

Financial synergies and the Organization of Bank Affiliates; A Theoretical Perspective on Risk and Efficiency*

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Abstract

We analyze theoretically banks' choice of organizational structures in branches or subsidiaries in the presence of government bailouts, default costs and - possibly - economies of scale as sources of financial synergies. We compare with stand-alone banks. Subsidiary and branch structures are characterized by different arrangements for internal insurance of affiliates against default risk. The cost of debt and leverage are endogenous. For moderate bailout probabilities, subsidiary structures, wherein the two entities provide mutual internal insurance under limited liability, have the highest private group value, but also the highest risk taking as measured by leverage, expected default costs and expected loss. The branch structure, wherein the two affiliates support each other until the whole bank fails, is generally burdened by greater default costs – in excess of bailout benefits – than the subsidiary structures. Stand-alone banks have the highest excess default costs. We explore also the impact on social values and policy implications of “ring-fencing” of affiliates.

KEYWORDS: bank organization, bank risk, financial synergies, endogenous leverage in banking, default costs, bailouts

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1 Introduction

Banks and other financial institutions face organizational choices with economic and legal dimensions when they operate across borders and across activities that are technologically separable. These organizational aspects of banking (in a broad sense) have entered the regulatory agenda after the 2007-2009 financial crisis in proposals for separation or “ring-fencing” of conventional commercial banking from other financial services within financial conglomerates, as well as separation of affiliates in different countries. Separation is an important element in “Living wills” as well. These “wills” refer to plans for rehabilitation and resolution of banks in distress. The exact meaning of separation or ring-fencing in different proposals is yet to be determined but it can refer to operational as well as financial separation. The ability of international banks and financial conglomerates to benefit from operational and financial synergies is likely to be strongly affected by the emerging regulatory structure.¹

The basic legal dimension of the organizational problem for a bank adding an affiliate is the choice between operating the affiliate as a separately incorporated subsidiary or as a legally integrated division without its own capital. The latter organizational choice is called a branch in the following. Affiliates within a branch structure rely on a common capital base and they support each other until they default jointly. Within subsidiary structures it is commonly assumed that each affiliate is financially independent with limited liability. This distinction between subsidiary and branch structures is insufficient for analysis of financial synergies in banking and related financial services because the two affiliates in a subsidiary structure can choose a level and direction of internal insurance against default of each subsidiary. In other words the subsidiaries are rarely financially independent.² Coinsurance within branch structures can also be affected by regulation that constrains mutual rescues.

If the level and direction of internal default insurance among bank affiliates is not taken into account, the affiliates are de facto financially independent and unable to exploit financial synergies. In this case the subsidiary structure would be rationalized only by their ability to take advantage of operational synergies in a comparison with two financially and operationally independent companies. Financial synergies would arise only in branch structures wherein the internal coinsurance is unlimited up to the point where the two branches default jointly.

The internal insurance aspects of organizational structures must be considered in an analysis of ring-fencing that may restrict each affiliate’s ability to

¹Traditionally universal banks were financial conglomerates supplying a variety of financial services in addition to deposit taking and direct lending within one legal entity. More recently the different activities within conglomerates have become legally separated in subsidiaries while often remaining strongly integrated both operationally and financially (Alexander, 2014).

²Internal default insurance arrangements can take the form of explicit guarantees of a subsidiary’s debt or a bank holding company’s responsibility for several subsidiaries’ debt. Internal insurance may also be more informal. For example, a parent firm facing distress can sell off subsidiaries in order to save itself or, if a subsidiary is facing distress, the parent can transfer assets to protect the bank’s brand name.

rescue another affiliate from default or mandate the insurance of one affiliate by other affiliates. We return to these regulatory issues in the concluding Section.

The main contribution of this paper is a model that allows analysis of positive and, possibly, negative financial synergies in a multi-affiliate bank. Banks may be organized in different ways, mainly because they may have different internal default insurance arrangements in the presence of default costs and a probability of state bailouts. The insurance arrangements affect the ability of the bank as a whole to reduce expected default costs and to take advantage of the probability of bailouts. We also consider that each affiliate may enjoy economies of scale in deposits although such economies of scale are controversial. An important aspect of the model is that it allows endogenous determination of both the interest rate on deposits and leverage in each affiliate.

We analyze private and social values along with leverage of a two-affiliate bank under four different internal insurance arrangements.³ The strongest internal insurance exists within a branch structure wherein the two affiliates rescue each other up to the point of joint default. The subsidiary structures we consider have mutual insurance or one-way insurance. In the mutual insurance case each subsidiary rescues the other one conditional on its own survival. In the one-way insurance case one affiliate rescues the other one conditional on its survival but rescues do not go the other way. Finally, we consider two stand-alone banks with financial independence in the sense that one will never rescue the other. Thus, they are not able to exploit financial synergies, which are the focus of our analysis.⁴

A preview of our main results is as follows. As long as default cost and probability of bailout parameters are similar across affiliates, subsidiary structures, and particularly those providing mutual internal insurance under limited liability, have the highest private group values, but also the highest risk taking, as measured by leverage and expected default costs. Although the advantage of the subsidiary structures from a private value point of view can be negated by a binding leverage constraint the difference between the privately optimized and the socially optimized leverage is particularly large for these structures. The branch structure, wherein the two affiliates support each other until the whole bank fails, becomes the structure of choice with the highest private value only if the bank can take advantage of differences in default cost and bailout parameters by incorporating itself in a favorable jurisdiction. Stand-alone banks have the highest private value only if leverage constraints are imposed on all structures and the probability of state bailouts is relatively high. If operational synergies differ across organizational structures these results must be adjusted accordingly. The results can also be modified by difference in the size of affi-

³The distinguishing characteristic of a bank relative to any corporation in this paper is, first, that there is a positive probability of a bailout by the government when a legal entity in its jurisdiction is insolvent. Second, we consider that banks may have economies of scale in debt above and beyond the tax shield.

⁴If they are under common ownership and control, they can exploit operational synergies. We do not incorporate such synergies in the model.

ates.⁵

A result of relevance for structural reform proposals is that in the presence of positive bailout probabilities the socially preferred internal insurance arrangement depends on whether capital requirements are binding or not.

We proceed as follows: related literature is reviewed in Section 2. The basic model for valuation of one stand-alone bank is set up in Section 3. Three factors are at work: the trade-off between the put option value of limited liability and default costs, the likelihood of a government bailout of depositors and returns to scale in deposits. They affect value and leverage. In Section 4 the bank expands by adding an affiliate as a subsidiary or a branch. Their coinsurance features are formulated in this Section. Analytical results are derived in Section 5 for relative private and social values of subsidiary, branch and stand-alone organizations under the assumptions that branches and subsidiaries are identical with respect to size and leverage, and that there are no scale effects. Analytical results with respect to leverage are derived as well. Numerical analysis of optimally levered banks follows in Section 6 where potential scale effects are introduced as well. Private and social values of different structures with and without leverage constraints are compared in this Section. In Section 7 we develop a proxy for potential systemic effects associated with the failure of banks with different organizational structures, the expected loss. The scope for "institutional arbitrage", i.e. taking advantage of differences in capital requirements, bailout probabilities and default costs across affiliates, through choice of organizational structure, is discussed briefly in Section 8. In the concluding Section 9 we summarize results and discuss implications for current reform proposals with respect to ring-fencing in international banks and financial conglomerates.

2 Background literature

Most of the literature discussed below, which compares organizational structures in banks and corporations, focus on the extremes of financially independent subsidiaries and merged financially integrated entities (branches). Two exceptions in the literature are Castiglionesi and Wagner (2012), who analyze efficiency properties of interbank insurance against liquidity problems, and Luciano and Nicodano (2014), who analyze financial synergies with one-way insurance against default within a corporate subsidiary structure relative to financially integrated (merged) structure. The subsidiary structure with mutual insurance seems to represent the norm for bank holding companies. In the US the "source

⁵Financial interdependence among subsidiaries may arise as a result of intra-corporate lending as well. We do not address this issue but focus entirely on internal default insurance arrangements. The basis for intra-corporate lending is usually internal information advantages or internal tax arbitrage (see, for example, Gertner, Scharfstein and Stein, 1994, and Shin and Stulz, 1998). In our model external lenders are fully informed. Tax arbitrage amounts to a form of transfer pricing. Taxes enter our model only as a potential source of economies of scale in debt. Intra corporate lending may implicitly be associated with default insurance of the borrowing subsidiary if the debt is expected to be forgiven in case of insolvency. (See also footnote 2)

of strength” doctrine implies that a bank holding company provides financial and managerial support to subsidiaries under stress.⁶

The analysis of risk and efficiency in bank organizations in this paper is related to several strands of literature. Dell’Ariccia and Marquez (2010) analyze theoretically the choice between branches and subsidiaries in banking. They model the choice as a trade-off between benefits of limited liability for a bank with a subsidiary in the presence of economic risk and protection against political risk (of expropriation) in a branch organization. In our terminology the subsidiaries in Dell’Ariccia and Marquez are stand-alone banks with limited liability. The benefit of branch organizations in terms of protection against expropriation risk can be viewed as an operational synergy. In the model below, we focus on financial synergies between affiliates with imperfectly correlated return distributions.

There is a strand of literature in corporate finance on financial synergies arising as a result of the merger of two firms. Leland (2007) and Banal-Estanol, Ottaviani and Winton (2012) show that when two stand-alone firms are merged into one legal entity the new firm cannot take advantage of limited liability but it can benefit from reduced default costs. Leland also considers tax-effects of the endogenous choice between debt and equity. Banal-Estanol et al. restrict the analysis to debt financing and the effects of the merger of two firms on default costs, while the merged firm re-optimizes leverage in the Leland paper. In the latter case, the merged firm always benefit from reduced default costs while a ‘contamination effect’ within the merged organization with fixed leverage can cause the default costs to rise above the default costs of the separately financed firms. Limited internal insurance between subsidiaries are not considered in these papers.

Luciano and Nicodano (2014) expands on the analysis in Leland (2007) and considers that a parent plus a subsidiary, which can be rescued by the parent, can economize on default costs relative to the merged firm. These papers also endogenize both leverage and the interest rate on debt.⁷ They focus on the organizational choice as a trade-off between default costs and tax-savings from debt financing. We build on the model in Luciano and Nicodano taking into account the probability of bailouts of banks. Furthermore, we consider the possibility of mutual rescue in subsidiary structures and financially independent banks as well as one-way rescue and merged affiliates (branches).

The literature on benefits and costs of financial conglomerates focus on operational synergies but financial synergies play an important role as well in

⁶The presence and effect of rescues, or internal insurance within banking groups, have been studied empirically by Bradley and Jones (2008), as well as by Ashcraft (2004) for the US. Bradley and Jones note that “the Federal Reserve Board (FRB) extended its longstanding position that a bank holding company serve as a source-of-strength to its subsidiary banks beyond the application process. Under the expanded policy, the FRB required that a holding company stand ready to provide troubled subsidiaries with both financial and managerial assistance in times of stress.”

⁷Freixas and Ma (2013) endogenize leverage as well in a bank with the purpose of analyzing a potential trade-off between competition and financial stability in different dimensions.

Freixas, Loranth and Morrison (2007).⁸ This paper considers a conglomerate wherein activities are risky to different degrees. The activities can be conducted in an integrated entity subject to one liability constraint—a branch bank in our terminology—or within a holding company structure with financially independent subsidiaries. In our terminology the subsidiaries are stand-alone banks that may benefit from operational synergies since they are under common ownership. The operational synergies originate in the ability to shift assets with different risk between the subsidiaries. The incentive to do this is created by weak market discipline in one of the activities that enjoys benefits from deposit insurance. In this setting capital requirements can be differentiated across subsidiaries to induce the bank to subject activities without deposit insurance to greater market discipline. The paper also considers stand-alone, independently owned financial institutions without ability to transfer assets between them. In our model focusing on financial synergies risk can only be shifted between subsidiaries through increased leverage in one entity. We introduce a probability of bailout for each legally separate entity instead of deposit insurance since implicit protection of banks’ creditors has become increasingly important.⁹ A result we have in common with Freixas et al is that optimal capital requirements must be differentiated across different organizational structures as well as different costs of default and state bailout schemes.

Kahn and Winton (2004) emphasize moral hazard incentives to shift risk to debt-holders who cannot observe the riskiness of the different activities within a financial conglomerate. Since the debt-holders know that they do not have risk-information, the financial institution can reduce its cost of funding by separating the financing of high-risk and low-risk activities into different entities with different leverage. The benefits associated with separation into subsidiaries accrue to financially independent subsidiaries without internal insurance in Kahn and Winton. In our model debt-holders know the risk characteristics of assets so that risk is reflected in interest rates. Benefits of subsidiary structures occur only as a result of internal insurance arrangements, which are not present in Kahn and Winton.

An empirical literature of relevance studies the flows of funds and organization of cross-border banks. Cross-border flows are documented empirically by Cetorelli and Goldberg (2012). Jeon et al. (2013) study the transmission of shocks generated by such flows. The choice of organization for affiliates is examined empirically in Cerutti et al. (2007). Ongena, Popov and Udell (2013) analyze empirically how cross-border banks shift risk between countries with

⁸There are a number of papers considering agency costs and governance problems associated with different organizational structures. These costs or benefits can be viewed as negative or positive operational synergies. See, for example, Boot and Schmeits (2000). Chemmanur and John (1996), Harr and Rønde (2006) and Kahn and Winton (2004). Agency problems also play a role in models of capital regulation and leverage when monitoring efforts are endogenized. Acharya, Mehran and Thakor (2013) analyze how capital regulation affect both risk-shifting and monitoring incentives.

⁹A recent Financial Stability Report from the IMF (2014) reports estimates of the implicit subsidization of debt financing generated by implicit protection of banks considered “too big to fail.”

different regulation. We return to these policy issues in the concluding Section.

3 The stand-alone bank

This Section models a single or stand-alone (*SA*) bank using as a starting point the structural model of Merton (1974) and introducing default costs and the possibility of external bailout in default. The bank produces a fixed amount of loans at time 0. It may obtain leverage by issuing deposits with an endogenous, competitively determined interest rate. We argue that this creates an incentive to raise leverage so as to exploit bailout, and a disincentive determined by default costs.

For the sake of simplicity, consider two points in time only, $t = 0, T$, and classify bank liabilities into deposits and equity. Deposits represent customer as well as interbank net deposits, borrowing from the Central Bank and issued bonds. Equity represents capital and reserves. In order to model deposits in a simple way, we assume that they take the form of zero-coupon debt. They can be withdrawn at maturity¹⁰ T . The face value of deposits is denoted by F . Deposits earn an interest rate which is implicitly determined by the fact that the (market) value of deposits at time 0, D_0 , is the present value of the expected payoff to depositors at time T : $F - D_0$ is the total amount of interests paid by the bank. Also the (market) value of equity at time 0, E_0 , is determined as the present value of the expected payoffs to the equity holders at T .

To simplify, we label as “loans” all the bank assets. We disregard interbank claims and consider as a unique entity proper loans and securities. In doing that we have in mind mainly commercial banks. The initial value of loans is denoted as $L_0 \in \mathbb{R}$. The value of loans at time T is a non-negative random variable - which we take to be continuous, for simplicity - denoted as $L(T)$. Loans can be traded at any time between 0 and T , and the market for loans is efficient (no transaction costs, no indivisibility). At time T the bank collects the value of loans $L(T)$. We capture possible scale effects in deposits as a fraction, k , of the interest cost of debt, $F - D_0$. This formulation can also be interpreted as an interest rate tax shield. Thereby, the bank’s total cash flows at T are expressed as:¹¹

$$\bar{L}(T) \triangleq L(T) + k(F - D_0),$$

where $k \geq 0$. We label these scale effects as “financial economies of scale” or “economies of scale in debt.” It is controversial whether there are such economies of scale in banking and other financial services except as a result of an interest tax shield from debt. We do not take a stand on the relevance of these financial

¹⁰Structural models of the type we are going to build have proven to be quite resilient to the possibility of liabilities’ repayment when the actual value of assets goes under a covenant level. This is why we do not introduce the hypothesis of a “bank run” when the value of deposits falls below a given threshold before T .

