Private Commitment Problems and Systemic Risk*

Alan D. Morrison† and Ansgar Walther‡

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Abstract

We show that correlation causes systemic risk even when there are no government bailouts. The risk derives from a soft budget constraint: banks would prefer to commit not to rescue a distressed peer, but interbank lenders prefer to renegotiate ex post. The optimal commitment is achieved only if distress is sufficiently correlated across the banking system. Our model therefore generates endogenous asset correlation across banks, interbank market freezes, and interbank exposures in debt and derivative markets. Capital regulation can alleviate these incentives. Moreover, with a well-designed lender of last resort, incentives to correlate disappear and banks voluntarily hold more capital.

1. Introduction

This paper presents an analysis of correlated distress in the financial sector that is consistent with three facts about the 2008–09 financial crisis. First, many banks were exposed to real estate markets, and so experienced simultaneous losses when those markets collapsed (Anginer and Demirgüç-Kunt 2014, Bernanke 2008). Second, financial institutions were closely connected in the interbank loan and derivatives markets, which caused losses to spill over (Peltonen, Scheicher, and Vuillemey 2014, Brunnermeier, Clerc, and Scheicher 2013, Yellen 2013).[1] Third, key funding markets dried up, so that it was hard for distressed banks to resolve their liquidity problems (Shin 2009, Afonso, Kovner, and Schoar 2011, Kuo, Skeie, Vickery, and Youle 2014, Covitz, Liang, and Suarez 2013, Gorton and Metrick 2012).

Common bank exposures, on both the asset and liability sides of the balance sheet, can therefore cause system-wide financial fragility. Policy makers have a legitimate interest in reducing

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†Saïd Business School, University of Oxford.

‡Department of Economics, University of Oxford.

[1] See also Iyer and Peydro (2011), demonstrating using Indian bank data that financial contagion can occur via interbank linkages.

this type of systemic risk (Brunnermeier, Crockett, Goodhart, Persaud, and Shin 2009, Haldane 2009). But, while some progress has been made in the quantification of systemic risk (Adrian and Brunnermeier 2011, Acharya, Pedersen, Philippon, and Richardson 2010), it is extremely hard to measure systemic risk in real time (Hansen 2012). Policy may be most effective when it attempts to address the incentives that foster systemic risk, rather than to tackle its direct causes head on.

Our paper deviates from recent work by Acharya and Yorulmazer (2007) and Farhi and Tirole (2012), which demonstrates that bankers have a natural incentive to ensure that they fail together when governments support the financial sector during systemic crises. We present an explanation for common bank exposures that does not rely upon state support and, hence, yields different policy implications and empirical predictions. We show that banks may choose correlated investments so as to resolve a time-inconsistency problem. Our paper explores this incentive and its implications for policy.

In our model, banks make an unobservable effort decision that affects the cash flow generated by their assets. Their moral hazard problem is most effectively resolved when they face a credible threat of closure. In general, closure threats are not credible when distressed banks can renegotiate and raise additional funding. Moreover, because distressed banks require peer monitoring, the only source of additional funding is the interbank market (for evidence that interbank relationships are an important source of funding for troubled banks, see Affinito (2012)).

Closure threats are therefore renegotiation proof if the interbank market is insufficiently liquid to meet the needs of distressed banks. The interbank market is illiquid if all banks experience simultaneous distress. A bank can achieve this by ensuring that its financial performance is correlated with that of its peers. In line with our opening paragraph, we identify two ways that banks achieve this correlation. First, they can select investments that are exposed to a common shock.

Second, they can sign contracts under which each bank's financial slack is contingent on its peers' performance, so that healthy banks have insufficient financial slack to provide interbank continuation finance. We show that this can be achieved by exposing banks to their peers' debt or to Credit Default Swaps. Thus, our model rationalizes the endogenous formation of interbank networks with mutual exposures in debt and derivative markets. These exposures complement asset correlations in solving a private incentive problem.

Incentives to correlate in our model do not derive from a desire to extract rent from an implicit government guarantee. Nevertheless, the privately optimal actions by banks generate systemic risk and trigger interbank market freezes. It is reasonable to assume that neither of these outcomes is socially desirable, although in this paper we do not model the externality that they generate. We consider two policy responses.

First, policy could attempt to render a closure threat unnecessary, by finding alternative ways to incentivize bankers. In our model, high enough levels of equity capital resolve bank moral hazard problems. So highly capitalized banks do not require a closure threat, and have no incentive to correlate with their peers. However, when equity capital is privately costly, for example because of tax shield effects, bankers prefer not to raise capital. If they were forced to do so then they would no longer have an incentive to select high asset correlations, and systemic risk would be reduced. The social value of this reduction may exceed its private costs.

Second, policy could render the closure threat incredible. We demonstrate that the classic Lender of Last Resort (LOLR) can accomplish this. We assume that the LOLR lends at actuarially fair rates. It is therefore prepared to lend only if it has the same peer monitoring skills as private banks, since, otherwise, its lending would be loss-making. When the LOLR can monitor, funds are available even when the banking sector is illiquid and, hence, banks cannot resolve time-inconsistency problems by selecting correlated investments. In that case, the only way to raise funds is voluntarily to raise sufficient equity capital to obviate the need for outside incentives. In other words, our model predicts that a credible LOLR results in endogenously higher bank capitalization.

Our LOLR result is novel, and suggests a less complicated means of increasing bank capitalization. But it hinges upon the efficiency with which central bank support is targeted. If the surplus generated by LOLR facilities is captured by bank creditors them they may cease to care about bank incentives altogether. If political or other pressures prevent private or central banks from capturing LOLR surpluses, it may be necessary to combine LOLR facilities with appropriately designed capital requirements.

Our analysis also identifies a role for specialist investors capable of replacing the management of distressed banks. If such investors provide effort incentives then banks need not rely upon systemic risk to do so. We suggest that bank vulture investors have a stabilizing influence upon financial markets and, hence, are socially valuable.

Our analysis yields several empirical results. First, there should be a positive association between the strength of the LOLR and banking sector capitalization. Similarly, because the LOLR solution is only open to fiscally strong states, fiscal capacity should be negatively associated with banking sector correlations.

