Sovereign yield spreads in Europe : credit-risk premia and the CDS-bond basis

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(*Preliminary version - please do not circulate*)

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1 Sovereign yield spreads: measuring credit risk

After the burst of latest financial crisis in Europe, long-term government bond yield spreads of Euro-area countries *vis-à-vis* Germany have become a major indicator to evaluate those sovereign entities' financial state of health; more precisely, yield spreads are used to measure *credit risk* associated to target country, whereas Germany is used as a credit-risk free benchmark.
The spreads we refer to are defined as the difference between ten-years interest rates; as long as credit conditions remained stable, up to 2008 at least, the non-zero observed spreads across different european states were normally attributed to country-specific factors. At the same time, rates remained globally aligned with each other and with european benchmark money-markets too.
When latest financial crisis reached Europe in 2010, upward shifts across country specific yields due to credit-risk related movements raised german obligations to the role of preferred investments across Europe. These excess-demand phenomena were due to the perception of Germany as a reliable credit-risk free borrower, a significant benchmark especially for those countries sharing common Euro-area factors with it.
Observing the rising importance of spreads' magnitude in recent years, severely affecting policy making too, we investigate the reliability of such a number when used to reckon european countries' creditworthiness.
In order to compute yield spreads, firstly a specific term structure of interest rates is retrieved for any of the countries, using the most liquid fraction of their correspondent sovereign obligations market.
The use of spread as a proxy for credit risk then relies on the assumption that country-specific yields embed a *credit-risk premium* and, in order to quantify it, the distance of that specific yield from the german one is measured.
Nonetheless, yields can be statistically explained using a wide range of different factors [5]; this suggests that credit risk could be only one of the determinants of Euro-area spreads.
Hence, we investigate whether each specific long-term sovereign rate includes other premia; by *other*, we mean any premium that is neither directly attributable to credit risk nor to systemic movements in the Euro-area, and thus still remains country-specific.
Exhibit 1(a) presents a stylized decomposition of sovereign yields across the Euro-zone: we separated yield driving factors into three distinct classes. Common practice is to take the difference with german rate so as to isolate the credit risk component; according to our decomposition, this is equivalent to both consider german yield not to embed any credit risk and cancel any eventual *basis* component.
The first issue to be tackled is to spin off yield portions attributable to credit risk *per-se*, so as to quantify the fraction which deserves different explanations; in order to decouple these components, we use credit derivatives.

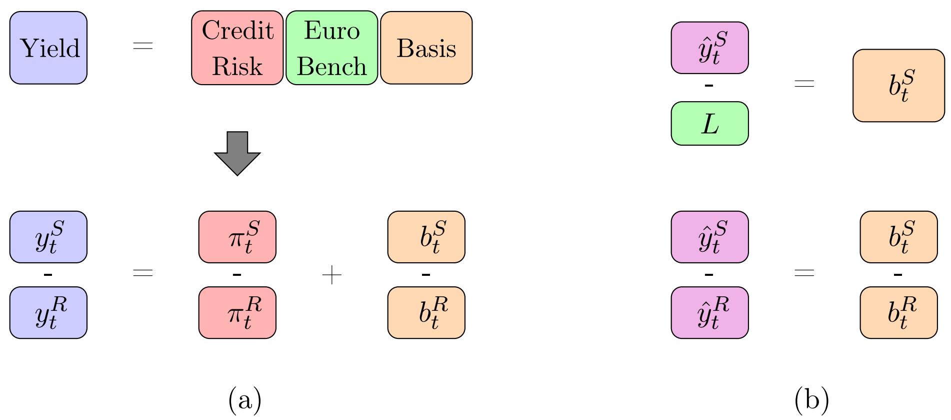

Exhibit 1: (a): European yield determinants and yield spreads; (b): CDS-bearing yield spread on Euribor (top) or between different countries (bottom) reveals the basis.

The basic vanilla instrument is *Credit Default Swap* (CDS): custom CDS-based hedging strategies exploits the countermonotonic motions of CDS spreads and bond yields.
As an example, consider a case of *credit crunch*: any combined long position on CDS and bond benefits from the rise in the derivative position and is affected by the rise in yields. Market efficiency sets the difference of these two variations to zero, so that hedging is completed; a rather detailed survey upon statistical hedging techniques built up with credit derivatives is [6].
We proposed to decompose yields and so we must be able to imply a *credit-risk free* long term yield for each specific country.
Retrieving yields of bond portfolios is a typical issue in *fixed-income analysis* [2]: fixed future cash flows are combined with current obligation price to retrieve a theoretical continuosly compounded yield-to-maturity. Term structures are implied using yields to different maturities and interpolating a set of time-continuous functions, which are by assumption described through a relativelty limited parameter set.
We define *CDS-bearing bond* a combined long position on a bond and the correspondent CDS; if we attach a proper (in terms of maturity) CDS to any of the bonds in the market, each of the portfolios we obtain is credit-risk free, so the implied term structure is a *credit-risk-free* term structure.
Exhibit 1(b) shows that, as the implied *CDS-bearing yields* are released of credit risk, the existence of the basis component can be proved by differencing over a benchmark Euro-area rate.
Claiming to be able to create *credit-risk-free* positions using CDS contracts, and that yields can be implied using fixed income techniques requires further explanations.

Indeed, before 2009, credit derivatives are specifically tailored to counterparties writing them, are traded over-the-counter and could thus suffer from lack of data or drains in liquidity; the standardization process [14] and [15] which interested European CDS market in late 2009 provides a correct and computationally efficient framework to pursue our objectives.
The *ISDA CDS-Bang* standardization process was aimed to regulate the CDS market which suffered from (and was cause of) several shortcomings during latest financial crisis; briefly, new contract features include the introduction of regulating authorities, namely a *Central Clearing House* (CCH), aimed at reducing both counterparty and liquidity risk, and a *Determination Committee* to take binding decisions on credit events deemed to trigger any of the relevant credit derivatives.
This new *standardized* market provides changes in specific contract features too: standard CDS have standard payment dates, standard maturities and fixed coupons. Moreover, an *upfront payment* made at inception is introduced, and could be viewed as a *spot price of protection*, floating in accordance with the microstructure of this new standard market.
We thus exploit the characteristics of these new contracts' cash flows so as to be able to use standard fixed-income methodologies for market-implied term structures; the presence of a CCH justifies our approach, in the sense that CDS-spreads should not contain any counterparty or liquidity risk premium, reflecting *exactly* credit risk.
These new contracts features make also the construction of credit risk-free portfolios an easy goal to achieve: merging CDS to bonds in the same term structure corresponds to melt sovereign yields and credit risk term structures into one.
The difference between long term *naked* yields and implied CDS-bearing yields then exactly quantifies credit risk premium; moreover, each country-specific term structure is credit-risk free, and thus, according to our decomposition, consists of a common Euro-zone component plus a country specific basis. Taking differences on a european benchmark, such as the Euribor rate, reveals this latter.
We can also test our decomposition by *spreading* CDS-bearing yields between two different countries: being both of them credit risk free, their difference should be a stationary market noise whether no basis were present. Notice that this latter approach does not depend on the particular Euro-area benchmark chosen.
The basis we imply is yet another form of the *CDS-bond basis*, which already captured the attention of central banks and regulators, see [4], [20].
Section 2 outlines the general methodology used to imply long-term yields to be compared for both naked and CDS bearings bond, briefly describing the features of the standardization process concerning our approach. Section 3 presents our results for Germany, Italy and Spain, these latter coupled together as sharing similar rating according to all major agencies; section 4 summarizes the results.