¹¹Recall that \triangleq means “equal by definition”.

scale effects; therefore, we work throughout the paper with $k = 0$ as well as $k > 0$.

Assume that both debt and equity holders are risk neutral. The bank distributes its cash flows to depositors and equity holders. Depositors are going to receive the face value of their deposits, F , either if this is greater or equal than the asset value $\bar{L}(T)$, or if it is smaller, $\bar{L}(T) < F$ and the state bails out depositors. There is a probability π that the state bails out the bank.¹² If $\bar{L}(T) < F$ and there is no bailout default is assumed to be costly. The asset value to be distributed to depositors is net of default costs. For the sake of simplicity, we take default costs to be proportional to total cash flows at T , $\alpha\bar{L}(T)$. The value to be distributed to depositors becomes the sum of what equity holders pay to debt holders and, in case of default, what they receive from the government in case of bailout, less default costs :

$$\min(F, \bar{L}(T)) + (F - \bar{L}(T))\mathbf{1}_{\{\bar{L}(T) < F, B\}} - \alpha\bar{L}(T)\mathbf{1}_{\{\bar{L}(T) < F, \bar{B}\}},$$

where $\mathbf{1}_{\{E\}}$ is the default indicator of event E , which is equal to one if and only if E occurs, B is the event of bailout, \bar{B} the event of no bailout.

The value of debt is the expected discounted value of the payoff from deposits:

$$D_0 = \exp(-rT) \times \left\{ \mathbb{E} \min(F, \bar{L}(T)) + \pi \mathbb{E} \max(F - \bar{L}(T), 0) - \alpha(1 - \pi) \mathbb{E} \left[\bar{L}(T) \mathbf{1}_{\{\bar{L}(T) < F\}} \right] \right\}$$

Following Merton (1974), it is easy to argue that the first term is the difference between the face value of deposits discounted and a put (price) on loans, with strike F . The second term is the so-called "default put": (the price of) a put option on L , with strike F , which is paid with probability π . The third represents expected default costs. Collecting the put terms, we have

$$D_0 = \exp(-rT) \left\{ \begin{array}{l} F - (1 - \pi) \mathbb{E} \max(0, F - \bar{L}(T)) \\ - \alpha(1 - \pi) \mathbb{E} \left[\bar{L}(T) \mathbf{1}_{\{\bar{L}(T) < F\}} \right] \end{array} \right\} \quad (1)$$

The payoffs to *equity holders* of the bank at T are

$$\max[\bar{L}(T) - F, 0].$$

Equity holders are long a call on loans' net value, with strike F . The equity value at time 0, E_0 , is then

$$E_0 = \exp(-rT) \mathbb{E} \max[\bar{L}(T) - F, 0]. \quad (2)$$

¹²Freixas *et al.* (2007) specifies two kinds of debt; insured deposits and non-insured loan funding. We assume here that all deposits are guaranteed because, in the current economic environment, implicit insurance of creditors of all types seems to be the rule rather than the exception. The implicit guarantees cannot be certain, however. This is why we add a parameter π .

The private value of the stand-alone bank GV_{SA} can be proven to be equal to the unlevered value L_0 , plus the bailout put minus default costs:

$$\begin{aligned}
 GV_{SA} &= D_0 + E_0 = \\
 &= L_0 + \underbrace{\exp(-rT)\pi\mathbb{E}\max(0, F - \bar{L}(T))}_{\text{bailout put}} - \underbrace{\alpha(1 - \pi)\exp(-rT)\mathbb{E}\left[\bar{L}(T)\mathbf{1}_{\{\bar{L}(T) < F\}}\right]}_{\text{default costs}}.
 \end{aligned} \tag{3}$$

Throughout the paper bank managers choose the (non-negative) face value of deposits in order to maximize GV_{SA} , which represents funds to managers disposal at time 0 comprehensive of dividends in E_0 and the deposits that equity holders cash in at time 0, in D_0 .¹³ Since bankruptcy costs and benefits from bailouts accrue to depositors, and therefore determine D_0 , they are taken into account when choosing F .¹⁴

If ever there are no default costs and no bailout, the bank value $D_0 + E_0$ reduces to the initial loan value L_0 : an irrelevance property of the Modigliani-Miller type holds in this case since leverage does not affect the bank value.

Stand-alone banks are defined here as entities that commit to no rescue. If we had *two stand-alone banks*, their total value would simply be the sum of their values. Their optimal face values of deposits would simply be the ones which maximize each affiliate's value separately. Leverage would be chosen independently by each affiliate.

Actually, the bank value specified so far is the *private* one. We are going to distinguish it from the *social value* SV_{SA} . The difference between social and private value is simply the *bailout put*, since this value represents a redistribution effect from the government to the depositors in the absence of externalities. As a consequence, the social value is the asset value minus default costs. In Section 6 below we discuss a broader concept of social value taking into consideration that there are potential systemic consequences as well as other costs of bank insolvencies, whether they are bailed out or not.

4 A bank with two affiliates

We model a bank with two affiliates, namely a home bank and a subsidiary or a branch, by specifying its financial synergies, which are affected by coinsurance between the affiliates in case of default. The two affiliates can be part of a cross-border bank or they can be producing different financial services as parts of a conglomerate bank organization. Either way we call one entity the home bank and the other entity we call subsidiary or branch. We move from the merger

¹³It should be clear from the payoffs to debt and equity - both in this case and the ones to follow - that we could equally well have taken deposits as given and solved for the amount of loans.

¹⁴Note also that the benefit of the bailout put is here revealed as a higher D_0 for a fixed F (lower deposit rate) while higher bankruptcy costs reduce D_0 (increase the deposit rate). A smaller difference between D_0 and F implies lower costs of leverage.

model of Leland (2007) and the parent-subsidiary case in Luciano and Nicodano (2014). We introduce the possibility of state bailouts in case of insolvency and financial returns to scale.

It is well-known that a merger of two imperfectly correlated activities within one firm can produce financial synergies by economizing on default costs. This type of synergy can be produced in a branch organization in our terminology. As noted by Banal-Estanol *et al.* (2012) there is also a negative side to merging the activities into a branch organization because there is “contamination” if one activity must come to the rescue of another loss-making activity, with the possible consequence that the whole merged firm defaults. This type of contamination can be reduced in a subsidiary organization wherein both entities enjoy limited liability. Therefore, the rescue of one subsidiary by another one can be interrupted if the latter would be threatened by the rescue costs. In the following financial synergies exist not only as a result of default costs but also because of a positive probability of bailout by the state of any separate legal entity enjoying limited liability. If the home bank organizes its affiliate as a branch the two entities do not separately enjoy limited liability, they default together and they are bailed-out together. The two entities provide internal guarantees for each other as long as there is capital in the bank as a whole. This distinguishes them from parent-subsidiaries, which can default separately. Indeed, if the bank organizes the affiliated activity in a subsidiary both entities enjoy limited liability. They can default individually and they can be bailed out individually. We consider two types of subsidiary organization. In one there is a mutual insurance. Each affiliate rescues the other affiliate conditional on the survival of the rescuing entity. The organization with mutual insurance is particularly relevant for affiliates belonging to a bank holding company. In the other, which is a subcase of the former, the home bank offers one-way insurance for the subsidiary (or the other way around). It intervenes in order to recapitalize (rescue) the subsidiary if ever the latter is unable to pay back its depositors, provided that rescuing the subsidiary does not trigger the home bank’s default.

If there are returns to scale, they may be different in the two organizational structures. This is represented by the magnitude of k_s , the returns to scale parameter for subsidiaries, versus k_b for branches. We have the following cash flows for subsidiaries and their parents:

$$L_i(T) + k_s(F_i - D_{0i}) \triangleq M_i(T), \quad i = s, h$$

and for branches

$$L_i(T) + k_b(F_i - D_{0i}) \triangleq N_i(T), \quad i = b, h$$

Later on, in order to compare the different structures, we set final loans in branches equal (in distribution) to loans in subsidiaries $L_b(T) = L_s(T)$. If not specified otherwise, but without loss of generality, apart from the fact that we exclude perfectly dependent loans, we also introduce the following technical assumption:

Assumption 1. The joint density of $L_h(T)$ and $L_b(T) = L_s(T)$ has non-null density over the whole positive orthant of \mathbb{R}^2 .¹⁵

The overall group value with two affiliates is

$$GV_j = D_{0h} + E_{0h} + D_{0a} + E_{0a}$$

where we have $a = s, b$, depending on whether we are in the subsidiary or branch structure, and $j = MR$ for the mutual rescue structure, OWR for the one-way-rescue structure, BR for the branch structure. We prove below that, similarly to the stand-alone case, the overall value is the sum of the home and affiliate unlevered value, plus their default puts, minus their default costs:

$$\begin{aligned}
GV_i = & P_{h0} + \underbrace{\pi \exp(-rT) \mathbb{E} \max \{ (0, F_h - P_h(T)) \mathbf{1}_{\{A\}} \}}_{\text{government bailout home}} + \\
& \underbrace{-\alpha(1 - \pi) \exp(-rT) \mathbb{E} [X \mathbf{1}_{\{A\}}]}_{\text{default cost home}} + \\
& + P_{a0} + \\
& + \underbrace{\pi \exp(-rT) \mathbb{E} \{ \max(0, F_a - P_a(T)) \mathbf{1}_{\{A'\}} \}}_{\text{government bailout affiliate (subsidiary or branch)}} + \\
& \underbrace{-\alpha(1 - \pi) \exp(-rT) \mathbb{E} [X' \mathbf{1}_{\{A'\}}]}_{\text{default cost affiliate (subsidiary or branch)}}
\end{aligned} \tag{4}$$

where P_{h0} and P_{a0} are the asset values of the home bank and its affiliate at time 0, and may take either the value M or N , namely M_h, M_a, N_h, N_a , according to the organizational form. Similarly for $P_h(T), P_a(T), a = s, b$. The cash flows associated with bailout and default depend on coinsurance arrangements in the event of default of either the home bank, A , or the affiliate, A' . X, X' are the cash flows that determine default costs of the home bank in A and of the affiliate in A' . Also these cash flows are different in the different arrangements, because mutual subsidization is compulsory in the branch case, while it is somewhat restricted in the subsidiary case. In order to specify the events and the payoffs, we need to describe how rescue occurs in the different organizations. This is the aim of the next two sections, for subsidiaries and branches respectively.

4.1 Rescue in the subsidiary organization

When two banks organize themselves in being a home bank and its subsidiary, they remain two separate legal entities, but the home bank may offer insurance against default of the subsidiary and vice versa. If the insurance consists in providing the subsidiary with assets at T , if the affiliate is in default, conditional on not endangering the safety of the home bank and vice versa, we have mutual insurance. If ever the guarantee works only from the home to the subsidiary (or from the subsidiary to the home bank), we have one-way insurance or rescue. If

¹⁵The Gaussian distribution on loan log-returns introduced later satisfies this hypothesis, which is needed only in order to simplify the discussion and avoid having events of null probability, as well as perfectly correlated returns (positively and negatively).

ever the gurantee works only from the home to the subsidiary or viceversa, we have one-way insurance or rescue.

Rescue of the subsidiary occurs if and only if the home bank is not in default or distress ($M_h(T) > F_h$), the subsidiary is in default - i.e. its asset value is below the default level ($M_s(T) < F_s$) and rescuing the subsidiary does not drive the home bank into default. Rescue means that, using its surplus $M_h(T) - F_h$, the home bank pays that part of the subsidiary deposits that are not covered by its own assets, $F_s - M_s(T)$. The home-bank can do this without facing default if its surplus is greater than the amount needed for rescue, $M_h(T) - F_h > F_s - M_s(T)$. The conditions for rescue of the subsidiary are

$$\left\{ \begin{array}{l} M_h(T) > F_h \\ M_s(T) < F_s \\ M_h(T) - F_h > F_s - M_s(T) \end{array} \right.$$

Since the first condition is always satisfied when the last is, the conditions can be reduced to the event

$$R \triangleq \left\{ \begin{array}{l} M_s(T) < F_s \\ M_h(T) - F_h > F_s - M_s(T) \end{array} \right. \quad (5)$$

Instead the subsidiary defaults if the following event A' occurs:

$$A' \triangleq \left\{ \begin{array}{l} M_s(T) < F_s \\ M_h(T) - F_h < F_s - M_s(T) \end{array} \right. \quad (6)$$

The state bails out the subsidiary with probability π .

Rescue of the home bank by the subsidiary takes place when the the latter is not in default and is not endangered by rescue:

$$R' \triangleq \left\{ \begin{array}{l} M_h(T) < F_h \\ M_s(T) - F_s > F_h - M_h(T) \end{array} \right.$$

If this rescue of the home bank does not take place the following default event A occurs:

$$A \triangleq \left\{ \begin{array}{l} M_h(T) < F_h \\ M_s(T) - F_s < F_h - M_h(T) \end{array} \right. \quad (7)$$

If A holds true, there is room for state bailout of the home bank, which occurs with probability π .

For given face value of deposits and initial loans of the home bank and subsidiary, the payoff to depositors of the subsidiary is the payoff to a stand alone bank, augmented by the conditional home-bank support $F_s - M_s(T)$ when rescue occurs. Since bailout comes after rescue by the home bank, the subsidiary debt before rescue and bailout is the one we obtained in the stand alone case. Default costs are paid only if there is default, no rescue and no bailout. We

have:

$$\begin{aligned}
D_{0s} = & \underbrace{+\exp(-rT) [F_s - \mathbb{E} \max(0, F_s - M_s(T))]}_{\text{value without bailout and rescue}} + & (8) \\
& \underbrace{+\exp(-rT) \mathbb{E} \{ [F_s - M_s(T)] \mathbf{1}_{\{R\}} \}}_{\text{rescue}} + \\
& \underbrace{+\pi \exp(-rT) \mathbb{E} \{ \max(F_s - M_s(T), 0) \mathbf{1}_{\{A'\}} \}}_{\text{government bailout}} \\
& \underbrace{- \exp(-rT) (1 - \pi) \alpha \mathbb{E} [M_s(T) \mathbf{1}_{\{A'\}}]}_{\text{default costs}}
\end{aligned}$$

Since the assets for rescue come from the home bank, rescue diminishes the equity value of the home-bank in comparison with (2). The equity value of the home bank becomes

$$\begin{aligned}
E_{0h} = & \exp(-rT) \mathbb{E} \max [M_h(T) - F_h, 0] \\
& - \exp(-rT) \mathbb{E} \{ [F_s - M_s(T)] \mathbf{1}_{\{R\}} \},
\end{aligned}$$

where the first term represents the home bank equity without a subsidiary.

