The Dynamics of Sovereign Credit Risk

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Abstract

This paper proposes a structural model for sovereign credit risk with endogenous sovereign debt and default policies. The model generates daily sovereign credit spreads using expected economic conditions, which are structurally extracted from the local stock market. The model-implied spreads explain most of the daily variation in sovereign spreads for emerging and European economies over the 2000-2011 period. Results are robust in- and out-of-sample, and hold before and during the European debt crisis. In contrast, global factors related to US equities, market uncertainty, and funding liquidity seem to play a marginal role in the explanation of sovereign spreads. This model also successfully replicates the level and the volatility of sovereign spread data.

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1 Introduction

The widespread rise in sovereign credit risk, recently illustrated by the 2010-2011 European sovereign debt crisis and the downgrade of the US debt in August 2011, has become a threat to global financial markets.[1] Investors thus pay increasing attention to the assessment of the risk embedded in sovereign debt, one of the largest asset classes in the world with $42 trillion in principal.[2] Understanding the time variation in sovereign credit risk is therefore of the utmost importance today. The aim of this paper is to develop a framework that provides useful guidance on the dynamics of sovereign credit risk and to investigate how this risk differs among emerging and European countries.

In this paper, I extend our knowledge of the pricing of sovereign default risk, with a specific analysis of the period that spans the recent 2010-2011 debt crisis. The contribution of the paper is twofold. First, it proposes a new structural model for sovereign credit risk that determines jointly the optimal sovereign debt policy and the timing of sovereign default. The model provides an analytical formula for sovereign credit spreads that can be easily compared to market spreads. Second, this study demonstrates the ability of the pricing model to explain the level and the daily variation in sovereign credit spreads for large emerging and European debt issuers.[3] Results are robust in- and out-of-sample, and hold before and during the 2010-2011 European debt crisis.

The theoretical framework consists of an economy with risk-neutral investors, a representative firm, and a government. The firm pays income taxes to the government, which are used to service sovereign debt. The government chooses the optimal level of debt and the default policy. The level of debt arises from a trade-off between the economic benefits of the debt issue (e.g., return on public investment) and the expected cost of default (e.g., economic contraction). Building on the contingent claims approach, sovereign default is triggered when the present value of firm income (the fundamentals) falls below an endogenous boundary, which characterizes the government's default policy. This policy is determined by the trade-off between the cost of default and the reduction in the level of debt service

[1] The financial press regularly reports the negative consequences of European sovereign credit risk on international stock markets. Recent articles include "Greek fears hit global stocks, bond spreads" (Reuters, April 8, 2010), "Global Markets Tumble on Continued Worries Over Europe's Debt Crisis" (The New York Times, July 11, 2011), and "Dow Drops 520 Amid New Europe Debt Concern" (Bloomberg, August 11, 2011), among others.

[2] Quarterly statistics on domestic and international sovereign debt can be found in Tables 12D and 16A of the BIS website (http://www.bis.org/statistics/secstats.htm).

[3] The emerging countries under consideration are Brazil, Colombia, Mexico, Peru, the Philippines, Russia, and Turkey, representing large government debt issuers in emerging markets with different geopolitical characteristics and levels of credit risk. The European countries under consideration are France, Greece, Ireland, Italy, Portugal, and Spain. The total value of government debt for these countries is $8.5 trillion of principal in 2010 (source: national sources and BIS). Emerging market debt consists of $2.95 trillion, while European debt equals $5.55 trillion.

through debt restructuring. Within the model, investors have perfect knowledge of the economy and government policies, and thus value sovereign debt and firm equity accordingly. Sovereign credit spreads and stock market prices jointly depend on the same fundamentals. Thus, the analytical solution for a country's stock market price allows to structurally infer the level of expected fundamentals, which can then be used for the computation of the sovereign spread. Importantly, the structural model explicitly accounts for the direct influence of sovereign default risk on the stock market price, through the economic contraction that a default would trigger.

The novel empirical approach of the paper is to generate sovereign credit spreads using information on expected fundamentals extracted structurally from local stock market prices. The market's expectations of future income embedded in the local stock market is particularly relevant for the pricing of sovereign credit risk.[4] This information is forward-looking and allows for the computation of sovereign credit spreads at any frequency. In comparison, data on government fiscal revenues or firm income are observable at low frequency and contain past information only.

The model parameters, which consist of the expected return on public investment and the level of macroeconomic risk, are estimated country-by-country to minimize the sum of the squared pricing errors between model credit spreads and those observed in the data. The sample period spans 2000-2011 for emerging markets and 2006-2011 for European countries.[5] The benchmark credit spreads for emerging countries are the Emerging Markets Bond Index (EMBI+) spreads computed by JP Morgan.[6] For European countries, the spreads are equal to the difference between JP Morgan Global Bond Index (GBI) yield-to-maturity of a country and a maturity-matched risk-free rate, which corresponds to the yield-to-maturity of a German government bond with the same maturity as the average maturity of the country's debt.

The estimation indicates that the model can successfully replicate the level and the volatility of sovereign spreads. The pricing errors are negligible, ranging from 2 basis points in emerging markets to

[4]The valuation of the local stock market also depends on the probability of a sovereign default. The structural model accounts for this effect. In addition, the relationship between stock prices and sovereign spreads is severely non-linear. Therefore, the role of the structural model also resides in breaking down this non-linearity to generate estimates of credit spreads that are linearly related - and thus easily comparable - to observed sovereign spreads.

[5]European sovereign spreads remain essentially constant over time (and close to zero) over the period 2000-2005. In contrast, the analysis of European sovereign credit risk is particularly interesting during the second half of the decade. This paper therefore focuses on the 2006-2011 period, which exhibits phases of stable and volatile sovereign credit spreads.

[6]JP Morgan, one of the major dealers in the Brady market, derives the credit spread implied in the price of each Brady bond. The company computes each country's EMBI+ index, which is a weighted average index of spreads using the country's most liquid Brady bonds. Focusing on Brady bonds rather than other debt instruments (e.g., sovereign credit default swap spreads) is likely to improve the analysis of credit risk as they present by far the best liquidity. For emerging markets, I only consider external debt denominated in US dollars because governments can monetize their debt when denominated in local currency.

3 basis points in Europe. The model-implied spreads match the level of observed sovereign spreads over quiet and crisis periods, and across countries. This is in contrast to the underestimation of the credit spread typically obtained from previous models. For example, the level of credit spreads predicted in Arellano (2008), Hilscher and Nosbusch (2010), and Yue (2010) is, respectively, 45%, 61%, and 26% of the level of observed sovereign spreads.

The model generates sovereign credit spreads comparable to those of the data with low government incentives for debt issuance. The estimated return on public investment is, on average, 6.2% in emerging countries and 4% in Europe before 2010. During the 2010-2011 European debt crisis, these estimates decrease to 4.2% in emerging countries and 3.7% in Europe. Emerging economies thus offer enhanced investment opportunities compared to European countries, while the recent crisis has lowered government incentives for debt issuance. The level of macroeconomic risk has an average estimate of 52% for emerging markets and 20% for European countries during the period that precedes the European crisis. The estimates are equal to 34% and 24% over the period 2010-2011, respectively. The difference in average estimates between the two groups of countries appears in line with the intuition that European countries generally present lower economic uncertainty than emerging countries. Interestingly, emerging markets have become safer over recent years, while the uncertainty related to European economies has increased during the 2010-2011 crisis.

A central result of the paper is that the model explains a substantial fraction of the variation across time in sovereign credit risk. Using country-by-country time-series estimations, the model-implied credit spreads explain 66% (before 2010) and 40% (2010-2011) of the daily variation in emerging spreads and 80% (before 2010) and 59% (2010-2011) of the daily variation in European spreads. The ability of the model to explain sovereign credit spreads holds with monthly data, with explanatory powers that remain remarkably similar, thereby illustrating the stability of the model.

An out-of-sample exercise provides further robustness for the pricing model. The procedure is to estimate the model over two years of data (500 trading days) with rolling-windows and generate sovereign spread forecasts at one month horizon (20 trading days). The structural model closely replicates the dynamics of sovereign spreads for all countries. More importantly, it outperforms the standard benchmark (i.e., random walk model) for most countries using different forecasting accuracy measures and horizons.

The model explains a large fraction of the time variation in sovereign credit spreads. Yet it does not explain all variation. Remolona et al. (2008) and Longstaff et al. (2011) suggest that variations in sovereign credit risk can be largely attributed to changes in common factors across countries, with country-specific information playing a secondary role only. They consider a number of measures from the equity and fixed income markets that can affect global investors, such as the return on the S&P500 index and the US Treasury rate. The option-implied volatility index (VIX), which captures a source of risk premium in the US equity market, also appears as a key factor in their explanation of sovereign credit risk. Pan and Singleton (2008) and Hilscher and Nosbusch (2010) similarly conclude on the importance of the VIX as a determinant of sovereign credit spreads.

Hence, what is the importance of market-wide factors relative to the model credit spreads in the explanation of sovereign credit spreads? To address this crucial question, I analyze the fraction of the total variation explained by a regression with additional global factors that is due solely to the variation in model spreads.[7] Following Longstaff et al. (2011), I refer to this measure as the local ratio. I consider as additional factors the implied volatility of the US equity market (VIX), the return on the S&P500 index, the 5-year US Treasury rate, and the slope of the US term structure, computed as the difference between the 30-year and the 3-month US Treasury rates. The average local ratio over the full sample period is 82% and 64% for emerging and European countries, respectively. The dynamics of sovereign credit risk is then explained primarily by the structural model, which accounts for a single source of country-specific information, with market-wide factors playing a secondary role only.[8] This finding may partially change interpretations of the results of Longstaff et al. (2011), as these authors conclude that credit risk movements are mostly attributed to changes in common factors across countries. Interestingly, both studies lead to similar findings when the focus is on countries with large and liquid equity and sovereign debt markets, such as those studied in the present paper.[9]

The data suggest that sovereign credit risk in Europe exhibits a behavior that stands apart from emerging markets. Factors related to measures of global market uncertainty (e.g., VIX) seem to contain relevant additional information for the pricing of sovereign credit risk in emerging countries. In contrast, European sovereign credit risk exhibits wide fluctuations that can only be explained by global funding liquidity and flight-to-quality measures (e.g., US Treasury market). The strong political and

[7] To calculate this measure, I first regress the observed sovereign spreads on model spreads, and then divide the adjusted R^2 from this regression by the adjusted R^2 from the regression with additional factors.

[8] Hilscher and Nosbusch (2010) draw on a similar conclusion using country-specific fundamentals measured by a country's terms of trade.

[9] Longstaff et al. (2011) compute the local ratio as the fraction of the total variation explained by local variables (e.g., stock market index, exchange rate, foreign reserves) for 26 countries and report an estimate of 43%. When limiting the analysis to the 7 emerging countries considered here, their average local ratio raises to 66%, thus reconciling the results of both studies. A possible explanation for their reported low estimate is the inclusion of small countries in the sample, for which illiquidity in the sovereign credit default swap market is certainly high (e.g., Panama, Qatar, Slovak, Venezuela).

international dimensions embedded in the European crisis seem to be natural candidates to explain why sovereign credit risk in European countries is particularly sensitive to those factors. The decision of the European Central Bank to intervene or not in the secondary bond market, the willingness of European banks to accept or not a voluntary restructuring, and the discussions between French and German leaders on the policies to prevent a contagion in the default crisis, for example, are likely to affect sovereign credit spreads beyond the level of uncertainty perceived in the US equity market.

The results of this paper help us better understand sovereign credit risk in emerging and European countries. First, a pricing model with endogenous debt and default policies can be conveniently applied to the data to generate credit spreads that closely match market spreads. Importantly, the model can be used for a variety of different countries. This paper thus contributes to a growing literature among finance researchers that tries to better quantify the risk of sovereign default. Second, the data show that the local stock market provides forward-looking information on firm income that is insightful for the pricing of daily sovereign credit risk. This approach offers an explanation of the substantial daily variation in sovereign credit spreads that cannot be explained by economic variables known to be determinants of sovereign credit risk, and only available quarterly or annually. Third, the results reveal that the time-variation in sovereign credit spreads is primarily driven by changes in expected domestic fundamentals, which affect the risk of sovereign default, and only secondarily by measures of global market sentiment, which affect the pricing of such risk. Finally, the data clearly indicate that the deterioration in local economic conditions has played a crucial role in the widening of European credit spreads during the recent period of distress. A better understanding of the international and the political nature of the European debt crisis could further improve our knowledge of how investors price the risk of sovereign default. Such an analysis is beyond the scope of the paper and is thus left for future research.

Related Literature

Various studies are related to the empirical analysis of the paper. Duffie, Pedersen, and Singleton (2003) and Pan and Singleton (2008) consider a reduced-form affine structure model to study weekly and daily spreads, respectively. Other studies investigate monthly credit spreads using a reduced-form contingent-claims analysis (Bodie, Gray, and Merton, 2007; Gapen et al., 2008), a structural model of the balance sheets (François, Hübner, and Sibille, 2011), a principal components analysis (Longstaff et al., 2011), or a linear regression model (Remolona, Scatigna, and Wu, 2008). Studies analyzing the dynamics of quarterly or annual credit spreads consider a dynamic stochastic equilibrium model (e.g., Arellano, 2008; Yue, 2010) or a panel-based approach (e.g., Hilscher and Nosbusch, 2010, and the references therein). An advantage of the proposed model is that it provides an intuitive theoretical framework for the analysis of credit spreads that can be applied at any frequency.

The recent credit risk literature provides the foundation for the empirical strategy of the paper. The approach is to consider credit spread levels, rather than credit spread changes.[10] As discussed in Doshi et al. (2011), this specification seems preferable for several reasons. First, credit spreads are ex ante expected return differentials and are thus expected to be stationary. Second, while the time-series of credit spreads are highly auto-correlated, they are mean-reverting and not characterized by long-term stochastic trends. Third, regressing changes of a stationary time-series rather than levels introduces noise into the estimation, which increases with the frequency of the data. There is indeed a large amount of noise at the daily level that vanishes when the frequency of the data decreases: the model's explanatory power of the time-variation in sovereign credit spread *changes* is twice as large with monthly data than with daily data.

On the theoretical side, this paper builds on the foundation laid by several strands of literature. One body of literature that was launched with the seminal contributions of Eaton and Gersovitz (1981) and Bulow and Rogoff (1989) addresses why sovereign lending takes place by focusing on the costs of future access to credit, trade, and financial markets, as well as on retaliatory actions by way of sanctions. However, these frameworks do not provide a clear understanding of why or when a government defaults. To address such important questions, a second body of literature driven by Hayri (2000), Gibson and Sundaresan (2001), Westphalen (2002), and Andrade (2009) offers models for the valuation of sovereign debt in the presence of strategic default using a contingent claims framework. Their approach follows the structural modeling developed in corporate finance, where default is modeled as the event that the debtor's cash-flows or asset-liability ratio falls below a certain cut-off level for the first time, as in Fischer, Heinkel, and Zechner (1989) and Leland (1994). While the timing of default is determined by the government, the debt policy is typically assumed exogenous. François, Hübner, and Sibille (2011) extend this literature with a model of renegotiation between the defaulting government and its lenders. They also highlight how interactions between the banking sector, firms, and a government help explain Brazilian sovereign spreads. In a different approach, Arellano (2008)

[10] Studies on bonds spreads relying on levels regression include Campbell and Taksler (2003), Cremers, Driessen, Maenhout, and Weibaum (2008), Zhang, Zhou, and Zhu (2009), Ericsson, Jacobs, and Oviedo (2009), and Doshi, Ericsson, Jacobs, and Turnbull (2011).

and Yue (2010) draw on dynamic stochastic equilibrium models to explain the dynamics of Argentinean spreads using quarterly GDP data. Results are conclusive but the frequency of the data under analysis remains low. Eventually, Bodie et al. (2007) and Gapen et al. (2008) highlight the importance of using higher-frequency, forward-looking data on the economy as provided by market prices. They use domestic debt price data to evaluate credit spreads on foreign debt. The approach considered in this paper combines these four bodies of literature.

