

Decomposing euro-area sovereign spreads: credit and liquidity risks*

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Abstract

This paper presents an arbitrage-free model of the joint dynamics of euro-area sovereign bond spreads. The latter reflect both credit and liquidity differences between national bonds. An innovative aspect of the approach lies in the modeling of intertwined credit- and liquidity-related crisis regimes. We find that a substantial share of the changes in euro-area yield differentials is liquidity-driven over the financial-crisis period. Once liquidity-pricing effects and risk premia are filtered out of the spreads, we obtain estimates of the actual default probabilities. These are significantly lower than their risk-neutral counterparts, which is consistent with the existence of a non-diversifiable euro-area sovereign credit risk.

JEL codes: E43, E44, E47, G12.

Keywords: default risk, liquidity risk, term structure of interest rates, regime switching, euro-area spreads.

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1 Introduction

One of the most spectacular symptoms of the crisis that began in mid-2007 is the dramatic rise in intra euro-area government-bond yield spreads. Whereas all euro-area sovereign 10-year bond yields were contained in a range of 50 bp between 2002 and 2007, the average spreads over Germany of only two countries were lower than 50 basis points in 2011, the debt-weighted mean being of about 250 bp. Since the inception of the euro in 1999 and the resulting elimination of exchange-rate risk, intra-euro-area spreads reflect the fluctuations of compensations demanded by investors for holding essentially two kinds of risks: credit and liquidity risks.¹ The credit risk is linked to the issuer's probability of default (PD). If investors assess that the PD of some indebted country is higher than in the past, the prices of the bonds issued by this country fall because the expected loss increases. Liquidity risk arises from the potential difficulty that one may have in selling the asset before its redemption (for instance if one is required to do so in distressed market conditions, where it is difficult to find a counterpart for trade relatively quickly). In many ways, the ongoing financial crisis has illustrated why, along with credit risk, liquidity risks matter and should not be underestimated (see Brunnermeier, 2009).

Disentangling credit and liquidity effects in bond prices is important in several respects. For instance, appropriate policy actions that may be needed to address a sharp rise in spreads depend on the source of the movement: if the rise in spreads reflects poor liquidity, policy actions should aim at improving market functioning. But if it is linked to credit concerns, the solvency of the debtors should be enhanced (see Codogno, Favero and Missale, 2003). Furthermore, optimal investment decisions would benefit from such a decomposition. In particular, those medium to long-term investors who buy bonds to hold them until redemption seek to buy bonds whose price is low because of poor liquidity, since it provides them with higher long-run returns than more liquid bonds with the same credit quality (see Longstaff, 2009).

In this paper, we develop a multi-issuer no-arbitrage affine term-structure framework to model the dynamics of bond spreads, with a twofold objective: to disentangle credit and liquidity components in euro-area sovereign spreads and to identify the part of these spreads corresponding to risk premia, defined as the part that would not be present if agents were risk-neutral. Risk premia

¹ Indeed, an overwhelming share of the euro-area sovereign debt is denominated in euros (see Eurostat, 2011).

are demanded by risk-averse investors to be compensated for non-diversifiable –or systematic– risk, and our results are supportive of the findings of Pan and Singleton (2008) and Longstaff et al. (2011) who point to the systematic nature of sovereign risk.² The resulting risk premia associated with sovereign credit quality implies that physical, or real-world, probabilities of default differ from their risk-neutral counterparts. Yet, the latter, derived from basic models like Litterman and Iben (1991), are extensively used by market practitioners, who refer to them as *implied default probabilities*.³ Our approach makes it possible to assess the deviations between the two kinds of PDs and we show that these can be substantial. In particular, these results are of significant interest in the current context where regulators want banks to model the *actual* default risk of even high-rated government bonds.⁴

In our framework, each country is characterized by a risk intensity which is the sum of a credit intensity and an illiquidity one. We propose an original use of regime-switching features to account for the joint dynamics of credit- and liquidity-related crises, the aim being to make the model consistent with theoretical approaches highlighting the potential interactions between these two kinds of risks.⁵ Credit- and liquidity-crisis regimes are key drivers of the countries' intensities, the latter being also affected by Gaussian shocks. In this framework, the spreads turn out to be linear combinations of the regime variables and of latent factors that follow Gaussian auto-regressive processes whose drifts depend on the regimes. Therefore, the model can be seen as a linear state-space model with regime switching.⁶ The countries' illiquidity intensities are driven by a single European liquidity-related factor. The identification of this factor is based on the exploitation of the term structure of the spreads between KfW (*Kreditanstalt für Wiederaufbau*), a German agency, and the *Bunds*, which are the bonds issued by the Federal Republic of Germany.

² Borri and Verdelhan (2011) propose a theoretical framework to investigate the implications of the investors' inability to hedge against correlated sovereign risks.

³ See e.g. Hull, Predescu and White (2005), Berd, Mashal and Wang (2003), Caceres, Guzzo and Segoviano (2010) or Berg (2009).

⁴ In early 2012, the European Union introduced new rules on trading-book capital, known as Basel 2.5. This package notably requires the banks to model the default risk of all sovereign entities for the first time. This contrasts with the special status that government bonds have enjoyed since the Basel Committee for Banking Supervision (BCBS) first proposed rules on the capital treatment of market risks in 1993. As stressed by Carver (*Risk Magazine*, 2012), these changes in regulation reveal the practitioners' lack of tools to extract actual default probabilities from market prices.

⁵ See e.g. Brunnermeier and Pedersen (2009) Brunnermeier and Pedersen [12] or Garleanu and Pedersen (2007) Garleanu and Pedersen [36].

⁶ Accordingly, we use Kim's (1994) algorithm to estimate the model parameters by maximizing the likelihood.

Indeed, the bonds issued by KfW, guaranteed by the Federal Republic of Germany, benefit from the same credit quality than the *Bunds* but are less liquid.⁷ Therefore, the KfW-*Bund* spread should be essentially liquidity-driven.⁸ The resulting liquidity-related factor significantly contributes to the dynamics of intra-euro spreads, supporting findings by Favero et al. (2010) or Manganeli and Wolswijk (2009).

The model is estimated on weekly data covering the last five years. These data consist of sovereign-bond yields associated with eight euro-area countries. Our estimation dataset is supplemented with survey-based forecasts. As evidenced by Kim and Orphanides (2012), this alleviates the downward small-sample bias in the persistence of the yields obtained with conventional estimation.⁹ Such biases typically result in too stable long-horizon expectations of yields and, as a consequence, overstate the variability of term premia. Generating reliable expectations is crucial given our goal of recovering historical –or actual, or real-world– probabilities of default from bond prices.

Our study contributes to the term-structure modeling literature in three main directions. First, we develop a regime-switching affine term-structure model –RS-ATSM hereinafter– that explicitly incorporates liquidity and default risks in a multi-country set up.¹⁰ This model relies on an original modeling of credit and liquidity crises based on interrelated switching regimes. Second, we bring this model to European data, shedding light on the fluctuations of intra-euro-area sovereign spreads over the last five years. Third, we investigate the potential of this RS-ATSM to generate term structures of PDs.

The remaining of this paper is organized as follows. Section 2 reviews related literature. Section 3 presents the model and details how bonds are priced in this framework. Section 4 deals with the choice and the construction of the data. Section 5 presents the estimation of the model and Section 6 examines the implication of the model in terms of liquidity and credit pricing. Section 7 summarizes the results and makes concluding remarks.

⁷ By abuse of language, we use here the term *Bunds* for the German sovereign bonds of any maturity although this name is usually used for ten-year bonds only.

⁸ See Schwarz (2009). This is also discussed in Subsection 4.2.

⁹ This way of reducing the bias is not the only one. In particular, Jardet, Monfort and Pegoraro (2009) use a “near-cointegrated framework” specification of the factors (averaging a stationary and a cointegrated specification).

¹⁰ Geyer, Kossmeier and Pichler (2004) present a multi-country ATSM. However, their model does not explicitly accommodate liquidity-pricing effects.

2 Related literature

There is compelling evidence that yields and spreads are affected by liquidity concerns¹¹. In particular, using euro-area data, Beber, Brandt and Kavajecz (2009) provide evidence of a nontrivial role in the dynamics of sovereign bond spreads, especially for low credit risk countries and during times of heightened market uncertainty.¹² In recent studies, some authors develop ATSM to breakdown several kinds of spreads into different components, including liquidity-related ones. These approaches are based on the assumption that there exists commonality amongst the liquidity components of prices of different bonds.¹³ For instance, Liu, Longstaff and Mandell (2006) use a five-factor affine framework to jointly model Treasury, repo and swap term structures. One of their factors is related to the pricing of the Treasury-securities liquidity and another factor reflects default risk.¹⁴ Feldhütter and Lando (2008) develop a six-factor model for Treasury bonds, corporate bonds and swap rates that makes it possible to decompose swap spreads into three components: a convenience yield from holding Treasuries, a credit-element associated with the underlying LIBOR rate, and a factor specific to the swap market. They find that the convenience yield is by far the largest component of spreads. Longstaff, Mithal and Neis (2005) use information in credit default swaps –in addition to bond prices– to obtain measures of the nondefault components in corporate spreads. They find that the nondefault component is time-varying and strongly related to measures of bond-specific illiquidity as well as to macroeconomic measures of bond-market liquidity.

To the best of our knowledge, the present paper is the first to explicitly incorporate liquidity-pricing effects in a no-arbitrage multi-country set-up. Despite the importance of sovereign credit risk in the financial markets, relatively little research proposing models of the joint dynamics of sovereign yields has appeared in the literature. Notable recent exceptions include Pan and Singleton (2008) and Longstaff et al. (2011). These two contributions point to an important

¹¹ See, e.g., Longstaff (2004), Landschoot (2004), Chen, Lesmond and Wei (2007), Covitz and Downing (2007) or Acharya and Pedersen (2005).

¹² Such a behaviour is captured in a theoretical framework by Vayanos (2004)[63].

¹³ See e.g. Chordia and Subrahmanyam (2000), Fontaine and Garcia (2009), Feldhütter and Lando, (2008), Longstaff, Mithal and Neis (2005), Liu, Longstaff and Mandell (2006) or Dick-Nielsen, Feldhütter and Lando (2011).

¹⁴ As noted by Feldhütter and Lando (2008), the identification of the liquidity and credit risk factors in Liu et al. relies critically on the use of the 3-month general-collateral repo rate (GC repo) as a short-term risk-free rate and of the 3-month LIBOR as a credit-risky rate. Liu et al. define the liquidity factor as the spread between the 3-month GC repo and the 3-month Treasury-bill yield (and is therefore observable). In each yield, their liquidity component is the share of the yield that is explained by this factor.

degree of commonality across sovereign credit risk. More precisely, they show that the risk premia included in sovereign credit spreads are substantial and covary importantly with economic measures of global event risk. According to Longstaff et al., an important source of commonality in sovereign credit spreads may be their sensitivity to the funding needs of major investors in the sovereign credit markets. This view implies that a better understanding of sovereign yield spreads requires models in which both credit and liquidity risks are explicitly taken into account. Such a model is presented in the next section.

Our paper also extends the literature that considers the introduction of regime-switching in ATSM. This literature is based on a strong evidence of regime switching in the dynamics of the term structure of interest rates (see Hamilton, 1988, Aït-Sahalia, 1996, Ang and Bekaert, 2002 or Davies, 2004 for spreads). Implied shifts in the interest-rate dynamics present a systematic risk to investors. The pricing of such a risk has already been empirically investigated within default-free ATSM incorporating Markov-switching.¹⁵ Building on the approaches introduced by Duffie and Singleton (1999) or Duffee (1999) to deal with credit risk in ATSM, Monfort and Renne (2012) explore the potential of Markov-switching in credit ATSM models.¹⁶ In the present paper, we propose an original use of regime-switching features to model interactions between credit-related and liquidity-related crises and their impact on bond pricing.

3 The model

We consider zero-coupon bonds issued by N debtors. These entities may default and their bonds are not perfectly liquid, both aspects having an impact on the bonds prices. Heuristically, a bondholder fears about the default of the bond's issuer –that would result in a early and reduced repayment of the bond– and about the risk of being hit by a liquidity shock.¹⁷ In the latter case, the bondholder is forced to precipitately liquidate her bond holdings and, in such circumstances, illiquid bonds are sold at a discount.

¹⁵ See Monfort and Pegoraro, 2007, Ang Bekaert and Wei, 2008 and Dai, Singleton and Yang, 2007.

¹⁶ Whereas Duffie and Singleton (1999) and Duffee (1999) present continuous-time credit ATSMs. Gouriéroux, Monfort and Polimenis, 2006 and Monfort and Renne (2012) consider discrete-time frameworks.

¹⁷ The liquidity shock may occur e.g. as a result of unexpected cash shortages, the need to rebalance a portfolio in order to maintain a hedging or diversification strategy, or a change in capital requirements (see He and Xiong, 2012).

Subsection 3.1 presents the notations and introduces default and liquidity intensities. The historical (respectively risk-neutral) dynamics of the model's variables is developed in Subsection 3.2 (3.3). The implications in terms of bond pricing are developed in Subsection 3.4.

3.1 Default events, liquidity shocks and associated intensities

At date t , each investor is provided with the new information $\tilde{w}_t = (r_t, z'_t, \tilde{\lambda}'_{d,t}, \tilde{\lambda}'_{\ell,t}, d'_t, \ell_t)'$ where r_t is the risk-free short-term rate, z_t is a crisis-regime variable, $\tilde{\lambda}_{d,t}$ is a N -dimensional vector containing the default intensities associated with the respective N debtors, d_t is a N -dimensional vector of binary variables $d_t^{(n)}$ indicating whether debtor n is in default at date t ($d_t^{(n)} = 1$, which is an absorbing state) or not ($d_t^{(n)} = 0$),¹⁸ $\tilde{\lambda}_{\ell,t}$ is the liquidity-shock intensity and ℓ_t is a binary variable indicating if the bondholder is affected by the liquidity shock at date t ($\ell_t = 1$) or not ($\ell_t = 0$).