Rescue of the home bank by the subsidiary affects the payoffs to the debt holders of the home bank positively since it entails a transfer to them if the event R' is true, as follows:

$$\begin{aligned}
D_{0h} = & \underbrace{+\exp(-rT) [F_h - \mathbb{E} \max(0, F_h - M_h(T))]}_{\text{value without bailout and rescue}} + & (9) \\
& \underbrace{+\exp(-rT) \mathbb{E} \{ [F_h - M_h(T)] \mathbf{1}_{\{R'\}} \}}_{\text{rescue received}} + \\
& \underbrace{+\pi \exp(-rT) \mathbb{E} \{ \max(F_h - M_h(T), 0) \mathbf{1}_{\{A\}} \}}_{\text{government bailout}} \\
& \underbrace{- \exp(-rT) (1 - \pi) \alpha \mathbb{E} [M_h(T) \mathbf{1}_{\{A\}}]}_{\text{default costs}},
\end{aligned}$$

Symmetrically, equity holders of the subsidiary are deprived of part of their cash flows if the rescue event R' is true:

$$\begin{aligned}
E_{0s} = & \exp(-rT) \mathbb{E} \max [M_s(T) - F_s, 0] \\
& - \exp(-rT) \mathbb{E} \{ [F_h - M_h(T)] \mathbf{1}_{\{R'\}} \},
\end{aligned}$$

In the mutual rescue structure the home bank chooses how many deposits to raise directly and through its subsidiary in order to maximize the overall value, GV_{MR} :

$$\max_{F_h, F_s} GV_{MR} = \max_{F_h, F_s} (D_{0h} + E_{0h} + D_{0s} + E_{0s}). \quad (10)$$

which can be written as in (4), provided that we define $X = M_h(T)$, $X' = M_s(T)$, $P_i = M_i$, $i = h, s$. The last formulation says that the home-bank plus subsidiary value is given by the sum of the asset values $M_h + M_s$ plus the bailout puts, minus their default costs. Rescue payments cancel out because they are paid by one stakeholder (equity owners of one affiliate) to debt holders of the other. This proves that the group value can be split as in the stand-alone case even though the events of default and bailout are different. With mutual rescue, the home-bank plus subsidiary value GV_{MR} is the sum of the asset values, which include returns to scale, $M_{h0} + M_{s0}$, plus the bailout puts for each bank minus their default costs, which are paid only in the absence of rescue by the other group member and in the absence of state bailout.

Assume now that, in the subsidiary case, rescue is unilateral and can go only from the home to the subsidiary. This means that the payoffs to the equity holders of the subsidiary and to depositors of the home bank are as in the stand-alone case. Their fair values are:

$$\begin{aligned} E_{0s} &= \exp(-rT)\mathbb{E} \max [M_s(T) - F_s, 0] \\ D_{0h} &= \exp(-rT) \times \\ &\times \left[F_h - (1 - \pi)\mathbb{E} \max(0, F_h - M_h(T)) - \alpha(1 - \pi)\mathbb{E} \left[M_h(T)\mathbf{1}_{\{M_h(T) < F_h\}} \right] \right]. \end{aligned} \quad (11)$$

The formulas for the subsidiary debt and home-bank equity of the mutual case remain in force, since their payoffs are not affected. It follows that the overall value of the subsidiary organization, with unilateral guarantee, is still given by (4), with $A = M_h(T) < F_h$, A', X, X' as above. The problem of the home-bank plus subsidiary is still of the type

$$\max_{F_h, F_s} GV_{OWR} = \max_{F_h, F_s} (D_{0h} + E_{0h} + D_{0s} + E_{0s}). \quad (12)$$

and, GV_{OWR} is still given by the sum of the asset values $M_h + M_s$ plus the government bailout put, minus their default costs, as specified in Appendix B. It is easy to prove that the home bank plus subsidiary value, GV_{OWR} , is the sum of the asset values, $M_{h0} + M_{s0}$, plus the bailout puts for each bank, minus their default costs. Default costs are paid by the home bank as in the *SA* case; they are paid by the subsidiary only if there are no rescue and no state bailout. A similar decomposition of value can be obtained if rescue goes only from the subsidiary to the home bank.

4.2 Rescue in the branch organization

Consider now the branch case. The returns to scale are the ones described by $N_h(T), N_b(T)$. The branch case is different from the mutual subsidiary case because there is no more limited liability of one bank versus the other. In the branch case, insolvency for the whole bank organization is the only possibility, although analytically we treat the two entities as separate with equity assigned to them. We expect this lack of limited liability to be a source of contamination

that affects branches negatively relative to subsidiary structures, exactly as the Sarig effect deprives mergers of value (see Sarig (1985), Leland (2007), Balan-Estanol *et al.* (2012)). Rescue is mutual, but is not conditional on survivorship of the guarantor. So, support from the home bank to the branch is offered whenever.

$$R_b \triangleq \begin{cases} N_h(T) > F_h \\ N_b(T) < F_b \end{cases}$$

while support in the other direction occurs when

$$R'_b \triangleq \begin{cases} N_h(T) < F_h \\ N_b(T) > F_b \end{cases}$$

These events substitute for R, R' . The transfer in the two events is respectively

$$\min(N_h(T) - F_h, F_b - N_b(T)),$$

and

$$\min(N_b(T) - F_b, F_h - N_h(T)).$$

It does not necessarily cover the difference between the face value of debt of the guaranteed company and its own cash flows, but it is the minimum between that difference and the extra-cashflows of the guarantor. There is the possibility of government bailout when the whole bank is insolvent. This happens when one affiliate is insolvent and the other entity is either insolvent or unable to rescue because the minimum above is $N_h(T) - F_h$ or $N_b(T) - F_b$. Whenever the state intervenes, there has been joint insolvency of the branch and home bank. If the state does not bailout there are default costs. The event in which bailout of the branch occurs is

$$A' \triangleq \begin{cases} N_b(T) < F_b \\ N_h(T) - F_h < F_b - N_b(T) \end{cases} \quad (13)$$

The event in which bailout of the home bank occurs is

$$A \triangleq \begin{cases} N_h(T) < F_h \\ F_h - N_h(T) > N_b(T) - F_b \end{cases} \quad (14)$$

The home bank can maximize the overall value by choosing how many deposits to raise directly and through its branch:

$$\max_{F_h, F_b} GV_{BR} = \max_{F_h, F_b} (D_{0h} + E_{0h} + D_{0b} + E_{0b}) \quad (15)$$

where the expressions for debts and equities are given in Appendix A. It can easily be shown that the overall value is still given by (4). The home-bank plus branch value is given again by the sum of the asset values $N_h + N_b$ plus the government bailout puts minus their default costs.

5 Comparing organizations without scale economies

We can compare analytically the different arrangements, when the level of deposits (in face value) and cash flows from loans (in distribution), as well as other parameters, are the same. Costs of debt remain endogenous. Some analytical results can also be derived for endogenous leverage. All through this Section we assume that there are no financial economies of scale.

We assume in Section 5.1 that the level of deposits is the same in different organizational structures. Equality of deposits in face value can be interpreted as equal capital requirements on all entities. In Section 5.2 we consider differences in bailout probabilities and default costs with the same constraint on leverage. In Section 5.3 we endogenize leverage as well and derive a few analytical results before turning to numerical analysis in Section 6. Appendices B and C provide the proofs of the propositions in Sections 5.1 and 5.3.

5.1 No returns to scale, equal parameters for bailouts and default costs

Introduce the following

Assumption 2. Let the affiliates have the same size, i.e. the same initial value and distribution of final loans ($L_{h0} = L_{a0}, L_h(T) = L_a(T), a = s, b$ in distribution), the same positive leverage ($F_h = F_a > 0, a = s, b$). Let the final loans value have any linear correlation, except one, and assume that there are no economies of scale ($k = 0$).

If we compare the one-way and the mutual rescue structures, we can conclude that

Proposition 1 *Under Assumption 2, unilateral conditional rescues in subsidiary-home-bank organizations are privately worse than mutual conditional rescues, if $\alpha > 0$ and $\pi = 0$, while they are better when $\pi = 1$ and $\alpha > 0$ or $\alpha = 0$. If $\alpha > 0$, there is a positive bailout probability π^* above which unilateral guarantees become better than mutual. Last, they have equal value in a neighborhood of $\alpha = \pi = 0$.*

So, independently of how many deposits are collected in the affiliate, if there is no bailout but default is dissipative ($\alpha > 0$ and $\pi = 0$), it is better to provide mutual rescues in case one affiliate becomes insolvent, instead of leaving it alone. Mutual rescue saves on default costs in the absence of state bailouts. When there is external bailout with certainty or when default is not dissipative ($\pi = 1$ and $\alpha > 0$ or $\alpha = 0$), mutual rescue is not a rational strategy. Above a given likelihood of state assistance (π^*), mutual rescue is not any more rational if default is dissipative ($\alpha > 0$). In this case, when the weight of rescuing is left to the external entity providing bailout, instead of being internalized, one way rescue is favored over mutual rescue.

If we compare the branch and the subsidiary with mutual rescue organizations, we get the following:

Proposition 2 *Under Assumption 2, the subsidiary organization (with mutual insurance) is privately preferable to the branch, for every $\alpha > 0$, when $0 \leq \pi < 1$; when $\alpha = 0$ or $\alpha > 0$ and $\pi = 1$, they have equal value.*

The proposition states that in all cases of dissipative default costs and some degree of uncertainty about the bailout, the bank will prefer the subsidiary organization even when there is commitment to mutual rescue.

Putting together propositions 1 and 2, it is easy to assess the following:

Corollary 3 *Under Assumption 2, if $\alpha > 0$, there exists a bailout probability π^* above which the subsidiary organization with unilateral insurance is privately preferred to the subsidiary with mutual insurance, which in turn is preferred or equivalent to the branch organization.*

The comparison between two stand alone banks and the same banks (in terms of assets) once they become affiliated within a group - with a one-way or mutual insurance - is quite straightforward too. In Appendix B we indeed prove that:

Proposition 4 *Under Assumption 2, organizing a bank as a stand-alone entity provides greater private value than organizing it as a subsidiary - with unilateral or mutual rescue - if default is not dissipative and there is a positive probability of bailout ($\alpha = 0$ and $\pi > 0$) or, for any level of default costs, if the probability of bailout is one ($\alpha \geq 0$, $\pi = 1$). The private value of the subsidiary organization is greater when default is dissipative but the probability of bailout is null ($\alpha > 0$ and $\pi = 0$). Under dissipative default costs, there is a positive bailout probability π^{**} above which stand alone banks are more valuable than unilaterally guaranteed subsidiaries. The stand alone organization becomes more valuable than the mutually guaranteed subsidiary when $\pi = \pi^{***}$, where $\pi^{***} \leq \max(\pi^*, \pi^{**})$.*

The visual representation of propositions 1-4 for the case when $\alpha > 0$ is also illustrated in Figures 1 and 2. These figures summarize the pairwise comparisons in the propositions. The difference between the two figures is that in Figure 2 the π -values that make two stand-alones more valuable than a subsidiary structure with one way rescue ($\pi = \pi^*$), two stand-alones more valuable than a subsidiary structure with mutual rescue ($\pi = \pi^{***}$), and the subsidiary structure with one way rescue more valuable than the subsidiary structure with mutual rescue ($\pi = \pi^{**}$), are ordered differently.

Insert here Figures 1 and 2

In both figures the private value $GV_j, j = OWR, MR, BR, SA$, is on the vertical axis while the probability of bailout, π , increases along the horizontal axis. In this and figures and tables to come, SA denotes the sum of two Stand-alone banks.

The shaded area for the value of the branch structure, GV_{BR} , reflects that we cannot order its value relative to the stand alone banks and the subsidiary with one way rescue for relatively low values of the bailout probability, π , when there are positive default costs. We know that when $\alpha = 0$ and $\pi = 0$ all organizational structures have the same value. Combining propositions 1 and 2 we can say that when ($\alpha > 0$ and $\pi = 0$) the mutual rescue subsidiary structure is privately more valuable than any of the other organizations but we cannot rank the branch organization relative to the one way rescue subsidiary and the stand-alone organizations.

As the bailout probability increases when there are positive default costs the value of the Stand-alone structure increases faster than the value of the one-way rescue subsidiary organization, which increases faster than the Mutual rescue subsidiary organization. At very high bailout probabilities the ordering of these three organizational structures have been reversed.

The ambiguity for the value of the branch structure relative to the one-way rescue subsidiary structure and the two stand-alones can be understood in terms of coinsurance and contamination as a result of unlimited rescue in the branch structure. Banal-Estanol, Ottaviani and Winton (2012) show how a branch structure in comparison with two stand-alone banks provides coinsurance that reduces default costs. However, contamination is also possible because large losses in one affiliate can drag the whole branch structure into default as a result of the unlimited coinsurance. This contamination effect does not occur in subsidiary structures and stand-alone banks under limited liability. Within our framework subsidiary structures have the advantage relative to stand-alone banks that they benefit from coinsurance gains in terms of default costs.¹⁶

Since the comparisons in this Section are based on the assumption that the face value of debt, F , is the same in each affiliate, value differences between structures are explained by differences in the interest rate on deposits, which reflect expected default costs and bailout expectations, default costs, which are different across organizations even with the same leverage because of the difference in guarantees, and the value of the bailout put to shareholders, which again kicks in differently because the inability to survive based on the group coinsurance only is different across different guarantees, even for the same leverage.

We can summarize the role of bailout probabilities and default costs in the ranking of the private values of the organizational structures as follows:

1. For a given $\alpha \geq 0$ a there is a relatively high probability of bailout, π , that favors the stand alone banks over the unilateral rescue subsidiary structure over the mutual rescue subsidiary structure. The branch structure approaches the mutual rescue subsidiary structure when the bailout probability approaches one.
2. At a relatively low probability of bailout, π , the mutual rescue subsidiary

¹⁶The value differences between the various organizations can be split into coinsurance gains and risk contamination, where the coinsurance gains are the reductions in default costs or the higher bailout revenues due to rescue, while risk contamination is the increase in default costs due to the fact that one entity can drag the other into default. An analysis of this split in all our structures can be obtained from the authors upon request.

is favored over the unilateral rescue subsidiary and over the stand-alone banks. The branch structure is also inferior to the mutual rescue structure but it cannot be ranked relative to the other two structures. The ranking of values at low bailout probabilities is explained primarily by the relative sensitivities to default costs.

3. The disadvantage of the branch structure relative to the subsidiary structures, which is caused by the relatively high expected default costs in branch structures, declines as the probability of bailouts increases since bailouts eliminate default costs.

We turn now to a comparison between social values of the different structures, $SV_j, j = OWR, MR, BR, SA$. Proofs of propositions can be developed as in Appendix B. The social value is defined as the group value minus the value of the bailout put.

Proposition 5 *Suppose that default is dissipative ($\alpha > 0$) and bailout does not occur with certainty ($\pi \neq 1$). Under Assumption 2, the social value of the mutual organization is higher than both the unilateral and branch ones. If $\pi = 1$ or $\alpha = 0$ they are indifferent.*

Proposition 6 *Suppose that default is dissipative ($\alpha > 0$) and bailout does not occur with certainty ($\pi \neq 1$). Under Assumption 2, the social value of the subsidiary structure (both mutual and unilateral) is higher than the stand-alone structure. If $\pi = 1$ or $\alpha = 0$ the social values are the same.*

These propositions are illustrated in Figure 3, where once again the shaded area refers to the ambiguity of the value of the branch structure relative to the one way subsidiary- and the stand-alone structures. The difference between Propositions 4 and 6 for stand-alone banks relative to subsidiary structures can be explained by the relatively high sensitivity of the value of the bailout put for stand-alone banks, which are never rescued internally. Thus, the favored organizational structure from a social point of view at a particular bailout probability becomes the one that economizes the most on default costs. The subsidiary with mutual rescue is always favored relative to all other structures while the relative social value of the branch structure is ambiguous relative to the subsidiary with unilateral rescue and two stand-alone banks.

Insert here Figure 3.

5.2 Differentiating default costs and bailout probabilities

Up to this point we have assumed that default costs and bailout probabilities are the same for the two affiliates. However, both default costs and bailout

probabilities may differ across jurisdictions as well as across types of financial activities. We will return to this issue in the numerical analysis with endogenous leverage below. Here we will simply note that differences in these institutional characteristics facing a bank create a possible advantage for the branch structure, in particular, because the bank structured as a branch organization faces default costs and bailout as one entity. On the other hand, affiliates of subsidiary structures and stand-alone banks may face different parameters if the two affiliates are located in different legal jurisdictions or if their activities are different. For example, traditional commercial banks may be more likely to be bailed out than investment banks.