The rest of the paper is organized as follows. Section 2 outlines a structural model for sovereign debt with endogenous debt and default policies. Section 3 brings the model to the data and examines the level and the time-variation in sovereign credit spreads. I conclude my analysis in Section 4.

2 Model

This section presents a model for sovereign credit risk with endogenous default and debt policies. Throughout the paper, capital markets are frictionless and all investors have perfect information. The default-free term structure is flat with an instantaneous risk-free rate r, at which investors may lend and borrow freely. All variables are measured in the same numeraire.

2.1 Environment

The economy consists of a government and a representative unlevered firm, which are both infinitely lived. The firm has an asset whose value is governed under the risk-neutral probability measure $\mathbb{Q}$ by the process

$$dV_t = \mu V_t dt + \sigma V_t dZ_{t,}^{\mathbb{Q}} \;\; V_0 > 0 \tag{1}$$

where μ and σ represent constant mean and volatility of the firm's asset value growth rate, and $Z_t^{\mathbb{Q}}$ is a Brownian motion defined on the probability space $(\Omega, \mathscr{F}, \mathbb{Q})$. The standard filtration of $Z_t^{\mathbb{Q}}$ is $F = \{\mathscr{F}_t : t \geq 0\}$. The value V essentially represents the discounted value of firm income X generated by the firm's economic activities:

$$V_t = \mathbb{E}^{\mathbb{Q}}\left[\int_t^{\infty} X_u e^{-r(u-t)} du\right] \tag{2}$$

The government issues an infinite maturity debt contract, characterized by a level D and a continuous debt service C until default. The government raises fiscal revenues by taxing firm income X at a constant rate τ, which are then used to service debt. Since sovereign debt contracts are not subject to enforceable law, the government may choose to strategically default on its debt obligations. The government defaults at time $T^D = \inf\{t \geq 0 \mid V_t \leq V^D\}$, when the value of the firm's activities falls to V^D.[11] I first evaluate the price of sovereign debt for a given default threshold V^D and then discuss the optimal default and debt policies in the following sections.

2.2 Sovereign Debt Pricing

The pricing formula of sovereign debt D does not require knowledge of investor tastes and must avoid arbitrage opportunities.[12] Using Itô's lemma, the value of the perpetual debt satisfies

$$rD = C + rVD_V + \frac{1}{2}\sigma^2 V^2 D_{VV} \tag{3}$$

where D_V and D_{VV} are the first and second derivatives, respectively, of the sovereign debt value D with respect to the asset value V. The solution to this ordinary differential equation is subject to a number of conditions. First, when the asset value V (and thus the value of fiscal revenues) tends to infinity, the value of the sovereign debt tends to the value of a risk-free debt. Second, when default occurs, the government and its lenders restructure the terms of the debt contract and agree on a reduction $\phi \in [0, 1]$ in the debt service.

The relevant boundary conditions are:

$$\lim_{V \to \infty} D(V) = \mathbb{E}^{\mathbb{Q}}\left[\int_0^{\infty} Ce^{-rt}dt\right] = \frac{C}{r} \tag{4}$$

$$\lim_{V \to V^D} D(V) = \frac{C(1-\phi)}{r} \tag{5}$$

where $1-\phi$ represents the recovery rate on the debt service C. The value of sovereign debt associated

[11]Sovereign defaults typically occur during economic downturns (see Reinhart, Rogoff, and Savastano, 2003; De Paoli, Hoggarth, and Saporta, 2006). While sovereign countries can certainly default several times, generalizing the framework to account for multiple defaults is left for future research.

[12]From a standpoint of market completeness, a portfolio strategy that holds a traded asset (e.g., equity) perfectly correlated (locally) with the firm's asset value V, debt D, and the risk-free asset can be modified to be riskless and thus avoid arbitrage by picking appropriate weights such that the expected return on the portfolio equals the risk-free rate r.

with the above boundary conditions (Equations 4 & 5) is:

$$D(V) = \underbrace{\mathbb{E}^{\mathbb{Q}}\left[\int_0^{T^D} Ce^{-rt}dt\right]}_{Debt\,service\,before\,default} + \underbrace{\mathbb{E}^{\mathbb{Q}}\left[\int_{T^D}^{\infty}(1-\phi)Ce^{-rt}dt\right]}_{Debt\,service\,after\,default} \tag{6}$$

$$= \frac{C}{r}\left[1 - \underbrace{\phi\left(\frac{V}{V^D}\right)^{\beta}}_{Default\,premium}\right] \tag{7}$$

where β is the negative root of the quadratic equation $\frac{1}{2}\sigma^2\beta(\beta-1)+r\beta-r=0$, which is given by

$$\beta = -\frac{2r}{\sigma^2} < 0 \tag{8}$$

The market value of sovereign debt $D(V)$ is equal to a riskless perpetual debt with continuous coupon C minus a default premium. This premium corresponds to the present value of the unrecovered value of the debt after default, where the Arrow-Debreu price of default $\mathbb{E}^{\mathbb{Q}}\left[e^{-rT^D}\right] = \left(\frac{V}{V^D}\right)^{\beta}$ has the interpretation of the present value of \$1 conditional on future default (i.e., V falling to V^D). Lenders anticipate the government's opportunistic behavior by reflecting the associated wealth extraction in the pricing of sovereign debt.

2.3 Sovereign Wealth

Sovereign debt issuance affects the economy in two ways. On one hand, it generates economic value, thereby providing the government the incentive to issue sovereign debt. For convenience, I assume that the government can invest the raised amount of debt to obtain a return on domestic investment r_g that is larger than the risk-free rate r.[13] On the other hand, more debt raises the risk that the government will be unable to service its debt in the future. This risk matters because default is costly for the economy.[14] If sovereign default occurs, the level of firm income is reduced by a fraction

[13] Two reasons justify this assumption. First, the government can have exclusive access to public investment offering high return. Public infrastructure investment is in general exclusively undertaken by state-owned enterprises or local governments. Legal restrictions can also play an important role in preventing foreign investors from capturing these returns. Second, the presence of financial constraints in the economy can drive a wedge between the return on public investment and the risk-free rate. Financial constraints can arise from the insufficient liquidity in emerging economies to finance domestic investment (e.g., Holmstrom and Tirole, 1998, Caballero and Krishnamurthy, 2005).

[14] There is ample evidence on the economic costs of a sovereign default (e.g., Reinhart, Rogoff, and Savastano, 2003; Sturzenegger and Zettelmeyer, 2006). In particular, debt repudiation can impede the ability of the defaulting government to trade (Rose, 2005; Martinez and Sandleris, 2011). This is pertinent, since trade is a significant part of economic activity (Frankel and Romer, 1999). In addition, default weakens the domestic financial system. As major creditors of

$\lambda \in [0, 1]$. The consideration of output cost in sovereign debt default follows the works of Arellano (2008), Andrade (2009), Hatchondo and Martinez (2009), Yue (2010), and Borri and Verdelhan (2011), among others. Avoiding such costs is the government's motivation in avoiding default.[15]

I define sovereign wealth $W(V)$ as the present value of future fiscal revenues net of the present value of future debt payments, as in Andrade (2009). I additionally consider the incentives for issuing debt. Sovereign wealth $W(V)$ is given by

$$\begin{aligned} W(V) &= \underbrace{\mathbb{E}^{\mathbb{Q}}\left[\int_0^\infty (\tau X_t - C)\, e^{-rt} dt\right]}_{\text{Fiscal revenues net of debt service}} + \underbrace{\alpha D(V)}_{\text{Incentives for debt issuance}} \quad (9) \\ &\quad \underbrace{+\mathbb{E}^{\mathbb{Q}}\left[\int_{T^D}^\infty C\phi e^{-rt} dt\right]}_{\text{Default gain}} - \underbrace{\mathbb{E}^{\mathbb{Q}}\left[\int_{T^D}^\infty \lambda\tau X_t e^{-rt} dt\right]}_{\text{Default costs}} \\ &= \tau V - \lambda\tau V^D \left(\frac{V}{V^D}\right)^\beta + \frac{(\alpha-1)\,C}{r}\left[1 - \phi\left(\frac{V}{V^D}\right)^\beta\right] \quad (10) \end{aligned}$$

where $\alpha = \mathbb{E}^{\mathbb{Q}}\left[\int_0^\infty (r_g - r)e^{-rt} dt\right] = \frac{r_g - r}{r}$ is the discounted benefit of issuing one unit of debt. The first term on the right-hand side of Equation 9 is the present value of future fiscal revenues net of debt payments in the absence of default. The second term captures the incentives for issuing debt. The last two terms represent the difference between the reduction in debt service and the loss upon default multiplied by the Arrow-Debreu price of this event.

The government's problem consists in solving for the optimal level of debt subject to the default policy. Solving the model backwards, I first determine the optimal default policy for a given level of debt, and then solve for the optimal level of debt.

2.4 Default Policy

I assume that households are the eventual beneficiaries of future government revenues. In a rational expectations model, the government chooses the default policy that maximizes sovereign wealth, such

the government, domestic banks may be prevented from providing liquidity and credit to the economy. Sovereign default crises have thus been associated with banking crises, which result in severe and prolonged recessions (De Paoli, Hoggarth, and Saporta, 2006; Gennaioli, Martin, and Rossi, 2011).

[15] Bulow and Rogoff (1989) suggest two additional reasons for a country to repay its foreign debt. First, lenders may be able to appropriate collateral. However, in the event of repudiation, the assets accessible to creditors are worth only a small fraction of the outstanding level of debt. Second, there may be a reputation effect that could impact future borrowing opportunities, though empirical support for such an effect is weak (Eichengreen, 1989; Gelos, Sahay, and Sandleris, 2011).

that the government and the country's households agree on the default policy.

The timing of default is a strategic decision for the government. The default policy, characterized by the default boundary V^D, is chosen to maximize sovereign wealth $W(V)$, such that the smooth-pasting condition $\frac{\partial W(V)}{\partial V} \mid_{V=V^D} = \tau(1-\lambda)$ is satisfied:[16]

$$V^{D*} = \frac{C\phi(\alpha-1)\beta}{\tau r \lambda (1-\beta)} \tag{11}$$

The debt reduction upon restructuring $C\phi$ provides the government the incentive to default. At the same time, the expected economic costs of default also rise with the willingness to default. The government considers this trade-off when determining its default policy.

Figure 1 provides a sensitivity analysis of the optimal default policy with respect to the model's key parameters. The government chooses a lower default boundary (i.e., low default probability) with greater default costs λ, debt issuance benefits α, economic risk σ, corporate tax rate τ, and the risk-free interest rate r. Finally, the asset value V^D at which the government chooses to default is proportional to the debt coupon C and the debt reduction in default ϕ.

Figure 1 [about here]

2.5 Debt Policy

I now examine the optimal level of sovereign debt, which arises from the trade-off between the economic benefits of issuing debt α and the costs of default λ. While the default boundary is selected once debt has been issued to maximize sovereign wealth $W(V)$, the optimal level of indebtedness is jointly determined by the government and the debholders at time $t=0$. The chosen level of debt service C maximizes sovereign wealth $W(V)$ after debt has been issued plus the value of debt $D(V)$, which is given by

$$C^* = \arg\max_{C \in \mathbb{R}+} W(V) + D(V) \mid_{t=0} \tag{12}$$

$$= \frac{\tau V_0 r \lambda (1-\beta)}{\phi(\alpha-1)\beta} \left(\frac{\phi(\alpha-\beta)}{\alpha} \right)^{1/\beta} \tag{13}$$

where V_0 is the asset value at $t=0$, at the time the debt policy is determined.

[16]The smooth-pasting condition ensures continuity in asset values, such as sovereign wealth and the stock market price, at the time of default (see Merton, 1973, and Dumas, 1991). The condition satisfies $\frac{\partial W(V)}{\partial V} \mid_{V=V^D} = \frac{\partial W(V)|_{V=V^D}}{\partial V}$.

I illustrate the determinants of the optimal debt coupon in Figure 1. The level of debt is proportional to the asset value V_0 and to the tax rate τ. Both measures translate into greater fiscal revenues, thus raising the optimal amount of borrowing. The government also chooses to increase its level of debt with greater benefits of issuing debt α. The effect of economic risk σ and the risk-free interest rate r is non-monotonic and follows a U-shape. Finally, government indebtedness increases with the expected default costs λ because greater costs of default lower the incentive to default (i.e., through a decrease in the default boundary V^D). A similar reasoning applies to the expected debt reduction upon default ϕ, which lowers the optimal debt level.

2.6 Asset Prices

The default boundary and the debt policy jointly determine the risk-neutral probability of a sovereign default, which can then be used for the valuation of asset prices in the economy.

2.6.1 Sovereign Credit Spread

I first analyze the sovereign credit spread, which measures the market's perception of default risk. The credit spread $CS(V)$ is defined as

$$CS(V) \equiv \frac{C}{D(V)} - r = r\left[\frac{1}{1-\frac{\alpha}{\alpha-\beta}\left(\frac{V}{V_0}\right)^{\beta}} - 1\right] \tag{14}$$

The determinants of sovereign credit risk are mostly consistent with what is expected (see Figure 2). The credit spread increases with the incentives for government indebtedness α and with macroeconomic risk σ.

At the optimal leverage ratio, the sovereign credit spread is insensitive to several variables, including the expected debt reduction ϕ upon default, the magnitude λ of the economic costs upon default, the tax rate, and the risk-free interest rate r. The government adjusts its debt policy to the country's characteristics in such a way that the policy cancels out the effect of these parameters on sovereign credit risk. For example, increased default costs might be thought to reduce a government's incentive to default and thus to lower the sovereign yield spread. Indeed they do, but only for a given level of sovereign debt. As default costs rise, the optimal level of debt increases, thus offsetting the initial effect on the default boundary. Similar reasoning applies to the remaining variables.

Figure 2 [about here]

2.6.2 Stock Market Price

The model allows for the computation of the value of equity, determined as the present value of future net corporate income. The value of equity in the economy depends on the probability of sovereign default, as the default costs translate into a reduction in firm income. Investors take this risk into account in the valuation of the aggregate stock price $S(V)$, which is given by

$$
\begin{aligned}
S(V) &= \underbrace{\mathbb{E}^{\mathbb{Q}}\left[\int_0^{T^D}(1-\tau)V_t e^{-rt}dt\right]}_{Net\,value\,before\,default} + \underbrace{\mathbb{E}^{\mathbb{Q}}\left[\int_{T^D}^{\infty}(1-\tau)\left(1-\lambda\right)V_t e^{-rt}dt\right]}_{Net\,value\,after\,default} \qquad (15)\\
&= (1-\tau)V\times \underbrace{\left[1-\frac{\lambda\alpha}{\phi\left(\alpha-\beta\right)}\left(\frac{V}{V_0}\right)^{\beta}\right]}_{Sovereign\,default\,risk\,discount} \qquad (16)
\end{aligned}
$$

Sovereign default risk lowers the valuation of future firm income, as the second term of Equation 16 is smaller than one. This prediction is in line with Andrade (2009). We can see that the magnitude λ of the economic costs upon default affects stock market prices, even though this measure has no influence the valuation of sovereign credit spreads. As illustrated in Figure 2, the value of the stock market is low when investors expect a severe contraction in firm income upon default (i.e., high λ). In addition, stock market prices are high when the risk-free rate r is high, the tax rate τ is low, the level of economic risk σ is low, and when the incentives for government indebtedness α is low. These parameters affect stock market valuation with an intuitive sign.