2 The yield curve

Although several different interpolation methods can be used to extract a benchmark term structure from government bond market, see for example [16] for a rather exhaustive review, the idea lying behind each of them is essentially the same.
At time t, it is assumed that target country A admits a term structure of its obligations market: that is, we assume that a zero-coupon-bond (zcb) exists for any maturity $T > t$. It is thus possible to define a time-varying map $\{P(t,T) : T \geq 0\}$ which represents the price of a zcb paying 1 at T.
Consider a single obligation, with maturity T_K; standard non-arbitrage assumptions imply that current bond *dirty* (bearing accrued coupon) price $p(t)$ must be equal to the sum of the obligation's discounted future cash flow:

$$p(t) = C \cdot \sum_{k=1}^{K} P(t, T_k) + \bar{p} \cdot P(t, T_K), \tag{1}$$

where $\mathcal{T} = \{T_k\}$ is the set of future payment dates, C is the bond's coupon and $\bar{p}$ is the *face value* paid at maturity T_K.
Each price can be used to retrieve the correspondent *yield-to-maturity*

$$y(t,T) = -\frac{\log(P(t,T))}{T-t} \qquad \forall \;\; T > t \; . \tag{2}$$

and this latter is in turn expressed as a function of the set of parameters θ_t; in this way, $y(t,T) = y(\theta_t, T)$ so that, using (9), a unique map is defined between the parameter set and the *term structure* $\{P(t,T)\}$ at time t.
Assuming face value is the same for any obligation, we can write

$$p(t) = G(y(t,\mathcal{T}), C) = G(y(\theta_t, \mathcal{T}), C) = G(\theta_t, \mathcal{T}, C) \tag{3}$$

where with $y(t,\mathcal{T})$ we indicate the unknown function $y(t,\cdot)$ evaluated on the grid defined by the fixed time schedule of the obligations.
Exhibit 2 outlines this procedure: current *clean* price plus accrued coupon (*floating*, as daily marked to market) are expressed as a combination of *fixed* future cash flows on a *fixed* time schedule.
Now consider at time t the set of the J_A obligations relevant for country A; specifically, we define the set of *dirty prices* $\{p_j^A(t)\}_{j=1}^{J_A}$, each with maturity T_j^A, coupon payment dates $\mathcal{T}_j = \{T_{kj}^A\}$ and fixed coupon $\{C_j^A\}$. We fix a country A and proceed without indexation to lighten notations.
Using (3), we collect bond prices in the vector $G(\theta_t) = (G^j(\theta_t, \mathcal{T}_j, C_j))_{j=1}^J$. The parameter vector θ_t^* is then defined as the solution to a weighted least squares problem:

$$\theta_t^* = \underset{\theta_t}{\operatorname{argmin}} \;\; (p(t) - G(\theta_t))' W(\theta_t)(p(t) - G(\theta_t)) \tag{4}$$

where $p(t) = (p_j(t))_{j=1}^J$ is the vector of observed prices for the obligations set and, for any matrix M, we denote with M' its transpose.

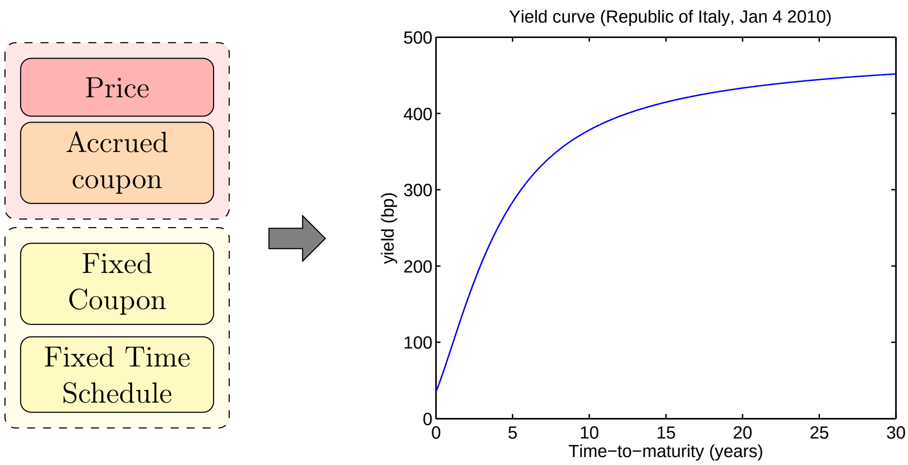

Exhibit 2: Yield curve: for each obligation, we combine floating quantities (left, red) with fixed ones (left, yellow) and use interpolation techniques to retrieve a yield curve (right).

The weighting matrix $W(\theta_t)$ can be constructed in different ways, but with the same principle of refining interpolation in those time intervals where most of the bonds' maturities are concentrated.
The function G typically depends on the interpolating functions: here we chose Nelson-Siegel method [17], as their algorithm and following refinements are the most used amongst central banks [1].
It is worth to stress that *no default probabilities are taken into account when computing yields*. Indeed, common practice is to assume that the obligations' market prices reflect any risk premium attributable to credit events that are likely to occur in the future.
Details of the method are provided in the appendix: it is however worth to mention here that θ_t in our setting is a four parameter vector $\theta_t = (\beta_{0t}, \beta_{1t}, \beta_{2t}, \lambda_t)$ and the yield is expressed through a three-dimension basis $(g_{0t}, g_{1t}, g_{2t}) \equiv (1, g_{1t}, g_{2t})$ as

$$y(t,T) = y(\theta_t, T) = \beta_{0t} + \beta_{1t} \cdot g_{1t}(\lambda_t, T) + \beta_{2t} \cdot g_{2t}(\lambda_t, T)$$

It is common to refer to the parameters respectively as *level*, *slope*, *curvature* and *scale* of the yield curve; the reasons behind this terminology are beyond the aim of this paper, but names are helpful in capturing different forms of shape changes of the curve along time. A detailed discussion can be found in [7].
A typical shape is shown in Exhibit 2: since throughout the paper we will focus on $T = 10$ year maturities yields, we will simply write y_t to indicate $y(\theta_t^*, 10)$, where for any *fixed* t the vector θ_t^* is defined in (4).

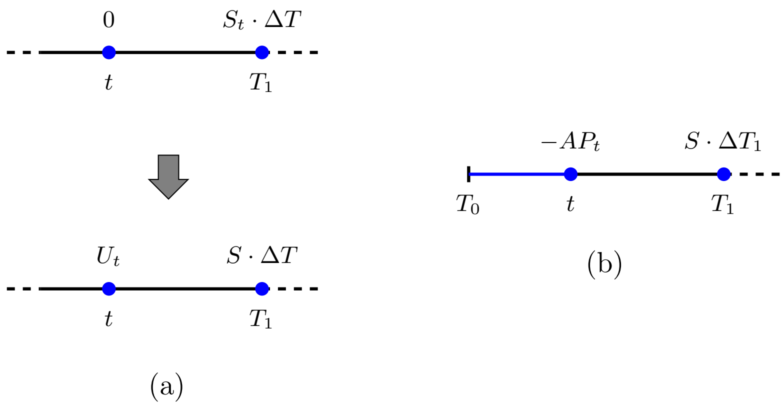

Exhibit 3: (a): Transition from time-variating spread S_t to time-variating upfront U_t plus fixed spread S; (b): Accrued first coupon $AP_t = -S \cdot (t - T_0)$, received by the protection buyer.

2.1 Standard CDS contracts: CDS-bearing yields

As mentioned in previous section, standardization of credit derivatives provides the correct background to perform our tests using market data. A complete description of the enhancements expected from this new standardized credit derivatives market is beyond the scope of this work; the interested reader can consult ISDA[1] official documentation [8] and [9], or refer to [3].
We assume the CCH injects sufficient liquidity in the market so that positions on any of the derivative contracts are not charged with liquidity premia.
Moreover, the clearing house reduces counterparty risk, which is now scattered among several big dealers, the same having easy access to credit market such as large investment institutions, which are also some of the major owners of sovereign debt across Europe.
Hence we *assume* that prices of CDS-contracts are unresponsive to counterparty risk and that there is sufficient liquidity whatever the maturity of the contract: CDS price credit risk.
Exhibit 3(a) shows the introduction of standard market coupons: a free entrance into a long credit derivative position attracted speculators, so the introduction of *upfront payments* contributed also to discourage purely speculative naked CDS positions.
This enforces our choice to perform this analysis from the perspective of a hedger, as this type of investor should be the most attracted by these market improvements; from a technical point of view, whether future coupons were priced daily, we would obtain future cash flows varying in time, and this would add another source of randomness and inaccuracy in the statistical results.

