

A Theoretical and Empirical Comparison of Systemic Risk Measures

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Abstract

We propose a theoretical and empirical comparison of several popular systemic risk measures that can be estimated from public data: *Marginal Expected Shortfall* (MES), *Systemic Risk Measure* (SRISK), and *Delta Conditional Value at Risk* (ΔCoVaR). First, we assume that the time-varying correlation completely captures the dependence between the firm and market returns and we show that (i) MES corresponds to the product of the market expected shortfall (market tail risk) and the firm beta (firm systematic risk) and that (ii) ΔCoVaR corresponds to the product of the firm VaR (firm tail risk) and the linear projection coefficient of the market return on the firm return. We also derive (iii) conditions under which different systemic risk measures lead to consistent rankings of systemically important financial institutions. Second, we relax our fundamental assumption and empirically validate our theoretical findings for a sample of US financial institutions. Collectively, our results indicate that the best systemic risk measure very much depends on the considered objective: MES helps little to *rank* systemically important financial institutions, whereas ΔCoVaR brings limited added value over and above VaR to *forecast* systemic risk. Overall, SRISK offers the best compromise between the too-big-to-fail and too-interconnected-to-fail dimensions.

Keywords: Marginal Expected Shortfall, CoVaR, systemic vs. systematic risk, tail risk

JEL classification: G01, G32.

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1 Introduction

The recent financial crisis has fostered extensive research on systemic risk, either on its definition, measurement, or regulation.¹ Of particular interest is the identification of the financial institutions that contribute the most to the overall risk of the financial system – the so-called *Systemically Important Financial Institutions* (SIFIs). The following definition has been proposed by the Financial Stability Board (2011): “SIFIs are financial institutions whose distress or disorderly failure, because of their size, complexity and systemic interconnectedness, would cause significant disruption to the wider financial system and economic activity.” As they pose a major threat to the system, regulators and policymakers from around the world have called for tighter supervision, extra capital requirements, and liquidity buffers for SIFIs (Financial Stability Board, 2011). This recent shift from focusing on single institutions’ risk in isolation (micro-prudential approach) to focusing on a firm contribution to system-wide risk (macro-prudential approach) is at the heart of the Basel 3 regulatory framework.

In practice, measuring the contribution of a given firm to the overall risk of the system is challenging. A first approach, called the supervisory approach, relies on firm-specific information concerning size, leverage, liquidity, interconnectedness, complexity, and substitutability. This approach is both quantitative and qualitative and can rely on proprietary data provided by the financial institution to the regulator (Financial Stability Board – International Monetary Fund – Bank for International Settlements, 2009, Basel Committee on Banking Supervision, 2011, Financial Stability Oversight Council, 2012, and Greenwood, Landier and Thesmar, 2012).

A second approach only relies on publicly available market data, such as stock or asset returns, option prices, or CDS spreads. Three prominent examples of such measures are the *Marginal Expected Shortfall* (MES) of Acharya et al. (2010), the *Systemic Risk Measure* (SRISK) of Brownlees and Engle (2011) and Acharya, Engle et Richardson (2012), and the *Delta Conditional Value-at-Risk* (Δ CoVaR) of Adrian and Brunnermeier (2011).² Very few crisis-related papers made a higher impact both in the academia and on the regulatory debate than this series of papers. Over the past four years, dozens of research papers have discussed, implemented, and sometimes generalized these systemic risk measures.³ They are nowadays regularly discussed in the financial press and other media. Some have been specifically recommended by Fed Chairman Bernanke, while others are computed in real-time for hundreds of financial institutions in the world and publicly disclosed

¹Bisias et al. (2012) survey 31 measures of systemic risk in the economic and finance literature.

²Other related papers include Elsinger et al. (2005), Huang et al. (2009), Gauthier et al. (2009), Manganelli et al. (2010), Drehmann and Tarashev (2011), Kritzman et al. (2011), and Billio et al. (2012).

³See for instance Adams et al. (2010), Cao (2010), Fong and Wong (2010), Danielson et al. (2011), Idier, Lamé, and Mésonnier (2011), Colletaz, Pérignon et Hurlin (2012), Ergun and Girardi (2012), and Lopez-Espinosa et al. (2012a,b).

on the Internet.⁴

These three systemic risk measures have some nice economic interpretation. First, the MES corresponds to a firm's expected equity loss when the market falls below a certain threshold over a given horizon, namely a 2% market drop over 1 day for the short-run MES, and a 40% market drop over six months for the long-run MES (LRMES). The basic idea is that the banks with the highest MES contribute the most to market declines; thus, these banks are the greatest drivers of systemic risk. Second, the SRISK measures the expected capital shortfall of an institution conditional on a crisis occurring. The intuition is that the firm with the largest capital shortfall will contribute the most to a crisis and therefore should be considered the most systemically risky. Third, the CoVaR corresponds to the VaR of the financial system conditional on a specific event affecting a given firm. The contribution of a firm to systemic risk (ΔCoVaR) is the difference between its CoVaR when the firm is, or is not, in financial distress.

In this paper, we propose a theoretical and empirical comparison of the major market-data based systemic risk measures (MES, SRISK, and ΔCoVaR) in a common framework. There are two main steps in our analysis. First, we assume that the dependence between firm and market returns is completely captured by the time-varying correlation. Under this assumption, we show that (i) MES corresponds to the product of the market's expected shortfall (market tail risk) and the firm beta (firm systematic risk), and that (ii) ΔCoVaR corresponds to the product of the firm VaR (firm tail risk) and the linear projection coefficient of the market return on the firm return. Furthermore, we derive (iii) conditions under which different systemic risk measures would lead to consistent rankings of financial institutions. Second, we relax the assumption that the dependence between firm and market returns is fully captured by time-varying correlation. Because of potential nonlinear dependencies, there is no closed form for the considered risk measures anymore. Hence, we propose an empirical comparison for a panel of 94 top US financial institutions over the period January 2000 - December 2010. For each measure, we propose two types of systemic risk analyses: a ranking of firms at one point in time (cross-sectional analysis) and a forecasting analysis for a given firm (time-series analysis).

Our empirical analysis delivers three main results. First, we show that different risk measures lead to identifying different SIFIs. On most days, there is not a single institution simultaneously identified as a top 10 SIFI by all three measures. Second, there is a strong empirical relationship between MES and firm beta, which implies that systemic risk rankings of financial institutions based on MES mirror rankings obtained by sorting firms on betas. Third, the empirical ΔCoVaR

⁴For recent media coverage, see Bloomberg Businessweek (2011), The Economist (2011), and Rob Engle's interview on CNBC (2011). The CoVaR paper was put on Ben Bernanke's recommended reading list about the financial crisis in September 2010, <http://blogs.wsj.com/deals/2010/09/02/ben-bernanke-labor-day-reading-list/?KEYWORDS=Brunnermeier>. For online computation of systemic risk measures, see the Stern-NYU's V-Lab initiative at <http://vlab.stern.nyu.edu/welcome/risk/>.

of a firm is strongly correlated with its VaR. Hence, ΔCoVaR brings limited added value over and above VaR to *forecast* systemic risk. The main take-away of our study is that the best systemic risk measure very much depends on the considered objective and that SRISK offers the best compromise between the *too-big-to-fail* and *too-interconnected-to-fail* dimensions.

Our paper makes several contributions to the academic literature on systemic risk. This paper is, to the best of our knowledge, the first to derive all the major market-data based systemic risk measures within a common framework. For all systemic risk measures, we derive some analytical forms that allow us to uncover the theoretical link between systemic risk and *standard* risks (systematic risk, tail risk, correlation, and beta), as well as firm characteristics (leverage and market capitalization). These expressions permit to quantify the added value of each systemic risk measure, both theoretically and empirically. Our conclusions not only have academic value, they also contribute to the current debate about enhanced systemic risk regulation in telling which measure should be used – and when they should not be.

The rest of the paper is organized as follows. Section 2 provides the general definitions of the three systemic risk measures and presents the common framework used for the comparison. Section 3 proposes a theoretical analysis of the MES, SRISK, and ΔCoVaR under the assumption that the dependence between firm and market return is completely captured by the time-varying correlation. In Section 4, we describe the data and present the main empirical findings. Section 5 summarizes and concludes.

2 Methodology

2.1 Definitions

In this section, we provide a formal definition for the considered systemic risk measures. We consider N firms and denote r_{it} the return of firm i at time t . Similarly, the market return is the value-weighted average of all firm returns, $r_{mt} = \sum_{i=1}^N w_{it}r_{it}$, where w_{it} denotes the relative market capitalization of firm i .

MES

First, the MES measures the marginal contribution of an institution i to systemic risk, as measured by the Expected Shortfall (ES) of the system. Originally proposed by Acharya et al. (2010), the MES was recently extended to a conditional version by Brownlees and Engle (2011). By definition, the ES at the $\alpha\%$ level is the expected return in the worst $\alpha\%$ of the cases, but it can be extended to the general case, in which the returns exceed a given threshold C . Formally,

the conditional ES of the system is defined as:⁵

$$ES_{mt}(C) = \mathbb{E}_{t-1}(r_{mt} | r_{mt} < C) = \sum_{i=1}^N w_{it} \mathbb{E}_{t-1}(r_{it} | r_{mt} < C). \quad (1)$$

Then, the MES corresponds to the partial derivative of the system ES with respect to the weight of firm i in the economy (Scaillet, 2004).⁶

$$MES_{it}(C) = \frac{\partial ES_{mt}(C)}{\partial w_{it}} = \mathbb{E}_{t-1}(r_{it} | r_{mt} < C). \quad (2)$$

The MES can be viewed as a natural extension of the concept of marginal VaR proposed by Jorion (2007) to the ES, which is a coherent risk measure (see Artzner et al., 1999). It measures the increase in the risk of the system (measured by the ES) induced by a marginal increase in the weight of firm i in the system. The higher the firm MES (in absolute value), the higher the individual contribution of the firm to the risk of the financial system.

SRISK

Second, the SRISK measure proposed by Brownlees and Engle (2011) and Acharya, Engle and Richardson (2012) extends the MES in order to take into account both the liabilities and the size of the financial institution. The SRISK corresponds to the expected capital shortfall of a given financial institution, conditional on a crisis affecting the whole financial system. In this perspective, the firms with the largest capital shortfall are assumed to be the greatest contributors to the crisis and are the institutions considered most systemically risky. The SRISK is defined as:

$$SRISK_{it} = \max[0 ; k D_{it} - (1 - k) W_{it} (1 - LRMES_{it})] , \quad (3)$$

where k is the prudential capital ratio, D_{it} is the quarterly book value of total liabilities, and W_{it} is the daily market capitalization or market value of equity. Note that the SRISK, which is positive by convention, is an increasing function of the liabilities and a decreasing function of the market capitalization.⁷ Then, the SRISK can be viewed as an implicit increasing function of the quasi-leverage (leverage thereafter) defined by the ratio of the book value of total liabilities to the market capitalization.

This systemic risk measure also considers the interconnection of a firm with the rest of the system through the LRMES. The LRMES denotes the long-run marginal expected shortfall: it

⁵We follow the original notations of the different authors: ES, MES, VaR, CoVaR and Δ CoVaR are typically negative. The only exception is the SRISK which is typically a positive number when a firm suffers from a capital shortage.

⁶To simplify the notation, we use MES_{it} (respectively ES_{it}) instead of $MES_{i,t|t-1}$ (respectively $ES_{i,t|t-1}$), but it should be understood as the conditional MES (respectively ES) computed at time t given the information available at time $t - 1$.