Finally, the evaluation of equity is high when investors expect a large debt reduction upon restructuring (i.e., high ϕ). The higher the expected loss to debtholders upon default ϕ, the lower the proceeds of the sovereign debt issue (i.e, low $D(V)$), which means a low government's incentives for debt issuance (i.e., low $\alpha D(V)$). A lower indebtedness level then translates into less sovereign credit risk, thus decreasing the probability of incurring a contraction in corporate income. As such, investors raise their evaluation of the stock market when they expect a greater debt reduction ϕ in default.

3 Empirical Predictions

This section brings the pricing model to the data and examines its ability to explain the dynamics of sovereign credit risk. The approach consists in using the model to generate daily credit spreads that I compare to observed sovereign bond spreads for emerging and European countries. The emerging countries under consideration are Brazil, Colombia, Mexico, Peru, the Philippines, Russia, and Turkey, while the European countries are France, Greece, Ireland, Italy, Portugal, and Spain. These countries represent large issuers of government debt, with a total value of $8,550 billion in principal in 2010. They also present different geopolitical characteristics and levels of credit risk.

The sample spans the period 2000-2011 for emerging markets and the period 2006-2011 for European countries. Data come from Datastream.

3.1 Methodology

Sovereign credit spreads capture a government's ability to service debt with income taxes. A conventional approach to generate model credit spreads would be to use data on (or proxies for) firm income or government fiscal revenues. However, such data are only observable at low frequency (i.e., quarterly or annually) and contain past information only. The consideration of this type of information is thus not particularly appropriate to predict sovereign credit spreads, which are forward-looking measures observed at a high frequency.

This paper proposes a novel approach, which consists in using stock market prices to infer the expected fundamentals of a country. The intuition is as follows. The computation of credit spreads requires information on expected future firm income in the economy, captured in the model by the asset value V. Local stock market prices provide such information, as the price of equity represents, by definition, the market's forward-looking measure of firm income. The consideration of the information embedded in daily stock market prices to infer the market's expectations of a country's fundamentals is thus key here. The extracted series of expected fundamentals then allow for the prediction of sovereign spreads, which are eventually compared to those observed in the data. This approach can be applied to analyze sovereign spreads at any frequency.

3.1.1 Data on Sovereign Credit Spreads

The benchmark credit spreads for emerging countries are the Emerging Markets Bond Index (EMBI+) spreads computed by JP Morgan. These spreads track the required returns of government debt instruments in emerging countries in excess of the return of a US Treasury bond of equal maturity. For European countries, I compute the daily spreads as the difference between JP Morgan Global Bond Index (GBI) yield-to-maturity in euro of a country and a maturity-matched risk-free rate, which corresponds to the yield-to-maturity of a German government bond with the same maturity as the average maturity of the country's debt.[17] Credit spreads are in US dollar for emerging countries and in euro for European countries.[18]

These spreads capture the risk premium that investors demand for bearing sovereign default risk above that implied by a riskless asset, thus appropriately reflecting a country's creditworthiness. Such data are preferred to sovereign credit default swap spreads, which present a relatively poor liquidity at the daily frequency over the sample period.

3.1.2 Expected Fundamentals

The analytical solution of a country's stock market price is used to structurally extract the time-series of expected fundamentals V for each country, which are then considered as input for the computation of daily sovereign credit spreads. The model developed in this paper explicitly accounts for the direct influence of sovereign default risk on the price of equity, through the path of future income. Therefore, the model ensures that causality analyzed in this paper goes from expected fundamentals to sovereign credit risk, and not the reverse.

Combining Equations 14 and 16, the level of expected fundamentals V can be written as a function of the stock market price $S(V)$ and the credit spread $CS(V)$, for each country i at any time t,

[17] I use a linear interpolation to obtain the yield of German government bonds with an average maturity that matches the average maturity of a country's Global Bond Index for bonds maturing after 10 years. The linear interpolation has two nodes. One node is the Government Bond Index for Germany for bonds with maturity between 1 and 10 years, and the other node is this index for bonds with maturity over 10 years. The approach follows Andrade and Chhaochharia (2011). I proceed to a daily interpolation, as the average maturity of Government Bond Indexes (i.e., the average life) varies continuously.

[18] I do not consider the UK and Switzerland in the analysis because these countries exclusively issue government debt in their respective currency (i.e, GBP and CHF). Subtracting a risk-free yield of the same currency is impossible as a corresponding riskless bond does not exist.

$$\frac{V_{i,t}}{V_{i,0}} = \frac{S_{Obs,i,t}}{S_{Obs,i,0}} \frac{1}{\left[1 - \frac{\lambda}{\phi} \frac{CS_{Obs,i,t}}{CS_{Obs,i,t}+r}\right]} \tag{17}$$

where $S_{Obs,i,t}$ and $CS_{Obs,i,t}$ denote the observed local stock market price and the observed sovereign spreads, respectively. The price of the daily local stock market, $S_{Obs,i,t}$, is obtained from the level of the Morgan Stanley Capital International (MSCI) local stock market index for each country. Values are in US dollar for emerging countries and in euro for European countries.

Equation 17 shows that high stock market prices are associated with high expected fundamentals. The second term on the right-hand side adjusts for the effect of sovereign default risk on stock market prices.

Model Parameters

I now discuss the parameter values used to extract the expected fundamentals. The relevant risk-free rate r for emerging countries is the average 10-year US Treasury rate, which is equal to 4.49% over the period 2000-2011. For the European countries, the risk-free rate r is equal to 3.85%, which corresponds to the average 10-year yield on German government bonds over the period 2006-2011. The expected loss ϕ to debtholders upon default is taken from the sovereign credit default swap market: 60% for European countries and 75% for emerging countries (www.cmavision.com). Finally, the magnitude of the expected economic contraction λ in sovereign default is taken from Andrade and Chhaochharia (2011). The authors use a structural model to compute the expected costs of sovereign default inferred in the valuation of equity markets. They obtain estimates of the expected contraction of 4.1% and 5% per year for emerging economies and European countries, respectively.

Expected Fundamentals and Observed Sovereign Spreads

Figure 3 (left panels) illustrates the tendency for sovereign credit spreads to rise dramatically when the economy performs poorly, as measured with the level of expected fundamentals extracted from local asset prices.[19] The right panels of Figure 3 suggest that the relationship between sovereign spreads and the market-implied fundamentals is highly non-linear. The model's success will be evaluated by

[19] For space consideration, I only provide figures for Brazil and Greece; two countries that differ substantially in terms of economic size, political risk, and industrial structure.

its ability to account for this non-linearity and generate a linear relationship between the model credit spreads and observed sovereign spreads.

Figure 3 [about here]

3.1.3 Model Estimation

The importance of avoiding an over-identification problem puts restrictions on the set of parameters to estimate. The estimated parameters are the return on public investment r_g, which determines the government's benefits of issuing debt α, and the level of economic uncertainty σ. These parameters capture investors' expectations of a country's investment return and economic risk. For each country, I estimate the model parameters by minimizing the sum of the squared pricing errors between model credit spreads and observed spreads over the period under study:

$$\left\{r_{gi}^{*}, \sigma_i^{*}\right\} = \arg\min_{r_{gi},\sigma_i} \sum_t \left(CS_{Obs,i,t} - CS_{Model,i,t}\right)^2 \tag{18}$$

with

$$CS_{Model,i,t} = r\left[\frac{1}{1-\frac{\alpha_i}{\alpha_i-\beta_i}\left(\frac{V_{i,t}}{V_{i,0}}\right)^{\beta_i}} - 1\right] \tag{19}$$

where $CS_{Model,i,t}$ denotes the credit spread implied by the model for country i at time t, $\alpha_i = \frac{r_{gi}-r}{r}$, and $\beta_i = -\frac{2r}{\sigma_i^2}$. The estimation is described in Appendix 5.4.[20] I break down the sample period into two subsamples to compare the results before and during the European debt crisis (2010-2011).

Parameter Estimates

Table I reports on the estimation of the model parameters, which exhibit substantial cross-country variation. First, the estimated rate of return r_g at which governments invest the proceeds of debt is, on average, 6.15% in emerging countries (Panel A) and 4.03% in Europe (Panel B) before 2010. During the 2010-2011 European debt crisis, the average return on public investment is 4.17% in emerging countries and 3.68% in Europe. These estimates indicate that emerging economies offer enhanced

[20]The local optimums are found for a large set of initial parameter values, thus suggesting robust estimates. The results reported in the paper use as starting values $r_g = 0.05$ and $\sigma = 0.4$.

investment opportunities compared to European countries and that the recent crisis has lowered government incentives for debt issuance. The country with the lowest expected public investment return during the crisis is, perhaps not surprisingly, Greece ($r_g = 2.79\%$). Overall, the data do not require high investment returns, and thus unreasonable incentives for debt issuance, to match the level of observed credit spreads.

Table I [about here]

The second parameter σ, which is related to the uncertainty of future economic conditions, has an average estimate of 51.8% for emerging markets and 19.7% for European countries during the period that precedes the European crisis. The estimates are equal to 34.4% and 23.8% over the period 2010-2011, respectively. The strong difference in average estimates between the two groups of countries appears in line with the intuition that European countries present lower economic uncertainty than emerging countries. Emerging markets have become safer over recent years, while the uncertainty related to European economies has increased during the 2010-2011 crisis. Interestingly, the market attributes a level of economic uncertainty in Greece during the crisis that is greater than for any of the emerging countries considered in this study. Finally, the estimation suggests that France is the safest country under analysis.

3.2 Observed vs. Model-implied Credit Spreads

This section compares the sample characteristics and the dynamics of the model credit spreads with those in the data.

Level of Credit Spreads

I provide the first two moments of the model-implied and observed sovereign spreads computed at the daily frequency in Table II for emerging markets and in Table III for European countries. First of all, it is apparent that there is substantial variation in spreads over time and across countries. The average pre-crisis sovereign spread is 47 basis points in Europe, while it is 399 basis points in emerging economies.[21] During the 2010-2011 crisis, the average sovereign spread rises to 373 basis points in Europe, while it decreases to 196 basis points in emerging economies. The average volatility

[21] Spreads in Europe and in emerging markets are not directly comparable, as they are not denominated in the same currency. The difference in the level of credit risk is nevertheless too substantial to be attributed to a currency effect.

in sovereign spreads is 6.6% and 36.2%, respectively, before the crisis, while it is 30.7% and 5.7%, respectively, during the European crisis. Sovereign credit risk has thus recently shifted from emerging markets to the European area.

Tables II and III [about here]

Despite the strong heterogeneity in sovereign spreads across countries and economic conditions, there is clear evidence that the model can closely replicate each country's level of sovereign spreads during both periods. The pricing errors are negligible, ranging from 3 basis points in Europe to 2 basis points in emerging markets. Hence, the model matches the observed spreads almost perfectly. This is in contrast to the underestimation of credit spreads typically observed in other studies. For example, the level of credit spreads predicted in Arellano (2008), Hilscher and Nosbusch (2010), and Yue (2010) is, respectively, 45%, 61%, and 26% of the level of observed sovereign spreads.

Correlations

The results in Table IV exhibit a very high correlation between the model-implied and the observed credit spreads. In emerging markets (Panel A), the average correlation is 0.81 before the crisis and 0.60 during the crisis. In Europe (Panel B), the average correlation is 0.89 and 0.76 before and during the crisis, respectively. The correlation is thus particularly strong over the pre-crisis period.

Table IV [about here]

The results suggest that the level of expected fundamentals embedded in a country's stock market provide insightful information for the valuation of this country's creditworthiness. In comparison, the naive linear correlation between a country's stock market prices and sovereign spreads is -0.72 for emerging countries and -0.80 in Europe in the pre-crisis period.[22] The pricing formula enhances the explanation of sovereign spreads for two reasons. First, the model breaks down the underlying non-linear relationship between stock market information and sovereign spreads into a linear relationship between model credit spreads and observed spreads. Indeed, the model-implied relationship between the estimated fundamentals and sovereign spreads closely fits the empirical relationship (Figure 3, right panels). Second, the structural estimation addresses the reverse causality issue that arises from the effect of sovereign default risk on stock market valuation.

[22] The correlation between sovereign spreads and the VIX is respectively 0.48 and 0.69 in emerging and European countries over the same period.

Analysis of the Dynamics

In Figure 4, I compare the daily dynamics of the credit spreads estimated from the model with the dynamics of sovereign spreads for Brazil and Turkey over the pre-crisis period (left panels). Figure 5 offers the same analysis for the 2010-2011 crisis period. The model credit spreads and the observed EMBI+ spreads evolve very similarly over both quiet and crisis periods; the model captures the turbulent 2001-2003 period in emerging debt markets, the substantial and steady narrowing of sovereign spreads over 2004-2007, and the peak in spreads after the failure of Lehman Brothers in September 2008. The weak US economy, along with the prospects of deflation and the slump in consumers' confidence, triggered a fall in world consumption over the last quarter of 2008 and the first quarter of 2009, which has negatively affected firm earnings in emerging countries. As the model predicts, lower expected fiscal revenues for governments have thus been accompanied by a high level in sovereign credit risk early 2009. The model also successfully matches the subsequent decline in spreads, which has been accompanied by a period of rapid economic recovery in emerging economies in 2010, as well as the rise in sovereign credit risk related to the fear of European defaults in 2011.

Figures 4 and 5 [about here]

The right panels of Figures 4 and 5 compare the model prediction with the observed sovereign spreads for Greece and Italy, two countries that drew the most attention over recent years. The model replicates the very narrow spreads before 2008, as well as the rapid rise in sovereign default risk that followed the US financial turmoil in 2008-2009. The model credit spreads also appear to fit the observed time-variation in European sovereign credit spreads remarkably well during the European debt crisis, particularly when Europe experiences an overall sharp increase in credit risk in 2011.

Model Fit

The analysis suggests that the model performs well in pricing sovereign credit spreads over both periods. The root mean square error (RMSE), computed as RMSE= $\sqrt{\frac{\sum_t (CS_{Obs,i,t} - CS_{Model,i,t})^2}{T}}$, measures the fit of the model. Overall, the RMSE is low despite the high volatility in sovereign credit spreads during the different periods of crisis. Table I reports an average RMSE for emerging countries equal to 122 and 26 basis points before and during the 2010-2011 crisis, respectively (Panel A). In Europe, the average RMSE is 15 and 99 basis points before and during the 2010-2011 crisis, respectively (Panel B). An analysis out-of-sample is discussed in Section 3.3.

3.2.1 Econometric Analysis

This section investigates the ability of the model spreads to explain the dynamics of observed sovereign spreads. The analysis relies on country-by-country time-series estimations to evaluate the quality of the pricing model.