The potential advantage of the branch structure can be understood intuitively by reference to Figures 1 and 2. In these figures it is assumed that both affiliates face the same default cost and bailout parameters. Also, the deposits are equal in the different affiliates. If the probability of bailout were different for two affiliates of subsidiary structures with mutual rescue with and stand-alone structures, the group values for two affiliates together would be the average of the individual affiliate-values at different bailout probabilities. The branch structure on the other hand could choose the most favorable bailout probability for the two affiliates jointly. Thus, by choosing the most favorable jurisdiction for the whole bank with its two branches this structure may be able to obtain a higher group value than mutual rescue subsidiary- and stand-alone structures in different jurisdictions. The same reasoning holds when default costs differ across jurisdictions.

The subsidiary structure with one-way rescue also has a potential advantage relative to the mutual rescue and the stand-alone structures if the bank has flexibility with respect to jurisdiction of the affiliate that provides one-way insurance for the other. It is clearly advantageous to place the rescuing affiliate in the high probability of bailout and low default cost jurisdiction.

With reference to Figure 3 it can be observed that maximizing group value of the branch structure by incorporating where the bailout probability is not necessarily optimal from a social point of view since a part of the group value benefit for the branch structure is due to the bailout put.

We return to these issues in Section 8 taking into account endogenous leverage and financial economies of scale as well.

5.3 Endogenous leverage.

In this sub-Section we derive a few analytical results with respect to leverage in different organizational structures in the absence of scale effects while the deposit interest rate remains endogenous and parameters across affiliates remain the same. Leverage is of concern since it affects default risk and, thereby, expected default costs and the expected value of bailouts. A more complete analysis follows in Section 6.

A first observation is that if $k = 0$ and there is perfect correlation between returns on assets, the subsidiary structure with mutual rescue, the branch structure and two stand alone banks become identical. In this case there are no ben-

efits of diversification with respect to default costs. As a result internal rescue policies cannot affect group values for these organizational structures if there is perfect correlation and bailout probabilities and default costs are the same. A second observation is that the subsidiary organization with one-way internal insurance can increase value by moving deposits from the non-insured affiliate to the internally insured affiliate. In other words, the subsidiary-structure with unilateral insurance can benefit from "debt diversity."

The benefit from debt diversity for the subsidiary structure with unilateral rescue does not require perfect correlation. For the sake of simplicity, let the loans of the two affiliates be independent. The following proposition, which is proven in Appendix C, holds:

Proposition 7 *Let $k = 0, \alpha > 0, 0 < \pi < 1$. Two banks with independent loans, which are optimally levered and decide to set up a unilateral insurance, create debt diversity at the margin. The guaranteed company is more levered than the guarantor and its overall value is greater than the one of two stand-alone banks.*

As concerns mutual conditional guarantees and branches, Appendix C says that, if default costs are small enough in comparison with bailout probability, optimal leverage is positive. Appendix C also provides conditions under which leverage is positive and greater for mutually insured subsidiary structures than for branch structures.

Appendix C observes that, as long as there are no returns to scale, and loans are equally distributed in the two affiliates, there is no incentive to have different face values of debt in the two affiliates of a subsidiary structure. It is not hard to imagine that, whenever we add financial returns to scale ($k > 0$), there will be incentives to create debt diversity in the cases of mutual rescue and branch organizations, even when their distribution before the economies of scale is the same. We will observe such debt diversity in the numerical analysis below.

6 Numerical analysis with endogenous leverage including returns to scale

In order to fully analyze the role of endogenous leverage and to incorporate the possibility of financial returns to scale we apply a numerical method for finding the face value of debt that maximizes the value to be distributed to the stakeholders of the bank. In sub-Section 6.1 we specify a distribution for returns on loans, G . A number of numerical cases are specified in sub-Section 6.2; each case represents a set of values for returns to scale, bailout probability, default costs and correlation between asset returns. The optimized market values for debt and equity, the value of the potential bailouts, expected default costs and consequently the private and social values of the bank will be calculated. Then in sub-Section 6.3 we compare rankings of the private and social values of

organizational structures with endogenous leverage with rankings of the same structures with exogenous leverage.

The number of numerical cases we analyze in this Section are restricted by the very computer-time consuming process of optimization with a specific set of parameters.

6.1 The model

We assume that log returns on loans Y , defined by $L(T) = L_0 \exp(Y)$, are Gaussian with mean $\mu = (r - \sigma^2/2)T$ and variance $\sigma^2 T$. In the case of one stand alone bank default costs are:

$$\alpha(1 - \pi) \exp(-rT) \mathbb{E} [L(T) \mathbf{1}_{\{L(T) < F\}}] = \alpha(1 - \pi) L_0 N(-d_1),$$

where N is the distribution function of the standard normal, and, as in the Black-Scholes formula,

$$d_1 = \frac{\ln(L_0/F) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}.$$

The bailout put is a plain vanilla Black-Scholes put on the asset value, with strike equal to F :

$$\pi [-L_0 N(-d_1) + F \exp(-rT) N(-d_2)],$$

where $d_2 = d_1 - \sigma\sqrt{T}$. The total value of the stand alone debt is

$$\begin{aligned} D_0 &= F \exp(-rT) - (1 - \pi) [-L_0 N(-d_1) + F \exp(-rT) N(-d_2)] - \alpha(1 - \pi) L_0 N(-d_1) \\ &= F \exp(-rT) [1 - (1 - \pi) N(-d_2)] + (1 - \pi)(1 - \alpha) L_0 N(-d_1), \end{aligned} \quad (16)$$

while the equity value at time 0, E_0 , is the plain vanilla Black-Scholes call on the asset value, namely

$$E_0 = L_0 N(d_1) - F \exp(-rT) N(d_2) \quad (17)$$

The total value of the bank is

$$\begin{aligned} D_0 + E_0 &= \\ &= F \exp(-rT) \pi N(-d_2) + L_0 [1 - \pi N(-d_1) - \alpha(1 - \pi) N(-d_1)]. \end{aligned}$$

So, in the stand-alone case we have a closed formula for the objective of the bank's maximization. With more than one bank, the maximization problems to be solved are obtained by introducing Gaussian returns in expression (12) for the unilateral rescue case for subsidiaries, expression (10) for the mutual rescue case for subsidiaries and expression (15) for the branch case. In each case the initial value of the loans from each entity is the same, $L_0 = 100$. The time horizon is set to five years, $T = 5$, and the instantaneous riskless rate is conventionally set to 5%, the percentage volatility is $\sigma = 20\%$ per year. Up to this point, the parameters resemble the calibration of Leland (2007), which was calibrated to non-financial BBB firms.

6.2 Numerical analysis of financial synergies

In order to compare financial synergies in different organizational structures when leverage is optimally chosen, we vary returns to scale, k , as well as default costs, α , and probability of bailout. To begin with these parameters are the same across affiliates in order to highlight how the different factors affect value and leverage. In all cases the correlation between asset returns in the two affiliates is 0.2 but as long as the correlation is less than one the qualitative results are robust.

In Tables 1-3 the banks maximize the private group value. In the tables we present the (private) group values (GV), the social values (SV), default costs, the values of the bailout puts for the whole bank, and leverage for each affiliate for each organization. Within the tables, each panel represents a set of numerical values for the parameters.

Insert here Tables 1-3

Comparing private group values

In Table 1 we begin by comparing the organizations at relatively low values of the parameters describing financial returns to scale ($k = 5\%$) and the probability of bailout ($\pi = 5\%$) in both affiliates. The default cost is 20 % in Panel 1 and increased to 50 % in Panel 2. Thereafter we will vary the probability of bailout in Table 2. In Panel 1 we see first that subsidiaries with unilateral as well as mutual rescue choose to shift most of the deposits to one subsidiary. "Debt diversity" exists even for small $k > 0$ in both the mutual rescue- and the one-way subsidiary structures. Financial returns to scale can be maximized by concentrating the leverage and, thereby, expected default costs, to one subsidiary. The stand-alone banks are by definition not able to take advantage of debt diversity. Debt diversity in the one-way and mutual rescue structures stands in contrast to the symmetry in the stand-alone structure and the near-symmetry in the branch structure as illustrated in Figure 4. The inability to concentrate default costs to one affiliate implies that the branch structure has less ability than subsidiaries to take advantage of returns to scale through debt diversity.

Insert here Figure 4

The group value (GV) is the highest in the mutual subsidiary structure because the group value of bailout puts is the highest. The differences between the different structures are small because the parameter values for default costs and bailout probability are low. The situation is illustrated in the extreme-left group of bars in Figure 5, which represents the private value of different organizations when $k = 5\%$, $\pi = 5\%$, $\alpha = 20\%$.

Insert here Figure 5

In Table 1 Panel 2 we push the default costs from 20% to 50%. The results are similar to those we obtained in the previous case. The group values

are slightly smaller than in the previous case. The mutual rescue structure is marginally "better" privately than the other structures. The asymmetry of leverage in the subsidiary cases remains with higher default costs but debt diversity declines. The higher exposure of the branch to high bankruptcy costs actually induces it to keep leverage relatively low. This structure cannot exploit the combination of returns to scale and limited liability. The value of the stand-alone structure remains relatively low because it is unable to take advantage of financial synergies including debt diversity. The situation is illustrated in the second group of bars from the left in Figure 5.

In Table 2 we raise the economies of scale parameter from 5 to 15 percent while keeping the default costs at 50%. Comparing, first, Panel 1 in Table 2 with Panel 2 in Table 1 only the parameter for financial returns to scale (k) is increasing. The advantage of the two subsidiary cases relative to the branch and stand-alone cases increases because of the ability of the subsidiaries to take advantage of the higher returns to scale through debt diversity without increasing default costs as much as in the branch case. This confirms that limited liability for each subsidiary enables the subsidiary structures to take advantage of the returns to scale to a greater extent than the branch, as well as the stand-alone structures. The latter remains inferior from a private point of view. (See Figure 5, third group of bars from the left.)

In Table 2, Panels 1-3, we raise the probability of bailout from a low of 5% in Panel 1 to 10% in Panel 2 and to 40% in Panel 3 while parameters for returns to scale and default costs remain the same in the three panels. The increase of the bailout probability to 10 percent does not change the picture dramatically although leverage increases in the subsidiary and the branch cases. The asymmetry of leverage between subsidiaries remains strong. The ranking of the group values remains the same (see last group of bars in Figure 5). The mutual subsidiary, in particular, is able to extract value from the bailout put while the branch with its greater exposure to bankruptcy does not.

The bailout probability is raised to 40 percent in Panel 3. Leverage in all the cases increases strongly. The numerical analysis captures the unique interior maximum. We know that with such a high default probability the interior maximum is dominated by the value when leverage goes to infinity. Indeed, when bailout is very likely there is an incentive for all possible structures to increase leverage as much as possible, boosting value. However, boosting leverage indefinitely cannot be considered a solution for the maximization problem. Here we report the maximum only. The result for each structure is driven by the exploitation of the relatively high bailout probability. Scale economies become nearly irrelevant from the point of view of debt diversity in this case. It can be seen that the group values are quite close to each other. The stand-alone banks achieve a group value comparable to the other structures by exploiting the bailout to a greater extent than the other structures, which use some degree of coinsurance. The mutual is still the optimal structure. However, the results illustrate that the high bailout probability make the different structures similar in private value when leverage is endogenous. This result holds even if $k = 0$ while other parameters remain the same as in table 2, Panel 3. They are illus-

trated in Table 3. Note that debt diversity disappears completely in this case except in the bank with unilateral rescue for reasons discussed in Section 5.1. Qualitatively the results are very similar to those shown in the previous panel where value differences are explained by the exploitation of the high bailout probability.

Comparing social values

We turn now to the social values associated with the different organizational structures. The concept of social value is defined in a narrow sense as above and, therefore, equal to the unlevered value less default costs. In the next Section we discuss additional aspects of social value.¹⁷

Referring back to Table 1 and Table 2, Panels 1 and 2 for moderate values of the bailout probability it can be seen that social values of the one way rescue structures (SV_{OWR}) are consistently the highest, while the private group values of the mutual rescue structures were consistently the highest. Unreported face values of debt show that the mutual rescue structure consistently pushes leverage higher than the one way rescue structure and, as a result, the value of default costs becomes higher for the mutual rescue structure. Higher default costs decrease value, so that the mutually guaranteed bank turns out to have a smaller social value than the one-way guaranteed bank. The branch structure never obtains the highest social value but it reaches a higher value than the mutual rescue structure in Table 1 as well as Table 2, Panel 2. This confirms that mutually guaranteed companies suffer in comparison with other structures, if we do not consider the bailout put value: their exploitation of leverage has a detrimental effect on expected default costs, which, being a deadweight loss, matter socially. It is not counterbalanced by the benefits of bailout. The exception, when leverage is optimized, occurs when the scale economies are relatively high (15%) and the bailout probability is low (5%) in Table 2, Panel 1.

Insert here Figures 6a and 6b

Figures 6a and 6b show the difference between private and social group values in the different cases described above. As we know, this difference is simply the value of the group default puts in the previous tables. The figures show that the differences between social and private values are relatively large for the mutual rescue case for the low bailout probabilities at 5 and 10 percent. The branch organization has the smallest difference because of its relatively greater sensitivity to default costs. The difference between social and private values increases dramatically for all organizations when the probability of bailout is increased to 40 percent. In relative terms the differences between the organizations become small. The stand-alone structure has the lowest private as well as social values in all cases due to its inability to adjust leverage to take advantage of financial synergies.

¹⁷We do not present results for choice of leverage for social value maximization. They can be obtained from the authors upon request. The analysis of the difference between optimized private and social values leads to similar results.

These results for relative social values in comparison with private values indicate that the case for constraining the leverage of the mutual rescue structure is particularly strong .

6.3 Comparing endogenous and constrained leverage

We comment first on the robustness of the analytical propositions in Section 4 with exogenous leverage in light of the analysis with endogenous leverage in the previous sub-Section. As a general proposition we can say that the ranking of the private group values of different structures as shown in Figures 1 and 2 remain robust in terms of the superiority of the subsidiary structure with mutual rescue as long as the bailout probability remains moderate. Under the same condition, the result that the subsidiary structure with one-way rescue was privately more valuable than the stand-alone structure is robust. We could not rank the branch structure relative to the one way rescue structure and the stand alone structure with exogenous leverage but with endogenous leverage the one-way rescue structure is more valuable privately than the branch structure, which is more valuable than the stand-alone structure. When the bailout probability became very high with exogenous leverage the ranking of the private values were reversed in Figures 1 and 2 but with endogenous leverage all structures exploited the high bailout probability. Nevertheless, the mutual insurance structure remained the most valuable privately.

Endogenizing leverage has a greater impact on the ranking of the social values of the different structures as expressed in Propositions 5 and 6 and illustrated in Figure 3. Proposition 5 stated that the mutual rescue organization is superior to branch and unilateral rescue organizations while Proposition 6 stated that the stand-alone structure provides less social value if default costs are positive. The inferiority of the stand-alone structure holds with endogenous leverage as well but the relative superiority of the mutual rescue structure does not, unless the bailout probability is high. As noted, the social value of the privately optimized mutual rescue structure is inferior to the privately optimized one way rescue structure for low and moderate values of the bailout probability, and inferior to the branch structure as well if scale economies are not too strong.

We have so far not found one case when the branch structure has the highest private value under the assumption that parameters for default costs and bailout probabilities are the same across affiliates. The relative advantage of the subsidiary structures is magnified by scale economies ($k > 0$). Our final numerical exercise asks how scale economies in combination with constrained leverage affect the ranking when we take into account that the leverage constraint for the branch structure applies on the two affiliates jointly.