⁷Brownlees and Engle (2011) define a short-run SRISK that is defined over one day: $SRISK_{it}^{SR} = \max[0 ; k D_{it} - (1 - k) W_{it} MES_{it}]$. In this paper, we focus on the long-run version of the SRISK because as it is the standard measure (see Stern-NYU's V-Lab).

corresponds to the expected drop in equity value the firm would experiment should the market fall by more than a given threshold within the next six months. Acharya, Engle and Richardson (2012) propose to approximate it using the daily MES (defined for a threshold C equal to 2%) as $LRMES_{it} \simeq 1 - \exp(18 \times MES_{it})$.⁸

ΔCoVaR

The third systemic risk measure is the ΔCoVaR of Adrian and Brunnermeier (2011). This measure is based on the concept of Value-at-Risk, denoted $\text{VaR}(\alpha)$, which is the maximum dollar loss within the $\alpha\%$ -confidence interval (see Jorion, 2007). Then, the CoVaR corresponds to the VaR of the market returns obtained *conditional* on some event $\mathbb{C}(r_{it})$ observed for firm i :

$$\Pr\left(r_{mt} \leq \text{CoVaR}_t^{m|\mathbb{C}(r_{it})} \mid \mathbb{C}(r_{it})\right) = \alpha. \quad (4)$$

The ΔCoVaR of firm i is then defined as the difference between the VaR of the financial system conditional on this particular firm being in financial distress and the VaR of the financial system conditional on firm i being in its median state. To define the distress of a financial institution, various definitions of $\mathbb{C}(r_{it})$ can be considered. Because they use a quantile regression approach, Adrian and Brunnermeier (2011) consider a situation in which the loss is precisely equal to its VaR :

$$\Delta\text{CoVaR}_{it}(\alpha) = \text{CoVaR}_t^{m|r_{it}=\text{VaR}_{it}(\alpha)} - \text{CoVaR}_t^{m|r_{it}=\text{Median}(r_{it})}. \quad (5)$$

A more general approach would consist in defining the financial distress of firm i as a situation in which the losses exceed its VaR (see Girardi and Ergun, 2011):

$$\Delta\text{CoVaR}_{it}(\alpha) = \text{CoVaR}_t^{m|r_{it} \leq \text{VaR}_{it}(\alpha)} - \text{CoVaR}_t^{m|r_{it}=\text{Median}(r_{it})}. \quad (6)$$

2.2 A Common Framework

The different systemic risk measures analyzed in this paper have been developed within very different frameworks. For instance, Adrian and Brunnermeier (2011) assume tail dependences and use a quantile regression approach to estimate the CoVaR and the ΔCoVaR . Differently, Brownlees and Engle (2011) model time-varying linear dependencies and use a multivariate GARCH-DCC model to fit the MES. Hence, their direct comparison is not straightforward since some empirical differences may be due to the estimation methods. In contrast, we derive all these risk measures within a unified theoretical framework to provide a level playing field. Formally, we consider the linear market model of Brownlees and Engle (2011):

⁸This approximation of the LRMES represents the firm expected loss at a 6 month-horizon, obtained conditional on the market falling by more than 40% within the next six months. For more details, see Acharya, Engle and Richardson (2012).

$$r_{mt} = \sigma_{mt} \varepsilon_{mt} , \quad (7)$$

$$r_{it} = \sigma_{it} \rho_{it} \varepsilon_{mt} + \sigma_{it} \sqrt{1 - \rho_{it}^2} \xi_{it} , \quad (8)$$

$$(\varepsilon_{mt}, \xi_{it}) \sim D. \quad (9)$$

The main features of this framework are the following. First, the return of firm i depends on the market return r_{mt} and on an orthogonal firm-specific component ξ_{it} . Second, the conditional standard deviations σ_{it} and σ_{mt} , and the conditional correlation ρ_{it} are supposed to be time-varying. Third, we assume that the process $\nu_t = (\varepsilon_{mt}, \xi_{it})'$ is *i.i.d.* and satisfies $\mathbb{E}(\nu_t) = 0$ and $\mathbb{E}(\nu_t \nu_t') = I_2$, a two-by-two identity matrix. Fourth, no particular assumption is made about the bivariate distribution D of the standardized innovations, which is assumed to be unknown. In this very flexible framework, there is only one additional assumption we need to make to be able to derive a theoretical expression for MES, SRISK, and ΔCoVaR :

Assumption A1: *We assume that the dependence between firm and market returns is fully captured by the time-varying conditional correlations ρ_{it} .*

This assumption means that the standardized innovations ε_{mt} and ξ_{it} are assumed to be independently distributed at time t . This is a rather mild assumption that is made, for instance, in the CAPM. In particular, we do not impose anything about the joint distribution of the returns (e.g., normal, Student's t). However, if there exists some nonlinear dependencies between firm and market returns, like tail dependence for instance, this assumption is violated. In order to generalize our conclusion, we will relax the A1 assumption in Section 4.

3 A Theoretical Comparison of Systemic Risk Measures

3.1 MES

Given Equations (7) to (9), the MES can be expressed as a function of the firm return volatility, its correlation with the market return, and the comovement of the tail of the distribution:

$$\begin{aligned} \text{MES}_{it}(C) &= \sigma_{it} \rho_{it} \mathbb{E}_{t-1} \left(\varepsilon_{mt} \mid \varepsilon_{mt} < \frac{C}{\sigma_{mt}} \right) \\ &\quad + \sigma_{it} \sqrt{1 - \rho_{it}^2} \mathbb{E}_{t-1} \left(\xi_{it} \mid \varepsilon_{mt} < \frac{C}{\sigma_{mt}} \right). \end{aligned} \quad (10)$$

MES is expressed as a weighted function of the tail expectation of the standardized market residual and the tail expectation of the standardized idiosyncratic firm residual. Under A1, the dependence between market and firm returns is completely captured by their correlation and, in turn, the conditional expectation $\mathbb{E}_{t-1} \left(\xi_{it} \mid \varepsilon_{mt} < \frac{C}{\sigma_{mt}} \right)$ is null. In order to facilitate the comparison with the ΔCoVaR , we consider a threshold C equal to the conditional VaR of the market return, which is defined as $\Pr [r_{mt} < \text{VaR}_{mt}(\alpha)] = \alpha$.

Proposition 1 *Under the A1 assumption, the MES of a given financial institution i is proportional to its systematic risk, as measured by its time-varying beta. The proportionality coefficient is the conditional expected shortfall of the market:*

$$MES_{it}(\alpha) = \beta_{it} ES_{mt}(\alpha) , \quad (11)$$

where $\beta_{it} = \text{cov}(r_{it}, r_{mt}) / \text{var}(r_{mt}) = \rho_{it} \sigma_{it} / \sigma_{mt}$ denotes the time-varying beta of firm i and $ES_{mt}(\alpha) = \mathbb{E}_{t-1}(r_{mt} \mid r_{mt} < VaR_{mt}(\alpha))$ is the conditional expected shortfall of the market.

The proof of Proposition 1 is in the Appendix A.⁹ This proposition has two main implications. First, *on a given date*, the systemic risk ranking of financial institutions based on MES (in absolute value) is strictly equivalent to the ranking that would be produced by sorting firms according to their betas. Indeed, since the system ES is not firm-specific, the greater the sensitivity of the return of a firm with respect to the market return, the more systemically-risky the firm is. Consequently, under A1, identifying SIFIs using MES is equivalent to considering the financial institutions with the highest betas. Second, *for a given financial institution*, the time profile of its systemic risk measured by its MES may be different from the evolution of its systematic risk measured by its conditional beta. Since the market ES may vary over time, forecasting the systematic risk of firm i may not be sufficient to forecast the future evolution of its contribution to systemic risk.

Note that Proposition 1 is robust with respect to the choice of the threshold C that determines the system crisis. For any threshold $C \in \mathbb{R}$, the MES is still proportional to the time-varying beta (see the proof in Appendix A). The only difference is that the proportionality coefficient, $\mathbb{E}_{t-1}(r_{mt} \mid r_{mt} < C)$, is different from the system ES if $C \neq VaR_{mt}(\alpha)$. However, this coefficient remains common to all firms.

3.2 SRISK

We show in Section 2 that SRISK is a linear function of the MES. As a result, a corollary of Proposition 1 is that SRISK can be expressed as a linear function of beta, liabilities, and market capitalization:

$$SRISK_{it} \simeq \max[0 ; k D_{it} - (1 - k) W_{it} \exp(18 \times \beta_{it} \times ES_{mt}(\alpha))] . \quad (12)$$

Under the A1 assumption, the SRISK is an increasing function of the systematic risk, as measured by the conditional beta. However, unlike with MES, systemic-risk rankings based on SRISK are

⁹For some particular distributions, both the ES and the MES of the market returns can be expressed in closed form. For instance, if ε_{mt} follows a standard normal distribution, then $VaR_{mt}(\alpha) = \sigma_{mt} \Phi^{-1}(\alpha)$ and $ES_{mt}(\alpha) = -\sigma_{mt} \phi(\Phi^{-1}(\alpha)) / \alpha$, where $\phi(\cdot)$ and $\Phi(\cdot)$, respectively, denote the standard normal probability distribution function and cumulative distribution function. Therefore, $MES_{it}(\alpha) = -\beta_{it} \sigma_{mt} \lambda(\Phi^{-1}(\alpha))$, where $\lambda(z) = \phi(z) / \Phi(z)$ denotes the Mills ratio.

not equivalent to rankings based on betas. SRISK-based rankings also depend on the liabilities and on the market capitalization of the financial institution.¹⁰ Recall that the system ES is typically a negative number, $ES_{mt}(\alpha) < 0$. Consequently, when $\beta_{it} > 0$, SRISK is a decreasing function of the market capitalization. Then, if we consider a given level of liabilities, SRISK increases with the leverage.

3.3 ΔCoVaR

Under A1, it is possible to express ΔCoVaR , defined for a conditioning event $\mathbb{C}(r_{it}) : r_{it} = \text{VaR}_{it}(\alpha)$, as a function of the conditional correlations and volatilities. Given Equations (7) to (9), we obtain the following result:

Proposition 2 *Under the A1 assumption, the ΔCoVaR of a given financial institution i is proportional to its tail risk, as measured by its VaR. The proportionality coefficient corresponds to the linear projection coefficient of the market return on the firm return.*

$$\Delta\text{CoVaR}_{it}(\alpha) = \gamma_{it} [\text{VaR}_{it}(\alpha) - \text{VaR}_{it}(0.5)] , \quad (13)$$

where $\gamma_{it} = \rho_{it}\sigma_{mt}/\sigma_{it}$ and $\text{VaR}_{it}(\alpha)$ denotes the $\text{VaR}(\alpha)$ of the firm return. If the marginal distribution of the returns is symmetric around zero, ΔCoVaR is strictly proportional to VaR :

$$\Delta\text{CoVaR}_{it}(\alpha) = \gamma_{it} \text{VaR}_{it}(\alpha) . \quad (14)$$

The proof of Proposition 2 is in the Appendix B.¹¹ The fact that the proportionality coefficient between ΔCoVaR and VaR is firm-specific has some strong implications. Let us, for instance, consider two financial institutions i and j , with $\text{VaR}_{it} < \text{VaR}_{jt}$. Given the relative correlations between the returns of firms i and j with the market return, which are denoted ρ_{it} and ρ_{jt} , and the volatilities σ_{it} and σ_{jt} , we could observe $\Delta\text{CoVaR}_{it} < \Delta\text{CoVaR}_{jt}$ or $\Delta\text{CoVaR}_{it} > \Delta\text{CoVaR}_{jt}$. This means that the most risky institution in terms of VaR is not *necessarily* the most systemically risky institution. In other words, *on a given date*, the systemic risk ranking over N financial institutions based on ΔCoVaR is not equivalent to a VaR -based ranking. In that sense, ΔCoVaR is not equivalent to VaR as already pointed out by Adrian and Brunnermeier (2011) in their Figure 1 (page 4). Indeed, they report a weak relationship between an institution's risk in isolation, measured by its VaR , and its contribution to system risk, measured by its ΔCoVaR . However, *for*

¹⁰Given Equation 8, the market value of the financial institution can be expressed as a function of the number of stocks at time t , denoted N_{it} , and the sequence of past volatilities, correlations and innovations : $W_{it} = N_{it} \times \exp \left[\int_{z=0}^t (\sigma_{iz}\rho_{iz}\varepsilon_{mz} + \sigma_{iz}\sqrt{1-\rho_{iz}^2}\xi_{iz}) + \log(P_{i0}) \right]$ where P_{i0} denotes the initial price of stock i .