[1]International Swaps and Derivatives association, *www.isda.com*

Credit derivatives market platforms continue however to quote CDS at time t in term of a varying spread S_t; ISDA provides a standard mechanism [12] to convert quoted spread to upfront payment. We refer to [3] for a detailed discussion, although it is worth to stress that the mechanism is *uniquely* defined for each participant in the standard market.

Upfront payment is immediately[2] due to the counterparty and can thus be interpreted as the *minimum premium demanded by the market to transfer credit risk*; this premium is settled by imposing an additional percentage of the obligation's face value to be attached to the initial price together with a set of (standard) fixed coupons to be paid in the future.

Notice also that this spot payment is a relevant against counterparty risk and collateral posting: if default occurs, U is the amount of money flowing immediately towards the protection seller, which could help him to liquidate his short CDS positions in case of sudden occurence of relevant credit events. This will also partially mitigate contagion effects on credit networks due to any of the counterparties' possible default.

Exhibit 3(b) shows the *first full coupon clause*; before standardization, a tricky mechanism determined when first payment was due, and how long it should be accrued. Standard contracts provides the buyer to pay full protection for a fixed accrual period (one quarter) between standard dates. He is then reimboursed of accrued premium $AP_t = S \cdot (T - t)$ which repays him for the credit risk protection period he did not benefit of; the premium is then subtracted from upfront price to determine[3] the *cash settlement amount* $u(t) = U_t - AP_t$. It is clear how this standardization process calls for the use of fixed-income tools: the cash flow of the CDS is now composed of a cash amount to be immediately paid, computed combining a spot price and the accrual of a coupon, to be added to a future cash flow which is deterministic both in term of payment dates and amounts, see Exhibit 4.

We could state that *the cash flow of the derivative has uniformed to that of its underlying*. It can be argued that future CDS payments are contingent on default, hence default probabilities should be taken into account as source of randomness in future cash flows, and used in pricing equations.

However, as mentioned before, that same assumption lies behind any of the standard methods used to imply yield spreads: we postulate the existence of a term structure reflecting credit risk within the yields and take differences with german yields exactly to isolate this component .

[2] Ignoring small effects coming from settlement dates and daily count conventions.

[3] Here, we underline that as long as a unique pricing mechanism is used by any dealer to imply upfront payments from quoted spread, we have a unique mapping from CDS quoted spread to upfront payments, so any discussion on correctedness on ISDA Standard CDS model is not meaningful here. Any investor willing to enter into a CDS position and looks at market quotes for protection determines the same *spot price* for credit risk $u(t)$, playing the role of dirty bond price $p(t)$.

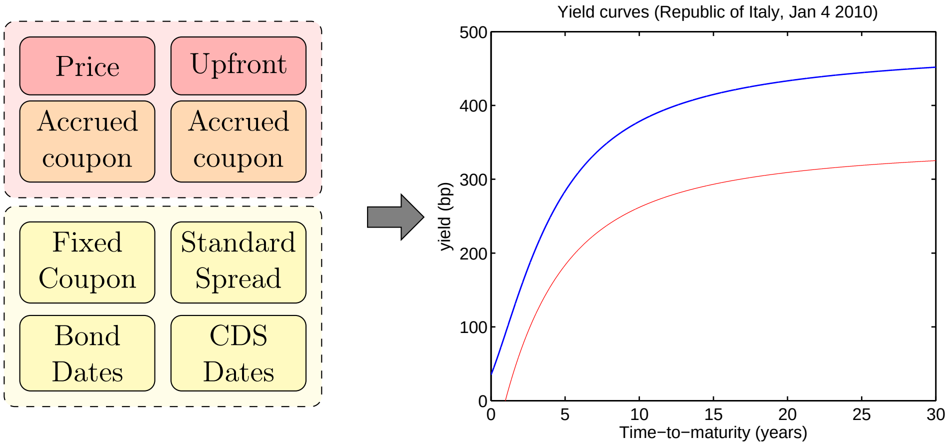

Exhibit 4: CDS-bearing yield curve: standardization allows us to rely exactly on the same methodology commonly used for unhedged yields. Again, we combine floating quantities (left, red) with fixed ones (left, yellow) and use interpolation techniques to retrieve a yield curve (right).

A combined position on CDS and obligation lead to a more costly bond in terms of initial price that will also repay the owner with lower coupons, due to the (quarterly) payment of CDS standard spread.
We chose to attach a CDS-contract to any of the relevant obligations contributing to the *naked* yield curve construction the way a highly-risk averse investor would. Indeed, standardized contracts are now traded with only ten different standard maturities: we decided to hedge the bond position with the CDS expiring on the first standard maturity which is able to cover the whole residual life of the bond.
This latter is an example of shortcomings stemming from standardized CDS market: for example a large premium must be paid for early expiring obligations, which must be covered up to the minimum standard available maturity, namely six months.
In this sense, credit derivatives could be relatively less attractive to investors that may prefer to enter into specifically tailored over-the-counter transactions.
We define *CDS-bearing bond* a combined long position on bond and standard CDS contract, the maturity of this latter selected as already mentioned; the marked-to-market value of the position at time t is

$$p(t) + u(t).$$

As for naked bond positions, standard non-arbitrage principles set this value equal to its discounted future cash flow.
We thus follow the same approach used for *naked* bond positions, and postulate the existence of a CDS-bearing term structure $\{\hat{P}(t, T)\}$ for each maturity $T > t$.

This credit risk-free zero-coupon curve is retrieved by numerical interpolation, using CDS-bearing positions across maturities as knots for this class of portfolios. We thus set:

$$p(t) + u(t) = C \cdot \sum_{T \in \mathcal{T}} \hat{P}(t,T) + \bar{p} \cdot \hat{P}(t,\bar{T}) - S \cdot \sum_{T \in \hat{\mathcal{T}}} \hat{P}(t,T)$$

where $\mathcal{T}$ is the set of bond payment dates, $\hat{\mathcal{T}}$ is the set of CDS payment dates and C and S are the fixed bond and cds coupon, respectively.
If default happens, the pricing equation presents a discontinuity, as post-default cash flows are no more due. This is however a shorthcoming that this methodology shares in common with measuring credit risk through sovereign yields, and since we are exactly testing this methodology, we are allowed to rely on the same assumptions.[4]
Using exactly the same procedure depicted for naked yields, we can imply a *CDS-bearing bond yield* $\hat{y}(t) = \hat{y}(\hat{\theta}^*_t, 10)$; details are provided in the appendix.
In the end, for any country A, we imply out of market observations two time series $\{y^A_t\}$ and $\{\hat{y}^A_t\}$; under the assumption that counterparty and liquidity risk can be set to zero thanks to standardization, the two yields differ one from the other because of credit risk. We define the *credit-risk (yield) premium* for country A at time t as

$$\pi^A_t = y^A_t - \hat{y}^A_t. \tag{5}$$

Notice that, as this new payments schedule is composed of an additional cost to bear at inception plus a sequence of negative coupons, $\hat{y}_t$ will be lower than y_t for any t, so π_t is a nonnegative process.
Considering a long-term benchmark risk-free rate for european countries, such as the 10-year Euribor rate $L_t = L(t, 10)$; we define the *CDS-bond basis* for country A as

$$b^A_t = \hat{y}^A_t - L_t \tag{6}$$

The choice of this benchmark could be criticised, but we underline that the whole CDS-conversion mechanism is based on this rate, hence this choice is justified.
If instead we compute the *CDS-bearing yield spread* for any pair of countries (A, B)

$$\hat{y}^A_t - \hat{y}^B_t \tag{7}$$

credit risk of both entities is hedged, so any deviation from zero of (7) measures the excess premium of country A with respect to country B which is not directly explained by credit risk, as both positions are credit risk-free by assumption.
If credit risk were the only driver of yield spread and CDS provided full hedging, CDS-bearing spreads should be equal to zero whatever the chosen pair of countries.

[4]In order to consider post-default losses in details, post-default scenario-analysis would be a possible rightful approach, but this goes beyond the scope of this work.

