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Optimizing Portfolios with Allocations to Insured Death Benefit

A Proposed Methodology for Evaluation

Robert G. Danielsen, Sergey S. Barabanov, John Schweers, and Michael F. Sullivan

Abstract

We articulate a new solution to the unique problem of merging actuarial concepts like mortality probability with traditional portfolio and investment concepts like mean/variance analysis in the context of assessing the use of insured death benefit to optimize the risk-adjusted returns of a portfolio. The uncertain duration of cash flows and timing of death benefits have historically made it difficult for researchers to treat life insurance as an asset class in portfolio optimization or allocation. Yet, various life insurance products have often been used in institutional and corporate strategies, as well as in high net worth individual and family portfolios, usually with a poor quantitative sense of the optimal level of life insurance. Using Monte Carlo simulations and actuarial techniques, we propose a method that allows an analysis of the benefits and costs of insured death benefit to long-term portfolio returns and volatility. The conclusion is that portfolios with insured death benefit can potentially yield significantly better risk-adjusted returns and future values.

An extensive body of the finance literature is devoted to various aspects of the asset allocation decision and portfolio optimization. Little attention, however, has been given to a very important yet overlooked asset class – life insurance and related products. Undoubtedly, this is mostly because it is not easy to compare insurance products with traditional asset classes. We attempt to fill this gap.

Both practitioners and academics would agree that investors are branching out to new or underrepresented asset classes. In fact, many alternative investments are becoming traditional choices in institutional or endowment portfolios. Life insurance is not a new or alternative asset, but has mostly been studied separately from the conventional portfolio allocation and optimization. Almost half a century ago, Yaari (1965) showed how life insurance and annuities can insure individuals against various uncertainties. In late 1960s, Samuelson (1969) and Merton (1969) developed optimization models for constant portfolio choice throughout individual lives without considering any human capital. Since then, most research focused on optimizing the choice between consumption and investment while hedging the loss of human capital (e.g., Merton, 1971, Jagannathan and Kocherlakota, 1996, Pliska and Ye, 2007, Nielsen and Steffensen, 2007, Kwak, Shin, and Choi, 2011). Modeling life insurance incorporating mortality assumptions has been done in the context of using death benefits to replace income for retired couples (Huebener et all, 2013). Pirvu and Zhang (2012) use mean-reverting Brownian motion to solve consumption, investment, and insurance allocation problems while, similar to this research, acknowledging terminal wealth and legacy issues; they do not, however, address the asset allocation decision and, therefore, do not treat life insurance as an asset class. Merton

(2003) underscores the importance of including human capital in portfolio allocation decision. Smith and Buser (1983) used traditional risk-return optimization framework to show that optimal insurance amount depends on the present value of human capital the risk-return profile of the insurance products. More recently, Chen, Ibbotson, Milevsky, and Zhu (2006) use five case studies to develop a framework to hedge human capital risk for an individual investor. This is probably the most notable attempt to approach insurance demand and portfolio choice jointly. Yet, it is limited to hedging individual human capital only.

While insurance is used to mitigate the risks of losing human capital, modern insurance products are designed for a substantially greater benefit than the present value of human capital. Tax optimization products are used by high net worth families, corporations, pension funds, and endowments. Often, non-profit endowments are named as beneficiaries of large insured death benefits. Yet, there appears to be no literature focusing on life insurance in portfolio optimization beyond the obvious hedging of human capital. To the best of our knowledge, this research is the first attempt to address the very important contribution of the asset class of insurance products.

The term "Insured Death Benefit" ("IDB") is used herein, rather than the more common "life insurance", in order to highlight that we are, at least at the outset, isolating the impact of death benefit, as opposed to other kinds of financial benefits that are common features in modern life insurance policies (e.g., cash surrender values, viatical settlements, waivers of premium on disability, annuitization factors, etc.). As will be seen, however, this does not preclude analyzing policies that include those features, especially policies whose significant cash value accumulations provide a means of financing of a policy's death benefit.

In the first instance, IDB is an insurance product, whose financial utility is as a hedge against a beneficiary's risk of the death of some other identified individual or individuals. Death may trigger liquidity needs such as survivor income or estate tax payments to protect illiquid assets. But death is different from most other casualty risk events, e.g., storm damage, accidents, etc., because the event is a certainty, only the timing is uncertain. It is also distinguishable because most IDB product is priced to provide a "return" on the premiums "invested", to reflect the fact that the death benefit is often received many years after the original purchase, as well as to help survivors meet financial targets and not just replace lost value. This return element suggests that IDB may be viewed as an investment, and not exclusively as an "insurance" product.

The quantification of the timing risk of death (mortality probability), has been studied for centuries, and is well understood (cf. Hald [1990]. Actuarial tables provide statistically reliable descriptions of mortality probability for many different sub groups in a population, using easily identified characteristics such as age, gender, smoker status, and current health data. In order to improve the predictive accuracy of mortality calculations, insurance carriers also often obtain medical examinations of the specific proposed insureds so as to incorporate precise medical data into the analysis, using internal, proprietary mortality tables, based on the carrier's own experience with the predictive ability of its medical underwriting process.

For the purposes of the discussion here, a distinct "asset class" may be understood to be any asset that has a market value, or otherwise available value, with a projected return and variance determined by reference to historical performance data or otherwise, which does not correlate perfectly to another class. There are, however, significant fundamental differences between IDB and traditional asset classes. It is these differences that require a different methodology for evaluating and optimizing the allocation of portions of a portfolio to IDB as an asset class.

