Liquidity Risk and Distressed Equity

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Abstract

I show theoretically and empirically that firms' endogenous cash holdings can help rationalize the low returns of distressed equity. In my model, levered firms with financing constraints can default because of liquidity or solvency, but firms seek to manage their cash to minimize liquidity risk. Using data on solvency, cash, and returns for US firms, I find evidence consistent with the model's predictions: (1) In all solvency levels, the average firm holds enough cash to cover short-term liabilities; (2) expected returns are humped and decreasing as solvency decreases because a less solvent firm has a higher fraction of its assets in cash; (3) expected returns are humped and decreasing as cash decreases because a less solvent firm demands less cash; (4) the outperformance of high-cash firms over low-cash firms increases as solvency decreases; and (5) the outperformance of high-solvency firms over low-solvency firms increases as cash decreases.

Keywords: Distress risk, equity returns, liquidity risk, cash holdings

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Introduction

> *"Cash is a bad investment over time. But you always want to have enough so that nobody else can determine your future, essentially."*
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> —Warren Buffet (November 12, 2009)

Recent studies of distressed equity rely on capital structure theory to rationalize why high default risk predicts low future returns, i.e. the "distress puzzle" documented by Dichev (1998), Griffen and Lemmon (2002), Campbell, Hilscher, and Szilagyi (2008), and others. Garlappi and Yan (2011) show that potential shareholder recovery upon resolution of distress increases the value of the default option and lowers expected returns near the default boundary. Opp (2013) argues that faster learning about firm solvency in aggregate downturns has the same effect, while McQuade (2013) argues that distressed equity hedges against persistent volatility risk and therefore commands lower expected returns.

In this paper, I argue that endogenous cash holdings can help rationalize the low returns of distressed equity. In contrast to extant studies of the puzzle, my model features levered firms with financing constraints that can default because of liquidity or solvency, but firms seek to manage their cash to minimize liquidity risk. I show that because an insolvent but liquid firm has a large fraction of its assets in cash, it also has a low conditional beta, which helps rationalize low expected returns. Using data on rated US firms for the period 1970-2013, I find empirical evidence consistent with the model's theoretical predictions.

The model features a levered firm generating uncertain earnings. The firm has common equity and coupon-bearing debt in its capital structure. Because of capital market frictions, the firm has no access to external financing and can in particular not issue additional equity to cover coupon payments. Default is costly. If the firm has no means to cover a coupon, it defaults due to a lack of liquidity. If the firm's asset value does not outweigh future coupons, it acts in the interest of equity holders and voluntarily defaults due to a lack of solvency. To offset liquidity risk, the firm seeks to retain some of its earnings as precautionary cash.[1] The firm's optimal cash-dividend policy is determined by the following trade-off: An extra dollar in dividends increases equity value conditional on survival, while an extra dollar in cash holdings increases the probability of survival. The asset level that triggers insolvency is determined by maximizing equity value.

In the model, the firm's optimal cash-dividend policy is to aim for a *target level of cash* that eliminates its liquidity risk (Proposition 1) and to only distribute dividends when its cash is at or above the target level (Proposition 2). The target cash level decreases as the firm becomes less solvent because a less solvent firm can only survive smaller earnings-shortfalls and thus demands less cash. Because any cash level different from the target is suboptimal, I consider the equilibrium case where cash is at its target level, meaning that the firm's liquidity risk is eliminated. In this case, I show that the firm optimally declares insolvency at a point where its assets consist mostly of cash (Proposition 3). In line with the model's optimal policies, I find for my sample of rated US firms that cash levels generally decline as firms become less solvent, but that even the least solvent firms hold cash levels that on average can cover their short-term liabilities.

The paper's main contribution is to shed new light on the distress puzzle by studying the expected equity returns of firms that optimally eliminate liquidity risk but may default due to a lack of solvency.

In equilibrium, the expected return on the liquid firm's equity is determined by its *conditional equity beta*, i.e. equity's time-varying sensitivity to systematic earnings-risk (Proposition 4). The model predicts that conditional beta is *humped and decreasing* in the probability of insolvency, i.e. initially high and upward-sloping but eventually low and downward-sloping (Proposition 5). The reason is the liquid firm's changing asset composition. When the firm is solvent, its asset value is mostly due to expected earnings, which makes its equity relatively sensitive to earnings risk. This implies a high conditional beta that increases as the firm becomes less solvent and therefore more levered. However, when the firm is close to insolvency, its asset value is mostly due to cash, which makes its equity relatively insensitive to earnings risk. This implies a low conditional beta that decreases as the firm becomes less solvent and therefore has a higher fraction of its assets in cash. In sum, this prediction provides a novel theoretical rationale of the distress puzzle for insolvent but liquid firms.

[1] There is ample empirical evidence suggesting that the precautionary motive is the most important determinant of corporate cash holdings and their secular growth since the 1980s—see, for instance, Opler, Pinkowitz, Stulz, and Williamson (1999), Bates, Kahle, and Stultz (2009), Acharya, Davydenko, and Strebulaev (2012), and references therein.

To empirically verify the model's rationale, I consider the following testable implications of its argument.

First, as stated above, the model predicts that expected returns are humped and decreasing as solvency decreases (Proposition 5). Second, the model also predicts that expected returns are humped and decreasing as cash levels decrease, because a less solvent firm demands less cash (Corollary 5.1). Third, the model predicts that the outperformance of high-cash firms over low-cash firms increases as solvency decreases, because firms with less cash are also less solvent while expected returns are humped and decreasing in decreasing solvency (Corollary 5.2). Finally, the model predicts that the outperformance of high-solvency firms over low-solvency firms increases as cash decreases, because less solvent firms demand less cash while expected returns are humped and decreasing in decreasing cash-levels (Corollary 5.3).

Consistent with the model's first and second theoretical predictions, the average returns of rated US firms are hump-shaped in decreasing solvency and cash-levels, which I find using conditional betas, cross-sectional Fama-MacBeth regressions, portfolio returns, alphas, and Sharpe/Information ratios. Furthermore, consistent with the third and fourth predictions, I find that a portfolio long high-cash firms and short low-cash firms earns increasing average returns and alphas as solvency-levels decrease, while a portfolio long high-solvency firms and short low-solvency firms earns increasing average returns and alphas as cash-levels decrease.

To summarize, I show in this paper that firms' endogenous cash holdings can help rationalize the low returns of distressed equity. My results suggest that there is no distress puzzle for insolvent but liquid firms.

Related literature

Chan and Chen (1991) and Fama and French (1995, 1996) were the first to suggest that asset pricing anomalies (relative to the classical CAPM) like the size and the value premia might be due to rational investors requiring higher returns as compensation for holding stocks with higher distress risk. Dichev (1998) was among the first to refute this by showing that higher default probability predicts lower, not higher, average future returns—i.e. the 'distress puzzle.' Subsequent empirical studies of the puzzle include Griffen and Lemmon (2002), Vassalou and Xing (2004), Campbell et al. (2008), and the references therein.

Garlappi, Shu, and Yan (2008) recognized that the empirical relation between average returns and probability of default is not monotonically decreasing but rather hump-shaped, and they rationalize this finding using simulations from a model in which shareholders can use their bargaining power to recover part of the firm's value upon resolution of distress. Extending these results, Garlappi and Yan (2011) show theoretically that shareholder recovery implies a hump-shaped relation between conditional betas and probability of default, and they find empirical support using estimated time-varying betas. The subsequent models of Opp (2013) and McQuade (2013) provide further theoretical rationals for these hump-shaped relations in models featuring shareholder learning and persistent volatility risk. I complement these theories by separating the liquidity and solvency components of default, and showing that in a model where precautionary cash is used to offset liquidity risk, there is a hump-shaped relationship between expected returns and probability of insolvency. Furthermore, I provide empirical evidence in support of my model's theoretical argument and predictions.

My model builds on the classical time-homogenous capital structure framework of e.g. Leland (1994), but augments it with a target cash level similar to Gryglewicz (2011), who extends the the classical pure-liquidity models of Jeanblanc-Picqué and Shiryaev (1995) and Radner and Shepp (1996) and studies the optimal capital structure of a firm facing liquidity and solvency concerns. A related framework is considered by Davydenko (2013), who studies whether corporate defaults are driven by insolvency or illiquidity. My model extends these frameworks by adding a systematic risk component to the firm's earnings process and studying the impact of equilibrium cash holdings on the expected returns of firms with varying levels of solvency.

Finally, this paper is also related to the literature on the determinants and implications of corporate cash holdings. Relevant empirical studies include Opler et al. (1999), Bates et al. (2009), Acharya et al. (2012), and Davydenko (2013). Relevant theoretical studies include Décamps, Mariotti, Rochet, and Villenueve (2011), and Bolton, Chen, and Wang (2011, 2013).

1 Model

This section develops an equity valuation model for a levered firm with financing constraints. The firm can default because of liquidity or solvency, but it seeks to retain some earnings as cash to offset its liquidity risk. The optimal policies for holding cash, paying out dividends, and declaring the firm insolvent are chosen so as to maximize the value of the firm's equity. I use the model to derive the optimal policies and equilibrium expected returns in Section 2.

The financially constrained firm

I consider a levered firm with uncertain earnings in a continuous-time economy with infinite time-horizon, $[0, \infty)$. Corporate earnings are taxed at the rate $\tau \in (0, 1)$ with a full loss offset, and the instantaneous risk-free interest rate, r, is assumed to be constant.

The firm's liabilities consist of common equity stock and, due to the tax benefit of debt, consol (infinite maturity) bonds with total coupon rate k per time unit. The number of shares outstanding and the total coupon level are assumed to be predetermined before time zero.

The firm's assets-in-place are fixed throughout its lifetime and continuously generate its earnings (revenue net of expenses). For my purpose of modeling cash holdings, it is convenient to specify the *stock* rather than the *flow* of earnings. The model's main state variable is therefore the firm's *cumulated earnings before interest and taxes* (EBIT) up to time t, which I denote X_t. I assume that X_t, under a physical probability measure, $\mathbb{P}$, is a geometric Brownian motion with dynamics

$$dX_t = \mu^{\mathbb{P}} X_t dt + \sigma X_t dW_t^{\mathbb{P}}. \tag{1}$$

Here, $W_t^{\mathbb{P}}$ is a standard $\mathbb{P}$-Brownian motion that drives the firm's total (idiosyncratic and systematic) earnings risk. *Instantaneous earnings* (per dt) are thus given by the increment dX_t, which can be either positive (a profit) or negative (a loss), depending on the realization of the total earnings shock, $dW_t^{\mathbb{P}}$.[2] The drift parameter, $\mu^{\mathbb{P}}$, is the $\mathbb{P}$-*expected growth rate* of earnings, while the diffusion parameter, σ, is the *volatility rate* of earnings.[3]

After taxes and coupons, the firm's instantaneous net earnings are given by $(1-\tau)(dX_t - kdt)$ and are at the discretion of equity holders before default. If the firm defaults, it is immediately liquidated. Bond holders have absolute-priority in default and recover a fraction of the market value of the firm's earnings-generating assets (i.e. the value of unlevered assets, as derived below)—the remainder is lost to bankruptcy costs.

I assume that due to capital market frictions, the firm has no access to external financing.[4] Coupon payments after time zero must therefore be internally financed. Should the firm have no means to cover a coupon payment, it default due to a lack of liquidity. This is called *liquidity default* and is driven by *short-term* or *financial* distress. On the other hand, if the value of the firm's earnings-generating assets does not outweigh its future coupon payments—meaning that its equity value becomes negative—the firm may choose to act in the interest of its equity holders and voluntarily default due to a lack of solvency. This is called *solvency default* and is driven by *long-term* or *economic* distress.

In an economy without financing constraints, liquidity default never occurs, because the firm finances any earnings-shortfalls by issuing additional equity—specifically in the form of negative dividends—as in e.g. Black and Cox (1976) and Leland (1994). Since this is possible as long as equity value is positive, only solvency default occurs. By contrast, when financing con-

[2] Modeling instantaneous earnings as the increment—instead of the level—of a stochastic process is common in models of liquidity management and related topics. See, for instance, Jeanblanc-Picqué and Shiryaev (1995), Radner and Shepp (1996), Demarzo and Sannikov (2006), Décamps et al. (2011), Gryglewicz (2011), Acharya et al. (2012), and Bolton et al. (2011, 2013).

[3] In contrast to Gryglewicz (2011), I specify cumulated earnings in (1) as a geometric Brownian motion rather than an arithmetic Brownian motion with a drift randomized over a known two-point distribution. This still implies a state-dependent value for the firm's earnings-generating assets, which ensures the existence of a strategic insolvency-trigger (see footnote 7), but simplifies the model's information structure and allows me to focus on systematic risk in my analysis of expected equity returns. Moreover, I show below that the implied value of earnings-generating assets is again a geometric Brownian motion, making the model consistent with classical capital structure models (see footnote 8).

[4] This could, for instance, be due to the debt-overhang problem of Myers (1977) or the information asymmetry problems of Leland and Pyle (1977) and Myers and Majluf (1984). The assumption of no external financing can be replaced by the milder assumption of sufficiently high issuance costs, but this does not alter the qualitative nature of the results as long as I) liquidation of the firm is costly, and II) only a fraction of future earnings can be pledged as collateral for external financing. See Acharya et al. (2012) for a further discussion.

straints prohibit additional external financing, the firm has a *precautionary motive* to reduce dividends and retain some earnings as cash, i.e. non-productive liquid assets, which serve to offset its liquidity risk.

The managers of the financially constrained firm are assumed to act in the interest of equity holders. They determine the firm's optimal cash-dividend policy by the following trade-off: An additional dollar in dividends increases equity value conditional on survival, while an additional dollar in cash holdings increases the probability of survival. The managers also determine the optimal policy for declaring the firm strategically insolvent. For simplicity, I assume that managers have full discretion over costless distribution of cash holdings to equity holders at any time before default, and that there are no bond covenants limiting such payouts. This makes the strategic insolvency-decision independent of the firm's cash. In reality, it may be difficult for bond holders to enforce covenants restricting payouts before default.

Cash holdings and dividend payouts

Conditional on survival, the firm collects earnings and pays bond holders the tax-deductible coupon. Net earnings can be paid out as dividends to equity holders or, to reduce liquidity risk, be retained as precautionary cash. I assume that cash holdings earn the risk-free rate, r, (for instance through investment in short-term marketable securities) and that equity holders choose payouts whenever they weakly prefer so.[5]

Let C_t be the firm's cash holdings at time t and let D_t be its cumulated dividend payouts up to time t. Then

$$dD_t = (1-\tau)(dX_t - k\,dt) - dC_t + rC_t\,dt. \tag{2}$$

Because the firm has no access to external financing, C_t and D_t have to remain nonnegative at all $t > 0$.[6] Liquidity default occurs when the firm runs out of cash, i.e. at $\tau_C = \inf_{t>0}\{C_t \leq 0\}$. Since expected earnings by (1) depend on X_t, solvency default occurs at an optimally chosen trigger, $\underline{X}^\star \geq 0$, i.e. at $\tau_X = \inf_{t>0}\{X_t \leq \underline{X}^\star\}$.[7]

Managers' optimization problem

The firm's managers choose the cash-dividend policy that maximizes the market value of equity. To calculate market values, I assume that the economy's stochastic discount factor, Λ_t, is given by the dynamics

$$d\Lambda_t = -r\Lambda_t dt - \lambda\Lambda_t dZ_t^{\mathbb{P}}. \tag{3}$$

Here, $Z_t^{\mathbb{P}}$ is a standard $\mathbb{P}$-Brownian motion that drives systematic earnings risk and has correlation ρ with the firm's total earnings risk, $W_t^{\mathbb{P}}$, as given in (1). The constant λ is then the market price of systematic earnings risk. In the appendix, I detail how (3) defines a risk-neutral pricing measure, $\mathbb{Q}$, under which X_t is a geometric Brownian motion with dynamics

$$dX_t = \mu^{\mathbb{Q}} X_t dt + \sigma X_t dW_t^{\mathbb{Q}}, \tag{4}$$

where $\mu^{\mathbb{Q}} = \mu^{\mathbb{P}} - \rho\sigma\lambda$ is the risk-neutral growth rate of earnings and $W_t^{\mathbb{Q}} = W_t^{\mathbb{P}} + \rho\lambda t$ is a standard $\mathbb{Q}$-Brownian motion. Assuming $r > \mu^{\mathbb{Q}} > 0$, the time-t market value of the firm's earnings-generating assets (i.e. the unlevered assets) is

$$U(X_t) = \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^\infty e^{-r(u-t)}(1-\tau)dX_u\right] = \frac{(1-\tau)\mu^{\mathbb{Q}}X_t}{r-\mu^{\mathbb{Q}}}, \tag{5}$$

which is again a geometric Brownian motion.[8] In the following, I assume $U(X_t) > (1-\tau)X_t$, which is equivalent to the restriction $\mu^{\mathbb{Q}} > \frac{1}{2}r$. Otherwise, it would be optimal for equity holders to liquidate the firm at time zero—see the discussion of "asset shifting" due to too low expected earnings in Acharya et al. (2012).

[5] The assumption that cash earns r simplistically implies a zero *carry-cost* of holding cash—such costs are, however, easily introduced in the present model without qualitatively altering the results.

[6] Usually, negative negative cash holdings are interpreted as the firm drawing on a credit line, while negative dividend payouts are interpreted as capital injections by equity holders. Since I assume no external financing beyond time zero, the firm has access to neither, and therefore its cash holdings and dividend payouts have to remain nonnegative at all times.

[7] The existence of the strategic insolvency trigger, $\underline{X}^\star$, follows from the assumption that X_t is a geometric Brownian motion. If X_t were an arithmetic Brownian motion with constant drift, say μ, earnings-generating asset value would be constant and the insolvency-decision would be determined at time zero by the sign of $\mu - k$.

[8] By Itô's Lemma and (4), $U(X_t)$ has the $\mathbb{Q}$-dynamics

$$dU(X_t) = \mu^{\mathbb{Q}}U(X_t)dt + \sigma U(X_t)dW_t^{\mathbb{Q}}.$$

This makes the specification of cumulated earnings in (4) (and, equivalently, in (1)) consistent with standard capital structure models assuming that the firm's unlevered asset value is a geometric Brownian motion—e.g. Black and Scholes (1973), Merton (1974), Black and Cox (1976), Leland (1994), Fan and Sundaresan (2000), Goldstein, Ju, and Leland (2001), Duffie and Lando (2001), etc.

The market value of the firm's equity is the $\mathbb{Q}$-expected, discounted value of future dividends until either form of default, plus a liquidation payout of any remaining cash holdings—the latter is a limiting consequence of the assumption of unrestricted payouts. Given the strategic insolvency trigger, $\underline{X}^\star$, and time-t levels of cumulated earnings and cash holdings, X_t and C_t, equity value is thus given by

$$E(X_t, C_t) = \sup_{(D_u)_{u\geq t}} \mathbb{E}_t^{\mathbb{Q}} \left[\int_t^{\tilde{\tau}} e^{-r(u-t)}\, dD_u + e^{-r(\tilde{\tau}-t)} C_{\tilde{\tau}} \right], \quad (6)$$

where $\tilde{\tau} = \tau_C \wedge \tau_X$ is the firm's liquidation time. The supremum is taken over all dividend payout policies, $(D_u)_{u\geq t}$, which are nonnegative, non-decreasing, satisfy (2), and are adapted to the filtration generated by the cumulated earnings-process in (4).

2 Theoretical results

This section derives the model's optimal policies and the equilibrium expected return on the firm's equity. The main goal is to show that, in equilibrium, the firm will eliminate its liquidity risk, and that this implies a hump-shaped relation between conditional equity beta and probability of insolvency. The theoretical results form the basis for the testable predictions studied in the paper's empirical part.

2.1 Optimal policies

I first derive the financially constrained firm's optimal policies for holding cash and paying out dividends. This will subsequently allow me to derive the equilibrium equity value and the optimal insolvency-policy.