3 Empirical results

We consider the sovereign bond market of three european countries, namely Germany, Italy and Spain. The aim of this choice is twofold: on the one hand, we can test how financial markets reflect the presumed credit-risk-free nature of german obligations; in our framework, this corresponds to compute the magnitude of the credit risk premium and compare naked and CDS-bearing yield. On the other hand, Italy and Spain share rather similar credit score across all major rating agencies, hence it is worth to compare the respective credit risk free portfolios and to test the presence of the basis *independently* from the chosen benchmark rate.
We take a sample period going from January 4th, 2010 to June 20th, 2013, using daily observations: as the CDS european market standardization started out in August 2009, we leave five months to the market to *digest* new features and become sufficiently liquid to justify a market-based analysis.
In principle, implied yield curves are computed and directly avaialble on all main databases: here, however, we are forced to imply yield curves on our own, as it is necessary to begin with the obligation prices to merge each of them with the CDS cash flow and retrieve $\hat{y}_t$.
Hence, we establish to use daily observation from the whole bond market of each of the countries; in order to choose the most liquid fraction, we decided to remove any of the obligation which price remains unchanged for more than ten working days.
Concerning CDS, we use daily observation of conventional spread S_t and convert them to cash settlements $u(t)$ using our own algorithms, based on ISDA standard methodology [12] and the following amendments [10] and [11].
The 10-year Euribor rate is not a spot rate but a swap one, hence a yield curve must be again implied using loglinear interpolation on the grid induced by semiannual swap coupons; an exhaustive description of the methodology can be found in [13].
The whole dataset is taken from Bloomberg database.
We imply a set of 903 daily obervations for both naked and CDS-bearing yields; in order to eliminate daily effects and harmonize our analysis in terms of variance, we take end of week average of previous observations and obtain:

$$\{y_t^A\}_{t=1}^T \quad \{\hat{y}_t^A\}_{t=1}^T \quad \{L_t\}_{t=1}^T$$

with $T = 177$ for each country $A \in \{GER, ITA, SPA\}$.
Credit risk premia and country specific basis are defined by taking yields differences

$$\{\pi_t^A\}_{t=1}^T = \{y_t^A - \hat{y}_t^A\}_{t=1}^T \quad \{b_t^A\}_{t=1}^T = \{\hat{y}_t^A - L_t\}_{t=1}^T$$

We start by analyzing the german obligations market, to test whether Germany is perceived as a credit-risk-free borrower.

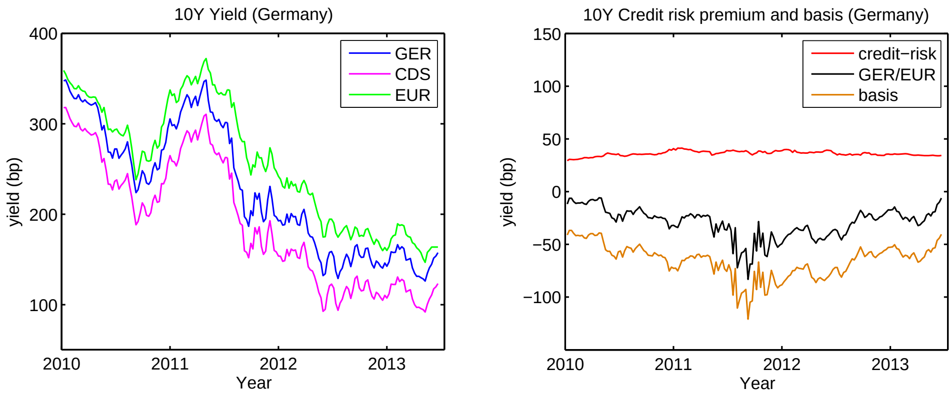

Exhibit 5: Naked yield y_t^{GER} and CDS-bearing yield $\hat{y}_t^{GER}$ compared to Euribor L_t (left); credit risk premium π_t^{GER}, basis b_t^{GER} and spread of german naked yield on Euribor $y_t^{GER} - L_t$ (right).

3.1 Germany: a reference benchmark

Exhibit 5 (left) presents both yields' evolution as implied by the german obligations market, together with the time series of Euribor rates. It is immediately evident how the three time series are following a similar pattern in time.

Statistical tables are provided in the appendix; unit is basis point. The peak of the three series is reached in early 2011: later on, the widely documented (see for example, [5] and references therein) *flight-to-quality* phenomenon becomes the main driver of german obligations market.

Yields drop down of nearly 200 basis point in less than one year, pushed by excess demand of investors willing to secure their money in the safest available investment. The average unhedged yield on the sample period is $\mathbb{E}\left(y^{GER}\right) = 227$ basis points, with a standard deviation of 70 basis points; the same standard deviation results from the process $\hat{y}_t$, suggesting CDS spread is not a major statistical driver of this latter process.

Table 1 summarizes descriptive statistics for the three processes; we notice that the Euribor rate, although displaying almost the same second moment, is on average 29 basis points higher than the naked interest rate.

Excess demand is again one possible explanation: indeed, while it is common to use Euribor as a theoretical benchmark rate, in practice only large investment institutions have access to that rate; thus a large portion of hedgers was forced to rely on the safety (in terms of credit risk) of german obligations.

The average credit risk premium is, by difference, $\mathbb{E}\left(\pi^{GER}\right) = 37$ bp; observe that credit risk premium cannot be zero, as any contract in the market must have a non-zero price, so that arbitrage opportunities are completely excised.

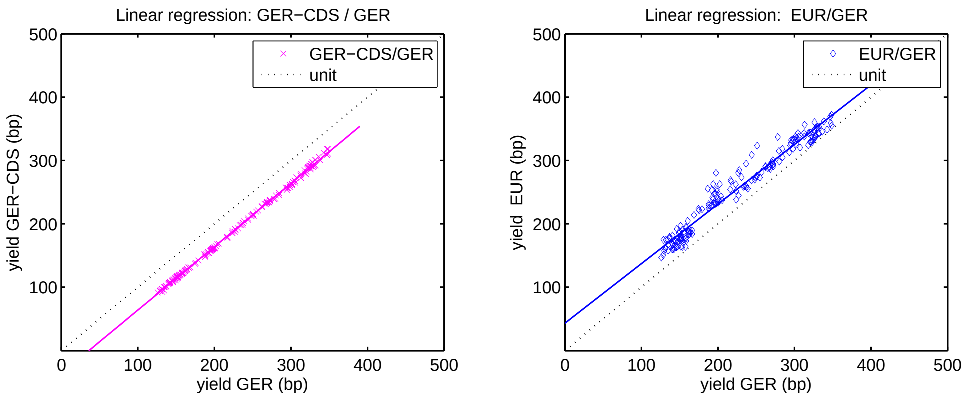

Exhibit 6: GER: Scatter plot and regression lines of y on $\hat{y}$ (left) and of y on L (right). The dotted line is the bisector of the first quarter.

Exhibit 5 (right) shows the credit risk premium π^{GER} and the basis b^{GER}; we chose also to represent the spread $y_t^{GER} - L_t$, so as to consider the difference between what (at least in principle) is a non-credit-risk-free position and the reference money-market rate; it is evident how credit risk premium is almost constant in time.
The correlation matrix displayed again in table 1 confirms this fact, as

$$\sigma_\pi^2 = \sigma_y^2 - 2\rho_{y\hat{y}}\sigma_y\sigma_{\hat{y}} + \sigma_{\hat{y}}^2 \quad ,$$

$\rho_{y\hat{y}} \approx 1$ and $\sigma_y \approx \sigma_{\hat{y}}$; the correlation of both yields with the Euribor rate is very high too, hovering around 98% and 97%, respectively.
The CDS-bond basis $b^{GER} = \hat{y}^{GER} - L$ is always negative: a downwward peak is recorded in late 2011, when deterioration in credit conditions in peripheral Euro-area countries exacerbated the demand for german obligations, thus lowering german yields also with respect to reference benchmark, for the reasons already mentioned.
Exhibit 6 plots the results from conducting linear regressions of CDS-bearing yields and Euribor rate on naked yields, while table 5 summarizes numerical results.
It is evident how statistical properties of y^{GER} and $\hat{y}^{GER}$ are the same: they differ by a constant which is the average cost of swapping default risk. Euribor rate is almost fully explained by german yields ($R^2 = 0.95$), and the intercept of the regression line represents the excess of 36 basis points between L and y^{GER}; in this sense, this analysis reveals that, on average, german yield is the mid-point between L and $\hat{y}^{GER}$.
We conclude that the perception of Germany as a risk free rate is fully justified by market-implied statistical analysis. Credit risk premium is constant, and covering a naked bond position with a CDS contract results in a useless waste of money.