In modern portfolio theory ("MPT"), the allocation of securities to various classes of investment is evaluated and optimized by using some form of combined risk and reward measurement, acknowledging that most investors will want to maximize returns only in the context of some control on volatility. Traditional mean/variance analysis therefore assigns to each identified investment class a periodic mean return and a periodic volatility, usually based on historical data. The portfolio itself then has a composite mean and variance that can be evaluated and manipulated pursuant to the well-understood tenets of MPT. By contrast, IDB does not have a periodic return. Instead, the internal rate of return to death benefit, at least in the case of a guaranteed death benefit, is fixed and knowable at every future point in time. But which future point in time will represent the relevant year for the return calculation (i.e., the year of death) cannot be known for sure in advance. This means that the variance in the return is not driven by periodic fluctuations in the capital markets and the relationship of the asset to those markets, but rather by the variability of the date of death.

Exhibit 1 illustrates the change in internal rate of return ("IRR") to the date of death over a period of several years for a typical policy. As can be seen, death in the early years results in extraordinarily high IRR's. This is driven by a number of factors, including:

1) the probability of death in the early years is lower relative to level premium pricing;

2) the present value of any given amount of death benefit is higher the sooner it is received; and,

3) the cumulative periodic premiums (the policy "investment") will obviously be smaller to the extent that the premiums are paid for a shorter period of time.[1]

The high early IRR's rapidly decline as a function of those dynamics, and flatten out over the longer period of the policy. Exhibit 2 magnifies the curve from Exhibit 1 to reveal the gradual change over the later years.

Because the IRR to death is driven primarily by the date of death, the variance in the IRR is a function of the probability distribution of mortality. The insurance company of course counts on experiencing far less variance in the composite performance of the policy portfolio because it employs the law of large numbers by insuring a large number of lives. But the beneficiary of an individual policy is left with a variance driven by the underlying mortality probability distribution. Of course, in a given strategic situation that involves multiple lives, the beneficiary of the multiple policies on multiple lives will experience a benefit in reduced variance for the same reasons as the insurance company.

By contrast, we can expect asset classes with a periodic return and variance to experience a continuous reduction in the variance of the cumulative return to a point in the future as that point is pushed further into the future. The fluctuating periodic returns will tend to produce a convergence of the observed mean on the expected mean as the number of periods increases, again as a function of the weak law of large numbers.

These differences between IDB and traditional asset classes as to what drives IRR and its variance imply that we must find a way to evaluate and optimize a portfolio consisting of both traditional asset classes and IDB that does not rely on traditional mean/variance techniques to describe risk-adjusted returns. But the fact that return and variance for IDB is driven by mortality probability, and not market fluctuations, should provide an opportunity under MPT for optimization based on the well-understood benefit of adding an asset class that has a low correlation to the return pattern of other classes in the portfolio.

Illiquidity

IDB also differs from other asset classes in its liquidity characteristics. Prior to the death of the insured, the death benefit is in effect an illiquid asset. However, unlike most other illiquid assets, if the death benefit is guaranteed for the duration of the life of the insured, then IDB is guaranteed to become liquid at some unknown point in the future; i.e., the insured's death. On the other hand, if the nature of the product is to provide a possibility of lapse prior to death (either by design or as a function of poor financial performance of the product), than the IDB carries the risk of never becoming a liquid asset in the portfolio.

The first concern here is that obviously the insured will never experience the liquidity of the death benefit. Secondly, the fact that otherwise liquid assets will have been allocated to the illiquid IDB (in the form of premiums), means that the cash flow capacity ("CFC") of the portfolio will be reduced, which in turn may affect all who derive current cash flow benefits from the portfolio, insureds and beneficiaries alike. Once again, mortality probability will be a factor if premium payments are ongoing because the premium payments will of course stop at death, resulting not only in the death benefit being added to the liquid assets of the portfolio, but terminating the negative effect of premium payments on the CFC as well. All of this means that minimum CFC requirements should be a constraint in optimizing the allocation of a portion of a portfolio to IDB.

Many factors will likely affect the portfolio's CFC, including the relationship of different premium payment patterns (e.g., lump sum, 10-pay, lifetime, etc.) and different underwriting circumstances (including, but not limited to, starting age), as well as the performance of the portfolio assets themselves, inflation rates, and other financial factors. This suggests that CFC may be modeled and evaluated as a constraint by calculating the probability from a simulation that a given level of draw down will cause a complete depletion of the portfolio by a selected date. For example, a 60-year old client with a beginning portfolio value of $10,000,000 may wish to be able to withdraw annually an amount equal to a present value of $250,000, and sustain those withdrawals to age 100 with a probability in excess of 95%. A simulation incorporating probability distributions for the variables described above, will determine whether the target probability of sustainability is attained.

Taking additional amounts out of the liquid portfolio to finance IDB will obviously further reduce the CFC of the portfolio. The issue is one of a constraint on the allocation to IDB; that is, what is the maximum amount that may be allocated to IDB that will not reduce the CFC below the target probability of sustainability?

Tax Differences

The income and transfer tax[2] (estate and gift tax) characteristics of IDB can, and often are, very different from those that apply to other asset classes. In the ordinary case, IDB is received income-tax free[3]. It is also easier to remove IDB from the insured's gross estate for estate-tax purposes[4]. The exclusion from estate taxation can derive from having the policy owned in a trust from which the insured derives no benefit and over which the insured retains no control. Such a trust arrangement has little effect on the financial utility of the IDB to the insured because the insured would derive no benefit from the IDB prior to death anyway[5]. While the limits of tax exclusions may mute these effects in some cases, the implication is that IDB and the other asset classes in a portfolio should be modeled on an after-tax basis in order to take into account any potential tax advantage inherent in the IDB.

