

Catastrophe Risk Transfer

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Abstract

Reinsurance and securitization are two risk transfer mechanisms used by insurers, especially in the catastrophe insurance market, but reinsurance significantly dominates securitization. We develop a signaling model to analyze the tradeoff between reinsurance and securitization for the transfer of insurance risks. Using the model, we provide a novel explanation for why traditional reinsurance still dominates the market for catastrophe risk transfer. When an insurer has private information about its portfolio, its choice between reinsurance and securitization serves as a signal of the risk of its portfolio. The insurer's choice reflects the tradeoff between the lower adverse selection costs of reinsurance due to the superior monitoring resources of reinsurers against the costs arising from reinsurers' market power. We show that Perfect Bayesian Equilibria of the signaling game have a partition form where the lowest risk insurers choose reinsurance, intermediate risk insurers choose partial securitization, and high risk insurers choose full securitization. An increase in the size of potential losses increases the trigger level of risk above which insurers choose securitization. Consequently, catastrophe risks, which are broadly characterized by losses of large magnitudes, are only securitized by very high risk insurers, thereby leading to the relative predominance of capital retention and reinsurance in catastrophe risk transfer. Our results also explain why catastrophe bonds have high premia and their credit ratings are usually below investment grade.

1 Introduction

Insurers with limited capital to completely cover the risks in their portfolios often exploit external risk transfer mechanisms such as reinsurance and securitization. While these risk-sharing mechanisms are used for all types of insurable risks, they are especially important in the case of catastrophe risks because of the large magnitudes of the potential losses involved. A strand of literature argues that securitization has a significant advantage over reinsurance because of the substantially higher available capital and risk-bearing capacity of capital markets (Durbin 2001). Nevertheless, an enduring puzzle is that reinsurance is still the dominant risk transfer mechanism for catastrophe risks. By the end of 2011, the outstanding risk capital of asset-backed-security catastrophe bonds amounted to \$12 billion, while the reinsurance capacity was \$470 billion. It is often argued that catastrophe bonds are too expensive suggesting that they are somehow “mispriced” relative to their payoffs. Further, catastrophe bonds typically receive “below investment grade” ratings even though catastrophe risks have low correlations with market risks (see Cummins (2008, 2012) for surveys).

We provide a novel explanation for the above stylized facts using a signaling model to analyze an insurer’s choice between reinsurance and securitization. When an insurer with private information about its portfolio faces a choice between reinsurance and securitization, its choice represents a signal of the nature of risks in its portfolio. The insurer’s choice trades off the lower adverse selection or information costs associated with reinsurance (because of the superior monitoring abilities of reinsurers) against the higher costs arising from reinsurers’ market power relative to competitive capital markets (Froot (2001)). We show that Perfect Bayesian Equilibria (PBE) of the signaling game have a partition form where an insurer chooses self-insurance if its risk is below a low threshold, reinsurance if its risk lies in an intermediate interval, and securitization if its risk is above a high threshold. The threshold risk level above which the insurer chooses securitization increases with the magnitude of potential losses in its portfolio. Given that catastrophe risks are usually characterized by “low frequency–high severity” losses, our results imply that an insurer is more likely to choose reinsurance to transfer catastrophe risk. Further, because an insurer only opts for securitization if its risk of potential losses is high, catastrophe bonds have high premia (relative to the ex ante expected losses) and have ratings below investment grade (Cummins (2008, 2012)). Importantly, our results suggest that the high costs and low credit ratings of catastrophe

securities are not due to any mispricing, but the rational incorporation of their inherent risks by capital markets based on the information they glean from insurers' risk transfer choices.

In our signaling model, a representative insurer with a limited amount of capital holds a portfolio of insurance risks. The insurer's probability of incurring losses belongs to an interval. The insurer incurs significant bankruptcy costs if it is unable to meet its liabilities, which provides incentives for it to transfer its risks. The insurer can choose to retaining its risks (that is, retention or self-insurance) or transfer them either partially or wholly through reinsurance or securitization. The insurer has private information about its risks so that there is adverse selection regarding its "type." Reinsurers have a significant information advantage over capital markets because they possess the resources to more effectively monitor insurers. For simplicity, we assume that reinsurers know an insurer's risk type and, therefore, do not face any adverse selection.¹ Consistent with Froot (2001), reinsurers have significant market power vis-a-vis insurers that allows them to extract additional rents relative to competitive capital markets that are represented by a markup over the actuarially fair premium. (The additional rents to reinsurers also compensate them for their monitoring costs.) The insurer's choice among retention, reinsurance and securitization reflects the tradeoff between the lower adverse selection costs associated with reinsurance and the rents extracted by reinsurers due to their market power.

To examine the robustness of our implications, we analyze two versions of our framework. In the first version, the insurer incurs *fixed* bankruptcy costs if it is unable to meet its liabilities. In the second version, it incurs *proportional* bankruptcy costs that are proportional to the magnitude of its losses. In both versions, the insurer's "risk" is determined by its probability of incurring a loss that exceeds its capital level so that it is unable to meet its liabilities.

In the model with fixed bankruptcy costs, we show that perfect Bayesian equilibria (PBE) of the signaling game have a "partition form" that is characterized by two thresholds. The insurer chooses retention if its risk is below the low threshold, reinsurance if its risk lies in the intermediate interval between the thresholds, and securitization if its risk is above the high threshold. The intuition for the equilibria is as follows. With fixed bankruptcy costs, the costs the insurer incurs are independent of the magnitude of its shortfall in meeting its liabilities. Consequently, it is

¹Our results are robust to allowing for adverse selection in reinsurance as long as its degree is less than that in securitization.

never optimal for the insurer to partially retain its risks, that is, it either chooses to retain all its risks or completely transfer them. If the insurer's risk is below a threshold, it prefers retention to avoid ceding rents to reinsurers due to their market power and the adverse selection costs associated with securitization. If its risk lies in an intermediate interval, it prefers reinsurance because securitization entails substantial adverse selection or "cross-subsidization" costs arising from asymmetric information about the insurer's risk type. If the insurer's risk is above a high threshold, it chooses securitization because the costs due to reinsurers' market power outweigh adverse selection costs. For intermediate risks, therefore, adverse selection costs dominate costs due to reinsurers' market power, while the reverse is true for high risks.

A shift in the potential loss distributions of insurers in the sense of "first order stochastic dominance" (FOSD) (that is, insurers' loss distributions change such that larger losses become more likely) increases the trigger risk level above which an insurer chooses securitization. Consequently, the likelihood of choosing securitization decreases. In the context of catastrophe risk, which is characterized by large magnitudes of potential losses, our results imply that an insurer chooses securitization if and only if its risk of potential losses is high, that is, reinsurance is more likely to be chosen as a risk transfer mechanism. Further, the prediction that only very high-risk insurers choose securitization is consistent with evidence that catastrophe bonds have high premia relative to their expected losses, and catastrophe-linked securities are usually issued with ratings below investment grade (Cummins (2008, 2012)).

In the model with proportional bankruptcy costs, an insurer's bankruptcy costs vary with the magnitude of its shortfall in meeting its liabilities. Consequently, it is always optimal for the insurer to transfer at least some portion of its risk either through reinsurance or securitization by choosing a retention level. We show that PBE of the model again have a partition structure, which depends on the level of reinsurers' market power. If reinsurers' market power is below a threshold, then the lowest risk insurers choose full reinsurance, the intermediate risk insurers choose separating securitization contracts with retention levels that decrease with their risk, while the highest risk insurers choose full pooling securitization. If reinsurers' market power is above the threshold, however, the equilibria are characterized by two intervals where the lower risk insurers choose separating partial securitization contracts, while the high risk insurers choose full securitization. When reinsurers' market power is sufficiently low, the costs of reinsurance are lower than the

signaling costs associated with (partial or full) securitization. To avoid the costs associated with reinsurers' market power, and the costs of subsidizing high-risk insurers, intermediate risk insurers signal their types by choosing separating securitization contracts that are characterized by retention levels that decline with their risk. For high-risk insurers, the costs of signaling are too high so that they choose to pool by offering full securitization contracts. When reinsurers' market power is high, however, the lowest risk insurers too prefer separating partial securitization contracts rather than reinsurance.

As in the model with fixed bankruptcy costs, an FOSD shift in the loss distributions of insurers increases the trigger risk level above which an insurer chooses securitization. Consequently, the implications regarding the transfer of catastrophe risks are qualitatively unchanged across the two versions of the model.

Our study relates to two branches of the literature that investigate insurers' choice between reinsurance and securitization, especially in the context of catastrophe risk transfer. The first branch examines the factors that affect the *demand* for insurance-linked securities. There are several reasons leading to the limited demand for insurance-linked securities. Bantwal and Kunreuther (2000) show that ambiguity aversion, loss aversion, myopic loss aversion, and fixed costs of education can account for the reluctance of institutional investors to enter the insurance catastrophe bonds market. Barrieu and Louberge (2009) demonstrate that one possible reason for the disappointing development of catastrophe bonds is the aversion to downside risk among investors and parameter uncertainty. In particular, investors might show aversion to the ambiguity regarding the dependence between the occurrence of a natural catastrophe and that of a market crash, although losses from catastrophes have been historically uncorrelated with financial market returns. They argue that there might be an increase in the demand for insurance-linked securities if insurers issue hybrid catastrophe bonds.

The second branch of the literature examines the factors that affect the *supply* of insurance-linked securities. Cummins and Trainor (2009) argue that the benefits of securitization relative to reinsurance increase when the magnitude of potential losses and the correlation of risks increase. Gibson *et al.* (2011) analyze the tradeoff between the costs and benefits of loss information aggregation procedures to determine the prevalent risk transfer form. They assume insurers not only take advantage of the relatively lower capital costs, but can also gather the loss information produced

either from reinsurers directly or through the price of the securities provided by investors in financial markets. When the loss volatility is higher, the information production costs of reinsurance are relatively lower. Better information, however, might allow insurers to allocate their capital more effectively and reduce the cost of capital. Thus reinsurance dominates when the loss volatility is higher. Hagendorff and Keasey (2012)'s empirical study shows support for the above predictions. Lakdawalla and Zanjani (2012) argue that catastrophe bonds can improve the welfare of insureds when reinsurers face contracting constraints on the distribution of assets in bankruptcy, and when they must insure a heterogeneous group of risks. Finken and Laux (2009) argue that, given low basis risk, catastrophe bonds with parametric triggers are insensitive to adverse selection. Their existence leads to less cross-subsidization among reinsurance contracts because catastrophe bonds can serve as an alternative risk transfer mechanism that is more attractive to low risk insurers who suffer from reinsurance contracts due to adverse selection.

We complement the above literature by developing a simple signaling model to analyze insurers' choice between reinsurance and securitization. An insurer's choice reflects the tradeoff between the lower adverse selection costs associated with reinsurance and the lower market power of investors in competitive capital markets. Our framework provides a novel explanation based on signaling considerations for the dominance of retention and reinsurance in the market for catastrophe risk transfer.

Although we focus on insurance risks for concreteness, our framework and results can be more broadly applied to analyze the sharing of all insurance risks (not just catastrophe risks), and the transfer of other types of risk such as credit risk by financial and non-financial firms. The model could also be potentially useful in the study of firms' choices between alternate modes of financing such as private versus public financing, and "informed" versus "arms length" financing. Rajan(1992) argues that a firm's choice between bank debt and arms-length debt is driven by the tradeoff between the benefits and costs arising from the informational advantage of the bank with respect to arms-length debt holders. Chemmanur and Fulghieri (1994) show that the dominance of bank loans for firms that face high likelihoods of financial distress arises due to banks' reputation for making the right "renegotiation" versus liquidation choices for financially distressed firms. Bolton and Freixas (2000) derive a model of financial markets and corporate finance with asymmetric information, where equity issues with dilution costs, bank debt with intermediation costs, and

bond financing with liquidation costs coexist in equilibrium.

Our paper also fits into the literature on the analysis of information revelation through the risk sharing arrangement. Leland and Pyle (1977) show that low risk transferers (or entrepreneurs in need of financing a risky project) would like to signal the information about their risk type through self-financing as long as the benefits from revealing their information exceed the signalling cost. Doherty and Thistle (1996) discuss the information gathering process in insurance markets by allowing for the absence of information about risk types even for some policy holders themselves. Schlee (2001) explores the value of information in efficient risk sharing contracts. He argues that better public information makes agents worse off under some conditions such as no aggregate risk, existence of some risk neutral agents (such as competitive investors), and existence of a representative agent. Our paper complements this area by comparing different information generation channels by different types of risk transferrers, i.e., primary insurers in our model. We examine two channels through which information is revealed: one is through costly monitoring performed by counterparties with market power; and the other one is through signaling to competitive counterparties with no market power. At a broad level, our results imply that information about low risk types is monitored by the risk bearer, information about intermediate risk types is signaled by the risk transferrers, and no information about high risk types is revealed in equilibrium.

2 Simple Model

An insurer with a limited amount of capital has a risky portfolio of insurable risks. The insurer is faced with the choice between retaining the risk (that is, self-insuring) or transferring the risk through reinsurance or securitization. We first develop our key implications and illustrate the intuition underlying them using a simple model with two insurer “types.” We generalize the model in subsequent sections.