¹¹Adrian and Brunnermeier (2011) derive the CoVaR and the ΔCoVaR under the normality assumption. They show that $\Delta\text{CoVaR}_{it}(\alpha) = \rho_{it}\sigma_{mt}\Phi^{-1}(\alpha)$ or equivalently $\gamma_{it}\sigma_{it}\Phi^{-1}(\alpha)$, where $\sigma_{it}\Phi^{-1}(\alpha)$ denotes the $\text{VaR}(\alpha)$ of the firm.

a given institution, ΔCoVaR is proportional to VaR. Consequently, forecasting the future evolution of the contribution of firm i to systemic risk is equivalent to forecasting its risk in isolation.

It is important to notice that Proposition 2 is robust with respect to the choice of a constant (unconditional) or time-varying (conditional) variance/covariance framework. It is also robust with respect to the form of the dependence between returns. Under A1, ΔCoVaR is a function of the linear dependence between the returns, which is measured by γ_{it} . As we will see in the next section, Adrian and Brunnermeier (2011) estimate the ΔCoVaR through quantile regression in order to capture the potential non-linear dependencies between the returns. But even in this case, ΔCoVaR remains proportional to the VaR, and the proportionality coefficient is constant over time.

Furthermore, Proposition 2 highlights two important differences between the MES, SRISK, and ΔCoVaR measures. First, MES is a *marginal* risk measure (defined by the first derivative of the system ES) whereas ΔCoVaR is an *incremental* risk measure (defined by the difference between two conditional VaRs). SRISK is a compromise between both approaches in the sense that it is based on a marginal measure of the return interconnectedness, through the MES, but also on nominal values such as liabilities and market capitalization. Second, MES depends on the linear projection coefficient of the firm return onto the market return, i.e., the beta, whereas ΔCoVaR is based on the linear projection coefficient of the market return on the firm return. This difference is quite important. Indeed, MES and SRISK are fundamentally linked to the sensitivity of the firm return to the market return. In contrast, ΔCoVaR captures the sensitivity of the market return with respect to the firm returns. Although both measures aim to measure systemic-risk contribution, their conditioning approaches are completely different. In the next section, we analyze the consequences of these differences on systemic-risk rankings and on the identification of the SIFIs.

3.4 Comparing Systemic-Risk Rankings

The main objective of any systemic risk analysis is to rank firms according to their systemic risk contribution and, in turn, identify the SIFIs. The key question is then to determine whether the different systemic risk measures lead to the same conclusion. A natural way to answer this question is to analyze their ratio.

Proposition 3 *Under the A1 assumption and for a given financial institution, the ratio between ΔCoVaR and MES is:*

$$\frac{\Delta\text{CoVaR}_{it}(\alpha)}{\text{MES}_{it}(\alpha)} = \frac{\text{VaR}_{it}(\alpha)}{\sigma_{it}^2} \times \frac{\sigma_{mt}^2}{\text{ES}_{mt}(\alpha)}, \quad (15)$$

if the marginal distribution of the firm return is symmetric. If the distribution is not symmetric,

the term $VaR_{it}(\alpha)$ is replaced by $VaR_{it}(\alpha) - VaR_{it}(0.5)$.

The $\Delta\text{CoVaR}/\text{MES}$ ratio is the product of two terms. The first term is firm-specific, whereas the second is common to all firms.¹² The fact that this ratio is firm-specific implies that the systemic risk rankings based on the two measures may not be the same. Consider two different financial institutions i and j such that i is *more* systemically risky than j according to ΔCoVaR , $\Delta\text{CoVaR}_{it} < \Delta\text{CoVaR}_{jt}$. It is possible to observe a situation where i is *less* risky than j according to the MES measure, $\text{MES}_{it} > \text{MES}_{jt}$. In other words, the SIFIs identified by the MES and by the ΔCoVaR may not be the same. Note that this result can be extended to the SRISK since the latter directly depends on MES.

Another advantage of our theoretical framework is that it allows us to derive conditions under which both rankings are convergent, respectively divergent.

Proposition 4 *A financial institution i is more systemically risky than an institution j according to the MES and the ΔCoVaR measures, $\text{MES}_{it}(\alpha) \leq \text{MES}_{jt}(\alpha)$ and $\Delta\text{CoVaR}_{it}(\alpha) \leq \Delta\text{CoVaR}_{jt}(\alpha)$, if:*

$$\rho_{it} \geq \max\left(\rho_{jt}, \frac{\rho_{jt}\sigma_{jt}}{\sigma_{it}}\right), \quad (16)$$

if the conditional distributions of the two standardized returns r_{it}/σ_{it} and r_{jt}/σ_{jt} are identical and location-scale.

The proof of Proposition 4 is in the Appendix C.¹³ Then under A1 and mild assumptions, the SIFIs identified according to the MES correspond to the SIFIs identified with ΔCoVaR , if the correlations of their returns with the market returns are larger than the correlations for the other financial institutions. The higher the correlation between the SIFI and the system returns, the more likely it is that both measures will lead to a convergent diagnostic. This result comes from the fact that correlation captures indifferently both the sensitivity of the system return with respect to the firm return (ΔCoVaR dimension) and the sensitivity of the firm return with respect to the system return (MES dimension).

The systemic risk rankings based on SRISK and ΔCoVaR can also be compared. In this case, the comparison depends on the liabilities and market capitalizations of the two firms. For simplicity, let us consider two financial institutions with the *same level of liabilities*.

¹²If we assume normality for the marginal distributions of ε_{mt} and ξ_{it} , this ratio has a closed form:

$$\frac{\Delta\text{CoVaR}_{it}(\alpha)}{\text{MES}_{it}(\alpha)} = -\left(\frac{\sigma_{mt}}{\sigma_{it}}\right) \frac{\Phi^{-1}(\alpha)}{\lambda(\Phi^{-1}(\alpha))}.$$

¹³If the conditional distributions are not identical and/or not location-scale, the corresponding condition has the same form and implies that the correlation ρ_{it} exceeds a given threshold (see Appendix C).

Proposition 5 *A financial institution i is more systemically risky than a financial institution j (with the same level of liabilities) according to the short-run SRISK and the ΔCoVaR measures, $\text{SRISK}_{it}(\alpha) \geq \text{SRISK}_{jt}(\alpha)$ and $\Delta\text{CoVaR}_{it}(\alpha) \leq \Delta\text{CoVaR}_{jt}(\alpha)$, if*

$$\rho_{it} \geq \rho_{jt} \quad \text{and} \quad W_{it} \leq W_{jt} \times \exp[18 \times ES_{mt}(\alpha) \times (\beta_{jt} - \beta_{it})] , \quad (17)$$

where W_{it} and W_{jt} denote the market capitalizations of both firms.

The proof of Proposition 5 is in the Appendix D. ΔCoVaR and SRISK provide a similar systemic risk ranking if and only if (i) the correlation of the riskiest firm with the system is higher than the correlation of the least risky institution and (ii) if the riskiest firm has the lowest market capitalization. Since both firms are assumed to have the same level of liabilities, this condition means that the rankings are similar if the riskiest financial institution has the *highest leverage*. In other words, if the SIFIs have a high leverage and are very correlated with the system, ΔCoVaR and SRISK will lead to the same conclusion. As soon as one of these conditions is violated, the ranking of the financial institutions will differ.

4 An Empirical Comparison of Systemic Risk Measures

The five propositions in Section 3 have been derived under the assumption that the dependence between firm and market returns is fully captured by the time-varying conditional correlations. In practice, however, the linear correlation does not always fully capture the dependence between financial asset returns (e.g., tail dependencies). When we relax the A1 assumption, it is no longer possible to derive analytical expressions for the MES, SRISK, and ΔCoVaR . For this reason, we propose an empirical comparison based on a sample of large US financial institutions. Specifically, we use the same sample as in Acharya et al. (2010) and Brownlees and Engle (2011), which contains all U.S. financial firms with a market capitalization greater than \$5 billion as of end of June 2007 (94 firms in total). For our sample period (January 3, 2000 - December 31, 2010), we extract daily firm stock returns, market value-weighted index returns, number of shares outstanding and daily closing prices from CRSP. Quarterly book value of total liabilities are from COMPUSTAT. Appendix E presents the list of all the sample firms.

4.1 SIFI or not SIFI?

The first step in our empirical analysis is to estimate the three systemic risk measures for all the financial institutions in our sample. The main finding from this preliminary analysis is that the *different* risk measures identify *different* SIFIs. For instance, Table 1 displays the tickers of the top 10 financial institutions according to their systemic risk contribution measured by the MES,

SRISK, and ΔCoVaR , respectively, for the last day of our sample period.¹⁴ On that day, there is not a single institution simultaneously identified as a SIFI by the three measures. Only two financial institutions (Bank of America and American International Group) are simultaneously identified by MES and SRISK, whereas ΔCoVaR identifies only three financial institutions (H&R Block, Marshall & Ilsley, and Janus Capital) in common with MES but none with SRISK. Furthermore, the SRISK-based top 10 list is clearly tilted towards the largest financial institutions (Bank of America, Citigroup, JP Morgan, etc.), whereas it is not necessarily the case for MES and ΔCoVaR . Indeed, these measures do not take into account the level of liabilities and the market capitalization of the financial institutions.

Table 1: Systemic Risk Rankings

Rank	MES	SRISK	ΔCoVaR
1	MBI	BAC	HRB
2	AIG	C	MI
3	MI	JPM	BEN
4	CBG	MS	CIT
5	RF	AIG	WU
6	LM	MET	AIZ
7	JNS	PRU	AXP
8	HRB	HIG	JNS
9	BAC	SLM	NYB
10	UNM	LNC	MTB

Notes: The column labeled MES displays the ranking of the top 10 financial institutions based on MES, listed from most to least risky. The following two columns display the top 10 financial institutions based on SRISK and ΔCoVaR , respectively. The ranking is for December 31, 2010.