We derive numerical solutions for optimized private values of each organization using the parameters in Table 3, Panel 2. T. The return to scale parameter is high ($k=15\%$), default costs are 50% and the probability of bailouts is moderate at 0.10. First, we impose capital requirements on the subsidiaries and the stand-alone banks equal to the optimized unconstrained face values of debt for each of the branch bank affiliates. Thus, the branch bank optimizes its total

face value of debt for the whole bank while the other banks are constrained for each legal entity. The face values of deposits for the branch bank's affiliates are 49 and 73. These face values are imposed on the subsidiary banks and the stand alone banks as well.

Table 4 shows that the subsidiary structure with mutual rescue still reaches the highest private group value. However, it is followed by the branch case, which now obtains a higher private value than the subsidiary with unilateral rescue.

Insert Table 4 here

The ranking of the four organizations in terms of social value is the same as the ranking in terms of private value but the differences between the two subsidiary organizations and the branch organization become smaller than in the unconstrained case. The social value of two stand-alone banks falls the most relative to the private value as a result of the high value of the bailout put for the stand-alone organization.

As a second numerical exercise we assumed that the total constrained face value of deposits of the branch structure (122), which it could allocate as desired between the affiliates, was applied in equal amounts (61) on the two subsidiary affiliated and the two stand-alone banks would be constrained to 61 for each affiliate. We do not show the results for this case but the group value of the branch bank gained relative to the other organizations under this capital requirement constraint. In this specific case the group value of the branch bank is very close to the value of the subsidiary bank with mutual rescue and remains higher than the subsidiary structure with one-way rescue.

As a general conclusion we can state that leverage constraints offset the advantage of the subsidiary structures relative to the branch structure if there are returns to scale. Even without returns to scale the relatively strong incentives of the subsidiary structures to create leverage when there is a probability of bailouts are negated. The relatively large difference between private and social values of subsidiary structures declines with leverage constraints.

7 Incorporating systemic risk; a preliminary view

We have so far assumed that the social value of a bank is the private group value minus the value of the bailout put. This social valuation does not incorporate the possibility of contagion in case there is a default and it does not take into account that bailouts may have its own social costs beyond the pure fiscal costs. A more complete formulation for social value would include these considerations. Contagion implies that the social costs of default are higher than the private default costs and social costs of bailout would imply that a fraction of the value of the bailout put would remain a social cost. The fear of contagion is actually the main reason why there is an expectation of bailouts.

As a preliminary analysis of systemic risk of a bank's default we assume that the social cost of contagion from the default is proportional to the expected

discounted loss to the bank’s creditors. This loss is what may threaten other banks in the financial system. Thus, we evaluate the expected discounted loss that leverage induces. This expected loss is measured by the difference between the value of the bank’s debt in the absence of default possibilities ($F \exp(-rT)$) and its actual no-arbitrage value, D_0 . We present this difference for different bank organizations as a basis for evaluation of their contributions to systemic risk.¹⁸

Table 5 shows the values of expected discounted losses for different organizations under the same assumptions about parameters as in Tables 1-4. In the subsidiary cases there is one column for each affiliate, as well as a column for the whole group, which can be compared with the columns for the branch organization and the stand-alone. There is only one column for the branch case since it can only default as one entity. To save space we include only one column for the stand-alone banks although two stand-alones can default separately. If one defaults the expected loss is half the number presented in the last column (2 total SA).

The comparison of the systemic risk associated with different organizational structures in Table 5 is to some degree to the disadvantage of subsidiary and stand-alone structures. If the correlation between the returns is less than one, the sum of the expected losses of separately incorporated affiliates is not exactly comparable to the expected losses within a branch structure. Nevertheless, the table provides a preliminary comparison of systemic risk associated with different structures.

Insert here Table 5

We can observe that the expected discounted loss for all organizations declines as default costs increase and increases as the probability of bailout increases. This pattern is consistent with the pattern for values of default puts in Tables 1-3. The increases in the expected discounted losses are dramatic when the bailout probability increases from 10 percent to 40 percent. This is the reason why, in Figure 7, where we present graphically the group expected losses we exclude the cases of high bailout probability.

Insert here Figure 7

Table 5 shows that the expected discounted loss is always the lowest for the branch bank in a comparison of whole groups. This result is consistent with the observation that leverage tends to be relatively low in branch banks (Tables 1 and 2). However, the difference relative to (total) subsidiary structures becomes smaller when the bailout probability increases. As noted, this result should be modified by the diversification of systemic risk within subsidiary and stand-alone banks.

Debt diversity affects the comparison between subsidiary and branch structures as well. In Table 5 the subsidiary organizations concentrate their debt

¹⁸Size is obviously one aspect of systemic risk. As stressed by Acharya (2009), correlation between bank returns is a source of systemic risk too.

in one affiliate as a result of financial economies of scale in all cases when the bailout probability is 5 and 10 percent. Figure 8 illustrates this phenomenon for some relevant parametrization. Binding capital requirements offset the incentives of subsidiary banks to diversify debt and, thereby, they contribute to diversification of systemic risk in subsidiary organization.

Insert here Figure 8

Finally, from a policy point of view capital requirements may be set with the objective of reducing risk of contagion from bank defaults. An analysis of optimal capital requirements is beyond the scope of this paper, however.

8 “Institutional arbitrage”

We noted in Sections 4.2 that the branch organization with constrained leverage could become superior in terms of group value if it were able to choose institutional characteristics such as default costs and bailout probability for the affiliates jointly. Subsidiary structures and stand-alone banks cannot take advantage of these differences if leverage is constrained. Similarly, if two jurisdictions have different capital requirements in place, the bank with branch structure can choose the country with low capital requirements as its legal jurisdiction and, thereby, obtain the low capital requirement for the whole bank.

Institutional arbitrage can be said to occur if the bank chooses organizational structure to maximize its value taking into account bailout probability, default costs and leverage constraints. A complete analysis of such arbitrage lies beyond the scope of this paper but, in addition to the case discussed in Section 4.2, we consider here informally the case when leverage is endogenous and when there are scale effects ($k > 0$). We do not consider differences in corporate tax rates, even though such differences could be captured as differences in scale effects, k .

Relaxing the constraint that leverage must be the same in the different affiliates in the absence of economies of scale ($k = 0$) implies that there are opportunities for institutional arbitrage for subsidiary structures as well. While the branch structure can choose leverage for the whole organization based on the most favorable bailout probabilities in combination with default costs, the financially integrated subsidiary structure can choose to concentrate debt in the affiliate facing the most favorable combination of bailout probability and default costs. Thus, as in Luciano and Nicodano’s (2014) analysis of one-way rescue in corporate structures debt diversity may arise in subsidiary banks as a result of differences in these institutional parameters even in the absence of economies of scale. If we also allow for scale effects there is an additional advantage for subsidiary structures since they have relatively strong incentives to diversify debt to exploit scale economies. Clearly, they would prefer to borrow where default costs are low and bailout probabilities are high. As noted, these advantages of subsidiary organizations are offset by capital requirements.

We cannot make a general statement about subsidiary banks being superior to branch banks when there are differences across jurisdictions or activities in

bailout probabilities and default costs, as well as economies of scale and capital regulation. The optimal organization depends on the specific combination of these factors.

We conclude by presenting one set of numerical solutions for leverage constrained subsidiary and branch structures when the branch can choose between high default costs combined with high bailout probability ($\alpha/\pi = .50/.20$) and low bailout probability combined with low default costs ($\alpha/\pi = .20/.05$). We choose these combinations since a high bailout probability is likely to be associated with high default costs but other combinations are possible. There are low economies of scale ($k = 0.05$) and capital requirement imposed on each legal entity is 50 for subsidiary banks, and 100 in total for the branch bank.

Insert here Table 6

Table 6 shows the chosen leverage in each affiliate. Only the individual affiliate of the branch bank is not constrained to $F = 50$. The two right hand columns show the branch structure subject to either high or low parameter values. Clearly the differences between the group values of different organizational structures are small; possibly because high bailout probabilities are value enhancing while high default costs have the opposite effect. The table shows that the branch structure with relatively low default costs and relatively low bailout probability has the highest value with the mutual rescue structure close. This result depends very much on the specific parameters but the point is that institutional arbitrage in the form of organizational choice is likely to occur in response to institutional characteristics in different jurisdictions.

9 Summary, policy implications and further research

We have analyzed how financial synergies within a bank with two affiliates depend on internal insurance arrangements within subsidiary and branch structures. The different structures were compared in terms of private value, social value, leverage and potential contribution to systemic risk. Two financially independent, stand-alone banks were considered, as well. There are no positive or negative financial synergies between two stand-alone banks even if they are subsidiaries of the same parent.

Subsidiary structures in our terminology have a degree of financial interdependence and characterized by either mutual or one-way insurance against default risk. The insurance in subsidiary structures is limited because it is conditional on the survival of the insuring affiliate. In branch structures, on the other hand, the mutual insurance is limited only by the survival of both affiliates as one corporate structure.

The different arrangements for internal insurance of the two affiliates influence value through differences in ability to exploit financial synergies. The

sources of these synergies under limited liability are reduced default costs, exploitation of a probability of state bailouts, and potential economies of scale in bank debt. There are by definition no financial synergies between two stand-alone banks but as a result of limited liability they can exploit a probability of state bailouts as well. There are conditions under which this structure achieves higher private value than the subsidiary and branch structures with some form of internal insurance.

Most of the analysis was conducted under the assumption that the parameters describing default costs, the probability of state bailouts, economies of scale and size were the same for the two affiliates. Thereby, we gain insight in the role different factors play in the creation of financial synergies and leverage but we also show how the different structures can take advantage of differences across affiliates in these parameters.

One general conclusion is that each organizational structure is privately preferred under some circumstances. From a social point of view the stand-alone banks are never preferred, however, since they cannot diversify with respect to default costs. Privately the stand-alone banks can be preferred when leverage is constrained and the bailout probability is very high.

The branch structure is also preferred only under specific circumstances. From a private valuation point of view the branch structure is preferred only in the presence of capital requirements if default costs and bailout probabilities are different across affiliates. This advantage is explained by the greater ability to take advantage of these differences by conducting "institutional arbitrage" through incorporation in a jurisdiction with relatively favorable bailout policies or default costs. Similarly, the multi-activity bank can increase its probability of bailout by including activities with relatively high likelihood of bailouts within a branch organization. From a social point of view the institutional arbitrage enhances value only if it occurs in response to differences in default costs while it is costly when it occurs in response to differences in bailout probabilities.

Apart from under the circumstances described the subsidiary structures with internal insurance arrangements are privately and socially preferred from the point of view of financial synergies. The general thrust of the comparisons among organizational structures with both endogenous and exogenous leverage is that the subsidiary structure with mutual insurance generates the highest private value for moderate levels of bailout probabilities while the one-way insurance arrangements obtain higher private value when bailout probabilities are relatively high and leverage constrained. From a social point of view the preference is different since much of the private value advantage is created by exploitation of expected bailouts. In particular, with endogenous leverage the one-way insurance structure is socially preferred for moderate levels of bailout probabilities while the mutual insurance structure is generally socially preferred when leverage is constrained. These contradictory results from private and social points of view are explained by the fact that expected bailouts have powerful effects on private incentives.

Expanding the concept of social value to take into account systemic risk generated by a bank's default strengthen the above contradiction between private

and social values for subsidiary structures with mutual insurance when leverage is endogenous. Under this assumption we found that the subsidiary structure with one-way rescue was superior from a social point of view in several cases followed by the branch structure ahead of the subsidiary structure with mutual insurance.

The general effect of leverage constraints in the form of capital requirements is that they reduce the private value advantages of the subsidiary structures since these constraints reduce their ability to take advantage of state bailouts as well as of economies of scale. Nevertheless, the subsidiary structure with mutual rescue always has the highest social value when capital requirements and parameters for default costs and bailouts are the same for all entities. For very high bailout probabilities the private values of stand-alone banks were the highest followed by the subsidiary structure with one-way insurance.

Generalizing further based on financial synergies alone in the presence of expected bailouts we expect to see the banking industry dominated by subsidiary structures with mutual insurance in the absence of leverage constraints. With leverage constraints in the form of capital requirements we expect to see more subsidiary structures with one way insurance at relatively high bailout probabilities and even stand-alone banks at very high bailout probabilities. Branch structures would also be observed as a result of institutional arbitrage when there are differences in bailout expectations and default costs.

Whether leverage constraints are binding or not the socially desirable organizational structure is often different from the privately optimizing structure and this difference is likely to be greater the higher is the bailout probability. This observation can be a rationale for regulatory intervention in organizational structures.

We noted in the introduction that there is ongoing reform work in the US, UK and the EU as a whole with respect to the organizational structure of international banks as well as financial conglomerates. The thrust of the reforms is to strengthen ring-fencing or the separation of commercial banking, in particular, from an operational as well as financial point of view as noted in Alexander (2014). We cannot comment on the costs and benefits of operational separation based on the analysis here but it seems that one rationale for operational separation is to reduce default costs of strongly integrated structures.¹⁹

Our analysis of financial synergies indicates that the direction of ring-fencing reforms should take opposite directions depending on the capital requirement regime. If such requirements are relatively uniform and binding the desirable organizational structure from a social point of view is the subsidiary structure with mutual insurance provided that subsidiaries can be separated operationally. Thus operational ring-fencing in combination with no ring-fencing in terms of internal insurance arrangements would seem to be optimal for financial conglomerates as well as for international banks.

¹⁹Carmassi and Herring (2013) notes that high default costs in the Lehman Brothers insolvency in 2008 was caused by the close integration of subsidiaries, which enabled Lehman to book assets in ways that obscured the true values of the different subsidiaries.

If, on the other hand, capital requirements are not binding, one-way insurance arrangements would seem to be superior from a social point of view in combination with operational separation that allows default costs to decline. In this case, the one way insurance should be directed at activities and jurisdictions with the highest default costs. If traditional commercial banking has the highest default costs it would make sense to ring-fence the commercial bank in the sense that it cannot insure other activities while other activities are allowed to insure the commercial bank.

With respect to international branch structures they can be efficient from a social point of view if there are substantial differences in default costs since institutional competition could encourage the development of less costly insolvency regimes. On the other hand, if international branch structures seek to incorporate where bailout probabilities are relatively large, financial separation is called for.

Further research

The analysis in this paper can be extended in several directions. First, as noted, leverage constraints can be optimized from a social point of view including the risk of contagion. Such an analysis requires specification of costs of contagion as well as costs of bailouts, which here have been considered a pure transfer. Cost of violating capital requirements should also be made explicit.

Second, once default costs, bailout probabilities, size and, possibly, tax rates differ across affiliates, optimal rescue policies (coinsurance) may differ from the ones considered above. Optimal rescue policies and optimal capital requirements are likely to depend on each other.

Third, mutual and one-way insurance within subsidiary structures may not be a commitment but rather probabilistic. If so, capital requirements and the probability of rescue would be interdependent within subsidiary structures. Allowing for probabilistic internal insurance implies that a game situation may arise between the regulator and the banks.

A fourth research agenda would be to compare theoretical predictions with empirical analysis of banks' choice of organizational structures and insurance arrangements across both countries and activities. This type of analysis would have to consider operational synergies in different structures as well.