The inability of financiers to commit not to refinance distressed banks lies at the heart of our analysis. This problem is closely related to the soft budget constraint literature, which studies related incentive effects in socialist economies where paternalistic governments cannot commit to close failing industries (for surveys, see Berglof and Roland (1995), Maskin (1999) and Kornai, Maskin, and Roland (2003)). That literature suggests that an important virtue of capitalist economies is that they harden budget constraints by delegating re-financing decisions to private sector institutions that do not internalize all of the costs of closure. But, as in our model, soft budget constraints can arise whenever a liquid financier perceives continuation to be ex post optimal (see Dewatripont and Maskin (1995) and Lóránth and Morrison (2012)). Our analysis extends previous work by demonstrating that systemic risk can be a response to soft budget constraints in developed financial markets.

Our work contributes to a large literature on correlation in the banking system. One class of models demonstrates that, when the return distribution on assets is fixed, bank liability choices can cause bank returns to be correlated. For example, Rochet and Tirole (1996) examine a model in which interbank lending occurs in order to channel deposits to banks with lending opportunities; Allen and Gale (2000) study a model in which interbank lending can enable efficient liquidity sharing between regions with different consumption shocks. In both models, interbank lending enhances efficiency, but does so at the undesirable cost of increasing systemic risk.

In contrast, Wagner (2010) shows that systemic risk can be an unintended consequence of efficient risk allocation within banks. In his work, banks diversify to limit their exposure to systematic risk; this has a dark side, in that, when all banks have diversified portfolios, their values become correlated, and systematic risk emerges.

Our work has features in common with both types of model. The correlation between bank assets is chosen endogenously in our set-up, and we demonstrate that our optimal contract can be implemented through endogenously chosen interbank exposures. But, in contrast to these papers, it is optimal for banks in our analysis to expose themselves to systemic risk since, without it, they would be unable to commit not to re-finance one another.

Our analysis is distinct from Allen and Gale's (1998) work, in which systemic risk is also optimal. In contrast to us, Allen and Gale study an incomplete contracting model, in which only common default facilitates the optimal state-contingent contract. We allow for complete state-contingent contracting; the friction that drives our results is an inability to commit not to renegotiate.

Another strand of the literature argues that systemic risk is an endogenous response to distortive regulation. Farhi and Tirole (2012) demonstrate that, when the authorities cannot credibly commit not to support a distressed banking sector, individual bankers have an incentive to take more liquidity risk, and so to expose the system to aggregate exposure. Similarly, Acharya and Yorulmazer (2007) show that a "too many to fail" problem, which arises because regulators are not prepared to allow the entire banking system to collapse, causing banks to herd. Our paper is in contrast to this literature, in that the effects that we examine arise in the absence of any regulation.

The literature on credit cycles in Macroeconomics, following Kiyotaki and Moore (1997) and Bernanke and Gertler (1989) that aggregate shocks can be amplified in economies with borrowing frictions. In related work, Lorenzoni (2008) establishes that this mechanism can generate socially excessive credit booms. Brunnermeier and Pedersen (2009) and Geanakoplos (2009) argue that rising downpayment requirements or "haircuts" further amplify aggregate crises. In these papers, aggregate shocks are taken as given. Our paper complements this literature by proposing a novel mechanism which generates aggregate risk endogenously.

2. Technological Set-up

We consider a three-date model of two banks. Time is indexed by $t \in \{0,1,2\}$, and banks by $i \in \{1,2\}$. Each bank has access to a large pool of investors, and every agent is risk-neutral. There is one consumption good. We write E for the total time 0 endowment of each bank.

Banks in our model are impatient: their marginal utility of consumption at time 1 or 2 is 1, and of consumption at time 0 is $1+\phi > 1$. ϕ can be thought of as capturing the relative cost of retaining inside equity over accessing external debt. The most obvious manifestation of this cost is in the relative tax treatments of debt and equity: as the tax advantage of debt increases, so, too, does the impatience parameter ϕ.

Investors are patient, and derive marginal utility 1 from consumption at any date. Investors have deep pockets. Gains from trade are therefore realized when banks borrow from investors.

Bank are endowed with a lending technology, which we discuss in the following two sections. Section 2.1 describes the operation of that technology in a world without financial frictions. Section 2.2 introduces the two types of frictions that drive our results.

2.1 *Bank projects*

At time 0, each bank is endowed with a project that requires a time 0 investment of I. The project generates a time 2 return R if it is brought to completion.

At time 1 the bank may face a time 1 reinvestment requirement, in which case its project can be completed only if the bank invests an additional $\rho > 0$. Partial time 1 investment of $x\rho$ is possible for $x \in [0,1]$: in that case, a fraction x of the project is completed. We follow Holmström and Tirole (1996) and refer to a reinvestment requirement as a *liquidity shock*.

We write $\omega_i \in \{0,\rho\}$ for the size of bank i's liquidity shock. Each bank experiences a time 1 liquidity shock with probability $\frac{1}{2}$. Banks can select their investment portfolios so as to achieve any desired correlation between their liquidity shocks.[2] Our measure of correlation β is defined as follows:

$$\mathbb{P}\left[\omega_i = \rho | \omega_j = \rho\right] = \beta.$$

Projects in our model have positive net present value (that is, $R - \rho/2 > I$), so that, in the absence of frictions, it would be optimal for banks to consume their endowment E, and to borrow to invest.

[2] We could model correlation selection explicitly by requiring banks to divide their investments between two sectors, one of which definitely experiences a liquidity shock. This approach would significantly complicate our analysis of financing frictions without generating any additional insights.

2.2 *Financial Frictions*

Our results follow from two financial frictions: moral hazard in project management and a cost of holding cash.

Moral hazard frictions

The first moral hazard friction is the bank's unobservable time 0 *maintenance effort* decision, which affects the probability that the bank experiences a liquidity shock. With maintenance effort the probability of a liquidity shock is $\frac{1}{2}$; when there is no maintenance effort the bank experience a private benefit $B > 0$ and a liquidity shock occurs with probability $\frac{1}{2} + \Delta$.

Projects that do not close at time 1 face an additional moral hazard problem. This problem generates effects that have been widely studied elsewhere (see, for example, Holmström and Tirole 1997); for simplicity, we employ the simplest reduced form that allows us to capture those effects. We assume that the project succeeds and returns $R > 0$ if and only if its bank exerts an additional *managerial effort*, and we assume that managerial effort is exerted only if generates sufficient rent to render it incentive compatible.

We think of maintenance and managerial effort as encompassing classic banking activities like borrower screening and collateral management (see, e.g., James (1987), Datta, Iskandar-Datta, and Patel (1999), Petersen and Rajan (1994), Boot (2000)).