Target cash holdings

To solve the manager's optimization problem (cf. (6)), note that higher cash holdings decrease liquidity risk but also dividend payouts. I therefore conjecture that an optimal cash-dividend policy involves a *target level of cash*, $\underline{C}(X)$, which is *the smallest amount of cash large enough to eliminate liquidity risk when cumulated earnings are at the level X.*

To characterize $\underline{C}(X)$, note that the cumulated dividend process in (2) is positive and non-decreasing at all $t > 0$ if and only if 1) its drift is nonnegative and 2) its volatility is zero. By these requirements, $\underline{C}(X)$ satisfies a differential equation coupled with a lower bound to all its solutions. The following proposition gives the smallest solution. (All proofs are in the appendix.)

Proposition 1 (Target cash level)**.** *Given the coupon, k, the strategic insolvency trigger, $\underline{X}^\star$, and the current level of cumulated earnings, X, the smallest level of cash large enough to eliminate the financially constrained firm's liquidity risk is given by*

$$\underline{C}(X) = (1-\tau)\left[X - \underline{X}^\star + \tfrac{k}{r}\right]. \quad (7)$$

Before interpreting the form in (7), note that since $X_t \geq \underline{X}^\star$ and cash holdings earn r per dt (cf. (2)), the interest on the target level is always sufficient to cover a coupon payment. Hence, when dividends have to remain nonnegative, the target cash level is the smallest amount of cash large enough to ensure a coupon payment even if a shock brings X_t down to $\underline{X}^\star$. Importantly, by holding at least the target level in cash, the financially constrained firm's liquidity risk is eliminated, implying that it only defaults due to a lack of solvency.[9]

To interpret the form in (7), note that since $(1-\tau)\frac{k}{r}$ can be interpreted as the firm's *current liabilities* (the present value of instantaneous after-tax coupons), the quantity $(1-\tau)(X - \underline{X}^\star)$ can be interpreted as the firm's *working capital*. In this sense, the target cash level is simply working capital plus current liabilities.

[9] An inspection of the proof of Proposition 1 reveals that the form in (7) is independent of the assumption that the cumulated earnings process, X_t, is a geometric Brownian motion (see (1) or (4)). In fact, the form of (7) is general as long as there exists a strategic solvency trigger, $\underline{X}^\star$. As argued in footnote 7, this is in particular the case when X_t is a geometric Brownian motion, but not if it were an arithmetic Brownian motion with constant drift.

Furthermore, an intuitive way to derive (7) without appealing to the specific dynamics of X_t is to note that for $\underline{C}(X_t)$ to eliminating liquidity risk for any $X_t \geq \underline{X}^\star$, it is reasonable to assume that $r\underline{C}(X_t)\,dt \geq (1-\tau)k\,dt$ for all $0 < t \leq \tau_X$. Given this, (2) implies that dD_t is nonnegative if and only if $d\underline{C}(X_t) \leq (1-\tau)dX_t$. Evaluating the latter expression at times τ_X and t and rearranging gives

$$\underline{C}(X_t) \geq \underline{C}(X_{\tau_X}) + (1-\tau)\left[X_t - X_{\tau_X}\right] \geq (1-\tau)\tfrac{k}{r} + (1-\tau)\left[X_t - \underline{X}^\star\right].$$

Choosing the smallest level satisfying this restriction gives (7).

Finally, note that $\underline{C}$ increases in X but decreases in $\underline{X}^\star$ because as the firm becomes less solvent (lower X or higher $\underline{X}^\star$), it can only withstand smaller earnings shocks before being declared insolvent, and therefore demands less cash. By the same logic, $\underline{C}$ decreases as $X - \underline{X}^\star$ decreases. Consequently, a firm that has cash at the target level but is close to insolvency holds as little cash as possible.

Optimal cash-dividend policy

Given Proposition 1's target cash level, I conjecture that the optimal cash-dividend policy adjusts cash holdings, C_t, towards the target level, $\underline{C}(X_t)$, as X_t fluctuates.

To see this, note that if $C_t < \underline{C}(X_t)$, it is suboptimal to pay out dividends, as this would make the firm vulnerable to liquidity risk, and all earnings should be retained until cash reaches the target level. Conversely, if $C_t > \underline{C}(X_t)$, dividend payouts are too low, in that cash is above the target level, and it is optimal to distribute the residual $C_t - \underline{C}(X_t)$ to equity holders.

I thus conjecture that the optimal cash-dividend policy, $(D_t^\star)_{t\geq 0}$, is given by

$$dD_t^\star = \begin{cases} 0 & \text{if} \quad C_t < \underline{C}(X_t) \\ \left[r\underline{C}(X_t) - (1-\tau)k\right] dt & \text{if} \quad C_t = \underline{C}(X_t) \\ C_t - \underline{C}(X_t) & \text{if} \quad C_t > \underline{C}(X_t). \end{cases} \tag{8}$$

The intermediate case—which follows by applying Itô's Lemma to the target cash level in (7) and using the dynamics of the dividend payout process in (2)—is of special interest and may also be written as

$$dD_t^\star = r(1-\tau)(X_t - \underline{X}^\star)\, dt. \tag{9}$$

When $C_t = \underline{C}(X_t)$, the form in (8) says that instantaneous dividends are the interest earned on cash holdings net of an after-tax coupon. The form in (9) says that this is equivalent to the interest earned on working capital, $(1-\tau)(X_t - \underline{X}^\star)$. It follows from either form that dividends equal zero at the strategic insolvency-trigger, $\underline{X}^\star$.

The following proposition asserts that the policy conjectured in (8) does in fact maximize equity value.

Proposition 2 (Optimal cash-dividend policy)**.** *The cash-dividend policy $(D_t^\star)_{t\geq 0}$ in (8) maximizes the financially constrained firm's equity value in (6).*

Intuitively, the cash-dividend policy in (8) maximizes equity value because it optimally exploits that an extra dollar in cash holdings is at least worth its face value to equity holders: $\frac{\partial E(X_t,C_t)}{\partial C} \geq 1$.[10] From (8), earnings are retained whenever an additional dollar in cash decreases liquidity risk (when $\frac{\partial E(X_t,C_t)}{\partial C} > 1$), while earnings are paid out whenever an additional dollar in cash no longer reduces liquidity risk (when $\frac{\partial E(X_t,C_t)}{\partial C} = 1$).

Equilibrium equity value and insolvency-policy

Proposition 2 implies that any cash level different from the target is suboptimal. Indeed, if $C_t \neq \underline{C}(X_t)$, the optimal cash-dividend policy in (8) dictates that all variations in C_t are due to the adjustment towards $\underline{C}(X_t)$. I therefore focus on the equilibrium case of $C_t = \underline{C}(X_t)$. In this case, the firm is *liquid* (i.e. its liquidity risk is eliminated) and only defaults when declared insolvent. This will allow me to derive the equilibrium equity value and the optimal insolvency-trigger.

The liquid firm's equity is a claim on its earnings that pays the continuous flow of dividends in (9) until strategic insolvency. Given the current level of cumulated earnings, X, let $E(X) = E(X, \underline{C}(X))$ be the market value of the liquid firm's equity. Using the $\mathbb{Q}$-dynamics of X_t in (4), an application of Itô's Lemma to the discounted gains process of equity gives that $E(X)$ solves the ordindary differential equation

$$rE(X) = \tfrac{1}{2}\sigma^2 X^2\, E_{XX}(X) \;+\; \mu^{\mathbb{Q}} X\, E_X(X) \;+\; r(1-\tau)(X - \underline{X}^\star). \tag{10}$$

As earnings become arbitrarily large, the value of the firm's total assets—i.e. cash holdings plus earnings-generating assets plus the tax-benefit of debt—converges towards the sum of risk-free equity and debt. This implies the *value matching* condition

$$E(X) \nearrow \underline{C}(X) + U(X) + \tau\tfrac{k}{r} - \tfrac{k}{r} \tag{11}$$

[10] This is because any excess cash above the level dictated by an optimal dividend policy can be paid out as a one-time dividend. More formally, under an optimal dividend policy, the value of equity with C in cash must be greater than or equal to the value of equity with $C - \epsilon$ in cash plus a dividend payout of ϵ: $E(X_t, C_t) \geq E(X, C-\epsilon) + \epsilon$. Rearranging and letting $\epsilon \to 0$ gives $\frac{\partial E(X_t,C_t)}{\partial C} \geq 1$.

for $X \nearrow \infty$, where $U(X) = \frac{(1-\tau)\mu^{\mathbb{Q}} X}{r-\mu^{\mathbb{Q}}}$ is the value of earnings-generating assets in (5). Here, the limit is the firm's total assets net of risk-free debt.

On the other hand, as earnings fall to the level of strategic insolvency, the value of earnings-generating assets evaporates and equity becomes claim on solely on the firm's cash. This, combined with the assumption of unrestricted payouts before default, implies the *limited liability* condition

$$E(X) \searrow \underline{C}(\underline{X}^\star) \quad \text{for} \quad X \searrow \underline{X}^\star. \tag{12}$$

Finally, the insolvency-trigger, $\underline{X}^\star$, is the level of cumulated earnings solving the *smooth-pasting* condition

$$\left.\frac{\partial E(X)}{\partial X}\right|_{X=\underline{X}^\star} = \left.\frac{\partial \underline{C}(X)}{\partial X}\right|_{X=\underline{X}^\star}, \tag{13}$$

which follows from the limited liability condition in (12). This states that insolvency is optimally declared when an extra dollar in earnings increases equity value by no more than the increase in cash holdings.

Combining the differential equation in (10) with the boundary conditions in (11)–(13) gives the following proposition.

Proposition 3 (Equity value and insolvency)**.** *Suppose that $r > \mu^{\mathbb{Q}} > 0$ and that cash holdings are at the target level, $C_t = \underline{C}(X_t)$. Then the following holds:*

(i) *Given the current level of cumulated earnings, X, the market value of the liquid firm's equity is*

$$E(X) = \underline{C}(X) + U(X) - (1-\tau)\tfrac{k}{r} + \left[(1-\tau)\tfrac{k}{r} - U(\underline{X}^\star)\right]\pi^{\mathbb{Q}}(X), \tag{14}$$

where $\pi^{\mathbb{Q}}(X) = \left(\frac{X}{\underline{X}^\star}\right)^{\phi^-}$ and where ϕ^- is given by

$$\phi^- = \frac{\sigma^2 - 2\mu^{\mathbb{Q}} - \sqrt{(\sigma^2 - 2\mu^{\mathbb{Q}})^2 + 8r\sigma^2}}{2\sigma^2} < 0.$$

(ii) *Given the coupon, k, the liquid firm's strategic insolvency-trigger is given by*

$$\underline{X}^\star = \frac{k}{r}\frac{r-\mu^{\mathbb{Q}}}{\mu^{\mathbb{Q}}}\frac{\phi^-}{\phi^- - 1}. \tag{15}$$

(iii) *At the strategic insolvency-trigger in (15), the liquid firm's cash holdings are worth more than its earnings-generating assets: $\underline{C}(\underline{X}^\star) > U(\underline{X}^\star)$.*

The equilibrium equity value in (14) is the sum of three terms. The first is the face value of cash holdings. The second is the present value of expected earnings (i.e. the market value of earnings-generating assets) net of future coupon payments. The third is the value of the option to default on the firm's debt when cumulated earnings fall to the strategic insolvency-trigger. The factor $\pi^{\mathbb{Q}}(X)$ goes to 0 as X approaches infinity and goes to 1 as X approaches $\underline{X}^\star$. It may thus, similar to Garlappi and Yan (2011), be interpreted as the firm's *instantaneous $\mathbb{Q}$-probability of insolvency*.

In the next section's analysis of equilibrium expected returns, I show how the third part of Proposition 3—namely, that the liquid but insolvent firm has most of its assets in cash—is closely connected to the fact that the liquid firm has a low conditional equity beta when it is close to insolvency.

2.2 Equilibrium expected returns

I now derive the equilibrium expected return on the firm's equity. The goal is to show that the equilibrium relation between the firm's conditional equity beta and its probability of insolvency is non-monotonic and hump-shaped. This, as I will argue, provides a theoretical rationale of the distress puzzle for insolvent but liquid firms. To verify the model's rationale empirically, I derive cross-sectional implications of its argument, which I will test in the paper's empirical part.

Expected returns and conditional beta

Applying Itô's Lemma to the equilibrium equity value, $E(X_t)$, and using the differential equation in (10), the excess return on the firm's equity, under $\mathbb{Q}$, is given by

$$\frac{dE(X_t) + dD_t^\star}{E(X_t)} - r\,dt = X_t \frac{E_X(X_t)}{E(X_t)}\sigma dW_t^{\mathbb{Q}}.$$

Using the translation $W_t^{\mathbb{Q}} = W_t^{\mathbb{P}} + \rho\lambda t$, it follows that the time-t conditional $\mathbb{P}$-expected instantaneous excess return on equity may be expressed as

$$\mathbb{E}_t^{\mathbb{P}}[r_{t+dt}^E] - r = \Omega^E(X_t)\rho\sigma\lambda, \tag{16}$$

where the key quantity $\Omega^E(X_t) = X_t \frac{E_X(X_t)}{E(X_t)} = \frac{X_t}{E(X_t)} \frac{\partial E(X_t)}{\partial X_t}$ is the *earnings-sensitivity* (or *elasticity*) *of the liquid firm's equity*. It measures the percentage change in equity value for a one percent change in earnings. A value above (below) 1 means that equity responds more (less) than proportionally to changes in earnings. The relation (16) says that given the correlated earnings volatility, $\rho\sigma$, and the market price of systematic earnings risk, λ, higher earnings-sensitivity implies higher expected return.

To determine the market price of systematic earnings risk, λ, I assume that there exists a traded, diversified portfolio with (cum-dividend) value M_t, which is instantaneously perfectly negatively correlated with the stochastic discount factor in (3). Its return, under $\mathbb{P}$, is then

$$\frac{dM_t}{M_t} = r^M \, dt + \sigma^M \, dZ_t^{\mathbb{P}}.$$

Since $Z_t^{\mathbb{Q}} = Z_t^{\mathbb{P}} + \lambda t$ and since the $\mathbb{Q}$-expected, instantaneous return on the portfolio has to be the risk-free rate, r, it follows that $\lambda = \frac{r^M - r}{\sigma^M}$, i.e. the expected excess return on the portfolio relative to its volatility, or its *Sharpe ratio*. Combining this with (16) gives

$$\mathbb{E}_t^{\mathbb{P}}[r_{t+dt}^E] - r = \Omega^E(X_t)\beta^U \,(r^M - r), \tag{17}$$

where $\beta^U = \frac{\rho\sigma}{\sigma^M}$ is the the correlated volatility of earnings relative to the volatility of the diversified portfolio, i.e. the firm's *unlevered* or *asset beta*.

The relation in (17) is a *conditional capital asset pricing model (conditional CAPM)*, stating that the time-t conditional expected excess return on the firm's equity is proportional to the expected excess return on the diversified portfolio. The proportionality factor is the product of the asset beta, β^U, and the earnings-sensitivity, $\Omega^E(X_t)$. In this sense, the product $\Omega^E(X_t)\beta^U$ is this model's *conditional equity beta* at time t.

The following proposition gives an expression for the earnings-sensitivity that highlights the marginal effects of cash, earnings, leverage, and the option to declare insolvency on the liquid firm's conditional beta.

Proposition 4 (Marginal effects on conditional beta)**.** *Given the time-t level of cumulated earnings, X_t, the earnings-sensitivity of the liquid firm's equity, $\Omega^E(X_t)$, can be written as*

$$\Omega^E(X_t) = 1 - \underbrace{\frac{C(X_t)}{E(X_t)}}_{\text{Cash-to-equity}} + \underbrace{(1-\tau)\frac{X_t}{E(X_t)}}_{\text{Earnings yield}} + \underbrace{(1-\tau)\frac{k/r}{E(X_t)}}_{\text{Debt-to-equity}} \underbrace{- (1-\phi^-)\frac{\left[(1-\tau)\frac{k}{r} - U(\underline{X}^\star)\right]\pi^{\mathbb{Q}}(X_t)}{E(X_t)}}_{\text{Insolvency option}}.$$

Proposition 4 benchmarks $\Omega^E(X_t)$ to 1, which would be its value if the firm was unlevered, i.e. if $k = 0$. The subsequent terms thus represent deviations in earnings-sensitivity relative to an unlevered firm.

The first term is the firm's *cash-to-equity ratio* and represents the value of the cash-claim owned by equity holders relative to the current value of their initial investment. The negative sign indicates that, everything else equal, higher cash-to-equity is associated with lower earnings-sensitivity. Intuitively, if a higher fraction of the equity claim is in cash as opposed to expected earnings, equity is less earnings-sensitive.

The second term is the firm's total after-tax *earnings yield* (the inverse of the *price-to-earnings ratio*) up to time t. It represents the value of the earnings-claim owned by equity holders relative to the current value of their initial investment. The positive sign indicates that higher earnings-yield is, everything else equal, associated with higher earnings-sensitivity. Intuitively, higher earnings-yield implies higher expected earnings which makes equity more earnings-sensitive.

The third term is the firm's after-tax *debt-to-equity ratio* and represents the present value of future coupon payments relative to the present value of equity. The positive sign indicates that, everything else equal, higher debt-to-equity is associated with higher earnings-sensitivity. Intuitively, higher debt-to-equity means that a higher fraction of earnings is directed towards coupon payments, which makes equity more earnings-sensitive.

The final term is the value of the *option to strategically declare the firm insolvent*. The negative sign indicates that a higher value for the insolvency-option is, everything else equal, associated with lower earnings-sensitivity. Intuitively, as the firm becomes less solvent, its earnings-generating assets decline in value, causing equity to converge to the value of cash holdings (cf. (12)), which makes it less earnings-sensitive.

In the next subsection, I use the marginal effects of Proposition 4 to study the equilibrium relation between conditional beta and probability of insolvency.

Conditional beta and solvency

This subsection determines the equilibrium relation between conditional beta and probability of insolvency and then uses it to rationalize the distress puzzle for insolvent but liquid firms.

Intuitively, the form of the of the earnings-sensitivity given in proposition Proposition 4 suggests that an increase in the probability of insolvency (which is equivalent a decrease in earnings) is associated with two opposing effects on conditional beta:

- higher cash-to-equity, lower earnings-yield, and higher value for the insolvency option, which drives conditional beta down;
- higher debt-to-equity (i.e. higher leverage), which drives conditional beta up.

Which of the two effects that dominates depends on the *level* of the firm's probability of insolvency, and this is closely related to its *asset-composition*, i.e. how much of the firm's (total) asset value that is due to earnings-generating assets and how much that is due to cash.

When the firm is solvent (i.e. when its probability of insolvency is low) its asset value is mostly due to earnings-generating assets. Therefore, an increase in probability of insolvency is associated with a relatively large decrease in asset value, which makes the firm more levered. This suggests that when the firm is solvent, the second effect dominates, and there is a positive relation between probability of insolvency and conditional beta.

However, when the firm is sufficiently close to insolvency (i.e. when its probability of insolvency is sufficiently high), its asset value is mostly due to cash holdings. An increase in the probability of insolvency will thus be associated with a relatively small decrease in asset value, but a relatively large increase in the fraction of equity value due to cash holdings (as opposed to expected earnings) and the value of the option to declare insolvency. This suggests that when the firm is sufficiently close to insolvency, the first effect dominates, and there is a negative relation between probability of insolvency and conditional beta.

The following proposition formalizes the above intuition based on Proposition 4. It shows that the relation between conditional beta and probability of insolvency depends on the firm's solvency level and is closely related to its asset-composition.

Proposition 5 (Conditional beta and solvency)**.** *Suppose $r > \mu^{\mathbb{Q}} > \frac{1}{2}r$. Then the following holds for the liquid firm's asset composition, its conditional beta, and its expected returns as the probability of insolvency, $\pi^{\mathbb{Q}}(X_t)$, varies.*

(i) *As $\pi^{\mathbb{Q}}(X_t) \to 0$, cash holdings are low relative to earnings-generating assets: $\underline{C}(X_t) < U(X_t)$.*
As $\pi^{\mathbb{Q}}(X_t) \to 1$, cash holdings are high relative to earnings-generating assets: $\underline{C}(X_t) > U(X_t)$.