Indeed, the variance of the CDS-bearing portfolio is unchanged, hence no effective hedging is performed, while the cost of protection lowers the return on the investment. The basis is negative becuase of the easier access to sovereign obligations of which most of the investors can benefit of; we can thus *assume* that investors does perceive a total lack of credit risk, and use y^{GER} as an alternative credit-risk free benchmark. The case of Germany suggests several features that a country-specific credit-risk-free rate $\hat{y}$ should exhibit when compared to the Euribor rate L: we expect a high correlation between the two as well as a large portion of explained variance in a linear regression setting, as measured by the R^2-statistic.
Moreover, we expect the CDS-bond basis to be negative, at least on average: indeed, financing at Euribor a hedged sovereign obligation position *must* in principle be costly. If this was not the case, those investment institutions which benefit of easy access to money market could finance themselves at L to buy sovereign debt covered with CDS-position through the CCH; if there were profits in doing so, huge amounts of obligations could be bought by such dealers, and concentration risk would be a major concern and a possible cause of severe systemic deficiencies.

3.2 Italy and Spain: hedging credit risk

Besides Germany, sovereign yields of other Euro-area countries present quite a different behavior from that of the Euribor benchmark; here, we chose to analyze long term interest rates of Italy and Spain, being these latter a glaring example of deterioration in credit conditions which severely affected sovereign obligations market.
The time series of yields, credit risk premia and basis are shown in exhibit 7 and 8 (left), while descriptive statistics are collected in tables 2 and 3.
Uncovered bond yields present two peaks in both countries, one in November 2011 and one between July and August 2012; the first is mainly caused by deterioration in credit conditions in Italy, while the second is driven by the crisis of Spanish banking sector, see again [5] for a rather detailed survey on this topic.
The average yield is 505 basis points in Spain, higher than that the italian, assessed at 461 basis points, although standard deviation is higher in Italy (79 versus 66 bp). CDS-bearing yields displays several interesting results: first of all, there's an alignment between the two average values, which now differ only from 23 basis points.
Secondly, the standard deviation becomes exactly the same across the two countries, confirming that CDS-based hedging is effective and moves yields to a common *credit-risk-free pattern*.
Observing the correlation matrix, we obtain similar results for both countries under examination: yields implied from unhedged positions are almost uncorrelated with CDS-bearing yields and show small negative correlations with Euribor rate.

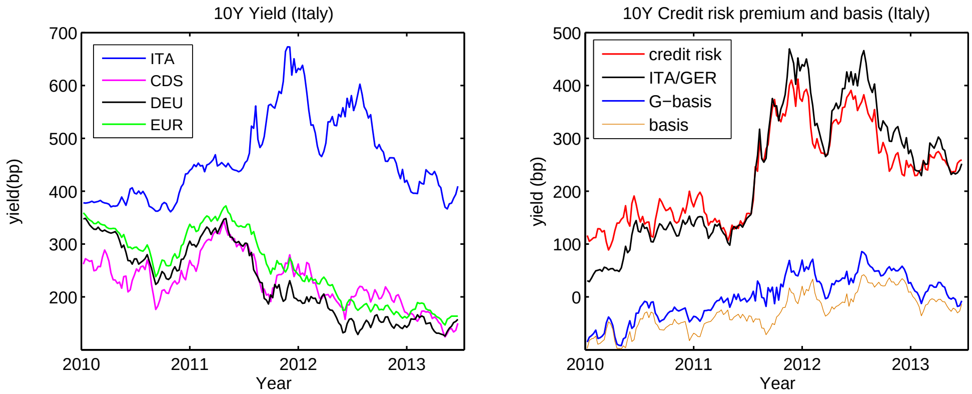

Exhibit 7: Naked yield y_t^{ITA} and CDS-bearing yield $\hat{y}_t^{ITA}$ compared to Euribor L_t and naked german yield y_t^{GER} (left); credit risk premium π_t^{ITA}, basis b_t^{ITA}, yield spread on Germany $y_t^{ITA} - y_t^{GER}$ and G-basis, the spread of CDS-bearing yield on naked german yield $\hat{y}_t^{ITA} - y_t^{GER}$ (right).

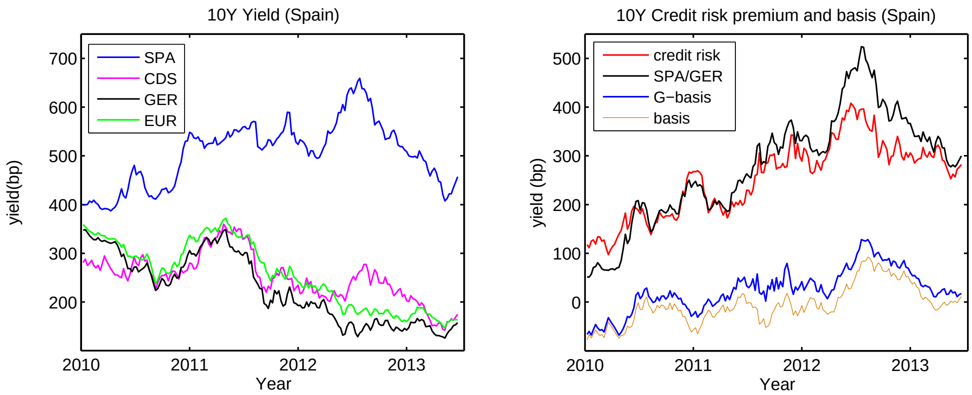

Exhibit 8: Naked yield y_t^{SPA} and CDS-bearing yield $\hat{y}_t^{SPA}$ compared to Euribor L_t and naked german yield y_t^{GER} (left); credit risk premium π_t^{SPA}, basis b_t^{SPA}, yield spread on Germany $y_t^{SPA} - y_t^{GER}$ and G-basis, the spread of CDS-bearing yield on naked german yield $\hat{y}_t^{SPA} - y_t^{GER}$ (right).

When credit risk is hedged, the resulting yields are highly correlated with Euribor rates, with $\rho = 0.86$ for Italy and $\rho = 0.82$ for Spain.
If we interpret covariance as a scalar product, we could imagine the existence of a *credit-risk free subspace* within the space of all portfolio yields, to which Euribor and german yield belong, and interpret CDS-bearing yields as the projection of bond yields onto this subspace. Indeed, correlation between hedged yields and any of the element of this subspace hugely rises, and yields converges to a statistically unique trajectory.
Table 4 shows how correlation between specific country portfolios changes when the naked position is hedged with the CDS contract; similarly to what happens with Euribor rate, Spain and Italy naked yields are negatively correlated with y^{GER}, and are instead positively correlated one with the other.
CDS-bearing yields are all highly correlated between themselves, yet still correlation is different from 1, confirming that hedging credit risk is not suffcient to perfectly align rates; extending our geometrical interpretation, we could assert that hedged portfolio yields can be written as a combination of a credit risk free component and a second component which does not lie on the credit-risk free subspace and thus must be attributed to other sources of randomness.
Exhibit 7 and 8 (left) shows the comparison between the credit risk premium as implied by our methodology to the standard yield spread *vis-à-vis* Germany.
This latter is higher than π when credit conditions are worse than average: it overestimate credit risk premium of up to 80 basis points for Italy and 128 bp for Spain, respectively. Yield spreads have also a (less pronounced) tendency to underestimate π when financial conditions are stabler.
On average, yield spreads hovers around 234 basis points for Italy and 278 basis points for Spain, while market implied credit risk premia based on CDS-hedging π is approximately the same (233 basis points) while is lower in Spain, assessing itself at 254 basis points.
We also observe a different behavior in the basis, either if computed as the spread on Euribor rate or as the spread on German naked yields; this latter similarity can be attributed to the high correlation between L and y^{GER}.
On average, the basis is still negative, approximately -30 basis points for Italy and -5 basis points for Spain; there's however an important remark worth to be mentioned. Indeed the basis becomes positive exactly when financial conditions of target country are heavily defective: borrowing at Euribor rate in order to hedge a sovereign bond position still pays a positive premium. One possible interpretation is that a spur to buy sovereign obligations is maintained when credit conditions are poor, in order to incentivize investment institutions to lend money to sovereign entities despite credit-risk related drawbacks.