MAV Values as a Methodology

As we have seen, mortality probability makes IDB an asset class with unique obstacles to overcome in order to understand how this class compares with, and is integrated with, other asset classes in a portfolio. A proposed solution is to use a Monte Carlo simulation incorporating the MAV[6] concept. The essence of this concept is that any death benefit strategy can be probabilistically valued at any point in the future as one of three values:

1) If the insured has not died, the cash value of the strategy, if any;

2) If the insured dies at the future point, the death benefit of the policy at the future point; or,

3) If the insured has died previous to the future point, the value of the previously-received death benefit, reinvested from the date of death to the future point at an assumed probable distribution of returns.

Thus,

$$MAV_{Value(n)} = \begin{cases} CSV_{(n)} & if\ A \\ DB_{(n)} & if\ B \\ DB_{pod}\left(\prod_{i=pod+1}^{n}\left(1 + r_{reinvestment(i)}\right)\right) & otherwise \end{cases}$$

Where:

A = insured is alive at the end of period(n)
B = insured dies in period(n), and assuming death at the end of the period
$CSV_{(n)}$ = net after-tax cash surrender value at end of period(n)
$DB_{(n)}$ = net after-tax death benefit at end of period(n)
DB_{pod} = net after-tax death benefit at the end of the period in which the insured dies
$r_{reinvestment(i)}$ = rate of return on reinvested death benefit in period(i)

(1)

A Monte Carlo simulation, incorporating the annual probability of death by use of relevant mortality tables will determine the death status (alive, dead, or dying in period), and thereby produce a MAV Value for each year of each iteration. The results can be summarized with usual statistical descriptions including the mean and standard deviation of the MAV Value for each year. In effect, the MAV Value becomes a probabilistic determination of the cash position of the strategy in any given year.

The question of which year to use for the comparison is the issue of "target year." In the case where the cash position of the strategy is relevant regardless of whether the insured is alive or dead (e.g., the need for buy/sell funding of a business partner's purchase of a partner's interest in a given year), then the given year is the appropriate target year for the MAV Value comparison. On the other hand, if one is comparing the at-death values of competing strategies, then the target year is the year of death, the MAV Value in which will vary from iteration to iteration based on mortality probability and the fluctuating value of the death benefit. In other words, with two or more insurance policies that have differing death benefits over time, the question of which policy will deliver greater value at death will often turn on the age at death. This problem is illustrated by Exhibit 3 in which the hypothetical death benefit of an increasing death benefit policy is compared with a level death benefit policy by plotting the death benefits over time against the probability density function of mortality. As can be seen, while the level-death benefit policy has the edge in death benefit volume in the early years, the increasing-death benefit policy in this example appears to have greater death benefit volume in those years that are more likely to be the year of death. A MAV Value calculation, using year of death as the target year, would reveal the correct answer probabilistically as it represents the mortality probability-adjusted value in the year of death.

In evaluating the MAV Values of a portfolio, it may be important for an advisor or owner to review every year over a long period because the issue is what is the likely cash position, year by year, in light of the financial demands on the portfolio, including cash flow requirements that may vary from time to time. For the purposes of the example, we have limited the data presentation to 30 years on the grounds that it is a relatively long period for the 70-year old in the example, and most financial projections tend to lose predictive precision in the minds of advisors and clients over longer time periods.

MAV Indexes

The utility of comparing MAV Values in the comparison of competing insurance strategies is incomplete because the volume of the death benefit for the subject policies will affect the results. This is particularly a problem if the year of death is the target year. Obviously a policy with a level death benefit of $10M dollars is going to have a greater MAV Value in the year of death than a policy with a level death benefit of $1M. But what if the policy with ten times the death benefit has only nine times the premium? Or, what if a policy with a lump sum premium payment is compared to a policy with an equal death benefit but level annual premiums? The answer again, will often turn on when the insured dies, the timing of which we cannot know for certain, but the probabilities of which are well understood.

These questions illustrate that often the issue in a comparison is not just the probable cash position of the strategies but the economic efficiency of the strategy, defined as the mortality probability-adjusted leverage of costs to benefits in the strategy. In order to provide a normalized factor for these purposes, an index can be calculated by adding the present values of the cash flows *out* of the policy during the continuance of the strategy (withdrawals, loans, etc.) to the present value of the MAV Value in the target year, and then dividing that result by the sum of the present values of cash flows *into* the strategy. The use of present values controls for the fact that cash flows in and out can take place at different points over a long period, while the use of the MAV Value incorporates the mortality probability that operates during the same time frame. Positive and negative cash flows are separated rather than netted to avoid the problems of calculation and interpretation if withdrawals exceed cash into the strategy resulting in no cash flows into the strategy, only cash flows out. In other words, the calculation of the MAV Index requires at least one cash flow *into* the strategy.

Thus,

$$MAV_{Index(n)} = \frac{MAV_{Value(n)}(1 - r_{discount})^{n} + \sum_{i=1}^{n} PCF_{(i)}\,(1 - r_{discount})^{i-1}}{\sum_{i=1}^{n} NCF_{(i)}\,(1 - r_{discount})^{i-1}}$$

Where:

$r_{discount}$ = present value discount rate
$PCF_{(i)}$ = positive cash flow received from the strategy at the beginning of period(i), other than the MAV Value
$NCF_{(i)}$ = negative cash flow paid into the strategy at the beginning of period(i)

(2)

However, in the context of portfolio design, the mean performance of the MAV Index is not a sufficient measure of optimization because the mean return must be penalized for volatility in order to account for risk. A risk-adjusted version of the index, a VMAV Index is calculated simply by dividing the mean MAV Index for the target year by the standard deviation of the MAV Index for the same target year.

