25%, and so on. The suggestion of this analysis is that, for countries starting from high debt levels and carrying a historically high sovereign risk factor, convergence to their truly s-s debt level may take a very long and painful way well beyond the SGP debt target.

5. Conclusions

Observational as well as statistical evidence concerning the intra-EMU explosion of interest-rate spreads on sovereign debts since 2009 indicate that they respond, *inter alia*, to both the evolution of each country's debt stock and its distance from that of the "safe haven" (i.e. Germany). In this paper I have put forward an analysis of public debt dynamics in a two-country context of portfolio choice which renders interest rates endogenous to the evolution of respective debt stocks.

The main analytical result is that debt dynamics is non linear. *Heterogeneity* of initial conditions and *interdependence* of debt dynamics via interest-rate spread are factors that entail different speed, fiscal efforts, reciprocal spillovers and chances of success of governments' convergence plans towards a common debt target. High-debt/high-risk countries may tend to diverge from low-debt/low-risk union fellows faster than implied by the basic linear model. Likewise, the former type of countries will face harder convergence plans towards the debt target. Moreover, these factors imply that this target may not be a stable steady state for all different countries, with the consequence that even when the Promised Land will be reached, some countries may be easily shocked away from it.

In the EMU context, these conclusions may also have normative implications to be further explored. One is that the almost exclusive focus on new budget rules as a means to regenerate investors' confidence in EMU sovereign debts may turn out to be insufficient against the divergence forces identified in this paper. A related implication is that the "principle of sovereignty", and the country-by-country approach that pinpoint the SGP

from its origin may lead to serious mismanagement of public debts and harmful "fallacies of composition" at the EMU level.

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Annex

Table 1. Statistics of interest-rate spreads with Germany in the EMU12, 2000-2011 (monthly yields on 10-year government bonds, percent)

	2000-2008		2009-2011	
	mean	s.d.	mean	s.d.
Austria	0.162	0.14	0.621	0.21
Belgium	0.196	0.15	0.967	0.45
Finland	0.133	0.12	0.388	0.17
France	0.100	0.07	0.484	0.16
Greece	0.414	0.28	6.802	4.68
Ireland	0.155	0.21	3.918	2.42
Italy	0.312	0.19	1.643	0.79
Luxemb.	0.165	0.27	0.399	0.91
Netherl.	0.096	0.09	0.335	0.39
Portugal	0.234	0.16	3.457	1.27
Spain	0.156	0.14	1.645	0.89
Area average	0.193	0.09	1.80	1.75

Source: ECB, Interest rate statistics, online database

Table 2. Statistics of public debts in the EMU12, 2000-2011 (GDP percent ratios, yearly data)

	2000		2008		2011	
	d_m	$\Delta_{m.GER}$	d_m	$\Delta_{m.GER}$	d_m	$\Delta_{m.GER}$
Austria	58.8	-0.9	62.6	-3.3	72.2	-9.6
Belgium	107.6	47.9	89.8	23.9	97.2	15.4
Finland	43.8	-15.9	34.1	-31.8	49.1	-32.7
France	57.3	-2.4	67.4	1.5	85.4	3.6
Germany	59.7	-	65.9	-	81.7	-
Greece	101.8	42.1	99.2	33.3	162.8	81.1
Ireland	37.7	-22	44.1	-21.8	108.1	26.4
Italy	109.2	49.5	105.8	39.9	120.5	38.7
Luxemb.	6.4	-53.3	13.5	-52.4	18.2	-63.5
Netherl.	53.8	-5.9	58.2	-7.7	64.2	-17.5
Portugal	55.9	-3.8	63.8	-2.1	101.6	19.8
Spain	59.2	-0.5	39.7	-26.2	69.6	-12.2
Area average	62.6		62.0		85.9	
Area s.d.	30.2		27.1		36.7	

d_m : debt/GDP of member m

$\Delta_{m.GER}$: ($d_m - d_{GER}$)

Source: AMECO database.