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Appendixes

A Values to stakeholders, branch case

This Appendix reports the values of debt and equity of the branch and its home bank. Payoffs to equity and bond holders resemble the subsidiary ones, reported in Section 4.1, whenever rescue or bailout occurs. They depart from them when insolvency occurs, because insolvency is always joint, and default costs are paid on both the assets of the bank originally in default and the assets transferred from its home or affiliate. We have the following expression for the branch debt:

$$\begin{aligned}
 D_{0b} = & \underbrace{+\exp(-rT) [F_b - \mathbb{E} \max(0, F_b - N_b(T))]}_{\text{value without bailout and rescue}} + \\
 & \underbrace{+\exp(-rT) \mathbb{E} \left\{ \min(N_h(T) - F_h, F_b - N_b(T)) \mathbf{1}_{\{R_b\}} \right\}}_{\text{rescue received}} + \\
 & \underbrace{+\pi \exp(-rT) \mathbb{E} \left\{ \max(F_b - N_b(T), 0) \mathbf{1}_{\{A'\}} \right\}}_{\text{government bailout}} + \\
 & \underbrace{- \exp(-rT) (1 - \pi) \alpha \mathbb{E} \left[[N_b(T) + \max(0, N_h(T) - F_h)] \mathbf{1}_{\{A'\}} \right]}_{\text{default costs}},
 \end{aligned}$$

the home-bank equity

$$\begin{aligned}
 E_{0h} = & \exp(-rT) \mathbb{E} \max[N_h(T) - F_h, 0] \\
 & - \exp(-rT) \mathbb{E} \left\{ \min(N_h(T) - F_h, F_b - N_b(T)) \mathbf{1}_{\{R_b\}} \right\},
 \end{aligned}$$

the home-bank debt

$$\begin{aligned}
 D_{0h} = & \underbrace{+\exp(-rT) [F_h - \mathbb{E} \max(0, F_h - N_h(T))]}_{\text{value without bailout and rescue}} + \\
 & \underbrace{+\exp(-rT) \mathbb{E} \left\{ \min(N_b(T) - F_b, F_h - N_h(T)) \mathbf{1}_{\{R'_b\}} \right\}}_{\text{rescue received}} + \\
 & \underbrace{+\pi \exp(-rT) \mathbb{E} \left\{ \max(F_h - N_h(T), 0) \mathbf{1}_{\{A\}} \right\}}_{\text{government bailout}} + \\
 & \underbrace{- \exp(-rT) (1 - \pi) \alpha \mathbb{E} \left[[N_h(T) + \max(0, N_b(T) - F_b)] \mathbf{1}_{\{A\}} \right]}_{\text{default costs}}.
 \end{aligned}$$

the branch equity

$$\begin{aligned}
 E_{0b} = & \exp(-rT) \mathbb{E} \max[N_b(T) - F_b, 0] \\
 & - \exp(-rT) \mathbb{E} \left\{ \min(N_b(T) - F_b, F_h - N_h(T)) \mathbf{1}_{\{R'_b\}} \right\},
 \end{aligned}$$

B Comparing organizations' values with exogenous debt

This Appendix proves first Propositions 1 and 2. It then proves proposition 4. There are no returns to scale ($k = 0$), so that $M_i = L_i, i = s, h$, both at time 0 and at time T , in all states of the world.

We start from Proposition 1.

Proof. Let us compare the values of the unilateral subsidiary arrangement and the mutual one, when the home bank and the affiliate have the same and positive level of deposits, cash flows of the affiliates are equally distributed and $k = 0$. They are respectively

$$\begin{aligned}
 & GV_{OWR} = M_{h0} \\
 & + \underbrace{\pi \exp(-rT) \mathbb{E} \max(0, F_h - M_h(T))}_{\text{government bailout home}} - \underbrace{\alpha(1 - \pi) \exp(-rT) \mathbb{E} [M_h(T) \mathbf{1}_{\{M_h(T) < F_h\}}]}_{\text{default cost home}} \\
 & \quad + M_{s0} + \\
 & \quad + \underbrace{\pi \exp(-rT) \mathbb{E} \{ \max(F_s - M_s(T), 0) \mathbf{1}_{\{A'\}} \}}_{\text{government bailout subsidiary}} \\
 & \quad - \underbrace{(1 - \pi) \alpha \exp(-rT) \mathbb{E} [M_s(T) \mathbf{1}_{\{A'\}}]}_{\text{default cost subsidiary}},
 \end{aligned} \tag{18}$$

and

$$\begin{aligned}
 & GV_{MR} = M_{h0} \\
 & + \underbrace{\pi \exp(-rT) \mathbb{E} \max \{ (0, F_h - M_h(T)) \mathbf{1}_{\{A\}} \}}_{\text{government bailout home w/ mutual support}} - \underbrace{\alpha(1 - \pi) \exp(-rT) \mathbb{E} [M_h(T) \mathbf{1}_{\{A\}}]}_{\text{default cost home w/ mutual support}} \\
 & \quad + M_{s0} + \\
 & \quad + \underbrace{\pi \exp(-rT) \mathbb{E} \{ \max(F_s - M_s(T), 0) \mathbf{1}_{\{A'\}} \}}_{\text{government bailout subsidiary}} \\
 & \quad - \underbrace{(1 - \pi) \alpha \exp(-rT) \mathbb{E} [M_s(T) \mathbf{1}_{\{A'\}}]}_{\text{default cost subsidiary}}.
 \end{aligned} \tag{19}$$

The former is smaller than the second if and only if the bailout put net of default costs in the home bank - with no support from the affiliate - is smaller than when the subsidiary intervenes, i.e.

$$\begin{aligned}
 & \underbrace{\pi \exp(-rT) \mathbb{E} \max(0, F_h - M_h(T))}_{\text{government bailout home}} - \underbrace{\alpha(1 - \pi) \exp(-rT) \mathbb{E} [M_h(T) \mathbf{1}_{\{M_h(T) < F_h\}}]}_{\text{default cost home}} < \\
 & \underbrace{\pi \exp(-rT) \mathbb{E} \max \{ (0, F_h - M_h(T)) \mathbf{1}_{\{A\}} \}}_{\text{government bailout home w/ mutual support}} - \underbrace{\alpha(1 - \pi) \exp(-rT) \mathbb{E} [M_h(T) \mathbf{1}_{\{A\}}]}_{\text{default cost home w/ mutual support}}.
 \end{aligned} \tag{20}$$

The event A is not empty, under hp 1. Whenever $0 < \pi < 1$, this makes the expectation which represents bailout on the left hand side greater than on the right hand side; the same for default costs (in absolute value), if $\alpha > 0, 0 \leq \pi < 1$. So, the difference on the left hand side can be greater, equal or smaller than the one in the right hand side. However, the overall inequality in (20) holds, for any positive value of α , if $\pi = 0$, while the opposite inequality holds for $\pi = 1$, any $\alpha > 0$, or $\alpha = 0$. The two sides are equal when $\alpha = \pi = 0$ and in a neighbourhood of it. Since the direction of the inequality (20) changes when π goes from 0 to 1 and α stays positive, and both its left and right-hand side are continuous in π , there is a positive bailout probability, which we call π^* , above which mutual guarantees become worse than unilateral. This concludes the proof. ■

Consider now Proposition 2.

Proof. Since $k = 0$ and cash flows from loans L_s and L_b are the same, $M_h = N_h$, $M_s = N_b$. Call the common value of the latter M . Notice also that the bailout events coincide for the two organizations, i.e. the sets A, A' coincide for the two organizations. Let us compare the values of the mutual subsidiary arrangement and the branch one, which are respectively

$$\begin{aligned}
& \underbrace{+\pi \exp(-rT) \mathbb{E} \max [(0, F_h - M_h(T)) \mathbf{1}_{\{A\}}]}_{\text{government bailout home}} \underbrace{- \alpha(1 - \pi) \exp(-rT) \mathbb{E} [M_h(T) \mathbf{1}_{\{A\}}]}_{\text{default cost home}} \\
& \quad + M_0 + \\
& \quad \underbrace{+\pi \exp(-rT) \mathbb{E} [\max(F - M(T), 0) \mathbf{1}_{\{A'\}}]}_{\text{government bailout subsidiary}} \\
& \quad \underbrace{- (1 - \pi) \alpha \exp(-rT) \mathbb{E} [M(T) \mathbf{1}_{\{A'\}}]}_{\text{default cost subsidiary}},
\end{aligned}$$

and

$$\begin{aligned}
& GV_{BR} = M_{h0} + \underbrace{\pi \exp(-rT) \mathbb{E} \max [(0, F_h - M_h(T)) \mathbf{1}_{\{A'\}}]}_{\text{government bailout home}} + \\
& \underbrace{- \alpha(1 - \pi) \exp(-rT) \mathbb{E} [[M_h(T) + \max(0, M(T) - F)] \mathbf{1}_{\{A\}}]}_{\text{default cost home}} + \\
& \quad + M_0 + \\
& \quad \underbrace{+\pi \exp(-rT) \mathbb{E} [\max(F - M(T), 0) \mathbf{1}_{\{A'\}}]}_{\text{government bailout branch}} + \\
& \underbrace{- (1 - \pi) \alpha \exp(-rT) \mathbb{E} [[M(T) + \max(0, M_h(T) - F_h)] \mathbf{1}_{\{A'\}}]}_{\text{default cost branch}}
\end{aligned} \tag{21}$$

The former is greater than the latter if and only if

$$\begin{aligned}
& \underbrace{-\alpha(1-\pi)\mathbb{E}[M_h(T)\mathbf{1}_{\{A\}}]}_{\text{default cost home s}} \\
& \underbrace{-(1-\pi)\alpha\mathbb{E}[M(T)\mathbf{1}_{\{A'\}}]}_{\text{default cost subsidiary}} > \\
& \underbrace{-\alpha(1-\pi)\mathbb{E}[[M_h(T) + \max(0, M(T) - F)]\mathbf{1}_{\{A\}}]}_{\text{default cost home b}} + \\
& \underbrace{-(1-\pi)\alpha\mathbb{E}[[M(T) + \max(0, M_h(T) - F_h)]\mathbf{1}_{\{A'\}}]}_{\text{default cost branch}}.
\end{aligned}$$

Default costs are paid only on the home or subsidiary cash flows in the subsidiary case, while they affect also the asset transfers from the affiliate ($\max(0, M(T) - F)$) or from the home bank ($\max(0, M_h(T) - F_h)$) in the branch organization. As a consequence, default costs in the subsidiary organization are smaller (in absolute value) than costs in the branch, for positive values of α and $0 \leq \pi < 1$, and the previous inequality is satisfied; it follows that the subsidiary organization is more valuable than the branch. When $\alpha = 0$, or $\pi = 1, \alpha > 0$, the two become indifferent. This concludes the proof. ■

Last, let us prove proposition 4, which compares two SA banks with a unilateral and mutual insurance, with no returns to scale from loans ($k = 0$).

Proof. Let us compare the values of the stand alone and unilateral subsidiary arrangement, when the home bank has the same and positive level of deposits in both cases. If we already name home and subsidiary the two affiliates when they are stand-alone banks, the values of the two arrangements are respectively

$$\begin{aligned}
& GV_{2SA} = M_{h0} \\
& \underbrace{+\pi \exp(-rT)\mathbb{E} \max(0, F_h - M_h(T))}_{\text{government bailout home as SA}} - \underbrace{\alpha(1-\pi) \exp(-rT)\mathbb{E} [M_h(T)\mathbf{1}_{\{M_h(T) < F_h\}}]}_{\text{default cost home as SA}} \\
& \quad + M_{s0} + \\
& \quad \underbrace{+\pi \exp(-rT)\mathbb{E} \{\max(F_s - M_s(T), 0)\}}_{\text{government bailout subsidiary as SA}} \\
& \quad - \underbrace{(1-\pi)\alpha \exp(-rT)\mathbb{E} [M_s(T)\mathbf{1}_{\{M_s(T) < F_s\}}]}_{\text{default cost subsidiary as SA}},
\end{aligned} \tag{22}$$

and

$$\begin{aligned}
& GV_{OWR} = M_{h0} \\
& + \underbrace{\pi \exp(-rT) \mathbb{E} \max(0, F_h - M_h(T))}_{\text{government bailout home}} - \underbrace{\alpha(1 - \pi) \exp(-rT) \mathbb{E} \left[M_h(T) \mathbf{1}_{\{M_h(T) < F_h\}} \right]}_{\text{default cost home}} \\
& \quad + M_{s0} + \\
& \quad + \underbrace{\pi \exp(-rT) \mathbb{E} \left\{ \max(F_s - M_s(T), 0) \mathbf{1}_{\{A'\}} \right\}}_{\text{government bailout subsidiary}} \\
& \quad - \underbrace{(1 - \pi) \alpha \exp(-rT) \mathbb{E} \left[M_s(T) \mathbf{1}_{\{A'\}} \right]}_{\text{default cost subsidiary}}.
\end{aligned} \tag{23}$$

The difference in value between the SA arrangement and the unilaterally-guaranteed group is

$$\begin{aligned}
& (GV_{2SA} - GV_{OWR}) \exp(rT) \\
& = + \underbrace{\pi \mathbb{E} \left\{ \max(F_s - M_s(T), 0) \right\}}_{\text{government bailout subsidiary as SA}} - \underbrace{\pi \mathbb{E} \left\{ \max(F_s - M_s(T), 0) \mathbf{1}_{\{A'\}} \right\}}_{\text{government bailout subsidiary}} \\
& \quad - \underbrace{(1 - \pi) \alpha \mathbb{E} \left[M_s(T) \mathbf{1}_{\{M_s(T) < F_s\}} \right]}_{\text{default cost subsidiary as SA}} + \underbrace{(1 - \pi) \alpha \mathbb{E} \left[M_s(T) \mathbf{1}_{\{A'\}} \right]}_{\text{default cost subsidiary}}.
\end{aligned} \tag{24}$$

Since the returns on loans satisfy hp 1, A' is not empty. This means that, in absolute value, both the bailout put and default costs are smaller in the unilateral case than in the SA one (notice that A' is a subset of $M_s(T) < F_s$). Since they show up with different signs, the trade-off between them depends on the parameters α and π . If $\alpha = 0$ and $\pi > 0$, only the first two terms in the above expression are non-null, and the value of the SA is greater than the value of a unilateral insurance. The same situation arises when $\pi = 1, \alpha \geq 0$. If $\pi = 0$ and $\alpha > 0$, only the last two terms in the above expression are non-null, and the value of the SA is smaller than the value of a unilateral insurance. Because of continuity of the above expression, it follows that there is a π^{**} above which the SA value becomes better than the unilateral one. Let us now compare SA and mutual, i.e.

$$\begin{aligned}
& GV_{2SA} = M_{h0} \\
& + \underbrace{\pi \exp(-rT) \mathbb{E} \max(0, F_h - M_h(T))}_{\text{government bailout home as SA}} - \underbrace{\alpha(1 - \pi) \exp(-rT) \mathbb{E} \left[M_h(T) \mathbf{1}_{\{M_h(T) < F_h\}} \right]}_{\text{default cost home as SA}} \\
& \quad + M_{s0} + \\
& \quad + \underbrace{\pi \exp(-rT) \mathbb{E} \left\{ \max(F_s - M_s(T), 0) \right\}}_{\text{government bailout subsidiary as SA}} \\
& \quad - \underbrace{(1 - \pi) \alpha \exp(-rT) \mathbb{E} \left[M_s(T) \mathbf{1}_{\{M_s(T) < F_s\}} \right]}_{\text{default cost subsidiary as SA}},
\end{aligned} \tag{25}$$

versus

$$\begin{aligned}
& \underbrace{+\pi \exp(-rT)\mathbb{E} \max\{(0, F_h - M_h(T))\mathbf{1}_{\{A\}}\}}_{\text{government bailout home w/mutual support}} - \underbrace{\alpha(1 - \pi) \exp(-rT)\mathbb{E} [M_h(T)\mathbf{1}_{\{A\}}]}_{\text{default cost home w/mutual support}} \\
& \quad + M_{s0} + \\
& \quad \underbrace{+\pi \exp(-rT)\mathbb{E} \{\max(F_s - M_s(T), 0)\mathbf{1}_{\{A'\}}\}}_{\text{government bailout subsidiary}} \\
& \quad \underbrace{-(1 - \pi)\alpha \exp(-rT)\mathbb{E} [M_s(T)\mathbf{1}_{\{A'\}}]}_{\text{default cost subsidiary}}.
\end{aligned} \tag{26}$$