Specification

An important feature of the empirical strategy is to consider credit spread levels, rather than credit spread changes. In line with a recent strand of studies,[23] this specification is preferable at the daily frequency for several reasons. First, credit spreads are *ex ante* expected return differentials and are thus expected to be stationary. Second, while the time-series of credit spreads are highly auto-correlated, they are mean-reverting and not characterized by long-term stochastic trends. Third, first differentiating a stationary time-series and regressing changes rather than levels introduces noise into the estimation. Such inefficiency tends to be acute for daily spread data.[24] Eventually, the analysis in levels calls for an empirical investigation of the structural model's ability to explain, simultaneously, the time-variation and the level of sovereign credit spreads. The econometric specification is as follows:

$$CS_{Obs,i,t} = \delta_i + \delta_i^{cs} CS_{Model,i,t} + \varepsilon_{i,t} \tag{20}$$

where I account for country-specific coefficients δ_i and δ_i^{cs} to allow for heterogeneous relations across countries, and $\varepsilon_{i,t}$ is the error term. I correct for heteroskedasticity with the Newey and West's non-parametric variance covariance estimator. Results are reported in Tables V and VI for the pre-crisis (before 2010) and the crisis periods, respectively.

Tables V and VI [about here]

[23] Existing studies on bonds spreads relying on levels regression include Campbell and Taksler (2003), Cremers, Driessen, Maenhout, and Weibaum (2008), Zhang, Zhou, and Zhu (2009), Ericsson, Jacobs, and Oviedo (2009), and Doshi, Ericsson, Jacobs, and Turnbull (2011).

[24] As discussed in Doshi et al. (2011), the choice between levels and difference regressions for credit spread analysis is a complex one, and no consensus has thus far emerged in the literature. Nevertheless, the choice of levels versus differences should depend on the frequency of the data. For monthly data, differentiating the data may be the best choice (see Section 3.4), whereas for daily spread data, analyzing levels may be preferable.

Test of the Model

If the model is correctly specified, the point estimates of the constant δ_i should be equal to zero on average, meaning that the model can adequately explain the level of sovereign spreads. It is indeed the case, as displayed in Tables V and VI for emerging markets (Panel A) and European countries (Panel B). In addition, the estimates δ_i^{cs} should be equal to unity under the correct specification. The point estimates for the coefficients are very close to one and are remarkably similar across countries and periods. More importantly, the t-statistics ($H0 : \delta_i^{cs} = 1$) indicate that we cannot reject the null hypothesis that any slope estimate δ_i^{cs} is equal to unity at 1%.

Goodness-of-fit Analysis

Tables V and VI also report goodness-of-fit measures for the linear regressions. Over the pre-crisis period, the regressions yield an average R^2 of 66% and 80% for emerging and European countries, respectively. The average R^2 is 40% and 59% for emerging and European countries, respectively, during the 2010-2011 crisis. Results show that the estimated credit spreads can explain a substantial fraction of the time variation in sovereign credit spreads. Therefore, information on the state of the economy, as obtained from the local stock market, is an essential ingredient of sovereign credit risk. In line with Longstaff et al. (2011), good news for the local stock market is also good news for sovereign credit spreads.[25]

Following Granger and Newbold (1974), the high R^2 of the linear regressions may be interpreted as a spurious relationship. However, the relationship is likely to be genuine for two reasons. First, sovereign credit spreads are expected return differentials and are thus typically stationary. Second, the model credit spreads are explicitly suggested by the theory, and it is therefore less likely that we uncover a spurious relationship in this setting.

3.2.2 Country-specific Information vs. Market-wide Factors

The model explains a large fraction of the time variation in sovereign credit spreads. Yet it does not explain all variation. Remolona et al. (2008) and Longstaff et al. (2011) suggest that variations in sovereign credit risk can be largely attributed to changes in common factors across countries, with country-specific information playing a secondary role only. They consider a number of measures from

[25]Longstaff et al. (2011) show that most countries exhibit a statistically significant effect of the local stock market return on sovereign credit spreads after controlling for 13 other factors.

the equity and fixed income markets that can affect global investors, such as the return on the S&P500 index and the US Treasury rates. The option-implied volatility index (VIX), which captures a source of risk premium in the US equity market, also appears as a key factor in their explanation of sovereign credit risk. Pan and Singleton (2008) and Hilscher and Nosbusch (2010) similarly conclude on the importance of the VIX as a determinant of sovereign credit spreads.

Full Specification

Let us see how these market-wide factors help explain the time variation in sovereign spreads. The regression now includes the VIX, the return on the S&P500 index (R_{SP500}), the 5-year US Treasury rate (UST), and the slope of the US term structure (UST_{slope}) computed as the difference between the 30-year and the 3-month US Treasury rates. I gradually include each of these factors in the analysis. The full regression is as follows:

$$CS_{Obs,i,t} = \delta_i + \delta_i^{cs} CS_{Model,i,t} + \gamma_i^{vix} VIX_t + \gamma_i^{sp} R_{SP500,t} + \gamma_i^{ust} UST_t + \gamma_i^{slope} UST_{slope,t} + \omega_{i,t} \quad (21)$$

where I correct for heteroskedasticity with the Newey and West's non-parametric variance covariance estimator. The focus of the analysis is primarily on the marginal explanatory power of these variables, and secondarily on the sign and the significance of the estimates.

Relative Importance of the Global Factors

What is the importance of these market-wide factors relative to the model credit spreads in the explanation of sovereign credit spreads? To address this crucial question, I analyze the fraction of the total variation explained by the full regression (i.e., with global factors) that is due solely to the model spreads. To calculate this "Local R^2" ratio, I divide the R^2 from the regression that only includes the model spreads by the adjusted R^2 from the full regression. The analysis is for the full sample, which spans 2000-2011 for emerging markets and 2006-2011 for European countries. Results are presented in Table VII.

Table VII [about here]

The model-implied spreads explain 69.6% (Panel A) and 51.6% (Panel B) of the total variation in sovereign spreads for emerging and European countries, respectively. When global factors are considered, the adjusted R^2 rise to 84.7% and 77.1%, respectively. These results indicate that such factors contain relevant additional information for the pricing of sovereign credit risk. However, the local ratios suggest that the model alone can explain 82.1% of the total variation in EMBI+ spreads and 64.4% of the time-variation in GBI spreads. Hence, the structural model accounts for a single source of information but captures a relatively important fraction of the time variation in sovereign credit spreads.

Discussion

Overall, the results suggest that the dynamics of sovereign credit risk seem to be explained primarily by country-specific information, with market-wide indicators playing a secondary role only.[26] This finding may stand in contrast to Longstaff et al. (2011), who conclude that sovereign credit risk is primarily driven by global factors. The authors compute a similar local ratio, which represents the fraction of the total variation explained by local variables (e.g., stock market index, exchange rate, foreign reserves), for 26 countries and report an estimate of 43%. However, using the results reported in the Table 3 of Longstaff et al. (2011), the average local ratio of the 7 emerging countries considered here equals 66%. This estimate reconciles the results of the two studies. A likely explanation for the difference in estimates is the inclusion of small countries in their sample, for which illiquidity in the sovereign credit default swap market is certainly high (e.g., Panama, Qatar, Slovak, Venezuela).

Therefore, the data indicate that the forward-looking information on firm income in a country, as embedded in local stock market prices, helps explain an important part of the time-variation in sovereign credit spreads; in particular for large sovereign debt issuers with liquid credit and equity markets.

Decomposition of the Global Factors

Table VII reports the significance and the sign of the relationships between the global factors and sovereign credit spreads for emerging markets in Panel A and for European countries in Panel B. A striking result that arises from the table is the strong difference in these relationships for the two groups of countries.

The VIX, which is a measure of global uncertainty in financial markets, widens emerging spreads but lowers European spreads. Similarly, the return on the S&P500 index has a positive effect on emerging

[26] Hilscher and Nosbusch (2010) draw on a similar conclusion using country-specific fundamentals measured by a country's terms of trade.

spreads and a negative effect on European spreads. The order of magnitude is similar for both groups. Countries thus differ considerably in their sensitivity to economic news related to the US equity market and to global market uncertainty.

The effect of the 5-year Treasury rate on sovereign credit risk also differs strongly across countries. The average effect of a 1% increase in the rate is to decrease European spreads by 1.12% and to increase emerging spreads by 0.28%. Eventually, the term structure of US interest rates does not provide systematic additional information to emerging markets but seems to affect European countries negatively, on average. A greater slope of the US term structure is thus associated with higher expected economic growth.

The table also provides the relative contribution of each global factor in the explanation of sovereign credit spreads, as measured by the increase in the adjusted R^2. The variables that carry important additional explanation of sovereign credit risk are the VIX and the 5-year Treasury rate. The slope of the US term structure also provides relevant information, but for European countries only.

Overall, the results show that sovereign credit risk responds to the economic conditions of the country, as predicted by the model, but also to global market factors. The sensitivity to such factors differs considerably across emerging and European economies. The residual variation in sovereign credit risk (i.e., unexplained by local information) is strongly related to measures of global market uncertainty (e.g., VIX) in emerging markets and global funding liquidity (e.g., US Treasury rate) in Europe.

3.3 Out-of-sample Analysis

This section provides an out-of-sample study over the full sample period, which spans 2000-2011 for emerging countries and 2006-2011 for European countries.

3.3.1 Estimation

The procedure is to estimate the model over two years of data (500 trading days) and generate sovereign spread forecasts at one month horizon (20 trading days). This step is repeated each month using a rolling-windows approach, so as to consider the latest 500 observations for the estimation of the model parameters. It is worth emphasizing that the out-of-sample credit spreads are computed using parameters only estimated with the information that precedes the forecasting period. In addition, these forecasts do not account for the information provided by the global factors discussed in Section 3.2.4.

Figure 6 offers a comparison of the credit spreads observed in the data with those predicted by the model out-of-sample. The left panels illustrate the case of Brazil and Turkey. The right panels display the case of Greece and Italy. It is obvious, even to the naked eye, that the structural model closely replicates the dynamics of sovereign spreads for all countries. The figure also indicates that the model tends to overestimate increases in sovereign credit risk in periods of severe distress (e.g., 2002 in Brazil, 2008 in Turkey, 2010 in Greece, and 2011 in Italy) but seems to quickly readjust its forecasts with new information. The overestimation period is thus relatively short.

Figure 6 [about here]

3.3.2 Time-variation in Model Parameters

The fit of the model credit spreads to the data is significantly improved out-of-sample. The reason for the improvement is that the rolling-windows allow the parameter estimates to vary over time. Figure 7 illustrates the time-variation of the return on public investment r_g and the level of economic uncertainty σ over the sample period.

Figure 7 [about here]

Emerging markets have experienced, on average, a downward trend in public investment return over the last 10 years. As these countries have become more developed over time, the productivity of public investments has decreased. The level of macroeconomic risk also exhibits a decline over the sample period. The level of uncertainty has exploded in 2008, during the peak of the financial turmoil in the US, but has rapidly declined to a low and rather constant level. In Europe, the estimate of public investment return has decreased over the period 2008-2010, which corresponds to the years of economic slowdown that most European countries have experienced. In contrast, the level of public investment return, which determines a government's motivation for debt issuance, has apparently recovered in 2011. The level of economic uncertainty has been low and stable over the period 2008-2009 but has sharply increased since mid-2010, when Europe entered the regional debt crisis territory. It is worth noting that the level of uncertainty, as implied by market prices, is at a similar level at the end of 2011 to that before the 2010-2011 crisis. At the same time, the model predicts an increase in sovereign credit risk in 2011, which is due to the deterioration of economic conditions in Europe. The low level of economic uncertainty towards the end of the period should thus not be associated with an improvement of the European debt crisis.

The model's parameter estimates vary over time for several reasons. First, new information becomes gradually available to investors. Second, economic conditions have dramatically changed over the last 10 years. An estimation with rolling-windows thus appears important to explain the dynamics of sovereign credit risk. Note that this approach is not inconsistent with the model's assumption of constant parameters, as values are kept constant over the forecasting period. Moreover, it is reasonable to account for the latest data, thus adjusting the model parameter values to the new information set.

3.3.3 Forecast Accuracy

Out-of-sample accuracy can be measured by two statistics: the root mean square error (RMSE) and the mean absolute error (MAE). Table VIII presents these statistics at a one-month horizon over the forecasting period, which spans 2002-2011 for emerging markets and 2008-2011 for European countries. The benchmark for the analysis of the results is the random walk model, which is used to generate the expected spreads at a one-month horizon over the forecasting period.

Table VIII [about here]

The model credit spread forecasts have very low RMSE and MAE for emerging market economies, respectively 3.9 and 2.1 basis points. Table VIII suggests that the structural model offers an improvement over the random walk model for emerging markets. For European countries, the structural model provides marginally higher RMSE and MAE. Overall, the model outperforms the benchmark for 7 and 9 countries (out of 13) when the RMSE and MAE measure is respectively considered.

The correlation between forecasted and observed sovereign credit spread changes is much higher for the structural model than for the random walk. The correlation equals, respectively, 0.29 and 0.02 for emerging countries and 0.27 and -0.01 for European countries. Therefore, the structural model appears insightful to predict the direction of sovereign credit spreads. Although not reported in Table VIII, results at 10 days and 30 days horizon remain very similar.

Eventually, this exercise offers two sets of results. First, the structural model can be conveniently applied to explain the level and the time-variation of sovereign credit spreads (see Figure 6), particularly when the parameter estimates account for the latest information available. Second, even though sovereign credit spreads probably do not follow a random walk exactly, they are difficult to forecast. In this regard, the sovereign credit market appears rather efficient.

3.4 Monthly Frequency

The results have thus far demonstrated the model's ability to explain sovereign credit spreads at the daily frequency. Yet the model is not restricted to a particular frequency of data, and can thus be used by market participants to evaluate sovereign credit risk with any kind of investment horizon. To illustrate the argument, I carry out an (in-sample) analysis at the monthly frequency.

The parameter estimates at the monthly frequency are very close to those obtained at the daily frequency.[27] Such consistency ensures that the pricing model is stable. Table IX reports the results of the regression in levels and in differences over different periods.

Table IX [about here]

The analysis offers a number of interesting results. First, the explanatory of the model remains remarkably stable when the frequency under consideration decreases. For example, Table IX shows that the model explains on average 66.3% and 83.2% of the time-variation in the levels of sovereign credit spreads in emerging and European countries, respectively, over the pre-crisis period. At the daily frequency, these estimates are on average 65.8% and 80%. The difference in explanatory power narrows during the 2010-2011 period. The consistency in explanatory power across frequencies ensures the robustness of the results. The analysis of the point estimates also indicates that the model cannot be rejected, as θ_i and θ_i^{cs} are not significantly different from zero and one, respectively, for all countries.

Analysis in Differences

At the monthly frequency, it becomes relevant to analyze credit spread changes, in addition to the levels, as in Longstaff et al. (2011). I thus additionally consider the following econometric specification for the analysis of credit spread changes:

$$\Delta CS_{Obs,i,t} = \delta_i + \delta_i^{cs} \Delta CS_{Model,i,t} + \varepsilon_{i,t} \tag{22}$$

where I correct for heteroskedasticity with the Newey and West's non-parametric variance covariance estimator.

The results show that the model can explain a substantial fraction of the time-variation in sovereign credit spread changes. For emerging markets, the R^2 measure is 38% and 52% at the monthly

[27]The parameter values are not reported for space consideration, but are available on request.

frequency (Panel A), before and during the European crisis, respectively. In comparison, the explanatory power equals 14% and 23% with daily data.[28] For European countries, the difference is also striking; the R^2 measure over both periods is 29% and 25% at the monthly frequency (Panel B), while it is 3% and 14% with daily data. Therefore, as suggested by Doshi et al. (2011), there seems to be a large amount of noise at the daily level that vanishes when the frequency of the data decreases.