Denoting by w_t the vector $w_t = (r_t, z'_t, \tilde{\lambda}'_{d,t}, \tilde{\lambda}'_{\ell,t})'$ and by $\underline{\tilde{w}}_t$ the cumulated information available at date t , i.e. $\underline{\tilde{w}}_t = (\tilde{w}_t, \tilde{w}_{t-1}, \dots, \tilde{w}_1)$, the conditional probability of default of debtor n is given by:

$$\mathbb{P} \left(d_t^{(n)} = 1 \mid w_t, d_{t-1}^{(n)} = 0, \underline{\tilde{w}}_{t-1} \right) = 1 - \exp \left(-\tilde{\lambda}_{d,t}^{(n)} \right),$$

which is close to $\tilde{\lambda}_{d,t}^{(n)}$ when this intensity is small. Let us consider a bond issued by debtor n with a residual maturity of h at date t . We denote by $B_{t,h}^{(n)}$ the non-default price of this bond. If debtor n defaults between date $t-1$ and date t , the bondholder is assumed to receive –from the borrower– a fraction ζ of the price that would have prevailed otherwise at date t . In other words, in the case of default, the recovery pay-off is $\zeta B_{t,h}^{(n)}$.

The conditional probability, for an investor, of being hit by the liquidity shock is:

$$\mathbb{P} \left(\ell_t = 1 \mid d_t, w_t, \underline{\tilde{w}}_{t-1} \right) = 1 - \exp \left(-\tilde{\lambda}_{\ell,t} \right).$$

In particular, this probability does not depend on $\underline{\ell}_{t-1}$ and \underline{d}_t . Upon the arrival of the liquidity shock ($\ell_t = 1$), the bond investor has to exit by selling her bond holdings at a fractional cost $1 - \theta^{(n)}$, that is, the proceed of the sale is then $\theta^{(n)} B_{t,h}^{(n)}$. A theoretical basis for such a fractional

¹⁸ We use parenthesis to distinguish country from exponentiation in the superscript.

cost can be found in Ericsson and Renault (2006)¹⁹

Conditionally on (w_t, \tilde{w}_{t-1}) , the $d_t^{(n)}$'s and ℓ_t are independent. However, conditionally on the past information \tilde{w}_{t-1} , the default events and the liquidity shocks are not independent because the associated intensities are correlated with each other.²⁰

For the sake of bond pricing, it will prove convenient to introduce the fractional-loss intensities $\lambda_{d,t}^{(n)}$ and $\lambda_{\ell,t}^{(n)}$ (see Appendix A). Intuitively, they correspond to the expected losses, conditional to w_t , associated with, respectively, the default of debtor n and the arrival of a liquidity shock (expressed as fractions of the price that would have prevailed, absent the default and/or the liquidity shock). Appendix A shows that these intensities are defined through:

$$\begin{cases} \exp(-\lambda_{d,t}^{(n)}) &= \exp(-\tilde{\lambda}_{d,t}^{(n)}) + \zeta \left(1 - \exp(-\tilde{\lambda}_{d,t}^{(n)})\right) \\ \exp(-\lambda_{\ell,t}^{(n)}) &= \exp(-\tilde{\lambda}_{\ell,t}) + \theta^{(n)} \left(1 - \exp(-\tilde{\lambda}_{\ell,t})\right). \end{cases} \quad (1)$$

When the $\tilde{\lambda}_{d,t}^{(n)}$'s and $\tilde{\lambda}_{\ell,t}$ are small, these equations are approximately $\lambda_{d,t}^{(n)} = (1 - \zeta)\tilde{\lambda}_{d,t}^{(n)}$ and $\lambda_{\ell,t}^{(n)} = (1 - \theta^{(n)})\tilde{\lambda}_{\ell,t}$.

Naturally, when the fractional recovery pay-offs (ζ and $\theta^{(n)}$) are equal to one, the fractional-loss intensities are null. Hence, when both kinds of losses are ruled out ($\zeta = \theta^{(n)} = 1$), the bonds issued by debtor n turn out to be risk-free bond. By contrast, when the recovery pay-offs are null, the fractional-loss intensities correspond to the conditional probabilities of default and to the probability of being hit by the liquidity shock, respectively.

3.2 Historical dynamics of w_t

3.2.1 Short rate, credit- and liquidity-related Markov chains

As in Pan and Singleton (2008) or Longstaff et al. (2011), we assume that the short-term risk-free interest rate is exogenous. Hence, we work conditionally to observed values of the r_t 's.

¹⁹ In their model, an investor hit by the liquidity shock must liquidate her bond holdings in a limited time (between t and t^+ , say). Then, she obtains a Poisson-distributed number K of offers from traders ($K \sim \mathcal{P}(\gamma^{(n)})$) and retains the best one, each offer being a random fraction ω_i ($i \in [1, K]$) of $B_{t,h}^{(n)}$, which can then be seen as the price she would get if $\gamma^{(n)}$ was infinite. Therefore the higher $\gamma^{(n)}$, the more liquid the bonds issued by n .

²⁰ This assumption appears in the ‘‘doubly stochastic’’ framework (see e.g. Duffie et al., 2005, Pan and Singleton, 2008 or Longstaff et al., 2011).

The joint dynamics of the recovery-adjusted default intensities ($\lambda_{d,t}^{(n)}$) and of the liquidity intensities ($\lambda_{\ell,t}^{(n)}$) crucially depends on an exogenous Markov chain z_t . The regime variable z_t is obtained by crossing two regime variables. A first regime variable $z_{\ell,t}$ defines the liquidity situation, which can be distressed ($z_{\ell,t} = [0, 1]'$) or not ($z_{\ell,t} = [1, 0]'$). A second regime variable $z_{c,t}$ represents the credit situation, the latter being either non-stressed ($z_{c,t} = [1, 0, 0]'$), distressed ($z_{c,t} = [0, 1, 0]'$) or severely distressed ($z_{c,t} = [0, 0, 1]'$).²¹ The credit/liquidity regime of the economy at date t is then summarized by the six-dimensional selection vector z_t , which is the Kronecker product of $z_{\ell,t}$ and $z_{c,t}$:

$$z_t = z_{\ell,t} \otimes z_{c,t}, \quad (2)$$

The vector z_t is valued in $\{e_1^{[6]}, \dots, e_6^{[6]}\}$, where $e_i^{[M]}$ denotes the M -dimensional vector whose all entries are equal to 0, except the i^{th} that is equal to 1.

Importantly, there may be causal relationships between $z_{\ell,t}$ and $z_{c,t}$. For instance, we allow for the probability of a change in the liquidity state to depend on the credit regime (and vice-versa). Formally, let us denote by Π the matrix of transition probabilities, whose (i, j) entry, denoted by $\pi_{i,j}$, corresponds to $p(z_{t+1} = e_j^{[6]} | z_t = e_i^{[6]})$. The entries of the row of this matrix summing to one, 30 parameters are required to specify this matrix. In order to keep the model parsimonious, some constraints are introduced. With these constraints, which are detailed in Appendix B, 11 parameters are required to specify the matrix Π .

3.2.2 Historical (\mathbb{P}) dynamics of the $\lambda_{d,t}^{(n)}$'s and the $\lambda_{\ell,t}^{(n)}$'s

The dynamics of the intensities $\lambda_{d,t}^{(n)}$ and $\lambda_{\ell,t}^{(n)}$ are connected through the regime variables. Consistently with the liquidity shock interpretation introduced in Subsection 3.1, we assume that the illiquidity intensities are driven by a single factor denoted by $\lambda_{\ell,t}$.²² This factor, as well as the credit-related ones, follow auto-regressive processes with drifts depending on the regime variables.

²¹ Preliminary modeling with a unique level of credit-distress regime led to a less satisfying fit of the data. That is why this additional level of credit distress (compared with the unique liquidity-distress regime) has been introduced in the framework.

²² However, this factor is not rigorously equal to the liquidity-shock intensity $\tilde{\lambda}_{\ell,t}$. Indeed, for this to be the case in a context where the $\theta^{(n)}$ are not time-varying, the $\lambda_{\ell,t}^{(n)}$'s should be the same up to a multiplicative factor. In other words, the $\alpha_{0,\ell}^{(n)}$'s in equation (4) should be equal to zero. That being so, we use these additional degrees of freedom to improve the model fit.

Formally:

$$\begin{cases} \lambda_{d,t}^{(n)} &= \mu_d^{(n)'} z_{d,t} + \rho_d \lambda_{d,t-1}^{(n)} + \sigma_d^{(n)} \varepsilon_{d,t}^{(n)} \\ \lambda_{\ell,t}^{(n)} &= \alpha_{0,\ell}^{(n)} + \alpha_{1,\ell}^{(n)} \lambda_{\ell,t} \end{cases} \quad \forall n \quad (3)$$

$$\lambda_{\ell,t} = \mu_{\ell}' z_{\ell,t} + \rho_{\ell} \lambda_{\ell,t-1} + \sigma_{\ell} \varepsilon_{\ell,t} \quad (4)$$

where the $\varepsilon_{d,t}^{(n)}$'s –some country-specific credit shocks– and the $\varepsilon_{\ell,t}$'s –some liquidity-related shocks– are i.i.d. $\mathcal{N}(0, 1)$. We denote by λ_t the $(N+1) \times 1$ vector containing the recovery-adjusted default intensities and the liquidity-related factor, i.e. $\lambda_t = [\lambda_{d,t}^{(1)}, \dots, \lambda_{d,t}^{(N)}, \lambda_{\ell,t}]'$, and by ε_t the associated innovations, i.e. $\varepsilon_t = [\varepsilon_{d,t}^{(1)}, \dots, \varepsilon_{d,t}^{(N)}, \varepsilon_{\ell,t}]'$. By abuse of notation, we may denote the entries of λ_t by $\lambda_{i,t}$ in the following.²³ Then, denoting by μ the $6 \times (N+1)$ matrix of drifts,²⁴ by Φ the matrix whose diagonal entries are ρ_d (N times) and ρ_{ℓ} , and by Σ the matrix whose diagonal entries are the $\sigma_d^{(n)}$'s and σ_{ℓ} , the dynamics of λ_t reads:

$$\lambda_t = \mu' z_t + \Phi \lambda_{t-1} + \Sigma \varepsilon_t \quad (5)$$

Equation (5) means that the conditional distribution of λ_t given $(r_t, z_t, \tilde{w}_{t-1})$ is $\mathcal{N}(\mu' z_t + \Phi \lambda_{t-1}, \Sigma^2)$, implying in particular that this distribution depends on \tilde{w}_{t-1} through λ_{t-1} only. Moreover, since r_t and z_t are exogenous, this implies that the distribution of w_t given \tilde{w}_{t-1} does not depend on $(\underline{d}_{t-1}, \underline{\ell}_{t-1})$, that is, (d_t, ℓ_t) does not Granger-cause w_t .

It can be seen that the $\lambda_{i,t}$'s are positively marginally skewed as soon as the μ vectors contain only positive entries. Moreover, the lower the standard deviations σ of the Gaussian shocks (in comparison with the drifts μ), the more often the $\lambda_{i,t}$'s are positive, which is important given their interpretations in terms of probabilities.

Furthermore, the instantaneous causality between z_t and λ_t implies that the variances of the $\lambda_{i,t}$'s, conditionally on \tilde{w}_{t-1} , depend on the regime variable z_{t-1} . More precisely, conditionally to \tilde{w}_{t-1} , the distributions of the $\lambda_{i,t}$'s are some mixtures of Gaussian distributions, thereby involving a form of heteroskedasticity in the innovations.²⁵

²³ $\lambda_{i,t} = \lambda_{d,t}^{(i)}$ for $i \leq N$ and $\lambda_{N+1,t} = \lambda_{\ell,t}$.

²⁴ The columns of this matrix are $\mu_d^{(1)}, \dots, \mu_d^{(N)}$, and μ_{ℓ} .

²⁵ Such a feature is discussed in Ang, Bekaert and Wei (2008).

3.3 Stochastic discount factor and risk-neutral (\mathbb{Q}) dynamics

We assume that the stochastic discount factor (s.d.f.) has the following expression:

$$M_{t-1,t} = \exp \left[-r_{t-1} - \frac{1}{2} \nu_t' \nu_t + \nu_t' \varepsilon_t + (\delta z_{t-1})' z_t \right] \quad (6)$$

where δ is a 6×6 matrix and where the entries of ν_t are affine in z_t and in the corresponding entries of λ_{t-1} , that is $\nu_{i,t} = \nu_{\lambda,i} \lambda_{i,t-1} + \nu'_{z,i} z_t$, say. ($\nu_{\lambda,i}$ is a scalar and $\nu_{z,i}$ is a vector.) The risk-sensitivity matrix δ and the vectors ν_t respectively price the regimes z_t and the (standardized) Gaussian innovations ε_t of λ_t . The fact that we must have $E_t(M_{t,t+1}) = \exp(-r_t)$ implies that the entries of δ are of the form $\ln(\pi_{ij}^*/\pi_{ij})$ where the π_{ij}^* are such that $\sum_j \pi_{ij}^* = 1$ for any i . Note that the variables (d_t, ℓ_t) do not appear in $M_{t-1,t}$, in other words, we assume that the risk aversion is completely captured by the pricing of the innovation process ε_t and the regime process z_t . It can be shown that, in such framework, the risk-neutral (\mathbb{Q}) dynamics of (z_t, λ_t) is of the same form as its historical counterpart.²⁶ More precisely, under \mathbb{Q} , z_t follows a time-homogenous Markovian chain whose dynamics is described by the matrix Π^* of transition probabilities $\{\pi_{ij}^*\}$ and, denoting by $\lambda_{i,t}$ the i^{th} entry of λ_t , we have:

$$\lambda_{i,t} = \mu_i^{*'} z_t + \rho_i^* \lambda_{i,t-1} + \sigma_i \varepsilon_t^* \quad (7)$$

where $\varepsilon_{i,t}^* \sim \mathcal{N}^{\mathbb{Q}}(0, 1)$, $\mu_i^* = \mu_i + \sigma_i \nu'_{z,i}$ and $\rho_i^* = \rho_i + \sigma_i \nu_{\lambda,i}$.

Let us turn to the risk-neutral dynamics of d_t and ℓ_t . As shown in Appendix C, the conditional distributions –given (w_t, \tilde{w}_{t-1}) – of these binary variables are the same functions of w_t under \mathbb{P} and \mathbb{Q} . In other words, for any n , $\tilde{\lambda}_{d,t}^{(n)}$ is the same process in both worlds, and the same is true for $\tilde{\lambda}_{\ell,t}$. This stems from the fact that the variables d_t and ℓ_t do not enter the s.d.f. (that depends on w_t only).²⁷ However, it is important to stress that while the intensities are the same processes under both measures, their \mathbb{Q} - and \mathbb{P} -dynamics are different (because the \mathbb{Q} - and \mathbb{P} -dynamics of (z_t, λ_t) differ). As a consequence, the probabilities of default are different under \mathbb{P} and \mathbb{Q} .