2.1 Insurer

A risk-neutral insurer has a restricted amount of capital W . The insurer’s portfolio has two possible realizations. In the “good” state, which occurs with probability $1 - p$, the portfolio suffers no loss and the insurer earns the premium A . However in the “bad” state, which occurs with probability

p , the portfolio suffers a loss and the insurer has to make the net payment B (total indemnity net of the premium). We assume that

$$W - B < 0,$$

so that the insurer's capital is not enough to cover the net loss payment in the bad state. The insurer incurs an *additional* deadweight bankruptcy cost C in the bad state if it is unable to fully cover the loss. We can either take the bankruptcy cost to be non-pecuniary or assume that it arises from the loss of future business opportunities. We assume a fixed bankruptcy cost C in this section. In Section 4, we alter the model to consider variable bankruptcy costs that increase with the magnitude of the insurer's shortfall in meeting its liabilities.

Hoerger *et al.*(1990) show that the demand for reinsurance might be created by the existence of bankruptcy costs even if the insurer is risk neutral. If the magnitude of underwriting losses and the correlations of risks are large, the risk-warehousing function of insurers may collapse. The presence of bankruptcy costs could motivate the insurer to hedge its underwriting losses through reinsurance or securitization. Given its linear objective function, it is optimal for the insurer to choose either reinsurance or securitization for its entire portfolio provided it chooses to transfer its risk. We assume this in the following.

The insurer has *private information* about the probability p so that there is adverse selection regarding the type p of the insurer. The loss probability p takes the value \bar{p} (the insurer is "high risk" using the standard terminology of insurance markets) with probability v and the value \underline{p} (the insurer is "low risk") with probability $1 - v$ where $\bar{p} > \underline{p}$. Throughout, we adopt the standard terminology of insurance markets where the "risk" of an insurer refers to its probability of incurring losses so that a "high risk" insurer is more likely to incur losses than a "low risk" one.

2.2 Reinsurance

We first analyze the case where insurers only have access to reinsurance. Reinsurers have an information advantage over investors in capital markets due to their specialized expertise and ability to monitor insurers (e.g. Jean-Baptiste *et al.*(2000)). To simplify matters, and to focus attention on the information advantage of reinsurers relative to capital markets, we assume that reinsurers have the monitoring technology to know the risk type of the insurer perfectly so that

they do not face any adverse selection. (Our results are robust to allowing for adverse selection in reinsurance as long as its degree is less than that in securitization.) Monitoring is, however, costly and reinsurers have nontrivial market power vis-a-vis insurers.

For simplicity, we assume that reinsurers have sufficient capital to fully insure the insurance company so that they do not face default risk.² Reinsurers usually have better diversification opportunities that may lower their default risks (*e.g.* Jean-Baptiste *et al.*(2000)). The main objective of our study is to compare the trade-off between the information advantage of reinsurers against the lower costs of risk-sharing with capital markets. Consequently, we avoid further complicating the analysis and obfuscating the intuition for our results by also introducing default risk for reinsurers.

Consistent with the arguments of Froot (2001), reinsurers have nonzero market power vis-a-vis insurers that we model by assuming that they must be guaranteed a proportional markup over the actuarially fair insurance premium that is determined by the parameter $\delta > 0$. The parameter, δ , indexes reinsurers' market power with higher values of δ corresponding to scenarios in which reinsurers have greater market power and can extract higher rents. The markup also incorporates the costs of reinsurers' monitoring technology, that is, reinsurance contracts also compensate the reinsurers for their monitoring costs.

Because reinsurance companies know the insurer's type, they offer distinguishing contracts $(\overline{A}_r, \overline{B}_r)$ for the high-risk insurer and $(\underline{A}_r, \underline{B}_r)$ for the low-risk insurer, where \overline{A}_r and \underline{A}_r are the reinsurance premia, and \overline{B}_r and \underline{B}_r are the net payments to the insurer in the bad state. In the following analysis, assume that the bankruptcy cost C is high enough that both high-risk and low-risk insurers would like to fully reinsure their underwriting risk. We later specify the condition on C that ensures the optimality of full reinsurance.

The optimal reinsurance contract $(\overline{A}_r, \overline{B}_r)$ for the high-risk insurer solves

$$\max_{\overline{A}_r, \overline{B}_r} (W + A - \overline{A}_r)(1 - \bar{p}) + (W - B + \overline{B}_r)\bar{p}$$

²According to the Guy Carpenter report, the total losses of the global property/casualty sector in 2011 exceeded \$ 100 billion, but shareholder funds exceeded \$ 160 billion. Consequently, the reinsurance sector continued to function normally despite the heavy losses in 2011.

such that

$$\overline{A}_r(1 - \overline{p}) - \overline{B}_r\overline{p} \geq \delta\overline{A}_r \quad (1)$$

$$W + A - \overline{A}_r \geq 0 \quad (2)$$

$$W - B + \overline{B}_r \geq 0, \quad (3)$$

where (1) is the participation constraint of the reinsurer. By (1), the parameter δ determines the markup over the actuarially fair premium that corresponds to $\delta = 0$. Higher values of δ correspond to greater market power because the reinsurer's markup is higher. The limited liability constraints, (2) and (3), incorporate the simplifying assumption that the insurers' bankruptcy cost is high enough that they would like full reinsurance (we relax this assumption later in our more general analysis).

Similarly, the optimal reinsurance contract $(\underline{A}_r, \underline{B}_r)$ for the low-risk insurer solves

$$\max_{\underline{A}_r, \underline{B}_r} (W + A - \underline{A}_r)(1 - \underline{p}) + (W - B + \underline{B}_r)\underline{p}$$

s.t.

$$\underline{A}_r(1 - \underline{p}) - \underline{B}_r\underline{p} \geq \delta\underline{A}_r$$

$$W + A - \underline{A}_r \geq 0$$

$$W - B + \underline{B}_r \geq 0$$

Lemma 1 (Reinsurance Contract). *Suppose that*

$$C > \frac{(B - W)\delta}{1 - \overline{p} - \delta}. \quad (4)$$

Both types of insurers choose full reinsurance. The reinsurance contract for the high-risk insurer is $(\overline{A}_r^, \overline{B}_r^*)$ where*

$$\overline{A}_r^* = \frac{\tilde{B}\overline{p}}{1 - \overline{p} - \delta}, \quad \overline{B}_r^* = \tilde{B}$$

The reinsurance contract for the low-risk insurer is $(\underline{A}_r^*, \underline{B}_r^*)$ where

$$\underline{A}_r^* = \frac{\tilde{B}p}{1 - p - \delta}, \quad \underline{B}_r^* = \tilde{B}$$

where

$$\tilde{B} = B - W \tag{5}$$

Condition (4) ensures that the bankruptcy cost C is high enough so that both high-risk and low-risk insurers would like to fully reinsure their underwriting risk rather than retain it. High-risk insurers, who are more likely to face bankruptcy, pay a higher premium.³ As expected, for all insurer types, both the reinsurance premium and the net payment increase as the net loss payment goes up keeping loss probabilities unchanged. In other words, higher underwriting loss payments result in greater demand for reinsurance and higher premia. We analyze the scenario when the bankruptcy cost takes arbitrary values when we examine the general setting in which insurers can choose retention, reinsurance or securitization.

2.3 Securitization

We now examine the case where insurers only have access to capital markets. An insurer's cost of transferring its risks is potentially reduced by the fact that capital markets are competitive. On the flip side, however, capital markets are marred by adverse selection since they cannot obtain the information about an insurer's risk type *ex ante*, that is, before it issues securities. We model the securitization game as a signaling game whose timing is as follows. An insurer offers a contract, (A_s, B_s) , where A_s is the premium received by the investors if there is no loss, and B_s is the net payment made by investors if a loss occurs. In general, the two types of insurers may offer the same contract or distinct contracts. Investors update their prior beliefs based on the offered contract and then either accept or reject it. The following lemma characterizes the unique Perfect Bayesian Equilibrium (PBE) of the signaling game when the bankruptcy cost is above a threshold. As in our analysis of reinsurance, we allow the bankruptcy cost to take arbitrary values when we examine the general setting where insurers can choose retention, reinsurance or securitization.

³It is easy to show that $A_r(p) = \frac{\tilde{B}p}{1-p-\delta}$ is an increasing function of p since $\frac{\partial A_r(p)}{\partial p} = \frac{\tilde{B}(1-\delta)}{(1-p-\delta)^2} > 0$.

Lemma 2 (Securitization Contract). *Suppose that*

$$C > \frac{\tilde{B}\nu(\bar{p} - \underline{p})}{\underline{p} [\nu(1 - \bar{p}) + (1 - \nu)(1 - \underline{p})]}. \quad (6)$$

In the unique PBE of the signaling game, both types of insurers fully transfer their risk and offer the same contract, (A_s^, B_s^*) , where*

$$A_s^* = \frac{\tilde{B}(\nu\bar{p} + (1 - \nu)\underline{p})}{\nu(1 - \bar{p}) + (1 - \nu)(1 - \underline{p})}, \quad B_s^* = \tilde{B}$$

The securitization contracts described by Lemma 2 are the same for insurers of both types. The presence of the fixed bankruptcy cost C makes *pooling contracts* optimal, where low-risk insurers subsidize the high-risk insurers.⁴ Low risk insurers still prefer securitizing their risk to retaining it as long as the bankruptcy cost C satisfies condition (6). In Section 4, we modify the model to allow for variable bankruptcy costs. In this modification, securitization contracts can be *separating*, that is, insurers with different risks may choose different risk retention levels that reveal their type.

2.4 Reinsurance versus Securitization

We now consider the more general case where insurers have access to both reinsurance markets and capital markets. The insurer's choice of reinsurance or securitization serves as a signal of its type. We derive Perfect Bayesian Equilibria (PBE) of the signaling game to derive an insurer's choices and its contracts. We first focus on the interesting case where the bankruptcy cost

$$C > \max \left[\frac{\tilde{B}\delta}{1 - \underline{p} - \delta}, \frac{\tilde{B}\nu(\bar{p} - \underline{p})}{(\nu(1 - \bar{p}) + (1 - \nu)(1 - \underline{p}))\underline{p}} \right]$$

so that an insurer faces a nontrivial choice between reinsurance and securitization. We later extend our results to accommodate an arbitrary bankruptcy cost.

Figure 1 shows the simple game. The dominant strategy for high-risk insurers is full securitization. In a pooling equilibrium where low-risk insurers also choose full securitization, they subsidize the high-risk insurers due to adverse selection. In a separating equilibrium where low-risk insurers

⁴The first best insurance-linked securities' premium paid by high-risk insurer is $\frac{\tilde{B}\bar{p}}{1 - \bar{p}}$, and paid by low-risk insurer is $\frac{\tilde{B}\underline{p}}{1 - \underline{p}}$. It is easy to see that $\frac{\tilde{B}\bar{p}}{1 - \bar{p}} > \tilde{A} > \frac{\tilde{B}\underline{p}}{1 - \underline{p}}$.

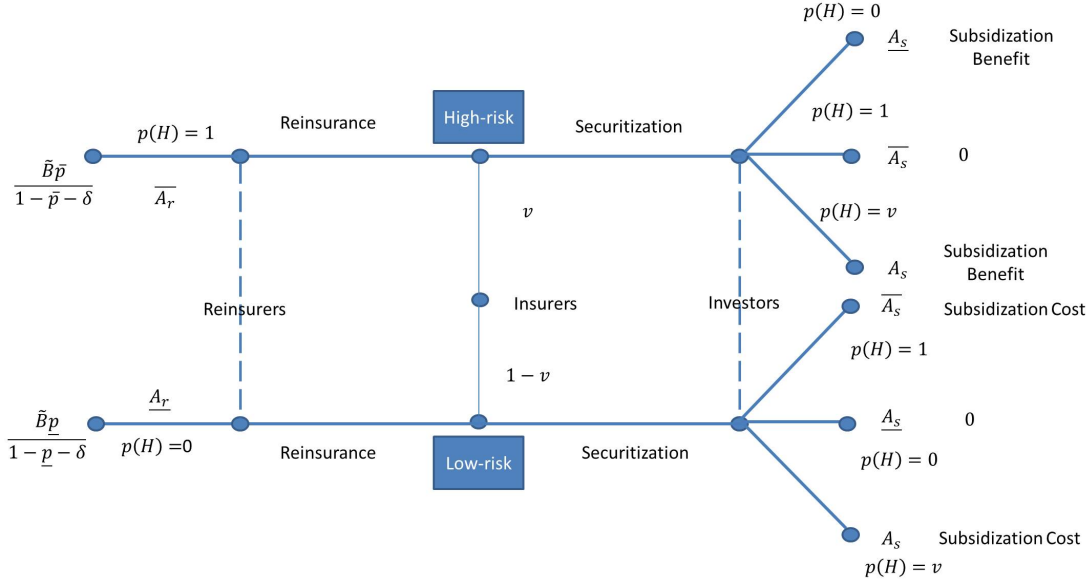


Figure 1: Simple Model

choose reinsurance, investors in the capital market perfectly know the high-risk insurers' type in which case full securitization is cheaper for them relative to full reinsurance because they do not incur the additional expected costs due to the market power of reinsurers. On the other hand, low-risk insurers have to compare the cost due to reinsurers' market power againsts the cost of subsidizing high-risk insurers due to adverse selection if they choose securitization. Consequently, a low-risk insurer's choice reflects the fundamental trade-off between the information advantage held by reinsurers and the higher market power of reinsurers relative to competitive capital markets. The following proposition shows that, if the reinsurer's market power is below a threshold, the unique PBE is a separating equilibrium in which the high-risk insurer chooses full securitization, while the low-risk insurer chooses full reinsurance.