The difference is

$$\begin{aligned}
& (GV_{2SA} - GV_{MR}) \exp(rT) = \\
& + \underbrace{\pi \mathbb{E} \max(0, F_h - M_h(T))}_{\text{government bailout home as SA}} - \underbrace{\pi \mathbb{E} \max\{(0, F_h - M_h(T))\mathbf{1}_{\{A\}}\}}_{\text{government bailout home w/mutual support}} \\
& - \underbrace{\alpha(1 - \pi) \mathbb{E} [M_h(T)\mathbf{1}_{\{M_h(T) < F_h\}}]}_{\text{default cost home as SA}} + \underbrace{\alpha(1 - \pi) \mathbb{E} [M_h(T)\mathbf{1}_{\{A\}}]}_{\text{default cost home w/mutual support}} \\
& + \underbrace{\pi \mathbb{E} \{\max(F_s - M_s(T), 0)\}}_{\text{government bailout subsidiary as SA}} - \underbrace{\pi \mathbb{E} \{\max(F_s - M_s(T), 0)\mathbf{1}_{\{A'\}}\}}_{\text{government bailout subsidiary}} \\
& - \underbrace{(1 - \pi)\alpha \mathbb{E} [M_s(T)\mathbf{1}_{\{M_s(T) < F_s\}}]}_{\text{default cost subsidiary as SA}} + \underbrace{(1 - \pi)\alpha \mathbb{E} [M_s(T)\mathbf{1}_{\{A'\}}]}_{\text{default cost subsidiary}}.
\end{aligned} \tag{27}$$

Since returns on loans satisfy hp 1, the event A (in which the home is not rescued by its subsidiary) is not empty. Within each line the first term is greater than the second, in absolute value; as above, let us analyze the difference in value by changing α and π . If $\alpha = 0$ and $\pi > 0$, only the first and third line in the above expression are non-null, and the value of the SA is greater than the value of a mutual insurance. The same situation arises when $\pi = 1, \alpha \geq 0$. The difference between the unilateral and mutual arrangement is

$$\underbrace{+\pi \exp(-rT)\mathbb{E} \max(0, F_h - M_h(T))}_{\text{government bailout home as SA}} - \underbrace{\pi \exp(-rT)\mathbb{E} \max\{(0, F_h - M_h(T))\mathbf{1}_{\{A\}}\}}_{\text{government bailout home w/mutual support}} > 0,$$

so that the SA arrangement is preferable to the unilateral, which in turn is better than the mutual (as we knew from the corresponding Proposition). If $\pi = 0$ and $\alpha > 0$, only the second and fourth lines are non-null, and the value of the SA is smaller than the value of a mutual insurance. It was also smaller than the unilateral one in that case. The difference between the unilateral and mutual arrangement is

$$\underbrace{-\alpha(1 - \pi) \exp(-rT)\mathbb{E} [M_h(T)\mathbf{1}_{\{M_h(T) < F_h\}}]}_{\text{default cost home as SA}} + \underbrace{\alpha(1 - \pi) \exp(-rT)\mathbb{E} [M_h(T)\mathbf{1}_{\{A\}}]}_{\text{default cost home w/mutual support}} < 0,$$

so that the stand alone is smaller than the unilateral and the latter is smaller than the mutual (as we knew from proposition 1). Because of continuity of the above expressions, there is a π^{***} above which the SA value becomes better than the mutual one, which is better than the branch. Using proposition 2 and the comparison between the unilateral and stand alone, such π^{***} is smaller or equal than the maximum between π^* and π^{**} . This concludes the proof. ■

C Proof of proposition 7, conditions for endogenous debt optimality and for higher leverage in mutual organizations

Suppose $k = 0$. The stand alone problem for the choice of the optimal face value of debt, which consists in maximizing $E_0 + D_0 - L_0$ in (3), after eliminating L_0 and the discount, can be restated as that of maximizing with respect to $F = b$ the following function:

$$\pi b \int_0^b dG_X(x) + c \int_0^b x dG_X(x),$$

where G_X is the distribution function of the loans, whose support is assumed to be the positive real line, $c \triangleq -(\pi + \alpha(1 - \pi))$. The FOC for this maximization is the equality between the marginal increase in value due to bailout and marginal default costs in case bailout does not occur. These are the left and right-hand-side of the following equality:

$$\pi G_X(b) = \alpha(1 - \pi)bg_X(b), \quad (28)$$

where g is the density corresponding to G . Let b^* be its solution.

Consider the case in which a unilateral insurance is provided from the home parent - for which the notation stays as above - to the subsidiary. Assuming that the loans of the two banks are independently distributed, the objective function of the group maximization, eliminating discounting, becomes $L_{0h} + L_{0s} + F(a, b)$ where

$$F(a, b) \triangleq \pi b \int_0^b dG_X(x) + c \int_0^b x dG_X(x) + \pi a \int_0^a \int_0^{h(y, a, b)} dG_X(x) dG_Y(y) + c \int_0^a \int_0^{h(y, a, b)} dG_X(x) y dG_Y(y), \quad (29)$$

G_Y is the distribution of the subsidiary's loans, a is the face value of its deposits and $h(y, a, b) \triangleq -y + a + b$. We want to demonstrate proposition 7.

Proof. We assume that the default cost and bailout parameter is the same for both banks, and that both were optimally levered before setting up the guarantee. Consider the differential of $F(a, b)$:

$$\frac{\partial F}{\partial a} da + \frac{\partial F}{\partial b} db.$$

We are going to prove that it will be worth moving a small amount of debt from the home to the subsidiary by showing that, starting from $b = b^*$, $a = a^*$, a decrease in the former's debt, $db < 0$, accompanied by a symmetric increase in the insured's debt, $da = -db > 0$, makes the differential positive:

$$\left\{ \begin{array}{l} [-\frac{\partial F}{\partial a} db + \frac{\partial F}{\partial b} db]_{a=a^*, b=b^*} > 0 \\ db < 0 \end{array} \right.$$

or

$$\left[\frac{\partial F}{\partial b} < \frac{\partial F}{\partial a} \right]_{a=a^*, b=b^*}. \quad (30)$$

Let us compute the derivatives:

$$\begin{aligned} \frac{\partial F}{\partial a} &= \pi \int_0^a \int_0^{h(y,a,b)} dG_X(x) dG_Y(y) + \\ &\quad -\alpha(1-\pi)ag_Y(a)G_X(h(y,a,b)) \\ &\quad +\pi a \int_0^a g_X(h(y,a,b))dG_Y(y) \\ &\quad +c \int_0^a g_X(h(y,a,b))ydG_Y(y), \end{aligned}$$

$$\begin{aligned} \frac{\partial F}{\partial b} &= \pi G_X(b) - \alpha(1-\pi)bg_X(b) + \\ &\quad +\pi a \int_0^a g_X(h(y,a,b))dG_Y(y) \\ &\quad +c \int_0^a g_X(h(y,a,b))ydG_Y(y). \end{aligned}$$

Simplifying and recalling that at b^* (28) holds, we get the following inequality, which is equivalent to (30):

$$\pi \int_0^a G_X(h(y,a,b))dG_Y(y) - \alpha(1-\pi)ag_Y(a)G_X(b) > 0. \quad (31)$$

Since $y < a$, $h > b$ and $G_X(h(y,a,b)) > G_X(b)$ in the first integral, the left-hand side of (31) is greater than

$$\begin{aligned} &G_X(b)\pi \int_0^a dG_Y(y) - \alpha(1-\pi)ag_Y(a)G_X(b) \\ &= G_X(b) [\pi G_Y(a) - \alpha(1-\pi)ag_Y(a)]. \end{aligned}$$

The last expression is equal to zero at $a = a^*$, where the FOC for the stand-alone debt (28) holds. As a consequence, (31) holds if the bank receiving insurance was initially at a^* . This proves that the difference of the group value is positive, i.e. that a group with unilateral rescue is more valuable than two stand-alone

banks, if they create debt diversity, with the guaranteed company more levered than the guarantor. This completes the proof. ■

In order to discuss the optima of mutually-guaranteed parent-subsidiaries and branches, let us introduce the following notation. The objective function for the mutual guarantee case, with independent loans, is such that it is equivalent to maximize $H(a, b)$

$$\begin{aligned} H(a, b) \triangleq & \pi b \int_0^{h(x,a,b)} \int_0^b dG_X(x)dG_Y(y) + c \int_0^{h(x,a,b)} \int_0^b xdG_X(x)dG_Y(y) \\ & + \pi a \int_0^a \int_0^{h(y,a,b)} dG_X(x)dG_Y(y) + c \int_0^a \int_0^{h(y,a,b)} ydG_X(x)dG_Y(y). \end{aligned}$$

H is symmetric in a and b , since loans are equally distributed, i.e. $G_X = G_Y$. So, when the same default cost and bailout probability apply to both affiliates, and $k = 0$, we should not have debt diversity. Let us now demonstrate the following

Lemma 8 *Let $k = 0, \alpha > 0, 0 < \pi < 1$. Let $G_X = G_Y$ be the distributions of loans of two affiliates ($X, Y = M_i, i = h, s$ or $X, Y = N_i, i = h, b$), let g_X, g_Y be their densities. If loans are independent between affiliates and the default cost rate α is smaller than*

$$\frac{\pi}{1 - \pi} \left(\frac{b \int_0^b dG_X(x)g_Y(b-x)}{\int_0^b xdG_X(x)g_Y(b-x)} - 1 \right)$$

then optimal leverage is positive in mutually-guaranteed affiliates and branches.

Proof. Consider the mutual case. The derivative with respect to a of the objective function, with independent loans, H , is

$$\begin{aligned} \frac{\partial H(a, b)}{\partial a} \triangleq & \pi b \int_0^b g_Y(a+b-x)dG_X(x) + c \int_0^b xg_Y(a+b-x)dG_X(x) \\ & + \pi a \left(g_Y(a) \int_0^b dG_X(x) + \int_0^a g_X(a+b-y)dG_Y(y) \right) \\ & + c \left(ag_Y(a) \int_0^b dG_X(x) + \int_0^a yg_X(a+b-y)dG_Y(y) \right) \\ & + \pi \int_0^a \int_0^{a+b-y} dG_X(x)dG_Y(y). \end{aligned}$$

When $a \rightarrow 0$, it becomes

$$\frac{\partial H(a, b)}{\partial a} \triangleq \pi b \int_0^b g_Y(b-x)dG_X(x) + c \int_0^b xg_Y(b-x)dG_X(x)$$

which is positive, indicating that optimal leverage, or the optimal a , is positive, if

$$\alpha(1 - \pi)/\pi < \frac{b \int_0^b dG_X(x)g_Y(b-x)}{\int_0^b x dG_X(x)g_Y(b-x)} - 1.$$

Rearranging, this happens for default costs α smaller than

$$\frac{\pi}{1 - \pi} \left(\frac{b \int_0^b g_Y(b-x)dG_X(x)}{\int_0^b x g_Y(b-x)dG_X(x)} - 1 \right) \quad (32)$$

So, if default costs are low enough, the maximum of H , if it exists, is not at the boundary, and the mutually-guaranteed bank is levered. Consider now branches. Their optimum problem is equivalent to maximizing $HH(a, b)$

$$HH(a, b) \triangleq H(a, b) - \alpha(1 - \pi) \times \left[\int_0^{h(x,a,b)} \int_0^b (y-a)dG_X(x)dG_Y(y) + \int_0^a \int_0^{h(y,a,b)} (x-b)dG_X(x)dG_Y(y) \right]$$

The derivative of HH is

$$\frac{\partial HH}{\partial a} \triangleq \frac{\partial H}{\partial a} - \alpha(1 - \pi) \times \left[\int_0^b (b-x)g_X(x)g_Y(a+b-x)dx + \int_0^a (a-y)g_X(a+b-y)g_Y(y)dy - \int_0^{a+b-x} \left(\int_0^b g_X(x)g_Y(y)dx \right) dy \right]$$

When $a \rightarrow 0$, then the derivatives of H and HH coincide and condition (32) guarantees that also branches are positively levered. ■

We are now ready to demonstrate the following proposition:

Proposition 9 *If $k = 0$ and the firm value is concave, then a subsidiary structure is optimally less levered than a branch structure if $\alpha > 0, 0 < \pi < 1$ and*

$$\left[2 \int_0^{a^*} (a^* - x)g(x)g(a+b-x)dx - \int_0^{2a^*-x} \left(\int_0^{a^*} g(x)g(y)dx \right) dy \right] < 0$$

where $a^* = b^*$ is the optimal face value of debt for mutually-guaranteed banks. If $\alpha = 0$ or $\pi = 1$ optimal leverage is the same.

Proof. In order to prove the first part of the proposition, i.e. to provide conditions under which the branch organization is more levered than the parent-subsidiary with mutual rescue, assume that both H and HH are concave in a and b . Then $\frac{\partial HH}{\partial a}$ is positive when H is at a maximum, with $\frac{\partial H}{\partial a} = 0$, with both $\alpha > 0$ and $0 < \pi < 1$, if

$$\left[\int_0^b (b-x)g_X(x)g_Y(a+b-x)dx + \int_0^a (a-y)g_X(a+b-y)g_Y(y)dy - \int_0^{a+b-x} \left(\int_0^b g_X(x)g_Y(y)dx \right) dy \right] < 0$$

at $a = a^*, b = b^*$. Since X and Y are equally distributed, and the optimal debts for the mutual are equal ($a^* = b^*$), the previous inequality can be written as

$$\left[2 \int_0^{a^*} (a^* - x)g(x)g(a + b - x)dx + \int_0^{2a^* - x} \left(\int_0^{a^*} g(x)g(y)dx \right) dy \right] < 0$$

If $\alpha = 0$ or $\pi = 1$ optimal leverage is the same since $H = HH$. This concludes the proof. ■

D Figures

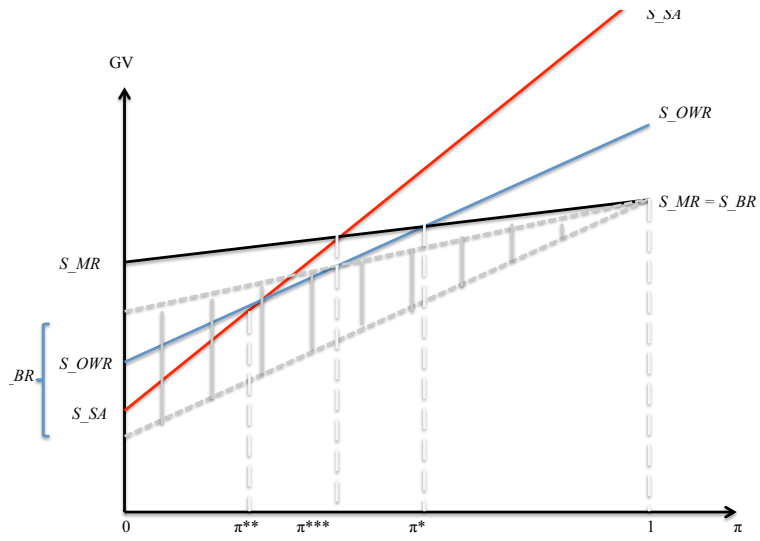


Figure 1: Group value(GV) and probability of bailout (π) for default costs > 0 .

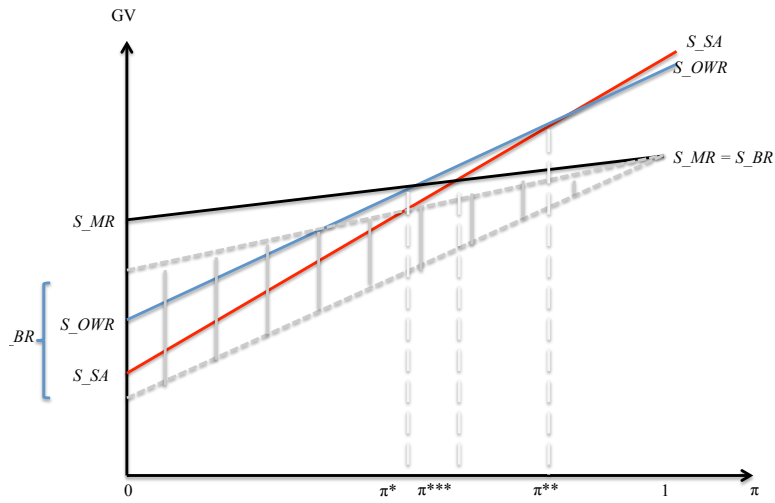


Figure 2: Group value(GV) and probability of bailout (π) for default costs > 0 .