²⁶ See Monfort and Renne (2012).

²⁷ Appendix C also shows that in that context, the fact that the distribution of w_t given \tilde{w}_{t-1} does not depend on $(\underline{d}_{t-1}, \underline{\ell}_{t-1})$ –i.e. that (d_t, ℓ_t) does not cause w_t – is true under both measures.

3.4 Bond pricing

In this framework, the price of a defaultable and illiquid zero-coupon bond issued by country n (not in default at date t) and with residual maturity h has a price at time t that is given by (see Appendix A):

$$B_{t,h}^{(n)} = E_t^{\mathbb{Q}} \left[\exp \left(-r_t - \dots - r_{t+h-1} - \lambda_{d,t+1}^{(n)} - \dots - \lambda_{d,t+h}^{(n)} - \lambda_{\ell,t+1}^{(n)} - \dots - \lambda_{\ell,t+h}^{(n)} \right) \right]. \quad (8)$$

where r_t is the return of a risk-free investment between t and $t+1$ and where $E_t^{\mathbb{Q}}$ is the conditional expectation given \tilde{u}_{t-1} in the risk-neutral world.

The short-term risk-free interest rate being exogenous, we have:

$$\begin{aligned} B_{t,h}^{(n)} &= E_t^{\mathbb{Q}} [\exp(-r_t - \dots - r_{t+h-1})] \times \\ &E_t^{\mathbb{Q}} \left[\exp \left(-\lambda_{d,t+1}^{(n)} - \dots - \lambda_{d,t+h}^{(n)} - \lambda_{\ell,t+1}^{(n)} - \dots - \lambda_{\ell,t+h}^{(n)} \right) \right]. \end{aligned} \quad (9)$$

Denoting by $y_{t,h}^{(n)}$ the yield-to-maturity of this bond, we obtain:

$$\begin{aligned} y_{t,h}^{(n)} &= -\frac{1}{h} \ln(B_{t,h}^{(n)}) \\ &= y_{t,h}^{(0)} - \frac{1}{h} \ln \left(E_t^{\mathbb{Q}} \left[\exp \left(-\lambda_{d,t+1}^{(n)} - \dots - \lambda_{d,t+h}^{(n)} - \lambda_{\ell,t+1}^{(n)} - \dots - \lambda_{\ell,t+h}^{(n)} \right) \right] \right) \end{aligned} \quad (10)$$

where $y_{t,h}^{(0)}$ denotes the yield to maturity of a risk-free zero-coupon bond of residual maturity h at date t . The vector (z_t, λ_t) being compound auto-regressive of order one under \mathbb{Q} , the second term on the right-hand side of (10) is linear in (z_t, λ_t) .²⁸ Therefore, the spread between the yield associated with the defaultable bond and the risk-free bond of the same maturity is of the form:

$$y_{t,h}^{(n)} - y_{t,h}^{(0)} = a_h^{(n)'} z_t + b_h^{(n)'} \lambda_t \quad (11)$$

where the $(a_h^{(n)'}, b_h^{(n)'})$ vectors are computed recursively.²⁹

²⁸ Appendix D derives the Laplace transform of (z_t, y_t) and shows that (z_t, y_t) is Compound auto-regressive of order one. Appendix E shows how to compute the multi-horizon Laplace transform of compound auto-regressive processes. See Darolles, Gouriéroux and Jasiak (2006) or Bertholon, Monfort and Pegoraro (2008) for in-depth presentations of compound auto-regressive –or Car– processes.

²⁹ The general recursive formulas are presented in Appendix E.

4 Data

4.1 Overview

The data are weekly (end of weeks), and cover the period from 1 June 2007 to 13 April 2012 (255 dates), encompassing the ongoing financial crisis. We consider the yield curves of eight euro-area countries: Austria, Belgium, Finland, France, Germany, Italy, the Netherlands and Spain. We exclude from the analysis those countries that were placed under EU-IMF programs during that period, namely Greece, Ireland and Portugal (in April 2010 for Greece, in November 2010 for Ireland and in May 2011 for Portugal). The choice of removing these countries from the analysis stems from the facts that (a) the three EU-IMF programs cover important shares of the total estimation period and that (b) these programs coincide with severe impairments of associated sovereign-debt markets, notably illustrated by a fall in primary-market activity.³⁰

Sovereign zero-coupon yields are extracted from Bloomberg. As will be detailed below, our estimation strategy also involves yields of bonds issued by KfW, a public German agency. The latter (zero-coupon) yields come from the Thomson Reuters Tick History database. The estimation dataset is completed by 12-month-ahead forecasts of 10-year sovereign yields for France, Germany, Italy, Spain and the Netherlands. These forecasts are the mean values of the respondents' forecasts by the Consensus Economics' expert panel. The survey is released around the middle of the month. Note that the survey implicitly targets yields-to-maturity of coupon bonds and not zero-coupon bonds. However, our zero-coupon yields remain very close to coupon yields over the estimation sample. The remaining discrepancy, of a few basis points, will be attributed to the deviation between the survey-based forecasts and the model-based ones (the $\xi_t^{(n)}$'s introduced in equation 13 below). This monthly series is converted into a weekly one using a cubic spline.

The risk-free rates are proxied by the Overnight Index Swap (OIS) rates. An OIS is a fixed-for-floating interest rate swap with a floating rate leg tied to the index of overnight interbank rates, that is the EONIA in the euro-area case.³¹ OIS have become especially popular hedging

³⁰ These impairments are illustrated by bid-ask spreads on government bonds. Based on bond prices extracted from the Thomson Reuters tick history database, the bid-ask spreads on 10-year bond issued by Greece, Ireland and Portugal were on average above 200 bp in 2011 (i.e. 2% of the face value, or 3% to 4% of the bond value) while they were lower than 40 bps for other euro-area countries.

³¹ For maturities higher than 12 months, OIS rates are homogenous to constant-maturity coupon yields. Therefore, we first have to convert Bloomberg-extracted OIS rates into zero-coupon yields. This is done using standard

and positioning vehicles in euro financial markets and grew significantly in importance during the financial turmoil of the last few years. The OIS curve is more and more seen by market participants as a proxy of the risk-free yield curve (see e.g. Joyce et al., 2011).³²

4.2 The KfW-*Bund* spread

Our identification of a liquidity-related latent factor is based on the yield spreads between German federal bonds and KfW agency bonds. The latter are less liquid than the sovereign counterparts, the so-called *Bunds*, but are explicitly and fully guaranteed against default by the German federal government.³³ Consequently, the spread between these two kinds of bonds can be seen as a measure of the German government bond-market liquidity premium demanded by investors. In the same spirit, Longstaff (2004) computes liquidity premia based on the spread between U.S. Treasuries and bonds issued by Refcorp, that are guaranteed by the U.S. Treasury.

Panel A of Figure 1 shows that the KfW-*Bund* spreads of different maturities are highly correlated. This suggests that a single factor may be adequate to model the term structure of these spreads. Here, it is important to check that this liquidity-pricing measure is not purely specific to Germany. To that purpose, we look at comparable liquidity-driven spreads –between government-guaranteed bonds and their sovereign counterparts– in alternative countries.³⁴ In France for instance, the CADES (Caisse d’amortissement de la dette sociale) issues bonds that are guaranteed by the French government. Panel B compares one of the KfW-*Bund* spreads with a CADES-OAT spread (OATs are French government-issued bonds) and displays spreads of government-guaranteed bank bonds –issued by the Dutch NIBC bank and the Austrian Raiffeisen Zentralbank– over their respective sovereign counterparts. This exercise points to a substantial degree of correlation among liquidity-driven spreads from different European countries.

bootstrapping methods.

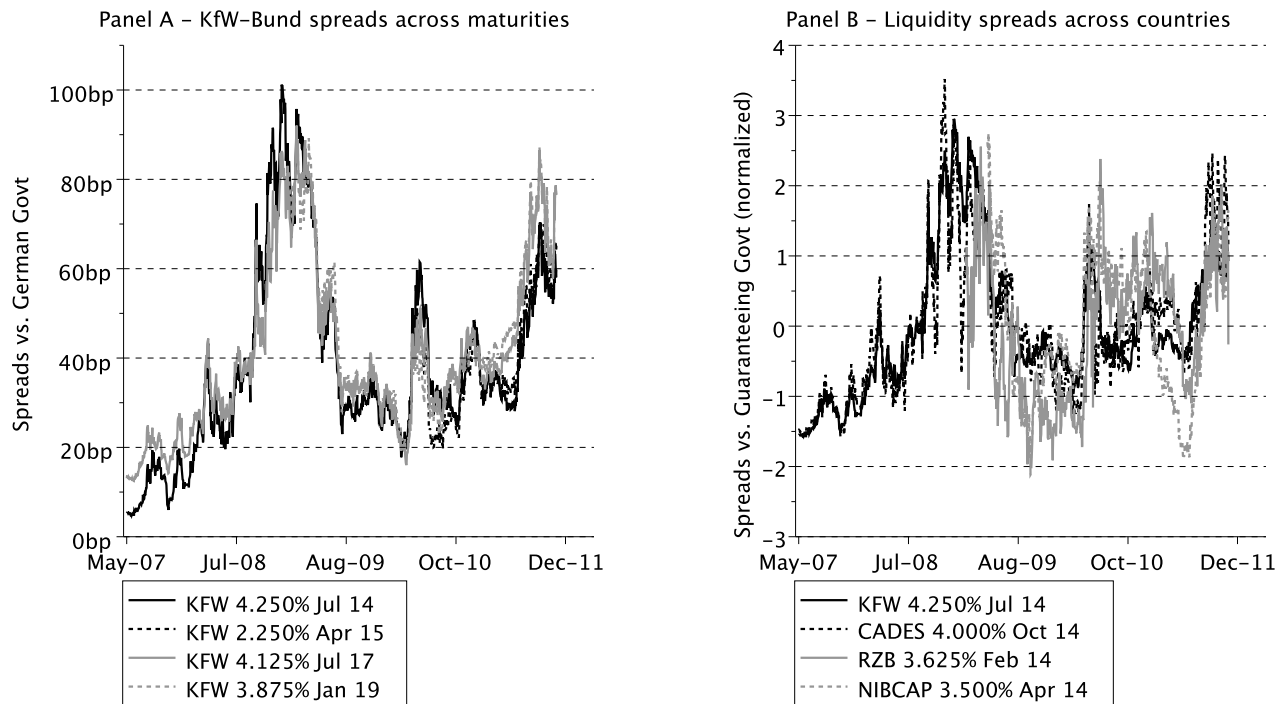
³² While OIS rates reflect the credit risk of an overnight rate, this may be regarded as negligible in most situations. Besides, even during financial-markets turmoils, the counterparty risk is limited in the case of a swap contract, due to netting and credit enhancement, including call margins (see Bomfin, 2003).

³³ An understanding between the European Commission and the German Federal Ministry of Finance (1 March 2002) stated that the guarantee of the Federal Republic of Germany will continue to be available to KfW. The three main rating agencies –Fitch, Standard and Poor’s and Moody’s– have assigned a triple-A rating to KfW (see KfW website http://www.kfw.de/kfw/en/KfW_Group/Investor_Relations/index.jsp). In addition, as the German federal bonds, KfW’s bonds are zero-weighted under the Basel capital rules. The relevance of the KfW-*Bund* spread as a liquidity proxy is also pointed out by McCauley (1999), the ECB, 2009 and is exploited by Schwarz (2009).

³⁴ Note that such alternative (term structures of) spreads are not available on our whole estimation period, that is why we use essentially KfW-*Bund* spreads to identify our liquidity factor within our econometric approach.

Fig. 1: Differentials between government and government-guaranteed bonds

Notes: The first plot shows the spreads between KfW bond yields and their sovereign counterparts. The second plot compares the spread between a KfW bond maturing in 2014 and its sovereign counterpart with other spreads between government-guaranteed European bonds and their respective sovereign counterparts: CADES', NIBCAP's and RZB's bonds are respectively guaranteed by the French, Dutch and Austrian governments (the spreads are demeaned and standardized). The yields come from Barclays Capital.



4.3 Euro-area government yields

Table 1 reports the correlations between the spreads vs. Germany for different countries over the sample period.³⁵ The results suggest that euro-area sovereign spreads are highly correlated across countries and across maturities (see also Favero, Pagano and von Thadden, 2010). According to these descriptive statistics, spreads' distributions are positively skewed and often leptokurtic. Table 2 presents a principal-component analysis of these spreads across countries. This analysis indicates that, for different maturities (2, 5 and 10 years), the first two principal components explain more than 95% of the spread variances across countries (75% for the first principal component alone). This highlights the importance of common sources of risk in euro-area sovereign spreads.

³⁵ German debt is often considered as a safe haven in terms of credit and liquidity quality. The German bond market is the only one in Europe that has a liquid futures market, which boosts demand for the German Bund compared to other euro area debt and bolsters its liquidity (see e.g. Pagano and von Thadden, 2004).

Tab. 1: Descriptive statistics of selected spreads

Notes: The table reports summary statistics for selected spreads (versus Germany). Two auto-correlations are shown (the 1-month and the 1-year auto-correlations). The underlying yields are continuously compounded and are in percentage annual terms. The lower panel of the table presents the covariances and the correlations (in italics) of the spreads. The data are weekly and cover the period from 1 June 2007 to 13 April 2012.

	France		Italy		Netherlands		Spain	
	2-year	10-year	2-year	10-year	2-year	10-year	2-year	10-year
Mean	0.198	0.345	1.188	1.388	0.127	0.237	1.073	1.286
Median	0.126	0.255	0.723	0.915	0.092	0.175	0.517	0.751
Standard dev.	0.216	0.321	1.319	1.391	0.109	0.173	1.161	1.238
Skewness	2.736	2.023	2.101	1.756	1.669	1.135	1.169	0.896
Kurtosis	12.245	6.944	7.331	5.432	5.511	3.546	3.73	2.69
Auto-cor. (lag 1)	0.957	0.979	0.984	0.99	0.922	0.963	0.979	0.987
Auto-cor. (lag 12)	0.838	0.894	0.916	0.936	0.782	0.876	0.908	0.926
<i>Correlations \ Covariances</i>								
France 2-yr yd	0.047	0.065	0.261	0.271	0.014	0.026	0.194	0.198
France 10-yr yd	0.938	0.103	0.392	0.425	0.018	0.043	0.307	0.336
Italy 2-yr yd	0.915	0.926	1.738	1.808	0.055	0.143	1.421	1.479
Italy 10-yr yd	0.899	0.952	0.986	1.933	0.059	0.162	1.486	1.597
Netherlands 2-yr yd	0.604	0.521	0.382	0.388	0.012	0.015	0.03	0.032
Netherlands 10-yr yd	0.706	0.775	0.626	0.675	0.787	0.03	0.101	0.119
Spain 2-yr yd	0.771	0.824	0.929	0.921	0.237	0.506	1.346	1.406
Spain 10-yr yd	0.741	0.845	0.906	0.928	0.235	0.559	0.979	1.532

Tab. 2: Principal component analysis of euro-area yield differentials

Notes: This table presents results of principal-component analyses carried out on the spreads versus Germany. There are three analyses that correspond respectively to three maturities: 2 years, 5 years and 10 years. For each PC analysis, the table reports the eigenvalues of the covariance matrices and the proportions of variance explained by the corresponding component (designated by "Prop. of var."). The data are weekly and cover the period from 1 June 2007 to 13 April 2012. the spread (versus Germany) of seven countries are included in the analysis (Austria, Belgium, Finland, France, Italy, Netherlands, Spain).