(ii) *As $\pi^{\mathbb{Q}}(X_t) \to 0$, conditional beta and expected returns are high: $\Omega^E(X_t) > 1$.*
As $\pi^{\mathbb{Q}}(X_t) \to 1$, conditional beta and expected returns are low: $\Omega^E(X_t) < 1$.

(iii) *As $\pi^{\mathbb{Q}}(X_t) \to 0$, conditional beta and expected returns are increasing in $\pi^{\mathbb{Q}}(X_t)$: $\frac{d\Omega^E(X_t)}{d\pi^{\mathbb{Q}}(X_t)} > 0$.*
As $\pi^{\mathbb{Q}}(X_t) \to 1$, conditional beta and expected returns are decreasing $\pi^{\mathbb{Q}}(X_t)$: $\frac{d\Omega^E(X_t)}{d\pi^{\mathbb{Q}}(X_t)} < 0$.

Proposition 5 shows that when the liquid firm is sufficiently solvent, earnings-generating assets make up most of its asset value, conditional beta is high (relative to an unlevered firm), and there is a positive relation between conditional beta and probability of insolvency. Intuitively, when the firm's asset-composition consists mostly of earnings-generating assets, its conditional beta is high (relative to an unlevered firm) and increases as the firm becomes less solvent and therefore more levered.

Conversely, the proposition shows that when the liquid firm is sufficiently close to insolvency, cash makes up most of its total asset value, conditional beta is low, and the relation between conditional beta and probability of insolvency turns negative. Intuitively, when the firm's asset-composition consists mostly of cash, its equity value decreases towards the value of cash, which is associated with a low conditional beta that decreases as the firm becomes less solvent and thus has an even larger fraction of its assets in cash.

Importantly, Proposition 5 shows that *in equilibrium, high-solvency firms have higher expected returns than low-solvency firms, and there is a hump-shaped relation between conditional beta and probability of insolvency*. This provides a novel theoretical rationale of the distress puzzle for insolvent but liquid firms: When liquid firms approach insolvency, their asset-composition tilts towards high cash relative to earnings-generating assets, which is associated with low and decreasing conditional betas and expected returns.

Conditional beta and cash; long-short portfolios

In this subsection, I derive additional testable implications of Proposition 5. First, I determine the equilibrium relation between conditional beta and cash. I then study how long-short portfolios perform in the cross-sections of solvency and cash levels.

First, because the target cash level, $\underline{C}(X)$, moves opposite the probability of insolvency, $\pi^{\mathbb{Q}}(X)$, parts (ii) and (iii) of Proposition 5 can be directly translated into equilibrium relations between conditional beta and cash.

Corollary 5.1 (Conditional beta and cash)**.** *Under the assumptions of Proposition 5, the following holds for the liquid firm's conditional beta and its expected returns as target cash holdings, $\underline{C}(X_t)$, vary:*

(i) *As $\underline{C}(X_t) \to \infty$, conditional beta and expected returns are high: $\Omega^E(X_t) > 1$.*

As $\underline{C}(X_t) \to \underline{C}(\underline{X}^\star)$, conditional beta and expected returns are low: $\Omega^E(X_t) < 1$.

(ii) *As $\underline{C}(X_t) \to \infty$, conditional beta and expected returns are decreasing in $\underline{C}(X_t)$: $\frac{d\Omega^E(X_t)}{d\underline{C}(X_t)} < 0$.*

As $\underline{C}(X_t) \to \underline{C}(\underline{X}^\star)$, conditional beta and expected returns are increasing in $\underline{C}(X_t)$: $\frac{d\Omega^E(X_t)}{d\underline{C}(X_t)} > 0$.

Corollary 5.1 is the dual of Proposition 5. It says that when equilibrium cash is sufficiently high, conditional beta is high (relative to an unlevered firm) and negatively related to cash. Intuitively, when target cash is sufficiently high, the firm is solvent, and this is associated with a high conditional beta that decreases as the firm demands higher cash and thus becomes more solvent. Conversely, the corollary says that as equilibrium cash approaches its lower boundary, conditional beta is low and positively related to cash. Intuitively, when target cash is sufficiently close to its lower boundary, the firm is close to insolvency, which is associated with a low conditional beta that increases as the firm demands higher cash and thus becomes more solvent. In sum, Corollary 5.1 says that *in equilibrium, high-cash firms have higher expected returns than low-cash firms, and there is a hump-shaped relation between conditional beta and decreasing cash levels*.

Next, consider two identical firms with time-t cash-level $\underline{C}(X_t)$. Suppose the first firm experiences a positive earnings-shock $x > 0$, while the second firm experiences a negative shock of the same magnitude, $-x$. The first firm will increase its cash to $\underline{C}(X_t + x)$, while the second will decrease its cash to $\underline{C}(X_t - x)$. Let r^{HC}_{t+dt} be the return of the high-cash firm and r^{LC}_{t+dt} be the return of the low-cash firm. Consider a *high-cash minus low-cash* (HCmLC) portfolio that buys the high-cash firm and shortsells the low-cash firm. Given x and using (17), the expected return of HCmLC is

$$\begin{aligned}\mathbb{E}^{\mathbb{P}}_t[r^{HCmLC}_{t+dt}] &= \mathbb{E}^{\mathbb{P}}_t[r^{HC}_{t+dt} - r^{LC}_{t+dt}] \\ &= (\Omega^E(X_t + x) - \Omega^E(X_t - x))\beta^U(r^M - r),\end{aligned}$$

i.e. the *expected return-spread* between firms with cash levels $\underline{C}(X_t + x)$ and $\underline{C}(X_t - x)$. The following corollary follows directly from part (iii) of Proposition 5.

Corollary 5.2 (High-cash minus low-cash)**.** *Under the assumptions of Proposition 5, and for a fixed $x > 0$, the following holds for the expected return of HCmLC:*

(i) *As $\pi^{\mathbb{Q}}(X_t) \to 0$, the expected return of HCmLC is negative and decreasing in $\pi^{\mathbb{Q}}(X_t)$:*

$\mathbb{E}^{\mathbb{P}}_t[r^{HCmLC}_{t+dt}] < 0$ *and* $\frac{d\,\mathbb{E}^{\mathbb{P}}_t[r^{HCmLC}_{t+dt}]}{d\pi^{\mathbb{Q}}(X_t)} < 0$.

(ii) *As $\pi^{\mathbb{Q}}(X_t) \to 1$, the expected return of HCmLC is positive and increasing in $\pi^{\mathbb{Q}}(X_t)$:*

$\mathbb{E}^{\mathbb{P}}_t[r^{HCmLC}_{t+dt}] > 0$ *and* $\frac{d\,\mathbb{E}^{\mathbb{P}}_t[r^{HCmLC}_{t+dt}]}{d\pi^{\mathbb{Q}}(X_t)} > 0$

Corollary 5.2 says that the performance of HCmLC is conditional on the solvency of the underlying firms. For sufficiently high levels of solvency, HCmLC earns negative expected returns that are lower for less solvent firms. The reason is that when the firms underlying HCmLC are sufficiently solvent, the high-cash firms have lower expected returns than the low-cash firms and the return-spread is more negative for less solvent firms. Conversely, for sufficiently low levels of solvency, HCmLC earns positive expected returns that are higher for less solvent firms. This is because when the firms underlying HCmLC are sufficiently close to insolvency, the high-cash firms have higher expected returns than the low-cash firms, and the return-spread becomes higher for less solvent firms. Importantly, the second part implies that *the outperformance of high-cash firms over low-cash firms increases as solvency decreases.*

Finally, for the dual of Corollary 5.2, consider two identical firms with time-t insolvency-probability $\pi^{\mathbb{Q}}(X_t)$. Suppose the first firm's insolvency-probability decreases to $\pi^{\mathbb{Q}}(X_t + x)$, while the second firm's insolvency-probability increases to $\pi^{\mathbb{Q}}(X_t - x)$. Let r^{HS}_{t+dt} be the return of the high-solvency firm and let similarly r^{LS}_{t+dt} be the return of the low-solvency firm. Consider a *high-solvency minus low-solvency* (HSmLS) portfolio with return $r^{HSmLS}_{t+dt} = r^{HS}_{t+dt} - r^{LS}_{t+dt}$. The following corollary follows directly from part (ii) of Corollary 5.1.

Corollary 5.3 (High-solvency minus low-solvency)**.** *Under the assumptions of Proposition 5, and for a fixed x, the following holds for the expected return of HSmLS:*

(i) *As* $\underline{C}(X_t) \to \infty$*, the expected return of HSmLS is negative and increasing in* $\underline{C}(X_t)$*:*
$\mathbb{E}^{\mathbb{P}}_t[r^{HSmLS}_{t+dt}] < 0$ *and* $\frac{d\mathbb{E}^{\mathbb{P}}_t[r^{HSmLS}_{t+dt}]}{d\underline{C}(X_t)} > 0.$

(ii) *As* $\underline{C}(X_t) \to \underline{C}(\underline{X}^\star)$*, the expected return of HSmLS is positive and decreasing in* $\underline{C}(X_t)$*:*
$\mathbb{E}^{\mathbb{P}}_t[r^{HSmLS}_{t+dt}] > 0$ *and* $\frac{d\mathbb{E}^{\mathbb{P}}_t[r^{HSmLS}_{t+dt}]}{d\underline{C}(X_t)} < 0.$

Corollary 5.3 says that the performance of HSmLS is conditional on the cash levels of the underlying firms. For sufficiently high levels of cash, HSmLS earns negative expected returns that increase as cash increases, while for sufficiently low levels of cash, HSmLS earns positive expected returns that decrease as cash increases. The intuition is similar to that of Corollary 5.2. Importantly, the second part implies that *the outperformance of high-solvency firms over low-solvency firms increases as cash levels decrease.*

Numerical illustration

Figure 1 illustrates the results of Proposition 5 and its cross-sectional implications in Corollaries 5.1–5.3. I consider a representative firm and economy given by the following parameters:

$$\mu^{\mathbb{Q}} = 0.04,\ \sigma = 0.15,\ k = \$4.5,$$
$$\tau = 0.15,\ r = 0.06,\ \rho = 0.33,\ \sigma^M = 0.05.$$

This choice of parameters implies that insolvency is triggered when cumulated earnings fall to $\underline{X}^\star = \$29.87$ (corresponding to $U(\underline{X}^\star) = \$50.79$ for the value of earnings-generating assets) and that the firm's unlevered asset beta is normalized to $\beta^U = \frac{\rho\sigma}{\sigma^M} = 1$. For instance, if the firm's time-t level of cumulated earnings is $X_t = \$58.82$, earnings-generating assets are worth $U(X_t) = \$100$, equity value is $E(X_t) = \$125.52$, the target cash level is $\underline{C}(X_t) = \$88.34$, the instantaneous $\mathbb{Q}$-probability of insolvency is $\pi^{\mathbb{Q}}(X_t) = 0.07$, and, finally, conditional beta is $\Omega^E(X_t)\beta^U = 1.17$.

The figure's top panels show the value of earnings-generating assets, target cash holdings, and conditional beta as functions of the probability of insolvency. For high levels of solvency (i.e. low levels of probability of insolvency) the target cash level is high, but the value of earnings-generating assets is higher. This is associated with a conditional beta above 1 (i.e. above the conditional beta of an unlevered firm) that is upwards-sloping in the probability of insolvency. However, as solvency decreases (i.e. as the probability of insolvency increases), the target cash level decreases, but the value of earnings-generating assets decreases by more and ultimately fall below the value of target cash. At the same time, conditional beta falls below 1 and becomes downwards-sloping in the probability of insolvency.

The lower left panel zooms in on the conditional beta as a function of probability of insolvency and indicates the long and short legs of the HCmLC portfolio. Because conditional beta initially increases in the probability of insolvency, the short leg of HCmLC has higher

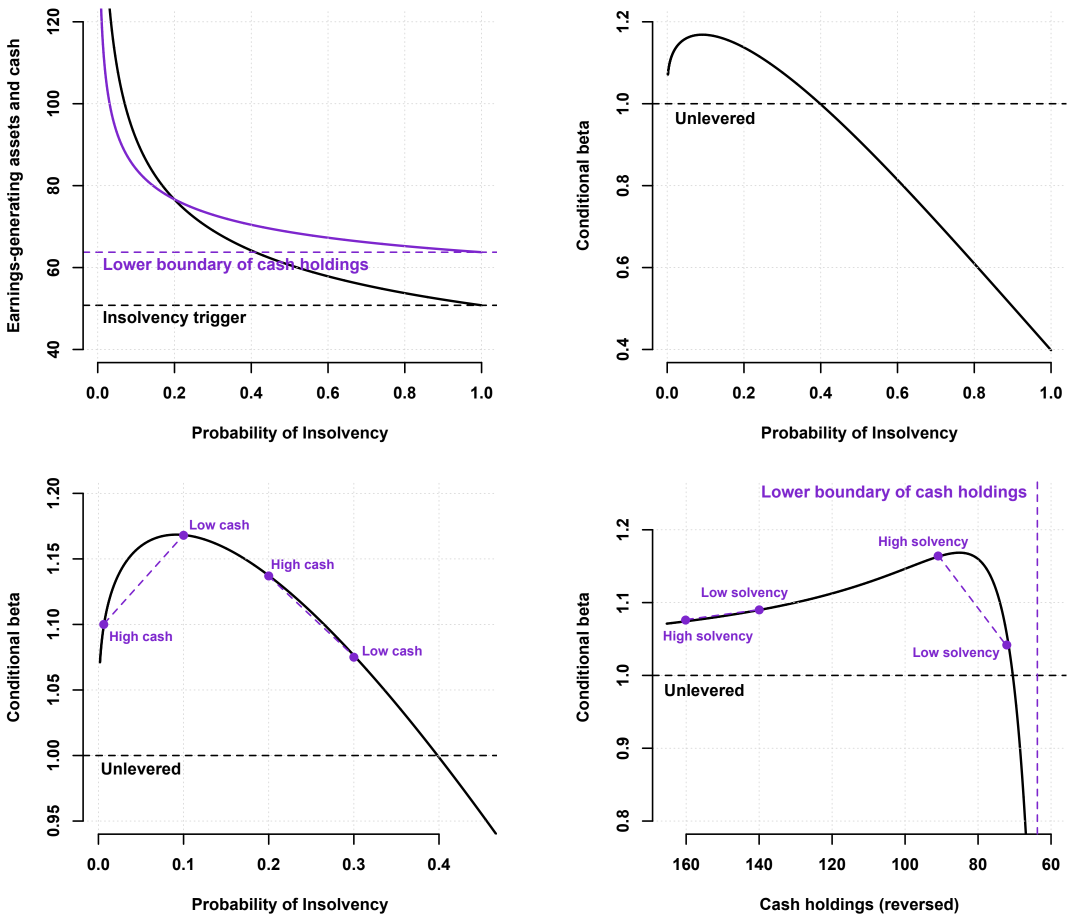

Figure 1. Asset-composition and conditional beta. This figure shows a numerical illustration of Corollary 5 and its cross-sectional implications. *Top left*: Earnings-generating asset value (black curve) and target cash holdings (purple curve) as a function of probability of insolvency with an indication of the strategic insolvency trigger (black dashed line) and the lower boundary of cash holdings (dashed purple line). *Top right:* Conditional beta as a function of the probability of insolvency with an indication of the conditional beta for an unlevered firm (dashed line). *Bottom left*: Conditional beta as a function of the probability of insolvency (from 0 to 0.4) with an indication of the high-cash minus low-cash strategy (dashed purple line segments). *Bottom right*: Conditional beta as a function of cash holdings with an indication of the high solvency minus low solvency strategy (dashed purple line segments). The parameters are $\mu^{\mathbb{Q}} = 0.04$, $\sigma = 0.15$, $k = \$4.5$, $\tau = 0.15$, $r = 0.06$, $\rho = 0.33$, and $\sigma^M = 0.05$.

expected returns than the long leg for high levels of solvency, so HCmLC has negative expected returns. This is, however, reversed for low levels of solvency, where the long leg of HCmLC has higher expected returns than the short leg.

Finally, the lower right panel shows conditional beta as a function of decreasing target cash levels and also indicates the long and short legs of the HSmLS portfolio.[11] Because target cash moves opposite the probability of insolvency, conditional beta is above 1 and downward-sloping in higher cash when target cash is high, but below 1 and upwards-sloping in higher cash when target cash is close to its lower boundary. Therefore, because conditional beta increases as cash decreases from high levels, the short leg of HSmLS has higher expected returns than the long leg for high levels of cash. The situation is, however, reversed for low levels of cash.

[11] Note that instead of plotting conditional beta as a function of the target cash level, one could equivalently use the target cash level divided by the firm's current liabilities, i.e. $\underline{C}(X)/(1-\tau)\frac{k}{r}$, which can be interpreted as the firm's *current ratio*. The resulting figure would be identical to the lower right panel of Figure 1, expect for the values on the horizontal axis, since the firm's current liabilities are constant.

3 Data and variables

This section presents the data and the variables that I use to test the model's predictions in Section 4. I use firm-level data on stock prices, accounting numbers, and credit ratings for US firms during the period 1970-2013. In the following, I first detail my sample construction and then define the measures of solvency and liquidity which I employ to in the analysis of equity returns.

3.1 Data

I search for data in the intersection of US industrial firms with stock prices in the CRSP database, accounting fundamentals in the Compustat North American database, and credit ratings or default records in the Moody's DRS (Default Risk Service) database.

For every US debt issuer in DRS' industrial category with an available third party identifier, I search for the corresponding security-level PERMNO-identifiers in the daily CRSP file and in the quarterly and yearly Compustat files, taking name changes, mergers, accusations, and parent-subsidiary relations into account, and excluding issuers which I cannot reliably match. I only include common stocks (CRSP's SHRCD 10-11) and I exclude utilities and financial firms (CRSP's SIC codes 4900-4999 and 6000-6999). The final sample has 3,947 unique firms, spanning 720,371 firm-months, and covers January 1970 to December 2013.

Because distress may ultimately result in a default or bankruptcy, I track these events for the firms in the sample. I identify a default- or bankruptcy event if it is recorded in DRS, or in CRSP (DLSTCD 400-490 or 574, or SECSTAT 'Q'), or in Compustat (DLRSN 2-3 or STALTQ 'TL'), and I count multiple events for the same firm occurring within a month as a single event. This results in a total of 874 events incurred by 683 firms, of which 529 events were identified solely through DRS, 134 solely through CRSP, and 137 solely through Compustat—the remaining 87 events were identified simultaneously by two or more sources.

I use the daily stock data to calculate market equity values, ME (the product of CRSP's PRC and SHROUT, both adjusted by their cumulative adjustment factors), and I accumulate daily log-returns (ln of 1 plus CRSP's simple return, RET) over a 21 trading day rolling window to obtain monthly returns at a daily frequency. I require at least 10 trading days to calculate a monthly return and I use delisting returns (CRSP's DLRET) whenever possible.

I use the quarterly accounting data to calculate balance-sheet based measures of firm solvency and liquidity as well as regression controls (to be detailed below). When possible, I substitute yearly accounting numbers for missing quarterly accounting numbers. I align the quarterly accounting data and the daily stock data as follows: On a given trading day, the corresponding accounting numbers are the latest ones available prior to that day. Except for returns, all variables are winsorized at the 1st and 99th percentile to remove the influence of divisions by near-zero denominators, recording errors, and statistical outliers.

3.2 Measuring solvency and liquidity

In essence, the model argues that because insolvent but liquid firms have a large fraction of their assets in cash, they also have low conditional betas, which helps rationalize low expected returns and thus the distress puzzle for such firms. I now present the variables which I employ to measure solvency and liquidity. These will constitute the 'sorting' or 'explanatory' variables in the analysis of conditional betas and expected returns in Section 4.

Solvency measures

Solvency is a firm's ability to honor its long-term debt obligations. To measure solvency, I primarily use Moody's senior unsecured long-term credit ratings. However, for robustness, and because numeric variables are better suited for regressions, I also use two balance-sheet based variables: Leverage and interest coverage.