Proposition 1 (Separating Equilibrium). *Suppose that $\delta < \frac{\nu(\bar{p}-\underline{p})}{\underline{p}+\nu(\bar{p}-\underline{p})}$. There exists a unique separating equilibrium in which the low-risk insurer chooses full reinsurance, while the high-risk insurer chooses full securitization.*

In the separating equilibrium described by the above proposition, the adverse selection problem is less severe than in the scenario where the insurer only has access to securitization. Based on the market's belief that only the high-risk insurer chooses securitization, investors only buy the securities at the high-risk insurers' issuing price. The high-risk insurer, therefore, takes advantage

of the lower cost of transferring its risk to perfectly competitive financial markets rather than ceding rents to the reinsurer due to its market power. In other words, under the market's posterior beliefs, it is clearly sub-optimal for the high-risk insurer to choose reinsurance because the reinsurer perfectly knows its type and it has to pay a higher premium compared with the premium it pays to capital markets. If the low-risk insurer mimics the high-risk insurer by choosing securitization, it incurs greater costs compared with the case where only securitization is available. The costs of mimicking the high-risk insurer through paying the high-risk insurer's premium exceeds the rent the low-risk insurer cedes to the reinsurer so that it chooses reinsurance. In summary, the "information advantage" of reinsurance is lower than the "market competitiveness" advantage of securitization for the high-risk insurer. On the other hand, the additional costs arising from the market power of the reinsurer are lower than the adverse selection costs due to securitization for low-risk insurer.

The following proposition shows that, if the reinsurer's market power exceeds a threshold, the unique PBE is a pooling equilibrium in which both types of insurer choose securitization.

Proposition 2 (Pooling Equilibrium). *Suppose that $\delta > \frac{\nu(\bar{p}-\underline{p})}{\underline{p}+\nu(\bar{p}-\underline{p})}$. There exists a unique pooling equilibrium in which both the high-risk and low-risk insurers choose full securitization.*

In the pooling equilibrium, the investors cannot learn the insurer's type. Consequently, the securitization contracts where the low-risk insurer subsidizes the high-risk insurer is more attractive to the high-risk insurer than reinsurance contracts where the reinsurer knows its type and the high-risk insurer has to cede rents due to the reinsurer's market power. For the low-risk insurer, the information costs of securitization contracts are lower than the costs it incurs from reinsurance if the reinsurer's market power exceeds a threshold.

In the above analysis, we have assumed that the bankruptcy cost

$$C > \max\left\{\frac{\tilde{B}\delta}{1-\underline{p}-\delta}, \frac{\tilde{B}\nu(\bar{p}-\underline{p})}{(\nu(1-\bar{p})+(1-\nu)(1-\underline{p}))\underline{p}}\right\}$$

that is, it is high enough that an insurer never chooses self-insurance/retention. We now generalize our results to allow for an arbitrary bankruptcy cost so that self-insurance may also be optimal.

Proposition 3 (Perfect Bayesian Equilibria for Arbitrary Bankruptcy Cost). *1. Suppose $C \leq$*

$$\frac{\tilde{B}\delta}{1-\underline{p}-\delta}.$$

(a) If $\frac{\tilde{B}\nu(\bar{p}-p)}{\nu(1-\bar{p})+(1-\nu)(1-p)} \leq C\underline{p}$, the unique PBE is a pooling equilibrium in which the both high-risk and low-risk insurers choose full securitization.

(b) If $\frac{\tilde{B}\nu(\bar{p}-p)}{\nu(1-\bar{p})+(1-\nu)(1-p)} \geq C\underline{p}$, the unique PBE is a separating equilibrium in which the low-risk insurer chooses full self-insurance, while the high-risk insurer chooses full securitization.

2. Suppose $C \geq \frac{\tilde{B}\delta}{1-\underline{p}-\delta}$.

(a) If $\frac{v(\bar{p}-p)}{v(1-\bar{p})+(1-v)(1-p)} \leq \frac{\delta}{1-\underline{p}-\delta}$ the unique PBE is a pooling equilibrium in which the both high-risk and low-risk insurers choose full securitization.

(b) If $\frac{v(\bar{p}-p)}{v(1-\bar{p})+(1-v)(1-p)} \geq \frac{\delta}{1-\underline{p}-\delta}$, the unique PBE is a separating equilibrium in which the low-risk insurer chooses full reinsurance, while the high-risk insurer chooses full securitization.

In each of the above propositions, if the bankruptcy cost is below a threshold $\max\{\frac{\tilde{B}\delta}{1-\underline{p}-\delta}, \frac{\tilde{B}\nu(\bar{p}-p)}{(\nu(1-\bar{p})+(1-\nu)(1-p))\underline{p}}\}$, full self-insurance becomes an attractive alternative relative to reinsurance for low-risk insurers. The high bankruptcy cost reduces insurers' incentive to choose full self-insurance. However, in all cases, high-risk insurers still prefer full securitization since they can always benefit from being subsidized by low-risk insurers.

The following propositions show the effects of the magnitude of the net loss payment B and the proportion ν of high-risk insurers on the equilibrium.

Proposition 4 (Effects of Loss Magnitude). *There exists two threshold magnitude of loss payment \hat{B}_1 and \hat{B}_2 where $\hat{B}_1 = \frac{C(1-\underline{p}-\delta)}{\delta} + W$, and $\hat{B}_2 = \frac{C\underline{p}(v(1-\bar{p})+(1-v)(1-p))}{v(\bar{p}-p)} + W$*

1. Suppose $\delta < \frac{\nu(\bar{p}-p)}{p+\nu(\bar{p}-p)}$. There is a threshold level of the net loss payment, \hat{B}_1 , where

$$\hat{B}_1 = \frac{C(1-\underline{p}-\delta)}{\delta} + W$$

such that,

(a) if $B \geq \hat{B}_1$, the equilibrium is separating where low-risk insurer chooses self-insurance, while the high-risk insurer chooses full securitization;

(b) if $B \leq \hat{B}_1$, the equilibrium is separating where low-risk insurer chooses full reinsurance, while the high-risk insurer chooses full securitization.

2. Suppose $\delta > \frac{\nu(\bar{p}-\underline{p})}{p+\nu(\bar{p}-p)}$. There is a threshold level of the net loss payment, \hat{B}_2 , where

$$\hat{B}_2 = \frac{C\underline{p}(v(1-\bar{p}) + (1-v)(1-\underline{p}))}{v(\bar{p}-\underline{p})} + W$$

such that

(a) if $B \geq \hat{B}_1$, the equilibrium is separating where low-risk insurer chooses self-insurance, while the high-risk insurer chooses full securitization;

(b) if $B \leq \hat{B}_2$, the equilibrium is pooling where both low-risk and high-risk insurers choose full securitization.

The above results show that, in the case where the market power for reinsurers is low enough, if the net loss payment exceeds a threshold, the low risk insurer prefers self-insurance. A higher loss payment raises the reinsurance rents undertaken by the low-risk insurer. If the loss payment exceeds a threshold, therefore, it is optimal for the low-risk insurer to choose self-insurance. In the case where the market power for reinsurers is high enough, if the net loss payment is below a threshold, the low-risk insurer prefers full securitization. However, a higher loss payment increases the adverse selection costs incurred by the low-risk insurer. Consequently, it is optimal for the low-risk insurer to self-insure if the loss payment exceeds a threshold.

The above results provide preliminary suggestions why catastrophe risk, which is characterized by large magnitudes of potential losses, is often either retained or transferred through traditional reinsurance markets rather than through securitization. The information advantage of reinsurance exceeds the disadvantage due to reinsurers' market power when potential catastrophic losses are high. Further, because only high-risk insurers choose securitization, they pay high premia in securities markets (relative to the ex ante expected loss determined by the average probability $\nu\bar{p} + (1-\nu)\underline{p}$), which could also explain why catastrophe-linked securities are usually expensive and credit ratings of catastrophe bonds are usually below investment grade. In the more general models with a continuum of insurer types that we analyze in the following sections, we obtain

sharper results that explain why retention and reinsurance dominate securitization in catastrophe risk markets.

The following proposition shows how the equilibrium is affected by the probability that the insurer is high-risk.

Proposition 5 (Effects of Probability of High Risk). *1. Suppose $C \leq \frac{\tilde{B}\delta}{1-\underline{p}-\delta}$, there is a threshold level of the probability of high risk, \hat{v}_1 , where*

$$\hat{v}_1 = \frac{C\underline{p}(1-\underline{p})}{(\tilde{B} + C\underline{p})(\bar{p} - \underline{p})}$$

such that,

- (a) if $v > \hat{v}_1$, the equilibrium is separating where the low-risk insurer chooses self-insurance, while the high-risk insurer chooses securitization, and*
- (b) if $v \leq \hat{v}_1$, the equilibrium is pooling where both high-risk and low-risk insurers choose securitization.*

2. Suppose $C \geq \frac{\tilde{B}\delta}{1-\underline{p}-\delta}$, there is a threshold level \hat{v}_2 , where

$$\hat{v}_2 = \frac{\delta\underline{p}(1-\underline{p})}{(1-\underline{p}-\delta + \underline{p}\delta)(\bar{p} - \underline{p})}$$

such that,

- (a) if $v > \hat{v}_2$, the equilibrium is separating where low-risk insurer chooses full reinsurance, while the high-risk insurer chooses full securitization, and*
- (b) if $v \leq \hat{v}_2$, the equilibrium is pooling where both high-risk and low-risk insurers choose full securitization.*

When the probability that the insurer is high risk is above a threshold, the information costs incurred by low-risk insurers due to pooling securitization are high enough that they either choose full reinsurance or full self-insurance.

The results of Proposition 5 also suggest an explanation for the observed spike in securitization transactions after major catastrophes such as Hurricane Katrina (e.g., see Cummins and Trainar

(2009)). Anecdotal evidence suggests that markets revised their perceptions of catastrophe risk upwards following Katrina (this would also be an intuitive outcome of a standard learning model). The above proposition implies that a higher average risk makes securitization more likely.

3 Extended Model with Continuum of Types

We now generalize the model to allow for the insurer's type p to be continuously distributed on $[0, 1]$. All other assumptions of the simple model remain the same. Investors in capital markets have a prior belief distribution about an insurer's type p that is represented by the cumulative distribution $F(p)$. As in our analysis of the simple model, we first derive the reinsurance and securitization contracts separately assuming that insurers only have access to one of the two mechanisms. We then analyze the insurer's choice among retention, reinsurance and securitization.

3.1 Reinsurance

As in the two-type model, reinsurers monitor the insurer and perfectly observe its type. The optimal contract for each insurer type maximizes its expected utility subject to meeting the reinsurer's reservation expected payoff $\delta \cdot A_r(p)$. Given the fixed bankruptcy cost C , it is easy to show that no insurer type chooses reinsurance if $\delta \geq \frac{C}{\bar{B}+C}$ because it is too expensive. Consequently, we consider the case where $\delta < \frac{C}{\bar{B}+C}$. Define $\hat{p} = 1 - \delta - \frac{\bar{B}}{C}\delta < 1$. If an insurer chooses reinsurance, the optimal reinsurance contract solves

$$\max_{A_r(p), B_r(p)} (W + A - A_r(p))(1 - p) + (W - B + B_r(p))p$$

such that

$$A_r(p)(1 - p) - B_r(p)p \geq A_r(p)\delta \tag{7}$$

$$W + A - A_r(p) \geq 0 \tag{8}$$

$$W - B + B_r(p) \geq 0 \tag{9}$$

The following lemma describes an insurer's decision on whether or not to transfer its risk through reinsurance and the optimal reinsurance contract if it chooses to do so.

Lemma 3 (Reinsurance Contract). *If $p > \check{p}$, the insurer chooses retention. If $p \in [0, \check{p}]$ the optimal reinsurance contract, $(A_r^*(p), B_r^*(p))$, is*

$$A_r^*(p) = \frac{\tilde{B}p}{1 - p - \delta}, \quad B_r^*(p) = B - W = \tilde{B}$$

The properties of the reinsurance contracts in the continuous-type case are similar to the two-type case. A higher loss probability raises the reinsurance premium. If the insurer's risk is higher than \check{p} , the expected bankruptcy cost is lower than the cost of reinsurance so that the insurer retains its risk. Because the bankruptcy cost is fixed, the insurer chooses full reinsurance if it opts to transfer its risks.

3.2 Securitization

As in the two-type model, an insurer offers a contract, (A_s, B_s) , where A_s is the premium received by the investors if there is no loss, and B_s is the net payment made by investors if a loss occurs. Because the bankruptcy cost is fixed and does not depend on the magnitude of the insurer's shortfall in the bad state, it is easy to see that separating securitization contracts are not incentive compatible. That is, it is better for an insurer to self-insure rather than choose a securitization contract with a nonzero retention level because it incurs the same bankruptcy cost in either case so that its expected payoff is the same. We conjecture that insurers with types in $[\check{p}, 1]$ choose securitization for some \check{p} . Let $\mu(\cdot)$ denote the posterior beliefs of capital markets regarding an insurer's type given that it has chosen securitization. Given that insurers with types in $[\check{p}, 1]$ choose securitization, the posterior probability measure is given by

$$d\mu(p) = \frac{dF(p)}{1 - F(\check{p})} \tag{10}$$

For the reasons discussed above, we focus on pooling securitization contracts. The following lemma characterizes the optimal contracts.