4 Conclusion

This study develops and estimates a structural model for sovereign credit risk that endogenizes government debt and default policies. The model is a novel and intuitive framework for the analysis of country creditworthiness that provides a closed-form solution for the computation of sovereign credit spreads. When the theory is applied to the data, this paper provides evidence that the model helps explain the level and the dynamics of sovereign credit spreads in emerging markets and European countries. The results are robust in- and out-of-sample, both before and during the recent European crisis. The success relies on the capacity to structurally extract a time-varying measure of expected fundamentals from the local stock market, which is found to be a central ingredient for the valuation of sovereign credit risk.

The results of this paper improve our understanding of why sovereign credit spreads exhibit considerable daily variation, while economic data known to be determinants of sovereign credit risk are only available quarterly or annually. In addition, the results indicate that market-wide factors play a rather marginal role in the explanation of sovereign credit risk. The time-variation in sovereign credit spreads is thus primarily driven by the model-implied spreads, using country-specific information embedded in daily stock market prices, and only secondarily by measures of global market uncertainty or funding liquidity.

Other dimensions are likely to play a role in the valuation of a country's creditworthiness, such as the level of political uncertainty embedded in the European debt crisis, the role of imperfect information on a country's fundamentals, or the fear of an international contagion in credit risk. While such analysis is beyond the scope of this study, it certainly provides an interesting agenda for future research.

[28]Details of these results are not reported but are available on request

5 Appendix

5.1 Default Policy

The first-order maximization of sovereign wealth $W(V)$ yields

$$\frac{\partial W(V)}{\partial V} = \tau - \tau\beta\lambda\left(\frac{V}{V^D}\right)^{\beta-1} - \frac{C\phi\left(\alpha-1\right)\beta}{rV^D}\left(\frac{V}{V^D}\right)^{\beta-1} \tag{23}$$

Using the smooth-pasting condition $\frac{\partial W(V)}{\partial V}\mid_{V=V^D}=\tau\left(1-\lambda\right)$, we have

$$\tau - \tau\beta\lambda - \frac{C\phi\left(\alpha-1\right)\beta}{rV^D} = \tau\left(1-\lambda\right) \tag{24}$$

which yields

$$V^{D*} = \frac{C\phi\left(\alpha-1\right)\beta}{\tau r\lambda\left(1-\beta\right)} \tag{25}$$

5.2 Debt Policy

The optimal level of indebtedness is determined to maximize the sum $W(X)+D(X)$, where

$$W(V)+D(V)\mid_{V^D=V^{D*}} = \tau V - \lambda C\varphi\left(\frac{V}{C\varphi}\right)^{\beta} + \frac{\alpha C}{r}\left[1-\phi\left(\frac{V}{C\varphi}\right)^{\beta}\right] \tag{26}$$

under the optimal default policy given by $V^{D*}=\varphi C$, with $\varphi=\frac{\phi(\alpha-1)\beta}{\tau r\lambda(1-\beta)}$.

The first-order maximization yields

$$\frac{\partial W(V)+D(V)\mid_{V^D=V^{D*}}}{\partial C} = -\lambda\varphi\left(\frac{V}{C\varphi}\right)^{\beta} + \frac{\alpha}{r}\left[1-\phi\left(\frac{V}{C\varphi}\right)^{\beta}\right] + \beta\left(\lambda\varphi+\frac{\alpha\phi}{r}\right)\left(\frac{V}{C\varphi}\right)^{\beta} \tag{27}$$

$$= 0 \tag{28}$$

After simplification, we have

$$\left(1-\beta\right)\left(\lambda\varphi+\frac{\alpha\phi}{r}\right)\left(\frac{V}{C\varphi}\right)^{\beta} = \frac{\alpha}{r} \tag{29}$$

Replacing $\varphi = \frac{\phi(\alpha-1)\beta}{\tau r\lambda(1-\beta)}$ in the above equation yields

$$\left(\frac{V\tau r\lambda\left(1-\beta\right)}{C\phi\left(\alpha-1\right)\beta}\right)^{\beta} = \frac{\alpha}{\phi\left(\alpha-\beta\right)} \tag{30}$$

Finally, the optimal debt coupon chosen at time $t = 0$ is given by the expression

$$C^{*} = \frac{V_0\tau r\lambda\left(1-\beta\right)}{\phi\left(\alpha-1\right)\beta}\left(\frac{\phi\left(\alpha-\beta\right)}{\alpha}\right)^{1/\beta} \tag{31}$$

5.3 Sovereign Credit Spread

The yield spread CS is defined as

$$CS(V) \equiv \frac{C^{*}}{D(V,C^{*},V^{D*})} - r = r\left[1-\phi\left(\frac{V}{V^{D*}}\right)^{\beta}\right]^{-1} - r \tag{32}$$

$$= r\left[\frac{1}{1-\frac{\alpha}{\alpha-\beta}\left(\frac{V}{V_0}\right)^{\beta}} - 1\right] \tag{33}$$

where the default policy at the optimal leverage is given by

$$V^{D*} = \frac{C^{*}\phi\left(\alpha-1\right)\beta}{\tau r\lambda\left(1-\beta\right)} = V_0\left(\frac{\phi\left(\alpha-\beta\right)}{\alpha}\right)^{1/\beta} \tag{34}$$

5.4 Estimation Procedure

The model credit spreads are generated using the following estimation procedure:

1. For country i, compute the time-series of expected fundamentals $\frac{V_{i,t}}{V_{i,0}}$ with the formula

$$\frac{V_{i,t}}{V_{i,0}} = \frac{S_{Obs,i,t}}{S_{Obs,i,0}}\frac{1}{\left[1-\frac{\lambda}{\phi}\frac{CS_{Obs,i,t}}{CS_{Obs,i,t}+r}\right]}$$

   where $CS_{Obs,i,t}$ and $S_{Obs,i,t}$ are the series of observed sovereign spreads (e.g., EMBI+ spreads) and local stock market prices (e.g., MSCI index), respectively. Parameter values λ, ϕ, and r are exogeneous and discussed in Section 3.1.1.

2. Use the time-series of expected fundamentals $\frac{V_{i,t}}{V_{i,0}}$ to generate a series of model spreads with the

formula

$$CS_{Model,i,t} = r\left[\frac{1}{1-\frac{\alpha_i}{\alpha_i-\beta_i}\left(\frac{V_{i,t}}{V_{i,0}}\right)^{\beta_i}}-1\right]$$

where $\alpha_i = \frac{r_{gi}-r}{r}$ and $\beta_i = -\frac{2r}{\sigma_i^2}$. Use, for example, $r_{gi} = 0.05$ and $\sigma_i = 0.4$ as starting values.

3. With standard numerical procedures, estimate the parameters r_{gi} and σ_i to minimize the squared pricing errors, given by $\sum_t (CS_{Obs,i,t} - CS_{Model,i,t})^2$. Note that for every change in parameter values, we need to compute a new series of $CS_{Model,i,t}$. This is done until the objective function is minimized.

4. Once the convergence is achieved, generate the series of model spreads using the final parameter values. Follow the same steps for each country.

References

[1] Andrade, S. C., 2009, "A Model of Asset Pricing under Country Risk," *Journal of International Money and Finance* 28(4), 671–695.

[2] Andrade, S. C. and V. Chhaochharia, 2011, "*How costly is sovereign default? Evidence from Financial markets*," Working Paper, University of Miami.

[3] Arellano, C., 2008, "Default Risk and Income Fluctuations in Emerging Economies," *American Economic Review* 98(3), 690–712.

[4] Bodie, Z., Gray, D. F. and R. C. Merton, 2007, "Contingent Claims Approach to Measuring and Managing Sovereign Credit Risk," *Journal of Investment Management* 5(4), 5–28.

[5] Borri, N. and A. Verdelhan, 2011, "*Sovereign Risk Premia*," Working Paper, MIT.

[6] Bulow, J. and K. Rogoff, 1989, "A Constant Recontracting Model of Sovereign Debt," *Journal of Political Economy* 97(1), 155–178.

[7] Caballero, R. J. and A. Krishnamurthy, 2005, "Exchange Rate Volatility and the Credit Channel in Emerging Markets: A Vertical Perspective," *International Journal of Central Banking* 1(1), 207–245.

[8] Campbell, J. Y. and G. Taksler, 2003, "Equity Volatility and Corporate Bond Yields," *Journal of Finance* 58(6), 2321–2349.

[9] Cohen, D. and J. Sachs, 1986, "Growth and External Debt under Risk of Debt Repudiation," *European Economic Review* 30(3), 529–560.

[10] Cremers, M. K. J., Driessen, J., Maenhout, P., and D. Weinbaum, 2008, "Individual Stock-Price Implied Volatility and Credit Spreads," *Journal of Banking and Finance* 32(1), 2706–2715.

[11] De Paoli, B., Hoggarth, G., and V. Saporta, 2006, "Costs of Sovereign Default," *Financial Stability Paper No. 1*, Bank of England.

[12] Doshi, H., Ericsson, J., Jacobs, K., and S. M. Turnbull, 2011, "On Pricing Credit Default Swaps With Observable Covariates," Working Paper, University of Houston.

[13] Duffie, D., Pedersen, L. H. and K. J. Singleton, 2003, "Modeling Sovereign Yield Spreads: A Case Study of Russian Debt," *Journal of Finance* 58(1), 119–159.

[14] Dumas, B., 1991, "Super Contact and Related Optimality Conditions," *Journal of Economic Dynamics and Control* 15(4), 675–685.

[15] Eaton, J. and M. Gersowitz, 1981, "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *Review of Economic Studies* 48(152), 288–309.

[16] Eichengreen, B., 1989, *"Til Debt Do Us Part: The U.S. capital Market and Foreign Lending, 1920-1955*," NBER Working Paper No 2394.

[17] Ericsson, J., Jacobs, K., and R. Oviedo, 2009, "The Determinants of Credit Default Swap Premiums," *Journal of Financial and Quantitative Analysis* 44(1), 109–132.

[18] Fischer, E., Heinkel, R., and J. Zechner, 1989, "Dynamic Capital Structure Choice: Theory and Tests," *Journal of Finance* 44(1), 19–40.

[19] François, P., Hübner, G. and J.-R. Sibille, 2011, "A Structural Balance Sheet Model of Sovereign Credit Risk," *Finance*, Forthcoming.

[20] Frankel, J. A. and D. Romer, 1999, "Does Trade Cause Growth?," *American Economic Review* 89(3), 379–399.

[21] Gapen, M., Gray, D., Lim, C. H. and Y. Xiao, 2008, "Measuring and Analyzing Sovereign Risk with Contingent Claims," *IMF Staff Papers* 55(1), 109–148.

[22] Gelos, R. G., Sahay, R. and G. Sandleris, 2011, "Sovereign Borrowing by Developing Countries: What Determines Market Access?," *Journal of International Economics* 83(2), 243–254.

[23] Gennaioli, N., Martin, A. and S. Rossi, 2011, "*Sovereign Default, Domestic Banks and Financial Institutions*," Working Paper, Universitat Pompeu Fabra.

[24] Gibson, R. and S. Sundaresan, 2001, *"A Model of Sovereign Borrowing and Sovereign Yield Spreads*," Working Paper, Columbia University.

[25] Granger, C. and P. Newbold, 1974, "Spurious Regressions in Econometrics," *Journal of Econometrics* 2(2), 111–120.

[26] Hatchondo, J., C., and L. Martinez, 2009, "Long-duration Bonds and Sovereign Defaults," *Journal of International Economics* 79(1), 117–125.

[27] Hayri, A., 2000, "Debt Relief," *Journal of International Economics* 52(1), 137–152.

[28] Hilscher, J. and Y. Nosbusch, 2010, "Determinants of Sovereign Risk: Macroeconomic Fundamentals and the Pricing of Sovereign Debt," *Review of Finance* 4(2), 235–262.

[29] Holmstrom, B. and J. Tirole, 1998, "Private and Public Supply of Liquidity," *Journal of Political Economy* 106(1), 1–40.

[30] Leland, H. E., 1994, "Corporate Debt Value, Bond Covenants, and Optimal Capital Structure," *Journal of Finance* 49(4), 1213–1252.

[31] Longstaff, F. A., Pan, J., Pedersen, L. H., and K. J. Singleton, 2011, "How Sovereign Is Sovereign Credit Risk," *American Economic Journal: Macroeconomics* 3(2), 75–103.

[32] Martinez, J. and G. Sandleris, 2011, "Is it Punishment? Sovereign Defaults and the Decline in Trade," *Journal of International Money and Finance* 30(6), 909–930.

[33] Merton, R. C. 1973, "The Theory of Rational Option Pricing," *Bell Journal of Economics* 4(1), 141–183.

[34] Pan, J. and K. Singleton, 2008, "Default and Recovery Implicit in the Term Structure of Sovereign CDS Spreads," *Journal of Finance* 63(5), 2345–2384.

[35] Reinhart, C., Rogoff, K., and M. A. Savastano, 2003, "Debt Intolerance," *Brooking Papers on Economic Activity* 1, 1–70.

[36] Reinhart, C. and K. Rogoff, 2009, "*This Time is Different: Eight Centuries of Financial Folly*," Princeton University Press.

[37] Remolona, E., Scatigna, M., and E. Wu, 2008, "The Dynamic Pricing of Sovereign Risk in Emerging Markets: Fundamentals and Risk Aversion," *Journal of Fixed Income* 17(4), 57–71.

[38] Rose, A., 2005, "One Reason Countries Pay their Debts: Renegotiation and International Trade," *Journal of Development Economics* 77(1), 189–206.

[39] Sturzenegger, F. and J. Zettelmeyer, 2006, *"Debt Defaults and Lessons from a Decade of Crises*," MIT Press.

[40] Westphalen, M., 2002*, "Valuation of Sovereign Debt with Strategic Defaulting and Rescheduling*," Working Paper, University of Lausanne.

[41] Yue, V. Z., 2010, "Sovereign Default and Debt Renegotiation," *Journal of International Economics* 80(2), 176–187.

[42] Zhang, B., Zhou, H., and H. Zhu, 2009, "Explaining Credit Default Swap Spreads with Equity Volatility and Jump Risks of Individual Firms," *Review of Financial Studies* 22(12), 5099–5131.