The long-term credit rating is a categorical measure of solvency, giving Moody's relative assessment of a firm's ability to honor its financial obligations with an original maturity of one year or more (Moody's Investors Service, 2014). Credit ratings incorporate not only firms' economic and financial characteristics, but also industry conditions and 'soft' information like experts' outlook. Moody's assigns 9 long-term credit ratings: Aaa, Aa, A, Baa, Ba, B, Caa, Ca, and C. Firms rated above Baa are considered 'investment grade,' while firms rated Baa and below are considered 'speculative grade.' Within speculative-grade firms, a rating of Caa or below corresponds to 'distressed,' with C-rated firms typically being in default. In some cases, I augment Moody's original rating categories with a D-category, corresponding to an identified default or bankruptcy.

I measure leverage using the (quasi) market leverage ratio, $LT/(LT+ME)$ (Compustat's total liabilities, LTQ, divided by the sum of total liabilities and market equity), but other leverage ratios produce similar results. This is a 'stock' variable measuring the firm's total liabilities as a fraction of its total market value. The closer the ratio is to 1, the higher the risk of 'accounting insolvency.' Although this does not automatically trigger default, a firm in solvency distress will likely be forced by its debt holders to take actions in response to its deteriorated solvency—this could be a restructuring of its operations, a renegotiation of its debt obligations, or even an involuntary bankruptcy filing.

As a final measure of solvency, I use the interest coverage ratio, OI/IX (Compustat's operating income or EBITDA variable, OIBDPQ, divided by interest expense, XINTQ). This is a 'flow' variable measuring the firm's ability to generate earnings in excess of its interest expense on a quarter-by-quarter basis. If interest coverage is below 1, the firm is currently not generating enough earnings to cover its interest expense, and a cash-flow based assessment of its asset value may thus indicate solvency distress. Because firms with negative operating income do not have meaningful interest coverage ratios, I set interest coverage to zero whenever operating income is negative.

Liquidity measures

Liquidity is a firm's ability to honor its short-term debt obligations. To measure liquidity, I use three balance-sheet based variables that compare a firm's stock of liquid assets (including cash and marketable securities) to its short-term liabilities (due within one year): The current ratio, the quick ratio, and the working capital ratio.

The current ratio, CA/CL (Compustat's current assets, ACTQ, divided by current liabilities, LCTQ), measures a firm's total stock of liquid assets relative to its short-term liabilities. A current ratio below 1 means that the firm's total stock of liquid assets is insufficient to meet its short-term liabilities. This is an indication of liquidity distress, since the firm will default in the short-term if it cannot generate sufficient earnings or obtain external financing to cover its liquidity short-fall.

The quick ratio, QA/CL (Compustat's current assets, ACTQ, minus inventories, INVTQ, the difference divided by current liabilities, LCTQ), measures a firm's assets that can 'quickly' be converted into cash at near book value in order to pay off short-term liabilities. The current- and quick ratios are similar, but the quick ratio does not regard inventories as 'quick' assets, and is thus more conservative. Like the current ratio, a quick ratio below 1 is an indication of liquidity distress.

Finally, the working capital ratio (Compustat's current assets, ACTQ, minus current liabilities, LCTQ, the difference divided by total assets, ATQ) measures a firm's net liquid assets as a percentage of its total book assets. It is thus the dollar-amount of total liquid assets in excess of current liabilities relative to total book assets. Therefore, a negative working capital ratio is an indication of liquidity distress.

Liquidity vs. solvency

Figure 2 shows how the liquidity measures vary with the solvency measures in the sample. The leftmost column shows liquidity across credit ratings (the 'total' measure of solvency); the middle column shows liquidity against market leverage (the 'stock' measure of solvency); and the rightmost column shows liquidity against interest coverage (the 'flow' measure of solvency). Two observations are important.

First, liquidity generally decreases as solvency decreases. More precisely, both average and median liquidity levels are hump-shaped in decreasing credit ratings (increasing up to Ba but decreasing thereafter) and almost monotonically decreasing in increasing leverage and decreasing interest coverage. Note, however, that investment grade firms (rated Aaa-A) still have higher liquidity than distressed and defaulted firms (rated Caa-C and D). The relatively modest reserves of liquid assets for investment grade firms is probably due to their high ratings, which mean that they need to worry less about financing constraints.

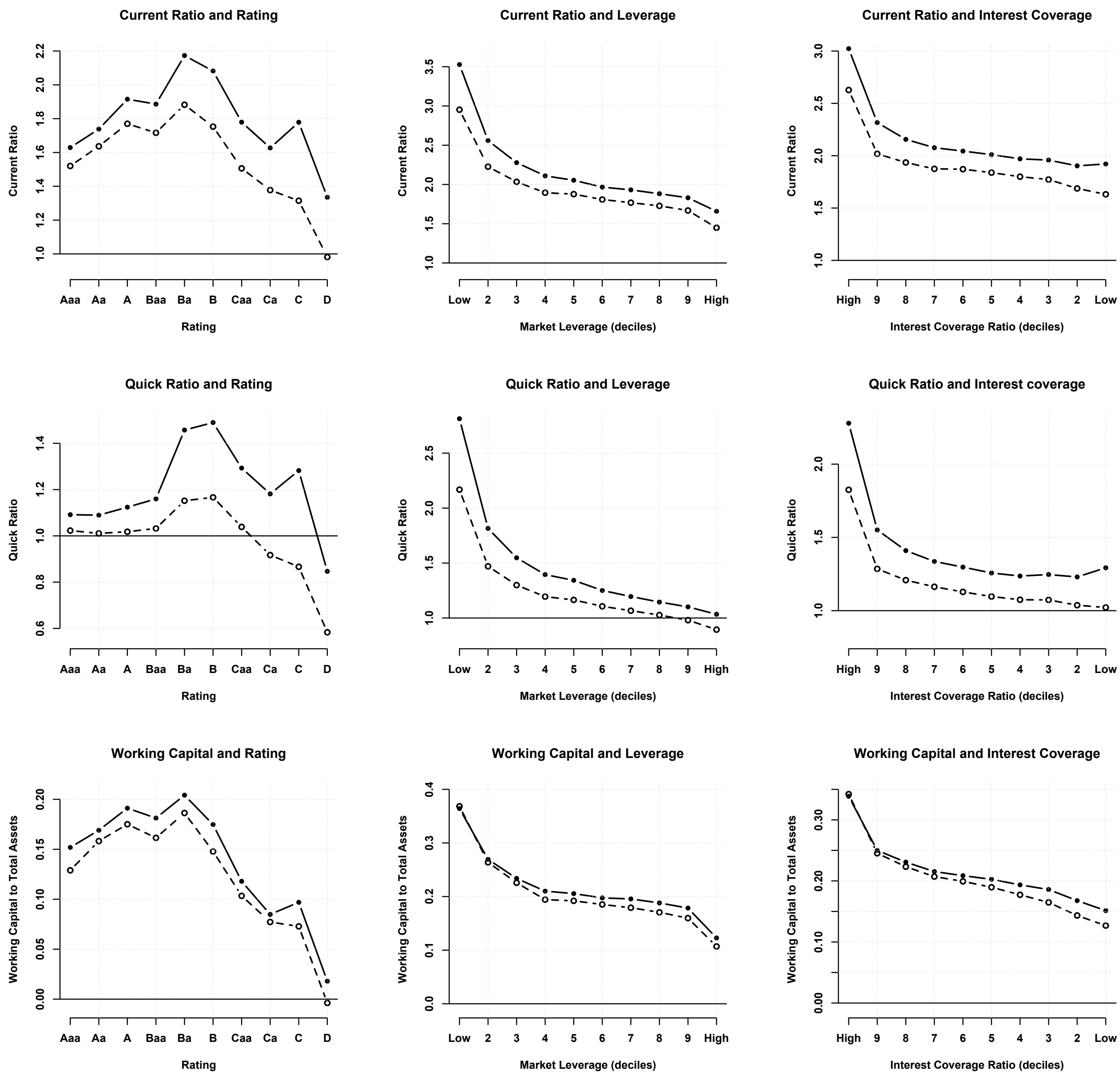

Figure 2. Liquidity measures sorted across solvency measures. This figure shows the three liquidity measures (current ratio, CA/CL, quick ratio, QA/CL, and working capital, WC/AT) plotted against the three solvency measures (Moody's long-term credit rating, deciles of market leverage, $LT/(LT + ME)$, and deciles of interest coverage, OI/IX). Solid lines indicate means within groups while dashed lines indicate medians. In all panels, a horizontal move to the right corresponds to lower solvency, while a vertical move downwards corresponds to lower liquidity.

Second, even the least solvent firms hold liquid assets in levels that on average can cover their current liabilities. The average firm rated Caa-C is still liquid and the same is true for the average firm in the highest leverage decile and the lowest interest coverage decile. Furthermore, even defaulted firms (rated D) are only illiquid according to the conservative quick ratio.

In sum, Figure 2 shows that the distribution of liquidity across solvency within the sample is closely related to the model's equilibrium policy for holding cash: Less solvent firms demand less cash for offsetting liquidity risk, but even the least solvent firms hold cash levels that allow them to service short-term liabilities (Proposition 1–2).

4 Empirical results

In this section, I investigate the model's rationale of the distress puzzle for insolvent but liquid firms by testing its four cross-sectional predictions.

In the model, I focus on the equilibrium case where cash is at the target level, i.e. when the firm is liquid. Hence, one could argue that the model's predictions should be tested in the subsample of liquid firms, i.e. firm-observations where liquid assets meet or exceed current liabilities. However, since Figure 2 suggests that even the least solvent firms in the sample are on average still liquid, I test the model's predictions using the entire sample. This biases my analysis against finding support for the model's rationale of the distress puzzle, but increases the available number of observations and alleviates concerns regarding data-mining.

The model predicts that conditional betas and expected returns are humped and decreasing as solvency decreases, because a less solvent firms has a higher fraction of its assets (Proposition 5). The dual of this prediction is that conditional betas and expected returns are humped and decreasing as cash decreases, because a firm with less cash is also less solvent (Corollary 5.1). In the model, there is a one-to-one correspondence between conditional betas and expected returns (cf. relation (17)). However, in practice, the relation may not be as clearcut: Conditional betas are subject to estimation noise and there is mixed evidence regarding the performance of conditional betas in asset pricing tests.[12] Therefore, I test these predictions separately for conditional betas and expected returns: First for conditional betas in the cross-section of solvency and cash-levels, and then for expected returns using 1) cross-sectional regressions of firm-level returns on the solvency and liquidity measures, and 2) portfolio returns and alphas formed on the solvency and liquidity measures.

The model also predicts that the outperformance of high-cash firms over low-cash firms increases as firms become less solvent (Corollary 5.2). I test this predictions using the returns and alphas of portfolios long high-cash and short low-cash firms (HCmLC portfolios) in the cross-section of solvency levels.

Finally, the model predicts that the outperformance of high-solvency firms over low-solvency firms increases as cash decreases (Corollary 5.3). For this prediction, I use the returns and alphas of portfolios long high-solvency and short low-solvency firms (HSmLS portfolios) in the cross-section of cash-levels.

4.1 Conditional betas

To test the predictions of Proposition 5 and Corollary 5.1 for conditional betas, Figure 3 plots estimated conditional betas across the solvency and liquidity measures.

The conditional betas are estimated at the firm-level at a monthly frequency by regressing daily excess returns on the daily excess returns of the CRSP value-weighted "market" index (available in the CRSP file or on prof. Ken French's website as the MKT-factor). The risk-free rate is proxied by the 1-month US T-bill rate. I require a minimum of 10 trading days to estimate a monthly beta and, to reduce the influence of outliers, I winsorize the monthly betas at the 1st and 99th percentile. Finally, following Vasicek (1973) and Frazzini and Pedersen (2014), I adjust firm i's estimated beta at month t, $\widehat{\beta}_{it}$, towards the cross-sectional average beta for the corresponding month, $\overline{\beta}_t$, by using the estimate

$$\widetilde{\beta}_{it} = w_{it}\widehat{\beta}_{it} + (1 - w_{it})\overline{\beta}_t.$$

Here, $w_{it} = 1 - v_i^2/(v_i^2 + v_t^2)$ is a Bayesian adjustment factor calculated using the time-series variance of the estimated betas for firm i, v_i^2, and the cross-sectional variance of the estimated betas at month t, v_t^2. It places more weight on the firm's beta estimates when their variance is small or when the cross-sectional variance is high.[13]

Figure 3 shows strong evidence in favor of the model's predictions that conditional betas are high for high-solvency and high-cash firms, but low for low-solvency and low-cash firms. The most solvent firms

[12] For instance, Lewellen and Nagel (2006) refute the early results of Jagannathan and Wang (1996) and others that conditional betas can explain asset-pricing anomalies. On the other hand, recent studies by Adrian and Franzoni (2009) and Bali, Engle, and Tang (2014) find that conditional betas do have have explanatory power in asset-pricing tests using more refined estimation procedures and tests.

[13] I have also produced a version of Figure 3 where I replace the firm-specific adjustment factors, w_{it}, with their average across firms and time, 0.54, and where the cross-sectional average beta is set as $\overline{\beta}_t = 1$ for all months. This mimics the simplified adjustment of conditional betas used by Frazzini and Pedersen (2014). The resulting figure is very similar to Figure 3.

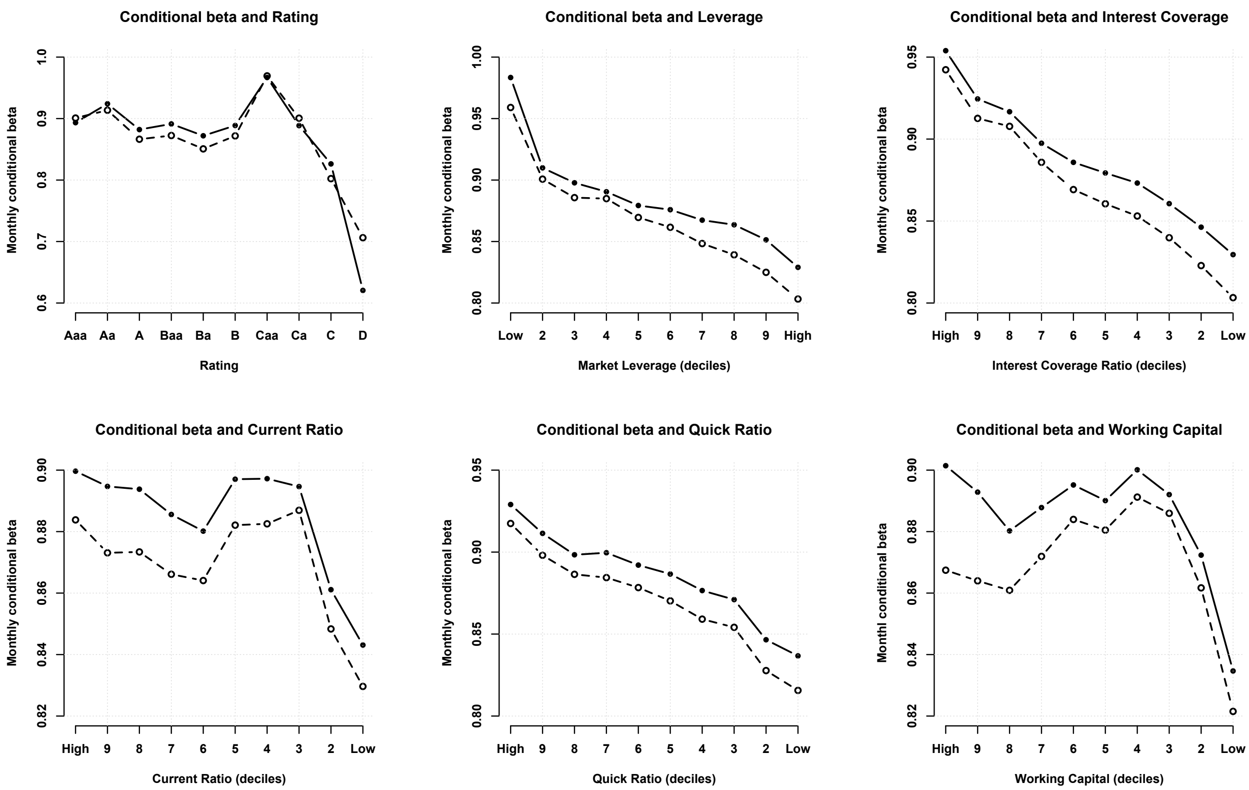

Figure 3. Conditional betas across solvency and liquidity measures. This figure shows conditional (time-varying) betas plotted against the three solvency measures (top panels: Moody's credit rating, deciles of market leverage, $LT/(LT + ME)$, and deciles of interest coverage, OI/IX) and the three liquidity measures (bottom panels: Deciles of current ratio, CA/CL, quick ratio, QA/CL, and working capital, WC/AT). Conditional betas are estimated at the firm-level at a monthly frequency by regressing daily excess returns on the daily excess returns of the CRSP value-weighted "market" index. The risk-free rate is proxied by daily values for the 1-month US T-bill rate. Each monthly beta is estimated using a minimum of 10 trading days. To reduce the influence of outliers, the monthly betas are winsorized at the 1st and 99th percentile and adjusted towards the cross-sectional mean monthly beta as in Vasicek (1973) and Frazzini and Pedersen (2014). Solid lines indicate means while dashed lines indicate medians. In the top (bottom) panels, a horizontal move to the right corresponds to lower solvency (liquidity).

(high credit rating, low leverage, or high interest coverage) have average monthly betas close to 1, while the least solvent firms have average monthly betas between 0.65 and 0.8. Similarly, the most liquid firms also have monthly betas close to 1, while the least liquid firms have monthly betas around 0.8. If higher conditional beta is associated with higher expected returns—at least on average—then this is also in favor of the model's predictions that high-solvency firms and high-cash firms have higher expected returns than, respectively, low-solvency firms and low-cash firms.

On the other hand, the figure shows mixed evidence for the model's prediction that conditional betas are hump-shaped in decreasing solvency and cash-levels, i.e. that conditional betas are initially upwards-sloping but eventually downwards-sloping as both solvency and cash-levels decrease. For the solvency measures, the monthly betas are hump-shaped as credit ratings deteriorate, but are almost monotonically decreasing as leverage increases and as interest coverage decreases. For the liquidity measures, the monthly betas are hump-shaped as current ratio and working capital ratio decrease, but are almost monotonically decreasing as the quick ratio decreases.

To summarize, Figure 3 strongly confirms the model's predictions that high-solvency firms and high-cash have higher conditional betas than, respectively, low-solvency firms and low-cash firms. This extends the findings of Garlappi and Yan (2011), in that the decrease in conditional betas is prevalent in both the solvency and liquidity dimensions of default risk. On the other hand, the figure shows mixed evidence for the predicted hump-shaped relations between conditional betas and decreasing solvency and cash levels.

Table 1. Fama-MacBeth regressions of firm-level returns on solvency and liquidity measures. This table shows Fama-MacBeth slope-coefficients ($\times 10^2$) from monthly cross-sectional regressions of firm-level returns on solvency and liquidity measures. At each calendar month in the sample, firm-level 1-month returns (the dependent variable) are regressed on lagged values of the explanatory variables (the independent variables). Market leverage is $LT/(LT + ME)$, interest coverage is OI/IX, current ratio is CA/CL, quick ratio is QA/CL where QA is current assets less inventories, and working capital is WC/AT where WC is current assets less current liabilities. All regressions include controls for firm size ($\log(ME)$), book-to-market ($\log(B/M)$), the return from 2 months ago until 1 month ago ($r_{t-2,t-1}$), and the return from 12 months ago until 2 months ago ($r_{t-12,t-2}$). Parentheses in subscript give the t-statistics for a null value of zero, based on standard errors that are adjusted for heteroskedasticity and autocorrelation as in Newey and West (1987) using a lag length of 12 months. Statistical significance at the 5% level is indicated in bold.