Lemma 4 (Securitization Contract). *Suppose that there is a unique \check{p} satisfying the following equation:*

$$C\check{p} = \tilde{B} \frac{\int_{\check{p}}^1 p\mu'(p)dp - \check{p}}{1 - \int_{\check{p}}^1 p\mu'(p)dp}, \tag{11}$$

where

$$R(p) = \frac{\int_p^1 t d\mu(t) - p}{1 - \int_p^1 t d\mu(t)}, \quad (12)$$

In the unique PBE of the signaling game, insurers with type p in the interval $[\tilde{p}, 1]$ fully transfer their risks and offer the same contract (A_s^*, B_s^*) , where

$$B_s^*(p) = B_s^* = \tilde{B}, \quad A_s^*(p) = A_s^* = \frac{B_s \int_{\tilde{p}}^1 t d\mu(t)}{1 - \int_{\tilde{p}}^1 t d\mu(t)}.$$

Insurers with types p below \tilde{p} choose full self-insurance.

The threshold, \tilde{p} , is the point of indifference between the cross-subsidization costs from pooling with higher types and the expected bankruptcy costs from retaining risk. Consider a candidate equilibrium where insurers with types greater than or equal to p offer pooling securitization contracts, while those with types less than p retain their risk. It is easy to show that the subsidization costs incurred by the insurer with type p from pooling with higher types are given by $\tilde{B}R(p)$, where the function $R(\cdot)$, which we refer to as the *subsidization ratio function*, is given by (12). The expected bankruptcy cost incurred by the insurer of type p if it retains its risks is given by Cp . Consequently, the indifference point, \tilde{p} , is determined by

$$C\tilde{p} = \tilde{B}R(\tilde{p}) \quad (13)$$

We assume that the ex ante distribution of possible insurer types, $F(\cdot)$, is such that the above equation has a unique solution, \tilde{p} . Because the subsidization costs incurred by insurer types greater than \tilde{p} decline with the type, it is optimal for all such insurers to pool by offering full securitization contracts. Given that \tilde{p} satisfies (13), the expected bankruptcy cost incurred by an insurer with type less than \tilde{p} is less than the subsidization costs incurred by choosing securitization.

3.3 Risk Transfer Equilibria

We now show that the PBE of the risk transfer game has the conjectured “partition” form as shown in Figure 2.

Proposition 6 (Partition Equilibrium). *Suppose $C > \frac{\tilde{B}\delta}{1-\tilde{p}-\delta}$. The unique PBE of the game is*

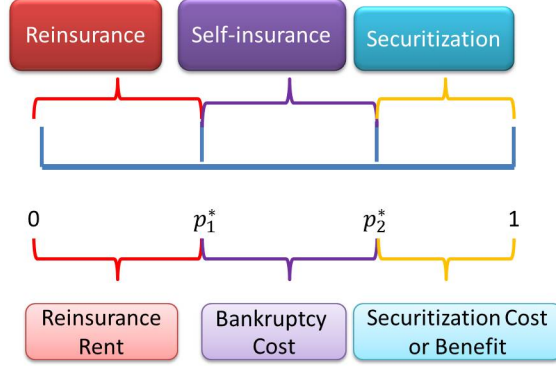


Figure 2: Conjecture of “Partition” Form

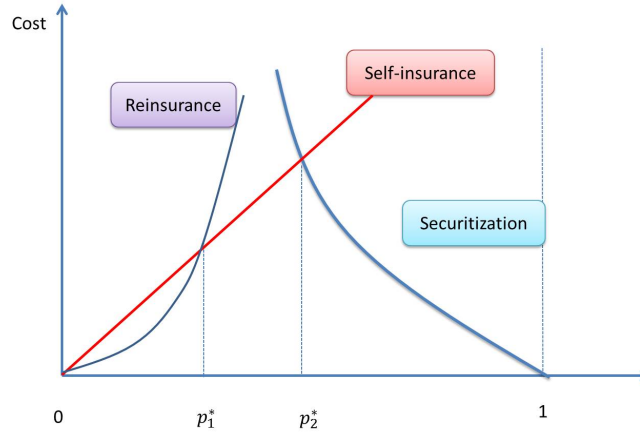


Figure 3: The cost of different risk transfer mechanisms

characterized as follows. There exist $p_1^*, p_2^* \in [0, 1]$ with $0 < p_1^* < p_2^* < 1$ such that insurers with types in the interval $[0, p_1^*]$ choose to fully self-insure, insurers with types in the interval $[p_1^*, p_2^*]$ choose full reinsurance, and insurers with types in the interval $[p_2^*, 1]$ choose full securitization where

$$p_1^* = 1 - \delta - \frac{\tilde{B}}{C}\delta, \quad p_2^* = \tilde{p}$$

and p_2^* is determined by the following equation:

Figure 3 shows the cost function of each risk transfer choice faced by insurers. For all types, the chosen form of risk transfer gives them the lowest expected cost among the three choices; self-insurance, reinsurance or securitization. The expected bankruptcy cost increases with an insurer’s type; the expected cost of reinsurance is fixed for all insurer types; and the expected cost of securitization decreases with an insurer’s type. Thus insurers with types in the interval $[0, p_1^*]$

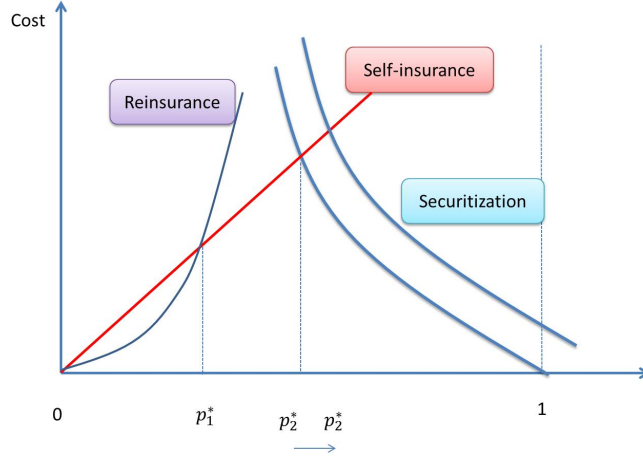


Figure 4: Loss payment shift or FOSD shift of the insurer types

choose self-insurance, intermediate-risk insurers with types in interval $[p_1^*, p_2^*]$ choose reinsurance, and high-risk insurers with types in the interval $[p_2^*, 1]$ choose securitization. The thresholds that determine the various subintervals are the “indifference” points. From the above, we get

$$\frac{dp_2^*}{dB_s} = \frac{R(p_2^*)}{C - \tilde{B}R'(p_2^*)} \quad (14)$$

The numerator of (14) is positive. Because $p_2^* = \tilde{p}$ and \tilde{p} is the unique solution of 13, we can show that the denominator of the R.H.S. of (14) is positive. Thus $dp_2/d\tilde{B} > 0$. In other words, p_2^* is an increasing function of \tilde{B} . Consequently, an increase in the magnitude of the insurer’s net loss payment increases the threshold risk level above which insurers choose securitization and the securitization subinterval shrinks.

A “first order stochastic dominance” shift in the distribution of insurer types $F(p)$ pushes up the subsidization cost function $R(p)$. The securitization subinterval in the PBE also shrinks. A “first order stochastic dominance” shift in the distribution of insurer types $F(p)$, therefore, also increases the threshold risk level above which insurers choose securitization. In other words, when the risks of insurers’ portfolios increase as a whole, reinsurance becomes *more attractive* relative to securitization.

As shown by Figure 4, an increase in the amount of net loss payment and a “first order stochastic dominance” shift in the distribution of insurer types $F(p)$ both shift up the expected cost of securitization since the cross-subsidization on securitization market is more severe. Consequently,

the upper threshold level of risk that determines the level at which insurers choose securitization increases since the relatively lower risk insurers find reinsurance less costly relative to securitization.

The above results sharpen those suggested by our analysis of the simple model in Section 2. Catastrophe risks are characterized by large magnitudes of potential losses. The fact that an increase in the magnitude of potential losses increases the trigger risk level above which securitization is chosen suggests that catastrophe risks are more likely to be retained by insurers or reinsured rather than securitized.

The above analysis assumes that the bankruptcy cost $C < \frac{\tilde{B}\delta}{1-\tilde{p}-\delta}$. In other words, the bankruptcy cost is higher than the market power cost for intermediate insurers. We now generalize our results to allow for an arbitrary bankruptcy cost.

Proposition 7 (Perfect Bayesian Equilibria for Arbitrary Fixed Bankruptcy Cost). *1. If $C < \frac{\tilde{B}\delta}{1-\delta}$, there exists \tilde{p} such that insurers with risk type below \tilde{p} choose full retention, while those with risk type above \tilde{p} choose full pooling securitization.*

2. If $\frac{\tilde{B}\delta}{1-\delta} < C < \frac{\tilde{B}\delta}{1-\tilde{p}-\delta}$, the equilibrium is characterized as in Proposition 6;

3. If $C > \frac{\tilde{B}\delta}{1-\tilde{p}-\delta}$, there exist $p_3^ \in [0, 1]$ with $0 < p_3^* < 1$ such that insurers with types in the interval $[0, p_3^*]$ choose full reinsurance, while insurers with types in the interval $[p_3^*, 1]$ choose full securitization.*

where \tilde{p} is determined by equation (13) and p_3^ is determined by $\frac{\delta}{1-\delta-p_3^*} = R(p_3^*)$*

The above proposition shows that reinsurance is dominated by retention if the fixed bankruptcy cost is lower than the threshold $\frac{\tilde{B}\delta}{1-\delta}$ so that all insurer types choose between retention and securitization. The lower risk insurers choose retention by avoiding the subsidization cost due to information asymmetry in capital markets, while higher risk insurers choose securitization due to the relatively lower cost of risk sharing. When the fixed bankruptcy cost is between the thresholds $\frac{\tilde{B}\delta}{1-\delta}$ and $\frac{\tilde{B}\delta}{1-\tilde{p}-\delta}$, the risk transfer choices of intermediate insurer types reflect the tradeoff between the market power for reinsurers and the fixed bankruptcy cost. Consequently, the equilibrium has a partition form with three subintervals. When the fixed bankruptcy cost is high enough, retention is dominated by either full reinsurance or full securitization. Subsequently, the equilibrium has a partition form with only two subintervals.

4 Variable Bankruptcy Cost

We now generalize the model to allow the bankruptcy cost to vary proportionally regarding to the insurer's risk retention level when it is not fully covered in the bad state. Assume the bankruptcy cost is a linear increasing function of the retention level with coefficient c (the incurred bankruptcy cost in the bad state is equal to $c \cdot (\tilde{B} - B_r)$ or $c \cdot (\tilde{B} - B_s)$), where $c > 1$. The maximum bankruptcy costs is $c \cdot \tilde{B}$ when the insurer retains all the risk is \tilde{B} . We assume $c\tilde{B} = C$ to compare with the results in the previous analysis. All other assumptions in Section 2 remain the same. As we demonstrated in previous sections, the separating partial securitization contracts may be the optimal choice for some insurer's type in the equilibrium since they benefit from sharing risk with investors in capital markets at the cost of retaining some risk to reveal their type.

4.1 Reinsurance

We now derive the case where insurers only have access to reinsurance market. The optimal reinsurance contract for each insurer type maximizes its expected utility subject to meeting the reinsurer's reservation expected payoff $\delta \cdot A_r(p)$ that represents its market power. To clarify, given the proportional bankruptcy cost associated to the risk retention level, the optimal reinsurance contract for an insurer with risk type $p \in [0, \tilde{p}]$ (where $\tilde{p} = 1 - \delta - \frac{\delta}{c}$ if $\delta < \frac{c}{1+c}$) solves:

$$\max_{A_r(p), B_r(p)} (W + A - A_r(p))(1 - p) + (W - B + B_r(p))p - c(\tilde{B} - B_r(p))$$

such that

$$A_r(p)(1 - p) - B_r(p)p \geq A_r(p)\delta \tag{15}$$

$$W + A - A_r(p) \geq 0 \tag{16}$$

$$W - B + B_r(p) \geq 0 \tag{17}$$

The following lemma captures the optimal reinsurance contract for each risk type p .

Lemma 5 (Reinsurance Contract). *The optimal reinsurance contract, $(A_r^*(p), B_r^*(p))$, for an in-*

surer of type $p \in [\hat{p}, 1]$ is

$$A_r^*(p) = \frac{\tilde{B}p}{1-p-\delta}, \quad B_r^*(p) = B - W = \tilde{B}$$

The above lemma characterizes the optimal reinsurance contracts, which is the same as the optimal contract with fixed bankruptcy costs for insurers who choose to buy reinsurance. The reinsurance contract has full coverages due to the linearity set-up including the linear market power function and the linear bankruptcy cost function.

4.2 Securitization

We now derive the case where insurers only have access to capital markets. The proportional bankruptcy cost provides low risk insurers the room to bear more risk. The insurer's choice of risk retention level, serving as a signal of its type, reduces the adverse selection cost due to information asymmetry. So lower risk insurers retain more risk than higher risk insurers; and very high risk insurers choose pooling reinsurance. Thus, the insurers' choice of securitization coverage reflects the tradeoff between adverse selection cost and the proportional bankruptcy cost. Consequently, the full securitization pooling contract is not optimal for the lower-risk insurers capable of signaling their type through retaining more risk. The following lemma characterizes the semi-pooling PBE of the securitization game.