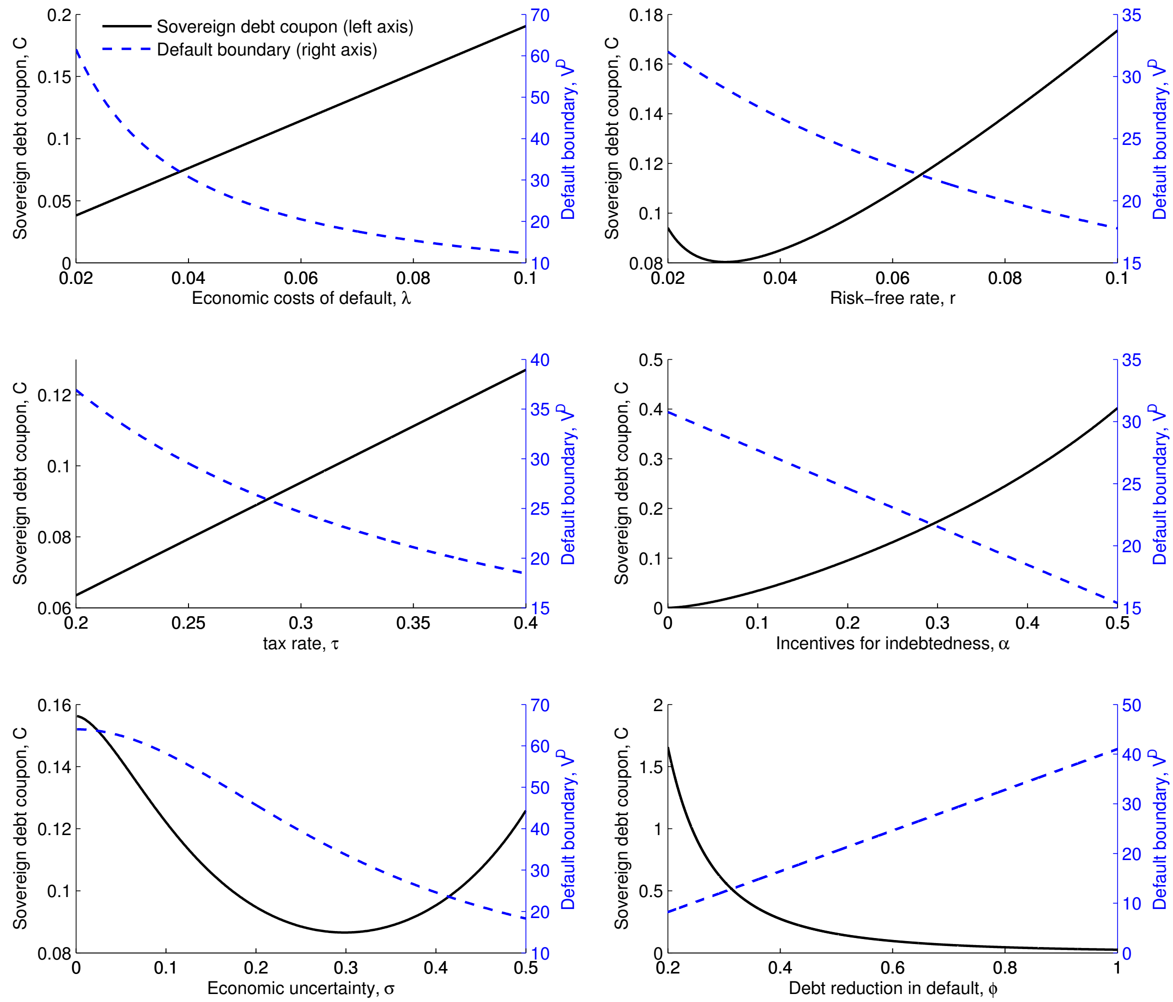

Figure 1: **Optimal Default and Debt Policies.** This figure illustrates how the model's main parameters determine the government's optimal default and debt policies. The default policy is computed with a fixed coupon payment ($C = 0.1$). Unless specified, the model's parameters are as follows: $V = V_0 = 1$, $\sigma = 0.4$, $r = 0.05$, $\tau = 0.3$, $\lambda = 0.05$, $\phi = 0.6$, and $\alpha = 0.2$.

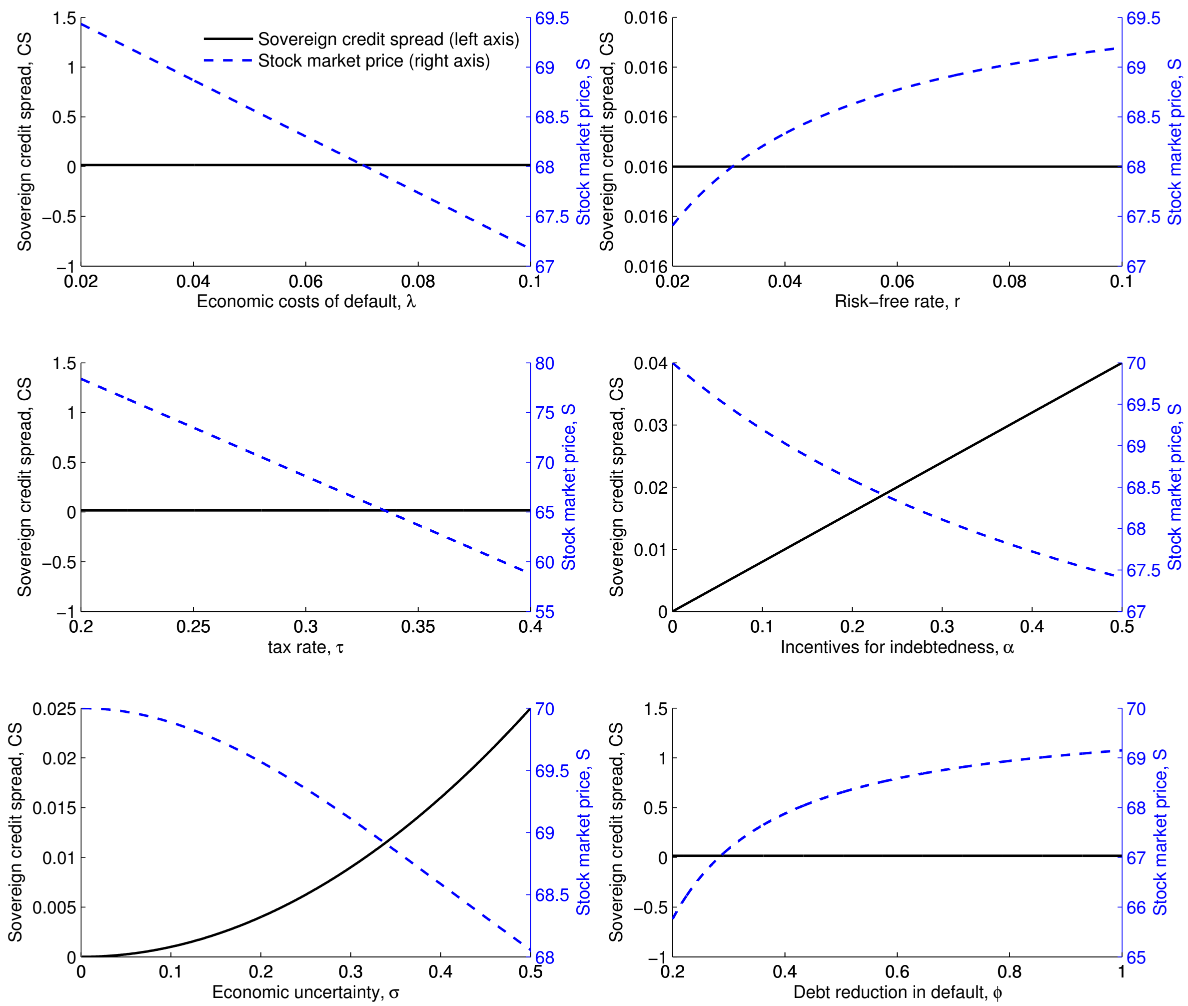

Figure 2: **Sovereign Credit Spread and Stock Market Price.** This figure illustrates how the model's main parameters affect sovereign credit risk and the valuation of the stock market. The sovereign credit spread is given by Equation 14 and the stock market price by Equation 16. Unless specified, the model's parameters are as follows: $V = V_0 = 100$, $\sigma = 0.4$, $r = 0.05$, $\tau = 0.3$, $\lambda = 0.05$, $\phi = 0.6$, and $\alpha = 0.2$.

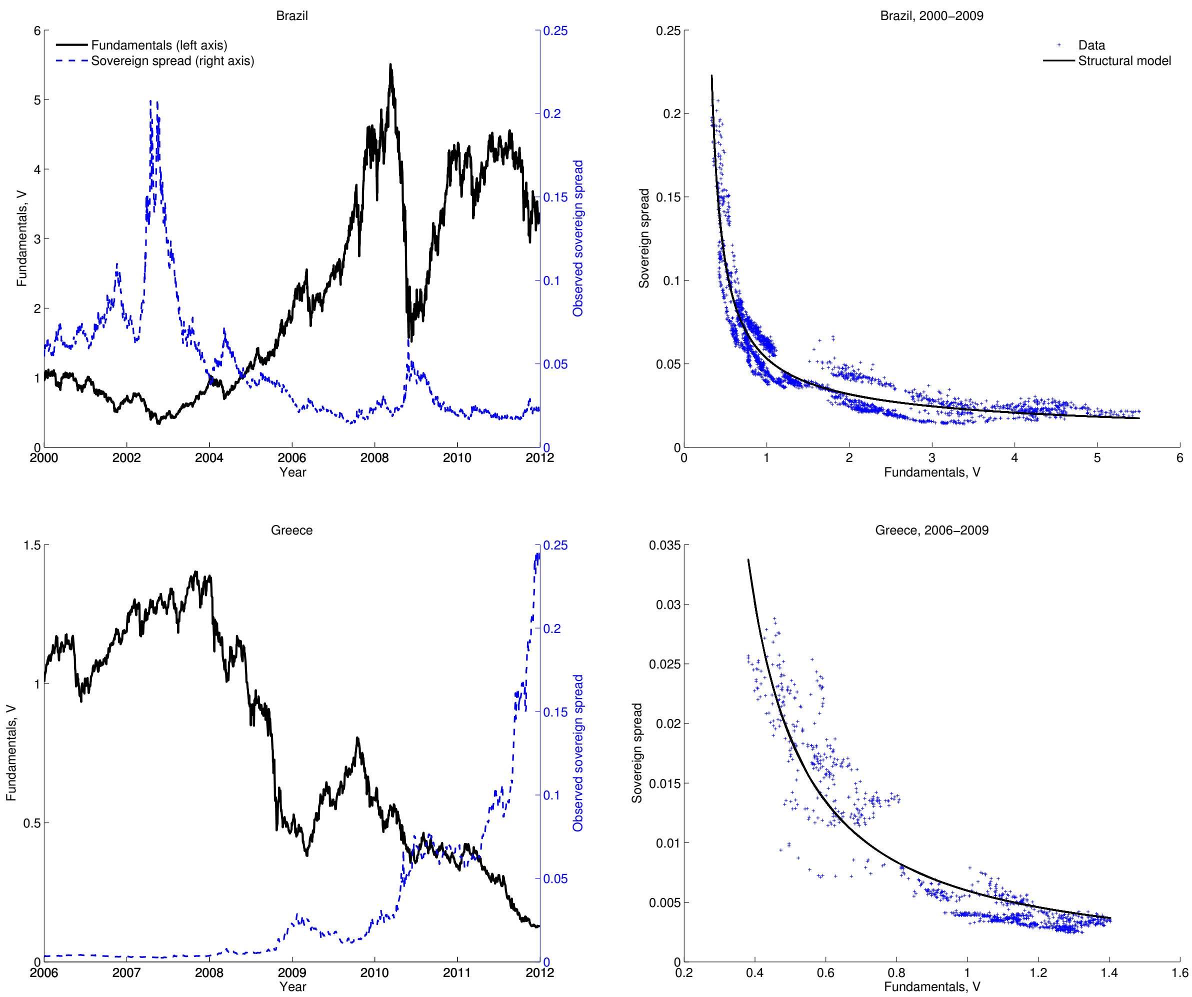

Figure 3: **Expected Fundamentals and Observed Sovereign Spreads.** The left panels plot the dynamics of daily observed spreads and the market-implied fundamentals for Brazil and Greece over the full sample period. The right panels illustrate their relationships over the period 2000-2009 for Brazil and the period 2006-2009 for Greece. The relationship in the data (blue crosses) is compared with that implied by the structural model (black line). The time-series of expected fundamentals, which capture forward-looking economic conditions, is extracted from local stock market prices (see Section 3.1.1). The sovereign spreads consist of the JP Morgan Emerging Market Bond Index (EMBI+) spreads and the Global Bond Index (GBI) spreads for Brazil and Greece, respectively.

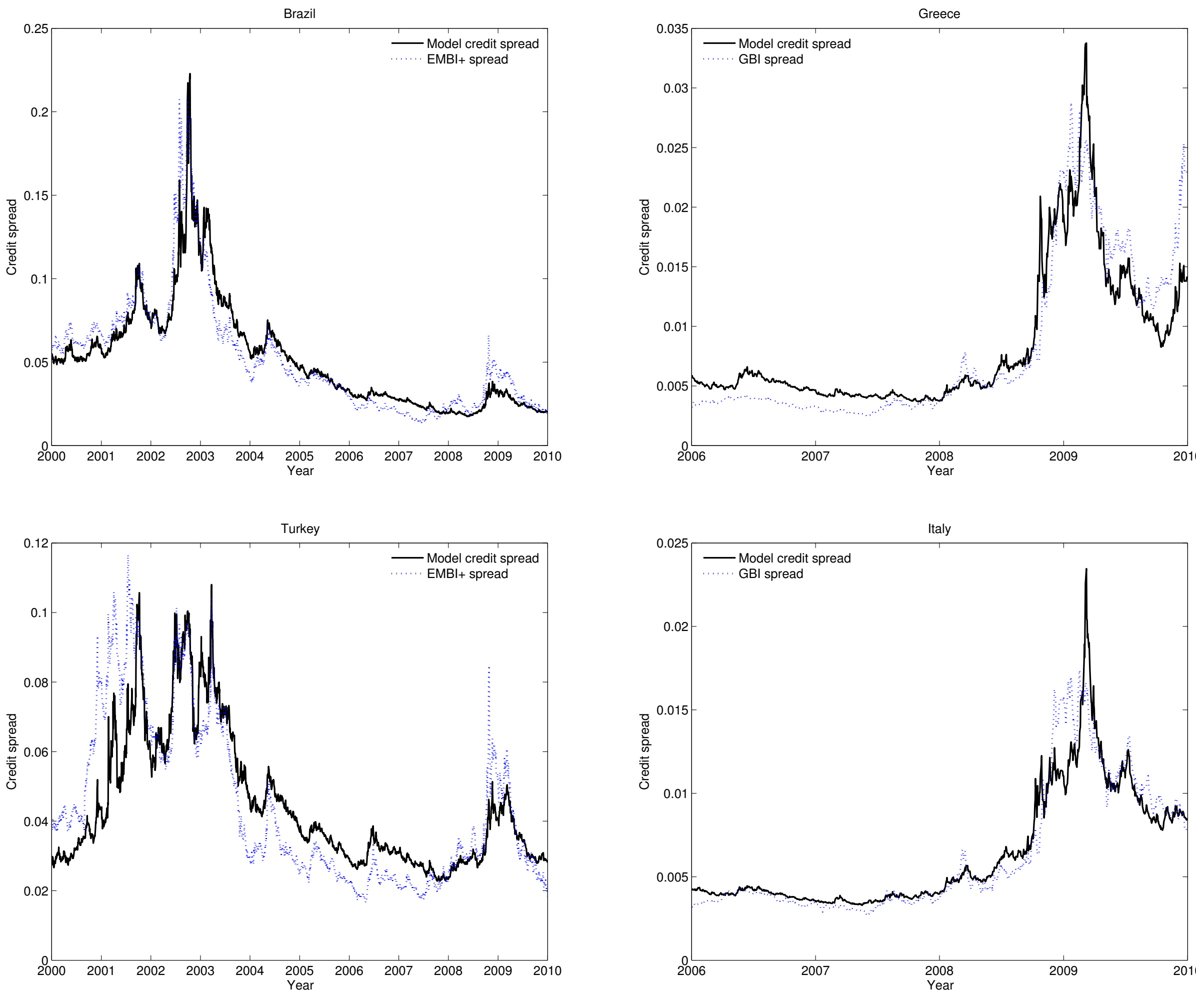

Figure 4: **Comparison of Observed and Model Credit Spreads - Pre-crisis Period.** The figure compares the model credit spreads and observed spreads for Brazil and Turkey over the period 2000-2009 (left panels), and for Greece and Italy over the period 2006-2009 (right panels). Both periods precede the 2010-2011 European debt crisis. The model credit spreads are computed using information on expected fundamentals extracted from local stock market prices (see Section 3.1) and the parameter values presented in Table I. The sovereign spreads for Brazil and Turkey consist of the JP Morgan Emerging Market Bond Index (EMBI+) spreads, whereas the Global Bond Index (GBI) spreads are used for Greece and Italy.