Component	1	2	3	4	5	6	7
<i>2-year spread</i>							
Eigenvalue	5.26	1.34	0.17	0.12	0.06	0.03	0.02
Prop. of var.	75%	19%	2%	2%	1%	0%	0%
Cumul. prop.	75%	94%	97%	98%	99%	100%	100%
<i>5-year spread</i>							
Eigenvalue	5.87	0.88	0.12	0.06	0.04	0.02	0.01
Prop. of var.	84%	13%	2%	1%	1%	0%	0%
Cumul. prop.	84%	96%	98%	99%	100%	100%	100%
<i>10-year spread</i>							
Eigenvalue	5.56	1.11	0.14	0.12	0.03	0.02	0.01
Prop. of var.	79%	16%	2%	2%	0%	0%	0%
Cumul. prop.	79%	95%	97%	99%	99%	100%	100%

5 Estimation

5.1 State-space form of the model

Our estimation is conducted by the maximum-likelihood method, in a single step. The likelihood function is approximated by means of the Kim's (1994) filter, that handles state-space models with Markov-switching.³⁶ The measurement equations of the model are of two kinds: a first set of equations relates observed spreads –stacked in a vector denoted by S_t – to modeled ones; a second one relates observed survey-based spreads' forecasts –stacked in a vector denoted by CF_t – to model-implied ones. Let us make these two sets of equations more precise.

Consistently with the fact that spreads versus Germany are the most scrutinized spreads in the euro-area sovereign bond market, the set of observed spreads consists of spreads versus German sovereign-bond yields. As for the German spreads included in the vector S_t , we take the yield differentials between Bunds' yields and (zero-coupon) OIS rates with comparable maturities (2, 5 and 10 years). These first measurement equations read:

$$S_t = Az_t + B\lambda_t + \xi_{S,t} \quad (12)$$

where the entries of the matrices A and B are respectively based on the $a_h^{(n)}$'s and the $b_h^{(n)}$'s appearing in equation (11). More precisely, consistently with the choice of the observed spreads, and replacing the German index (1) (say) by GER , the entries of A and B corresponding to German yields are respectively the a_h^{GER} 's and the b_h^{GER} with the appropriate maturities h .³⁷ Those entries of A and B corresponding to other debtors ($n > 1$) are of the form $a_h^{(n)} - a_h^{GER}$ and $b_h^{(n)} - b_h^{GER}$. The vector $\xi_{S,t}$ contains i.i.d. normally-distributed pricing errors.

This set of measurement equations is augmented with equations linking survey-based 12-month-ahead forecasts of spreads to their model-based equivalent. Four spreads are considered: the yield differentials between Dutch, French, Italian and Spanish 10-year bonds and their German

³⁶ See also Kim and Nelson (1999). The algorithm has been slightly adapted for this application. In particular, for each iteration of the algorithm, in the updating step, we prevent the algorithm from resulting in values of the i th unobserved variables $\lambda_{i,t}$ that would be below $-2\sqrt{\sigma_i^2/(1-\rho_i^2)}$. Note that $\sigma_i^2/(1-\rho_i^2)$ would be the unconditional variance of the i th unobserved factor $\lambda_{i,t}$ if μ_i was null (since the vector μ_i is positive, the unconditional mean of the i th factor is higher than zero, implying that the unconditional probability of having $\lambda_{i,t} < 0$ is lower than 2.5%).

³⁷ More rigorously, the vector $b_h^{(1)'}$ defines one line of the B matrix.

counterparts. These equations read:

$$CF_t^{(n)} = E_t^{\mathbb{P}} \left(y_{t+h,H}^{(n)} - y_{t+h,H}^{GER} \right) + \xi_t^{(n)}, \quad n \in \{2, 3, 4, 5\} \quad (13)$$

where $H = 52 \times 10$, $h = 52$, the $\xi_t^{(n)}$'s are i.i.d. normally-distributed measurement errors and where the model-based forecasts $E_t^{\mathbb{P}}(y_{H,t+h}^{(n)} - y_{H,t+h}^{GER})$ are easily derived using equation (11) and:

$$\begin{cases} E_t^{\mathbb{P}}(\lambda_{t+h}) &= [\mu\Pi^h + \Phi\mu\Pi^{h-1} + \dots + \Phi^{h-1}\mu\Pi] z_t + \Phi^h \lambda_t \\ E_t^{\mathbb{P}}(z_{t+h}) &= \Pi^h z_t. \end{cases} \quad (14)$$

Appendix B presents and discusses different constraints that are imposed on the parameter estimates. In particular, it details the relationship between λ_t^{KfW} and λ_t^{GER} that is aimed at identifying the liquidity-related factor $\lambda_{\ell,t}$.

5.2 Estimation procedure and results

The log-likelihood function is highly non-linear in the underlying model parameters. Therefore, good starting values are required to achieve convergence in a reasonable computing time.³⁸ In a first step, we estimate the model using data associated with a subset of debtors, namely Germany, KfW, Italy and France. In a second step, the parameters defining the dynamics of the risk intensities of the remaining countries are estimated successively, one country after the other, taking the other parameters as given. In the final stage, all the parameters are (re)estimated jointly.

The approach results in a satisfying fit of the data. Modeled spreads versus observed ones are displayed in Figure 2 (grey lines for observed spreads, dotted lines for modeled spreads). The average of the measurement-error standard deviations is around 18 basis points (across 27 time series: 3 maturities for 9 entities).

Figure 3 compares survey-based forecasts of four spreads with their model-implied counterparts. These spreads are the yield differentials between Dutch, French, Italian and Spanish 10-year bonds on the one hand and the German 10-year ones on the other hand. These plots suggest that the

³⁸ Optimizations are based on iterative uses of quasi-Newton and Nelder-Mead algorithms (as provided by the Scilab software).

model is able to reproduce most of the survey-based forecasts' fluctuations.

Table 3 and Table 4 present the parameter estimates. The standard deviations of these estimates are based on the outer product of the first derivative of the likelihood function. Important differences arise in the parameters across countries. Naturally, those countries that have been characterized by the highest rises in spreads are more affected by the crises regimes. Notably, in an intense credit crisis regime, the drift of the the Italian credit-related factor is 70 times larger than in a less intense credit crisis (this is obtained by comparing the entries of the line " μ_c " with those of the line " μ_{cc} " in Table 3). It can also be noted that the auto-regressive coefficients (the ρ 's) of the different factors are higher under the risk-neutral measure than under the historical one. This suggests that credit and liquidity intensities factors are more persistent under the risk-neutral measure than under the historical one. Note that another source of persistence originates from the regime-switching features: indeed, low switching probabilities generate persistence in the processes that depend on these regimes (the $\lambda_{i,t}$'s here). These transition probabilities are discussed in the next section.

Tab. 3: Parameter estimates (1/2)

Notes: This table reports the estimates of the parameters defining the dynamics of the intensities under the historical and the risk-neutral measures. It is completed by an additional table (Table 4) that presents the matrices of transition probabilities under both measures. The estimation data are weekly and span the period from 1 June 2007 to 13 April 2012. Standard errors are reported in parentheses below the coefficient estimates. ***, ** and * respectively denote significance at the 1%, 5% and 10% significance level. The standard deviations of these estimates are based on the outer product of the first derivative of the likelihood function. The entries of the $\mu_c^{(n)}$ vectors (see equation 3) that correspond to the two crises regimes are reported in the lines " μ_c " and " μ_{cc} " of the table. The entries of the $\mu_c^{(n)}$ vectors (see equation 3) that correspond to the non-credit-crisis regime is assumed to be zero (consistently with the existence of periods of very low spreads). It can be checked that $\alpha_{1,\ell}^{KfW} = \alpha_{1,\ell}^{GER} + 1$; this constraint is imposed in order to identify the liquidity-related factor (see B.1).

	KfW	Germany	Spain	Italy	France	Netherlands	Belgium	Finland	Austria
Specification of the liquidity intensities									
$\alpha_{0,\ell}$	-0.058*	-0.24***	-0.025	-0.24***	-0.24***	-0.24***	-0.24***	-0.16***	-0.24***
	(0.03)	(0.027)	(0.28)	(0.027)	(0.027)	(0.027)	(0.027)	(0.047)	(0.027)
$\alpha_{1,\ell}$	0.73***	-0.27***	0.52	1.1*	0.084	0.41***	0.53*	0.43***	0.63***
	(0.045)	(0.045)	(0.45)	(0.6)	(0.104)	(0.051)	(0.27)	(0.075)	(0.091)
Historical dynamics of the intensities λ									
μ_c	0.0101***	0.0101***	0.0052***	0.0051***	0.0051***	0.0103***	0.0051***	0.0085***	0.0105***
	(0.00087)	(0.00087)	(0.00044)	(0.00044)	(0.00044)	(0.00052)	(0.00044)	(0.0015)	(0.0018)
μ_{cc}	0.0102***	0.0102***	0.27***	0.38***	0.092***	0.0104***	0.24***	0.013***	0.071***
	(0.00087)	(0.00087)	(0.032)	(0.05)	(0.007)	(0.00052)	(0.022)	(0.0041)	(0.0078)
ρ_c	0.99***	-	-	-	-	-	-	-	-
	(0.00031)	-	-	-	-	-	-	-	-
σ_c	0.0025***	0.0025***	0.02***	0.022***	0.0051***	0.0028***	0.012***	0.0031***	0.0057***
	(0.000027)	(0.000027)	(0.00022)	(0.00024)	(0.000056)	(0.000031)	(0.00013)	(0.000034)	(0.000063)
μ_ℓ	0.076***	-	-	-	-	-	-	-	-
	(0.0027)	-	-	-	-	-	-	-	-
ρ_ℓ	0.9***	-	-	-	-	-	-	-	-
	(0.0043)	-	-	-	-	-	-	-	-
σ_ℓ	0.0101***	-	-	-	-	-	-	-	-
	(0.0002)	-	-	-	-	-	-	-	-
Risk-neutral dynamics of the intensities λ									
μ_c^*	0.0043*	0.0043*	0.0022*	0.0022*	0.0022*	0.0024**	0.0024**	0.0022*	0.0025*
	(0.0023)	(0.0023)	(0.0012)	(0.0012)	(0.0012)	(0.0012)	(0.0012)	(0.0013)	(0.0012)
μ_{cc}^*	0.0044*	0.0044*	0.009	0.0083	0.0047	0.0025**	0.0023**	0.0026**	0.0048
	(0.0023)	(0.0023)	(0.012)	(0.013)	(0.0043)	(0.0012)	(0.0012)	(0.0013)	(0.0044)
ρ_c^*	1***	1***	1***	1***	1***	1***	1***	1***	1***
	(0.00012)	(0.00012)	(0)	(0)	(0)	(0)	(0.00054)	(0)	(0)
μ_ℓ^*	0.00059	-	-	-	-	-	-	-	-
	(0.00037)	-	-	-	-	-	-	-	-
ρ_ℓ^*	1***	-	-	-	-	-	-	-	-
	(0)	-	-	-	-	-	-	-	-
Measurement errors									
σ_{perr}	0.093***	0.06***	0.49***	0.53***	0.12***	0.068***	0.28***	0.075***	0.14***
	(0.00051)	(0.00033)	(0.0027)	(0.0029)	(0.00068)	(0.00037)	(0.0016)	(0.00041)	(0.00076)

Tab. 4: Parameter estimates (2/2)

Notes: The table reports the estimates of the regime-transition probabilities, i.e. the matrices Π and Π^* . Note that only 11 parameters are used to define the 36 entries of each of these two matrices (see Subsection 3.2.1 and Appendix B). Standard errors of the estimates, based on the outer-product approximation of the Information matrix, are reported in parentheses below the coefficient estimates. ***, ** and * respectively denote significance at the 1%, 5% and 10% significance level. NL: no liquidity crisis, L: liquidity crisis, NC: no credit crisis, C: (non intense) credit crisis, CC: intense credit crisis. Each line of the table indicates the probabilities of switching from one regime (defined by the first column) to another (defined by the second line of the table).