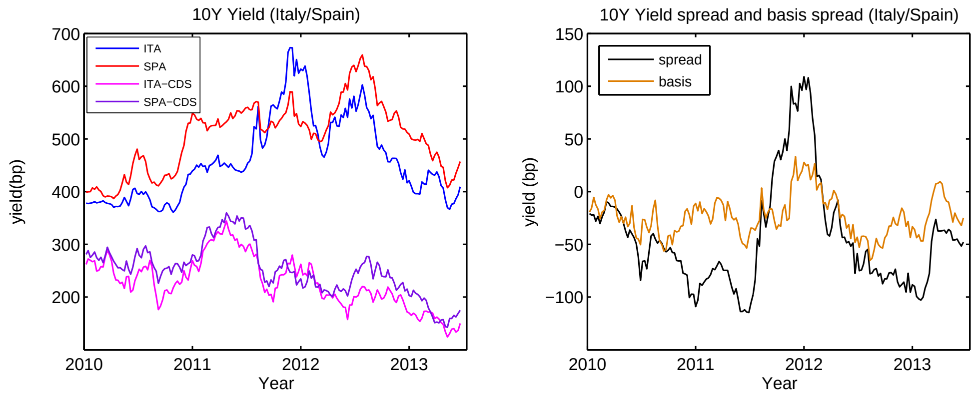

Exhibit 9: Naked yields y_t^{ITA} and y_t^{SPA} compared to the correspondent CDS-bearing yields (left); naked yield spread $y_t^{ITA} - y_t^{SPA}$ and basis spread $b_t^{ITA} - b_t^{SPA} = \hat{y}_t^{ITA} - \hat{y}_t^{SPA}$ (right).

Another possible interpretation is that, despite all new features offered by standard contracts, when market conditions are heavily averse, investors do not feel completely hedged with credit derivatives, still scared by the possibility that the CCH is not able to bear the aggregated risk stemming from large exposures to sovereign CDS. The CDS-bond basis we defined can thus be used as an indicator of credit conditions in the country under exhamination: indeed we can recognize a correspondence between its positive peaks and severly adverse market news; for example, the highest value of the basis is realized at the end of 2011 in Italy and in mid-2012 in Spain. Notice that minor movements within the positive region of the basis can be recognized also when relatively minor news affect target country's obligation markets, like the turmoil following political elections in Italy in March 2013, when the basis moves in the positive region, and reaches approximately 35 basis points.
Exhibit 9 displays a direct comparison between the two countries: it is of particular interest to explore both naked and CDS-bearing yield spread. It is clear how naked yield spread straight reflects fears of the investor on each particular market: computing $y^{ITA} - y^{SPA}$, we observe a positive peak of almost 100 basis points in late 2011 while the turmoil deriving from Spanish banking sector keeps this spread negative for most of 2012, with a downward peak in July 2012.
In principle, whether all our assumptions were true, we should observe a poorly varying spread between the two credit-risk-free portfolios, so as to be able to assimilate to a white noise process the difference

$$\hat{y}^{ITA} - \hat{y}^{SPA} = b^{ITA} - b^{SPA}. \tag{8}$$

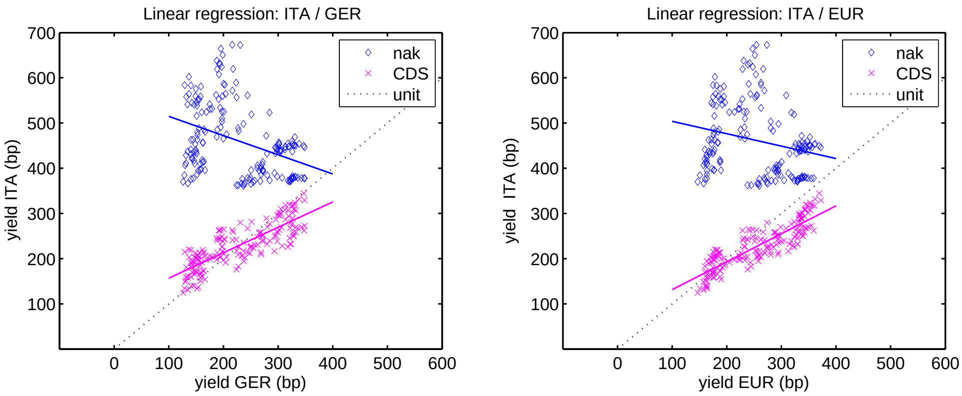

Exhibit 10: ITA: Scatter plot and regression lines of y on $\hat{y}$ (left) and of y on L (right). The dotted line is the bisector of the first quarter.

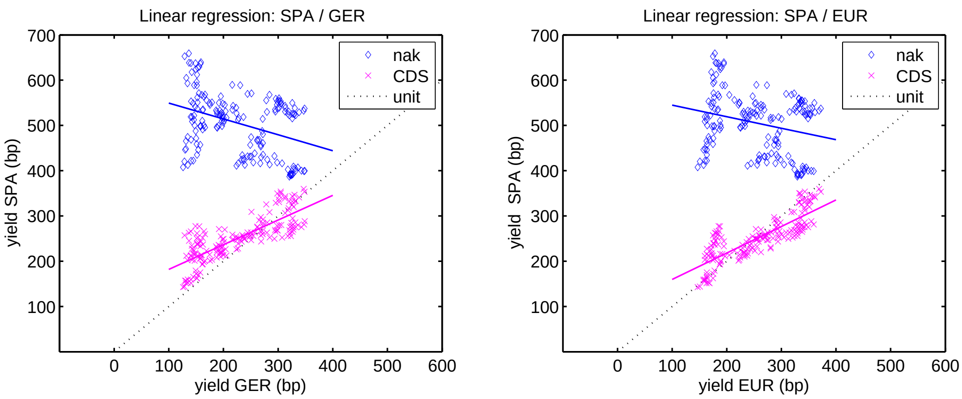

Exhibit 11: SPA: Scatter plot and regression lines of y on $\hat{y}$ (left) and of y on L (right). The dotted line is the bisector of the first quarter.

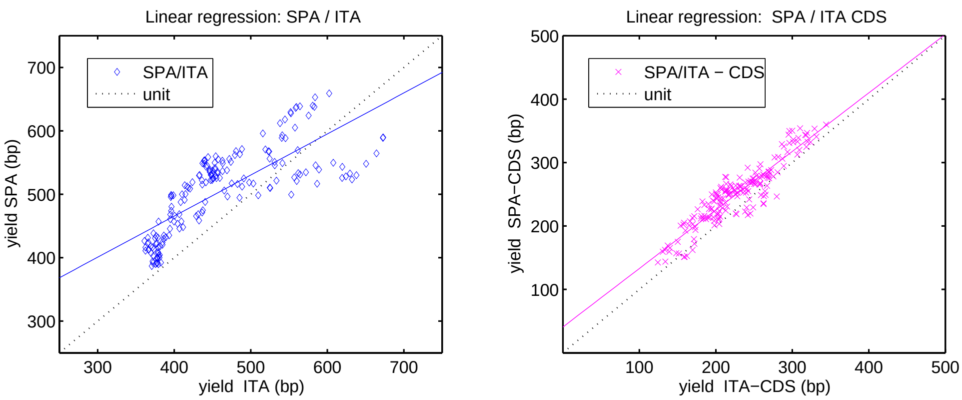

Exhibit 12: Scatter plot and regression lines of y^{SPA} on y^{ITA} (left) and of $\hat{y}^{SPA}$ on $\hat{y}^{ITA}$. The dotted line is the bisector of the first quarter.