The findings about diverging rankings is not specific to any particular date. Indeed, out of 2,767 days in our sample, there are 1,263 days (45.7%) during which *none* of the 94 financial institution is jointly included in the top 10 ranking of the three risk measures. Figure 1 shows the daily percentage of concordant pairs between the top 10 SIFIs identified by the different risk measures. On average, the percentage of concordant pairs between MES and SRISK is 18.9%, which means that, only two SIFIs out of ten are common to both measures. Over our 11-year sample, this percentage ranges between 0% and 60%; the latter being at the peak of the crisis in October 2008. The figures are much lower for SRISK and ΔCoVaR , with on average 9.9% of concordant pairs. The highest level of similarity is obtained for MES and ΔCoVaR , with an average percentage of

¹⁴The ΔCoVaR is estimated with a quantile regression as proposed by Adrian and Brunnermeier (2011). See Appendix E for the full name of each financial institution.

concordant pairs of 43%.¹⁵

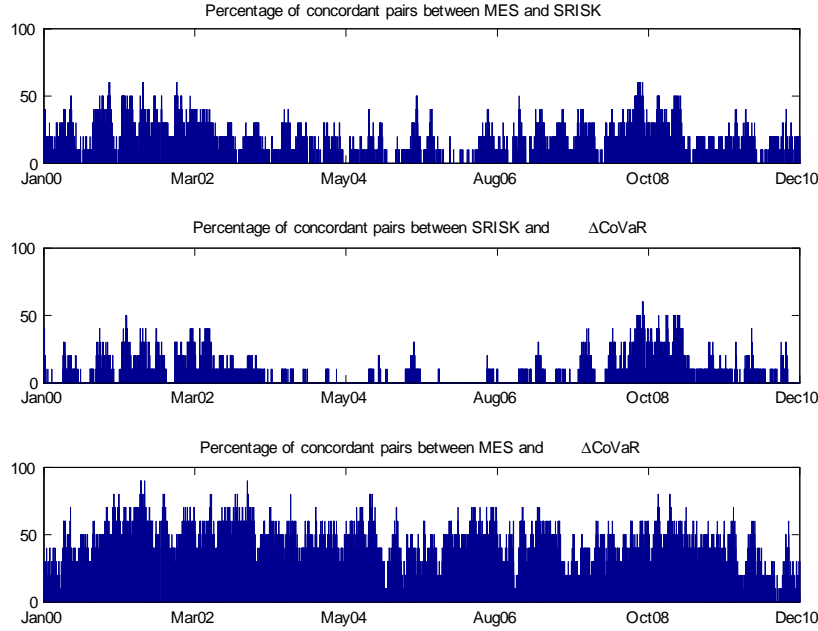


Figure 1: These figures show the daily percentage of concordant pairs between the top 10 financial institutions according to the MES and SRISK (top figure), the SRISK and ΔCoVaR (middle figure), and the ΔCoVaR and MES (bottom figure), respectively. The estimation period is from 01/03/2000 to 12/31/2010.

Even if these systemic risk measures are divergent, they each deliver a consistent ranking *for a given institution*. Indeed, for each measure, we compute the Kendall rank-order correlation coefficient between the systemic risk ranking obtained at time t and the one obtained at time $t - 1$. The average correlations are 91.3% for MES, 97.7% for SRISK, and 93.4% for ΔCoVaR , and are always statistically significant. This result indicates that the rankings produced by these measures are stable through time. This is a nice property to have since it would make little sense for a measure to regularly classify a bank as SIFI on one day, and as non-SIFI on the following day. Therefore, the divergence of the systemic risk rankings is not due to the instability of a particular measure but instead to their fundamental differences.

¹⁵Differently, short-run and long-run SRISK provide very similar rankings. The percentage of concordance is generally higher than 50% and was particularly high during the 2007-2010 crisis period.

4.2 Main Forces Driving MES-Based Systemic Risk Rankings and Forecasts

Estimation

When the A1 assumption is relaxed, the MES can no longer be expressed as the product of the market ES and the time-varying beta of the institution. Indeed, the conditional tail expectation $\mathbb{E}_{t-1}(\xi_{it} \mid \varepsilon_{mt} < C/\sigma_{mt})$ in the expression of the MES (Equation 10) can differ from zero. This term captures the tail-spillover effects from the financial system to the financial institution that are not captured by the correlation. Additionally, if both marginal distributions of the standardized returns are unknown, then the conditional expectation $\mathbb{E}_{t-1}(\varepsilon_{mt} \mid \varepsilon_{mt} < C/\sigma_{mt})$ is also unknown. Consequently, both tail expectations must be estimated. To do so, we follow Brownlees and Engle (2011) and use a nonparametric kernel estimation method (Scaillet, 2005). We consider an unconditional threshold C equal to the unconditional VaR of the system, denoted $VaR_m(\alpha)$.¹⁶ Then, if the standardized innovations ε_{mt} and ξ_{it} are *i.i.d.*, the nonparametric estimates of these tail expectations are given by:

$$\widehat{\mathbb{E}}_{t-1}(\varepsilon_{mt} \mid \varepsilon_{mt} < \kappa) = \frac{\sum_{t=1}^T K\left(\frac{\kappa - \varepsilon_{mt}}{h}\right) \varepsilon_{mt}}{\sum_{t=1}^T K\left(\frac{\kappa - \varepsilon_{mt}}{h}\right)}, \quad (18)$$

$$\widehat{\mathbb{E}}_{t-1}(\xi_{it} \mid \varepsilon_{mt} < \kappa) = \frac{\sum_{t=1}^T K\left(\frac{\kappa - \varepsilon_{mt}}{h}\right) \xi_{it}}{\sum_{t=1}^T K\left(\frac{\kappa - \varepsilon_{mt}}{h}\right)}, \quad (19)$$

where $\kappa = VaR_m(\alpha)/\sigma_{mt}$, $K(x) = \int_{-\infty}^{x/h} k(u) du$, $k(u)$ is a kernel function, and h is a positive bandwidth parameter. Following Scaillet (2005), we fix the bandwidth at $T^{-1/5}$ and choose the standard normal probability distribution function as a kernel function, i.e., $k(u) = \phi(u)$. The final elements needed to compute the MES are the conditional variance and correlation estimates. As in Brownlees and Engle (2011), we model the conditional variances σ_{it}^2 and σ_{mt}^2 according to a TGARCH specification (Rabemananjara and Zakořian, 1993) and use a DCC model (Engle, 2002) for the time-varying correlations ρ_{imt} . The model is estimated in two steps using Quasi Maximum Likelihood.

We obtain daily MES estimates for all sample firms between January 3, 2000 and December 31, 2010. Using debt and equity data, we then compute the SRISK for each firm on each day. Based on these series, we conduct two types of analyses: (i) a cross-sectional analysis for all financial institutions, which allows us to rank systemic financial institutions and (ii) a time-series analysis, which allows us to forecast the systemic risk contribution of a given firm.

¹⁶Results obtained with $C = VaR_{mt}(\alpha)$, where $VaR_{mt}(\alpha)$ denotes the conditional VaR, are similar and available upon request.

Cross-Sectional Analysis and Systemic Risk Ranking

Under A1, we show in Section 3 that the MES of a firm linearly depends on the firm beta. We now investigate whether this result holds true when we relax the A1 assumption. The scatter plot in Figure 2 compares the average MES, $\overline{MES}_i(\alpha) = T^{-1} \sum_{t=1}^T |MES_{it}(\alpha)|$, to the average beta, $\bar{\beta}_i = T^{-1} \sum_{t=1}^T \beta_{it}$, for the 61 firms that have been continuously traded during our sample period.¹⁷ The coverage rate α is fixed at 5%, and the threshold C is fixed to the unconditional market daily VaR at 5%, which is equal to 2% in our sample.

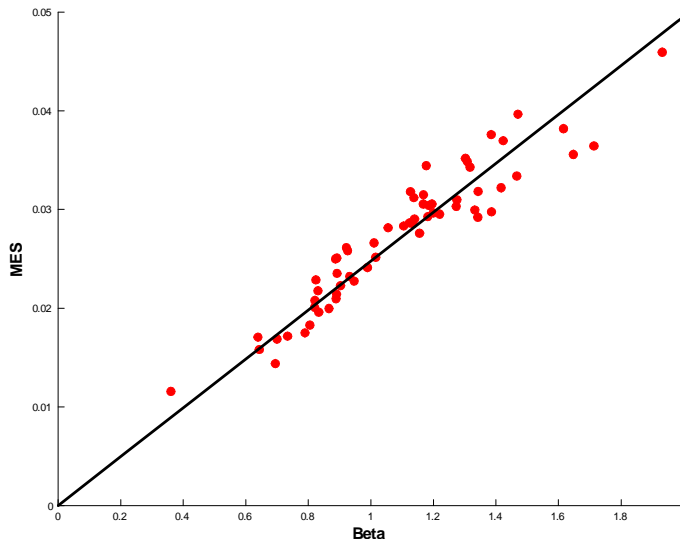


Figure 2: The scatter plot shows the strong cross-sectional link between the time-series average of the MES at 5% estimated for each institution (y -axis) and its beta (x -axis). The beta corresponds to the average of the time-varying beta β_{it} . Each point represents a financial institution and the solid line is the OLS regression line with no constant. The estimation period is from 01/03/2000 to 12/31/2010.

This scatter plot confirms the strong relationship between MES (y -axis) and firm beta (x -axis). Even when Proposition 1 is no longer valid, the OLS estimated slope coefficient (0.0248) is extremely close to the unconditional ES of the market at 5%, 0.0252 or 2.52% (see Equation 11).¹⁸ The main implication of this result is that systemic risk rankings of financial institutions based on their MES tend to mirror rankings obtained by sorting firms on betas. We illustrate this point in Table 2 in which we display the tickers of the top 10 financial institutions based on MES (second column) and betas (third column) on December 31, 2010. We see that seven out of the ten highest beta firms are also identified among the top 10 SIFIs according to their MES. This

¹⁷The data requirement allows us to estimate the average ES of the market return over the same period for all firms.

¹⁸Similar results (not reported) are obtained when we consider unconditional (constant) betas rather than conditional betas, or when we consider the firm MES and beta at a given point in time rather than averages.

very high matching score is by no means specific to this date as shown in Figure 4 and remains pervasive during the entire sample period.¹⁹

Table 2: Systemic Risk Rankings: MES, Beta and SRISK

Rank	MES	β	SRISK	LVG	LTQ	MV
1	MBI	MBI	BAC	SLM	BAC	JPM
2	AIG	LM	C	HIG	JPM	WFC
3	MI	JNS	JPM	LNC	C	C
4	CBG	MI	MS	MS	WFC	BAC
5	RF	CBG	AIG	PRU	GS	GS
6	LM	AIG	MET	MET	MS	BRK
7	JNS	ACAS	PRU	GNW	MET	USB
8	HRB	AMTD	HIG	BAC	AIG	AXP
9	BAC	BAC	SLM	AIG	PRU	MET
10	UNM	ATFC	LNC	RF	HIG	MS
Pairs	MES	β	SRISK	LVG	LTQ	MV
β	7	–	2	2	2	1
SRISK	2	2	–	7	8	5
LVG	3	2	7	–	6	2
LTQ	2	2	8	6	–	7
MV	1	1	5	2	7	–

Notes: In the upper panel, the column labeled MES displays the ranking of the top 10 financial institutions based on MES, listed from most to least risky. The following five columns display the top 10 financial institutions based on conditional beta (β), SRISK, Leverage (LVG), Liabilities (LTQ), and Market Value (MV) of equity, respectively. In the lower panel, we report the number of concordant pairs between two risk measures. The ranking is for December 31, 2010.