Thus,

$$VMAV_{Index(n)} = \frac{\mu MAV_{Index(n)}}{\sigma MAV_{Index(n)}} \tag{3}$$

The VMAV Index therefore provides a quick means to compare the mortality and risk-adjusted performance of alternative death-triggered insurance policies or strategies, regardless of whether death has yet occurred[7].

Side Funds

In the formulation of MAV Values as described above, we can incorporate a side fund investment, with its own return distribution, affecting the calculation of both the cash value and the death benefit at a future point, so as to be able to model the comparison and integration of alternative investments with the insurance element of the strategy. A portfolio with non-insurance asset classes can be seen as a side fund, setting up a comparison of the side fund itself to a side fund coupled with an IDB funded with allocations from the side fund. Thus, by looking at the MAV Values for the two portfolios in the comparison, we can see year by year, and for any given target year, the mortality-adjusted, probable value. This method provides a platform within the context of mean/variance analysis for examining all of the factors that might affect portfolio design when considering the incorporation of IDB as a potential asset allocation.

Example

It must be stressed that any example of the MAV methodology does not indicate anything conclusive about particular insurance products or categories of products because the number and range of variables is so great, including but not limited to the subjective constraints such as the cash flow capacity of a portfolio or the risk tolerance of the portfolio's owners.

For purposes of the example, we have used the following assumptions and factors in the model:

Beginning Portfolio Value: $50,000,000

Portfolio Returns
Annual Probability Distribution:Normal
Full Historical[8]
Mean = .09048
Std Dev = .13706
25% Periodic Return Reduction[9]
Mean = .05757
Std Dev = .12667
25% Periodic Return Premium[10]
Mean = .12338
Std Dev = .15058
Rebalancing: Annual

Tax Rates
Ordinary Income: .40
Capital Gain: .20
Estate Tax: .40

Present Value Discount Rate: .03

Mortality Table:
2008 VBT Male UCS 46 RR70 Nonsmoker ANB
Discount = 30%

Insured: Male
70 year old
Non smoker
Preferred Underwriting Category
Life exp. per table and discount= 20.55 years

IDB Guaranteed Universal Life
Death Benefit Guaranteed for Life
Death Benefit = $25,000,000
Level Annual Premium = 681,235
Premium Funding from All Classes Ratably

The model was simulated in six instances:

1) Full historical return probabilities/IDB and securities subject to estate taxation

2) Full historical return probabilities/Securities but not IDB subject to estate taxation

3) 25% Periodic Return Reductions/ IDB and securities subject to estate taxation

4) 25% Periodic Return Reductions/ Securities but not IDB subject to estate taxation

5) 25% Periodic Return Premium/ IDB and securities subject to estate taxation

6) 25% Periodic Return Premium/ Securities but not IDB subject to estate taxation

The goal is not only to illustrate a MAV analysis, but to isolate the relative effects of the potential estate tax differences between IDB and securities, and the benefit of IDB in markets with lower (or higher)-than-expected returns.

Exhibit 4 lays out the simulation results for instance (1) - full historical return probabilities/IDB and securities subject to estate taxation. In both the portfolio with or without an IDB allocation, the mean MAV Value for each year is divided by its standard deviation in order to provide a risk-adjusted factor. The risk-adjusted factor for the portfolio with an IDB allocation is then divided by the risk-adjusted factor for the non-IDB portfolio in order to determine the risk-adjusted advantage, if any, of the IDB portfolio. To the extent the factor is greater than 1.0000, the portfolio with an IDB allocation has a better risk-adjusted MAV Value for that year. Conversely, if the factor is less than 1.0000, the portfolio without an IDB allocation has a better risk-adjusted MAV Value for the year.

The fluctuation of the risk-adjusted factor from year to year illustrates the interplay of mortality probability and return sequence risk with increasing cumulative premiums and changing present value death benefits. Given the nature of MAV calculations, this model could accommodate the integration of such variables with any pattern of premiums, cash values, and death benefits over time.

In this particular instance, the IDB portfolio starts with an advantage, which declines over several years and then is recovered, declining again as it approaches the end of the 30-year period. We interpret this to mean that given the assumptions in this instance, the volatility of the core portfolio has a greater impact then the cumulative premium in the early years, but the cumulative premium increases in effect until the probability of death increases sufficiently to overcome the disadvantage (i.e., the probability that the IDB has been received and factored into the MAV has increased sufficiently). Eventually, the rate of the return on the policy declines enough that the possibility of survival to later years with such lower returns precipitates a slight decline in the advantage.

Note that the mean return of the IDB portfolio never eclipses the mean return of the non-IDB portfolio during the illustrated duration of 30 years, which means that any advantage is derived from the lower volatility of the portfolio with an IDB allocation.[11]

Exhibit 5 compares the results of the six instances based on different return and tax characteristics. From this comparison, at least for the assumptions in this case, we may derive that:

1) The avoidance of estate tax on the IDB, if achievable, can create a significant advantage for the portfolio with an IDB allocation.

2) Because the return performance of the IDB has no connection to market fluctuations, an IDB allocation can provide significant downside-risk protection in the case of disappointing returns in the core portfolio.

3) The lower volatility of IDB returns to a given future point can provide significantly superior risk-adjusted returns.