		Historical dynamics						
		at date t :	NL-NC	NL-C	NL-CC	L-NC	L-C	L-CC
at date $t-1$	NL-NC		0.9994*** (0.00097)	0.00041 (0.00097)	0 -	0.0002*** (0)	0 (0)	0 -
	NL-C		0.00014*** (0)	0.98*** (0.0081)	0.000045*** (0)	0*** (0)	0.022*** (0.0081)	0*** (0)
	NL-CC		0 -	0.59*** (0.12)	0.39*** (0.12)	0 -	0.013** (0.0052)	0.0087* (0.0046)
	L-NC		0.19** (0.09)	0.032 (0.03)	0 -	0.66*** (0.14)	0.11*** (0.038)	0 -
	L-C		0.065** (0.028)	0.22*** (0.063)	0.076** (0.036)	0.11*** (0.032)	0.39*** (0.093)	0.13** (0.056)
	L-CC		0 -	0.22*** (0.063)	0.14** (0.064)	0 -	0.38*** (0.106)	0.25*** (0.081)
			Risk-neutral dynamics					
		at date t :	NL-NC	NL-C	NL-CC	L-NC	L-C	L-CC
at date $t-1$	NL-NC		0.94*** (0.045)	0.025 (0.016)	0 -	0.036 (0.045)	0.00094 (0.0012)	0 -
	NL-C		0.000059*** (0)	0.91*** (0.091)	0.000042*** (0)	0 (0)	0.092 (0.091)	0 (0)
	NL-CC		0 -	0.103 (0.12)	0.8*** (0.19)	0 -	0.0105 (0.021)	0.082 (0.073)
	L-NC		0.0089 (0.055)	0.00036 (0.0023)	0 -	0.95*** (0.069)	0.039 (0.034)	0 -
	L-C		0 (0)	0.000052*** (0)	0 (0)	0.094 (0.103)	0.87*** (0.13)	0.036 (0.04)
	L-CC		0 -	0 (0)	0.000053*** (0)	0 -	0.11 (0.14)	0.89*** (0.14)

Tab. 5: Conditional probabilities of transition

Notes: The table reports selected probabilities of transition. These probabilities are some combinations of those presented in Table 4. “Credit crisis” refers to one of the two credit-crisis regimes (intense or not). For instance, the probability of staying in a credit-crisis regime (intense or not) when in a liquidity crisis (at $t - 1$) is 82% under the historical measure. \mathbb{P} (resp. \mathbb{Q}) correspond to the historical (resp. risk-neutral) measure. Standard errors are reported in parentheses below the coefficient estimates. ***, ** and * respectively denote significance at the 1%, 5% and 10% significance level.

		at date t		at date $t - 1$	
				no liquidity crisis	liquidity crisis
		no credit crisis	credit crisis	no credit crisis	credit crisis
under \mathbb{P}	liquidity crisis	0.0002*** (0)	0.022*** (0.0081)	0.78*** (0.12)	0.64*** (0.09)
under \mathbb{Q}	liquidity crisis	0.036 (0.046)	0.092 (0.091)	0.99*** (0.057)	1*** (0)
under \mathbb{P}	credit crisis	0.00041 (0.00097)	0.99986*** (0)	0.14** (0.064)	0.82*** (0.051)
under \mathbb{Q}	credit crisis	0.026 (0.016)	0.99994*** (0)	0.039 (0.034)	0.91*** (0.103)

6 Interpretation

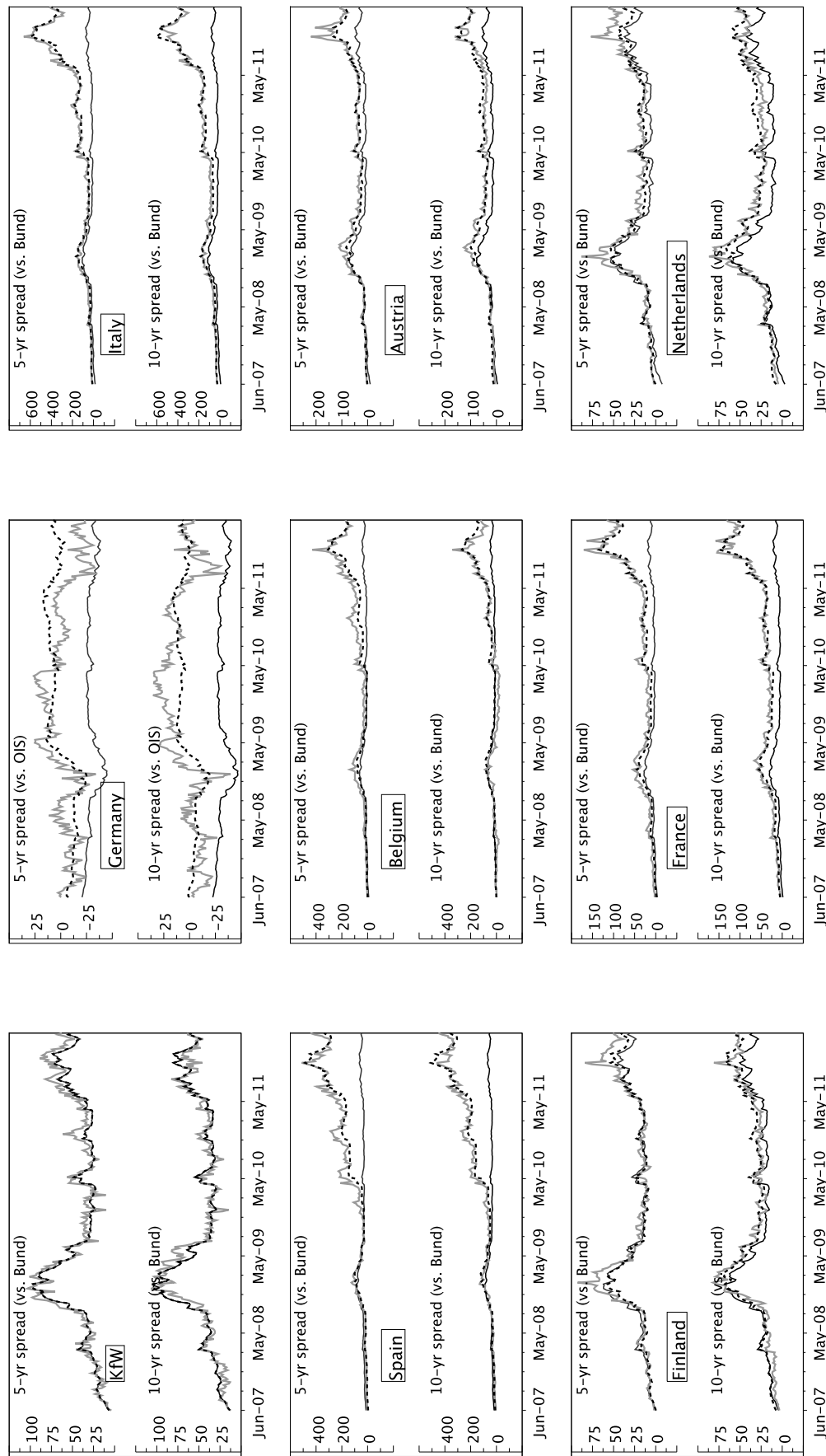
6.1 Credit and liquidity crises

The upper two panels in Figure 4 present the smoothed probabilities of being in the different crisis regimes.³⁹ The first period of liquidity crisis begins with the collapse of Bear Sterns in March 2008. This period is relatively short (a few weeks). By contrast, the second liquidity-crisis period, that begins with the Lehman Brothers’ bankruptcy in September 2008 lasted about six months. The premise of the so-called euro-area crisis (April 2010) and the latest development of the same crisis (starting in mid-2011) are also identified as liquidity-crisis periods. Turning to the credit crises, one can distinguish two long stress periods: November 2008 to mid-2009 and April 2010 to the end of the sample (April 2012). Within these credit-crisis phases, several peak periods of severe market stress –indicated by black-shaded areas in the second panel of Figure 4– are observed, notably in Autumn 2011. The lower panels in Figure 4 display the (smoothed) estimates of the unobserved factors λ_t . For instance, looking at the first and the third panel, one can observe the influence of

³⁹ The smoothed probabilities are obtained by applying Kim’s (1994) filter. While filtered probabilities, as of date t , use only information available up to date t , smoothed probabilities exploit all sample information.

Fig. 2: Actual vs. model-implied spreads

Notes: These plots compare observed (light-grey solid lines) and model-implied (dotted lines) spreads. For all entities, except Germany, the spreads are yield differentials between zero-coupon 5- and 10-year zero-coupon yields and their German counterparts. For Germany, the yield differentials are against (zero-coupon) overnight-index-swap rates. The black solid line is the model-implied contribution of the liquidity intensity $\lambda_{\ell,t}^{(n)}$ to the spreads (these contributions are computed as the spread that would prevail if the credit parts $\lambda_{c,t}^{(n)}$ of the debtor intensities were equal to zero). For KfW (upper-left plot), the fact that the dotted line and the black solid line are confounded results from the identification of the liquidity factor $\lambda_{\ell,t}$.



the occurrence of liquidity crises on the liquidity-related factor $\lambda_{\ell,t}$.

Besides, the estimated specifications of the regimes' dynamics are meaningful. The historical (risk-neutral) dynamics is described by the matrix Π (the matrix Π^*) reported in Table 4. It appears that the adverse states of the world are more long-lived under the risk-neutral measure than under the historical one, which tends to give rise to risk premia associated with those bad states of the world. To set an example, while the probability of remaining in the most adverse regime (liquidity crisis and severe credit crisis) is of 25% under the historical measure, it is of 89% under the risk-neutral one. So as to facilitate the interpretation of the transition probabilities, Table 5 presents selected combinations of these. More precisely, it gives the probabilities of switching to liquidity- or credit-crisis regimes conditional on the existence of a crisis at the previous period, ruling out the distinction between the intense and the less intense credit crises. These probabilities illustrate the causality between the two kinds of crises. Indeed, the probability of switching to a credit (resp. liquidity) crisis is significantly higher when there is a liquidity (resp. credit) crisis at the previous period. For instance, the probability of switching to a credit crisis between date $t - 1$ and date t is of 14% (resp. 0.04%) if there is (resp. not) a liquidity crisis at date $t - 1$.

6.2 Liquidity intensity and pricing

In our model, a single factor ($\lambda_{\ell,t}$) drives liquidity pricing in euro-area bond yields. The first panel in Figure 5 illustrates the striking comovements between our estimated liquidity factor and another proxy of liquidity pricing, the bid-ask spreads associated with French 10-year benchmark bonds.

The lower part Figure 5 is aimed at relating the countries' sensitivities to the liquidity-related factor ($\alpha_{1,\ell}^{(n)}$, see equation 3) to national marketable-debt characteristics: (a) the countries' debt outstanding and (b) the average bid-ask spreads of the countries 10-year benchmark bonds. The first scatter plot of Figure 5 shows that, leaving Italy aside, there seems to be a negative relationship between these sensitivities and the debt outstanding. In spite of the large size of the tradable debt issued by the Italian government, Italy's intensity appears to be particularly sensitive to the liquidity factor.⁴⁰ The second scatter plot (bottom-right panel in Figure 5) points to a positive relationship between the countries' sensitivities and the bid-ask spreads.

⁴⁰ To some extent, such a finding is consistent with the results of Chung-Cheung, de Jong and Rindi (2004) according to which transitory costs would be more important in the Italian market, dominated by local traders.

In order to gauge the relative importance of the liquidity-related part of the spreads, we have computed the spreads (versus German yields) that would prevail if the default intensities were equal to zero. Figure 2 presents the resulting spreads (black solid lines).⁴¹ The liquidity-related parts of the spreads turn out to account for a substantial part of the changes in spreads, especially for the less indebted countries (the Netherlands and Finland). The German plot reveals that the high liquidity of the German Bunds translates into negative spreads versus swap rates. Such negative spreads can be attributed to the so-called convenience yield of holding government-issued securities and/or to flight-to-liquidity phenomena taking place amid the financial crisis.⁴² While the liquidity factor was explaining the main part of the spreads' fluctuations for most of the countries in the post-Lehman period, the part of the spreads explained by credit-related factors became predominant for several countries (Austria, Belgium, France, Italy and Spain) over the last year of the sample.

6.3 Default probabilities

In the remaining of the paper, we show how our results can be exploited to compute the default probabilities implied by the yield data. In the spirit of Litterman and Iben (1991), various methodologies that are widely used by practitioners or market analysts end up with risk-neutral PDs (see, e.g. Chan-Lau, 2006). Our framework makes it possible to investigate the potential differences that exist between the latter and their historical, or real-world, counterparts. As stated above (see Subsection 3.3), while the intensities of default are the same processes under both measures, the \mathbb{P} - and \mathbb{Q} -probabilities of default are not the same because the \mathbb{P} - and \mathbb{Q} -dynamics of these processes differ.

⁴¹ Due to non-linearity effects, the sum of this counterfactual spreads and those that would be obtained, alternatively, by switching off the liquidity intensities are not strictly equal to the complete modeled spreads. However, the differences are visually imperceptible.

⁴² See Felthütter and Lando, 2008 or Liu, Longstaff and Mandell 2006 for empirical studies and discussions of convenience yield on U.S. data. Flight-to-liquidity effects in the euro area sovereign bond market are investigated by Beber, Brandt and Kavajecz 2009.

In our framework, the actual PD between time t and time $t + h$ is given by

$$\begin{aligned}
\mathbb{P}\left(d_{t+h}^{(n)} = 1 \mid \tilde{w}_t, d_t^{(n)} = 0\right) &= E_t^{\mathbb{P}}\left(\mathbb{I}_{\{d_{t+h}^{(n)}=1\}} \mid d_t^{(n)} = 0\right) \\
&= 1 - E_t^{\mathbb{P}}\left(\mathbb{I}_{\{d_{t+h}^{(n)}=0\}} \mid d_t^{(n)} = 0\right) \\
&= 1 - E_t^{\mathbb{P}}\left(\exp(-\tilde{\lambda}_{d,t+1}^{(n)} - \dots - \tilde{\lambda}_{d,t+h}^{(n)})\right). \tag{15}
\end{aligned}$$

We are then left with the computation of the survival probability $E_t^{\mathbb{P}}(\exp(-\tilde{\lambda}_{d,t+1}^{(n)} - \dots - \tilde{\lambda}_{d,t+h}^{(n)}))$. Recall that $\exp(-\lambda_{d,t}^{(n)}) = \exp(-\tilde{\lambda}_{d,t}^{(n)}) + \zeta[1 - \exp(-\tilde{\lambda}_{d,t}^{(n)})]$. When $\lambda_{d,t}^{(n)}$ is small, the first order approximation leads to:

$$\tilde{\lambda}_{d,t}^{(n)} \simeq \frac{1}{1 - \zeta} \lambda_{d,t}^{(n)}. \tag{16}$$

Up to this approximation, the survival probability is a multi-horizon Laplace transform of a compound auto-regressive process of order one. In the same way as for the yields, the recursive algorithm detailed in Appendix E can be used in order to compute these probabilities. In the computation, we use a constant recovery rate of 50%, which corresponds to the average of the recovery rates observed for sovereign defaults over the last decade (see Moody's, 2010).

Figure 6 shows the model-based 5-year probabilities of default (i.e. the probabilities that the considered countries will default during the next 5 years). One-standard-deviation bands are also reported. These standard deviations take two kinds of uncertainty into account: (1) the smoothing errors that are associated with the Kim's (1994) smoothing algorithm used to estimate the intensities λ_t and (2) the uncertainty stemming from the parameters' estimation (MLE).⁴³

Figure 7 presents the model-implied term-structure of PDs as of 9 May 2008 and 30 December 2011. This Figure illustrates the dramatic changes in the term-structure of PDs that took place over these 3 years. For all countries and especially for the more indebted ones, the term-structure of the PDs is much higher and steeper in late 2011 than in Spring 2008.