Unlike with MES, systemic risk rankings based on SRISK reflect extra information over and above systematic risk. Table 2 shows that, as of December 31, 2010, only one high-beta bank (Bank of America) is identified as a SIFI by the SRISK measure. This result is due to the fact that the SRISK also reflects the leverage of the firm. The scatter plot in Figure 3 contrasts the average SRISK and beta for all sample firms. The relationship remains clearly positive, but largely less robust than the one obtained with MES (Figure 2).

Accounting for information about market capitalization and liabilities in the definition of the systemic risk measure leads to including some of the largest financial institutions in the SIFI list. This result is in line with the *too-big-to-fail* paradigm, whereas the MES tends to be naturally attracted by interconnected institutions (through the beta), which is more in line with the *too-*

¹⁹The beta measures the linear dependencies between the returns of the firms and of the financial system. Differences in rankings between MES and beta are due to differences observed in these dependencies among firms.

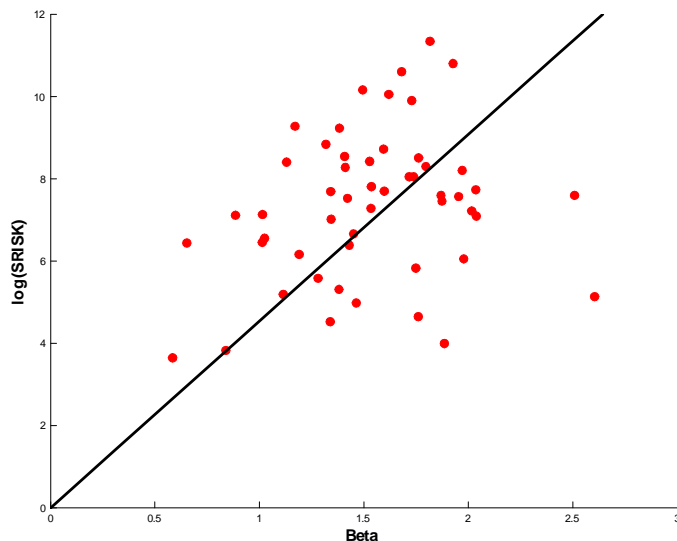


Figure 3: The scatter plot displays the average SRISK estimated for each institution (y -axis) against its beta (x -axis). The beta corresponds to the average of the time-varying beta β_{it} . Each point represents an institution and the solid line is the OLS regression line with no constant. The estimation period is from 01/03/2000 to 12/31/2010.

interconnected-to-fail paradigm (Markose et al., 2010). However, we see in Table 2 that identifying SIFIs with SRISK is not equivalent to simply considering a threshold on the level of debt, firm size or leverage. In that sense, the SRISK is a nice compromise between the *too-big-to-fail* and *too-interconnected-to-fail* dimensions. We illustrate this property of SRISK in Figure 4.

The middle panel of Figure 4 displays the daily percentage of concordance between the top 10 financial institutions according to SRISK and beta, while the bottom panel displays a similar graph for SRISK and leverage. Unlike for MES-beta (top panel, 85.1% match), the matching is far from perfect for SRISK-beta, with an average percentage of concordant pairs of 23.3%. However, SRISK rankings are not mechanically determined by leverage (71.4% match on average); especially during the crisis both correlations and MES vary across firms. The minimum percentage of concordant pairs between SRISK and leverage during the 2008 crisis is equal to 20%: during this period only two top-leverage firms are identified as top SIFI by the SRISK.

Time-Series Analysis and Systemic Risk Forecasting

Another application of systemic risk modeling techniques is to study, and ideally forecast, the dynamics of the contribution of *a particular financial institution* to the risk of the system. In this section, we study the main forces driving MES and SRISK through time.

We have shown that under the A1 assumption, the MES is defined as the product of the market ES and the firm beta. Since the market ES is not constant through time, forecasting the conditional

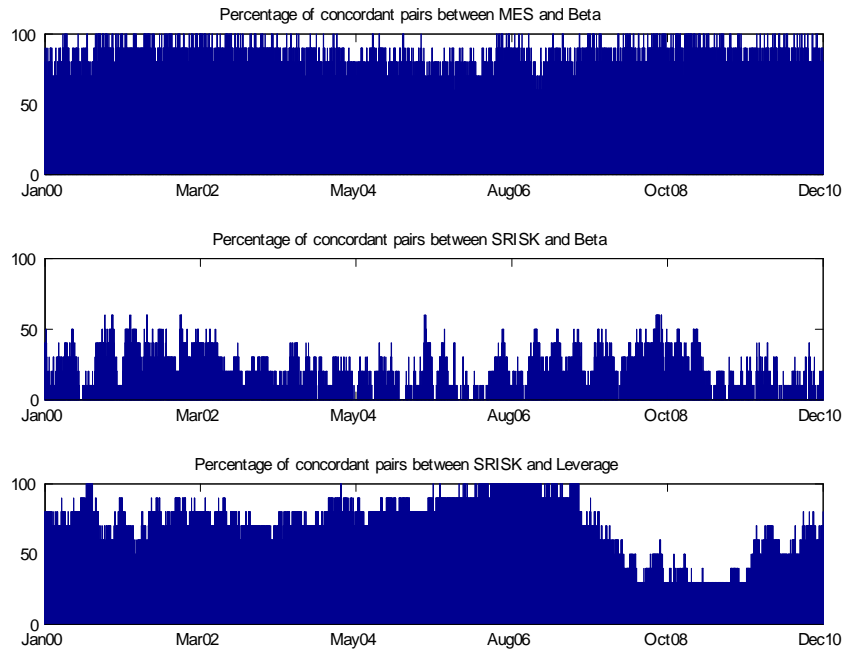


Figure 4: These figures show the daily percentage of concordant pairs between the top 10 financial institutions according to the MES and beta (top figure), the SRISK and beta (middle figure), and the SRISK and leverage (bottom figure), respectively. The estimation period is from 01/03/2000 to 12/31/2010.

beta may not be sufficient to forecast the MES. We consider as an example Bank of America, which is generally the top SIFI according to SRISK in our sample. The top panel in Figure 5 displays the in-sample time profile of its MES and of its conditional beta.²⁰ We notice that the evolution of MES and beta can be very different at times. For instance, MES keeps increasing between 2007:Q4 and 2009:Q4 whereas the beta decreases after the 2008 crisis. So, during this period, measuring the systemic risk of Bank America by focusing solely on its beta would have been misleading. The next two panels plot the evolution of the SRISK with beta (middle panel) and leverage (lower panel). We see that, unlike beta and leverage, SRISK can be seen as a leading indicator of systemic risk.

²⁰Similar results are obtained for out-of-sample forecasts.

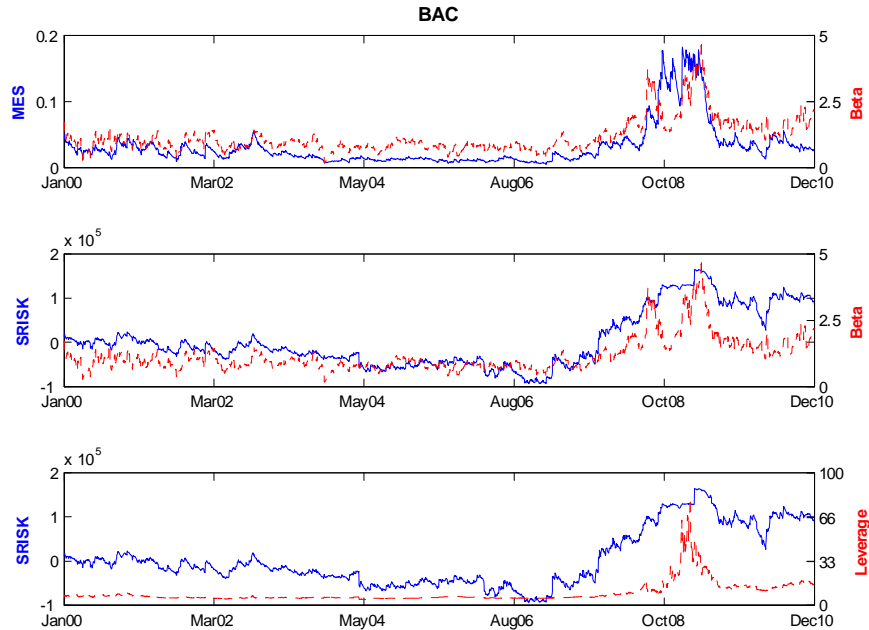


Figure 5: The top figure displays the 5%-MES (blue solid line, left y -axis) and the conditional beta (red dashed line, right y -axis) of Bank of America. The middle figure shows its SRISK (blue solid line, left y -axis) and its conditional beta (red dashed line, right y -axis). The bottom figure displays its SRISK (blue solid line, left y -axis) and its leverage (red dashed line, right y -axis). The estimation period is from 01/03/2000 to 12/31/2010.

4.3 Main Forces Driving ΔCoVaR -Based Systemic Risk Rankings and Forecasts

Estimation

When we relax the A1 assumption, ΔCoVaR can no longer be expressed as a simple function of the correlation between returns. For any conditioning event $\mathbb{C}(r_{it}) : r_{it} = c_t, \forall c_t \in \mathbb{R}$, the CoVaR satisfies:

$$\int_{-\infty}^{\text{CoVaR}_t^{m|c_t}} f_{r_i, r_m}(x, c_t) dx = \alpha \int_{-\infty}^{\infty} f_{r_i, r_m}(x, c_t) dx, \quad (20)$$

where $f_{r_i, r_m}(x, y)$ denotes the joint distribution of (r_{it}, r_{mt}) . There is no closed form for the CoVaR, but it can be estimated in various ways including a copula function, a time-varying second-order moments model, or by bootstrapping past returns. Adrian and Brunnermeier (2011) suggest to use a standard quantile regression (Koenker and Bassett, 1978) of the market return on a particular firm return for the α -quantile:

$$r_{mt} = \mu_{\alpha}^i + \gamma_{\alpha}^i r_{it}. \quad (21)$$

For a conditioning event $\mathbb{C}(r_{it}) : r_{it} = \text{VaR}_{it}(\alpha)$, where $\text{VaR}_{it}(\alpha)$ denotes the conditional

VaR of the i^{th} financial institution The CoVaR defined by:

$$\Pr \left(r_{mt} \leq CoVaR^{m|VaR_{it}(\alpha)} \mid r_{it} = VaR_{it}(\alpha) \right) = \alpha, \quad (22)$$

is estimated by $CoVaR^{m|VaR_{it}(\alpha)} = \hat{\mu}_\alpha^i + \hat{\gamma}_\alpha^i VaR_{it}(\alpha)$, where $\hat{\mu}_\alpha^i$ and $\hat{\gamma}_\alpha^i$ denote the estimated parameters of the quantile regression.²¹ A similar result is obtained for the CoVaR defined for the median state of the institution, $CoVaR_t^{m|median(r_i)} = \hat{\mu}_\alpha^i + \hat{\gamma}_\alpha^i VaR_{it}(0.5)$. Then, by definition, the $\Delta CoVaR$ is equal to:

$$\Delta CoVaR_{it}(\alpha) = \hat{\gamma}_\alpha^i (VaR_{it}(\alpha) - VaR_{it}(0.5)). \quad (23)$$

As for MES and SRISK, we get an estimated $\Delta CoVaR$ time-series for each of the 94 sample firms and we propose two types of analyses: (i) a cross-sectional analysis and (ii) a time-series analysis.