Optimization

The following assumptions, based upon the above-described case, were used to illustrate optimizing[12] with this methodology:

No periodic return modification from historical values
IDB is not taxable
CFC constraint = $1,500,000 present value
Optimized for IDB risk-adjusted factor in year 30

The results are summarized in Exhibit 6, and indicate that with these assumptions there is a level of IDB that will provide better risk-adjusted portfolio values. As expected, at least in this case, the CFC constraint changes the results.

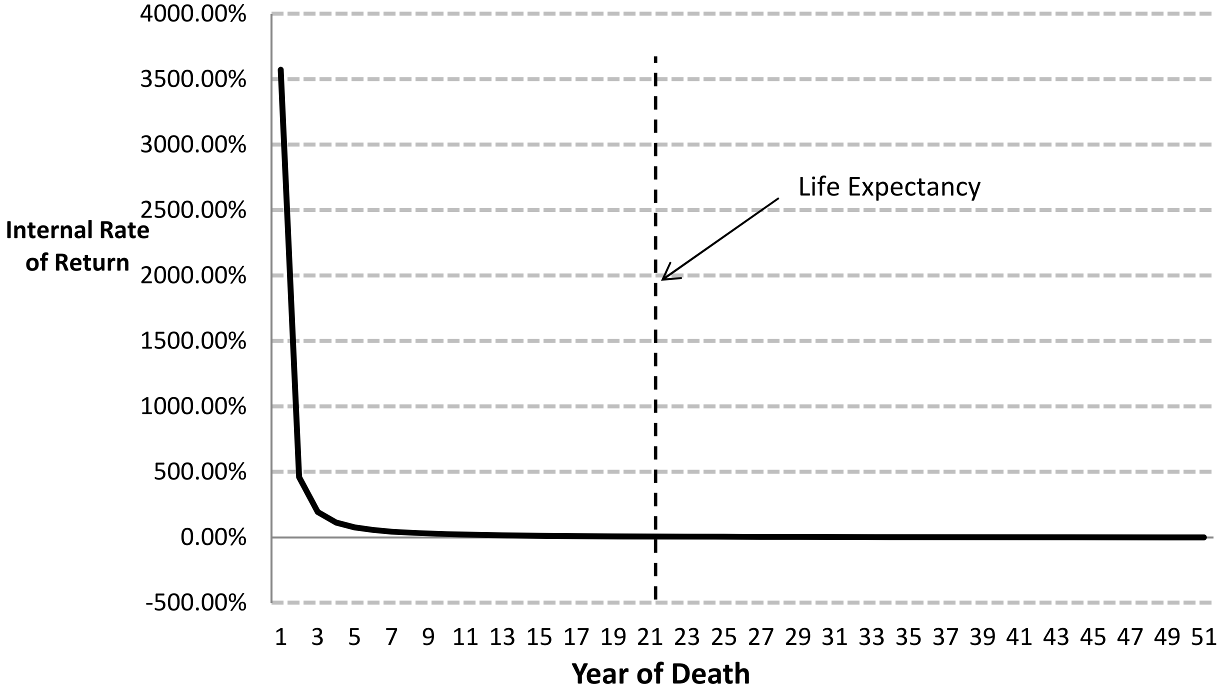
Conclusion

The use of MAV Value calculations provides a methodology for evaluating the use of IDB allocations that maps the traditional mean/variance analysis onto the special characteristics of IDB. It is clear that it is possible for a portfolio allocation to IDB to provide superior risk-adjusted outcomes. However, any use of IDB should always be seen in the context of its utility as an insurance product as well. For those instances where there is need for the early death-risk protection, the impact of the IDB on the risk-adjusted outcomes of the portfolio, while not the primary motivation for the insurance allocation, may provide an additional motivation for the purchase decision, or at least indicate that the purchase decision is not otherwise financially inadvisable. The use of the CFC constraint will not only guard against a portfolio that is "insurance poor," but is an important exercise by itself for any portfolio advisor, and should result in focusing the owners of the portfolio on probable cash flow as a way to understand and evaluate the risks of their portfolio design.

Clearly, there are many elements of this methodology that would benefit from further research. The type of IDB product used, the use of cash value accumulations in IDB products, different funding patterns, different underwriting characteristics, the use of multiple insureds, and the use of different measures of optimization are only a beginning list of the fertile areas for review of the impact of portfolio allocations to IDB in order to optimize risk-adjusted outcomes.