Independent variables	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Market leverage	$\mathbf{-0.89}_{(\mathbf{-3.04})}$	$1.39_{(1.70)}$								
[Market leverage]^2		$\mathbf{-2.50}_{(\mathbf{-2.94})}$								
log(Interest coverage)			$\mathbf{0.48}_{(\mathbf{12.12})}$	$\mathbf{1.38}_{(\mathbf{12.44})}$						
[log(Interest coverage)]^2				$\mathbf{-0.17}_{(\mathbf{-10.48})}$						
log(Current ratio)					$\mathbf{0.23}_{(\mathbf{3.18})}$	$\mathbf{0.73}_{(\mathbf{6.63})}$				
[log(Current ratio)]^2						$\mathbf{-0.38}_{(\mathbf{-5.60})}$				
log(Quick ratio)							$\mathbf{0.27}_{(\mathbf{4.05})}$	$\mathbf{0.29}_{(\mathbf{4.43})}$		
[log(Quick ratio)]^2								$\mathbf{-0.12}_{(\mathbf{-3.36})}$		
Working capital									$\mathbf{1.13}_{(\mathbf{4.31})}$	$\mathbf{3.01}_{(\mathbf{6.55})}$
[Working capital]^2										$\mathbf{-3.85}_{(\mathbf{-5.16})}$
$\log(BE/ME)$	$\mathbf{0.47}_{(\mathbf{6.02})}$	$\mathbf{0.45}_{(\mathbf{5.76})}$	$\mathbf{0.42}_{(\mathbf{5.52})}$	$\mathbf{0.37}_{(\mathbf{4.89})}$	$\mathbf{0.35}_{(\mathbf{4.56})}$	$\mathbf{0.33}_{(\mathbf{4.39})}$	$\mathbf{0.36}_{(\mathbf{4.89})}$	$\mathbf{0.35}_{(\mathbf{4.87})}$	$\mathbf{0.37}_{(\mathbf{4.88})}$	$\mathbf{0.34}_{(\mathbf{4.67})}$
$\log(ME)$	$0.05_{(1.11)}$	$0.03_{(0.82)}$	$-0.08_{(-1.83)}$	$\mathbf{-0.13}_{(\mathbf{-3.16})}$	$0.07_{(1.60)}$	$0.06_{(1.38)}$	$0.07_{(1.64)}$	$0.07_{(1.52)}$	$0.09_{(1.89)}$	$0.07_{(1.58)}$
$r_{t-2,t-1}$	$\mathbf{-2.00}_{(\mathbf{-3.42})}$	$\mathbf{-2.08}_{(\mathbf{-3.60})}$	$\mathbf{-2.42}_{(\mathbf{-4.15})}$	$\mathbf{-2.58}_{(\mathbf{-4.46})}$	$\mathbf{-1.83}_{(\mathbf{-3.02})}$	$\mathbf{-1.86}_{(\mathbf{-3.07})}$	$\mathbf{-1.86}_{(\mathbf{-3.10})}$	$\mathbf{-1.89}_{(\mathbf{-3.16})}$	$\mathbf{-1.87}_{(\mathbf{-3.10})}$	$\mathbf{-1.90}_{(\mathbf{-3.14})}$
$r_{t-12,t-2}$	$\mathbf{1.42}_{(\mathbf{6.05})}$	$\mathbf{1.38}_{(\mathbf{5.95})}$	$\mathbf{0.90}_{(\mathbf{3.79})}$	$\mathbf{0.75}_{(\mathbf{3.16})}$	$\mathbf{1.46}_{(\mathbf{5.96})}$	$\mathbf{1.44}_{(\mathbf{5.87})}$	$\mathbf{1.45}_{(\mathbf{5.99})}$	$\mathbf{1.45}_{(\mathbf{6.00})}$	$\mathbf{1.43}_{(\mathbf{5.83})}$	$\mathbf{1.41}_{(\mathbf{5.74})}$

4.2 Fama-MacBeth regressions

I now test the predictions of Proposition 5 and Corollary 5.1 for expected returns using predictive cross-sectional regressions of firm-level returns on solvency and liquidity. In the following, I consider both linear and linear-quadratic regression specifications. A linear specification allows me to test whether the correlation between expected returns and a given solvency or liquidity measure is as predicted by the model. A linear-quadratic specification allows me to conduct a simple but direct test of a hump-shaped relation by testing whether the coefficient on the quadratic term is negative.

Specifically, I consider linear and linear-quadratic regression specifications of the forms

$$\text{L:} \qquad r_{it}^E = \gamma_0 + \gamma_1 V_{i,t-1} + \text{controls} + \epsilon_{it},$$
$$\text{L-Q:} \qquad r_{it}^E = \tilde{\gamma}_0 + \tilde{\gamma}_1 V_{i,t-1} + \tilde{\gamma}_2 V_{i,t-1}^2 + \text{controls} + \tilde{\epsilon}_{it}.$$

Here, r_{it}^E is firm i's 1-month return at month t, $V_{i,t-1}$ is a given measure of the firm's solvency or liquidity at month $t-1$, while ϵ_{it} and $\tilde{\epsilon}_{it}$ are mean-zero error terms. In the linear specification, the parameter γ_1 estimates the marginal effect of V on expected (next-month) returns. In the linear-quadratic specification, the parameter $\tilde{\gamma}_1$ estimates the marginal effect of V when V is close to zero, while the parameter $\tilde{\gamma}_2$ estimates the curvature of the relation between expected returns and V. Hence, a hump-shaped relation between expected returns and V can be tested by testing whether $\tilde{\gamma}_2 < 0$.

Table 1 shows results from monthly Fama and MacBeth (1987) regressions of firm-level 1-month returns on lagged values for the two numeric solvency measures and the three liquidity measures. I log-transform non-negative variables that do not have a naturally bounded distribution. All specifications include 1-month lagged controls for firm size ($\log(ME)$) and book-to-market equity ($\log(BE/ME)$), as well as the firm's return from 2 months ago until a month ago ($r_{t-2,t-1}$, a control for short-term-reversal) and the firm's return from 12 months ago until 2 months ago ($r_{t-12,t-2}$, a control for momentum).[14] The table reports average monthly slope-coefficients with t-statistics based on standard errors that are adjusted for heteroskedasticity and autocorrelation as in Newey and West (1987) using a lag length of 12 months.

Specification (1) shows that higher market leverage (i.e. lower solvency measured in 'stock' terms) is associated with significantly lower expected returns. Specification (2) shows that the curvature of the relation between expected returns and market leverage is significantly negative—that is, if the relation between expected returns and market leverage can be reasonably approximated by a linear-quadratic function, then this function has a significantly negative second derivative. That market leverage has a positive but insignificant linear-effect in the linear-quadratic specification means that when market leverage is close to zero, an increase in market leverage is associated with a positive but insignificant increase in expected returns.

Specification (3) shows that higher interest coverage (i.e. higher solvency measured in 'flow' terms) is associated with significantly higher expected returns. Interestingly, the t-statistic on the effect of interest coverage is 12 standard deviations away from zero and over twice as large as the t-statistic on the effect of book-to-market equity from the same specification. Specification (4) shows that the curvature of the relation between expected returns and interest coverage is significantly negative, and that, when interest coverage is close to zero, an increase in interest coverage is associated with significantly higher expected returns.

Specification (5) shows that higher current ratio (i.e. higher liquidity) is associated with significantly higher expected returns, while specification (6) shows that the curvature of the relation between expected returns and current ratio is significantly negative. The same is true for the quick ratio in specifications (7) and (8), and for

[14] Book-to-market equity is book equity, BE, divided by market equity, ME. Similar to Fama and French (1993), I calculate quarterly book equity as stockholders' equity (Compustat's SEQQ) plus deferred taxes and investment tax credit (Compustat's TXDITCQ) minus preferred stock (Compustat's PSTKQ). I exclude negative BE-values and, following Novy-Marx (2013), I use 6-month lagged ME to avoid taking unintentional positions in momentum. The return from 12 months ago until 2 months ago is calculated by compounding daily log-returns, and similarly for the return from 2 months ago until a month ago.

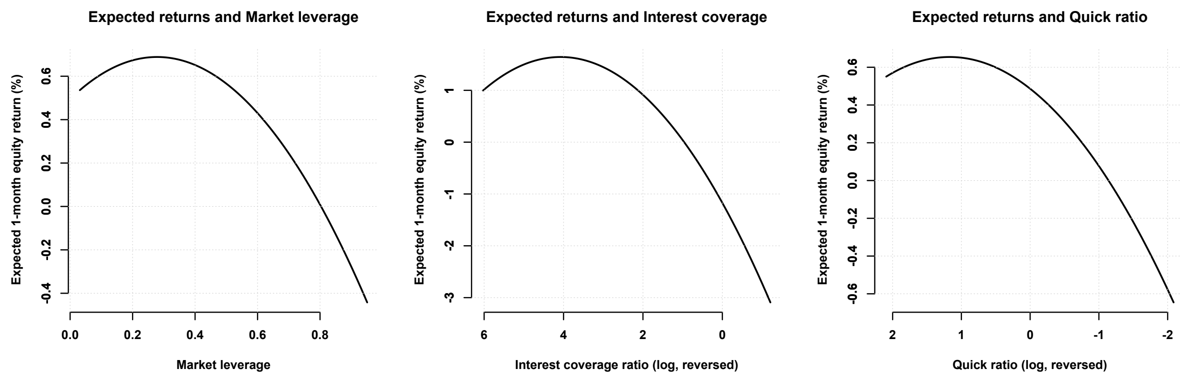

Figure 4. Estimated relations from linear-quadratic regressions of returns on the solvency and liquidity measures. This figure shows the estimated relations from the linear-quadratic regressions of firm-level returns on the solvency and liquidity measures in Table 1. *Left panel*: Estimated relation between 1-month expected returns and lagged market leverage from specification (2). *Middle panel*: Estimated relation between 1-month expected returns and lagged (log) interest coverage from specification (4). *Right panel*: Estimated relation between 1-month expected returns and lagged (log) quick ratio from specification (8). In all panels, the control variables from the regression specifications are fixed at their sample means and the range of the horizontal axis is the observed range in the sample.

the working capital ratio in specification (9) and (10).

To summarize, the the linear specifications provide strong support for the model's predictions that firms with higher solvency (i.e. lower leverage or higher interest coverage) and higher cash (i.e. higher values for the liquidity measures) have higher expected returns. Furthermore, the linear-quadratic specifications show support for a hump-shaped relation between expected returns and the measures of solvency and liquidity. As an illustration of the latter point, Figure 4 shows the estimated relations from the linear-quadratic regressions of returns on market leverage in specification (2); returns on interest coverage in specification (4); and returns on quick ratio in specification (8) [the corresponding plots for the current ratio and the working capital ratio are very similar to the one for the quick ratio, and are omitted here for brevity]. The estimated relations are seen to be the empirical counterparts of the model's implied relations in the lower panels of Figure 1.

4.3 Portfolio returns and alphas

The tests of the two previous subsections were based on estimated conditional betas and Fama-MacBeth regression coefficients. Since both are potentially sensitive to estimation noise, misspecified parametric forms, and an over-weighting of small firms, I now test Proposition 5 and Corollary 5.1 using the realized returns and alphas of value-weighted portfolios.

Table 2 shows average excess returns and 1-4 factor alphas (i.e. 'risk-adjusted' returns) as well as Sharpe and Information ratios for portfolios formed on the three solvency measures, while Table 3 shows the same for portfolios formed on the three liquidity measures.

At the beginning of each calendar month, I sort firms into portfolios according to their solvency or liquidity levels for the previous month. The portfolios are value-weighted, refreshed every calendar month, and rebalanced every calendar month to maintain value-weighting. Following Asness, Moskowitz, and Pedersen (2013) and Frazzini and Pedersen (2014), I construct the portfolio weights using ranks—in this case ranks of the previous month's market equity values. Using ranks instead of raw market equity values mitigates the extremely skewed distribution of market equity values in the cross-section (which over-weights large firms) and implies a much higher degree of diversification, since more firms are given a nonzero weight and the weights are less extreme. Specifically, the weight of firm i at month t in portfolio $J(t)$ is given by

$$w_{it}^{J} = \frac{\text{rank}(ME_{it}) \cdot 1_{(i \in J(t))}}{\sum_{i \in J(t)} \text{rank}(ME_{it})}.$$

The tables report time-series averages of monthly portfolio returns in excess of the 1-month US T-bill rate. To reduce the influence of outliers, I exclude each portfolio's highest and lowest realized return. The corresponding alphas are the intercepts from monthly time-series regressions of excess returns on the "market" factor (MKT); the size (SMB) and value (HML) factors of Fama and French (1993); and the momentum factor (UMD) of Carhart (1997). Finally, the reported Sharpe

(Information) ratios are calculated as the annualized average excess return (annualized 4-factor alpha) divided by the annualized volatility of excess returns (annualized residual standard error from the 4-factor time-series regression).

Panel A of Table 2 shows the results for the portfolios formed on credit ratings. Consistent with the model's prediction that high-solvency firms have higher expected returns than low-solvency firms, it is seen that investment grade firms (ratings above Baa) have positive and significant excess returns and alphas, while speculative grade and distressed firms have insignificant and even significantly negative excess returns and alphas. The Sharpe and Information ratios are also clearly higher for investment grade firms compared to speculative grade and distressed firms. Furthermore, and consistent with the model's prediction that expected returns are hump-shaped in decreasing solvency levels, it is seen that excess returns, alphas, and Sharpe/Information ratios increase slightly as ratings deteriorate from Aaa to Aa, and then decreasing monotonically as ratings deteriorate from Aa to C.

Panels B and C of Table 2 show the results for the portfolios formed on deciles of market leverage and interest coverage. Once again, high-solvency firms (low leverage deciles or high interest coverage deciles) outperform low-solvency firms. For the portfolios formed on market leverage, the outperformance is strongest in excess returns and Sharpe ratios because the alphas are mostly insignificant, but the magnitudes of the alphas and Information ratios follow the same pattern as the magnitudes of the excess returns and Sharpe ratios. For the portfolios formed on interest coverage, the outperformance is, however, prevalent in excess returns, alphas, and Sharpe/Information ratios. Finally, it is seen that the performance-measures are hump-shaped in increasing leverage and decreasing interest coverage—the only exception is the Information ratio for the portfolios formed on interest coverage, which is monotonically decreasing as interest coverage decreases.

Table 3 shows the results for the portfolios formed on the deciles of the three liquidity measures. Consistent with the model's prediction that high-cash firms have higher expected returns than low-cash firms, it is seen that firms in the highest deciles outperform firms in the lowest deciles for all three liquidity measures. The outperformance is strongest in excess returns and Sharpe ratios because the alphas are mostly insignificant, but the magnitudes of the alphas and Information ratios follow the same pattern as the magnitudes of the excess returns and Sharpe ratios. Furthermore, and consistent with the model's prediction that expected returns are hump-shaped in decreasing cash-levels, it is seen that excess returns, alphas, and Sharpe/Information ratios increase from as liquidity declines from high to mid-levels, but decrease as liquidity declines from mid to low-levels.

To summarize, the tests based on portfolio returns and alphas confirm the previous tests based on conditional betas and cross-sectional regressions and show strong support for the predictions of Proposition 5 and Corollary 5.1—namely, that because insolvent but liquid firms have a large fraction of their assets in cash but also demand less cash, expected returns are initially high and upwards-sloping but eventually low and downwards-sloping as both solvency and cash-levels decrease.

A note on unconditional betas

For completeness, Tables 2 and 3 also report each portfolio's realized CAPM-beta (with a t-statistic for a null value of 1).

For the portfolios formed on the solvency measures, the unconditional betas are either U-shaped or monotonically increasing as solvency levels decrease. Similarly, for the portfolios formed on the liquidity measures, the unconditional betas are decreasing or slightly U-shaped decreasing cash-levels. This is in stark contrast to the hump-shaped or decreasing conditional betas shown in Figure 3. Now, because the conditional betas more precisely reflect firms' capital structure, this discrepancy between conditional and unconditional betas suggests that the unconditional betas are only capturing the part of the exposure to systematic risk that is due to less solvent and less liquid firms' higher levels of leverage (cf. the middle column of Figure 2). Importantly, the unconditional betas seem to ignore other, important aspects of firms' capital structure—in particular, the presence of cash used to offset liquidity risk.

Table 2. Monthly excess returns of portfolios formed on solvency measures. This table shows excess returns, alphas, and related quantities for portfolios formed on solvency measures. At the beginning of each calendar month, I assign firms into portfolios according to credit ratings (Panel A), deciles of market leverage (Panel B), or deciles of interest coverage ratio (Panel C) for the previous month. The portfolios are value-weighted using the ranks of the previous month's market equity values, refreshed every calendar month, and rebalanced every calendar month to maintain value-weighting. The highest and lowest realized return is excluded for each portfolio. Excess return is the time-series average of the monthly portfolio returns in excess of the 1-month US Treasury bill rate. CAPM alpha and CAPM beta (unconditional) are the intercept and slope estimates from a time-series regression of monthly excess returns on the excess returns of the value-weighted CRSP "market" index (MKT). Three- and four-factor alphas are the intercepts from time-series regressions of monthly excess returns on the three Fama and French (1993) factors (MKT, SMB, and HML) or these three factors and the Carhart (1997) factor (UMD). Sharpe ratio is the annualized average excess return divided by the annualized volatility of the excess returns. Information ratio is the annualized 4-factor alpha divided by the annualized residual standard error from the 4-factor time-series regression. Parentheses in subscript give *t*-statistics. For the excess returns and alphas, the null value is zero, while it is one for the unconditional beta. Statistical significance at the 5% level is indicated in bold.