Finally, it is worth noting that even when taking into account the uncertainty regarding the estimated real-world PDs, the gap between these and their risk-neutral counterparts is significant in many cases, particularly for the most recent years (see Figure 6). Nevertheless, as stated above,

⁴³ The computation of these standard errors is inspired from Hamilton (1986). It relies on the assumption that the two kinds of errors (smoothing and MLE) are independent from each other.

risk-neutral PDs are extensively used by market practitioners and analysts. This mainly stems from the fact that risk-neutral PDs are relatively easy to compute, using basic methods inspired by the one proposed by Litterman and Iben (1991)⁴⁴. To illustrate, Figure 8 compares the PDs estimates derived from our model with alternative estimates, as of the end of 2011. Two kinds of alternative estimates are considered: (a) PDs that are based on the Moody’s credit ratings and the associated matrix of long-run credit-rating-migration probabilities and (b) risk-neutral probabilities computed by CMA Datavision (2011). Figure 8 shows that our estimates lie somewhere between the two others.⁴⁵ In addition, it appears that our risk-neutral PDs (the triangles) are relatively close to the risk-neutral CDS-based ones computed by CMA.⁴⁶

7 Conclusion

In this paper, we present a multi-country no-arbitrage model of the joint dynamics of euro-area sovereign spreads. At the heart of our model is an innovative approach capturing the intertwined dynamics of credit- and liquidity-related crises by a joint modeling of two kinds of switching regimes. These crises are key drivers of, respectively, credit and illiquidity intensities associated with the different issuers.

Using euro-area spread data covering the last five years, we estimate such intensities for eight euro-area countries. The resulting fit is satisfying, the standard deviations of the yields pricing errors –across countries and maturities– being of 18 basis points. Interestingly, we provide evidence of causal relationships between credit and liquidity crises periods.

Our approach makes it possible to exhibit the part of the spreads reflecting liquidity-pricing effects. A key assumption is that the country-specific illiquidity intensities perfectly comoves, that is, that there exists a single European liquidity-pricing factor. The identification of the latter is based on the term structure of the yield differentials between the bonds issued by KfW (a German

⁴⁴ In particular, these methods do not care about liquidity-pricing effects.

⁴⁵ Credit-rating-based PDs are extremely small (for instance: 6.10^{-6} for a AAA-rated countries, 6.10^{-4} for a A-rated countries). This reflects the fact that transition probabilities are based on past 25-year history of rating changes, during which quick sovereign downgrades were infrequent (contrary to during the current crisis period).

⁴⁶ The remaining differences between the latter two risk-neutral estimates may be attributed to (i) the fact that we consider spreads w.r.t. Germany in our methods while the CMA’s method involves “absolute” CDS, (ii) the absence of treatment of liquidity-pricing effects in the CMA methodology (while empirical evidence suggests that CDS contain liquidity premia, see Buhler and Trapp, 2008) or also to (iii) the measurement errors of our approach (see Figure 2).

agency) and the German sovereign bonds (the *Bunds*). Indeed, KfW's liabilities are explicitly and unconditionally guaranteed by the Federal Republic of Germany. Therefore, the KfW-*Bund* spread should be essentially liquidity-driven. Our results indicate that a substantial part of intra-euro spreads is liquidity-driven.

Given some assumptions regarding the recovery process, our framework makes it possible to decompose the credit part of the spreads between actual, or real-world, probabilities of default on the one hand and risk premiums on the other hand. To that respect, our results suggest that actual PDs are often significantly lower than their risk-neutral counterparts. According to these results, relying on risk-neutral PDs to assess the market participants expectations regarding future sovereign defaults would be misleading.

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A Pricing of defaultable bonds

For the sake of notational convenience, we drop the issuer subscript n in this Appendix.

Let us consider the price $B_{T-1,1}$, at date $T-1$, of a one-period bond issued by the debtor (before $T-1$). If the debtor is not in default at $T-1$, then:

$$B_{T-1,1} = \exp(-r_{T-1})E^{\mathbb{Q}} \left[(1 - d_T + \zeta d_T)(1 - \ell_T + \theta \ell_T) \mid \underline{\tilde{w}}_{T-1} \right]$$

($\underline{\tilde{w}}_{T-1}$ containing the information $d_{T-1} = 0$)

$$\begin{aligned} &= \exp(-r_{T-1})E^{\mathbb{Q}} \left[E^{\mathbb{Q}} \left((1 - d_T + \zeta d_T)(1 - \ell_T + \theta \ell_T) \mid w_T, \underline{\tilde{w}}_{T-1} \right) \mid \underline{\tilde{w}}_{T-1} \right] \\ &= \exp(-r_{T-1})E^{\mathbb{Q}} \left[\left\{ \exp \left(-\tilde{\lambda}_{d,T}^{(n)} \right) + \zeta \left(1 - \exp \left(-\tilde{\lambda}_{d,T}^{(n)} \right) \right) \right\} \times \right. \\ &\quad \left. \left\{ \exp \left(-\tilde{\lambda}_{\ell,T} \right) + \theta \left(1 - \exp \left(-\tilde{\lambda}_{\ell,T} \right) \right) \right\} \mid \underline{\tilde{w}}_{T-1} \right]. \end{aligned}$$

The last equality is obtained by using the conditional independence of d_t and ℓ_t and the expressions of the conditional \mathbb{Q} -distributions of d_t and ℓ_t (that are the same as their historical counterparts, as shown in Appendix C). From that, using the definitions of $\lambda_{d,T}$ and $\lambda_{\ell,T}$ given in formula (1), it follows that:

$$\begin{aligned} B_{T-1,1} &= \exp(-r_{T-1})E^{\mathbb{Q}} \left[\exp(-\lambda_{d,T} - \lambda_{\ell,T}) \mid \underline{\tilde{w}}_{T-1} \right] \\ &= \mathcal{B}(r_{T-1}, z_{T-1}, \lambda_{T-1}) \quad (\text{say}) \end{aligned}$$

The fact that $E^{\mathbb{Q}} \left[\exp(-\lambda_{d,T} - \lambda_{\ell,T}) \mid \underline{\tilde{w}}_{T-1} \right]$ is a function of (z_{T-1}, λ_{T-1}) originates from the assumptions on the distribution of (z_T, λ_T) given $\underline{\tilde{w}}_{T-1}$, in particular the non-causality from (d_t, ℓ_t) to (z_t, λ_t) under \mathbb{Q} (see Appendix C).

Let us then consider $B_{T-2,2}$, we have:

$$B_{T-2,2} = \exp(-r_{T-2})E^{\mathbb{Q}} \left[(1 - d_{T-1} + \zeta d_{T-1})(1 - \ell_{T-1} + \theta \ell_{T-1}) \mathcal{B}(r_{T-1}, z_{T-1}, \lambda_{T-1}) \mid \underline{\tilde{w}}_{T-2} \right],$$

($\underline{\tilde{w}}_{T-2}$ containing the information $d_{T-2} = 0$).

Conditioning first by $(w_{T-1}, \underline{\tilde{w}}_{T-2})$ and using the fact that $\mathcal{B}_{T-1}(z_{T-1}, \lambda_{T-1})$ only depends on w_{T-1} , we get:

$$B_{T-2,2} = E^{\mathbb{Q}} \left[\exp(-r_{T-2} - \lambda_{d,T-1} - \lambda_{\ell,T-1}) \mathcal{B}(r_{T-1}, z_{T-1}, \lambda_{T-1}) \mid \underline{\tilde{w}}_{T-2} \right].$$

Replacing $\mathcal{B}(r_{T-1}, z_{T-1}, \lambda_{T-1})$ by $E^{\mathbb{Q}} \left[\exp(-r_{T-1} - \lambda_{d,T} - \lambda_{\ell,T}) \mid \underline{\tilde{w}}_{T-1} \right]$ and using the fact that $\exp(-r_{T-2} - \lambda_{d,T-1} - \lambda_{\ell,T-1})$ is function of $\underline{\tilde{w}}_{T-1}$, we get:

$$\begin{aligned} B_{T-2,2} &= E^{\mathbb{Q}} \left[E^{\mathbb{Q}} \left(\exp(-r_{T-2} - \lambda_{d,T-1} - \lambda_{\ell,T-1} - r_{T-1} - \lambda_{d,T} - \lambda_{\ell,T}) \mid \underline{\tilde{w}}_{T-1} \right) \mid \underline{\tilde{w}}_{T-2} \right] \\ &= E^{\mathbb{Q}} \left[\exp(-r_{T-2} - \lambda_{d,T-1} - \lambda_{\ell,T-1} - r_{T-1} - \lambda_{d,T} - \lambda_{\ell,T}) \mid \underline{\tilde{w}}_{T-2} \right]. \end{aligned}$$

Applying this methodology recursively leads to equation (8).

B Parameter constraints

B.1 Econometric identification of the liquidity factor $\lambda_{\ell,t}$

As documented in 4.2, the bonds issued by KfW and those issued by the German government embed the same credit risks but are not equally exposed to the liquidity-related factor. Therefore, the sum of the recovery-adjusted default intensity and the liquidity intensity of KfW is given by:⁴⁷

$$\begin{aligned}\lambda_t^{KfW} &= \lambda_{d,t}^{GER} + \lambda_{\ell,t}^{KfW} \\ &= \lambda_{d,t}^{GER} + \alpha_{0,\ell}^{KfW} + \alpha_{1,\ell}^{KfW} \lambda_{\ell,t},\end{aligned}\tag{17}$$

that is, the risk intensities of KfW and the Federal Republic of Germany differ only through α_{ℓ}^{KfW} . We impose $\alpha_{1,\ell}^{KfW} = \alpha_{1,\ell}^{GER} + 1$ so as to identify the liquidity-related factor $\lambda_{\ell,t}$ (without loss of generality).

B.2 Specification of the matrix of transition probabilities Π

Here, we present the specification of the matrix Π that defines the dynamics of z_t , which is the Kronecker product of the liquidity-crisis variable $z_{\ell,t}$ and of the credit-crisis variable $z_{c,t}$. First, we assume that there is no instantaneous causality between $z_{c,t}$ and $z_{\ell,t}$, meaning that conditionally to z_{t-1} , $z_{c,t}$ and $z_{\ell,t}$ are independent. Second, whereas the switching probabilities of the liquidity-regime variable $z_{\ell,t}$ between date $t-1$ and date t may be influenced by the existence of a credit crisis at date $t-1$, it does not depend on the *distinction* between the two credit-crisis levels.⁴⁸ Third, the probabilities of switching from the severe credit-crisis state ($z_{c,t-1} = e_3^{[3]}$) to the no-credit-stress regime ($z_{c,t} = e_1^{[3]}$) is zero, as well as the opposite. That is, the first credit-crisis level ($z_{c,t} = e_2^{[3]}$) acts as an intermediary regime between the two others. Fourth, the probability of remaining in the severe-credit-crisis state does not depend on $z_{\ell,t}$. With these restrictions, 11 parameters are required to define the matrix Π .

B.3 The size of the Gaussian shocks

The standard deviations of the Gaussian shocks entering equations (3) and (4) are constrained to make sure that the regime variables z_t are the main sources of the spreads fluctuations. If such constraints are not imposed, most of the spread fluctuations tend to be accounted for by the Gaussian shocks. This phenomenon, that reflects that Gaussian shocks are more flexible than the discrete-numbered regimes to fit the spreads, has two undesirable implications within our framework. First and foremost, the higher the standard deviation of the Gaussian shocks, the higher the frequency of generating/estimating negative intensities. Second, the lower the importance of the regime variables, the less information about the relationships between liquidity- and credit-crises the estimation is brought to reveal. Accordingly, we constrain the parameters to be such that a limited part of the (unconditional) fluctuations of the intensities is accounted for by Gaussian shocks. Practically, we impose the following constraints on the parameter estimates: $\sigma_i / \sqrt{1 - \rho_i^2} \leq 10\% \tilde{\sigma}_i$, where $\tilde{\sigma}_i$ is the sample standard deviation of the (observed) spreads associated with entity i (and where σ_i is expressed in the same unit as the spreads). This calibration implies unconditional distribution of the intensities that is consistent with mainly positive intensities. Alternative estim-

⁴⁷ For the sake of clarity, we slightly modify the notations by replacing the (n) subscripts by “KfW” and “GER”.

⁴⁸ Formally, $p(z_{\ell,t} | z_{\ell,t-1}, z_{c,t-1} = e_2^{[3]}) = p(z_{\ell,t} | z_{\ell,t-1}, z_{c,t-1} = e_3^{[3]})$.

ation (with ratios of 5% and 20%) suggest that the qualitative results presented above are fairly robust to changes in the 10% ratio.

B.4 The auto-regressive coefficient ρ_c

Under the historical measure, the auto-regressive coefficient ρ_c is assumed to be constant across countries. This choice is related to our use of survey-based forecasts to address the downward bias in the estimated persistency of the factors. Kim and Orphanides (2012) have shown that using survey-based forecasts of yields makes it possible to overcome this bias. However, we have survey-based forecasts of spreads vs. Germany for only four countries (France, Italy, Spain and the Netherlands). Under the assumption that the persistence of the credit factor is common across countries, the information content of available survey-based forecasts benefits the parameterizations of all debtors' intensities.

B.5 The standard deviations of the pricing errors

For a given debtor, the standard deviations of the pricing errors (gathered in the vector $\xi_{S,t}$, see equation 12) are assumed to be the same across maturities. However, they differ across countries, proportionally to the standard deviations of the observed spreads (the proportionality coefficient being estimated by the MLE). There are two exceptions: First, given the crucial role of the KfW-Bund spreads in the identification of the liquidity factor $\lambda_{\ell,t}$, the proportionality coefficient associated with the pricing errors of the KfW-Bund spreads is twice lower than the others (to make sure that the liquidity factor properly fits the KfW-Bund spread). Second, given its different nature, the standard deviation of the Bund-OIS pricing-errors is twice larger than the others. (What we want to fit in the first place are the highly scrutinized spreads versus Germany).

C Relationship between the risk-neutral and historical intensities

In this Appendix, we show that the default intensities are the same processes under both measures (\mathbb{P} and \mathbb{Q}) when the stochastic discount factor depends on w_t only.