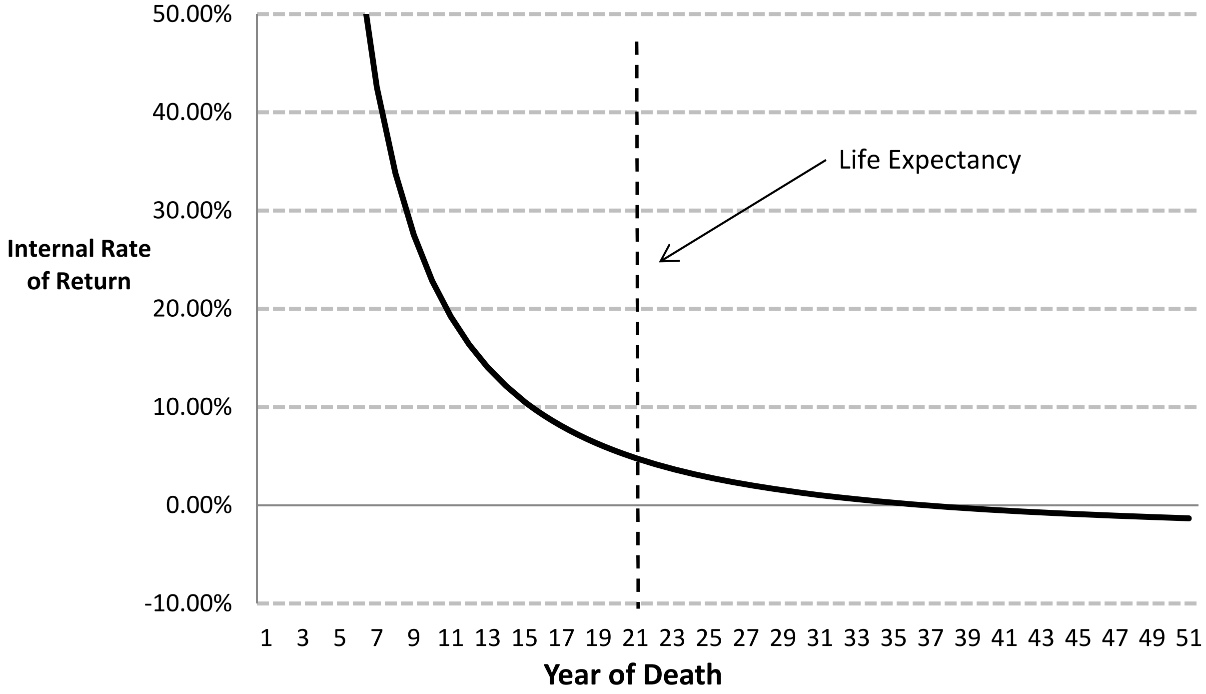
Since insurance is designed to mitigate the risk of rare catastrophes and disasters, further investigation of the role of insurance products in optimal portfolios beyond the human capital hedging may also contribute to our understanding of the common financial puzzles, such as the excess volatility puzzle and high price of deep out-of-the-money puts.

Exhibit 1



Internal Rate of Return to Net Death Benefit in Year of Death [Full Scale]

Exhibit 2



Internal Rate of Return to Net Death Benefit in Year of Death [Reduced Scale]

Exhibit 3

Level of Net Death Benefit vs Mortality Probability

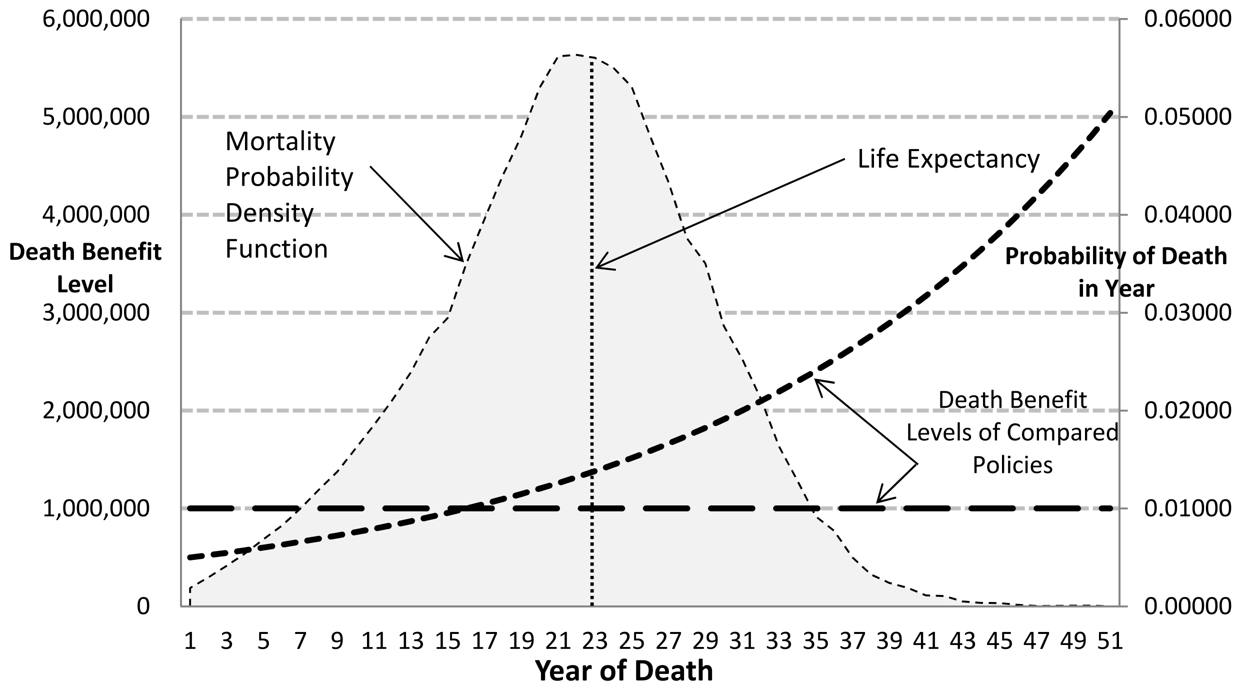


Exhibit 4

MAV Results – 0% Periodic Return Modification – Death Benefit Taxable

Beginning Portfolio Value:	50,000,000
□ Portfolio Return:	.096152
□ Portfolio Return:	.134620
Death Benefit:	25,000,000/Taxable
Annual Premium:	681,235