Table 3: Systemic Risk Rankings: $\Delta CoVaR$ and VaR

Rank	$\Delta CoVaR$	DCC- $\Delta CoVaR$	VaR
1	HRB	TMK	MBI
2	MI	TROW	MI
3	BEN	EV	AIG
4	CIT	AFL	RF
5	WU	AMP	HRB
6	AIZ	PRU	SNV
7	AXP	SEIC	HBAN
8	JNS	MS	BAC
9	NYB	JNS	FITB
10	MTB	CINF	JNS
Pairs	$\Delta CoVaR$	DCC- $\Delta CoVaR$	VaR
DCC- $\Delta CoVaR$	1	–	1
VaR	3	1	–

Notes: In the upper panel, the column labeled $\Delta CoVaR$ displays the ranking of the top 10 financial institutions based on $\Delta CoVaR$ estimated by quantile regression, listed from most to least risky. The following two columns display the top 10 financial institutions based on DCC- $\Delta CoVaR$ estimated by GARCH-DCC model and VaR, respectively. The lower panel reports the number of concordant pairs between two systemic measures. The ranking is for December 31, 2010.

²¹The conditional VaR itself is not estimated through a quantile regression; rather, it is deduced from the QML estimated conditional variances issued from a TGARCH model. If we assume that the marginal distribution of the standardized firm returns is a location-scale distribution, the conditional VaR satisfies $VaR_{it}(\alpha) = F_i^{-1}(\alpha) \hat{\sigma}_{it}$, where $F_i(\cdot)$ denotes the true distribution of the standardized returns r_{it}/σ_{it} and $\hat{\sigma}_{it}^2$ is the estimated conditional variance. Because the quantile $F_i^{-1}(\alpha)$ is unknown, we estimate it by its empirical counterpart.

Cross-Sectional Analysis and Systemic Risk Ranking

Under A1, we have shown that ΔCoVaR is proportional to the VaR and that the proportionality coefficient is firm-specific. The main implication of this result is that the riskiest institution in terms of VaR is not necessarily the systemically riskiest institution. If we relax the A1 assumption and use a quantile regression to estimate the ΔCoVaR (Equation 23), we get a similar result: ΔCoVaR remains proportional to the VaR and the relationship between both risk measures is firm specific. The only difference is that the proportionality coefficient γ_{α}^i , issued from the quantile regression (Equation 21), is constant over time, whereas this was not the case in our time-varying second-order moment framework (see Proposition 2). However, the systemic risk ranking based on ΔCoVaR is not equivalent to a VaR-based ranking. Table 3 displays the tickers of the top 10 financial institutions according to their ΔCoVaR (second column) and VaR (fourth column) at 12/31/2010. Among the top 10 SIFIs identified by the ΔCoVaR , only three of them belong to the top 10 VaR group: H&R Block, Marshall & Ilsley, and Janus Capital. In that sense, the Conditional VaR is not equivalent to the VaR as correctly pointed out by Adrian and Brunnermeier (2011).

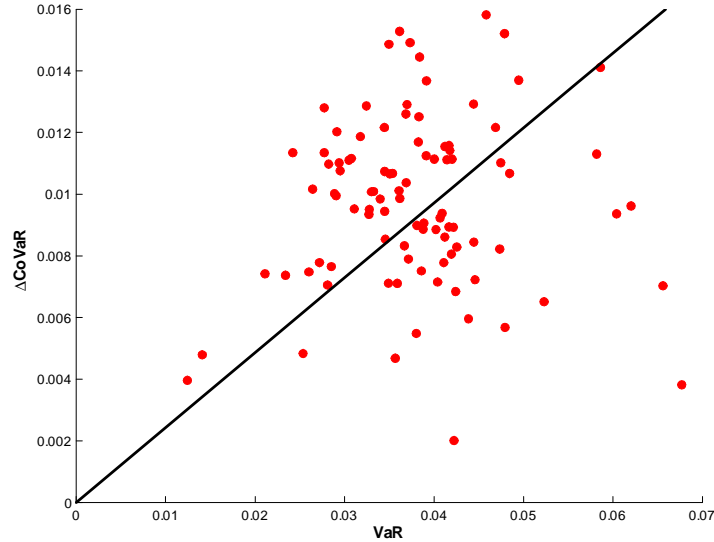


Figure 6: The scatter plot shows the cross-sectional link between the time-series average of the ΔCoVaR estimated for each institution (y -axis) and its VaR at 5% (x -axis). Each point represents an institution and the solid line is the OLS regression line with no constant. The estimation period is from 01/03/2000 to 12/31/2010.

This result is robust to the choice of a particular date. Indeed, the average concordant pairs rate of the daily rankings based on VaR and ΔCoVaR is only equal to 45% for the period 2000-2010. It is also robust to the choice of the estimation method. For instance, Table 3 displays the ranking

based on the ΔCoVaR computed from the estimated time-varying second-order moments issued from the DCC model used in subsection 4.2, i.e., $\Delta\text{CoVaR}_{it}(\alpha) = \hat{\gamma}_{it}(VaR_{it}(\alpha) - VaR_{it}(0.5))$, where $\hat{\gamma}_{it} = \hat{\rho}_{it}\hat{\sigma}_{mt}/\hat{\sigma}_{it}$.²² Only one institution (Janus Capital) appears jointly in the top ten riskiest financial institutions according to both the DCC- ΔCoVaR and the VaR. Note that the difference between the rankings issued from quantile regression and the DCC model is due to the nature of the $\Delta\text{CoVaR}/\text{VaR}$ ratio: time-varying γ_{it} in the DCC framework vs. constant γ_{α}^i in the quantile regression.

Another way to illustrate the weak link between institution's risk in isolation and institution's contribution to systemic risk consists in comparing the averages $\overline{\Delta\text{CoVaR}_i} = T^{-1} \sum_{t=1}^T \Delta\text{CoVaR}_{it}(\alpha)$ and $\overline{VaR_i} = T^{-1} \sum_{t=1}^T VaR_{it}(\alpha)$ for the 94 sample firms. The scatter plot of Figure 6 shows the cross-sectional relationship that exists between the two measures. This figure, similar to that proposed by Adrian and Brunnermeier (Figure 1, page 4) with another sample, points out the great dispersion of both measures and confirms that ΔCoVaR is not VaR.

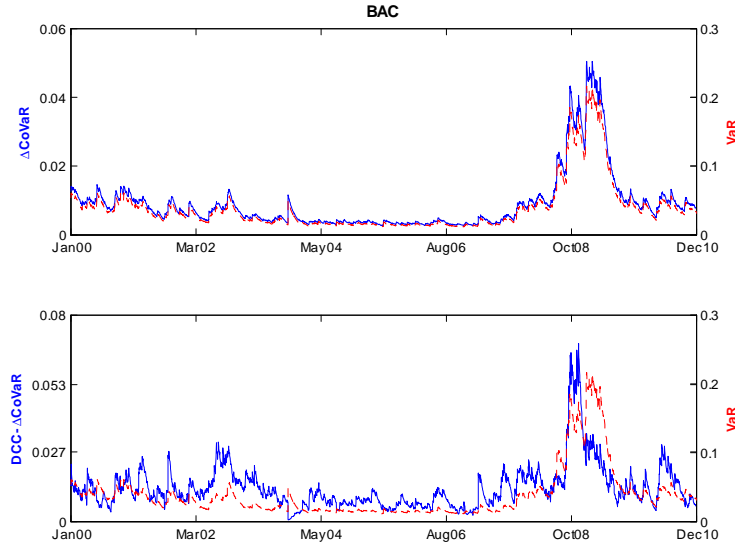


Figure 7: The top figure displays the ΔCoVaR (blue solid line, left y -axis) and the 5%-VaR (red dashed line, right y -axis) of Bank of America. The bottom figure displays the DCC- ΔCoVaR (blue solid line, y -axis) and the 5%-VaR (red dashed line, right y -axis). The estimation period is from 01/03/2000 to 12/31/2010.

Time-Series Analysis and Systemic Risk Forecasting

The ΔCoVaR is not VaR conclusion is more questionable when it comes to forecasting systemic risk. Figure 7 compares the dynamics of the ΔCoVaR and VaR of Bank of America over the entire

²²This last estimate does not require any distributional assumption for the firm return because the DCC is estimated by QML, and we use the empirical quantile of the standardized returns to estimate the VaR.

sample period; the coverage rate α for both risk measures is set at 5%. We see that the two lines match almost perfectly and there is a theoretical reason for this. Indeed, with quantile regression, ΔCoVaR is strictly proportional to the VaR. So, for a given financial institution, ΔCoVaR is nothing else but VaR.²³ This finding implies that, within the ΔCoVaR framework, forecasting the contribution of a firm to the risk of the system is completely equivalent to the tail risk of this firm.

This conclusion is robust to the estimation method used. When one considers the DCC- ΔCoVaR (bottom panel of Figure 7), the correlation between VaR and ΔCoVaR is less than one, but remains very high (0.73). The fact that the correlation is not equal to one is due to the time variability of the proportionality coefficient γ_{it} in the DCC model. In that sense, a time-varying coefficient approach, based either on a multivariate GARCH model or Kalman filter, should be preferred to model the dependencies between firm and market returns.

5 Conclusion

In this paper, we have theoretically and empirically compared several popular systemic risk measures that can be estimated from public data: Marginal Expected Shortfall (MES), Systemic Risk Measure (SRISK), and Delta Conditional Value at Risk (ΔCoVaR). We have derived these popular risk measures into a common framework in order to make the comparison easier and to identify the main forces driving each risk measure. For all systemic risk measures, we derive some analytical expressions that allow us to uncover the theoretical link between systemic risk and standard risks (systematic risk, tail risk, correlation, and beta), as well as firm characteristics (leverage and market capitalization). We show for a sample of large US financial institutions that our decomposition of systemic risk measures remains true in practice.

Which systemic risk measure should we use then? This paper demonstrates that it very much depends on the objective.

First, if the goal is to identify the most systematically important firms and to rank them, two measures should be preferred: SRISK and ΔCoVaR . We have shown, both theoretically and empirically, that MES has a strong tendency to reproduce the ranking based on beta. Out of the two preferred measures, SRISK has one clear advantage: it explicitly depends on market capitalization and liabilities, which leads to systematically identify some of the biggest financial institutions as SIFIs.

Second, if the goal is to forecast the contribution of a particular firm to the global risk of the financial system, MES and SRISK are the favorite measures. Indeed, we have shown that the ΔCoVaR of a given firm is largely determined by its VaR. Thus, forecasting the contribution of

²³Adrian and Brunnermeier (2011) extend the quantile regression (21) by including other lagged state variables M_{t-1} , but conditionally to these variables, the result still holds.

a firm to the systemic risk is equivalent to forecasting the risk generated by its own activities, considered in isolation.