The minimal rent required to induce a bank to perform managerial effort depends upon whether the bank's project is *monitored*. Consistent with this assumption, recent work by Coco, Gomes, and Martins (2009) demonstrates that banks receive cheaper funding from peers with whom they have a close relationship. Monitoring is a scarce skill, which requires specialist banking knowledge. We assume that monitoring can only be performed by another bank. Suppose that a fraction $\mu \in [0, x]$ of a bank's project is monitored where, as above, x is the proportion of the project that remains after time 1. Then the minimum rent that induces the bank to exert managerial effort is $\mu f + (x - \mu)F$, where $0 < f < F$.

Monitoring must be incentivized: it is incentive compatible for a bank to monitor a fraction μ of its peer's project if and only if it earns rent of at least $M\mu$, where $M > 0$.

The moral hazard frictions affect the joint distribution of liquidity shocks. Nature draws the time 1 state $\omega = (\omega_1, \omega_2) \in \{0, \rho\}^2$ according to the distribution laid out in Figure 1.

As in Section 2.1, the distribution of liquidity shocks depends upon the parameter $\beta \in (0, 1)$; the distribution described in Section 2.1 is illustrated in Panel (a) of the Figure. With moral hazard, the distribution also depends upon bank's maintenance effort decision. Shirking by bank i increases the probability of a liquidity shock by Δ. The probability shifts are scaled to preserve the marginal distribution of bank j's liquidity shock, which is unaffected by bank i's effort.[3]

[3] In particular, the probability shift is scaled by $1 - \beta$ when the bank j experiences a liquidity shock, and by β when it does not. This is not the unique scaling factor which preserves marginal distributions, but it is useful because

	$\omega_2 = 0$	$\omega_2 = \rho$
$\omega_1 = 0$	$\frac{\beta}{2}$	$\frac{1-\beta}{2}$
$\omega_1 = \rho$	$\frac{1-\beta}{2}$	$\frac{\beta}{2}$

(a) Both banks exert maintenance effort.

	$\omega_2 = 0$	$\omega_2 = \rho$
$\omega_1 = 0$	$\frac{\beta}{2} - \beta\Delta$	$\frac{1-\beta}{2} - (1-\beta)\Delta$
$\omega_1 = \rho$	$\frac{1-\beta}{2} + \beta\Delta$	$\frac{\beta}{2} + (1-\beta)\Delta$

(b) Bank 1 (row) shirks.

	$\omega_2 = 0$	$\omega_2 = \rho$
$\omega_1 = 0$	$\frac{\beta}{2} - \beta\Delta$	$\frac{1-\beta}{2} + \beta\Delta$
$\omega_1 = \rho$	$\frac{1-\beta}{2} - (1-\beta)\Delta$	$\frac{\beta}{2} + (1-\beta)\Delta$

(c) Bank 2 (column) shirks.

	$\omega_2 = 0$	$\omega_2 = \rho$
$\omega_1 = 0$	$\frac{\beta}{2} - \beta\Delta$	$\frac{1-\beta}{2} - (1-\beta)\Delta$
$\omega_1 = \rho$	$\frac{1-\beta}{2} - (1-\beta)\Delta$	$\frac{\beta}{2} + (2-\beta)\Delta$

(d) Both banks shirk.

Figure 1. **Liquidity shock distribution**. The joint distribution of liquidity shocks is determined by the time 0 maintenance effort decision of each bank.

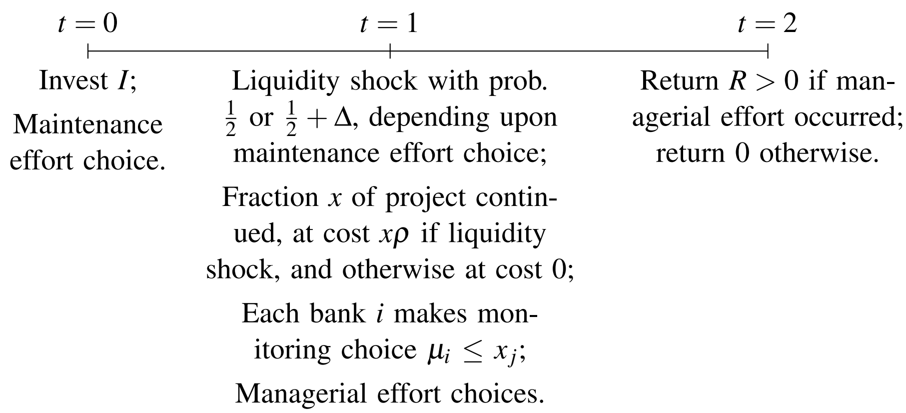

Figure 2. **Model timeline**. Banks make a time 0 maintenance effort choice that affects the likelihood that they experience a liquidity shock. If they continue past time 1 then their return depends upon a time 1 managerial effort choice.

Costs of Holding Cash

The second financial friction is that it is expensive in our model to hold cash. This is true in practice both because it is tax inefficient (for example, see Soo Kim, Mauer, and Sherman (1998) and Riddick and Whited (2009)), and because cash is easier to divert, either by investing it into poor projects or by simply stealing it (for example, see Jensen (1986) and Burkart and Ellingsen (2004)). We capture this cost by assuming that that cash held at date 0 is partially diverted: only a fraction $(1-\eta)$ of any cash held at time 0 will remain on the balance sheet at time 1. All of our results hold in the limit as $\eta \to 0$ from above.

The model timeline is illustrated in Figure 2.

it ensures ensure non-negativity of probabilities for all $\beta \in (0,1)$. Using different scaling factors would not affect our qualitative results.

2.3 *Parametric assumptions*

We impose three parametric assumptions. First, recall that bank i requires rent M in order to monitor bank j's project, and that bank j requires rent f to induce managerial effort if it is monitored, and rent $F > f$ if it is not. We assume that the aggregate rent requirement is lower when bank i monitors bank j:

$$M + f < F. \tag{A1}$$

Second, we assume that a bank with a project and monitoring obligations cannot raise sufficient deposits to cover a liquidity shock ex post. We demonstrate below that this requirement is met if Equation (A2) is satisfied:

$$R - \rho < f + M. \tag{A2}$$

Third, we assume that a bank without monitoring obligations can raise ex post interbank funding to cover a liquidity shock. This is true precisely when Equation (A3) is satisfied:

$$R - \rho \geq f. \tag{A3}$$

3. Contracting

In this Section we describe the contracting environment faced by banks and their investors.

Bank i can raise funds at time 0 from a pool of local investors, and also from bank j. We assume that neither bank can raise funds directly from the other's investors.[4] There are therefore two types of investment contracts in our model:

Definition 1.