Panel A

Credit Rating	Aaa	Aa	A	Baa	Ba	B	Caa	Ca	C
Excess return (avg. mon. %)	$\mathbf{0.72}_{(\mathbf{3.62})}$	$\mathbf{0.83}_{(\mathbf{4.35})}$	$\mathbf{0.75}_{(\mathbf{3.44})}$	$\mathbf{0.62}_{(\mathbf{2.58})}$	$0.42_{(1.44)}$	$-0.17_{(-0.46)}$	$\mathbf{-1.31}_{(\mathbf{-2.58})}$	$\mathbf{-2.09}_{(\mathbf{-2.07})}$	$\mathbf{-5.68}_{(\mathbf{-3.22})}$
CAPM alpha (mon. %)	$\mathbf{0.45}_{(\mathbf{3.47})}$	$\mathbf{0.48}_{(\mathbf{4.75})}$	$\mathbf{0.31}_{(\mathbf{4.40})}$	$0.14_{(1.44)}$	$-0.14_{(-1.08)}$	$\mathbf{-0.84}_{(\mathbf{-3.82})}$	$\mathbf{-2.21}_{(\mathbf{-6.55})}$	$\mathbf{-2.97}_{(\mathbf{-3.20})}$	$\mathbf{-6.31}_{(\mathbf{-3.65})}$
Three-factor alpha (mon. %)	$\mathbf{0.40}_{(\mathbf{3.52})}$	$\mathbf{0.39}_{(\mathbf{4.24})}$	$\mathbf{0.22}_{(\mathbf{3.34})}$	$-0.01_{(-0.11)}$	$\mathbf{-0.27}_{(\mathbf{-2.65})}$	$\mathbf{-0.96}_{(\mathbf{-5.36})}$	$\mathbf{-2.41}_{(\mathbf{-8.34})}$	$\mathbf{-3.19}_{(\mathbf{-3.58})}$	$\mathbf{-6.23}_{(\mathbf{-3.64})}$
Four-factor alpha (mon. %)	$\mathbf{0.34}_{(\mathbf{2.98})}$	$\mathbf{0.34}_{(\mathbf{3.69})}$	$\mathbf{0.28}_{(\mathbf{4.19})}$	$0.07_{(0.86)}$	$-0.18_{(-1.82)}$	$\mathbf{-0.78}_{(\mathbf{-4.37})}$	$\mathbf{-2.07}_{(\mathbf{-7.37})}$	$\mathbf{-2.66}_{(\mathbf{-2.99})}$	$\mathbf{-6.34}_{(\mathbf{-3.67})}$
Sharpe Ratio (ann.)	0.56	0.67	0.53	0.40	0.22	−0.07	−0.44	−0.39	−0.80
Information Ratio (ann.)	0.48	0.59	0.67	0.14	−0.29	−0.70	−1.30	−0.58	−0.93
CAPM beta (unconditional)	$\mathbf{0.73}_{(\mathbf{-9.79})}$	$\mathbf{0.81}_{(\mathbf{-8.79})}$	$1.01_{(0.36)}$	$\mathbf{1.09}_{(\mathbf{4.27})}$	$\mathbf{1.28}_{(\mathbf{10.01})}$	$\mathbf{1.48}_{(\mathbf{10.20})}$	$\mathbf{1.66}_{(\mathbf{9.20})}$	$\mathbf{1.59}_{(\mathbf{3.08})}$	$1.18_{(0.52)}$
Months	496	503	505	505	504	505	415	340	196

Panel B

Market Leverage	Low	2	3	4	5	6	7	8	9	High
Excess return (avg. mon. %)	$0.44_{(1.66)}$	$\mathbf{0.57}_{(\mathbf{2.47})}$	$\mathbf{0.68}_{(\mathbf{3.05})}$	$\mathbf{0.65}_{(\mathbf{2.82})}$	$\mathbf{0.72}_{(\mathbf{3.06})}$	$\mathbf{0.78}_{(\mathbf{3.12})}$	$\mathbf{0.76}_{(\mathbf{2.88})}$	$\mathbf{0.69}_{(\mathbf{2.39})}$	$0.42_{(1.31)}$	$-0.38_{(-0.99)}$
CAPM alpha (mon. %)	$-0.13_{(-1.23)}$	$0.05_{(0.66)}$	$\mathbf{0.17}_{(\mathbf{2.47})}$	$0.13_{(1.68)}$	$\mathbf{0.20}_{(\mathbf{2.22})}$	$\mathbf{0.24}_{(\mathbf{2.28})}$	$0.20_{(1.64)}$	$0.07_{(0.49)}$	$-0.25_{(-1.53)}$	$\mathbf{-1.10}_{(\mathbf{-5.04})}$
Three-factor alpha (mon. %)	$0.14_{(1.86)}$	$0.11_{(1.69)}$	$\mathbf{0.17}_{(\mathbf{2.48})}$	$0.04_{(0.57)}$	$0.04_{(0.55)}$	$0.03_{(0.30)}$	$-0.07_{(-0.74)}$	$\mathbf{-0.24}_{(\mathbf{-2.51})}$	$\mathbf{-0.61}_{(\mathbf{-5.11})}$	$\mathbf{-1.48}_{(\mathbf{-8.81})}$
Four-factor alpha (mon. %)	$0.09_{(1.22)}$	$0.08_{(1.11)}$	$\mathbf{0.19}_{(\mathbf{2.78})}$	$0.08_{(1.15)}$	$0.12_{(1.53)}$	$0.13_{(1.62)}$	$0.10_{(1.13)}$	$-0.02_{(-0.22)}$	$\mathbf{-0.28}_{(\mathbf{-2.82})}$	$\mathbf{-1.06}_{(\mathbf{-7.55})}$
Sharpe Ratio (ann.)	0.25	0.38	0.46	0.43	0.46	0.47	0.44	0.36	0.20	−0.15
Information Ratio (ann.)	0.19	0.18	0.44	0.18	0.24	0.26	0.18	−0.03	−0.45	−1.19
CAPM beta (unconditional)	$\mathbf{1.21}_{(\mathbf{8.71})}$	$\mathbf{1.10}_{(\mathbf{6.06})}$	$\mathbf{1.06}_{(\mathbf{3.81})}$	$\mathbf{1.10}_{(\mathbf{5.71})}$	$\mathbf{1.10}_{(\mathbf{4.83})}$	$\mathbf{1.14}_{(\mathbf{6.09})}$	$\mathbf{1.18}_{(\mathbf{6.82})}$	$\mathbf{1.27}_{(\mathbf{9.17})}$	$\mathbf{1.38}_{(\mathbf{10.49})}$	$\mathbf{1.54}_{(\mathbf{11.46})}$
Months	520	520	520	520	520	520	520	520	520	520

Panel C

Interest Coverage	High	9	8	7	6	5	4	3	2	Low
Excess Return (avg. mon. %)	$\mathbf{0.93}_{(\mathbf{3.77})}$	$\mathbf{1.02}_{(\mathbf{4.47})}$	$\mathbf{0.97}_{(\mathbf{4.35})}$	$\mathbf{0.94}_{(\mathbf{4.23})}$	$\mathbf{0.87}_{(\mathbf{3.87})}$	$\mathbf{0.80}_{(\mathbf{3.34})}$	$\mathbf{0.62}_{(\mathbf{2.45})}$	$0.45_{(1.64)}$	$0.04_{(0.12)}$	$\mathbf{-1.20}_{(\mathbf{-3.39})}$
CAPM alpha (mon. %)	$\mathbf{0.39}_{(\mathbf{4.53})}$	$\mathbf{0.52}_{(\mathbf{6.76})}$	$\mathbf{0.48}_{(\mathbf{6.03})}$	$\mathbf{0.46}_{(\mathbf{5.76})}$	$\mathbf{0.37}_{(\mathbf{4.52})}$	$\mathbf{0.29}_{(\mathbf{2.93})}$	$0.08_{(0.76)}$	$-0.13_{(-1.02)}$	$\mathbf{-0.61}_{(\mathbf{-4.33})}$	$\mathbf{-1.94}_{(\mathbf{-10.90})}$
Three-factor alpha (mon. %)	$\mathbf{0.55}_{(\mathbf{7.79})}$	$\mathbf{0.53}_{(\mathbf{7.19})}$	$\mathbf{0.44}_{(\mathbf{5.51})}$	$\mathbf{0.34}_{(\mathbf{4.61})}$	$\mathbf{0.25}_{(\mathbf{3.43})}$	$0.11_{(1.30)}$	$-0.12_{(-1.51)}$	$\mathbf{-0.36}_{(\mathbf{-3.96})}$	$\mathbf{-0.86}_{(\mathbf{-8.92})}$	$\mathbf{-2.23}_{(\mathbf{-18.06})}$
Four-factor alpha (mon. %)	$\mathbf{0.56}_{(\mathbf{7.87})}$	$\mathbf{0.53}_{(\mathbf{7.02})}$	$\mathbf{0.46}_{(\mathbf{5.70})}$	$\mathbf{0.37}_{(\mathbf{5.01})}$	$\mathbf{0.31}_{(\mathbf{4.19})}$	$\mathbf{0.18}_{(\mathbf{2.22})}$	$-0.03_{(-0.41)}$	$\mathbf{-0.21}_{(\mathbf{-2.46})}$	$\mathbf{-0.70}_{(\mathbf{-7.60})}$	$\mathbf{-1.96}_{(\mathbf{-17.44})}$
Sharpe Ratio (ann.)	0.57	0.68	0.66	0.64	0.59	0.51	0.37	0.25	0.02	−0.52
Information Ratio (ann.)	1.25	1.11	0.90	0.79	0.66	0.35	−0.07	−0.39	−1.20	−2.76
CAPM beta (unconditional)	$\mathbf{1.17}_{(\mathbf{8.94})}$	$\mathbf{1.08}_{(\mathbf{4.58})}$	$\mathbf{1.05}_{(\mathbf{2.77})}$	$\mathbf{1.05}_{(\mathbf{2.81})}$	$\mathbf{1.05}_{(\mathbf{2.52})}$	$\mathbf{1.10}_{(\mathbf{4.54})}$	$\mathbf{1.16}_{(\mathbf{7.13})}$	$\mathbf{1.25}_{(\mathbf{9.07})}$	$\mathbf{1.34}_{(\mathbf{11.19})}$	$\mathbf{1.53}_{(\mathbf{13.76})}$
Months	520	520	520	520	520	520	520	520	520	520

Table 3. Monthly excess returns of portfolios formed on liquidity measures. This table shows excess returns, alphas, and related quantities for portfolios formed on liquidity measures. At the beginning of each calendar month, I assign firms into portfolios according to deciles of current ratio (Panel A), deciles of quick ratio (Panel B), or deciles of working capital ratio (Panel C) for the previous month. The portfolios are value-weighted using the ranks of the previous month's market equity values, refreshed every calendar month, and rebalanced every calendar month to maintain value-weighting. The highest and lowest realized return is excluded for each portfolio. Excess return is the time-series average of the monthly portfolio returns in excess of the 1-month US Treasury bill rate. CAPM alpha and CAPM beta (unconditional) are the intercept and slope estimates from a time-series regression of monthly excess returns on the excess returns of the value-weighted CRSP "market" index (MKT). Three- and four-factor alphas are the intercepts from time-series regressions of monthly excess returns on the three Fama and French (1993) factors (MKT, SMB, and HML) or these three factors and the Carhart (1997) factor (UMD). Sharpe ratio is the annualized average excess return divided by the annualized volatility of the excess returns. Information ratio is the annualized 4-factor alpha divided by the annualized residual standard error from the 4-factor time-series regression. Parentheses in subscript give t-statistics. For the excess returns and alphas, the null value is zero, while it is one for the unconditional beta. Statistical significance at the 5% level is indicated in bold.

Panel A

Current ratio	High	9	8	7	6	5	4	3	2	Low
Excess Return (avg. mon. %)	0.36$_{(1.16)}$	**0.55**$_{\mathbf{(1.99)}}$	**0.61**$_{\mathbf{(2.30)}}$	**0.69**$_{\mathbf{(2.75)}}$	**0.66**$_{\mathbf{(2.62)}}$	**0.71**$_{\mathbf{(2.88)}}$	**0.66**$_{\mathbf{(2.73)}}$	**0.69**$_{\mathbf{(3.04)}}$	**0.61**$_{\mathbf{(2.59)}}$	0.21$_{(0.81)}$
CAPM alpha (mon. %)	**−0.30**$_{\mathbf{(-2.31)}}$	−0.06$_{(-0.63)}$	0.04$_{(0.37)}$	0.15$_{(1.54)}$	0.11$_{(1.20)}$	0.17$_{(1.95)}$	0.13$_{(1.50)}$	**0.20**$_{\mathbf{(2.30)}}$	0.10$_{(1.04)}$	**−0.36**$_{\mathbf{(-3.26)}}$
Three-factor alpha (mon. %)	−0.14$_{(-1.47)}$	−0.07$_{(-0.89)}$	−0.05$_{(-0.68)}$	0.02$_{(0.27)}$	−0.01$_{(-0.08)}$	0.04$_{(0.55)}$	0.00$_{(-0.01)}$	0.05$_{(0.68)}$	−0.06$_{(-0.71)}$	**−0.54**$_{\mathbf{(-5.84)}}$
Four-factor alpha (mon. %)	−0.07$_{(-0.73)}$	0.02$_{(0.24)}$	0.06$_{(0.81)}$	0.11$_{(1.44)}$	0.11$_{(1.42)}$	0.14$_{(1.90)}$	0.08$_{(1.13)}$	0.13$_{(1.76)}$	0.00$_{(0.02)}$	**−0.45**$_{\mathbf{(-4.89)}}$
Sharpe Ratio (ann.)	0.18	0.30	0.35	0.42	0.40	0.44	0.41	0.46	0.39	0.12
Information Ratio (ann.)	−0.12	0.04	0.13	0.23	0.22	0.30	0.18	0.28	0.00	−0.78
CAPM beta (unconditional)	**1.38**$_{\mathbf{(13.36)}}$	**1.27**$_{\mathbf{(12.45)}}$	**1.24**$_{\mathbf{(10.81)}}$	**1.17**$_{\mathbf{(8.34)}}$	**1.18**$_{\mathbf{(8.75)}}$	**1.16**$_{\mathbf{(8.15)}}$	**1.13**$_{\mathbf{(6.77)}}$	**1.05**$_{\mathbf{(2.87)}}$	**1.07**$_{\mathbf{(3.56)}}$	**1.20**$_{\mathbf{(8.14)}}$
Months	520	520	520	520	520	520	520	520	520	520

Panel B

Quick ratio	High	9	8	7	6	5	4	3	2	Low
Excess Return (avg. mon. %)	0.40$_{(1.28)}$	**0.61**$_{\mathbf{(2.19)}}$	**0.71**$_{\mathbf{(2.74)}}$	**0.71**$_{\mathbf{(2.83)}}$	**0.66**$_{\mathbf{(2.64)}}$	**0.68**$_{\mathbf{(2.75)}}$	**0.68**$_{\mathbf{(2.81)}}$	**0.60**$_{\mathbf{(2.57)}}$	**0.50**$_{\mathbf{(2.05)}}$	0.27$_{(1.07)}$
CAPM alpha (mon. %)	**−0.27**$_{\mathbf{(-2.03)}}$	0.01$_{(0.06)}$	0.14$_{(1.54)}$	0.16$_{(1.79)}$	0.12$_{(1.25)}$	0.14$_{(1.54)}$	0.16$_{(1.68)}$	0.09$_{(0.97)}$	−0.02$_{(-0.21)}$	**−0.27**$_{\mathbf{(-2.28)}}$
Three-factor alpha (mon. %)	−0.06$_{(-0.63)}$	0.05$_{(0.64)}$	0.08$_{(1.07)}$	0.05$_{(0.70)}$	−0.02$_{(-0.26)}$	−0.01$_{(-0.15)}$	−0.01$_{(-0.18)}$	−0.09$_{(-1.10)}$	**−0.21**$_{\mathbf{(-2.60)}}$	**−0.46**$_{\mathbf{(-4.75)}}$
Four-factor alpha (mon. %)	0.00$_{(-0.04)}$	0.14$_{(1.76)}$	**0.17**$_{\mathbf{(2.27)}}$	0.14$_{(1.89)}$	0.06$_{(0.83)}$	0.08$_{(0.96)}$	0.08$_{(1.11)}$	0.01$_{(0.13)}$	−0.12$_{(-1.49)}$	**−0.34**$_{\mathbf{(-3.50)}}$
Sharpe Ratio (ann.)	0.19	0.33	0.42	0.43	0.40	0.42	0.43	0.39	0.31	0.16
Information Ratio (ann.)	−0.01	0.28	0.36	0.30	0.13	0.15	0.18	0.02	−0.23	−0.56
CAPM beta (unconditional)	**1.40**$_{\mathbf{(13.75)}}$	**1.30**$_{\mathbf{(12.96)}}$	**1.21**$_{\mathbf{(10.57)}}$	**1.18**$_{\mathbf{(8.95)}}$	**1.17**$_{\mathbf{(8.14)}}$	**1.14**$_{\mathbf{(6.95)}}$	**1.12**$_{\mathbf{(6.00)}}$	**1.08**$_{\mathbf{(3.86)}}$	**1.12**$_{\mathbf{(5.51)}}$	**1.13**$_{\mathbf{(5.16)}}$
Months	520	520	520	520	520	520	520	520	520	520

Panel C

Working capital ratio	High	9	8	7	6	5	4	3	2	Low
Excess Return (avg. mon. %)	0.56$_{(1.78)}$	**0.63**$_{\mathbf{(2.24)}}$	**0.67**$_{\mathbf{(2.52)}}$	**0.68**$_{\mathbf{(2.67)}}$	**0.64**$_{\mathbf{(2.58)}}$	**0.61**$_{\mathbf{(2.52)}}$	**0.67**$_{\mathbf{(2.92)}}$	**0.57**$_{\mathbf{(2.37)}}$	**0.56**$_{\mathbf{(2.44)}}$	0.17$_{(0.66)}$
CAPM alpha (mon. %)	−0.13$_{(-0.97)}$	0.02$_{(0.20)}$	0.08$_{(0.82)}$	0.12$_{(1.25)}$	0.09$_{(1.02)}$	0.07$_{(0.80)}$	**0.17**$_{\mathbf{(2.02)}}$	0.05$_{(0.56)}$	0.07$_{(0.75)}$	**−0.39**$_{\mathbf{(-3.60)}}$
Three-factor alpha (mon. %)	0.07$_{(0.79)}$	0.02$_{(0.25)}$	0.00$_{(-0.03)}$	0.03$_{(0.34)}$	0.00$_{(0.00)}$	−0.05$_{(-0.65)}$	0.02$_{(0.23)}$	−0.13$_{(-1.58)}$	−0.10$_{(-1.22)}$	**−0.55**$_{\mathbf{(-6.04)}}$
Four-factor alpha (mon. %)	0.12$_{(1.38)}$	0.13$_{(1.60)}$	0.10$_{(1.25)}$	0.14$_{(1.82)}$	0.11$_{(1.46)}$	0.06$_{(0.87)}$	0.08$_{(1.13)}$	−0.04$_{(-0.45)}$	−0.07$_{(-0.85)}$	**−0.47**$_{\mathbf{(-5.09)}}$
Sharpe Ratio (ann.)	0.27	0.34	0.38	0.41	0.39	0.38	0.44	0.36	0.37	0.10
Information Ratio (ann.)	0.22	0.25	0.20	0.29	0.23	0.14	0.18	−0.07	−0.13	−0.81
CAPM beta (unconditional)	**1.42**$_{\mathbf{(14.35)}}$	**1.31**$_{\mathbf{(13.09)}}$	**1.22**$_{\mathbf{(10.33)}}$	**1.18**$_{\mathbf{(8.89)}}$	**1.16**$_{\mathbf{(8.29)}}$	**1.13**$_{\mathbf{(6.61)}}$	**1.08**$_{\mathbf{(4.19)}}$	**1.11**$_{\mathbf{(5.43)}}$	**1.07**$_{\mathbf{(3.40)}}$	**1.18**$_{\mathbf{(7.49)}}$
Months	520	520	520	520	520	520	520	520	520	520

4.4 Long-short portfolios

The analyses of Subsections 4.1-4.3 test the model's predictions for each of the cross-sections of solvency and cash levels separately. In this subsection, I test the model's predictions for the combination of the two cross-sections by studying the performance of long-short portfolios in the two cross-sections.

High-cash minus low-cash across solvency

The model predicts that the outperformance of high-cash firms over low-cash firms increases as solvency decreases, because firms with higher cash are also more solvent firms while expected returns are humped and decreasing in decreasing solvency. Hence, a high-cash minus low-cash (HCmLC) portfolio that is long high-cash firms and short low-cash firms should have increasing expected returns as the underlying firms become less solvent (Corollary 5.2).

To test this prediction, Table 4 shows the performance of HCmLC portfolios across credit ratings. To ensure a sufficient number of firms in each portfolio, I re-code the original 9 credit ratings into the following three groups: Aaa-A, Baa-B, and Caa-C. Similar to the construction of long-short portfolios in e.g. Fama and French (1993), I form the HCmLC portfolios using conditional sorts: First into the three credit rating portfolios, and then into three liquidity portfolios, where the liquidity breakpoints are the 30th and the 70th percentile. I then calculate the return-spread for each credit rating group as the difference between the value-weighted returns of the top 30% "high cash" firms and the bottom 30% "low cash" firms. As in Subsection 4.3, I use the ranks of the market equity values to construct the portfolio weights, and, to reduce the influence of outliers, I exclude the highest and lowest realized return for each portfolio. The results for HCmLC portfolios formed using market leverage or interest coverage as the measure of solvency produce very similar results and are omitted here for brevity.

Consistent with the models prediction, the HCmLC portfolios have monotonically increasing average returns, 1-4 factor alphas, and Sharpe/Information ratios as credit ratings deteriorate. For firms rated Aaa-A, the HCmLC portfolios have insignificant average returns between -0.09% and -0.07% on a monthly basis, insignificant alphas in the same range, and low annualized Sharpe/Information ratios between -0.13 and 0.06. As the credit ratings deteriorate into Baa-B, the returns and alphas increase somewhat but remain insignificant. Finally, for the riskiest firms rated Caa-C, the HCmLC portfolios have large and significantly positive average returns between 1.84% and 2.81% on a monthly basis, significant alphas in the same range, and fairly large annualized Sharpe/Information ratios between 0.58 and 0.86.

High-solvency minus low-solvency across cash

The dual of Corollary 5.2 is that the outperformance of high-solvency firms over low-solvency firms increases as cash-levels decrease, because less solvent firms demand less cash while expected returns are humped and decreasing in decreasing cash-levels. Hence, a high-solvency minus low-solvency (HSmLS) portfolio that is long high-solvency firms and short low-solvency firms should have increasing expected returns as the underlying firms hold less cash (Corollary 5.3).