Lemma 6 (Securitization Contract). *let $\mathcal{P} = \{p^* : R(p^*) < cp^*; c - (c + \frac{1}{1-p^*}) (1 - \frac{R(p^*)}{cp^*}) + \frac{1}{1-\int_{p^*}^1 t d\mu(t)} \geq 0\}$. $\forall p^* \in \mathcal{P}$, the optimal securitization contract, $(A_s^*(p), B_s^*(p))$, is*

- for an insurer of type $p < p^*$

$$B_s^*(p) = \tilde{B} \left(1 - \frac{R(p^*)}{cp^*}\right) \left(\frac{1-p^*}{p^*}\right)^{\frac{1}{c}} \left(\frac{p}{1-p}\right)^{\frac{1}{c}},$$

$$A_s^*(p) = \tilde{B} \left(1 - \frac{R(p^*)}{cp^*}\right) \left(\frac{1-p^*}{p^*}\right)^{\frac{1}{c}} \left(\frac{p}{1-p}\right)^{\frac{1}{c}+1}$$

- for an insurer of type $p > p^*$

$$B_s^*(p) = B_s^* = \tilde{B}, \quad A_s^*(p) = A_s^* = \frac{\tilde{B} \int_{p^*}^1 t d\mu(t)}{1 - \int_{p^*}^1 t d\mu(t)}$$

where

$$\mu(p) = \frac{F(p) - F(p^*)}{1 - F(p^*)}$$

The above lemma describes the multiple semi pooling PBE of the securitization game. There exists a threshold $p^* \in \mathcal{P}$ such that the insurers whose risk level is below p^* choose the separating securitization contracts with partial coverage, while the insurers whose risk level is above p^* choose the pooling securitization contracts with full coverage. The semi-pooling PBE of the game is driven by the existence of the proportional bankruptcy cost. The bankruptcy cost associated with partial separating securitization trades off the adverse selection cost embedded in the pooling securitization contract. To clarify, lower-risk insurers suffer the adverse selection cost by subsidizing the high risk insurers when they choose pooling securitization. However, partial securitization provides an alternative way for lower-risk insurers to gain from the reduction of information asymmetry in spite of some additional bankruptcy cost. When bankruptcy cost is positively associated with the risk retention level, full retention is dominated either by separating securitization or by pooling securitization in the PBE since risk sharing is more efficient than full retention. As the analysis in the case with fixed bankruptcy cost, we next derive the PBE of the case where insurers have both reinsurance markets and capital markets.

4.3 Partition Equilibrium

We now show the PBE of the signaling game have a partition form as in the following proposition when insurers have access to both reinsurance and securitization.

Proposition 8 (Partition Equilibrium). *let a set*

$$\mathcal{U} = \{p_1^* : p_1^* < 1 - \delta - \frac{\delta}{c}\}$$

For any $p_1^ \in \mathcal{U}$, $p_2^* \in \mathcal{L} = \{p_2^* : c - \left(c + \frac{1}{1-p_2^*}\right) \left(1 - \frac{R(p_2^*)}{cp_2^*}\right) + \frac{1}{1-\int_{p_2^*}^1 td\mu(t)} \geq 0\}$ is determined by the following equation*

$$1 - \frac{\delta}{1 - p_1^* - \delta} \left(\frac{1 - p_1^*}{p_1^*}\right)^{\frac{1}{c}} \left(\frac{p_2^*}{1 - p_2^*}\right)^{\frac{1}{c}} = \frac{R(p_2^*)}{cp_2^*}.$$

1. Suppose $\delta < \frac{c}{1+c}$. The PBE of the game are characterized as follows.

- a. There exist $p_1^*, p_2^* \in [0, 1]$ with $0 < p_1^* < p_2^* < 1$ such that insurers with types in the interval $[0, p_1^*]$ choose full reinsurance, insurers with types in the interval $[p_1^*, p_2^*]$ choose separating partial securitization, and insurers with types in the interval $[p_2^*, 1]$ choose pooling full securitization, for any $p_1^* \in \mathcal{U}$ and $p_1^* < p_2^*$
- b. There exists $p_3^* \in [0, 1]$ with $0 < p_2^* < p_3^* < p_1^* < 1$ such that insurers with types in the interval $[0, p_3^*]$ choose full reinsurance and insurers with types in the interval $[p_3^*, 1]$ choose pooling full securitization if $\forall p_1^* \in \mathcal{U}$ and $p_2^* < p_1^*$
2. Suppose $\delta > \frac{c}{1+c}$. The PBE of the game are characterized to be a partition with two subintervals as in Lemma 6.

The above proposition suggests that full reinsurance dominates partial risk sharing for low risk insurers given the reinsurer's proportional market power is lower than the threshold level $\frac{1}{1+c}$. For intermediate insurers, they would like to share partial risk in the capital market at the cost of some moderate bankruptcy cost; however, they would switch to either full reinsurance or full risk sharing as the bankruptcy cost increases. Nevertheless, all types of insurers prefer risk sharing to reinsurance, including fully sharing and partial sharing, when reinsurers' proportional market power is extremely high.

5 Conclusions

We develop a signaling model to analyze insurers' choice between reinsurance and securitization. In our signaling model, an insurer's choice serves as a signal of the quality of risks in its portfolio when it has a private information about the portfolio. The insurer's choice reflects the tradeoff between the lower adverse selection costs associated with reinsurance against the lower market power of investors in competitive capital markets. We show that the unique Perfect Bayesian Equilibrium of the signaling game has a partition form where lowest risk insurers choose reinsurance, intermediate risk insurers choose self-insurance, and highest risk insurers choose securitization. An increase in the magnitude of potential losses in the portfolio increases the threshold level of risk above which insurers choose securitization. In particular, our results imply that catastrophe risk, which is characterized by "high severity" losses is only securitized by very high risk insurers. Further, because

only highest risk insurers choose securitization, they pay high premia in securities markets, which could explain why catastrophe-linked securities are usually expensive, and why catastrophe securities receive “below investment grade” ratings. Our results, therefore, provide an novel alternate explanation for the relative predominance of reinsurance in the market for catastrophe risk transfer and the high cost of catastrophe bonds.

Appendix

Proof of Lemma 1

Proof. Assume initially that full reinsurance is optimal. For the high-risk insurer, (3) should be binding. The net reinsurance payment \overline{B}_r in the bad state is, therefore,

$$\overline{B}_r = B - W$$

Let

$$\tilde{B} = B - W$$

Thus

$$\overline{B}_r = \tilde{B}$$

The insurer's maximization problem is equivalent to minimizing

$$\overline{A}_r(1 - \overline{p}) - \overline{B}_r\overline{p}$$

which implies that (1) must be binding,

$$\overline{A}_r(1 - \overline{p}) - \overline{B}_r\overline{p} = \overline{A}_r\overline{p}$$

and we can solve for the premium for high-risk insurer \overline{A}_r

$$\overline{A}_r = \frac{\tilde{B}\delta}{1 - \overline{p} - \delta}$$

The optimal contract for low-risk insurer can be derived similarly.

The net cost of reinsurance for high risk insurers equals the reinsurers' market power $\delta\overline{p}$, while the net cost for low risk insurers equals $\delta\underline{p}$. Condition (4) immediately ensures that full reinsurance is, indeed, optimal for both types of insurers. \square

Proof of Lemma 2

Proof. Consider first a candidate separating equilibrium $(\overline{A}_s, \overline{B}_s); (\underline{A}_s, \underline{B}_s)$, where $(\overline{A}_s, \overline{B}_s)$ is the contract offered by the high-risk insurer and $(\underline{A}_s, \underline{B}_s)$ is the contract offered by the low-risk insurer. We must have

$$\tilde{B} - \overline{B}_s \geq 0, \tag{18}$$

$$\tilde{B} - \underline{B}_s \geq 0. \tag{19}$$

Because the high-risk insurer incurs an additional bankruptcy cost C if it does not fully cover its losses in the bad state, it is clearly optimal for it to fully transfer its risk so that $\overline{B}_s = \tilde{B}$. Since capital market investors must at least break even, we must also have

$$(1 - \overline{p})\overline{A}_s - \overline{p}\overline{B}_s \geq 0; (1 - \underline{p})\underline{A}_s - \underline{p}\underline{B}_s \geq 0. \tag{20}$$

Since $\overline{B}_s = \tilde{B}$, $(1 - \overline{p})\overline{A}_s = \overline{p}\tilde{B}$.

If the low-risk insurer were to also purchase full insurance, then we would similarly have $(1 - \underline{p})\underline{A}_s = \underline{p}\tilde{B}$. In this case, however, the menu $(\overline{A}_s, \overline{B}_s); (\underline{A}_s, \underline{B}_s)$ would not be incentive compatible because the high-risk insurer strictly gains by deviating and offering the low-risk insurer's contract.

Consequently, the low-risk insurer must retain some risk in which case it incurs the additional bankruptcy cost, C in the bad state. Further, it is clearly optimal for the participation constraint of investors to be binding so that $(1 - \underline{p})A_s = \underline{p}B_s$. It is then easy to see that the expected payoff of the low-risk insurer regardless of its choice of retention level is

$$\underline{EU}_s^{separating} = (1 - \underline{p})(W + A) + \underline{p}(W - B) - \underline{p}C. \quad (21)$$

Now consider a candidate pooling equilibrium where both insurers offer the contract, (A_s^*, B_s^*) . Because an insurer incurs the bankruptcy cost C if it does not fully cover its loss, condition (6) ensures that a full transfer of risk is optimal. Since investors' participation constraint must be binding for optimality, it easily follows that

$$[\nu(1 - \bar{p}) + (1 - \nu)(1 - \underline{p})]A_s^* = [\nu\bar{p} + (1 - \nu)\underline{p}]\tilde{B}. \quad (22)$$

Further, the expected payoff of the low risk insurer is

$$\begin{aligned} \underline{ER}_s^{pooling} &= (1 - \underline{p})(W + A) + \underline{p}(W - B) - ((1 - \underline{p})A_s^* - \underline{p}B_s^* \\ &= (1 - \underline{p})(W + A) + \underline{p}(W - B) - \frac{\tilde{B}\nu(\bar{p} - \underline{p})}{\nu(1 - \bar{p}) + (1 - \nu)(1 - \underline{p})}. \end{aligned} \quad (23)$$

Comparing (23) with (21), and using (22), condition (6) immediately implies that the low risk insurer's expected payoff from the pooling contract exceeds its expected payoff from the separating contract. Consequently, there can be no separating equilibrium.

The pooling equilibrium can be supported by the following "worst case" off-equilibrium beliefs. If either insurer deviates and offers a contract different from (A_s^*, B_s^*) , investors assume that the insurer is high-risk with probability one. □

Proof of Proposition 1

Proof. Insurers faces a nontrivial choice between reinsurance and securitization if the bankruptcy cost is greater than the threshold $\max[\frac{\tilde{B}\delta}{1 - \underline{p} - \delta}, \frac{\tilde{B}\nu(\bar{p} - \underline{p})}{(\nu(1 - \bar{p}) + (1 - \nu)(1 - \underline{p}))\underline{p}}]$. Low-risk insurers incur additional bankruptcy cost C if they are not fully covered by the securitization contract to signal their type to the investors. However, the high signaling cost prevents low-risk insurers from retaining any level of risk. Thus, the pooling securitization contract, where low-risk insurers subsidize high-risk insurers, survives in the equilibrium. In this case, securitization is a dominant choice for high-risk insurers compared to reinsurance since they suffer from the cost due to reinsurers' bargaining power $\delta \cdot \bar{A}_r$, but they are subsidized by low-risk insurers. Therefore, there are two equilibrium candidates.

First, we prove that there exists a separating equilibrium in which high-risk insurer chooses full securitization while low-risk insurer chooses full reinsurance if $\delta < \frac{\nu(1 - \underline{p})(\bar{p} - \underline{p})}{\underline{p}(\nu(1 - \bar{p}) + (1 - \nu)(1 - \underline{p})) + \nu(\bar{p} - \underline{p})}$ holds.

We conjecture that in the equilibrium low-risk insurers choose full reinsurance, while high-risk insurers choose full securitization. Then, investors view the insurers in capital markets to be high risk and require higher premium. Since reinsurers can monitor insurers' type, the reinsurance contract for high-risk insurer is the same as in Lemma 1 where

$$\underline{A}_r = \frac{\tilde{B}\underline{p}}{1 - \underline{p} - \delta} \quad \underline{B}_r = \tilde{B}$$

Thus the expected utility of the low-risk insurers purchasing full reinsurance is

$$\begin{aligned}
\underline{EU}_r &= (W + A - \underline{A}_r)(1 - \underline{p}) + (W - B + \underline{B}_r)\underline{p} \\
&= W + (A(1 - \underline{p}) - B\underline{p}) - (\underline{A}_r(1 - \underline{p}) - \underline{B}_r\underline{p}) \\
&= W + (A(1 - \underline{p}) - B\underline{p}) - \frac{\tilde{B}\underline{p}\delta}{1 - \underline{p} - \delta}
\end{aligned}$$

The low-risk insurers are viewed to be high risk if they choose full securitization, which requires higher premium and provide the expected payoff $\underline{EU}_s^{deviate}$ such that

$$\overline{B}_s = \tilde{B} \quad \overline{A}_s = \frac{\tilde{B}\bar{p}}{1 - \bar{p}} \quad (24)$$

$$\Rightarrow \underline{EU}_s^{deviate} = W + (A(1 - \underline{p}) - B\underline{p}) - \frac{\tilde{B}(\bar{p} - \underline{p})}{1 - \bar{p}} \quad (25)$$

In comparison, the expected payoff of the high-risk insurers choosing full securitization, \overline{EU}_s , is

$$\underline{EU}_s = W + (A(1 - \bar{p}) - B\bar{p}) \quad (26)$$

However, it is easy to show the expected payoff of the high-risk insurers choosing full reinsurance, $\overline{EU}_r^{deviate}$, is lower than the choice of full securitization since reinsurers can monitor all insurers with the positive bargaining power.