1. *Bank i's* retail deposit contract *is a tuple (D_i, d_{1i}, d_{2i}) stipulating the payment D_i that bank i's investors make to bank i at time 0, and the repayments d_{1i} and d_{2i} that the bank makes to its investors at times 1 and 2;*
2. *Bank i's* interbank deposit contract *is a tuple (B_i, b_{1i}, b_{2i}) stipulating the time 0 payment B_i by bank j to bank i at time 0 and time 1 and 2 repayments b_{1i} and b_{2i} by bank i to bank j.*

The retail and interbank deposit contracts must obey time 0 and time 1 budget constraints.

Time 0 budget constraint. We write δ_i for bank i's time 0 dividend payment. Its time 0 budget constraint is therefore given by Equation (1):

$$E + D_i + B_i = I + \delta_i + C_i + B_j, \tag{1}$$

where C_i is bank i's time 0 cash holding. We write $A_i = E - \delta_i$ for bank i's endowment net of its time 0 dividend; A_i is bank i's equity stake in its project. Equation (1) can then be re-written as

[4] This assumption renders the analysis clearer, but is without loss of generality: bank i can access bank j's borrowers indirectly by borrowing from bank j.

follows:

$$A_i + D_i + B_i = I + C_i + B_j. \tag{2}$$

Time 1 budget constraint. We write C_i' for bank i's time 1 cash holding. Its time 1 budget constraint is then

$$d_{1i} + b_{1i} + \omega_i x_i + C_i' = C_i(1-\eta). \tag{3}$$

A bank has positive net present value precisely when it exerts maintenance effort and managerial effort. Therefore, we require both types of effort to be incentive compatible. We write v_i for the date 2 return that bank i derives from its project and retained cash, net of all repayments:

$$v_i = (R-\omega_i)x_i + C_i(1-\eta) - (d_{1i}+d_{2i}) - (b_{1i}+b_{2i}) + (b_{1j}+b_{2j}). \tag{4}$$

Monitoring Incentive Compatibility. Bank i will monitor a proportion μ_i of bank j's project if bank i earns rent at least $\mu_i M$; bank i requires rent $x_i F + \mu_j(F-f)$ if it is to exert managerial effort in its own project, where μ_j is the level of cross-monitoring performed by bank j. Bank i will cross-monitor and exert managerial effort if the sum of both rent requirements is less than bank i's total return, v_i:

$$v_i \geq x_i F - \mu_j(F-f) + \mu_i M. \tag{5}$$

Maintenance Effort Incentive Compatibility. A bank chooses to exert maintenance effort if and only if the marginal revenue generated by maintenance effort exceeds its cost, B. It is immediate from Figure 1 that this requirement is equivalent to the following conditions:

$$\begin{aligned} \beta\left(v_1(0,0)-v_1(\rho,0)\right)+(1-\beta)\left(v_1(0,\rho)-v_1(\rho,\rho)\right) &\geq \frac{B}{\Delta}; \\ \beta\left(v_2(0,0)-v_2(0,\rho)\right)+(1-\beta)\left(v_2(\rho,0)-v_2(\rho,\rho)\right) &\geq \frac{B}{\Delta}. \end{aligned} \tag{6}$$

Individual rationality. When the incentive constraints (5) and (6) are satisfied, investors invest only if the following break-even constraint is satisfied:

$$\mathbb{E}[d_{1i}+d_{2i}] \geq D_i. \tag{7}$$

The expectation of Equation (7) is taken when the maintenance effort constraints (6) are satisfied: that is, under the probability measure of panel (a) in Figure 1.

Limited Liability. We assume that banks have no recourse to investors' other assets. Deposit contracts therefore satisfy Equation (8):

$$\begin{aligned} d_{2i} &\geq 0; \\ d_{1i}+d_{2i} &\geq 0. \end{aligned} \tag{8}$$

We assume that all cash flows, the state of nature ω, the correlation parameter β and the continuation decisions x_1, x_2 are verifiable. Maintenance effort, managerial effort, and monitoring decisions are not verifiable. Banks are therefore able to commit at time 0 to state-contingent actions. Those commitments and the terms of all interbank deposit contracts are public knowledge when contracts are signed, so that banks are therefore able to modify the terms of one investment contract in order to achieve better terms in another. In short, all parties effectively sign up to a *grand contract* at time 0.

Definition 2. *A date* 0 grand contract K *comprises:*

1. *State contingent continuation decisions* x_1 *and* x_2*;*
2. *Retail deposit contracts* (D_i, d_{1i}, d_{2i}) *for* $i = 1, 2$*;*
3. *Interbank deposit contracts* (B_i, b_{1i}, b_{2i}) *for* $i = 1, 2$*;*
4. *Bank equity investments* (A_1, A_2)*;*
5. *A correlation parameter* β*,*

and satisfies:

1. *The time* 0 *and* 1 *budget constraints* (1) *and* (3)*;*
2. *The monitoring incentive compatibility constraint* (5)*;*
3. *The maintenance effort incentive compatibility constraint* (6)*;*
4. *The individual rationality constraints* (7)*;*
5. *The limited liability condition* (8)*;*
6. *The following* monitoring feasibility constraint*:*

$$\mu = (\mu_1, \mu_2) \leq \min\{x_i, x_j\}. \tag{9}$$

The grand contract is designed by the banks at time 0 and is offered on a take-it-or-leave it basis to investors.

4. Optimal contracts

4.1 *Benchmark case*

In this Section we consider optimal grand contracts when the only contracting friction is moral hazard; in Section 4.2 we consider the effects of limited liability and contract renegotiation. We write $\mathscr{C}$ for the set of grand contracts. We identify Pareto optimal grand contracts: that is, allocations that achieve optimality subject to the frictions identified in Section 2.2. The banks choose an element of $\mathscr{C}$ that maximizes the sum of their lifetime expected utilities (we prove later that this element exists).[5] Hence, the contracting problem is

$$V = \max_{K \in \mathscr{C}} \sum_i \mathbb{E}[v_i] + (1+\phi)(E - A_i). \tag{P}$$

By Equation (4), we have

$$v_1 + v_2 = \sum_i [(R - \omega_i)x_i - \eta C_i] + C_1 + C_2 - \sum_i (d_{1i} + d_{2i}). \tag{10}$$

Taking the expectation of Equation (10) and noting that the participation constraints (7) must bind in equilibrium, we have

$$\sum_i \mathbb{E}[v_i] = \sum_i (\mathbb{E}[(R - \omega_i)x_i] - \eta C_i) + \sum_i (C_i - D_i) \tag{11}$$

$$= \sum_i (\mathbb{E}[(R - \omega_i)x_i] - \eta C_i + A_i - I), \tag{12}$$

where Equation (12) follows from Equation (11) by summing the time 0 budget constraint (2) over i.