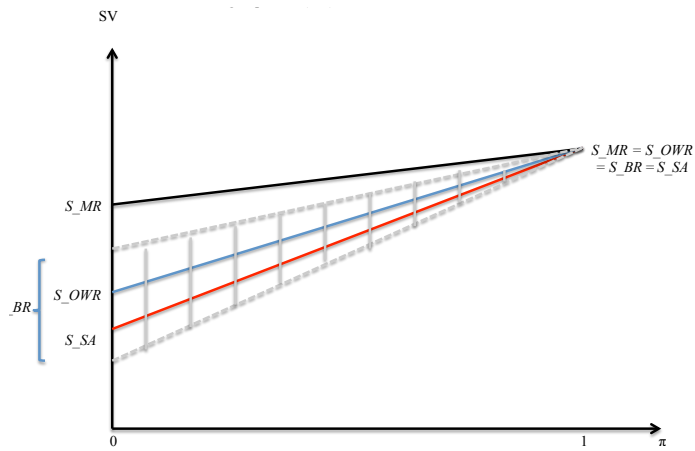


Figure 3: Social value (SV) and probability of bailout (π) for $\alpha > 0$ assuming banks maximize group value (GV).

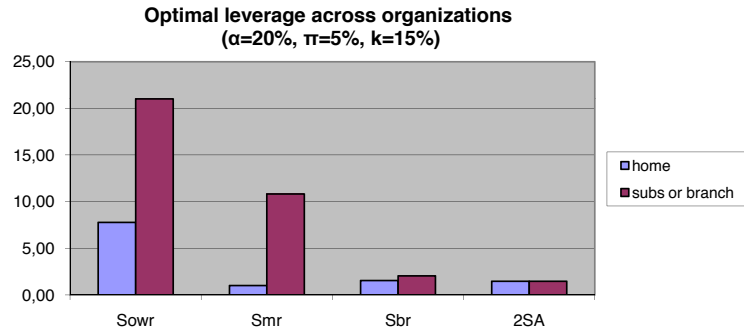


Figure 4: Optimal leverage across organizations.

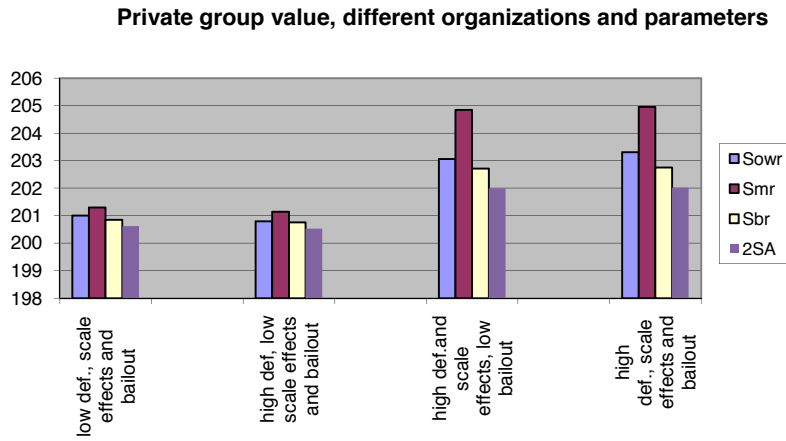


Figure 5: Private group value in different organizations, different parameter combinations, bailout probability smaller than 40%.

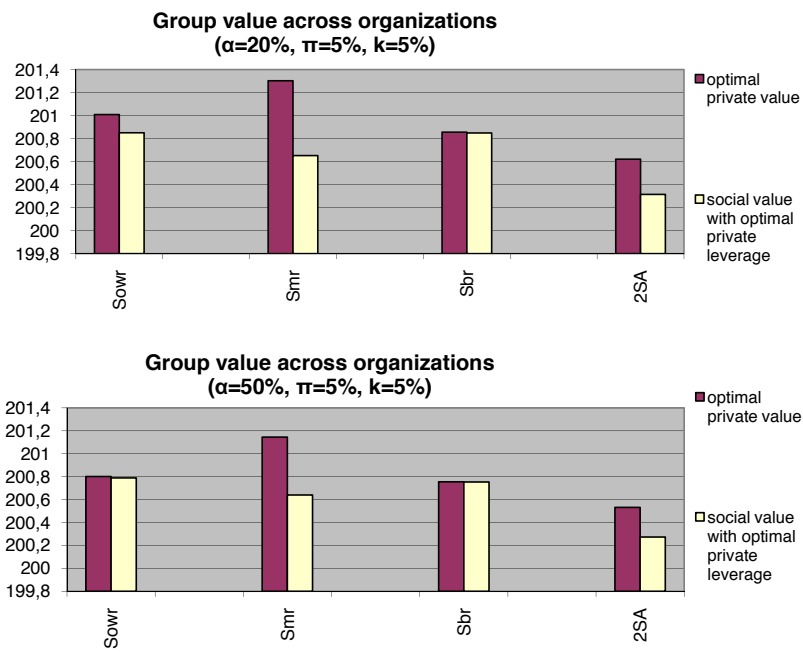


Figure 6a: Group value across organizations, private and social.

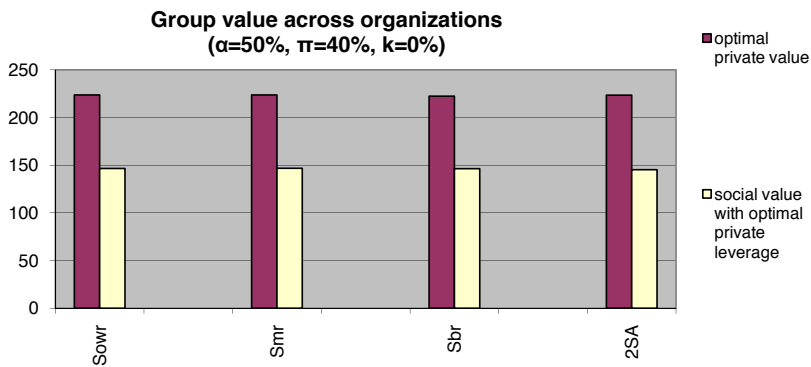
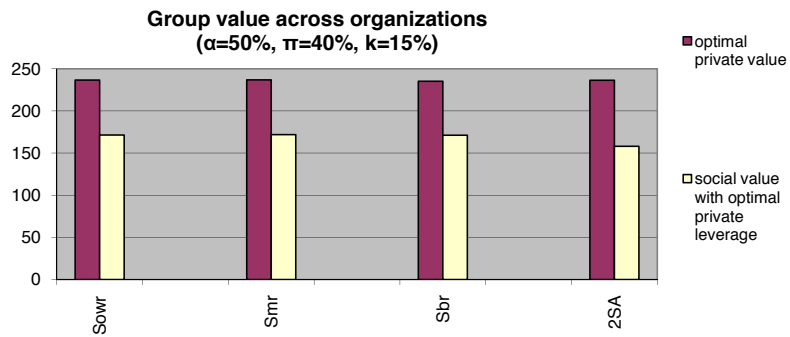
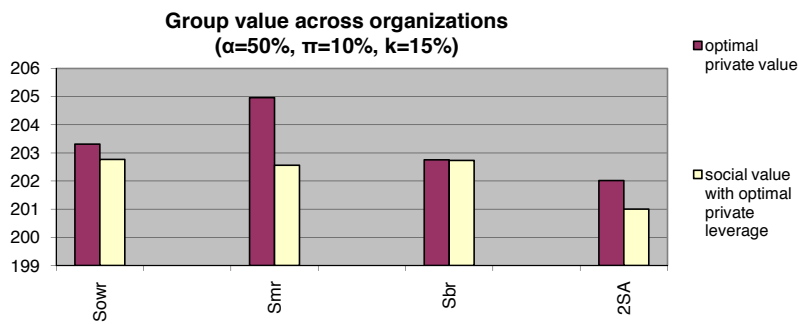
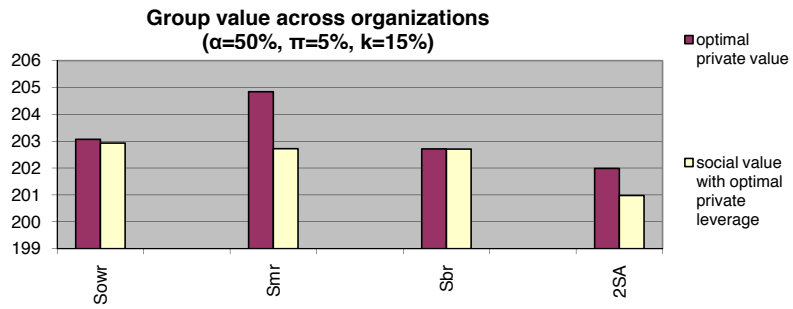


Figure 6b: Group value across organizations, private and social.

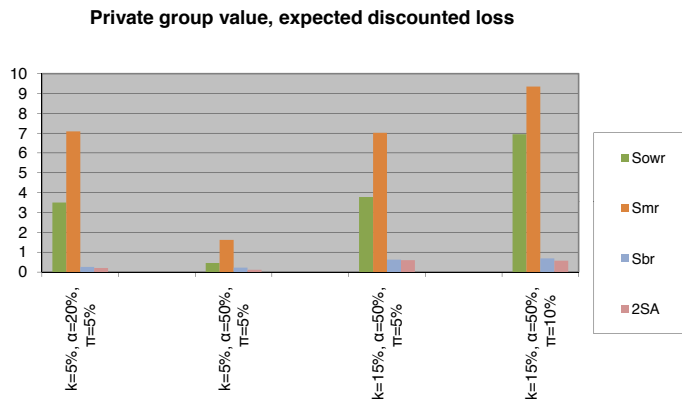


Figure 7: Private values, expected discounted loss.

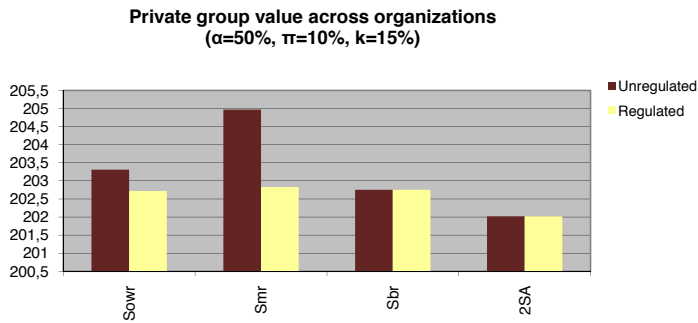


Figure 8: Private group value with and without capital requirements (regulated and unregulated case respectively), different organizations.

Table 1 Private value maximization. $K=5\%$, $\pi=5\%$, $\text{corr}=0.2$
 (low scale effects); Max GV and
 SV in bold

Panel 1: $\alpha=20\%$	S_OWR		S_MR		S_BR		2 SA
GV	201.01		201.30		200.86		200.62
Def. costs	0	0.48	0.03	0.96	0.03	0.10	0.11
Group put	0.16		0.65		0.01		0.31
SV	200.85		200.65		200.85		200.32
Lev (V/E)	7.77	21.01	1.01	10.84	1.56	2.05	1.48
Panel 2: $\alpha=50\%$							
GV	200.80		201.15		200.76		200.53
Def. costs	0	0.14	0	0.48	0.03	0.07	0.09
Group put	0.02		0.51		0.00		0.26
SV	200.79		200.64		200.75		200.27
Lev (V/E)	1	3.90	1.01	5.47	1.48	1.76	1.39

Table 2 Private value maximization. $k=15\%$, $\alpha=50\%$, $\text{corr}=0.2$
(high scale effects)

Panel 1: $\alpha=20\%$	S_OWR		S_MR		S_BR		2 SA
GV	203.07		204.85		202.72		201.99
Def. costs	0	1.13	0	2.08	0.10	0.37	0.47
Group put	0.14		2.12		0.01		1.01
SV	202.93		202.72		202.71		200.98
Lev (V/E)	1.01	7.52	2.56	9.89	1.59	2.18	1.54
Panel 2: $\pi=10\%$							
GV	203.31		204.97		202.75		202.02
Def. costs	0	2.05	0	2.75	0.11	0.40	0.44
Group put	0.54		2.41		0.02		1.02
SV	202.77		202.56		202.73		201.00
Lev (V/E)	1.01	10.13	1.01	11.82	1.61	2.21	1.54
Panel 3: $\pi=40\%$							
GV	236.64		236.93		235.31		236.35
Def. costs	29.61	31.20		29.61	29.95	29.95	60.40
Group put	65.13		64.94		64.06		78.23
SV	171.51		172.00		171.25		158.12
Lev (V/E)	44.74	56.39	56.94	56.94	124.67	124.67	44.30

Table 3 Private value maximization. $k=0\%$, $\pi=40\%$, $\text{corr}=0.2$, $\alpha=50\%$
(no scale effects)

	S_OWR		S_MR		S_BR		2 SA
GV	223.56		223.73		222.46		223.39
Def. costs	26.60	26.92	26.60	26.60	26.87	26.87	53.84
Group put	77.08		76.92		76.20		78.15
SV	146.49		146.81		146.27		145.24
Lev (V/E)	61.04	76.77	77.15	77.15	203.76	203.76	60.77

Table 4 Capital constrained. $k=15\%$, $\pi=10\%$, $\text{corr}=0.2$, $\alpha=50\%$
 (high scale effects)

	S_OWR		S_MR		S_BR		2 SA
GV	202.72		202.83		202.75		202.02
Def. costs	0.00	0.23	0.11	0.34	.11	.40	0.44
Group put	0.03		0.04		0.02		1.02
SV	202.69		202.80		202.73		201.00
Lev (V/E)	1.48	2.22	1.61	2.21	1.61	2.21	1.54

Table 5 Expected loss for different parameters
 $F \exp(-r T) - D0$

	uni, EL home	uni, EL subs	uni, total EL	mutual, EL home	mutual, EL subs	mutual, total EL	total EL branch	2* total SA
k=5% α=20% π=5%	0	3.5	3.5	0	7.10	7.10	0.26	0.20
k=5% α=50% π=5%	0	0.47	0.47	0	1.63	1.63	0.23	0.12
k=15% α=50% π=5%	0	3.78	3.78	0	7.03	7.03	0.64	0.61
k=15% α=50% π=10%	0	6.94	6.94	0.00	9.36	9.36	0.69	0.58
k=15% α=50% π=40%	79.19	78.31	157.50	78.31	78.31	156.63	155.98	158.38
k=0% α=50% π=40%	84.84	84.28	169.12	84.28	84.28	168.57	168.02	169.68

Table 6 Face values of debts of Home (Fh) and affiliate (Fi), Group value (GV) and Social value (SV) with different default costs (α) and probability of bailout (π) for the two legal entities. Subsidiary entities are constrained to Face value =50, Branch bank to a total of 100. $k=0.05$

	S_OWR (α/π) HOME=.20/.20 (α/π) SUBS=(.50/.05)	S_OWR (α/π) HOME=.20/.05 (α/π) SUBS=(.50/.20)	S_MR (α/π)=.20/.05 and (.50/.20)	S_BR α/π = .20/.05	S_BR α/π = .50/.20
FH/FI, I=S,B	41/50	43/50	50/50	42/58	42/48
GV	200.71	200.73	200.81	200.82	200.79
SV	200.71	200.72	200.80	200.82	200.77