MAV Values

	[------------------Portfolio w/IDB---------]			[---------------Portfolio wo/IDB---------]			Risk Adjusted Advantage of Portfolio w/IDB
Year	μ	σ	μ/σ	μ	σ	μ/σ	
1	53,767,735	6,762,251	7.9512	54,481,529	6,911,806	7.8824	1.0087
2	57,862,473	10,357,111	5.5867	59,336,368	10,685,210	5.5531	1.0061
3	62,341,474	13,891,200	4.4878	64,623,748	14,463,204	4.4681	1.0044
4	67,215,213	17,598,446	3.8194	70,351,841	18,442,007	3.8148	1.0012
5	72,475,653	21,383,209	3.3894	76,509,585	22,552,076	3.3926	0.9991
6	78,220,003	25,632,531	3.0516	83,204,672	27,204,928	3.0584	0.9978
7	84,328,947	29,817,932	2.8281	90,309,421	31,855,719	2.8350	0.9976
8	90,932,809	34,389,107	2.6442	97,949,159	36,936,698	2.6518	0.9971
9	98,129,750	39,806,831	2.4651	106,214,160	42,936,036	2.4738	0.9965
10	105,928,305	45,601,456	2.3229	115,131,410	49,474,558	2.3271	0.9982
11	114,260,853	52,137,564	2.1915	124,600,284	56,780,603	2.1944	0.9987
12	123,259,574	59,387,866	2.0755	134,762,316	64,968,758	2.0743	1.0006
13	132,809,215	67,142,549	1.9780	145,484,441	73,781,679	1.9718	1.0031
14	142,903,191	75,718,264	1.8873	156,715,114	83,439,141	1.8782	1.0048
15	153,793,642	85,047,307	1.8083	168,748,363	93,977,806	1.7956	1.0071
16	165,162,953	94,662,382	1.7448	181,177,539	104,841,726	1.7281	1.0096
17	177,330,267	106,189,926	1.6699	194,358,608	117,893,462	1.6486	1.0129
18	190,215,869	118,861,403	1.6003	208,214,358	132,244,276	1.5745	1.0164
19	203,269,819	130,693,229	1.5553	221,993,872	145,585,836	1.5248	1.0200
20	217,032,745	144,763,294	1.4992	236,322,861	161,336,455	1.4648	1.0235
21	230,753,873	158,362,475	1.4571	250,377,920	176,365,792	1.4197	1.0264
22	245,764,067	173,345,800	1.4178	265,640,119	192,882,794	1.3772	1.0294
23	261,961,592	191,014,126	1.3714	281,979,738	211,796,839	1.3314	1.0301
24	277,821,966	205,359,346	1.3529	297,699,231	226,997,722	1.3115	1.0316
25	295,698,908	222,269,088	1.3304	315,399,514	244,222,913	1.2914	1.0301
26	315,732,178	243,290,514	1.2978	335,481,094	266,097,843	1.2607	1.0294
27	336,848,386	261,189,360	1.2897	356,741,850	284,876,421	1.2523	1.0299
28	360,309,827	283,671,264	1.2702	380,165,286	307,680,605	1.2356	1.0280
29	386,104,739	310,772,044	1.2424	405,899,773	335,103,138	1.2113	1.0257
30	415,428,906	341,981,042	1.2148	435,577,922	366,733,934	1.1877	1.0228

Exhibit 5

Risk-Adjusted MAV Value Advantage -
Risk-Adjusted Factor with IDB / Risk-Adjusted Factor without IDB

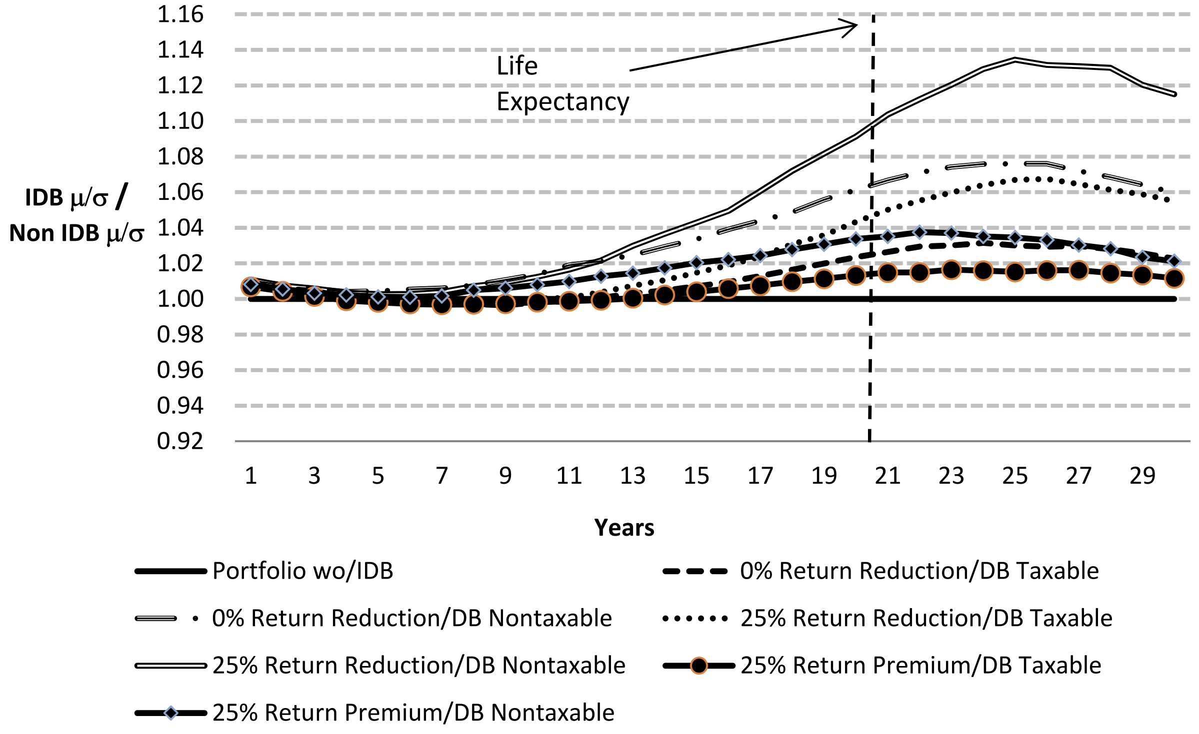


Exhibit 6 **Optimization Results – 0% Periodic Return Modification**
– Death Benefit Taxable
– CFC Annual Constraint = $1,500,000 (PV)
– Optimized for IDB risk-adjusted advantage in yr 30