As to low-risk insurers, if

$$\delta < 1 - \frac{\underline{p}}{\bar{p}} \quad (27)$$

then, $\underline{EU}_r > \underline{EU}_s^{deviate}$. As a result, low-risk insurers will not deviate from full reinsurance.

Therefore, suppose condition (27) holds, there exists an equilibrium in which low-risk insurers choose full reinsurance, high-risk insurers choose full securitization.

Now we conjecture that both types insurers choose full securitization. Similarly, the high bankruptcy cost prohibits low-risk insurers from retaining some risk to signal their level of risk. Thus, the full pooling securitization contracts are offered by both types of insurers in capital markets, in which case the low-risk insurers subsidize the high-risk insurers. To clarify, low-risk insurers pay more than the actuarial fair contract, while high-risk insurers pay less than the actuarial fair one.

The securitization contract is marred with adverse selection issues which are given in Lemma 2, but the reinsurance contracts are same as in Lemma 1.

Then we define the off-equilibrium beliefs. If high-risk insurers deviate to choose reinsurance, they are offered the reinsurance contracts $(\overline{A}_r, \overline{B}_r)$ by reinsurers due to their ability to monitor the insurers' type. Similarly, if low-risk insurers switch to reinsurance, they are offered the reinsurance contracts $(\underline{A}_r, \underline{B}_r)$.

High-risk insurers never deviate as a result of the subsidization from low-risk insurers. For low-risk insurers, the expected payoff when choosing full securitization, \underline{EU}_s , is

$$W + (A(1 - \underline{p}) - B\underline{p}) - \frac{\tilde{B}v(\bar{p} - \underline{p})}{v(1 - \bar{p}) + (1 - v)(1 - \underline{p})}$$

and the expected payoff when purchasing full reinsurance $\overline{EU}_r^{deviating}$ is

$$W + (A(1 - \underline{p}) - B\underline{p}) - \frac{\tilde{B}\underline{p}\delta}{1 - \underline{p} - \delta}$$

Thus, if the condition

$$\delta > \frac{\nu(\bar{p} - \underline{p})}{\underline{p} + \nu(\bar{p} - \underline{p})} \quad (28)$$

holds, low-risk insurers prefer the pooling full securitization even though they subsidize the high-risk insurers, which is still provides them higher payoff.

Therefore, there exists a pooling equilibrium in which both high-risk and low-risk insurer chooses the pooling full securitization contract if condition (28) holds.

However, it is easy to show that

$$\frac{\tilde{B}v(\bar{p} - \underline{p})}{v(1 - \bar{p}) + (1 - v)(1 - \underline{p})} < \frac{\tilde{B}(\bar{p} - \underline{p})}{1 - \bar{p}}$$

that is

$$1 - \frac{\underline{p}}{\bar{p}} < \frac{\nu(\bar{p} - \underline{p})}{\underline{p} + \nu(\bar{p} - \underline{p})}$$

Thus we check the uniqueness of the equilibrium.

If

$$1 - \frac{\underline{p}}{\bar{p}} < \delta < \frac{\nu(\bar{p} - \underline{p})}{\underline{p} + \nu(\bar{p} - \underline{p})}$$

The expected payoff of the low-risk insurers purchasing full reinsurance is \underline{EU}_r in the separating equilibrium, and the expected payoff of the low-risk insurers choosing full securitization is \underline{EU}_s in the pooling equilibrium. Thus, the pooling equilibrium dominates the separating equilibrium when the condition (28) holds.

So there exists only one pooling equilibrium in which both high-risk and low-risk insurer chooses the full pooling securitization contract if $1 - \frac{\underline{p}}{\bar{p}} < \delta < \frac{\nu(\bar{p} - \underline{p})}{\underline{p} + \nu(\bar{p} - \underline{p})}$.

Therefore, there exists a unique separating equilibrium in which the low-risk insurer chooses full reinsurance, while the high-risk insurer chooses full securitization if $\delta < \frac{\nu(\bar{p} - \underline{p})}{\underline{p} + \nu(\bar{p} - \underline{p})}$. \square

Proof of Proposition 2

Proof. See Proof of Proposition 1 \square

Proof of Proposition 3

Proof. The expected payoff of the low-risk insurers choosing reinsurance \underline{EU}_r is

$$\underline{EU}_r = W + (A(1 - \underline{p}) - B\underline{p}) - \frac{\tilde{B}\underline{p}\delta}{1 - \underline{p} - \delta}$$

, the expected payoff of the low-risk insurers choosing self-insurance \underline{EU}_{self} is equal to the expected payoff when the low-risk insurers choose partial coverage securitization, which is

$$\underline{EU}_{self} = W + (A(1 - \underline{p}) - B\underline{p}) - C\underline{p},$$

and the expected payoff of the low-risk insurers choosing full securitization is

$$\underline{EU}_s^{full} = W + (A(1 - \underline{p}) - B\underline{p}) - \frac{\tilde{B}\nu(\bar{p} - \underline{p})}{\nu(1 - \bar{p}) + (1 - \nu)(1 - \underline{p})}$$

For the high-risk insurers, the dominant risk management strategy is full securitization since they can benefit from the subsidization of low-risk insurers due to their private information about their type.

1. If $C \leq \frac{\tilde{B}\delta}{1 - \underline{p} - \delta}$, low-risk insurers never purchase full reinsurance since the expected payoff is lower than that of self-insurance. Thus the choice of low-risk insurers depends on the cost comparison between full self-insurance and full securitization.

If $\frac{\tilde{B}\nu(\bar{p} - \underline{p})}{\nu(1 - \bar{p}) + (1 - \nu)(1 - \underline{p})} < C\underline{p}$, the expected payoff of full securitization exceeds the expected payoff of self-insurance. Thus, the unique pooling equilibrium where both high-risk and low-risk insurers choose full securitization is achieved. However, if $\frac{\tilde{B}\nu(\bar{p} - \underline{p})}{\nu(1 - \bar{p}) + (1 - \nu)(1 - \underline{p})} > C\underline{p}$, full securitization provides higher expected payoff than self-insurance. Thus, the unique separating equilibrium where high-risk insurers choose full securitization and low-risk insurers choose self-insurance is achieved.

2. If $C > \frac{\tilde{B}\delta}{1 - \underline{p} - \delta}$, low-risk insurers never self-insure the catastrophe risk or securitize partial risk. The proof of PBE is the same as the proof in Proposition 1 and Proposition 2

□

Proof of Proposition 4

Proof. 1. If $\delta < \frac{\nu(\bar{p} - \underline{p})(1 - \underline{p})}{\underline{p}(1 - \bar{p}) - \underline{p}\nu(\bar{p} - \underline{p}) + \nu(\bar{p} - \underline{p})(1 - \underline{p})}$,

The optimal strategy for high-risk insurers is always full securitization. Thus low-risk insurers choose among self-insurance, reinsurance and securitization.

If $B > \hat{B}_1$, we have $\frac{(B - W)\nu(\bar{p} - \underline{p})}{\nu(1 - \bar{p}) + (1 - \nu)(1 - \underline{p})} > \frac{(\hat{B}_1 - W)\nu(\bar{p} - \underline{p})}{\nu(1 - \bar{p}) + (1 - \nu)(1 - \underline{p})} > \frac{(\hat{B}_2 - W)\nu(\bar{p} - \underline{p})}{\nu(1 - \bar{p}) + (1 - \nu)(1 - \underline{p})} = C\underline{p}$. According to the proof of Proposition 3, we can show that low-risk insurers prefer self-insurance to full securitization. Also $C\underline{p} = \frac{(\hat{B}_1 - W)\delta}{1 - \underline{p} - \delta} < \frac{(B - W)\delta}{1 - \underline{p} - \delta}$ implies that low-risk insurers prefer self-insurance to reinsurance. Consequently, the dominate strategy for low-risk insurers is self-reinsurance.

If $B < \hat{B}_1$, we have $C\underline{p} = \frac{(\hat{B}_1 - W)\delta}{1 - \underline{p} - \delta} > \frac{(B - W)\delta}{1 - \underline{p} - \delta}$. Thus low-risk insurers prefer full reinsurance to self-insurance. Also $\frac{(B - W)\delta}{1 - \underline{p} - \delta} < \frac{(B - W)\nu(\bar{p} - \underline{p})}{\nu(1 - \bar{p}) + (1 - \nu)(1 - \underline{p})}$ implies that low-risk insurers prefer full reinsurance to full securitization. Therefore, the dominate strategy for low-risk insurers is full reinsurance when the loss payment magnitude is below the threshold \hat{B}_1

1. If $\delta > \frac{\nu(\bar{p} - \underline{p})(1 - \underline{p})}{\underline{p}(1 - \bar{p}) - \underline{p}\nu(\bar{p} - \underline{p}) + \nu(\bar{p} - \underline{p})(1 - \underline{p})}$,

The optimal strategy for high-risk insurers is always full securitization. Thus low-risk insurers choose among self-insurance, reinsurance and securitization.

If $B > \hat{B}_2$, we have $\frac{(B-W)\nu(\bar{p}-p)}{\nu(1-\bar{p})+(1-\nu)(1-p)} > \frac{(\hat{B}_2-W)\nu(\bar{p}-p)}{\nu(1-\bar{p})+(1-\nu)(1-p)} = C\underline{p}$. According to the proof of Proposition 3, we can show that low-risk insurers prefer self-insurance to full securitization. Also $C\underline{p} = \frac{(\hat{B}_1-W)\delta}{1-p-\delta} < \frac{(\hat{B}_2-W)\delta}{1-p-\delta} < \frac{(B-W)\delta}{1-p-\delta}$ implies that low-risk insurers prefer self-insurance to reinsurance. Consequently, the dominate strategy for low-risk insurers is self-reinsurance.

If $B < \hat{B}_2$, we have $C\underline{p} = \frac{(\hat{B}_2-W)\nu(\bar{p}-p)}{\nu(1-\bar{p})+(1-\nu)(1-p)} > \frac{(B-W)\delta}{1-p-\delta}$. Thus low-risk insurers prefer full securitization to self-insurance. Also $\frac{(B-W)\delta}{1-p-\delta} > \frac{(\hat{B}_2-W)\nu(\bar{p}-p)}{\nu(1-\bar{p})+(1-\nu)(1-p)}$ implies that low-risk insurers prefer full securitization to full reinsurance. Therefore, the dominate strategy for low-risk insurers is full securitization when the loss payment magnitude is below the threshold \hat{B}_2 \square

Proof of Proposition 5

Proof. The proof is similar to proof of Proposition 4

The first threshold equalize the cost of self-insurance and the cost of full securitization, which is

$$\hat{v}_1 = \frac{C\underline{p}(1-p)}{(\tilde{B} + C\underline{p})(\bar{p}-p)}$$

The second threshold equalize the cost of full reinsurance and the cost of full securitization, which is

$$\hat{v}_2 = \frac{\delta\underline{p}(1-p)}{(1-p-\delta-\delta\underline{p})(\bar{p}-p)}$$

\square

Proof of Lemma 3

Proof. Assume full reinsurance is optimal. For the insurer of type p , condition (9) is binding. So

$$B_r^*(p) = B - W = \tilde{B}$$

The insurer's maximization problem is equivalent to minimizing $A_r(p)(1-p) - B_r p$ which implies that condition (7) is binding, and we can solve for the premium

$$A_r^*(p) = \frac{\tilde{B}p}{1-p-\delta}$$

Consequently, the existence of fixed bankruptcy cost leads full reinsurance to be actually optimal. \square

Proof of Lemma 4

Proof. Consider first a candidate separating equilibrium $(A_s(p), B_s(p))$, where $(A_s(p), B_s(p))$ is the securitization contract offered by the insurer of type p . Because the insurer incurs an additional fixed bankruptcy cost C if it does not fully cover its loss in the bad state, it is clearly optimal for it to fully transfer its risk so that $B_s^*(p) = \tilde{B}$. The capital market investors break even, thereby leading the participation condition for investors on capital markets to bind, that is

$$A_s^*(p) = \frac{p\tilde{B}}{1-p}$$

However, $(A_s^*(p), B_s^*(p))$ is not incentive compatible because the higher risk insurers are strictly better off by deviating and offering the lower risk insurers' contract. Subsequently, the lower risk insurers must retain some risk by bearing the additional bankruptcy cost. Assume all insurers can signal their type by retaining different level of risk; that is, $B_s^*(p) < \tilde{B}$. Then, the expected cost of partial securitization is 0 when the investors update their belief of the insurer's type and accept actuarial fair price, and the additional expected cost of the remaining risk is pC . Yet the separating partial securitization contract $(A_s^*(p), B_s^*(p))$ is not incentive compatible due to the fact that the higher-risk insurers gain from offering the lower-risk insurers' contract. Consequently, separating partial securitization is not an equilibrium.