Table 3. Mean-variance determinants of the interest-rate spread with Germany 2000-11. EMU12 countries

2000-08	σ_m^2	σ_{mGER}	$\sigma_m^2 - \sigma_{mGER}$	$\sigma_{GER}^2 - \sigma_{mGER}$	spread (%)	ρ'
Austria	0.450	0.398	0.052	-0.034	0.162	0.0306
Belgium	0.455	0.399	0.056	-0.035	0.196	0.0256
Finland	0.447	0.398	0.049	-0.034	0.133	0.0332
France	0.383	0.371	0.012	-0.006	0.100	0.0869
Germany	0.365	-	-	-	--	
Greece	0.561	0.421	0.140	-0.057	0.414	0.0235
Ireland	0.460	0.391	0.069	-0.027	0.155	0.0344
Italy	0.391	0.361	0.030	0.004	0.312	0.1026
Luxemb.	0.426	0.360	0.067	0.005	0.165	0.4522
Netherlands	0.401	0.379	0.022	-0.015	0.096	0.0447
Portugal	0.446	0.393	0.053	-0.028	0.234	0.0471
Spain	0.454	0.399	0.055	-0.035	0.156	0.0318
2009-11	σ_m^2	σ_{mGER}	$\sigma_m^2 - \sigma_{mGER}$	$\sigma_{GER}^2 - \sigma_{mGER}$	spread	ρ'
Austria	0.196	0.167	0.029	0.033	0.621	-1.4209
Belgium	0.151	0.020	0.131	0.180	0.967	-1.0280
Germany	0.199	-	-	-	-	1.1985
Spain	0.486	-0.133	0.619	0.332	1.645	-0.0739
Finland	0.210	0.190	0.021	0.010	0.388	-
France	0.115	0.129	-0.014	0.071	0.484	0.0025
Greece	20.172	-1.384	21.556	1.583	6.802	0.0095
Ireland	4.973	-0.281	5.253	0.480	3.918	0.0362
Italy	0.468	-0.146	0.614	0.346	1.643	0.0419
Luxemb.	0.482	0.282	0.201	-0.082	0.399	0.7090
Netherlands	0.209	0.193	0.015	0.006	0.335	0.0056
Portugal	7.336	-0.749	8.085	0.948	3.457	0.1207

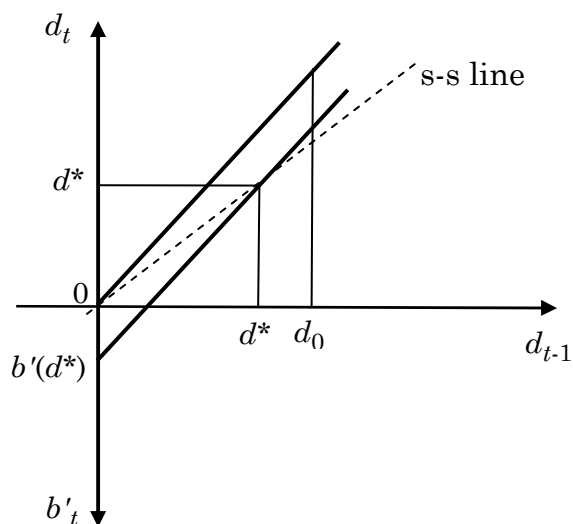
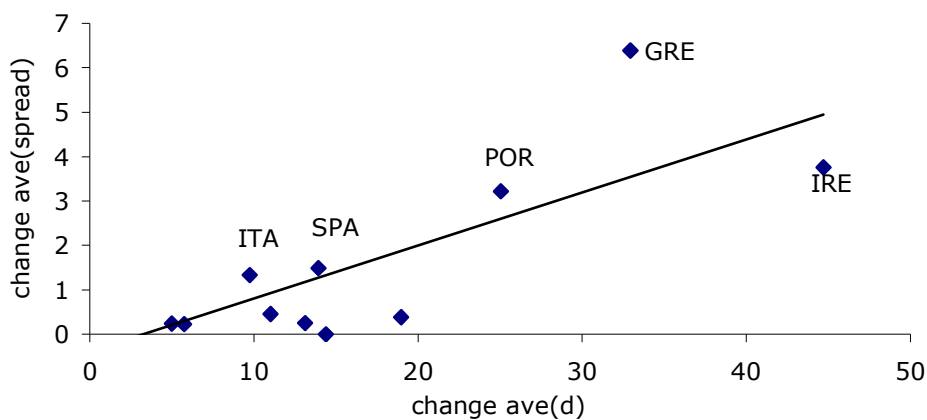
Source: elaborations on ECB, Interest rate statistics, online database;