We can use Equation (12) to write problem (P) as follows:

$$V = \max_{K \in \mathscr{C}} \sum_i \mathbb{E}[(R - \omega_i)x_i] - 2I - \eta \sum_i C_i - \phi \sum_i A_i + (1+\phi)2E. \tag{P_1}$$

Equation (P_1) lays bare the incentives faced by banks. The first two terms in the equation constitute the project's Net Present Value as a function of the continuation decision x. The third term is the cost of holding cash, and the fourth is the cost of inside equity; the final term is a constant.

We present a formal analysis of the optimal grand contract in the proof of Proposition 1 in the Appendix; we present here some intuition for our results. First, because cash holdings are unambiguously costly and do not affect the incentive constraints, we must have $C_1 = C_2 = 0$.

Second, it is clear from equation (P_1) that the internal equity holdings A_1 and A_2 should be set as low as possible. Those holdings are constrained by the need to provide bankers with adequate incentives. To see how, note that the maintenance effort incentive constraint (6) can be written as follows:

$$\begin{aligned} \mathbb{E}[v_1] &\geq \frac{1}{2}\left[\frac{B}{\Delta} + v_1(\rho,\rho) + v_1(\rho,0)\right]; \\ \mathbb{E}[v_2] &\geq \frac{1}{2}\left[\frac{B}{\Delta} + v_2(\rho,\rho) + v_2(0,\rho)\right]. \end{aligned} \tag{13}$$

[5] An alternative approach would be explicitly to model the non-cooperative game used to select a grand contract, and to identify the Nash equilibria of that game. Our approach generates a simpler and cleaner analysis, and does not require us to take a view on the precise nature of the interaction between banks. Moreover, we anticipate that a non-cooperative analysis would yield similar results; we discuss this in greater depth below.

The constraints (13) establish the minimum amount of rent required to create ex ante incentives for maintenance rent. Summing the constraints, substituting into Equation (12) and rearranging yields Equation (14):

$$\sum_i A_i \geq 2I - \sum_i \mathbb{E}\left[(R-\omega_i)x_i\right] + \frac{B}{\Delta} + \frac{1}{2}\left(v_1(\rho,\rho) + v_1(\rho,0) + v_2(\rho,\rho) + v_2(0,\rho)\right). \tag{14}$$

It is clear from Equation (P_1) that, because banks are impatient ($\phi > 0$), inside equity is costly: consequently, Equation (14) binds. The final bracketed term in Equation (14) comprises the payoffs achieved by banks 1 and 2 in case of liquidity shocks. Incentives can be sharpened, and inside equity holdings can be reduced, by closing banks upon liquidity shocks, so that the continuation payoffs are zero. Early closure is inefficient, because $R > \omega_i$; it is therefore selected only when bank impatience ϕ is sufficiently high. For low ϕ, banks elect instead to maintain high levels of inside equity and not to close liquidity-shocked banks.

Proposition 1. *Provided*

$$\phi > \phi^* \equiv \frac{1}{\frac{f+M}{R-\rho} - 1}, \tag{15}$$

under the optimal grand contract, banks are closed at time 1 *if and only if they experience a liquidity shock.*

Note that, by Assumption (A2), $\phi^* > 0$. Moreover, neither closure nor high equity holdings relies upon the Limited Liability constraint (8), and the proof of Proposition 1 demonstrates that this constraint is slack at the optimum.

The intuition for Proposition 1 is straightforward. At date 0 banks wish to find the cheapest way to commit themselves to perform maintenance effort. There are two ways to provide the necessary effort incentive. First, the bank could elect to reduce its continuation value after a liquidity shock. This approach requires inefficient closure, because the bank requires a minimum stake to induce managerial effort. Alternatively, the bank can retain sufficient inside equity A to induce effort. But inside equity is costly, because bankers are impatient. The latter effect outweighs the first, so that the threat of closure is a more efficient incentive mechanism, when the cost ϕ to the banker of inside equity is higher than the threshold value ϕ^*.

It is immediate from Equation (P_1) that any grand contract is characterised by x, C, and A. We can use this fact to derive the following characterization of optimal grand contracts:

Corollary 1. *Let K be a solution to the program (P) that achieves value V, and suppose that $\phi > \phi^*$, so that banks are closed after liquidity shocks under K. Then K satisfies the following* *conditions:*

$$x_i(\omega) = \begin{cases} 1, & \text{if } \omega_i = 0; \\ 0, & \text{otherwise.} \end{cases} \tag{16}$$

$$C_i = 0; \tag{17}$$

$$\sum_i A_i = 2I - \left(R - \frac{B}{\Delta}\right). \tag{18}$$

Proof. Equation (16) follows from Proposition 1 and the fact that $\phi > \phi^*$, Equation (17) from the fact that it is never optimal under Program (P) to hold cash, and Equation (18) is obtained by letting Equation (14) bind and noting that, when liquidity-shocked banks are closed, $\sum_i \mathbb{E}[(R-\omega_i)x_i)] = R$.

4.2 *Renegotiation, limited liability and systemic risk*

Section 4.1 characterizes the optimal contract when perfect commitment is possible. We now relax that assumption, by allowing the parties to a grand contract to renegotiate it at time 1. The optimal time 0 contract must therefore be renegotiation proof.

Definition 3. *Let K be a grand contract. A time* 1 contract renegotiation $\hat{K}$ *of K is a grand contract obtained by making the following modifications to K:*

1. *Changing the continuation decisions (x_1, x_2) to $(\hat{x}_1, \hat{x}_2)$;*
2. *Changing the retail deposit contract payments (d_1, d_2) to $(\hat{d}_1, \hat{d}_2)$;*
3. *Changing the interbank deposit contract payments (b_1, b_2) to $(\hat{b}_1, \hat{b}_2)$;*
4. *Changing the monitoring profiles (μ_1, μ_2) to $(\hat{\mu}_1, \hat{\mu}_2)$.*

We write $\hat{\mathscr{C}}(K)$ for the set of contract renegotiations of K, and we write $\hat{v}_i$ for the continuation payoff of bank i after contract renegotiation.

Note in particular that any grand contract renegotiation must satisfy the time 1 budget constraint (3), the monitoring incentive compatibility constraint (5), and the monitoring feasibility constraint (9).