Proof. Recalling that $\tilde{w}_t = (w'_t, d'_t, \ell_t)'$, we have:

$$f_{d,\ell}^{\mathbb{Q}}(d_t, \ell_t | w_t, \tilde{w}_{t-1}) = \frac{f^{\mathbb{Q}}(\tilde{w}_t | \tilde{w}_{t-1})}{f_w^{\mathbb{Q}}(w_t | \tilde{w}_{t-1})}. \quad (18)$$

Using the definition of the s.d.f., the numerator of (18) can be expressed as:

$$\begin{aligned} f^{\mathbb{Q}}(\tilde{w}_t | \tilde{w}_{t-1}) &= M_{t-1,t} \exp(r_{t-1}) f^{\mathbb{P}}(\tilde{w}_t | \tilde{w}_{t-1}) \\ &= M_{t-1,t} \exp(r_{t-1}) f^{\mathbb{P}}(d_t, \ell_t | w_t, \tilde{w}_{t-1}) f_w^{\mathbb{P}}(w_t | \tilde{w}_{t-1}) \end{aligned} \quad (19)$$

Since the s.d.f. $M_{t-1,t}$ depends on w_t only, the integration of both sides of (19) w.r.t. (d_t, ℓ_t) leads to:

$$f_w^{\mathbb{Q}}(w_t | \tilde{w}_{t-1}) = M_{t-1,t} \exp(r_{t-1}) f_w^{\mathbb{P}}(w_t | \tilde{w}_{t-1}). \quad (20)$$

Using (19) and (20) to compute the r.h.s. of (18), we obtain $f_{d,\ell}^{\mathbb{Q}}(d_t, \ell_t | w_t, \tilde{w}_{t-1}) = f_{d,\ell}^{\mathbb{P}}(d_t, \ell_t | w_t, \tilde{w}_{t-1})$. \square

Moreover, since we have assumed that $f_w^{\mathbb{P}}(w_t | \tilde{w}_{t-1}) = f_w^{\mathbb{P}}(w_t | \underline{w}_{t-1})$ and since $M_{t-1,t} \exp(r_{t-1})$ does not depend on $(\underline{d}_{t-1}, \underline{\ell}_{t-1})$, equation (20) implies that the same is true for $f_w^{\mathbb{Q}}(w_t | \tilde{w}_{t-1})$. In

other words, since we assume that if (d_t, ℓ_t) does not Granger cause w_t under \mathbb{P} , the same is true in the risk-neutral world.

D Laplace transform of (z_t, y_t)

The risk-neutral Laplace transform of (z_t, λ_t) conditional to the information available in time $t-1$ is:

$$\varphi_{t-1}^{\mathbb{Q}}(u, v) = \exp(v' \Phi^* \lambda_{t-1} + [l_1, \dots, l_J] z_{t-1}), \quad (21)$$

where $l_i = \log \sum_{j=1}^J \pi_{ij}^* \exp \{u_i + v' \mu^* e_j + \frac{1}{2} v' \Sigma \Sigma' v\}$ and where e_j is the j^{th} column of the 6×6 identity matrix. Therefore, (z_t, λ_t) is compound auto-regressive of order one –denoted by Car(1)– under the risk-neutral measure.

Proof. We have

$$\begin{aligned} \varphi_{t-1}^{\mathbb{Q}}(u, v) &= E_{t-1}^{\mathbb{Q}}(\exp[u' z_t + v' \lambda_t]) \\ &= E_{t-1}^{\mathbb{Q}}\left(\exp\left[u' z_t + v' \mu^{*'} z_t + v' \Phi^* \lambda_{t-1} + v' \Sigma \varepsilon_t\right]\right) \\ &= E_{t-1}^{\mathbb{Q}}\left(E_{t-1}^{\mathbb{Q}}\left\{\exp\left[u' z_t + v' \mu^{*'} z_t + v' \Phi^* \lambda_{t-1} + v' \Sigma \varepsilon_t\right] \mid z_t\right\}\right) \\ &= \exp(v' \Phi^* \lambda_{t-1}) E_{t-1}^{\mathbb{Q}}\left(\exp\left\{u' z_t + v' \mu^{*'} z_t\right\} \times E_{t-1}^{\mathbb{Q}}(\exp\{v' \Sigma \varepsilon_t \mid z_t\})\right) \\ &= \exp(v' \Phi^* \lambda_{t-1}) E_{t-1}^{\mathbb{Q}}\left(\exp\left\{u' z_t + v' \mu^{*'} z_t\right\} \times \frac{1}{2} v' \Sigma \Sigma' v\right) \\ &= \exp(v' \Phi^* \lambda_{t-1} + [l_1, \dots, l_J] z_{t-1}). \end{aligned}$$

where the l_i 's are given above. \square

E Multi-horizon Laplace transform of a Car(1) process

Let us consider a multivariate Car(1) process Z_t and its conditional Laplace transform given by $\exp[a'(s)Z_t + b(s)]$. Let us further denote by $L_{t,h}(\omega)$ its multi-horizon Laplace transform given by:

$$L_{t,h}(\omega) = E_t \left[\exp(\omega'_{H-h+1} Z_{t+1} + \dots + \omega'_H Z_{t+h}) \right], \quad t = 1, \dots, T, \quad h = 1, \dots, H,$$

where $\omega = (\omega'_1, \dots, \omega'_H)$ is a given sequence of vectors. We have, for any t ,

$$L_{t,h}(\omega) = \exp(A'_h Z_t + B_h), \quad h = 1, \dots, H,$$

where the sequences $A_h, B_h, h = 1, \dots, H$ are obtained recursively by:

$$\begin{aligned} A_h &= a(\omega_{H-h+1} + A_{h-1}) \\ B_h &= b(\omega_{H-h+1} + A_{h-1}) + B_{h-1}, \end{aligned}$$

with the initial conditions $A_0 = 0$ and $B_0 = 0$.

Proof. The formula is true for $h = 1$ since:

$$L_{t,1}(\omega) = E_t(\omega'_H Z_{t+1}) = \exp[a'(\omega_H)Z_t + b(\omega_H)]$$

and therefore $A_1 = a(\omega_H)$ and $B_1 = b(\omega_H)$.

if it is true for $h - 1$, we get:

$$\begin{aligned} L_{t,h}(\omega) &= E_t \left[\exp(\omega'_{H-h+1} Z_{t+1}) E_{t+1}(\omega'_{H-h+2} Z_{t+2} + \dots + \omega'_H Z_{t+H}) \right] \\ &= E_t \left[\exp(\omega'_{H-h+1} Z_{t+1}) L_{t+1,h-1}(\omega) \right] \\ &= E_t \left[\exp(\omega'_{H-h+1} Z_{t+1} + A_{h-1} Z_{t+1} + B_{h-1}) \right] \\ &= \exp \left[a(\omega'_{H-h+1} + A_{h-1}) Z_t + b(\omega'_{H-h+1} + A_{h-1}) + B_{h-1} \right] \end{aligned}$$

and the result follows. \square

Fig. 3: Model-based vs. survey-based forecasts

Notes: This Figure compares 12-month-ahead survey-based forecasts of the spreads (circles) versus Germany with model-based forecasts (solid line; the expectations are conditional to on the current values of the state variables w_t). Source of the survey-based forecasts: *Consensus forecasts*.

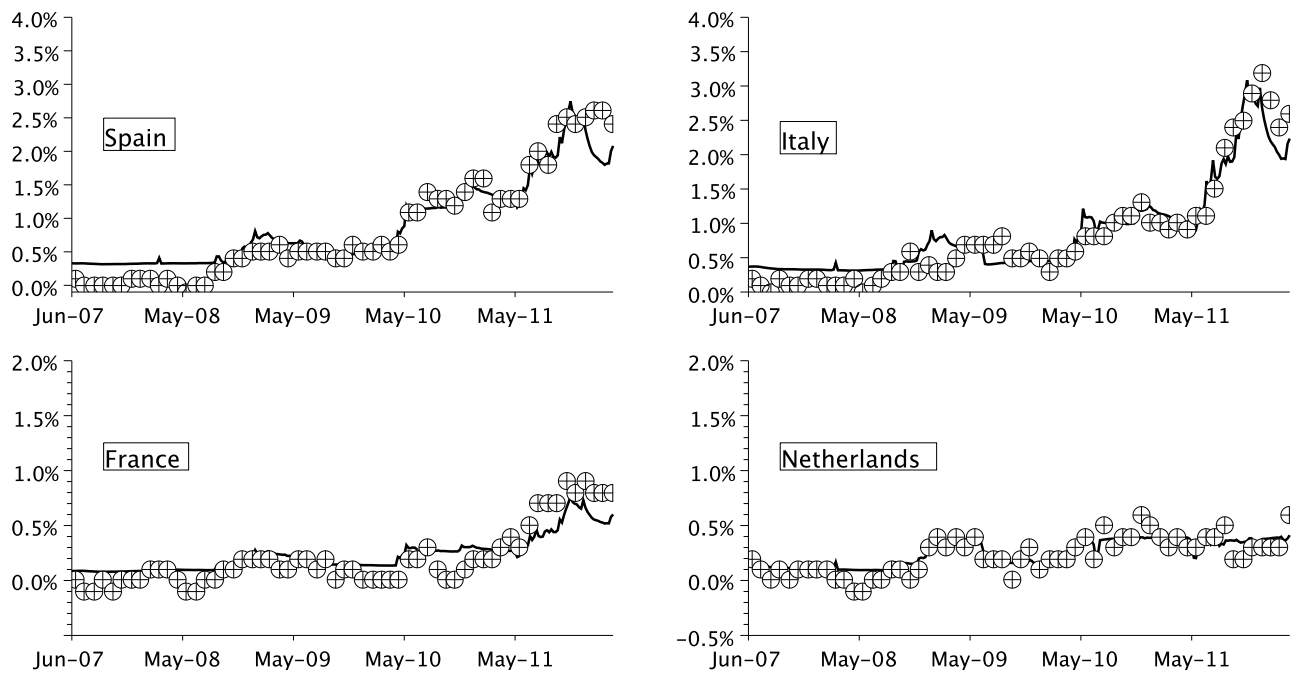


Fig. 4: Estimated regimes and intensities

Notes: The first two panels display the (smoothed) probabilities of being in the crises regimes (Kim's (1994) algorithm). More precisely, there are two liquidity regimes (normal and crisis) and three credit regimes (normal, distress and severe distress). The last three panels show the nine unobserved factors (λ_t). The dynamics of the liquidity factor $\lambda_{\ell,t}$ (resp. credit factors $\lambda_{c,t}^{(n)}$) depends on the liquidity regime (resp. the credit regime) but not on the credit regime (resp. the liquidity regime).

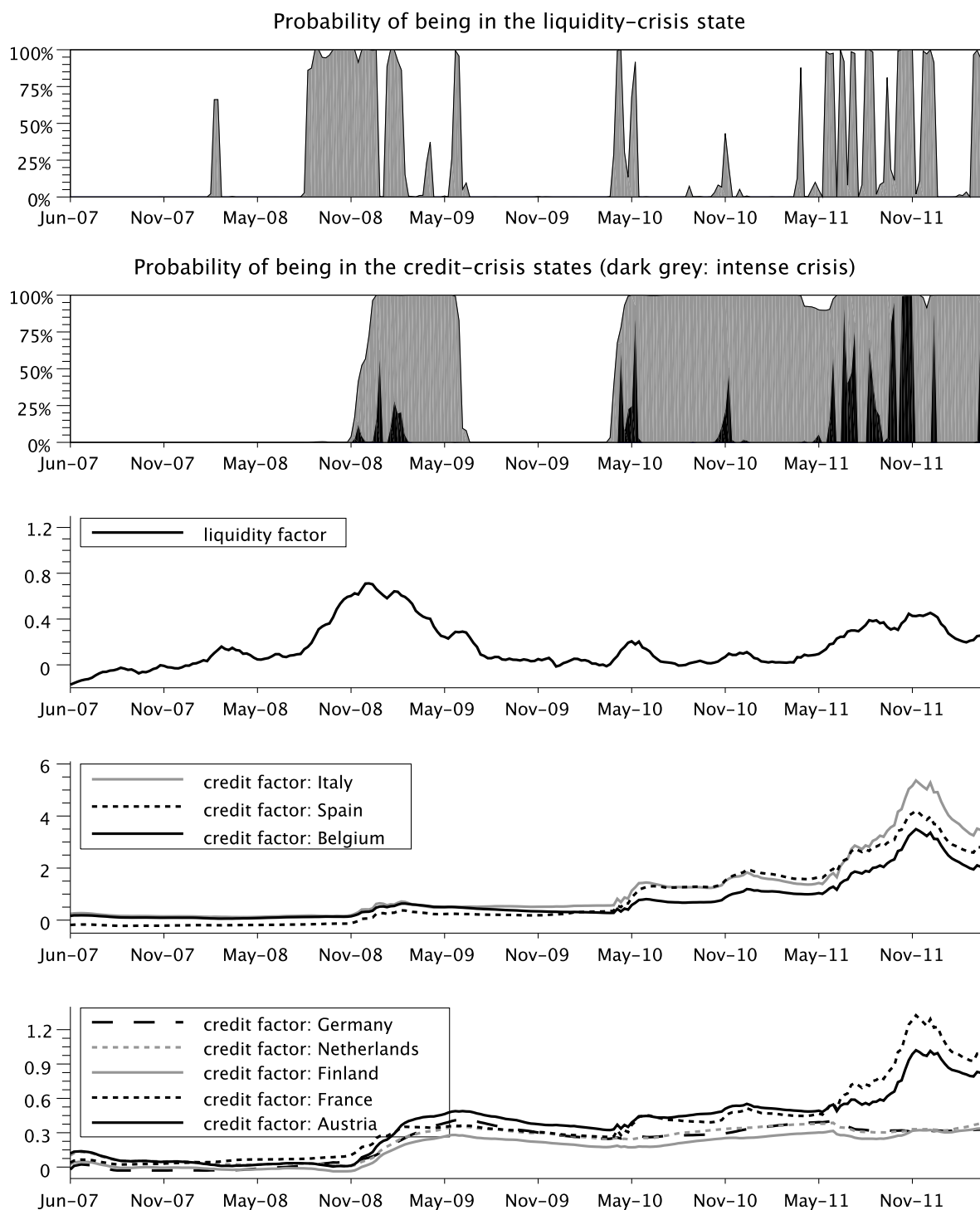


Fig. 5: Sensitivity to the liquidity factor versus debt outstanding

Notes: The upper panel plots the estimated liquidity-related factor $\lambda_{\ell,t}$ together with the bid-ask spread on French 10-year benchmark bonds (source: Thomson Reuters Tick History database). In the first scatter plot, the coordinates of the countries correspond to (x -coordinates) the sensitivities $\mu_{1,\ell}^{(n)}$ of their risk intensities to the European liquidity factor $\lambda_{\ell,t}$ (these sensitivities are reported in the upper part of Table 3) and (y -coordinates) their total marketable sovereign debt (as of the end of 2009, Source: Eurostat). In the first scatter plot, the abscissa of the countries are the same than for the previous plot, and the y -coordinates are the average (2010-2012) of the bid-ask spreads of 10-year sovereign bonds (source: Thomson Reuters Tick History database).