Our results also have some key implications in terms of capital procyclicality. Indeed, US and international banking regulators have recently called for adding some extra capital requirements for SIFIs (Financial Stability Board, 2011). Ideally, the increase in regulatory capital should take place during the run-up phase to the crisis and not at the outbreak of the crisis. Our investigations suggest that the MES, as it depends on beta and market tail risk, is unlikely to comply with this property. Indeed, during a crisis, both beta and market tail risk increase very quickly, and so would a MES-based capital charge. A similar evolution is to be expected with ΔCoVaR . The latter is tightly linked to the firm VaR and the correlation between firm and market returns, which both surge during a crisis. Differently, the main driving forces of SRISK are the six-month ahead MES and leverage. By construction, long-run simulated risk measures are less procyclical than their instantaneous counterparts and the leverage of large financial institutions tend to build up before a financial crisis (Kalemli-Ozcan, Sorensen and Yesiltas, 2011). As a result, SRISK is likely to act as a leading indicator of financial crises, rather than being triggered by them. This theoretical argument is consistent with the actual evolution of the SRISK prior to the 2008 crisis (see Figure 5 and the Stern-NYU's V-Lab website).

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Appendix A: Proof of Proposition 1 (MES)

Proof. Under assumption A1, for any conditioning event C , we have:

$$MES_{it}(C) = \sigma_{it}\rho_{imt}\mathbb{E}_{t-1}\left(\varepsilon_{mt} \mid \varepsilon_{mt} < \frac{C}{\sigma_{mt}}\right), \quad (\text{A1})$$

or equivalently:

$$MES_{it}(C) = \sigma_{it}\rho_{imt}\mathbb{E}_{t-1}(\varepsilon_{mt} \mid r_{mt} < C). \quad (\text{A2})$$

Let $\beta_{it} = \text{cov}(r_{it}, r_{mt}) / \text{var}(r_{mt}) = \rho_{imt}\sigma_{it} / \sigma_{mt}$ denote the time-varying beta of firm i . Combining β_{it} with Equation (A2), we obtain:

$$\begin{aligned} MES_{it}(C) &= \beta_{it}\sigma_{mt}\mathbb{E}_{t-1}(\varepsilon_{mt} \mid r_{mt} < C) \\ &= \beta_{it}\mathbb{E}_{t-1}(r_{mt} \mid r_{mt} < C). \end{aligned} \quad (\text{A3})$$

The MES is expressed as the product between the time-varying beta and the truncated expectation of the market return for any given threshold C . By definition, the conditional expected shortfall of the market return $ES_{mt}(\alpha)$ corresponds to the truncated expectation of the market return for a given threshold equal to the conditional VaR (Jorion, 2007), $C = VaR_{mt}(\alpha)$:

$$ES_{mt}(\alpha) = \mathbb{E}_{t-1}(r_{mt} \mid r_{mt} < VaR_{mt}(\alpha)). \quad (\text{A4})$$

Then, the MES defined for the specific event $C = VaR_{mt}(\alpha)$, denoted $MES_{it}(\alpha)$, is simply expressed as the product of time-varying firm beta and expected shortfall of the market return:

$$MES_{it}(\alpha) = \beta_{it}ES_{mt}(\alpha). \quad (\text{A5})$$

■

Appendix B: Proof of Proposition 2 (ΔCoVaR)

Proof. We consider two cases: a general case with $\rho_{it} \neq 0$ and a particular case with $\rho_{it} = 0$. Given Equations (5) and (6), if $\rho_{it} \neq 0$ then the market return can be expressed as:

$$r_{mt} = \frac{\sigma_{mt}}{\sigma_{it}\rho_{it}} r_{it} - \frac{\sigma_{mt}\sqrt{1-\rho_{it}^2}}{\rho_{it}} \xi_{it}. \quad (\text{B1})$$

Under assumption A_1 , we have $\text{cov}(\xi_{it}, \varepsilon_{mt}) = 0$. For each conditioning event form $\mathbb{C}(r_{it}) : r_{it} = c$, CoVaR is defined as follows:

$$\Pr\left(r_{mt} \leq CoVaR_t^{m|r_{it}=c} \mid r_{it} = c\right) = \alpha, \quad (\text{B2})$$

or equivalently:

$$\Pr\left(\xi_{it} \leq \frac{\rho_{it}}{\sigma_{mt}\sqrt{1-\rho_{it}^2}} \left(\frac{\sigma_{mt}}{\sigma_{it}\rho_{it}}c - CoVaR_t^{m|r_{it}=c}\right) \mid r_{it} = c\right) = 1 - \alpha. \quad (\text{B3})$$

In the special case where the conditional mean function of ξ_{it} is linear in r_{it} , the first two conditional moments of ξ_{it} given $r_{it} = c$ can be expressed as:

$$\begin{aligned} \mathbb{E}(\xi_{it} \mid r_{it} = c) &= \frac{\text{cov}(\xi_{it}, r_{it})}{\sigma_{it}^2} \times c \\ &= \frac{\sigma_{it}\sqrt{1-\rho_{it}^2}}{\sigma_{it}^2} \times c \\ &= \frac{\sqrt{1-\rho_{it}^2}}{\sigma_{it}} \times c, \end{aligned} \quad (\text{B4})$$

$$\begin{aligned}
\mathbb{V}(\xi_{it} \mid r_{it}) &= \mathbb{V}(\xi_{it}) - \mathbb{V}_{r_{it}}[\mathbb{E}(\xi_{it} \mid r_{it})] \\
&= \mathbb{V}(\xi_{it}) \times \left[1 - \left(\frac{\text{cov}(\xi_{it}, r_{it})}{\sigma_{it}^2} \right)^2 \sigma_{it}^2 \right] \\
&= 1 - \left(\frac{\sigma_{it} \sqrt{1 - \rho_{it}^2}}{\sigma_{it}^2} \right)^2 \sigma_{it}^2 \\
&= \rho_{it}^2.
\end{aligned} \tag{B5}$$

Consider $G(\cdot)$ the conditional (location-scale) demeaned and standardized cdf of ξ_{it} such that:

$$\mathbb{E} \left[\frac{1}{\rho_{it}} \left(\xi_{it} - \frac{\sqrt{1 - \rho_{it}^2}}{\sigma_{it}} \times c \right) \mid r_{it} = c \right] = 0, \tag{B6}$$

$$\mathbb{V} \left[\frac{1}{\rho_{it}} \left(\xi_{it} - \frac{\sqrt{1 - \rho_{it}^2}}{\sigma_{it}} \times c \right) \mid r_{it} = c \right] = 1. \tag{B7}$$

Thus, Equation (B3) is expressed as:

$$\frac{1}{\rho_{it}} \left[\frac{\rho_{it}}{\sigma_{mt} \sqrt{1 - \rho_{it}^2}} \left(\frac{\sigma_{mt}}{\sigma_{it} \rho_{it}} c - \text{CoVaR}_t^{m|r_{it}=c} \right) - \frac{\sqrt{1 - \rho_{it}^2}}{\sigma_{it}} \times c \right] = G^{-1}(1 - \alpha).$$

By rearranging these terms, we write the general expression of the CoVaR:

$$\text{CoVaR}_t^{m|r_{it}=c} = -\sigma_{mt} \sqrt{1 - \rho_{it}^2} G^{-1}(1 - \alpha) + \frac{\rho_{it} \sigma_{mt}}{\sigma_{it}} c. \tag{B8}$$

The CoVaR defined for the conditioning event $\mathbb{C}(r_{it}) : r_{it} = \text{median}(r_{it})$, has a similar expression:

$$\text{CoVaR}_t^{m|r_{it}=\text{median}(r_{it})} = -\sigma_{mt} \sqrt{1 - \rho_{it}^2} G^{-1}(1 - \alpha) + \frac{\rho_{it} \sigma_{mt}}{\sigma_{it}} F^{-1}(0.5). \tag{B9}$$

where $F(\cdot)$ denotes the marginal cdf of the firm return. Then, for each conditioning event form $\mathbb{C}(r_{it}) : r_{it} = c$, the ΔCoVaR is defined as:

$$\begin{aligned}
\Delta\text{CoVaR}_{it}(c) &= \text{CoVaR}_t^{m|r_{it}=c} - \text{CoVaR}_t^{m|r_{it}=\text{median}(r_{it})} \\
&= \frac{\rho_{it} \sigma_{mt}}{\sigma_{it}} \times [c - \text{median}(r_{it})]
\end{aligned} \tag{B10}$$

$$= \gamma_{it} \times [c - \text{median}(r_{it})], \tag{B11}$$

where $\gamma_{it} = \rho_{it} \sigma_{mt} / \sigma_{it}$ denotes the time-varying linear projection coefficient of the market return on the firm return. If the marginal distribution of r_{it} is symmetric around zero, then $F^{-1}(0.5) = 0$, and we have:

$$\Delta\text{CoVaR}_{it}(c) = \frac{\rho_{it} \sigma_{mt}}{\sigma_{it}} \times c = \gamma_{it} \times c. \tag{B12}$$

As in Adrian and Brunnermeier (2011) the ΔCoVaR , denoted $\Delta\text{CoVaR}_{it}(\alpha)$, defined for a conditioning event $\mathbb{C}(r_{it}) : r_{it} = \text{VaR}_{it}(\alpha)$ is:

$$\Delta\text{CoVaR}_{it}(\alpha) = \gamma_{it} \times [\text{VaR}_{it}(\alpha) - \text{VaR}_{it}(0.5)], \tag{B13}$$

or

$$\Delta\text{CoVaR}_{it}(\alpha) = \gamma_{it} \times \text{VaR}_{it}(\alpha), \tag{B14}$$

if the marginal distribution of the firm return is symmetric around zero.

Now, we consider the case where $\rho_{it} = 0$, the bivariate process becomes:

$$r_{mt} = \sigma_{mt} \varepsilon_{mt} , \quad (\text{B15})$$

$$r_{it} = \sigma_{it} \xi_{it} , \quad (\text{B16})$$

$$(\varepsilon_{mt}, \xi_{it}) \sim D, \quad (\text{B17})$$

where $\nu_t = (\varepsilon_{mt}, \xi_{it})'$ satisfies $\mathbb{E}(\nu_t) = 0$ and $\mathbb{E}(\nu_t \nu_t') = I_2$, and D denotes the bivariate distribution of the standardized innovations. It is straightforward to show that:

$$\Pr \left(r_{mt} \leq \text{CoVaR}_t^{m|r_{it}=\text{VaR}_{it}(\alpha)} \mid r_{it} = \text{VaR}_{it}(\alpha) \right) = \Pr \left(r_{mt} \leq \text{CoVaR}_t^{m|r_{it}=\text{VaR}_{it}(\alpha)} \right) = \alpha.$$

Hence, we have $\text{CoVaR}_{it}(\alpha) = \sigma_{mt} F_m^{-1}(\alpha)$ and $\Delta \text{CoVaR}_{it}(\alpha) = 0$, where $F_m(\cdot)$ denotes the cdf of the marginal distribution of the standardized market return. ■

Appendix C: Proof of Proposition 4 (Rankings MES- Δ CoVaR)

Proof. First, given Equation (13), the inequality $\Delta \text{CoVaR}_{it}(\alpha) \leq \Delta \text{CoVaR}_{jt}(\alpha)$ is then equivalent to:

$$\frac{\rho_{it}}{\sigma_{it}} \times [\text{VaR}_{it}(\alpha) - \text{VaR}_{it}(0.5)] \leq \frac{\rho_{jt}}{\sigma_{jt}} \times [\text{VaR}_{jt}(\alpha) - \text{VaR}_{jt}(0.5)]. \quad (\text{C1})$$

If we assume that the conditional distribution of the firm return is a location scale distribution, then $\text{VaR}_{it}(\alpha) = \sigma_{it} F_i^{-1}(\alpha)$ where $F_i^{-1}(\alpha)$ denotes the conditional α -quantile of the standardized returns r_{it}/σ_{it} . The inequality becomes:

$$\rho_{it} \times [F_i^{-1}(\alpha) - F_i^{-1}(0.5)] \geq \rho_{jt} \times [F_j^{-1}(\alpha) - F_j^{-1}(0.5)]. \quad (\text{C2})$$

For simplicity, we assume that the two conditional distributions for firms i and j are identical, i.e., $F_i^{-1}(\cdot) = F_j^{-1}(\cdot) = F^{-1}(\cdot)$. The difference $F^{-1}(\alpha) - F^{-1}(0.5)$ is typically a negative number, so the inequality $\Delta \text{CoVaR}_{it}(\alpha) \leq \Delta \text{CoVaR}_{jt}(\alpha)$ can be reduced to the simple condition $\rho_{it} \geq \rho_{jt}$.