We assume that renegotiation occurs precisely when it can generate a Pareto improvement among banks and investors. We make no assumption about bargaining power during the time 1 renegotiation process. Our results therefore encompass set-ups in which one party makes take-it-or-leave-it offers, as well as the standard Nash and Shapley Value bargaining solutions.

Definition 4. *A grand contract K is* renegotiation proof (RP) *if and only if there is no renegotiation $\hat{K} \in \hat{\mathscr{C}}(K)$ such that for all i and all ω, $\hat{v}_i \geq v_i$ and $\hat{d}_{1i} + \hat{d}_{2i} \geq d_{1i} + d_{2i}$ with at least one strict inequality for some i and ω.*

We write

$$S(x,\hat{x}) = \sum_i (\hat{x}_i - x_i)(R - \omega_i) \tag{19}$$

for the (state-contingent) surplus created by renegotiating continuation decisions from x to $\hat{x}$. Clearly, a grand contract is renegotiation proof if its continuation decisions are interim efficient, so that no such surplus can be created. In a frictionless model, this condition would also be necessary, because every interim inefficient contract would be renegotiated. However, some inefficient contracts can survive renegotiation in our model, because a renegotiation $\hat{K}$ is possible only if the revised monitoring level $\hat{\mu}$ is incentive compatible. Lemma 1 establishes necessary and sufficient conditions for this to be the case.

Lemma 1. *A grand contract K is renegotiation proof if and only if for all $\hat{x}$ and $\hat{\mu}$ satisfying the monitoring feasibility constraint* (9)*, the following is satisfied in all states ω such that $S(x,\hat{x}) > 0$:*

$$S(x,\hat{x}) < \sum_i \max\left\{\hat{x}_i F - \hat{\mu}_j(F-f) + \hat{\mu}_i M - v_i, 0\right\}. \tag{20}$$

The intuition for Equation (20) is straightforward. Bank i requires total rent $\hat{x}_i F - \hat{\mu}_j(F-f) + \hat{\mu}_i M$ to perform the monitoring and managerial effort called for by a grand contract renegotiation $\hat{K}$. The total *additional* rent requirement for the renegotiation is therefore given by the right hand side of Equation (20); renegotiation cannot occur if this figure exceeds the surplus $S(x,\hat{x})$ generated by the renegotiation.

When renegotiation is possible, any payoff that can be achieved by a grand contract K that is subsequently renegotiated to $\hat{K}$ can also be achieved by grand contract $\hat{K}$. Hence, it is without loss of generality in this case to impose the requirement that grand contracts be renegotiation proof. The time 0 contracting problem when time 1 contract renegotiation is possible can therefore be written as follows:

$$V^{RP} = \max_{K \in \mathscr{C}^{RP}} \sum_i \mathbb{E}[v_i] + (1+\phi)(E - A_i), \tag{P^{RP}}$$

where $\mathscr{C}^{RP}$ is the set of renegotiation proof grand contracts.

We now determine under what circumstances program (P^{RP}) yields a lower maximum welfare than program (P); in other words, when the parties' inability to commit not to renegotiate a grand contract is welfare-diminishing.

Lemma 2. *Suppose that the grand contract K solves program* (P)*. Then K is renegotiation proof if and only if Condition* (21) *and the parametric restriction* (22) *are both satisfied:*

$$v_1(0,\rho) = v_2(\rho,0) = F; \tag{21}$$

$$f + \frac{M}{z} > R - \rho, \tag{22}$$

where $z = (F - f)/M > 1$ is a measure of the efficiency of the monitoring technology.

The intuition for Lemma 2 is as follows. First, recall that the optimal contract requires bank 1 to close and bank 2 to continue in state $(\rho, 0)$. But, if bank 2 is able to offer at least partial refinancing to bank 1 then it will do so. This type of renegotiation can only be ruled out if bank 2 has no financial slack in its continuation value, so that it cannot credibly commit to monitor bank 1. This is achieved by setting bank 2's value in state $(\rho, 0)$ at the minimum level required to ensure that it continues its own project, as in Condition (P^{RP}).

Now note that, by Assumption (A2), the parameter restriction (22) is guaranteed to be true when $z = 1$. In other words, Equation (22) is a requirement that monitoring not be too efficient. We need to impose this in order to rule out a pathological two-step renegotiation under which bank 1 first monitors bank 2 in order to *create* slack in 2's continuation value, and bank 2 then partially refinances and monitors bank 1.

We now present our key result: renegotiation and limited liability reduce the profitability of banks unless projects are *sufficiently correlated* across banks.

Proposition 2. *Suppose that $\phi > \phi^*$, so that liquidity-shocked banks are closed under the solution K to the benchmark problem (P). When contracts can be renegotiated, the optimal contract K can be achieved if the parametric restriction (22) is satisfied by setting the correlation parameter β to exceed $\beta^* = \frac{B/\Delta - F}{R - F}$.*

When $\phi > \phi^*$, so that the optimal bank contract calls for banks to be closed if they experience a liquidity shock, Proposition 2 establishes that the correlation parameter β is critically important when renegotiation is possible: the optimal contract can be achieved only if the project correlation β is sufficiently high.

The intuition for this result is as follows. Recall that the financial slack of well-performing banks must be reduced so as to prevent contract renegotiation with distressed banks. Hence, the optimal contract requires depositors to receive a high payment in the states where their bank is not liquidity shocked, but the other bank is. The bank's optimal profit can be realised only if depositors receive a low payment in the "boom" states where neither bank is liquidity shocked; the size of the payment in these boom states is decreasing in the probability that they arise and, hence, in β. But the payment to bank depositors in boom states is bounded by limited liability; this requirement places a lower bound upon the correlation parameter β in the optimal contract.

If cash holdings are costless, the optimal contract can be achieved without high correlation. Suppose that each bank can hold a large amount of cash at date 0, funded by retail deposits. The renegotiation-proof contract then specifies that bank 1 (say) repays the cash with interest in state $(0, \rho)$, but pays back less than the full amount of cash in state $(0, 0)$. The interest rate in state $(0, \rho)$ can be specified to satisfy condition (21) without violating limited liability. However, we are not aware of such contracts being written in practice, and Proposition 2 goes through for an arbitrarily small cost of holding cash.

5. Implementation

The analysis of Section 4 identifies optimal state-contingent contracts. In this Section, we show how these contracts can be implemented with standard financial instruments.

In each of the implementations that we consider, each bank contributes inside equity A and raises D of term deposits at date 0, with a promised fixed repayment d in the future.[6]

Term deposits achieve first best when it is possible to rule out renegotiation. In that case, each bank includes covenants in its deposit contracts that preclude any interbank loans at date 1, and so enforce closure threats.