As a test of this prediction, Table 5 shows the performance of HSmLS portfolios implemented using credit ratings across the liquidity measures. The credit ratings are again re-coded into the three (Aaa-A, Baa-B, and Caa-C), and the HSmLS portfolios are again formed using conditional sorts: First into into three liquidity portfolios, where the liquidity breakpoints are the 30th and the 70th percentile, and then into the three credit rating portfolios. I then calculate the return-spread for each liquidity group as the difference between the value-weighted returns on the Aaa-A rated 'high-solvency' firms and the Caa-C rated 'low solvency' firms. I again use the ranks of the market equity values to construct the portfolio weights and, to reduce the influence of outliers, I exclude the highest and lowest realized return for each portfolio. The results for HSmLS portfolios formed using market leverage or interest coverage as the measure of solvency produce very similar results and are omitted here for brevity.

Consistent with the models prediction, the HSmLS portfolios have generally increasing average returns, 1-4 factor alphas, and Sharpe/Information ratios as the liquidity measures decrease. For firms with the highest cash levels, the HSmLS portfolios have high and significant average returns between 1.26% and 1.99% on a monthly basis, even higher alphas, and annualized Sharpe/Information ratios between 0.50 and 0.93. While this performance is relatively strong, it is in fact dwarfed by the performance of the HSmLS portfolio for firms with the lowest cash levels, where average returns are between 3.56% and 3.63% on a monthly basis, alphas are even higher, and annualized Sharpe/Information ratios are between 1.09 and 1.62. Furthermore, the increase in returns, alphas, and Sharpe/Information ratios is monotonic for the quick ratio and the working capital ratio.

5 Concluding remarks

This paper has shown theoretically and empirically that endogenous cash holdings can help rationalize the low returns of distressed equity.

I have presented a model in which levered firms with financing constraints can default because of liquidity or solvency, but firms seek to manage their cash to minimize liquidity risk. Using data on solvency, cash, and returns for US firms, I have documented empirical evidence consistent with the model's theoretical predictions: (1) In all solvency levels, the average firm holds enough cash to cover short-term liabilities; (2) expected returns are humped and decreasing as solvency decreases because a less solvent firm has a higher fraction of its assets in cash; (3) expected returns are humped and decreasing as cash decreases because a less solvent firm demands less cash; (4) the outperformance of high-cash firms over low-cash firms increases as solvency decreases; and (5) the outperformance of high-solvency firms over low-solvency firms increases as cash decreases.

In conclusion, my results suggest that there is no distress puzzle for insolvent but liquid firms.

Table 4. High-cash minus low-cash portfolios across credit ratings This table shows returns, alphas, and related quantities for high-cash minus low-cash (HCmLC) portfolios that are long high-cash firms and short low-cash firms within credit rating groups. At the beginning of each calendar month, I assign firms into 3 portfolios according to their credit rating (Aaa-A, Baa-B, and Caa-C) and then into three portfolios according to the 30th and 70th percentile of current ratio (Panel A), quick ratio (Panel B), or working capital ratio (Panel C). The portfolios are value-weighted using the ranks of the previous calendar month's market equity values, refreshed every calendar month, and rebalanced every calendar month to maintain value-weighting. The highest and lowest realized return is excluded for each portfolio. Return is the time-series average of the monthly returns for the portfolio that is long the "high cash" firms and short the "low cash" firms. CAPM alpha and CAPM beta (unconditional) are the intercept and slope estimates from a time-series regression of monthly excess returns on the excess returns of the value-weighted CRSP "market" index (MKT). Three- and four-factor alphas are the intercepts from time-series regressions of monthly excess returns on the three Fama and French (1993) factors (MKT, SMB, and HML) or these three factors and the Carhart (1997) factor (UMD). Sharpe ratio is the annualized average excess return divided by the annualized volatility of the excess returns. Information ratio is the annualized 4-factor alpha divided by the annualized residual standard error from the 4-factor time-series regression. Parentheses in subscript give *t*-statistics for a null value of zero. Statistical significance at the 5% level is indicated in bold.

Panel A: **Current Ratio**

	Aaa-A High cash minus low cash	**Baa-B** High cash minus low cash	**Caa-C** High cash minus low cash
Return (avg. mon. %)	$-0.07_{(-0.56)}$	$0.15_{(1.20)}$	$\mathbf{2.81_{(5.02)}}$
CAPM alpha (mon. %)	$-0.14_{(-1.08)}$	$0.07_{(0.57)}$	$\mathbf{2.81_{(4.99)}}$
Three-factor alpha (mon. %)	$-0.05_{(-0.38)}$	$0.19_{(1.79)}$	$\mathbf{2.85_{(5.01)}}$
Four-factor alpha (mon. %)	$0.05_{(0.41)}$	$\mathbf{0.23_{(2.13)}}$	$\mathbf{2.74_{(4.72)}}$
Sharpe Ratio (ann.)	−0.09	0.18	0.86
Information Ratio (ann.)	0.06	0.34	0.84
CAPM beta (unconditional)	$\mathbf{0.15_{(5.47)}}$	$\mathbf{0.18_{(6.97)}}$	$-0.01_{(-0.07)}$
Months	504	505	405

Panel B: **Quick Ratio**

	Aaa-A High cash minus low cash	**Baa-B** High cash minus low cash	**Caa-C** High cash minus low cash
Return (avg. mon. %)	$-0.09_{(-0.82)}$	$0.09_{(0.76)}$	$\mathbf{1.84_{(3.39)}}$
CAPM alpha (mon. %)	$-0.13_{(-1.28)}$	$0.01_{(0.10)}$	$\mathbf{1.73_{(3.18)}}$
Three-factor alpha (mon. %)	$-0.02_{(-0.17)}$	$\mathbf{0.20_{(1.98)}}$	$\mathbf{2.07_{(3.86)}}$
Four-factor alpha (mon. %)	$-0.01_{(-0.13)}$	$0.18_{(1.71)}$	$\mathbf{2.19_{(4.02)}}$
Sharpe Ratio (ann.)	−0.13	0.12	0.58
Information Ratio (ann.)	−0.02	0.27	0.72
CAPM beta (unconditional)	$\mathbf{0.11_{(5.07)}}$	$\mathbf{0.19_{(7.54)}}$	$0.19_{(1.58)}$
Months	503	504	404

Panel C: **Working Capital Ratio**

	Aaa-A High cash minus low cash	**Baa-B** High cash minus low cash	**Caa-C** High cash minus low cash
Return (avg. mon. %)	$-0.07_{(-0.56)}$	$0.15_{(1.20)}$	$\mathbf{2.81_{(5.02)}}$
CAPM alpha (mon. %)	$-0.14_{(-1.08)}$	$0.07_{(0.57)}$	$\mathbf{2.81_{(4.99)}}$
Three-factor alpha (mon. %)	$-0.05_{(-0.38)}$	$0.19_{(1.79)}$	$\mathbf{2.85_{(5.01)}}$
Four-factor alpha (mon. %)	$0.05_{(0.41)}$	$\mathbf{0.23_{(2.13)}}$	$\mathbf{2.74_{(4.72)}}$
Sharpe Ratio (ann.)	−0.09	0.18	0.86
Information Ratio (ann.)	0.06	0.34	0.84
CAPM beta (unconditional)	$\mathbf{0.15_{(5.47)}}$	$\mathbf{0.18_{(6.97)}}$	$-0.01_{(-0.07)}$
Months	504	505	405

Table 5. High-solvency minus low-solvency across liquidity measures. This table shows returns, alphas, and related quantities high-solvency minus low-solvency (HSmLS) portfolios that are long firms rated Aaa-A and short firms rated Caa-C within liquidity groups. At the beginning of each calendar month, I assign firms into 3 portfolios according to the 30th and 70th percentile of current ratio (Panel A), quick ratio (Panel B), or working capital ratio (Panel C), and then into three groups according to credit ratings (Aaa-A, Baa-B, and Caa-C). The portfolios are value-weighted using the ranks of the previous calendar month's market equity values, refreshed every calendar month, and rebalanced every calendar month to maintain value-weighting. The highest and lowest realized return is excluded for each portfolio. Return is the time-series average of the monthly returns for the portfolio that is long the "high solvency" firms and short the "low solvency" firms. CAPM alpha and CAPM beta (unconditional) are the intercept and slope estimates from a time-series regression of monthly excess returns on the excess returns of the value-weighted CRSP "market" index (MKT). Three- and four-factor alphas are the intercepts from time-series regressions of monthly excess returns on the three Fama and French (1993) factors (MKT, SMB, and HML) or these three factors and the Carhart (1997) factor (UMD). Sharpe ratio is the annualized average excess return divided by the annualized volatility of the excess returns. Information ratio is the annualized 4-factor alpha divided by the annualized residual standard error from the 4-factor time-series regression. Parentheses in subscript give *t*-statistics for a null value of zero. Statistical significance at the 5% level is indicated in bold.

Panel A: **Current Ratio**

	High cash Aaa-A minus Caa-C	**Medium cash** Aaa-A minus Caa-C	**Low cash** Aaa-A minus Caa-C
Return (avg. mon. %)	$\mathbf{1.99}_{(4.18)}$	$\mathbf{0.96}_{(2.16)}$	$\mathbf{3.56}_{(6.37)}$
CAPM alpha (mon. %)	$\mathbf{2.31}_{(4.98)}$	$\mathbf{1.37}_{(3.27)}$	$\mathbf{4.03}_{(7.49)}$
Three-factor alpha (mon. %)	$\mathbf{2.46}_{(5.68)}$	$\mathbf{1.52}_{(3.84)}$	$\mathbf{4.28}_{(8.87)}$
Four-factor alpha (mon. %)	$\mathbf{2.23}_{(5.09)}$	$\mathbf{1.11}_{(2.84)}$	$\mathbf{3.79}_{(7.89)}$
Sharpe Ratio (ann.)	0.74	0.37	1.09
Information Ratio (ann.)	0.93	0.51	1.41
CAPM beta (unconditional)	$\mathbf{-0.54}_{(-5.31)}$	$\mathbf{-0.69}_{(-7.48)}$	$\mathbf{-0.77}_{(-6.41)}$
Months	388	401	413

Panel B: **Quick Ratio**

	High cash Aaa-A minus Caa-C	**Medium cash** Aaa-A minus Caa-C	**Low cash** Aaa-A minus Caa-C
Return (avg. mon. %)	$\mathbf{1.69}_{(3.65)}$	$\mathbf{1.88}_{(3.83)}$	$\mathbf{3.62}_{(6.80)}$
CAPM alpha (mon. %)	$\mathbf{2.15}_{(4.89)}$	$\mathbf{2.28}_{(4.82)}$	$\mathbf{4.04}_{(7.85)}$
Three-factor alpha (mon. %)	$\mathbf{2.29}_{(5.54)}$	$\mathbf{2.27}_{(5.14)}$	$\mathbf{4.38}_{(9.83)}$
Four-factor alpha (mon. %)	$\mathbf{2.01}_{(4.83)}$	$\mathbf{1.81}_{(4.13)}$	$\mathbf{4.07}_{(9.09)}$
Sharpe Ratio (ann.)	0.64	0.68	1.16
Information Ratio (ann.)	0.87	0.77	1.62
CAPM beta (unconditional)	$\mathbf{-0.70}_{(-7.23)}$	$\mathbf{-0.65}_{(-6.36)}$	$\mathbf{-0.70}_{(-5.98)}$
Months	397	379	412

Panel C: **Working Capital Ratio**

	High cash Aaa-A minus Caa-C	**Medium cash** Aaa-A minus Caa-C	**Low cash** Aaa-A minus Caa-C
Return (avg. mon. %)	$\mathbf{1.26}_{(2.86)}$	$\mathbf{1.49}_{(3.22)}$	$\mathbf{3.63}_{(6.31)}$
CAPM alpha (mon. %)	$\mathbf{1.57}_{(3.68)}$	$\mathbf{2.01}_{(4.65)}$	$\mathbf{4.06}_{(7.26)}$
Three-factor alpha (mon. %)	$\mathbf{1.75}_{(4.40)}$	$\mathbf{2.15}_{(5.30)}$	$\mathbf{4.24}_{(8.47)}$
Four-factor alpha (mon. %)	$\mathbf{1.59}_{(3.93)}$	$\mathbf{1.82}_{(4.49)}$	$\mathbf{3.78}_{(7.56)}$
Sharpe Ratio (ann.)	0.50	0.56	1.10
Information Ratio (ann.)	0.71	0.81	1.37
CAPM beta (unconditional)	$\mathbf{-0.53}_{(-5.58)}$	$\mathbf{-0.81}_{(-8.50)}$	$\mathbf{-0.71}_{(-5.66)}$
Months	399	400	398

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Appendices

A Additional details and proofs

This appendix gives additional details of the model's development that were omitted from the main text as well as the proofs.

A.1 Change of measure

The firm's cumulated earnings and the economy's stochastic discount factor were given by the $\mathbb{P}$-dynamics in (1) and (3), i.e.

$$dX_t = \mu^{\mathbb{P}} X_t dt + \sigma X_t dW_t^{\mathbb{P}}$$

and

$$d\Lambda_t = -r\Lambda_t dt - \lambda \Lambda_t dZ_t^{\mathbb{P}},$$

where $W_t^{\mathbb{P}}$ and $Z_t^{\mathbb{P}}$ are standard $\mathbb{P}$-Brownian motions with correlation ρ. There thus exists a standard $\mathbb{P}$-Brownian motion, $\widetilde{Z}_t^{\mathbb{P}}$, independent of $Z_t^{\mathbb{P}}$, such that

$$W_t^{\mathbb{P}} = \rho Z_t^{\mathbb{P}} + \sqrt{1-\rho^2}\, \widetilde{Z}_t^{\mathbb{P}}.$$

Now, suppose $\Lambda_0 = 1$, let $L_t = e^{rt}\Lambda_t$, and note that L_t may be written as

$$L_t = \exp\left(-\tfrac{1}{2}\boldsymbol{\lambda}'\boldsymbol{\lambda}\, t - \boldsymbol{\lambda}'\, \mathbf{Z}_t^{\mathbb{P}}\right),$$

where $\boldsymbol{\lambda} = (\lambda, 0)'$ and $\mathbf{Z}_t^{\mathbb{P}} = (Z_t^{\mathbb{P}}, \widetilde{Z}_t^{\mathbb{P}})'$. Fix T and define a risk-neutral pricing measure $\mathbb{Q}$ by $\frac{d\mathbb{Q}}{d\mathbb{P}} = L_T$. Girsanov's Theorem then gives that the process $\mathbf{Z}_t^{\mathbb{Q}} = (Z_t^{\mathbb{Q}}, \widetilde{Z}_t^{\mathbb{Q}})$, defined by

$$\mathbf{Z}_t^{\mathbb{Q}} = \mathbf{Z}_t^{\mathbb{P}} + \boldsymbol{\lambda} t,$$

is a standard $\mathbb{Q}$-Brownian motion.

Finally, let $W_t^{\mathbb{Q}} = \rho Z_t^{\mathbb{Q}} + \sqrt{1-\rho^2}\, \widetilde{Z}_t^{\mathbb{Q}}$, and note that $W_t^{\mathbb{Q}}$ is standard $\mathbb{Q}$-Brownian motion which may be written as

$$W_t^{\mathbb{Q}} = \rho\left(Z_t^{\mathbb{P}} + \lambda t\right) + \sqrt{1-\rho^2}\, \widetilde{Z}_t^{\mathbb{P}} = W_t^{\mathbb{P}} + \rho\lambda t.$$

It thus follows that the $\mathbb{Q}$-dynamics of the firm's cumulated earnings process may be written as

$$dX_t = \mu^{\mathbb{Q}} X_t dt + \sigma X_t dW_t^{\mathbb{Q}},$$

where $\mu^{\mathbb{Q}} = \mu^{\mathbb{P}} - \rho\sigma\lambda$, exactly as in (4).

A.2 Proof of Proposition 1

Suppose cash holdings are of the form $C_t = \underline{C}(X_t)$ for some twice continuously differentiable function $\underline{C}(X)$. It follows by Itô's Lemma along with (2) and (4) that

$$\begin{aligned} dD_t \;=\; & \Big[r\underline{C}(X_t) + (1-\tau)(\mu^{\mathbb{Q}} X_t - k) \\ & -\underline{C}_X(X_t)\mu^{\mathbb{Q}} X_t - \tfrac{1}{2}\underline{C}_{XX}(X_t)\sigma^2 X_t^2\Big]dt \\ & +\Big[(1-\tau)\sigma X_t - \underline{C}_X(X_t)\sigma X_t\Big]dW_t^{\mathbb{Q}}, \quad (18) \end{aligned}$$

where subscripts denote partial derivatives. Since the firm has no access to external financing beyond time 0, it can, in particular, not issue additional equity. This implies that the cumulated dividend process has to be non-decreasing for all $t > 0$, which is satisfied if and only if i) the drift-term in (18) is nonnegative and ii) the volatility-term in (18) is zero.

The requirement that the volatility-term of dD_t has to be zero implies the simple differential equation $\underline{C}_X(X) = (1-\tau)$, which has the general solution

$$\underline{C}(X) = (1-\tau)X + L$$

for some constant L. Plugging this solution into the drift-term of dD_t, and imposing the requirement that the drift has to be nonnegative, it follows that the solution has to satisfy

$$\underline{C}(X) \geq (1-\tau)\tfrac{k}{r}$$

for all X. To solve for the constant L, note that since $\underline{C}_X(X) = (1-\tau) > 0$, the target cash level is increasing in X. As the insolvency trigger $\underline{X}^\star$ is a lower bound for X, it follows that $\underline{C}(X) \geq \underline{C}(\underline{X}^\star) \geq (1-\tau)\frac{k}{r}$ for all X. Combining this inequality with the general solution then gives that the constant L must satisfy

$$L \geq -(1-\tau)\underline{X}^\star + (1-\tau)\tfrac{k}{r} = (1-\tau)\left[-\underline{X}^\star + \tfrac{k}{r}\right].$$

By choosing L as low as possible (i.e. the value where the inequality is binding), the expression for the solution given in Proposition 1 follows.

The remaining part of the proof is to assert that the target cash level, $\underline{C}(X)$, is twice continuously differentiable in current cumulated earnings, X. If $X' \geq \underline{X}^\star$ is a discontinuity point for $\underline{C}(X)$, then $\underline{C}(X)$ must jump downwards at X', or else it would not have been the *target* cash level an (infinitesimal) instance before X'. However, even a downwards jump at X' would imply that $\underline{C}(X)$ is not the target cash level an instance before X'. Hence, $\underline{C}(X)$ must be continuous for all X.

On the other hand, if $\underline{C}(X)$ is continuous but non-differentiable at some $X' \geq \underline{X}^\star$, then $\underline{C}(X)$ would for all $X \neq X'$ satisfy the same differential equation as before. However, this differential equation and the associated inequalities imply a solution that is twice continuously differentiable for all X. □

A.3 Proof of Proposition 2

To prove that the conjectured dividend policy in (8), $dD_t^\star$, is optimal, it must first be shown that it attains the maximal equity value in (6), $E(X_t, C_t)$, and, second, that no other dividend policy implies a higher equity value—that is, it must be shown that

I) $E(X_t, C_t) = \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^{\tilde{\tau}} e^{-r(u-t)}\, dD_u^\star + e^{-r(\tilde{\tau}-t)} C_{\tilde{\tau}}\right]$, and that

II) $E(X_t, C_t) \geq \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^{\tilde{\tau}} e^{-r(u-t)}\, dD_u + e^{-r(\tilde{\tau}-t)} C_{\tilde{\tau}}\right]$ for all nonnegative, non-decreasing, adapted dividend processes, D_t.

In both parts, the proof will make use of the following two general results.