Table 4. The SCP paths of debt and primary balance GDP ratios

Case 1		Case 2			Case 3		
Country A		Country A		Country B	Country A		Country B
spread	primary bal./GDP	spread	primary bal./GDP	primary bal./GDP	spread	primary bal./GDP	primary bal./GDP
4.4%	6.3%	4.0%	5.9%	2.1%	4.0%	5.9%	6.1%
4.2%	6.0%	3.8%	5.7%	2.2%	3.9%	5.7%	6.2%
4.0%	5.8%	3.7%	5.5%	2.2%	3.8%	5.5%	6.4%
3.8%	5.5%	3.5%	5.3%	2.3%	3.6%	5.4%	6.6%
3.6%	5.3%	3.4%	5.1%	2.4%	3.5%	5.2%	6.7%
3.4%	5.1%	3.2%	4.9%	2.5%	3.4%	5.1%	-1.1%
3.2%	4.9%	3.0%	4.8%	2.6%	3.2%	4.9%	-1.1%
3.0%	4.7%	2.9%	4.6%	2.6%	3.0%	4.7%	-1.1%
2.8%	4.5%	2.7%	4.5%	2.7%	2.8%	4.6%	-1.1%
2.6%	4.4%	2.6%	4.3%	2.8%	2.6%	4.4%	-1.1%
2.4%	0.2%	2.4%	0.2%	-1.1%	2.4%	0.2%	-1.1%
2.4%	0.2%	2.4%	0.2%	-1.1%	2.4%	0.2%	-1.1%

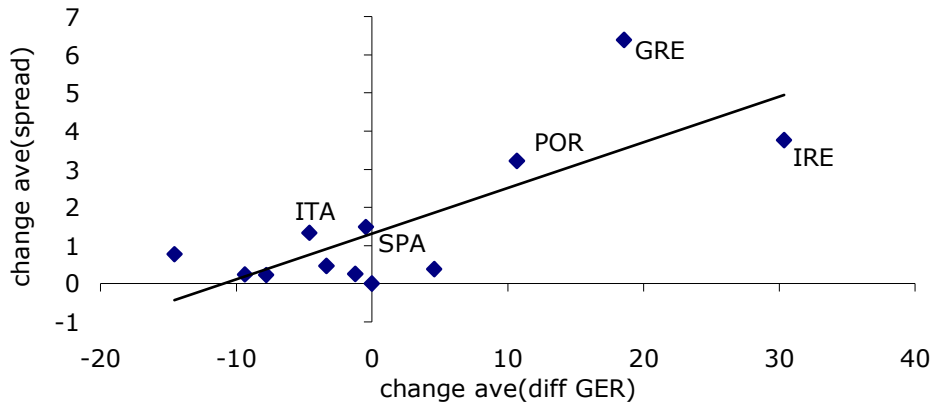
Table 5. Effects of exogenous shocks to country A's debt path

	Debt/GDP	Primary surplus/GDP	Change of debt/GDP	Change of spread	Corrective change of primary surplus/GDP
Baseline	60%	0.2%			
-1% n			+0.8%	+0.04%	+0.58%
+1% i_B			+0.8%	+0.04%	+0.57%
+1% d_A			+1.3%	+0.06%	+1.3%

Figure 1. Debt dynamics with $i > n$ **Figure 2. Cross-country relationship between the change in the average debt/GDP ratio and in the average monthly interest-rate spread with the 10-year German *Bund* from 2000-08 to 2009-11, EMU12 countries.**

Source: elaborations on Table 1 and 2

Figure 3. Relationship between the change in the average monthly interest-rate spread and the change in the excess debt/GDP ratio with Germany from 2000-08 to 2009-11, EMU12 countries.



Source: elaborations on Table 1 and 2

Figure 4. Debt dynamics with endogenous interest-rate spread