Term deposits on their own do not achieve first best when renegotiation is possible. In that case, covenants that rule out interbank date 1 lending will not be enforced, because it is in everyone's interest to avoid closure *ex post*. As demonstrated in Lemma 2, it is necessary in this case to reduce the financial slack of well-performing banks in states where the other bank is performing badly. This can be achieved by creating a network of mutual exposures between banks, using either credit default swaps or interbank debt.

Our implementation analysis is consistent with summary interbank exposure data before the crisis. In 2007, the notional value of outstanding credit default swaps was between \$45 and \$62 trn, against \$5 trn of outstanding corporate bonds (Brunnermeier 2009, pp. 96–7). One of the reasons for this excess was banks' purchase of offsetting credit protection against one another (Brunnermeier 2009, pp. 97–8). Babus (2015) argues that this type of mutual insurance occurs endogenously so as to mitigate contagion risk in the financial sector. But, as Allen and Gale (2000) demonstrate, network structure is important: inadequately connected networks are more subject to systemic risk than unconnected ones, although Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015) argue that dense interconnections worsen contagion when shocks are big enough; several authors suggest that the network topology prior to the 2008–09 financial crisis served to increase systemic risk, rather than to dampen it. For example, Haldane (2009) notes that Lehman's gross CDS exposures were about eight times its balance sheet size, and suggests that the associated network of interbank claims served as a conduit for financial contagion, and Hale, Kapan, and Minoiu (2014) demonstrate using a panel of banks from 110 countries that more connected banks were less profitable from 1997–2012. Our analysis below indicates that financial institutions may have selected systemically risky interbank exposures so as to commit not to refinance and, hence, to implement their (privately) optimal contract.

[6] Alternatively, banks could raise short-term deposits that are rolled over at date 1. Provided that depositors can realise a liquidation value $\varepsilon > 0$, depositors strictly prefer to withdraw at date 1, so that states with closure trigger bank runs.

5.1 *Credit default swaps*

Suppose that, in addition to raising term deposits, each bank sells a *credit default swap* on the other bank at date 0; we denote by CDS_i the credit default swap that bank i sells. Under the terms of CDS_i, bank i is paid P at date 0, and contracts to pay out n at date 2 if bank $j \neq i$ defaults at date 1. Bank i defaults on CDS_i when it is not solvent. Recall that in the optimal contract, a bank is closed if and only if it experiences a liquidity shock. Hence, for example, CDS_1 only pays out in state $(0,\rho)$.

The CDS trade serves to reduce the continuation value of healthy banks whenever the other bank experiences a liquidity shock. When bank i does not experience a liquidity shock, its time 1 continuation value is $R-d$ if bank j is not shocked, and is $R-d-n$ when bank j experiences a shock. By Lemma 2, provided $R-d-n=F$ renegotiation does not occur when only one bank experiences a liquidity shock.

Proposition 3. *Suppose that $\phi > \phi^*$. Then there exist a simple deposit contract (D,d) and a CDS contract (P,n) which are renegotiation-proof and implement the optimal grand contract. The deposit contract respects the limited liability of investors ($d>0$) if and only if $\beta > \beta^\star$.*

The banks set $\beta > \beta^*$ for the reasons outlined after Proposition 2. When one bank experiences a liquidity shock and the other does not, renegotiation proofness requires the unshocked bank to have a low enough continuation value; it is only possible to so this without violating the limited liability constraint when the return correlation β is high enough.

5.2 *Interbank debt*

Now suppose that, in addition to raising term deposits, each bank writes an interbank debt contract. Each bank transfers B to the other at date 0, and promises to repay b at date 2. Each bank honors its repayment promise only if it does not experience a liquidity shock; the payment b is made to the other bank if it is solvent, and to its investors if it is not.

When neither bank experiences a liquidity shock, each pays the other b, so that neither experiences a cost. If only one bank is liquidity shocked, the other's continuation value is reduced by its repayment b. As with Credit Default Swaps, the interbank contract can be designed to ensure that Equation (21) is satisfied and, hence, that first best is achieved with renegotiation.

Proposition 4. *Suppose that $\phi > \phi^*$. Then there exist a simple deposit contract (D,d) and an interbank contract (B,b) that are renegotiation-proof and implement the optimal grand contract. The deposit contract respects the limited liability of investors ($d>0$) if and only if $\beta > \beta^\star$.*

6. Policy responses

Proposition 2 demonstrates that it can be privately optimal for banks to select highly correlated investments so as to provide adequate managerial effort incentives. But this choice increases the probability that all banks will fail simultaneously. That is, the privately optimal decisions of banks could raise the probability that all banks fail. Such a systemic banking failure is socially very damaging, and so policy makers may wish to intervene to correct banks' incentives to create systemic risk.[7]

If banks' choices were perfectly observable, governments could simply make correlation illegal. Casual empiricism suggests that this is a practical impossibility. We consider the more challenging case where regulators cannot observe correlation choices, and so are forced to rely on indirect strategies.

In our model, banks optimally create project correlation so as to render closure threats credible, and so to generate cheap managerial effort incentives. Policy makers can therefore remove the incentive to create systemic risk in two ways. First, they can force banks to adopt an alternative device to incentivize managerial effort. Second, they can render closure threats in liquidity shocked states incredible, and so induce banks to select alternative incentive mechanisms. We demonstrate below that the first of these approaches is achieved via bank capital requirements, and that the second is achieved through a properly designed Lender of Last Resort policy.

6.1 *Bank capital requirements*

Suppose that a regulator imposes a minimum inside equity requirement on all banks:

$$A_i \geq A_{min}. \tag{23}$$

We assume that this rule is perfectly enforceable. Banks then design their optimal grand contract as before, but subject to Condition (23). Then, provided A_{min} is high enough, banks opt for high managerial effort to protect their "skin in the game," irrespective of the correlation parameter β.

Proposition 5. *Suppose that $\phi > \phi^*$, banks are constrained by* (23)*, and that A_{min} satisfies*

$$A_{min} \geq \frac{B}{2\Delta} + f + M - \left(R - \frac{\rho}{2} - I\right). \tag{24}$$

If the cost of holding cash η is sufficiently small, then, regardless of whether date 1 *contract renegotiation is possible, banks are indifferent between all $\beta \in (0,1)$.*

[7] At the cost of greater notational complexity, we could present this argument using an explicit regulator objective function that included the private value realized in the banking sector as well as a social cost of simultaneous closure of both banks, for example due to fire sale effects, as in Lorenzoni (2008). The policy responses that we present in this Section would be optimal in such an analysis. Alternatively, the private cost ϕ of inside equity might represent a transfer rather than a social cost. This would be the case if ϕ reflected the corporate tax advantage of debt.