Now consider a candidate pooling equilibrium where the insurers whose type p is above the threshold level \tilde{p} choose pooling securitization, and those whose type is below the threshold level choose self-insurance. The full securitization contract is optimal for the higher risk insurers since they incur the fix bankruptcy cost if not covered fully in the bad state. Thus, $B_s^*(p) = B_s^* = \tilde{B}$, and the break even condition of investors require

$$A_s^*(p) = A^* = \tilde{B}R(\tilde{p})$$

where $R(\tilde{p})$ is the subsidization ratio function.

$$R(\tilde{p}) = \frac{\int_{\tilde{p}}^1 td\mu(t) - \tilde{p}}{1 - \int_{\tilde{p}}^1 td\mu(t)}$$

Thus, the expected payoff of insurers choosing full securitization is

$$EU_s^{pool}(p) = W + (A(1-p) - Bp) - \tilde{B} \frac{\int_{\tilde{p}}^1 td\mu(t) - p}{1 - \int_{\tilde{p}}^1 td\mu(t)}$$

The expected payoff of insurers with risk level above the threshold \tilde{p} if they choose self-insurance is

$$EU^{deviate}(p) = W + (A(1-p) - Bp) - pC$$

It is easy to show that insurers with risk level above \tilde{p} will never deviate from the pooling securitization if

$$C\tilde{p} = \tilde{B}R(\tilde{p})$$

Similarly, the expected payoff of insurers with risk level below the threshold \tilde{p} choose self-insurance is

$$EU(p) = W + (A(1-p) - Bp) - pC$$

However, the expected payoff when they deviate to full securitization is

$$\begin{aligned}
EU(p)^{deviate} &= W + (A(1-p) - Bp) - \underbrace{\frac{\tilde{B}(\int_{\tilde{p}}^1 td\mu(t) - p)}{1 - \int_{\tilde{p}}^1 td\mu(t)}}_{\text{an decreasing function of } p} \\
&< W + (A(1-p) - Bp) - \underbrace{\frac{\tilde{B}(\int_{\tilde{p}}^1 td\mu(t) - \tilde{p})}{1 - \int_{\tilde{p}}^1 td\mu(t)}}_{=C\tilde{p}} \\
&= W + (A(1-p) - Bp) - C\tilde{p} \\
&< \underbrace{W + (A(1-p) - Bp) - Cp}_{EU(p)}
\end{aligned}$$

Thus, the insurers whose risk level below the threshold never deviate.

Therefore, there is a unique pooling securitization equilibrium in which the insurers whose risk level is above \tilde{p} choose full securitization, while the insurers whose risk level is below \tilde{p} choose full retention. \square

Proof of Proposition 6

Proof. Assume that when $C > \frac{\tilde{B}\delta}{1-\tilde{p}-\delta}$, insurers with types in the interval $[0, p_1^*]$ choose to full retention, insurers with types in the interval $[p_1^*, p_2^*]$ choose full reinsurance, and insurers with types in the interval $[p_2^*, 1]$ choose full securitization, where $p_1^* = 1 - \delta - \frac{\tilde{B}}{C}\delta$ and $p_2^* = \tilde{p}$

Then we have to check whether insurers would deviate from the conjecture.

For insurers with types in the interval $[0, p_1^*]$, the expected payoff of the insurer choosing full reinsurance is

$$EU^{re}(p) = W + (A(1-p) - Bp) - \frac{\tilde{B}p\delta}{1-p-\delta} \quad \forall p \in [0, p_1^*]$$

The expected payoff of the insurer when it switch to fully self-insure is

$$EU_{sf}(p) = W + (A(1-p) - Bp) - Cp \quad \forall p \in [0, p_1^*]$$

Since $0 = Cp_1^* - \frac{\tilde{B}p_1^*\delta}{1-p_1^*-\delta} > Cp - \frac{\tilde{B}p\delta}{1-p-\delta}$, thus, $Cp > \frac{\tilde{B}p\delta}{1-p-\delta}$. Consequently, the insurer does not want to switch

Similarly, the expected payoff of the insurer when it switch to full securitization is

$$EU^{poolingse} = W + (A(1-p) - Bp) - \frac{\tilde{B}(\int_{p_2^*}^1 td\mu(t) - p)}{1 - \int_{p_2^*}^1 td\mu(t)} \quad \forall p \in [0, p_1]$$

Because $Cp \leq Cp_1 = \frac{\tilde{B}(\int_{p_2^*}^1 td\mu(t) - p_2^*)}{1 - \int_{p_2^*}^1 td\mu(t)} < \frac{\tilde{B}(\int_{p_2^*}^1 td\mu(t) - p)}{1 - \int_{p_2^*}^1 td\mu(t)}$, the insurer does not want to switch.

For insurers with types in the interval $[p_1, p_2]$, the expected payoff of the insurer choosing full retention is

$$EU_{re}(p) = W + (A(1-p) - Bp) - Cp \quad \forall p \in [p_1^*, p_2^*]$$

Then, the expected payoff of the insurer when it switch to full reinsurance is

$$EU^{sf}(p) = W + (A(1-p) - Bp) - \frac{\tilde{B}p\delta}{1-p-\delta} \quad \forall p \in [p_1^*, p_2^*]$$

Since $0 = Cp_1^* - \frac{\tilde{B}p_1^*\delta}{1-p_1^*-\delta} < Cp - \frac{\tilde{B}p\delta}{1-p-\delta} \quad \forall p \in [p_1^*, p_2^*]$, thus, $Cp < \frac{\tilde{B}p\delta}{1-p-\delta}$. Consequently, the insurer does not want to switch to full reinsurance.

Similarly, the expected payoff of the insurer when it switch to full securitization is

$$EU^{poolingse} = W + (A(1-p) - Bp) - \frac{\tilde{B}(\int_{p_2^*}^1 t d\mu(t) - p)}{1 - \int_{p_2^*}^1 t d\mu(t)} \quad \forall p \in [p_1^*, p_2^*]$$

Because $Cp \leq Cp_2 = \frac{\tilde{B}(\int_{p_2^*}^1 t d\mu(t) - p_2^*)}{1 - \int_{p_2^*}^1 t d\mu(t)} < \frac{\tilde{B}(\int_{p_2^*}^1 t d\mu(t) - p)}{1 - \int_{p_2^*}^1 t d\mu(t)}$, the insurer does not want to switch to full securitization.

For insurers with types in the interval $[p_2^*, 1]$, the expected payoff of the insurer choosing full securitization is

$$EU^{poolingse}(p) = W + (A(1-p) - Bp) - \frac{\tilde{B}(\int_{p_2^*}^1 t d\mu(t) - p)}{1 - \int_{p_2^*}^1 t d\mu(t)} \quad \forall p \in [p_2^*, 1]$$

Then, the expected payoff of the insurer when it switch to full self-insure is

$$EU^{sf}(p) = W + (A(1-p) - Bp) - Cp \quad \forall p \in [p_2^*, 1]$$

Since $\frac{\tilde{B}(\int_{p_2^*}^1 t d\mu(t) - p)}{1 - \int_{p_2^*}^1 t d\mu(t)} \leq \frac{\tilde{B}(\int_{p_2^*}^1 t d\mu(t) - p_2^*)}{1 - \int_{p_2^*}^1 t d\mu(t)} = Cp_2^* < Cp$, the insurer does not want to switch.

Similarly, the expected payoff of the insurer when it switch to full reinsurance is

$$EU^{re} = W + (A(1-p) - Bp) - \frac{\tilde{B}p\delta}{1-p-\delta} \quad \forall p \in [p_2^*, 1]$$

Since $\frac{\tilde{B}(\int_{p_2^*}^1 t d\mu(t) - p)}{1 - \int_{p_2^*}^1 t d\mu(t)} < Cp < \frac{\tilde{B}p\delta}{1-p-\delta} \quad \forall p \in [p_2^*, 1]$, the insurer does not want to switch.

Therefore, the conjecture is a partition PBE when the fixed bankruptcy cost is below the threshold level $\frac{\tilde{B}\delta}{1-\tilde{p}-\delta}$ \square

Proof of Proposition 7.

Proof. 1. If $C < \frac{\tilde{B}\delta}{1-\delta}$, that is $\delta > \frac{C}{\tilde{B}+C}$, reinsurance is dominated by self-insurance. Then the proof is the same as proof of Lemma 6.

2. If $\frac{\tilde{B}\delta}{1-\delta} < C < \frac{\tilde{B}\delta}{1-\tilde{p}-\delta}$ The partition equilibria is characterized as in Proposition 6.

3. If $C > \frac{\tilde{B}\delta}{1-\tilde{p}-\delta}$, self-insurance is too expensive to be chosen as an optimal strategy by insurers. Then there exists a partition form with two subintervals. We conjecture that there exists p_3^* such that insurers whose risk level is below p_3^* choose full reinsurance, while insurers whose risk level is above p_3^* choose full securitization, where p_3^* is determined by $\frac{\delta}{1-\delta-p_3^*} = R(p_3^*)$. Then we have to check whether insurers would deviate from the conjecture.

For insurers with types in the interval $[0, p_3^*]$, the expected payoff of insurers choosing full reinsurance is

$$EU_r(p) = W + (A(1-p) - Bp) - \frac{\tilde{B}p\delta}{1-p-\delta}$$

The expected payoff of insurers choosing pooling securitization is

$$EU_s^{deviate}(p) = W + (A(1-p) - Bp) - \frac{\tilde{B}(\int_{p_3^*}^1 td\mu(t) - p)}{1 - \int_{p_2^*}^1 td\mu(t)}$$

Since for any $p \in [0, p_3^*]$,

$$\frac{\tilde{B}p\delta}{1-p-\delta} < \frac{\tilde{B}p_3^*\delta}{1-p_3^*-\delta} = \frac{\tilde{B}(\int_{p_3^*}^1 td\mu(t) - p_3^*)}{1 - \int_{p_3^*}^1 td\mu(t)} < \frac{\tilde{B}(\int_{p_3^*}^1 td\mu(t) - p)}{1 - \int_{p_3^*}^1 td\mu(t)}$$

Then, $EU_r(p) > EU_s^{deviate}(p)$. Thus insurers with types in the interval $[0, p_3^*]$ will not deviate. Similarly, the expected payoff of insurers with types in the interval $[p_3^*, 1]$ choosing securitization is

$$EU_s(p) = W + (A(1-p) - Bp) - \frac{\tilde{B}(\int_{p_3^*}^1 td\mu(t) - p)}{1 - \int_{p_2^*}^1 td\mu(t)}$$

The expected payoff of the insurers who deviate to choose reinsurance is

$$EU_r^{deviate} = W + (A(1-p) - Bp) - \frac{\tilde{B}p\delta}{1-p-\delta}$$

Since for any $p \in [p_3^*, 1]$, we have

$$\frac{\tilde{B}(\int_{p_3^*}^1 td\mu(t) - p)}{1 - \int_{p_3^*}^1 td\mu(t)} < \frac{\tilde{B}(\int_{p_3^*}^1 td\mu(t) - p_3^*)}{1 - \int_{p_2^*}^1 td\mu(t)} = \frac{\tilde{B}p_3^*\delta}{1-p_3^*-\delta} < \frac{\tilde{B}p\delta}{1-p-\delta}$$

Thus, $EU_s(p) > EU_r^{deviate}$. Therefore, insurers with types in the interval $[p_3^*, 1]$ will not deviate.

Consequently, the conjectured partition form with two subintervals is indeed a PBE of the game. □

Proof of Lemma 5

Proof. The maximization problem is equivalent to the following problem:

$$\max_{B_r(p)} W + (A(1-p) - Bp) + \left(c - \frac{\delta}{1-p-\delta} \right) pB_r(p) - c\tilde{B}$$

If $c > \frac{\delta}{1-p-\delta}$, then $B_r(p) = \tilde{B}$, which indicates full reinsurance. If $c < \frac{\delta}{1-p-\delta}$, then $B_r(p) = 0$, which indicates no reinsurance.

Therefore, when $\delta < \frac{c}{1+c}$, there exists \hat{p} where $\hat{p} = 1 - \delta - \frac{\delta}{\hat{p}}$ such that insurers whose risk type is below \hat{p} will choose reinsurance with full coverage. Consequently, the reinsurance premium is $A_r(p) = \frac{\tilde{B}p}{1-p-\delta}$

□

Proof of Lemma 6

Proof. Conjecture that for any $p^* \in \mathcal{P}$, there is a two-partition equilibrium such that insurers with types above p^* choose pooling full securitization, while those with types below p^* choose separating partial securitization.

Lower risk insurers choose the optimal risk retention level $(\tilde{B} - B_s(p))$ to signal their type. The incentive compatible level of $B_s(p)$ is solved by:

For any $p \leq p^*$

$$\max_{\tilde{p}} W + (A(1-p) - Bp) - (A_s(\tilde{p})(1-p) - B_s^{sep}(\tilde{p})p) - c(\tilde{B} - B_s^{sep}(\tilde{p}))p$$

s.t.

$$A_s(\tilde{p})(1-\tilde{p}) - B_s^{sep}(\tilde{p})\tilde{p} \geq 0 \tag{29}$$

$$\tag{30}$$

The constraint (29) is binding, and the above problem is equivalent to

$$\min_{\tilde{p}} \frac{B_s^{sep}(\tilde{p}-p)}{1-\tilde{p}} + c(\tilde{B} - B_s^{sep}(p))p$$

F.O.C is

$$\frac{(B_s^{sep'}(\tilde{p})(\tilde{p}-p) + B_s^{sep}(\tilde{p}))(1-\tilde{p}) + B_s^{sep}(\tilde{p})(\tilde{p}-p)}{(1-\tilde{p})^2} - cB_s^{sep'}(\tilde{p})p = 0$$

at $\tilde{p} = p$ is

$$B_s^{sep}(p) = B_s^{sep'}(p)cp(1-p)$$

The general solution of above ordinary differential equation is

$$B_s^{sep}(p) = \exp(\lambda) \left(\frac{p}{1-p} \right)^{\frac{1}{c}}$$

where λ is constant of the general solution.