First, note that a general dividend process, D_t, is not necessarily continuous, meaning that it, and the corresponding cash process, C_t, may have jumps. Still, a general D_t may be decomposed into the sum of its purely continuous part and its jumps: $D_t = D_t^c + \sum_{s \leq t}(D_s - D_{s-})$. Using this along with the generalized Itô's Lemma and the dynamics of X_t and C_t in (2) and (4), it follows that the discounted gains process of equity, as defined by $G_t = e^{-rt}E(X_t, C_t) + \int_0^t e^{-rs} dD_s$, has the dynamics

$$\begin{aligned} dG_t \;=\;& e^{-rt}\left[-rE(X_t, C_{t-}) + \mathcal{A}E(X_t, C_{t-})\right] dt \\ &+ e^{-rt}\left[1 - E_C(X_t, C_{t-})\right] dD_t \\ &+ e^{-rt}\sigma X_t \left[E_X(X_t, C_{t-}) + (1-\tau)E_C(X_t, C_{t-})\right] dW_t^{\mathbb{Q}} \\ &+ e^{-rt}\left[E(X_t, C_t) - E(X_t, C_{t-})\right. \\ &\qquad \left. - E_C(X_t, C_{t-})\,(C_t - C_{t-})\right]. \end{aligned} \tag{19}$$

Here, the infinitesimal operator, $\mathcal{A}E(X, C)$, is given by

$$\begin{aligned} \mathcal{A}E(X, C) \;=\;& \mu^{\mathbb{Q}} X E_X(X, C) \\ &+ \left[rC + (1-\tau)(\mu^{\mathbb{Q}} X - k)\right] E_C(X, C) \\ &+ \tfrac{1}{2}\sigma^2 X^2 E_{XX}(X, C) \\ &+ \tfrac{1}{2}(1-\tau)^2\sigma^2 X^2 E_{CC}(X, C) \\ &+ (1-\tau)\sigma^2 X^2 E_{XC}(X, C), \end{aligned}$$

while the jumps of the cash process, by (2), are given by $C_t - C_{t-} = -(D_t - D_{t-})$ for all t. Since equity is a traded asset, G_t is a (possibly non-continuous) $\mathbb{Q}$-martingale, which will impose restrictions on the first two terms in (19) depending on the specific form of D_t.

Second, it follows by definition of the equity value function in (6) that

$$E(X_t, C_t) \to C_{\tilde{\tau}} \quad \text{for} \quad t \to \tilde{\tau}. \tag{20}$$

Proof of Part I)

Assume that $dD_t = dD_t^\star$ for all t, and consider, in turn, the three regions implied by the conjectured dividend policy in (8).

First, when $0 < C_t < \underline{C}(X_t)$, dividend payouts are given by $dD_t^\star = 0$ for all t, which is continuous and implies that C_t is also continuous. By (19), this implies that G_t is a $\mathbb{Q}$-martingale if and only if $-rE + \mathcal{A}E = 0$. Combining this with the implied form of (19) then gives

$$\begin{aligned} &d(e^{-rt}E(X_t, C_t)) \\ &\quad = e^{-rt}\sigma X_t \left[E_X(X_t, C_t) + (1-\tau)E_C(X_t, C_t)\right] dW_t^{\mathbb{Q}}. \end{aligned}$$

Assuming that the derivatives of the equity value function are sufficiently integrable, it follows by integrating this expression between t and $v \wedge \tilde{\tau}$ for any $v \geq t$ and taking expectations that

$$e^{-rt}E(X_t, C_t) = \mathbb{E}_t^{\mathbb{Q}}\left[e^{-r(v\wedge\tilde{\tau})}E(X_{v\wedge\tilde{\tau}}, C_{v\wedge\tilde{\tau}})\right].$$

Since v was arbitrary, it follows by letting $v \to \infty$ and applying (20) that

$$E(X_t, C_t) = \mathbb{E}_t^{\mathbb{Q}}\left[e^{-r(\tilde{\tau}-t)} C_{\tilde{\tau}}\right],$$

which proves part I) in the first region.

Next, when $C_t = \underline{C}(X_t)$, dividend payouts are given by $dD_t^\star = r\underline{C}(X_t) - (1-\tau)k\, dt = r(1-\tau)(X_t - \underline{X}^\star)\, dt$, which again is continuous. In this region, equity value is given by $E(X, \underline{C}(X)) = E(X)$ and satisfies the ODE in (10)—hence, $-rE + \mathcal{A}E = 0$. Furthermore, Proposition 3 and its proof (see Section A.4 of this appendix) give that the solution to (10) is separable in X and C and linear in C, which implies $E_C(X, \underline{C}(X)) = 1$. Combining these results with the implied form of (19), integrating between t and $v \wedge \tilde{\tau}$, and taking expectations, one then arrives at

$$E(X_t, C_t) = \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^{v\wedge\tilde{\tau}} e^{-r(u-t)} dD_u^\star + e^{-r((v\wedge\tilde{\tau})-t)} E(X_{v\wedge\tilde{\tau}}, C_{v\wedge\tilde{\tau}})\right].$$

Letting $v \to \infty$ and applying (20) thus proves part i) for the second region.

Finally, when $C_t > \underline{C}(X_t)$, dividend payouts are $dD_t^\star = C_t - \underline{C}(X_t)$ and thus not continuous. However, since this by (8) occurs if and only if $C_{t-} = \underline{C}(X_t)$, it holds that $E(X_t, C_{t-}) = E(X_t, \underline{C}(X_t))$. Therefore, by Proposition 3 and its proof, equity value at time $t-$ is separable in X and C and linear in C, which gives $E_C(X_t, C_{t-}) = E_C(X_t, \underline{C}(X_t)) = 1$. Combining these results with (19), G_t is a martingale if and only if

$$\begin{aligned} &rE(X_t, C_{t-}) + \mathcal{A}E(X_t, C_{t-}) \\ &\quad = -rE(X_t, \underline{C}(X_t)) + \mathcal{A}E(X_t, \underline{C}(X_t)) = 0, \end{aligned}$$

while the jump-term of (19) becomes

$$\begin{aligned} &E(X_t, C_t) - E(X_t, C_{t-}) - E_C(X_t, C_{t-})(C_t - C_{t-}) \\ &\quad = C_t - \underline{C}(X_t) - (C_t - \underline{C}(X_t)) = 0. \end{aligned}$$

Using the implied form of (19) and repeating the steps of the proof in the second region thus also proves part i) in the third region.

Proof of Part ii)

From part 1), the dividend policy in (8) attains the maximal equity value in (6), $E(X_t, C_t)$. The task is now to show that no other nonnegative, non-decreasing, adapted dividend process implies a higher equity value. Let $D_t = D_t^c + \sum_{s\leq t}(D_s - D_{s-})$ be such a dividend process and let C_t be its corresponding cash process with jumps $C_t - C_{t-} = -(D_t - D_{t-})$ for all t. Plugging this form of D_t into (19), the discounted gains process under the dividend policy D_t has the dynamics

$$\begin{aligned} dG_t &= e^{-rt}\left[-rE(X_t, C_{t-}) + \mathcal{A}E(X_t, C_{t-})\right] dt \\ &\quad + e^{-rt} dD_t - e^{-rt} E_C(X_t, C_{t-})\, dD_t^c \\ &\quad + e^{-rt}\sigma X_t\left[E_X(X_t, C_{t-}) + (1-\tau)E_C(X_t, C_{t-})\right] dW_t^{\mathbb{Q}} \\ &\quad + e^{-rt}\left[E(X_t, C_t) - E(X_t, C_{t-})\right]. \end{aligned}$$

By the proof of part i), the maximal equity value satisfies $-rE(X_t, C_t) + \mathcal{A}E(X_t, C_t) = 0$ for all X and C. Hence, once more assuming that the deviates of the maximal equity value function are sufficiently integrable, it follows by integrating the last expression between t and $v \wedge \tilde{\tau}$, taking expectations, and rearranging, that

$$\begin{aligned} E(X_t, C_t) &= \mathbb{E}_t^{\mathbb{Q}}\left[e^{-r((v\wedge\tilde{\tau})-t)} E(X_{v\wedge\tilde{\tau}}, C_{v\wedge\tilde{\tau}})\right] \\ &\quad + \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^{v\wedge\tilde{\tau}} e^{-r(u-t)} E_C(X_u, C_{u-})\, dD_u^c\right] \\ &\quad - \mathbb{E}_t^{\mathbb{Q}}\left[\sum_{t\leq u\leq v\wedge\tilde{\tau}} e^{-r(u-t)}\left[E(X_t, C_t) - E(X_t, C_{t-})\right]\right]. \end{aligned}$$

Since it in general holds that $E_C(X, C) \geq 1$ (see footnote 10), the middle-term in the above expression can be bounded from below. Furthermore, the same property and the relation $C_t - C_{t-} = -(D_t - D_{t-}) \leq 0$ (since the dividend process is non-decreasing) imply

$$-\left[E(X_t, C_t) - E(X_t, C_{t-})\right] \geq D_t - D_{t-},$$

which thus bounds the last term. Therefore,

$$\begin{aligned} E(X_t, C_t) &\geq \mathbb{E}_t^{\mathbb{Q}}\left[e^{-r((v\wedge\tilde{\tau})-t)} E(X_{v\wedge\tilde{\tau}}, C_{v\wedge\tilde{\tau}})\right] \\ &\quad + \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^{v\wedge\tilde{\tau}} e^{-r(u-t)} dD_u^c\right] \\ &\quad + \mathbb{E}_t^{\mathbb{Q}}\left[\sum_{t\leq u\leq v\wedge\tilde{\tau}} e^{-r(u-t)}(D_u - D_{u-})\right] \\ &= \mathbb{E}_t^{\mathbb{Q}}\left[e^{-r((v\wedge\tilde{\tau})-t)} E(X_{v\wedge\tilde{\tau}}, C_{v\wedge\tilde{\tau}})\right] \\ &\quad + \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^{v\wedge\tilde{\tau}} e^{-r(u-t)} dD_u\right], \end{aligned}$$

where the last equality uses $D_t = D_t^c + \sum_{s\leq t}(D_s - D_{s-})$. Letting $v \to \infty$ and applying (20) thus proves part ii), which completes the proof. □

A.4 Proof of Proposition 3

Proof of Part (i)

The ODE (10) has the general solution

$$\begin{aligned} E(X) &= (1-\tau)r\left[\frac{X}{r-\mu^{\mathbb{Q}}} - \frac{\underline{X}^{\star}}{r}\right] + M_1 X^{\phi^+} + M_2 X^{\phi^-} \\ &= \underline{C}(X) + U(X) - (1-\tau)\tfrac{k}{r} + M_1 X^{\phi^+} + M_2 X^{\phi^-}, \end{aligned}$$

where the second equality follows from the form of the target cash level, $\underline{C}(X)$, in (7), and the value of earnings-generating assets, $U(X)$, in (5). Here, M_1 and M_2 are real-valued constants to be determined by boundary conditions, while the exponents $\phi^+ > 1$ and $\phi^- < 0$ are given by

$$\phi^{\pm} = \frac{\sigma^2 - 2\mu \pm \sqrt{(\sigma^2-2\mu)^2 + 8r\sigma^2}}{2\sigma^2}.$$

Since $\phi^+ > 1$, the value matching condition (11) implies $M_1 = 0$. Given this, the limited liability condition (12) implies

$$M_2 = (\underline{X}^{\star})^{-\phi^-}\left[(1-\tau)\tfrac{k}{r} - U(\underline{X}^{\star})\right].$$

By plugging the constants into the general solution, and defining $\pi^{\mathbb{Q}}(X) = (X/\underline{X}^{\star})^{\phi^-}$, the expression for $E(X)$ in the proposition follows.

Proof of Parts (ii) and (iii)

The expression for $\underline{X}^{\star}$ given in (15) easily follows by solving the smoothing pasting condition in (13). Combining the solution for $\underline{X}^{\star}$ with the expressions for $\underline{C}(X)$ in (7) and $U(X)$ in (5), it follows that

$$\frac{\underline{C}(\underline{X}^{\star})}{U(\underline{X}^{\star})} = \frac{(1-\tau)\frac{k}{r}}{\frac{(1-\tau)\mu^{\mathbb{Q}}\underline{X}^{\star}}{r-\mu^{\mathbb{Q}}}} = \frac{\phi^- - 1}{\phi^-} > 1,$$

since $\phi^- < 0$ implies $\frac{\phi^-}{\phi^- - 1} \in (0,1)$. This completes the proof. □

A.5 Proof of Proposition 4

Using the expression in (3) for the equilibrium equity value, it follows by differentiating that

$$\begin{aligned} \Omega^E(X_t) &= X_t\frac{E_X(X_t)}{E(X_t)} \\ &= \frac{X_t}{E(X_t)}\Big[(1-\tau) + (1-\tau)\tfrac{\mu^{\mathbb{Q}}}{r-\mu^{\mathbb{Q}}} \\ &\qquad + \left[(1-\tau)\tfrac{k}{r} - U(\underline{X}^{\star})\right]\tfrac{\phi^-\pi^{\mathbb{Q}}(X_t)}{X_t}\Big] \\ &= \frac{1}{E(X_t)}\Big[E(X_t) - \underline{C}(X_t) + (1-\tau)X_t + (1-\tau)\tfrac{k}{r} \\ &\qquad -(1-\tau)(1-\phi^-)\left[\tfrac{k}{r} - \tfrac{\mu^{\mathbb{Q}}\underline{X}^{\star}}{r-\mu^{\mathbb{Q}}}\right]\pi^{\mathbb{Q}}(X_t)\Big], \end{aligned}$$

where the last equality follows by the expressions for $E(X)$ and $\underline{C}(X)$ in (14) and (7). The form of $\Omega^E(X_t)$ given in the proposition then follows by dividing $E(X_t)$ into each term in the parentheses. □

A.6 Proof of Corollary 5

The proofs of parts (ii) and (iii) will make use of the following results about the earnings-sensitivity, $\Omega^E(X_t)$.

First, using the expression for $\underline{C}(X)$ in (7), the sensitivity can be compactly written as

$$\Omega^E(X_t) = 1 + \frac{(1-\tau)\underline{X}^{\star} - (1-\phi^-)\,M\,\pi^{\mathbb{Q}}(X_t)}{E(X_t)}, \quad (21)$$

where $M = \left[(1-\tau)\frac{k}{r} - U(\underline{X}^{\star})\right]$.

Second, by differentiating the form of $\Omega^E(X_t)$ in (21) with respect to X_t, it follows that

$$\begin{aligned} \frac{\partial\Omega^E(X_t)}{\partial X_t} &= -\frac{(1-\phi^-)\,M\,\frac{\phi^-\pi^{\mathbb{Q}}(X_t)}{X_t}}{E(X_t)} \\ &\quad -\frac{E_X(X_t)\left[(1-\tau)\underline{X}^{\star} - (1-\phi^-)\,M\,\pi^{\mathbb{Q}}(X_t)\right]}{E^2(X_t)} \\ &= -\frac{(1-\phi^-)M\,\frac{\phi^-\pi^{\mathbb{Q}}(X_t)}{X_t}}{E(X_t)} - \frac{\Omega^E(X_t)(\Omega^E(X_t)-1)}{X_t}, \end{aligned}$$

where the last equality uses the definition $\Omega^E(X_t) = X_t\frac{E_X(X_t)}{E(X_t)}$ as well as (21). Hence, by the chain rule, the parametric derivative of $\Omega^E(X_t)$ with respect to $\pi^{\mathbb{Q}}(X_t)$ is given by

$$\frac{d\Omega^E(X_t)}{d\pi^{\mathbb{Q}}(X_t)} = \frac{\partial\Omega^E(X_t)}{\partial X_t}\left(\frac{\partial\pi^{\mathbb{Q}}(X_t)}{\partial X_t}\right)^{-1} = -\frac{(1-\phi^-)M}{E(X_t)} - \frac{\Omega^E(X_t)(\Omega^E(X_t)-1)}{\phi^-\pi^{\mathbb{Q}}(X_t)}. \quad (22)$$

Proof of Part (i)

Suppose $\pi^{\mathbb{Q}}(X_t) \to 0$. Then $X_t \to \infty$, so the expressions for $\underline{C}(X_t)$ in (7) and $U(X_t)$ in (5) give that

$$\lim_{\pi^{\mathbb{Q}}(X_t)\to 0} \underline{C}(X_t) = \lim_{\pi^{\mathbb{Q}}(X_t)\to 0} U(X_t) = \infty,$$

while

$$\lim_{\pi^{\mathbb{Q}}(X_t)\to 0} \frac{\frac{\partial \underline{C}(X_t)}{\partial X_t}}{\frac{\partial U(X_t)}{\partial X_t}} = \frac{r-\mu^{\mathbb{Q}}}{\mu^{\mathbb{Q}}} < 1$$

since $r > \mu^{\mathbb{Q}} > \frac{1}{2}r$ by assumption. Therefore, by l'Hôpital's rule,

$$\lim_{\pi^{\mathbb{Q}}(X_t)\to 0} \frac{\underline{C}(X_t)}{U(X_t)} < 1.$$

Suppose now that $\pi^{\mathbb{Q}}(X_t) \to 1$. Then $X_t \to \underline{X}^\star$, so

$$\lim_{\pi^{\mathbb{Q}}(X_t)\to 1} \frac{\underline{C}(X_t)}{U(X_t)} = \frac{(1-\tau)\frac{k}{r}}{\frac{(1-\tau)\mu^{\mathbb{Q}}\underline{X}^\star}{r-\mu^{\mathbb{Q}}}} = \frac{\phi^- - 1}{\phi^-} > 1,$$

by the expression for $\underline{X}^\star$ in (15).

Proof of Part (ii)

First, suppose $\pi^{\mathbb{Q}}(X_t) \to 0$. Since $\phi^- < 0$, Proposition 3 gives that as $X \to \infty$, $\pi^{\mathbb{Q}}(X)$ approaches zero faster than $E(X)$ approaches infinity. Combined with the form of $\Omega^E(X_t)$ in (21), it thus follows that

$$\lim_{\pi^{\mathbb{Q}}(X_t)\to 0} \Omega^E(X_t) > 1. \quad (23)$$

Next, by the limited liability condition (12) and the expression for $\underline{C}(X)$ in (7), it follows that

$$\lim_{\pi^{\mathbb{Q}}(X_t)\to 1} \Omega^E(X_t) = 1 + \frac{(1-\tau)\underline{X}^\star - (1-\phi^-)\,M}{\underline{C}(\underline{X}^\star)} = \frac{r-\mu^{\mathbb{Q}}}{\mu^{\mathbb{Q}}}\frac{\phi^-}{\phi^- - 1} < 1, \quad (24)$$

because the assumption $\mu^{\mathbb{Q}} > \frac{1}{2}r$ implies $\frac{r-\mu^{\mathbb{Q}}}{\mu^{\mathbb{Q}}} < 1$ and $\phi^- < 0$ implies $\frac{\phi^-}{\phi^- - 1} \in (0,1)$. Note that $\mu^{\mathbb{Q}} > \frac{1}{2}r$ is a sufficient but not necessary condition for (24) to hold.

Proof of Part (iii)

Consider the expression in (22). As $\pi^{\mathbb{Q}}(X_t) \to 0$, $E(X_t) \to \infty$, implying that the first term vanishes. By (23), the numerator of the second term is asymptotically positive, while $\phi^- < 0$ implies that the denominator is negative. The second term is therefore as a whole asymptotically positive, implying in total that

$$\lim_{\pi^{\mathbb{Q}}(X_t)\to 0} \frac{d\Omega^E(X_t)}{d\pi^{\mathbb{Q}}(X_t)} > 0.$$

Finally, consider again the expression in (22). As $\pi^{\mathbb{Q}}(X_t) \to 1$, $E(X_t) \to \underline{C}(\underline{X}^\star)$ by the limited liability condition in (12), which implies that the first term is asymptotically negative. By (24), the numerator of the second term will be asymptotically negative, while $\phi^- < 0$ implies that the denominator is negative. The second term is therefore as a whole asymptotically negative, implying in total that

$$\lim_{\pi^{\mathbb{Q}}(X_t)\to 1} \frac{d\Omega^E(X_t)}{d\pi^{\mathbb{Q}}(X_t)} < 0.$$

This completes the proof. □