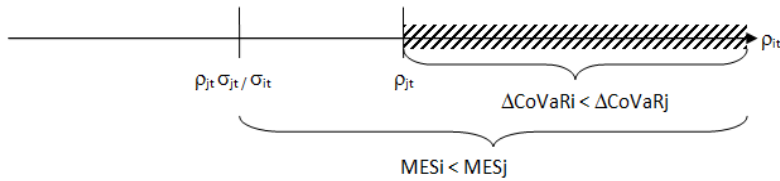
$$\Delta \text{CoVaR}_{it}(\alpha) \leq \Delta \text{CoVaR}_{jt}(\alpha) \iff \rho_{it} \geq \rho_{jt}. \quad (\text{C3})$$

Second, the inequality $\text{MES}_{it}(\alpha) \leq \text{MES}_{jt}(\alpha)$, means that $\beta_{it} \geq \beta_{jt}$ since the system's ES is negative, $ES_{mt} < 0$. Given the definition of conditional beta, this inequality is equivalent to the condition $\sigma_{it} \rho_{it} \geq \sigma_{jt} \rho_{jt}$.

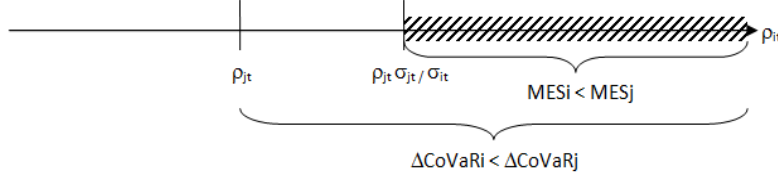
$$\text{MES}_{it}(\alpha) \leq \text{MES}_{jt}(\alpha) \iff \sigma_{it} \rho_{it} \geq \sigma_{jt} \rho_{jt}. \quad (\text{C4})$$

We have simultaneously $\text{MES}_{it}(\alpha) \leq \text{MES}_{jt}(\alpha)$ and $\Delta \text{CoVaR}_{it}(\alpha) \leq \Delta \text{CoVaR}_{jt}(\alpha)$ when conditions (C3) and (C4) are satisfied. Given the relative values of the volatilities, two cases can be studied separately.

Case a: $\sigma_{it} \geq \sigma_{jt}$. The conditions (C3) and (C4) are satisfied if $\rho_{it} \geq \rho_{jt}$.



Case b: $\sigma_{it} \leq \sigma_{jt}$. The conditions (C3) and (C4) are satisfied if $\rho_{it} \geq \rho_{jt}\sigma_{jt}/\sigma_{it}$.



Then, the systemic risk rankings (MES and ΔCoVaR) of both financial institutions are identical when we have:

$$\rho_{it} \geq \max\left(\rho_{jt}, \frac{\rho_{jt}\sigma_{jt}}{\sigma_{it}}\right). \quad (\text{C5})$$

If the two conditional distributions $F_i(\cdot)$ and $F_j(\cdot)$ are different, but location-scale, this condition becomes:

$$\rho_{it} \geq \max\left(\rho_{jt}, \rho_{jt} \frac{[F_j^{-1}(\alpha) - F_j^{-1}(0.5)]}{[F_i^{-1}(\alpha) - F_i^{-1}(0.5)]}\right), \quad (\text{C6})$$

and if they are not location-scales it is:

$$\rho_{it} \geq \max\left(\rho_{jt}, \rho_{jt} \frac{\sigma_{it} [VaR_{jt}(\alpha) - VaR_{jt}(0.5)]}{\sigma_{jt} [VaR_{it}(\alpha) - VaR_{it}(0.5)]}\right). \quad (\text{C7})$$

■

Appendix D: Proof of Proposition 5 (Rankings SRISK- ΔCoVaR)

Proof. Given Equation (12) of the SRISK, the financial institution i is riskier than firm j if:

$$k D_{it} - (1 - k) W_{it} \exp(18 \times \beta_{it} \times ES_{mt}(\alpha)) \geq k D_{jt} - (1 - k) W_{jt} \exp(18 \times \beta_{jt} \times ES_{mt}(\alpha)).$$

For simplicity, we consider two firms with the same level of liabilities, $D_{it} = D_{jt}$. Then, the inequality $SRISK_{it}(\alpha) \geq SRISK_{jt}(\alpha)$ is equivalent to:

$$W_{it} \exp(18 \times \beta_{it} \times ES_{mt}(\alpha)) \leq W_{jt} \exp(18 \times \beta_{jt} \times ES_{mt}(\alpha)). \quad (\text{D1})$$

As shown in Appendix C, under some mild assumptions, we have:

$$\Delta\text{CoVaR}_{it}(\alpha) \leq \Delta\text{CoVaR}_{jt}(\alpha) \iff \rho_{it} \geq \rho_{jt}. \quad (\text{D2})$$

The systemic risk ranking given by the SRISK and the ΔCoVaR are convergent when conditions (D1) and (D2) are satisfied. These conditions can be expressed as constraints on both the correlation and the market value of the riskiest firm i :

$$\rho_{it} \geq \rho_{jt} \quad \text{and} \quad W_{it} \leq W_{jt} \exp[18 \times ES_{mt}(\alpha) \times (\beta_{jt} - \beta_{it})]. \quad (\text{D3})$$

■

Proof. A similar result can be obtained for short-run SRISK. Given the expression of the short-run SRISK, firm i is riskier than firm j if:

$$k D_{it} - (1 - k) W_{it} \beta_{it} ES_{mt} \geq k D_{jt} - (1 - k) W_{jt} \beta_{jt} ES_{mt}, \quad (\text{D4})$$

For simplicity, we consider two firms with the same level of liabilities, $D_{it} = D_{jt}$. Then, the inequality $SRISK_{it}^{SR}(\alpha) \geq SRISK_{jt}^{SR}(\alpha)$ is equivalent to

$$W_{it}\beta_{it}ES_{mt} \leq W_{jt}\beta_{jt}ES_{mt}. \quad (D5)$$

Since the ES of the market return is typically negative and is not firm-specific and since $\beta_{zt} = \rho_{zt}\sigma_{zt}/\sigma_{mt}$ for $z = \{i, j\}$, this condition can be expressed as:

$$SRISK_{it}^{SR}(\alpha) \geq SRISK_{jt}^{SR}(\alpha) \iff W_{it}\sigma_{it}\rho_{it} \geq W_{jt}\sigma_{jt}\rho_{jt}. \quad (D6)$$

As shown in Appendix C, under some mild assumptions, we have:

$$\Delta CoVaR_{it}(\alpha) \leq \Delta CoVaR_{jt}(\alpha) \iff \rho_{it} \geq \rho_{jt}. \quad (D7)$$

The systemic risk ranking given by the short-run SRISK and the $\Delta CoVaR$ are convergent when conditions (D6) and (D7) are satisfied. These conditions can be expressed as constraints on both the correlation and the market value of the riskiest firm i :

$$\rho_{it} \geq \rho_{jt} \quad \text{and} \quad W_{it} \geq W_{jt} \frac{\rho_{jt}\sigma_{jt}}{\rho_{it}\sigma_{it}}, \quad (D8)$$

or equivalently:

$$\rho_{it} \geq \rho_{jt} \quad \text{and} \quad W_{it} \geq W_{jt}(\beta_{jt} - \beta_{it}). \quad (D9)$$

This result illustrates the difference between the short-run SRISK and the SRISK. The first measure is an increasing function of the market value, whereas the second is a decreasing one. Thus, the systemic ranking of the $\Delta CoVaR$ and the short-run SRISK will be similar if and only if the riskiest firm has the highest market capitalization, and consequently the lowest leverage. ■

Appendix E: Dataset

Tickers and Company Names by Industry Groups

Depositories (29)		Insurance (32)	
BAC	Bank of America	ABK	Ambac Financial Group
BBT	BB&T	AET	Aetna
BK	Bank of New York Mellon	AFL	AFLAC
C	Citigroup	AIG	American International Group
CBH	Commerce Bancorp	AIZ	Assurant
CMA	Comerica Inc.	ALL	Allstate Corp.
HBAN	Huntington Bancshares	AOC	Aon Corp.
HCBK	Hudson City Bancorp	BKLY	W.R. Berkley Corp.
JPM	JP Morgan Chase	BRK	Berkshire Hathaway
KEY	Keycorp	CB	Chubb Corp.
MI	Marshall & Ilsley	CFC	Countrywide Financial
MTB	M&T Bank Corp.	CI	CIGNA Corp.
NCC	National City Corp.	CINF	Cincinnati Financial Corp.
NTRS	Northern Trust	CNA	CNA Financial Corp.
NYB	New York Community Bancorp	CVH	Coventry Health Care
PBCT	Peoples United Financial	FNF	Fidelity National Financial
PNC	PNC Financial Services	GNW	Genworth Financial
RF	Regions Financial	HIG	Hartford Financial Group
SNV	Synovus Financial	HNT	Health Net
SOV	Sovereign Bancorp	HUM	Humana
STI	Suntrust Banks	LNC	Lincoln National
STT	State Street	MBI	MBIA
UB	Unionbancal Corp.	MET	MetLife
USB	US Bancorp	MMC	Marsh & McLennan
WB	Wachovia	PFG	Principal Financial Group
WFC	Wells Fargo & Co	PGR	Progressive
WM	Washington Mutual	PRU	Prudential Financial
WU	Western Union	SAF	Safeco
ZION	Zions	TMK	Torchmark
		TRV	Travelers
		UNH	UnitedHealth Group
		UNM	Unum Group
Broker-Dealers (10)		Others (23)	
AGE	A.G. Edwards	ACAS	American Capital
BSC	Bear Stearns	AMP	Ameriprise Financial
ETFC	E*Trade Financial	AMTD	TD Ameritrade
GS	Goldman Sachs	AXP	American Express
LEH	Lehman Brothers	BEN	Franklin Resources
MER	Merill Lynch	BLK	BlackRock
MS	Morgan Stanley	BOT	CBOT Holdings
NMX	Nymex Holdings	CBG	C.B. Richard Ellis Group
SCHW	Schwab Charles	CBSS	Compass Bancshares
TROW	T. Rowe Price	CIT	CIT Group
		CME	CME Group
		COF	Capital One Financial
		EV	Eaton Vance
		FITB	Fifth Third Bancorp
		FNM	Fannie Mae
		FRE	Freddie Mac
		HRB	H&R Block
		ICE	Intercontinental Exchange
		JNS	Janus Capital
		LM	Legg Mason
		NYX	NYSE Euronext
		SEIC	SEI Investment Company
		SLM	SLM Corp.