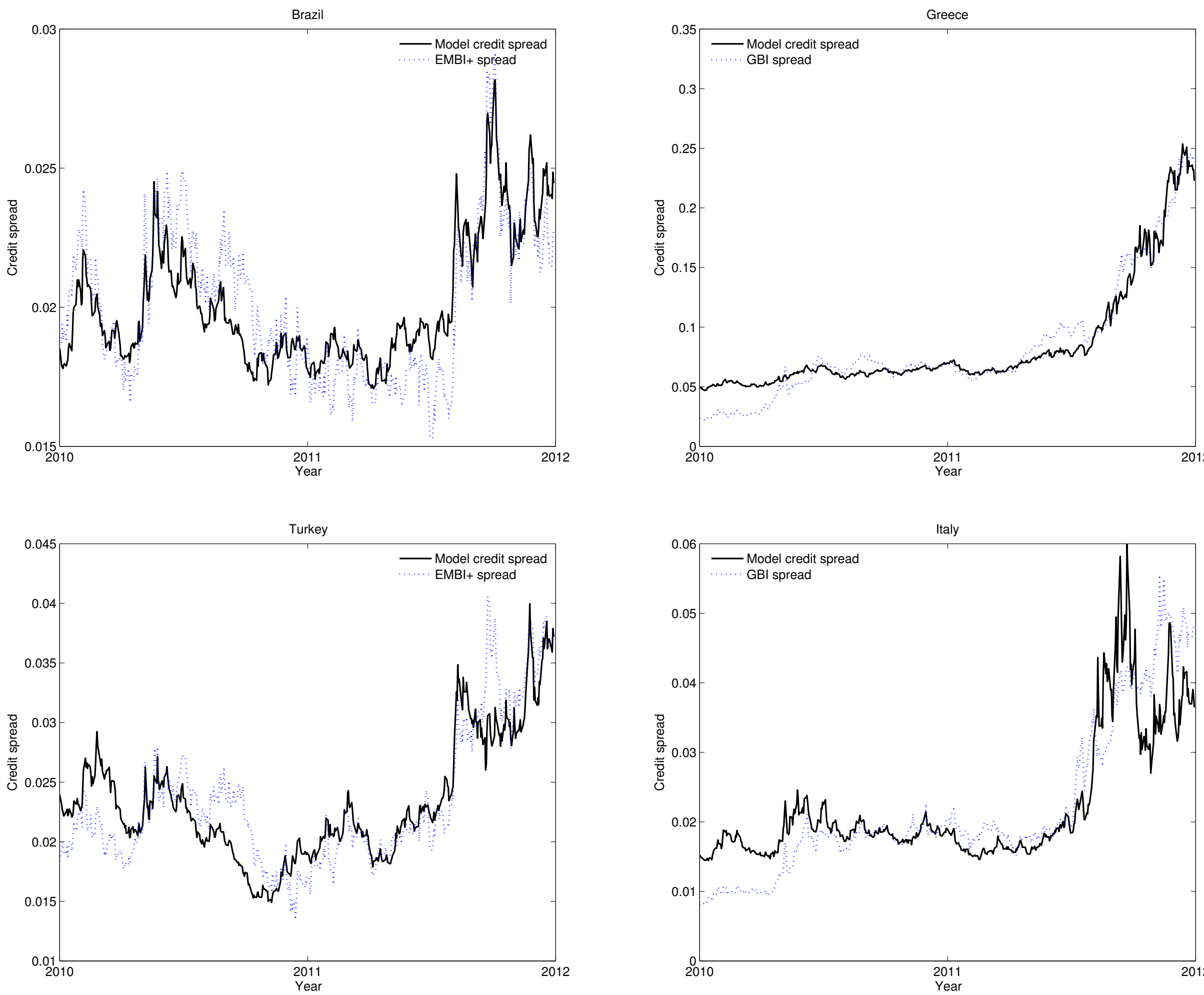

Figure 5: **Comparison of Observed and Model Credit Spreads - European Debt Crisis.** The figure compares the model credit spreads and observed spreads for Brazil and Turkey (left panels), and for Greece and Italy (right panels) over the period 2010-2011. This period characterizes the European debt crisis. The model credit spreads are computed using information on expected fundamentals extracted from local stock market prices (see Section 3.1) and the parameter values presented in Table I. The sovereign spreads for Brazil and Turkey consist of the JP Morgan Emerging Market Bond Index (EMBI+) spreads, whereas the Global Bond Index (GBI) spreads are used for Greece and Italy.

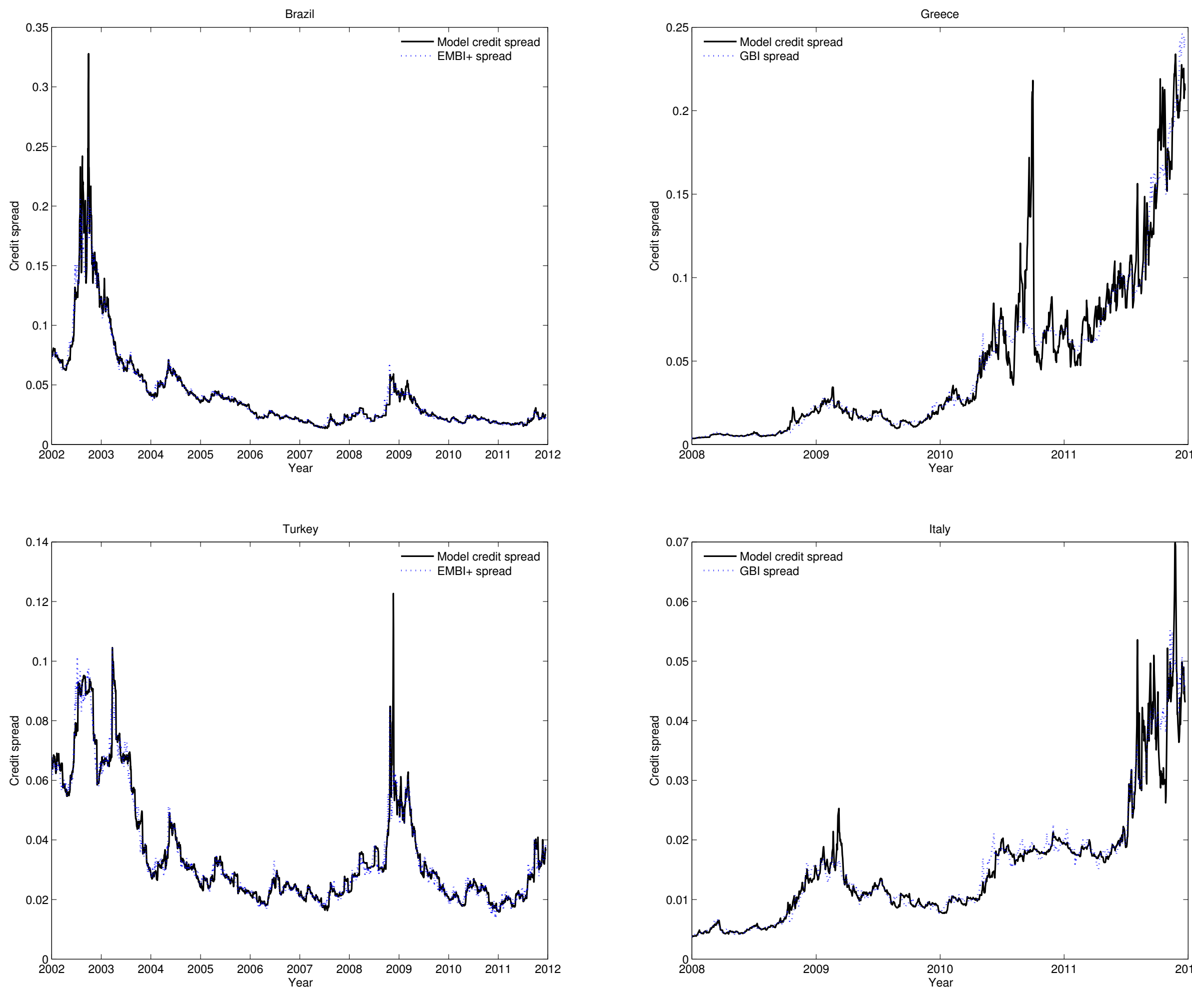

Figure 6: **Out-of-sample Comparison of Observed Spreads and Model Credit Spread Forecasts.** The figure compares the model credit spread forecasts and the observed sovereign spreads for Brazil, Turkey, Greece, and Italy over the full sample period. The model credit spread forecasts are computed at a one month horizon, using information on expected fundamentals extracted from local stock market prices. The model is estimated using rolling windows over the two years that precede the forecasting period. The procedure is described in Section 3.3. The sovereign spreads for Brazil and Turkey consist of the JP Morgan Emerging Market Bond Index (EMBI+) spreads (left panels), whereas the Global Bond Index (GBI) spreads are used for Greece and Italy (right panels).

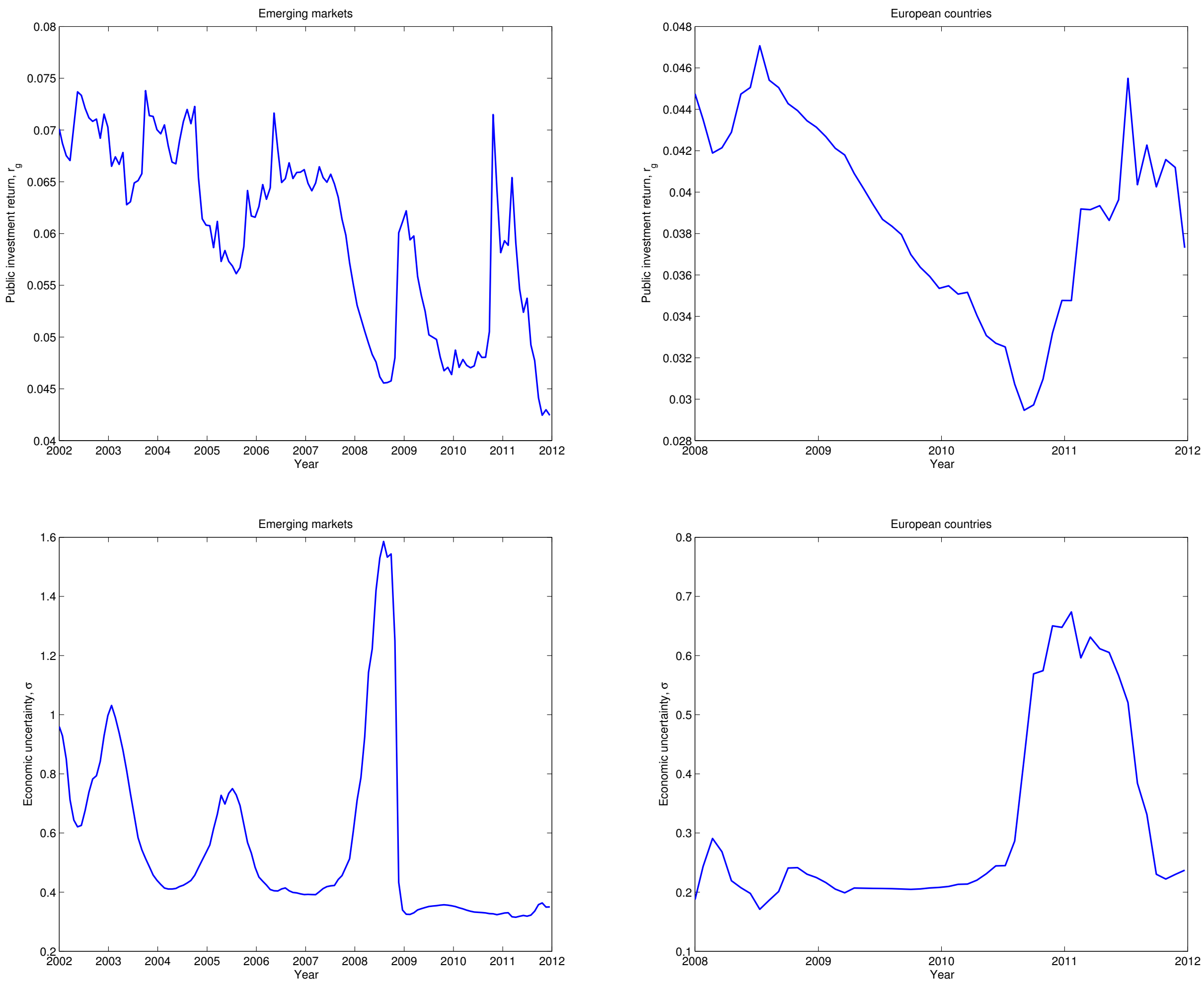

Figure 7: **Model Parameters Estimated using Rolling-windows.** The figure displays the time-series of the monthly parameter values estimated with 500-days rolling-windows. The procedure is described in Section 3.3. The left panels provide the average parameter estimates across emerging markets and the right panels provide the average estimates across European countries. The expected public investment return r_g is illustrated in the top panels. Economic uncertainty σ is illustrated in the bottom panels.

Table I : **Estimated Parameters.** This table presents the parameter values estimated from the equation $\{r^*_{gi}, \sigma^*_i\} = \arg\min_{r_{gi},\sigma_i} \sum_t (CS_{Obs,i,t} - CS_{Model,i,t})^2$ over the sample period, which spans 2000-2011 for emerging countries (Panel A) and 2006-2011 for European countries (Panel B). The estimation disentangles the European debt crisis period (2010-2011) and the pre-crisis period (up to 2009). $CS_{Obs,i,t}$ and $CS_{Model,i,t}$ stand for the observed sovereign spreads and the credit spreads estimated by the model, respectively, for country i at time t. The last columns report the root mean square error (RMSE) over the estimation periods computed as RMSE= $\sqrt{\frac{\sum_t (CS_{Obs,i,t} - CS_{Model,i,t})^2}{T}}$. The frequency of the data is daily and the values are annualized.

	Return on public investment r_g		Economic uncertainty σ		RMSE (bps)	
Panel A: emerging markets						
	2000-2009	2010-2011	2000-2009	2010-2011	2000-2009	2010-2011
Brazil	0.0647	0.0422	0.4790	0.3011	113.0	14.7
Colombia	0.0567	0.0390	0.6391	0.3706	95.3	25.6
Mexico	0.0523	0.0417	0.5299	0.2968	57.7	19.1
Peru	0.0659	0.0354	0.4791	0.4622	93.5	27.3
The Philippines	0.0533	0.0378	0.5450	0.4086	100.4	32.3
Russia	0.0829	0.0470	0.4317	0.2957	267.3	37.3
Turkey	0.0545	0.0490	0.5186	0.2748	126.4	28.8
Average	0.0615	0.0417	0.5175	0.3443	121.9	26.4
Panel B: European countries						
	2006-2009	2010-2011	2006-2009	2010-2011	2006-2009	2010-2011
France	0.0368	0.0316	0.1507	0.0956	3.8	21.6
Greece	0.0426	0.0279	0.2363	0.4823	25.2	135.1
Ireland	0.0379	0.0410	0.2116	0.2729	19.2	161.3
Italy	0.0407	0.0367	0.2343	0.1870	12.2	55.2
Portugal	0.0425	0.0476	0.1860	0.2087	13.5	165.4
Spain	0.0415	0.0361	0.1615	0.1821	13.7	55.9
Average	0.0403	0.0368	0.1967	0.2381	14.6	99.1

Table II : **Statistics of the Model and Observed Credit Spreads for Emerging Countries.** This table provides descriptive statistics for the model credit spreads and JP Morgan Emerging Market Bond Index (EMBI+) spreads for emerging countries. The analysis disentangles the European debt crisis period (2010-2011) and the pre-crisis period (up to 2009). The frequency of the data is daily and the values are annualized.