	Without CFC Constraint	CFC Constraint 1,500,000
Death Benefit	39,406,906	11,860,708
Annual Premium	1,073,815	323,197
Mean Portfolio Value (30 year)		
with IDB	1,016,593,071	727,835,186
without IDB	1,052,816,178	735,753,819
Standard Deviation		
with IDB	881,175,319	669,395,803
without IDB	941,191,497	680,283,404
Risk-Adjusted Factor (30 year)		
with IDB	1.1537	1.0873
without IDB	1.1186	1.0815
IDB Advantage	1.0314	1.0053

ENDNOTES

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[1] Obviously, a single premium, or fixed period of payments, would alter the results.

[2] Reference is made to the income, estate, and gift tax laws of the United States (Title 26, U.S. Code "IRC"). For any given insured, owner, or beneficiary, such laws may or may not apply depending on such factors as citizenship and residency. For the same reasons, other countries' tax laws may alternatively apply.

[3] cf. IRC §101

[4] cf. IRC §2042

[5] Consideration of this issue is different if the policy has significant cash value accumulations that the owner may otherwise be able to access during life.

[6] MAV^{sm} values, indexes, and processes are protected by US Patent No. 8,175,900. The MAV^{sm} calculations for this article were produced by Financial Logic LLC, under an exclusive license for the use of the process.

[7] While in this formulation a VMAV Index would not be calculable if the standard deviation of the MAV Index were zero, such a case would be exceptional in the real world of IDB products and strategies.

[8] Based on a composite portfolio of 60% equities/40% bonds as measured by the annualized composite monthly returns of the S&P 500 and Merrill Lynch Corporate Master Bond Index from January, 1990, through December, 2009

[9] Based on reducing each composite monthly return of the portfolio from January, 1990, through December, 2009, by 25%

[10] Based on increasing each composite monthly return of the portfolio from January, 1990, through December, 2009, by 25%

[11] In the case of 25% reduced periodic returns, the portfolio with an IDB in fact has a 6.9% greater mean return, reflecting the IDB's lack of sensitivity to market fluctuations and protection on the downside.

[12] Optimization of the models was based on a combination solver using primarily a genetic algorithm in conjunction with a Monte Carlo simulation in each iteration

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REFERENCES

Chen, Peng, Ibbotson, Roger G., Milevsky, Moshe A. and Zhu, Kevin X., 2006. "Human Capital, Asset Allocation, and Life Insurance," *Financial Analysts Journal*, Vol. 62, No. 1, January/February: 97-109.

Hald, Anders, "History of Probability and Statistics and Their Applications before 1750," Hoboken, NJ, John Wiley and Sons, Inc., 2003.

Hubener, Andreas, Raimond Maurer, and Ralph Rogalla, 2013. "Optimal Portfolio Choice with Annuities and Life Insurance for Retired Couples**,"** Review of Finance , forthcoming.

Jagannathan, Ravi, and N.R. Kocherlakota. 1996. "Why Should Older People Invest Less in Stocks Than Younger People?" *Federal Reserve Bank of Minneapolis Quarterly Review*, vol. 20, no. 3, (Summer):11–23.

Kwak, Minsuk.; Yong Hyun Shin, and U Jin Choi, "Optimal investment and consumption decision of a family with life insurance," *Insurance: Mathematics and Economics, Volume 48, Issue 2, March 2011, Pages 176-188.*

Nielsen, Peter Holm & Steffensen, Mogens, 2008. "Optimal investment and life insurance strategies under minimum and maximum constraints," *Insurance: Mathematics and Economics*, vol. 43(1), August: 15-28.

Merton, Robert C., 1969. "Lifetime Portfolio Selection under Uncertainty: The Continuous-Time Case." *The Review of Economics and Statistics,* 51, August: 247-257.

Merton, Robert C., 1971. "Optimum Consumption and Portfolio Rules in a Continuous-Time Model. *Journal of Economic Theory* 3, December: 373-413.

Merton, Robert C. 2003. "Thoughts on the Future: Theory and Practice in Investment Management," *Financial Analysts Journal,* Vol. 59, no. 1 (January/February): 17-23.

Pirvu, Traian A., Huayue Zhang, 2012. "Optimal investment, consumption and life insurance under mean-reverting returns: The complete market solution," *Insurance: Mathematics and Economics,* Volume 51, Issue 2, September: 303-309.

Pliska, Stanley.R., Ye, Jinchun, 2007. "Optimal life insurance purchase and consumption/investment under uncertain lifetime," *Journal of Banking and Finance*, 31, 1307–1319.

Samuelson, Paul. 1969. "Lifetime Portfolio Selection by Dynamic Stochastic Programming." *Review of Economics and Statistics,* vol. 51, no. 3 (August):239–246.

Smith, Michael L., and Stephen A. Buser. 1983. "Life Insurance in a Portfolio Context." *Insurance: Mathematics and Economics*, vol. 2, no. 3 (July):147–157.

Yaari, M.E. 1965. “Uncertain Lifetime, Life Insurance, and the Theory of the Consumer.” *Review of Economic Studies*, vol. 32, no. 2 (April):137–150.