The cost function of insurers with type p choosing separating partial securitization is

$$C_s^{sep}(p) = c \left(\tilde{B} - \exp(\lambda) \left(\frac{p}{1-p} \right)^{\frac{1}{c}} \right)$$

Insurers with type p^* are indifference between separating partial securitization and pooling full securitization. Thus λ are determined by

$$C_s^{sep}(p^*) = \tilde{B}R(p^*)$$

So

$$\exp \lambda = \tilde{B} \left(1 - \frac{R(p^*)}{cp^*} \right) \left(\frac{1-p^*}{p^*} \right)^{\frac{1}{c}}$$

$\forall p^* \in \mathcal{P}$ guarantee the existence of λ

We now check whether insurers will deviate from our initial conjecture.

For insurers with type $p \in [0, p^*]$, the expected payoff of choosing incentive compatible partial securitization is

$$EU_s^{sep}(p) = W + (A(1-p) - Bp) - c \left(\tilde{B} - B_s^{sep}(p) \right) p$$

The expected payoff of insurers choosing incentive compatible partial securitization is always higher than full retention since they can benefit from risk sharing in the capital markets.

the expected payoff of insurers switching to full securitization is

$$EU_s^{deviatepool}(p) = W + (A(1-p) - Bp) - \tilde{B} \frac{\int_{p^*}^1 td\mu(t) - p}{1 - \int_{p^*}^1 td\mu(t)}$$

Let

$$G(p) = cp \left(\tilde{B} - B_s^{sep}(p) \right) - \tilde{B} \frac{\int_{p^*}^1 td\mu(t) - p}{1 - \int_{p^*}^1 td\mu(t)}$$

where

$$B_s^{sep}(p) = \tilde{B} \left(1 - \frac{R(p^*)}{cp^*} \right) \left(\frac{1-p^*}{p^*} \right)^{\frac{1}{c}} \left(\frac{p}{1-p} \right)^{\frac{1}{c}}$$

Since $G(p)$ is a concave function of p , when we have

$$\frac{\partial G(p)}{\partial p} \Big|_{p < p^*} \geq \frac{\partial G(p)}{\partial p} \Big|_{p = p^*}$$

If

$$\begin{aligned} \frac{\partial G(p)}{\partial p} \Big|_{p=p^*} &= c \left(\tilde{B} - \tilde{B} \left(1 - \frac{R(p^*)}{cp^*} \right) \right) - \frac{\tilde{B} \left(1 - \frac{R(p^*)}{cp^*} \right)}{1-p^*} - \frac{\tilde{B}}{1 - \int_{p^*}^1 td\mu(t)} \\ &= c - \left(c + \frac{1}{1-p^*} \right) \left(1 - \frac{R(p^*)}{cp^*} \right) + \frac{1}{1 - \int_{p^*}^1 td\mu(t)} \end{aligned}$$

For any $p^* \in \mathcal{P}$, where $c - \left(c + \frac{1}{1-p^*} \right) \left(1 - \frac{R(p^*)}{cp^*} \right) + \frac{\tilde{B}}{1 - \int_{p^*}^1 td\mu(t)} > 0$, we have $\frac{\partial G(p)}{\partial p} > 0$ for any

$p < p^*$. So $G(p)$ is an increasing function of p . Thus $G(p) \leq G(p^*) = 0$, that is

$$cp \left(\tilde{B} - B_s^{sep}(p) \right) < \tilde{B} \frac{\int_{p^*}^1 td\mu(t) - p}{1 - \int_{p^*}^1 td\mu(t)}$$

So insurers will not deviate .

Similarly, for insurers with type $p \in [p^*, 0]$, the expected payoff of choosing pooling full securitization is

$$EU_s^{pool}(p) = W + (A(1-p) - Bp) - \tilde{B} \frac{\int_{p^*}^1 td\mu(t) - p}{1 - \int_{p^*}^1 td\mu(t)}$$

the expected payoff of mimicking lower-risk insurers of type p^* is

$$EU_s^{deviatesep}(p) = W + (A(1-p) - Bp) - c \left(\tilde{B} - B_s^{sep}(p^*) \right) p$$

So insurers will not deviate because $c \left(\tilde{B} - B_s^{sep}(p^*) \right) p > c \left(\tilde{B} - B_s^{sep}(p^*) \right) p^* = \tilde{B} \frac{\int_{p^*}^1 td\mu(t) - p^*}{1 - \int_{p^*}^1 td\mu(t)} > \tilde{B} \frac{\int_{p^*}^1 td\mu(t) - p}{1 - \int_{p^*}^1 td\mu(t)}$.

Therefore for any conjectured $p^* \in \mathcal{P}$, we can get λ to solve the following

$$\begin{aligned} B_s^{sep}(p) &= B_s^{sep'}(p)cp(1-p) \\ &= \tilde{B} \left(1 - \frac{R(p^*)}{cp^*} \right) \left(\frac{1-p^*}{p^*} \right)^{\frac{1}{c}} \left(\frac{p}{1-p} \right)^{\frac{1}{c}} \\ A_s^*(p) &= \tilde{B} \left(1 - \frac{R(p^*)}{cp^*} \right) \left(\frac{1-p^*}{p^*} \right)^{\frac{1}{c}} \left(\frac{p}{1-p} \right)^{\frac{1}{c}+1} \end{aligned}$$

□

Proof of Proposition 8

Proof.

1. a. Suppose $\delta < \frac{1}{1+c}$, for any $p_1^* \in \mathcal{U}$ and $p_1^* < p_2^*$,

Given the conjecture of a partition equilibrium where insurers with type in the range $[0, p_1^*]$ choose full reinsurance, insurers with types in the range $[p_1^*, p_2^*]$ choose separating partial securitization, and insurers with types in the range $[p_2^*, 1]$ choose pooling full securitization, for any $p_1^* \in \mathcal{U}$. We now check whether this conjecture is indeed an equilibrium for any given $p_1^* \in \mathcal{U}$

For insurers with types in the range $[0, p_1^*]$, the expected payoff of full reinsurance is

$$EU_r(p) = W + (A(1-p) - Bp) - \frac{\tilde{B}\delta p}{1-p-\delta}$$

If they deviate to choose partial securitization, the expected payoff is

$$EU_{deviatesep}(p) = W + (A(1-p) - Bp) - c(\tilde{B} - B_s^{sep}(p))p$$

For given p_1^* , the incentive compatible partial securitization coverage for the insurer with type p is

$$B_s^{sep}(p) = \tilde{B} \left(1 - \frac{\delta}{(1-p_1^* - \delta)c} \right) \left(\frac{1-p_1^*}{p_1^*} \right)^{\frac{1}{c}} \left(\frac{p}{1-p} \right)^{\frac{1}{c}}$$

If they deviate to choose pooling full securitization, the expected payoff is

$$EU_{deviatepool}(p) = W + (A(1-p) - Bp) - \tilde{B} \frac{\int_{p_2}^1 td\mu(t) - p}{1 - \int_{p_2}^1 td\mu(t)}$$

Let

$$\Phi(p) = \frac{\tilde{B}\delta p}{1-p-\delta} - cp \left(\tilde{B} - B_s^{sep}(p) \right)$$

It is easy to show that $\Phi(p)$ is a convex function.

Since $\Phi(0) = \Phi(p_1^*) = 0$, then, we can have $\Phi(p) \leq 0 \forall p \in [0, p_1^*]$. That is

$$\frac{\tilde{B}\delta p}{1-p-\delta} < c \left(\tilde{B} - B_s^{sep}(p) \right) p$$

Let

$$\Psi(p) = cp \left(\tilde{B} - B_s^{sep}(p) \right) - \tilde{B} \frac{\int_{p_2}^1 td\mu(t) - p}{1 - \int_{p_2}^1 td\mu(t)}$$

Then we have

$$\frac{\partial \Psi(p)}{p} \Big|_{p < p_2^*} > \frac{\partial \Phi(p)}{p} \Big|_{p = p_2^*}$$

and we have

$$\frac{\partial \Phi(p)}{p} \Big|_{p = p_2^*} = c - \left(c + \frac{1}{1-p_2^*} \right) \left(1 - \frac{R(p^*)}{cp_2^*} \right) + \frac{1}{1 - \int_{p_2}^1 td\mu(t)}$$

For any $p_1^* \in \mathcal{L}$, we have $\frac{\partial \Phi(p)}{p} \Big|_{p = p_2^*} > 0$. Thus $\frac{\partial \Phi(p)}{p} > 0$ for all $p < p_2^*$. Therefore, $\Phi(p)$ is an increasing function of p . So we have $\Phi(p) \leq 0$ for all $p < P_2^*$

That is

$$cp \left(\tilde{B} - B_s^{sep}(p) \right) < \tilde{B} \frac{\int_{p_2}^1 td\mu(t) - p}{1 - \int_{p_2}^1 td\mu(t)}$$

Since

$$\frac{\tilde{B}\delta p}{1-p-\delta} < c\left(\tilde{B} - B_s^{sep}(p)\right)p < \tilde{B} \frac{\int_{p_2}^1 td\mu(t) - p}{1 - \int_{p_2}^1 td\mu(t)}$$

thus

$$EU_r(p) > EU_{deviatesep}(p), \quad EU_r(p) > EU_{deviatepool}(p)$$

Therefore insurers with types in the range $[0, p_1^*]$ will not deviate to choose either separating partial securitization or pooling full securitization.

For insurers with types in the range $[p_1^*, p_2^*]$, the expected payoff of full reinsurance is

$$EU_r(p) = W + (A(1-p) - Bp) - \frac{\tilde{B}\delta p}{1-p-\delta}$$

If they deviate to choose full reinsurance, the expected payoff is

$$EU_{deviatesep}(p) = W + (A(1-p) - Bp) - c(\tilde{B} - B_s^{sep}(p))p$$

If they deviate to choose pooling full securitization, the expected payoff is

$$EU_{deviatepool}(p) = W + (A(1-p) - Bp) - \tilde{B} \frac{\int_{p_2}^1 td\mu(t) - p}{1 - \int_{p_2}^1 td\mu(t)}$$

Since $\Phi(p)$ is a convex function with $\Phi(0) = \Phi(p_1^*) = 0$, we can show $\Phi(p) > 0$ for $p > p_1^*$, that is,

$$\frac{\tilde{B}\delta p}{1-p-\delta} > cp\left(\tilde{B} - B_s^{sep}(p)\right)$$

Thus

$$EU_r(p) > EU_{deviatesep}(p)$$

Also, when $p < p_2^*$, we have

$$c(\tilde{B} - B_s^{sep}(p))p < \tilde{B} \frac{\int_{p_2}^1 td\mu(t) - p}{1 - \int_{p_2}^1 td\mu(t)}$$

therefore,

$$EU_r(p) > EU_{deviatepool}(p)$$

Therefore insurers with types in the range $[p_1^*, p_2^*]$ will not deviate to choose either separating partial securitization or pooling full securitization.

Similarly, we can show that insurers with types in the range $[p_2^*, 1]$ will not deviate.

Therefore, the conjecture is indeed a PBE of the game.

- b. If for given $p_1^* \in \mathcal{U}$ such that $p_1^* > p_2^*$, partial securitization choice is always dominated by either reinsurance or pooling securitization. Subsequently, the PBE is characterized as a partition with two subintervals.

Thus the threshold indifference point p_3^* satisfies:

$$\frac{\tilde{B}\delta p_3^*}{1 - p_3^* - \delta} = \tilde{B}R(p_3^*)$$

For the insurers with risk type in the interval $[0, p_3^*]$, the expected payoff of choosing full reinsurance is

$$EU_r(p) = W + (A(1-p) - Bp) - \frac{\tilde{B}\delta p}{1 - p - \delta}$$

If they deviate to choose full securitization, the expected payoff is

$$EU_s^{pool}(p) = W + (A(1-p) - Bp) - \tilde{B} \frac{\int_{p_2^*}^1 t d\mu(t) - p}{1 - \int_{p_2^*}^1 t d\mu(t)}$$

$$\text{Let } \Pi(p) = \frac{\tilde{B}\delta p}{1 - p - \delta} - \tilde{B} \frac{\int_{p_2^*}^1 t d\mu(t) - p}{1 - \int_{p_2^*}^1 t d\mu(t)}$$

It is easy to show that $\Pi(p)$ is an increasing function of p and $\Pi(p_3^*) = 0$.

Thus, for $p < p_3^*$, $\Pi(p) < 0$, that is, $\frac{\tilde{B}\delta p}{1 - p - \delta} < \tilde{B} \frac{\int_{p_2^*}^1 t d\mu(t) - p}{1 - \int_{p_2^*}^1 t d\mu(t)}$.

So

$$EU_r(p) > EU_{deviatepool}(p)$$

Thus insurers with risk type lower than p_3^* choose reinsurance.

For $p > p_3^*$, $\Pi(p) > 0$, that is, $\frac{\tilde{B}\delta p}{1 - p - \delta} > \tilde{B} \frac{\int_{p_2^*}^1 t d\mu(t) - p}{1 - \int_{p_2^*}^1 t d\mu(t)}$.

So

$$EU_s^{pool}(p) > EU_{deviaterein}(p)$$

Thus insurers with risk type greater than p_3^* choose pooling full securitization.

Thus insurers with risk type in the interval $[0, p_3^*]$ will not deviate. Similarly, it is easy to show that insurers with risk type in the interval $[p_3^*, 1]$ will not deviate. Therefore, the conjecture is a PBE.

2. Suppose $\delta > \frac{1}{1+c}$. Reinsurance is too expensive to be an optimal choice of the insurers. Thus, the PEB is characterized the same as in Lemma 6.

□

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