	Pre-crisis (2000-2009)		Debt crisis (2010-2011)	
	Mean	Standard Deviation	Mean	Standard Deviation
Brazil				
Model spreads	0.0525	0.5111	0.0201	0.0362
EMBI+ spreads	0.0525	0.5429	0.0201	0.0431
Colombia				
Model spreads	0.0411	0.2735	0.0173	0.0273
EMBI+ spreads	0.0410	0.3148	0.0173	0.0473
Mexico				
Model spreads	0.0210	0.0754	0.0155	0.0226
EMBI+ spreads	0.0210	0.1187	0.0155	0.0379
Peru				
Model spreads	0.0364	0.2825	0.0182	0.0167
EMBI+ spreads	0.0366	0.3120	0.0182	0.0464
The Philippines				
Model spreads	0.0390	0.1316	0.0197	0.0202
EMBI+ spreads	0.0389	0.2146	0.0197	0.0548
Russia				
Model spreads	0.0461	0.4639	0.0229	0.0535
EMBI+ spreads	0.0446	0.6600	0.0229	0.0781
Turkey				
Model spreads	0.0447	0.3030	0.0233	0.0811
EMBI+ spreads	0.0445	0.3703	0.0233	0.0913
Average				
Model spreads	0.0401	0.2916	0.0196	0.0364
EMBI+ spreads	0.0399	0.3619	0.0196	0.0570

Table III : **Statistics of the Model and Observed Credit Spreads for European Countries.** This table provides descriptive statistics for the model credit spreads and JP Morgan Global Bond Index (GBI) spreads for European countries. The analysis disentangles the European debt crisis period (2010-2011) and the pre-crisis period (up to 2009). The frequency of the data is daily and the values are annualized.

	Pre-crisis (2006-2009)		Debt crisis (2010-2011)	
	Mean	Standard Deviation	Mean	Standard Deviation
France				
Model spreads	0.0008	0.0108	0.0039	0.0354
GBI spreads	0.0008	0.0123	0.0039	0.0498
Greece				
Model spreads	0.0083	0.0928	0.0849	0.7564
GBI spreads	0.0081	0.1068	0.0839	0.8142
Ireland				
Model spreads	0.0074	0.1356	0.0491	0.1333
GBI spreads	0.0072	0.1413	0.0489	0.2967
Italy				
Model spreads	0.0064	0.0551	0.0222	0.1441
GBI spreads	0.0063	0.0607	0.0220	0.1766
Portugal				
Model spreads	0.0039	0.0381	0.0435	0.2372
GBI spreads	0.0039	0.0445	0.0433	0.3596
Spain				
Model spreads	0.0021	0.0223	0.0220	0.1071
GBI spreads	0.0020	0.0324	0.0219	0.1425
Average				
Model spreads	0.0048	0.0591	0.0376	0.2356
GBI spreads	0.0047	0.0663	0.0373	0.3066

Table IV : **Correlation between Model and Observed Credit Spreads.** The table provides the correlation coefficients between daily model credit spreads and daily observed spreads, both in levels and in changes. The analysis covers emerging countries (Panel A) and European countries (Panel B). The analysis disentangles the European debt crisis period (2010-2011) and the pre-crisis period (up to 2009). The observed sovereign spreads consist of the JP Morgan Emerging Market Bond Index (EMBI+) spreads for emerging countries and of the Global Bond Index (GBI) spreads for European countries.

	Correlation in levels		Correlation in changes	
Panel A: emerging markets				
	2000-2009	2010-2011	2000-2009	2010-2011
Brazil	0.944	0.843	0.614	0.553
Colombia	0.878	0.519	0.190	0.346
Mexico	0.639	0.604	0.454	0.549
Peru	0.881	0.370	0.194	0.457
The Philippines	0.676	0.365	0.214	0.178
Russia	0.772	0.656	0.209	0.608
Turkey	0.842	0.867	0.511	0.499
Average	0.805	0.603	0.341	0.456
Panel B: European countries				
	2006-2009	2010-2011	2006-2009	2010-2011
France	0.870	0.727	0.063	0.301
Greece	0.930	0.966	0.222	0.278
Ireland	0.977	0.514	0.097	0.281
Italy	0.949	0.871	0.333	0.543
Portugal	0.877	0.686	0.128	0.272
Spain	0.744	0.785	0.131	0.509
Average	0.891	0.758	0.162	0.364

Table V : **Test of the Model, Pre-crisis Period.** The table provides the coefficient estimates of the regression $CS_{Obs,i,t} = \delta_i + \delta_i^{cs} CS_{Model,i,t} + \varepsilon_{i,t}$ over the pre-debt crisis period, which covers 2000-2009 for emerging countries (Panel A) and 2006-2009 for European countries (Panel B). The frequency of the data is daily. The heteroskedasticity consistent standard errors are corrected for serial correlation using the Newey and West's non-parametric variance covariance estimator, and are reported in parentheses. The symbols *, **, and *** indicate the coefficients' significance at the 90, 95, and 99% confidence levels, respectively.

	Constant δ	Model CS δ^{cs}	t-stat $Ho: \delta_i^{cs} = 1$	R^2	Observations
Panel A: emerging markets					
Brazil	-0.000	1.003***	0.083	0.892	2609
	(0.002)	(0.036)			
Colombia	-0.000	1.010***	0.286	0.771	2609
	(0.001)	(0.036)			
Mexico	-0.000	1.007***	0.131	0.409	2609
	(0.001)	(0.053)			
Peru	0.001	0.973***	-0.861	0.776	2609
	(0.001)	(0.031)			
The Philippines	-0.004	1.102***	1.568	0.456	2609
	(0.003)	(0.065)			
Russia	-0.006***	1.098***	1.385	0.596	2609
	(0.002)	(0.071)			
Turkey	-0.001	1.029***	0.832	0.709	2609
	(0.002)	(0.035)			
Average	-0.002	1.032***	0.489	0.658	2609
	(0.002)	(0.047)			
Panel B: European countries					
France	0.000	0.994***	-0.093	0.757	1044
	(0.000)	(0.061)			
Greece	-0.001	1.071***	1.010	0.865	1044
	(0.000)	(0.071)			
Ireland	-0.000	1.018***	0.700	0.954	1044
	(0.000)	(0.025)			
Italy	-0.000	1.046***	0.637	0.900	1044
	(0.000)	(0.072)			
Portugal	-0.000	1.023***	0.299	0.769	1044
	(0.000)	(0.077)			
Spain	-0.000	1.080***	0.749	0.553	1044
	(0.000)	(0.107)			
Average	-0.000	1.039***	0.550	0.800	1044
	(0.000)	(0.069)			

Table VI : **Test of the Model, Crisis Period.** The table provides the coefficient estimates of the regression $CS_{Obs,i,t} = \delta_i + \delta_i^{cs} CS_{Model,i,t} + \varepsilon_{i,t}$ over the 2010-2011 European debt crisis period. The table provides results for emerging countries (Panel A) and European countries (Panel B). The frequency of the data is daily. The heteroskedasticity consistent standard errors are corrected for serial correlation using the Newey and West's non-parametric variance covariance estimator, and are reported in parentheses. The symbols *, **, and *** indicate the coefficients' significance at the 90, 95, and 99% confidence levels, respectively.

	Constant δ	Model CS δ^{cs}	t-stat $Ho : \delta_i^{cs} = 1$	R^2	Observations
Panel A: emerging markets					
Brazil	-0.000	1.004***	0.061	0.709	521
	(0.001)	(0.070)			
Colombia	-0.000	1.011***	0.062	0.268	521
	(0.003)	(0.185)			
Mexico	-0.000	1.014***	0.099	0.364	521
	(0.002)	(0.141)			
Peru	-0.000	1.027***	0.096	0.135	521
	(0.005)	(0.282)			
The Philippines	0.000	0.989***	-0.045	0.132	521
	(0.005)	(0.236)			
Russia	0.001	0.957***	-0.319	0.429	521
	(0.003)	(0.133)			
Turkey	0.001	0.976***	-0.334	0.751	521
	(0.002)	(0.073)			
Average	0.000	0.997***	-0.054	0.398	521
	(0.003)	(0.160)			
Panel B: European countries					
France	-0.000	1.024***	0.128	0.528	521
	(0.001)	(0.186)			
Greece	-0.004	1.039***	1.047	0.933	521
	(0.004)	(0.038)			
Ireland	-0.007	1.144***	0.715	0.263	521
	(0.011)	(0.201)			
Italy	-0.002	1.068***	0.533	0.758	521
	(0.002)	(0.127)			
Portugal	-0.002	1.040***	0.363	0.470	521
	(0.005)	(0.111)			
Spain	-0.001	1.045***	0.426	0.616	521
	(0.003)	(0.105)			
Average	-0.003	1.060***	0.535	0.594	521
	(0.004)	(0.128)			

Table VII : **Analysis of the Global Factors.** The table provides the coefficient estimates of the regression $CS_{Obs,i,t} = \delta_i + \delta_i^{cs} CS_{Model,i,t} + \gamma_i^{vix} VIX_t + \gamma_i^{sp} R_{SP500,t} + \gamma_i^{ust} UST_t + \gamma_i^{slope} UST_{slope,t} + \omega_{i,t}$. The global factors are gradually added in the regression. The sample period spans 2000-2011 for emerging countries (Panel A) and 2006-2011 for European countries (Panel B). The frequency of the data is daily. The table reports the averages across emerging and European countries. The heteroskedasticity consistent standard errors are corrected for serial correlation using the Newey and West's non-parametric variance covariance estimator, and are reported in parentheses. The symbols *, **, and *** indicate the coefficients' significance at the 90, 95, and 99% confidence levels, respectively.

Average across countries							
Constant	Model CS	VIX	Return SP500	UST 5Y	UST Slope	$\overline{R}^2$	Local $\overline{R}^2$
γ	δ^{cs}	γ^{vix}	γ^{sp}	γ^{ust}	γ^{slope}		
Panel A: emerging markets							
-0.002	1.032***					0.696	
(0.001)	(0.041)						
-0.012***	0.969***	0.055***				0.808	0.868
(0.001)	(0.038)	(0.004)					
-0.012***	0.969***	0.055***	0.025***			0.809	0.867
(0.001)	(0.038)	(0.004)	(0.013)				
-0.024***	0.914***	0.074***	0.041***	0.280***		0.846	0.822
(0.003)	(0.035)	(0.007)	(0.012)	(0.058)			
-0.024***	0.923***	0.074***	0.041***	0.281***	-0.001	0.847	0.821
(0.004)	(0.040)	(0.007)	(0.012)	(0.090)	(0.047)		
Panel B: European countries							
-0.003	1.150***					0.516	
(0.002)	(0.167)						
0.001	1.532***	-0.041***				0.573	0.875
(0.002)	(0.205)	(0.008)					
0.001	1.542***	-0.043***	-0.035**			0.574	0.873
(0.002)	(0.206)	(0.008)	(0.017)				
0.036***	0.829***	-0.059***	-0.045***	-0.739***		0.722	0.686
(0.005)	(0.175)	(0.008)	(0.014)	(0.089)			
0.056***	0.737***	-0.059***	-0.040***	-1.116***	-0.283***	0.771	0.644
(0.008)	(0.156)	(0.009)	(0.014)	(0.152)	(0.078)		

Table VIII : **Out-of-sample Forecasts.** The table provides an out-of-sample comparison of the model credit spreads and observed spreads. The sample period spans 2000-2011 for emerging countries (Panel A) and 2006-2011 for European countries (Panel B). The model is estimated over rolling-windows of 500 trading days (2 years). Forecasts are generated for the following 20 trading days (1 month horizon). Out-of-sample accuracy is measured by the root mean square error (RMSE) and the mean absolute error (MAE). Results are compared with the expected spreads generated by a random walk model over the foreacasting period. The frequency of the data is daily.

	Root mean square forecast errors (bps)		Mean forecast errors (bps)		Correlation in differences	
	Model	Random walk	Model	Random walk	Model	Random walk
Panel A: emerging markets						
Brazil	6.68	7.22	2.52	3.60	0.483	0.064
Colombia	4.04	4.38	2.40	2.60	0.151	-0.004
Mexico	1.98	2.99	1.09	1.53	0.487	-0.044
Peru	3.54	3.98	2.27	2.48	0.130	0.024
The Philippines	3.01	4.21	1.89	2.23	0.125	-0.050
Russia	4.06	3.83	2.13	2.07	0.301	0.023
Turkey	3.92	5.03	2.11	2.86	0.355	0.101
Average	3.89	4.52	2.06	2.48	0.290	0.016
Panel B: European countries						
France	1.00	1.04	0.44	0.49	0.301	-0.034
Greece	14.94	7.88	6.51	4.07	0.174	0.057
Ireland	12.95	4.51	4.27	2.76	0.050	-0.013
Italy	2.51	2.28	1.24	1.36	0.546	-0.077
Portugal	11.33	3.07	3.28	1.82	0.038	0.082
Spain	2.59	2.51	1.39	1.46	0.488	-0.091
Average	7.55	3.55	2.86	1.99	0.266	-0.013

Table IX : **Analysis with Monthly Data.** The table provides the coefficient estimates of the regression in levels $CS_{Obs,i,t} = \delta_i + \delta_i^{cs} CS_{Model,i,t} + \varepsilon_{i,t}$ and in differences $\Delta CS_{Obs,i,t} = \delta_i + \delta_i^{cs} \Delta CS_{Model,i,t} + \varepsilon_{i,t}$ with monthly data. The analysis disentangles the European debt crisis period (2010-2011) and the pre-crisis period (up to 2009) for emerging countries (Panel A) and European countries (Panel B). The heteroskedasticity consistent standard errors are corrected for serial correlation using the Newey and West's non-parametric variance covariance estimator, and are reported in parentheses. The symbols *, **, and *** indicate coefficients' significance at the 90, 95, and 99% confidence levels, respectively.

	Constant δ	Model CS δ^{cs}	t-stat $Ho: \delta_i^{cs} = 1$	R^2	Observations
Panel A: emerging markets					
Levels					
2000-2009	-0.002	1.030***	0.125	0.663	120
	(0.006)	(0.150)			
2010-2011	0.000	0.996***	-0.018	0.393	24
	(0.005)	(0.270)			
Differences					
2000-2009	-0.000	1.191***	0.422	0.380	120
	(0.000)	(0.210)			
2010-2011	-0.000	1.895***	2.083	0.515	24
	(0.000)	(0.378)			
Panel B: European countries					
Levels					
2006-2009	-0.000	1.036***	0.622	0.832	48
	(0.000)	(0.069)			
2010-2011	-0.002	1.039***	0.211	0.595	24
	(0.011)	(0.205)			
Differences					
2006-2009	-0.000	0.543***	-5.991	0.288	48
	(0.000)	(0.091)			
2010-2011	0.001	0.401***	-4.035	0.246	24
	(0.001)	(0.165)			