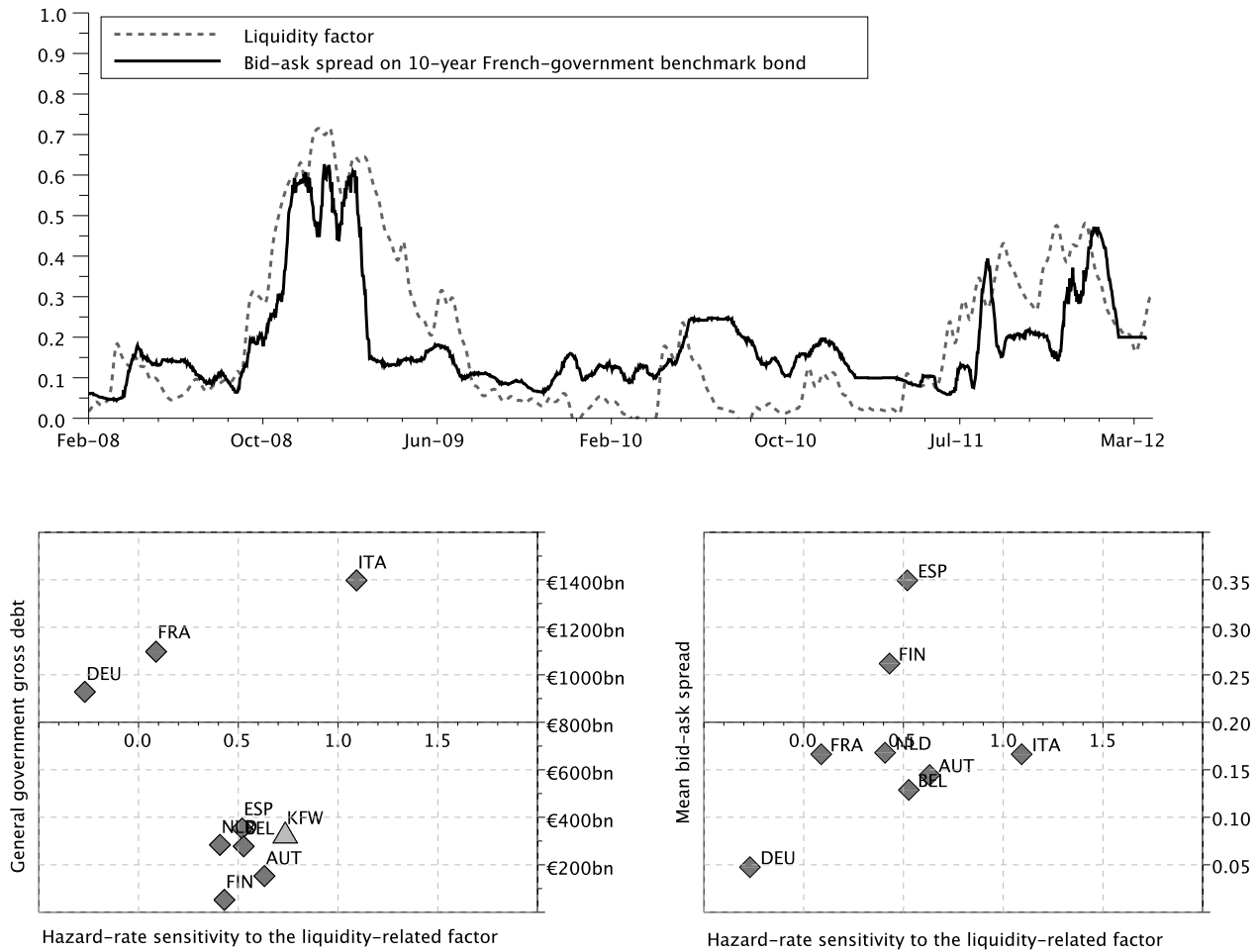


Fig. 6: Default probabilities estimates (5-year horizon)

Notes: These plots display the model-implied 5-year default probabilities computed under both measures (risk-neutral: dotted line, historical: solid line). Formally, the dotted line corresponds to the time series of $E_t^Q(\mathbb{1}\{d_{t+5yrs}^{(n)} = 1\} | d_t^{(n)} = 0)$, where E_t denotes the expectation (under the historical measure) conditional to the information available at time t (see Section 6.3 for the computation of these default probabilities). The black solid line represents $E_t^Q(\mathbb{1}\{d_{t+5yrs}^{(n)} = 1\} | d_t^{(n)} = 0)$. One-standard-deviation bands are reported. These standard deviation bands account for smoothing errors (associated to Kim's smoothing algorithm, 1994) as well as uncertainty related to the parameter estimates, following Hamilton's (1986) approach. The y -axis scales differ across countries.

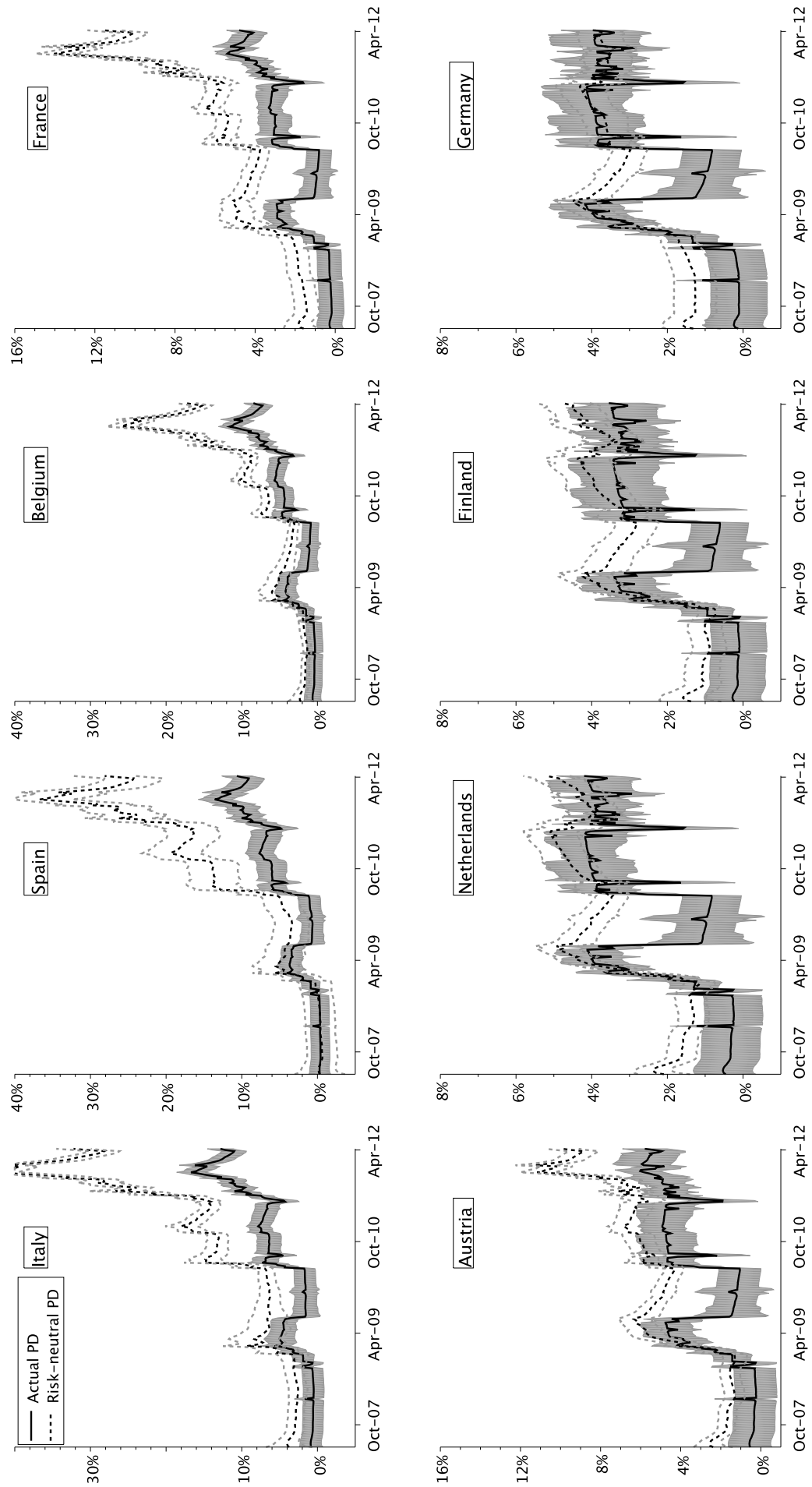


Fig. 7: Term structure of default probabilities

Notes: These plots display the term structure of the default probabilities for two distinct dates. Formally, for date t and debtor n , the plot shows $E_t^{\mathbb{P}}(\mathbb{1}_{\{d_{t+h}^{(n)} = 1\}} | d_t^{(n)} = 0)$ for h between 1 month and 10 years (where $E_t^{\mathbb{P}}$ denotes the expectation under the historical measure conditional to the information available at time t). The grey lines delimit the ± 1 standard deviation area. These standard deviations account for smoothing errors (associated to Kim's smoothing algorithm, 1994) as well as uncertainty related to the parameter estimates, following Hamilton's (1986) approach. The y -axis scales differ across countries.

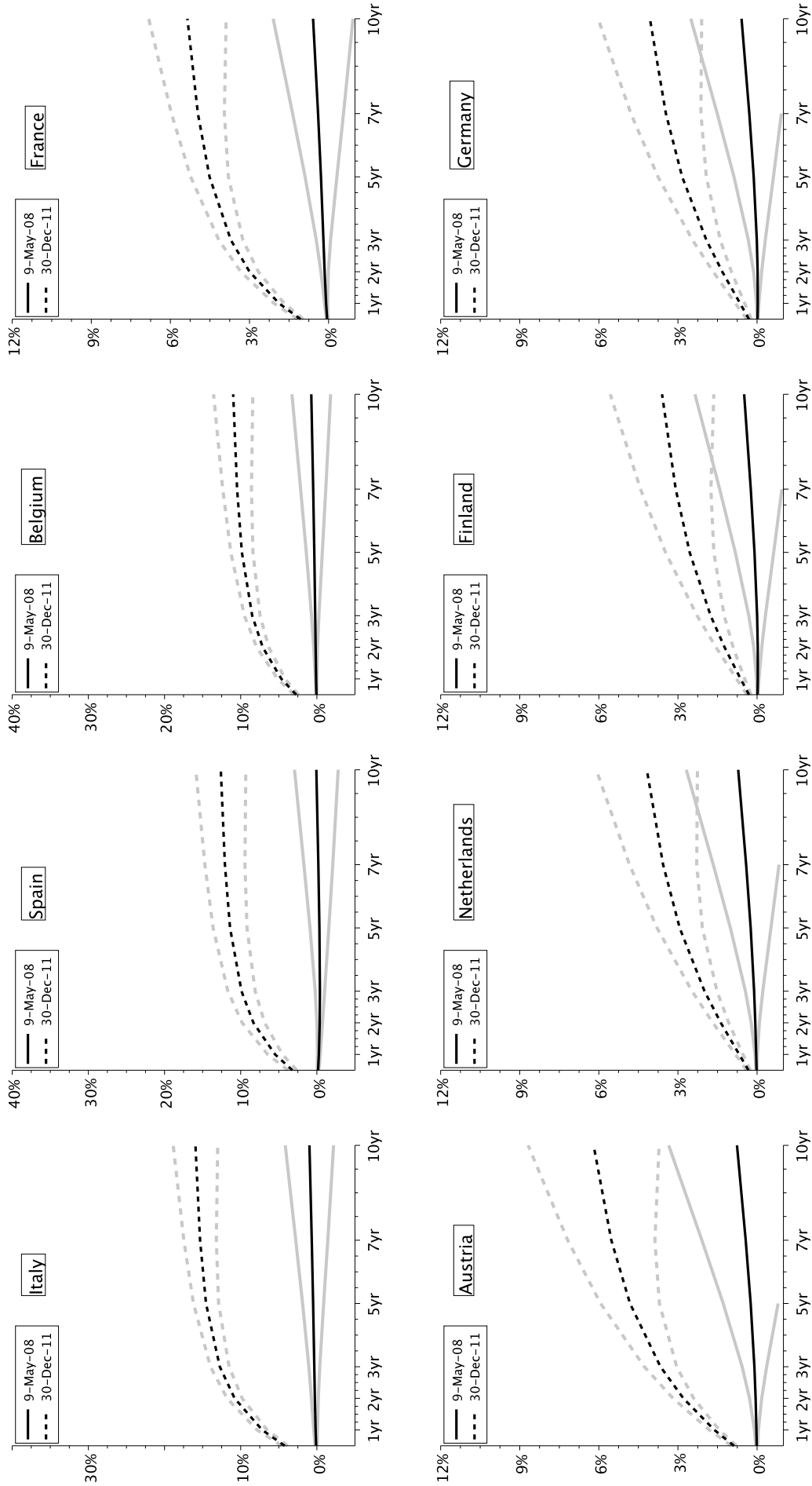


Fig. 8: Default probabilities estimates (5-year horizon)

Notes: This plot displays different estimates of probabilities of default (PD) of 8 euro-area governments (as of 30 December 2011). The squares and the triangles correspond to outputs of our model. While the squares indicate "real-world" PDs (i.e. the default probabilities obtained under the physical, or historical, measure), the triangles are risk-neutral PDs. The vertical black bars associated with squares delineate the ± 2 standard-deviation area. These standard deviations account for smoothing errors (associated to Kim's smoothing algorithm, 1994) as well as uncertainty related to the parameter estimates, following Hamilton's (1986) approach. The circles indicate the PDs computed by CMA, using an industry standard model and proprietary CDS data from CMA Datavision (2011) [24]. The diamonds correspond to PDs that derive from (a) the Moody's ratings of the countries (as of 2011Q4) and (b) the matrix of credit-rating-migration probabilities given by Moody's (2010) [58].