Instead, the basis spread (8) is on average -23 basis points, but again up and downward peaks track the country which is suffering feeble financial conditions at the moment.
Exhibit 10, 11 and 12 plots observed yields together with least squares lines, while regression results are presented in the appendix.
Particularly, observing the R^2 statistic, we notice the large portion of variance which is explained when regression is performed between Euribor and CDS-bearing yields (tables 6 and 7) and among these latter (table 8), while their unhedged counterparties remain markedly scattered around regression lines.

4 Summary and conclusions

In this work, we discuss the reliability of long term yield spreads of Euro-area countries *vis-à-vis* Germany when used to measure credit-risk associated to sovereign borrowers, and explore eventual additional premia enclosed into interest rates.
We postulate that determinants of yields can be separated in three broad classes, namely credit-risk premia, country-specific basis and a common benchmark rate, and develop a methodology to isolate each of them within market implied long term yields. In particular, to isolate credit risk premium, we use Credit Default Swaps.
The standardization of credit derivatives market is the rightful background in which to perform our analysis: by relying on the introduction of a clearing house as a feature eliminating counterparty and liquidity risk embedded in such contracts, we assume that CDS spreads reflect credit-risk only.
Moreover, the *CDS Bangs* uniformed the cash flow of a CDS to that of its underlying; each contract now provides a standard schedule of payment dates and maturities, a prefixed standard coupon and a spot *upfront* payment.
Exploting these new features, it is possible to combine a long position on any obligation within target sovereign market with a Credit Default Swap covering the residual life of the obligation, simply by merging the cash flows of the two financial positions. We define each resulting position a CDS-bearing bond, and assume the existence of a term structure for such portfolios in order to imply a credit-risk free yield curve, comparing the yields we obtain with those implied through uncovered bond positions.
Observe that this approach is not consistent if applied to zero-upfront CDS, as future payments are contingent on daily market quotes, hence standard fixed income techniques are not directly appliable.
We use Euribor 10 year rate, bootstrapped from swap rates, as the standard Euro-area benchmark and retrieve the CDS-bond basis by difference; statistical analysis *under the physical measure* using market data is then performed for a sample period of three years and a half, from January 2010 to June 2013.

The results we obtain are significant especially for those big dealers having easy access to both credit and money market. Specifically, we observe that credit risk premium in Germany is almost constant, and that Euribor rate and german long term yield are too highly correlated to be statistically distinguished, this latter being on average lower than the former, due to flight to quality phenomena which interested Germany. Borrowing at Euribor to buy a naked german position is costly, and it would be even more costly to attach a CDS-position to the obligation, thus confirming the perception of Germany as a credit risk free entity.
We performed this same analysis for Italy and Spain: here credit risk is the main driver of yields, although common yield spreads *vis-à-vis* Germany tend to overprice credit-risk premium when financial conditions weightly deteriorate.
The negative correlation displayed by naked yields with respect to each risk free rate (Euribor or german yield) turns into a highly positive coefficient when a long CDS position is combined with each obligation, while variances are reduced, confirming the good performances of statistical hedging.
A geometric interpretation of this latter fact is that hedging procedures correspond to a projection of yields on a *credit risk-free* subspace: CDS-bearing yields obtained in this way are highly correlated one with each other as well as with Euribor rate.
Still, their variability is not completely explained, as evidenced by linear regressions, thus we must infer statistical properties of the basis; we observe that the basis is negative on average, but becomes positive during periods of severe financial distress of the country under examination.
This fact is even more evident if we consider the spread between hedged positions in Italy and Spain: indeed, by differencing these latter, we expect to find white noise distributed residuals attributable to specific market microstructures. Instead, we find the basis is on average greater in Spain, but each troubling market event moves the basis in the direction of the country which is suffering it.
One possibility is concluding that investors did not not perceive the set of new features in standard credit derivatives market to have completely eliminated collateral risks, such as counterparty risk and systemic crisis, so that our underlying assumption that CDS spread reflects only credit risk is not correct.
Another possible explanation is that, in those period of crisis, sovereign borrowers needs to increase public debt in order to repay accrued interest and thus investors, even if credit-risk is hedged, demand for an additional premium to switch their portfolios composition and include that country's distressed obligations.

Appendix A : Nelson-Siegel method

At time t, the yield curve $\{y(t,T) : T \geq t\}$ is defined as the image of the function

$$T \mapsto y(t,T)$$

mapping $T \geq t$ to the yield of a zero-coupon-bond (zcb) with maturity T.
Whether a zcb had been available for any T, and quoted $P(t,T)$ on the market, the yield would have been computed according to its definition:

$$y(t,T) = -\frac{\log(P(t,T))}{T-t} \qquad \forall \;\; T > t \; . \tag{9}$$

Since this is clearly not the case, the yield curve can be retrieved out of the most liquid fraction of the government bond market using a nonlinear method, namely Nelson-Siegel method [17], which will be explored in detail in what follows.
The method requires, as inputs, current bonds prices together with their fixed coupons and payment dates, including reimbursment of face value at maturity.
It is worth to stress that no default probabilities are taken into account when computing yield spreads. Indeed, common practice is to assume that the obligations' market prices reflect any risk premium attributable to credit events which could eventually occur in the future.
For a generic country A, we consider at time T the *dirty*[5] prices $\{p_j(t)\}_{j=1}^{J_A}$, each with maturity T_j, coupon payment dates $\{T_{kj}\}$ and fixed coupon $\{C_j\}$.
Let $P(t,T)$ denote the (unknown) zcb price (or *discount factor*) at time t with maturity T; if the market is arbitrage free, then

$$p_j(t) = \sum_{k=1}^{K_j} C_j \cdot P(t, T_{kj}) + \bar{p} \cdot P(t, T_{K_j,j}) \tag{10}$$

for any obligation $j \in \{1 \dots J\}$, where $\bar{p}$ is the bond face value.[6]
Observe that left member of (10) is due to the bond issuer at inception while the right one is contingent on credit risk: future coupons will not be paid, for example, in case of default. In order to imply the yield curve, we interpolate the *zero-coupon curve* $\{P(t,T)\}$ and then use equation (9).
Nelson-Siegel interpolation is based on the projection of zero-coupon price on a three-functions basis, which allows us to describe that curve in terms of a small parameters set; interpolation knots are the set of all coupon payments dates

$$\{T_{kj}\} \quad i = 1 \dots \;\; K_j \;, \;\; j = 1 \dots J.$$

[5] The prices including the accrued payment on the bond's next coupon; the payment is zero only on any bond's coupon payment date.

[6] Again, for the sake of notation's simplicity, we simply write C_j but consider it as the annual coupon multiplied by the year fraction between two consecutive coupon dates.

The choice of the functions to be used is mainly due to statistical analysis [17], which evidenced that most of the observed shapes in the zero-coupon curve across years were efficiently reproduced by setting:

$$
\begin{aligned}
y(t,T) =& \beta_{0t} + \beta_{1t}\left[\frac{\lambda_t}{\tau}\exp\left(-\frac{\tau}{\lambda_t}\right)\right] + \beta_{2t}\left[\frac{\lambda_t}{\tau}\exp\left(-\frac{\tau}{\lambda_t}\right) - \exp\left(-\frac{\tau}{\lambda_t}\right)\right] \\
=& \beta_{0t}\cdot g_0(\tau) + \beta_{1t}\cdot g_1(\tau) + \beta_{2t}\cdot g_2(\tau)
\end{aligned} \tag{11}
$$

for all $\tau \geq 0$, where $\tau = T - t$ and the parameter vector $\theta_t = (\beta_{0t}, \beta_{1t}, \beta_{2t}, \lambda_t)$ has to be *dynamically* (for each t) implied from market quotes.
For each j, we write $p_j(\theta_t)$ to indicate the right member of (10) with θ_t as unknown in any of the $P(t, T_{ij})$; the vector

$$
p(\theta_t) = \big(p_1(\theta_t)\dots p_J(\theta_t)\big)'
$$

collects all model-implied prices of relevant obligations at time t.
There's quite a relevant stream of literature proposing different methods in order to imply the parameter vector θ_t; the strategy is essentially the same, that is minimizing the square distance between model-implied price vector and observed price vector $p = (p_1, \dots p_J)'$,with a correction embedded in a weighting matrix $W = W(\theta_t)$.
Different choices of W characterize different methods; here, we chose a diagonal weighting matrix

$$
W(\theta_t) = \begin{pmatrix} w_1(\theta_t) & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & w_J(\theta_t) \end{pmatrix}
$$

with

$$
w_j(\theta_t) = \frac{\dfrac{1}{D_j(\theta_t)}}{\displaystyle\sum_{l=1}^{J}\frac{1}{D_l(\theta_t)}} \qquad l = 1\dots J \tag{12}
$$

where D_j is the *duration* of the $j-th$ obligation, defined as

$$
D_j = \sum_{k=1}^{K_j} \frac{P(t, T_{kj})}{\displaystyle\sum_{k=1}^{K_j} P(t, T_{kj})} \cdot (T_{kj} - t) \tag{13}
$$

We then select the correct parameter

$$
\theta_t^* = \text{argmin} \ \ (p - p(\theta_t))' \cdot W(\theta_t) \cdot (p - p(\theta_t)) \tag{14}
$$

and construct the zero-coupon curve for any T using θ_t^* and (11). The last step is to use (9) to imply the yield curve $\{y(t,T)\}$ for any t.

Appendix A1 : CDS-bearing yield curve

A practical implementation of this method demands for a set of approximation due to the structure of the standardized market:

- In principle, the CDS-bearing bond purchaser is not covered for the portion of dirty price exceeding face value; while this is rarely a problem for zero coupon bonds, longer-dated bonds with coupons exceeding those of more recently issued obligations of several basis points could be severely overpriced with respect to face value. This however is a consequence of standardization: CDS contracts are traded with custom underlying notional where upfront payment is computed as percentage of face value N.
- CDS are traded with standard maturities and standard scheduled payments; for any trade date t, the choice of the CDS is limited to a set of ten maturities covering a quarter plus thirty years at most[7]. We chose to combine any of the available bonds with a cds hedging the whole life of the obligation, by selecting the first available CDS maturity after bond's final payment date. When residual life of the obligation exceeds the longest CDS maturity available, we simply select the 30 year contract.

We stress that such simplifications are not driven by computational needs but are instead direct consequence of market standardization: as CDS lost their specifically-tailored nature, an investor willing to hedge credit risk is facing these issues, and is obliged to enter into these *imperfect* hedges.
At time t, we take relevant obligations for country A priced $\{p_1 \dots p_J\}$ and combine each of them with their correspondent CDS. Among the ten available standard CDS maturities $\{\tilde{T}_h\}$ we select, for the j-th obligation,

$$\tilde{T}_j = min\{\tilde{T}_h | \tilde{T}_h \geq T_{I_j j}\}.$$

The present value of the CDS-bearing bond is equal to its discounted future cash flow:

$$p_j(t) + u_j(t) = C_j \sum_{k=1}^{K_j} \hat{P}(t, T_{kj}) + N \cdot \hat{P}(t, T_{K_j}) - S \cdot \sum_{h=1}^{H_j} \hat{P}(t, \tilde{T}_{hj}) \qquad (15)$$

where $u_j = N(U_j(t) - AP01(t))$, U_j is the implied upfront (%) and $AP01(t)$ is the reimbursment due to the first-full-coupon clause for target CDS maturity $\tilde{T}_j = \tilde{T}_{H_j j}$, while $\{\tilde{T}_{hj}\}_{h=1}^{H_j}$ are CDS standard dates, when spread S is paid to protection seller.[8]
As observed for bonds, the right term of (15) is immediately paid while the left one is contingent on future credit events.

[7]If for example we choose a 30-year CDS on March 20th 2010, protection will last on June 20th 2040, thirty years later than first coupon payment, so it will cover bonds with a residual maturity of approximately 30.25 years.

[8]Again, we consider it as implicitely multiplied by the year fraction occurring between two consecutive payment dates.

Proceeding as in section 2, we retrieve a *CDS-bearing zero-coupon curve* $\{\hat{P}(t,T)\}$ on the timegrid

$$\{T_{ij}\} \cup \{\tilde{T}_{hj}\} \quad i = 1, \dots I_j \quad , \quad h = 1 \dots H_j \quad , \quad j = 1 \dots J$$

minimizing

$$(\hat{p}(\hat{\theta}_t) - p_j(t) - u_j(t))' W(\hat{\theta}(t))(\hat{p}(\hat{\theta}_t) - p_j(t) - u_j(t))$$

and then use equation (9) to retrieve the CDS-bearing yield $\{\hat{y}(t,T)\}$ for each t.

Appendix B: Statistical tables

	y_t	$\hat{y}_t$	L
μ	227	190	256
σ	70	70	67

ρ	y_t	$\hat{y}_t$	L
y_t	1	0.99	0.98
$\hat{y}_t$		1	0.97
L			1

Table 1: GER - Sample means, standard deviations (left) and correlations (right).

	y_t	$\hat{y}_t$	L
μ	461	228	256
σ	79	48	67

ρ	y_t	$\hat{y}_t$	L
y_t	1	0.08	-0.23
$\hat{y}_t$		1	0.86
L			1

Table 2: ITA - Sample means, standard deviations (left) and correlations (right).

	y_t	$\hat{y}_t$	L
μ	505	251	256
σ	66	48	67

ρ	y_t	$\hat{y}_t$	L
y_t	1	0.14	-0.26
$\hat{y}_t$		1	0.82
L			1

Table 3: SPA - Sample means, standard deviations (left) and correlations (right).

ρ	y^{GER}	y^{ITA}	y^{SPA}
y^{GER}	1	-0.37	-0.37
y^{ITA}		1	0.77
y^{SPA}			1

ρ	$\hat{y}^{GER}$	$\hat{y}^{ITA}$	$\hat{y}^{SPA}$
$\hat{y}^{GER}$	1	0.81	0.78
$\hat{y}^{ITA}$		1	0.91
$\hat{y}^{SPA}$			1

Table 4: Correlation between naked yields (left) and CDS-bearing yields (right).

$\hat{y}_t$	y_t	L_t
α	-36	-72
(t-stat)	(-61)	(-15)
β	1.00	1.02
(t-stat)	(410)	(59)
R^2	0.99	0.95

Table 5: GER - Regression coefficients of $\hat{y}_t$ on y_t (left) and on L_t (right).

$\hat{y}_t$	y_t	L_t
α	206	69
(t-stat)	(9.6)	(9.6)
β	0.04	0.62
(t-stat)	(1.06)	(23)
R^2	0.00	0.75

Table 6: ITA - Regression coefficients of $\hat{y}_t$ on y_t (left) and on L_t (right).

$\hat{y}_t$	y_t	L_t
α	199	100
(t-stat)	(7.2)	(12)
β	0.10	0.58
(t-stat)	(1.88)	(19)
R^2	0.02	0.67

Table 7: SPA - Regression coefficients of $\hat{y}_t$ on y_t (left) and on L_t (right).

y_t^{SPA}	y_t^{ITA}	$\hat{y}_t^{ITA}$
α	207	479
(t-stat)	(11)	(20)
β	0.64	0.11
(t-stat)	(11)	(1.2)
R^2	0.60	0.01

$\hat{y}_t^{SPA}$	y_t^{ITA}	$\hat{y}_t^{ITA}$
α	263	41
(t-stat)	(12)	(5.8)
β	-0.03	0.92
(t-stat)	(-0.56)	(31)
R^2	0.00	0.84

Table 8: Regression coefficients of y_t^{SPA} on y_t^{ITA} and $\hat{y}_t^{ITA}$ (left table) and regression coefficients of $\hat{y}_t^{SPA}$ on y_t^{ITA} and on $\hat{y}_t^{ITA}$ (right table).

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