When banks have sufficient inside equity at stake, they exert managerial effort whether or not they run the risk of closure after a liquidity shock. Indeed, closure is value-destructive, and can never be optimal: provided the cost η of holding cash against correlated liquidity shocks is sufficiently low, banks never close at time 1. And, when the threat of closure is not needed to provide incentives, banks are indifferent between correlation levels, β.

6.2 *Lender of last resort*

When banks select correlated investment returns, it is to reduce the likelihood that a date 1 lender with the requisite monitoring skills will be able to provide continuation finance to a liquidity-shocked bank. The resultant incentive effects would therefore be destroyed if banks could be sure that a deep-pocketed investor with specialist monitoring skills would be present at time 1. This is the role of a Lender of Last Resort (LOLR).

The classic exposition of the LOLR is due to Bagehot (1873 [1999]); for a recent discussion of the LOLR in the wake of the 2008–09 financial crisis, see Gorton and Metrick (2013). The standard explanation for the LOLR is that, by ensuring that banks have sufficient liquidity, it can prevent that value destruction that would ensue if the bank was forced to liquidate assets at fire-sale prices in the face of a run by retail or wholesale depositors. But, in providing emergency funds, the LOLR ensures that distressed banks remain open when they have a positive net value. Hence, the LOLR re-introduces the soft budget constraint problem that bank correlation is intended to prevent. Banks are therefore forced to find alternative devices to incentivize managerial monitoring. They respond by increasing their capitalization.

We model the LOLR as follows. The LOLR makes a state-contingent transfer L_i to bank i at date 1, in return for repayment l_i at date 2. The LOLR monitors λ_i of bank i's project; projects can only be monitored once, so $\lambda_i \leq x_i - \mu_j$. With LOLR monitoring, the managerial incentive compatibility constraint (5) is rewritten as follows:

$$v_i \geq x_i F - (\mu_j + \lambda_i)(F - f) + \mu_i M. \tag{25}$$

Equation (25) replaces (5) in the grand contract definition. We capture the classical prescription the lenders of last resort should only lend against good collateral by assuming that the LOLR only participates in a grand contract if it breaks even. Moreover, we assume that the LOLR does not need to earn a monitoring rent from its bank lending, because it places a very high social value on bank survival.

With this definition of the LOLR, banks are always able to attract date 1 funding and, as a result, bank incentives to select correlated investments disappear:

Proposition 6. *Suppose that $\phi > \phi^*$. When there is a Lender of Last Resort, banks are indifferent between all $\beta \in (0,1)$. Every bank raises inside equity A_{min}, where A_{min} is defined by Equation*

(24).

The LOLR renders closure threats incredible and, as a result, banks have no incentive to create systemic risk in order to incentivize their managerial effort. Bankers are forced to use capital requirements to commit to managerial effort and, hence, capital levels rise without any need for direct regulation.

Many authors have suggested that a LOLR could generate moral hazard problems (for a brief survey, see Carlson and Wheelock (2012)); we demonstrate in the following Section that, when a LOLR provides an implicit subsidy, this fear is well-founded. But Proposition 6 demonstrates that a correctly designed LOLR renders the banking system more stable without creating moral hazard. This stability comes at a cost to bankers, because, absent a LOLR, they would prefer to be undercapitalized and to create systemic risk. But, to the extent that the LOLR addresses a social problem, we can think of the costs that imposes upon bankers as a form of Pigouvian tax.

6.3 *Vulture Investors*

Recall from the discussion following Proposition 1 that banks select high asset return correlations, and so generate systemic risk, because it is impossible to find an alternative way to commit at time 0 to remove the manager of a liquidity shocked bank. A specialist investor able to take control of a liquidity-shocked bank and to remove its management would provide maintenance effort incentives and, hence, would obviate the need for systemic risk. "Vulture investors," which specialise in exercising control rights over distressed institutions, perform exactly this role. For evidence that vulture investments realise value, see Guo, Hotchkiss, and Song (2011) and Hotchkiss and Mooradian (1997). Hence, and in contrast to views expressed by some press commentators, we identify a positive social role for vulture investors in distressed banks. Specifically, our model suggests that legislation to outlaw corporate vulture investment might actually increase systemic risk in the banking sector.

7. Bailouts and correlation

Section 6.2 demonstrates that state support in the form of a LOLR results in better capitalized banks and lower systemic risk. This effect arises because the LOLR breaks even and, hence, does not pay a subsidy to banks. Of course, when a state subsidy is available, banks have an incentive to pursue it. As Acharya and Yorulmazer (2007) and Farhi and Tirole (2012) note, when the subsidy is more likely to be paid at times of systemic failure, banks have a strong incentive to select correlated investments.

We can capture this effect in our model. To do so, define a *bailout* to be a real transfer S made by the state to each bank whenever both experience a liquidity shock. We do not model the state's preferences, or the interaction of S with bank choices: for an analysis of these issues, see Acharya and Yorulmazer (2007).

Proposition 7. *Suppose that $\phi > \phi^*$, and that the state provides a bailout of S to each bank when both experience a liquidity shock. Then the correlation parameter β is equal to 1 under any optimal grand contract. Moreover, there exists a threshold bailout $S^\star$ such that banks make no maintenance effort and raise zero inside equity whenever $S > S^\star$. The threshold $S^\star$ is lower, and maintenance effort becomes less likely, when there is a lender of last resort.*

The intuition for Proposition 7 is as follows.

First, a bailout S augments the value of any grand contract by the expected value of state support, which is

$$2Pr[\omega = (\rho,\rho)]S = S\beta.$$

Because this expression is increasing in β, the optimal choice is to set $\beta = 1$. Hence, bailouts strictly increase the incentive to correlation, because, absent bailouts, banks are indifferent between all $\beta > \beta^\star$ in the absence of bailouts.

Second, date 0 maintenance effort has two offsetting effects. First, it increases the expected payoff to banks. Second, is decreases the expected value of the state's bailout by

$$2(2-\beta)\Delta S.$$

When S is large enough, the second effect dominates and banks make no maintenance effort.

Third, inside equity is costly and banks hold it only to commit to managerial effort. Hence, when $S > S^*$ and when banks perform no maintenance effort, there is no role for inside equity.

Fourth, when there is a LOLR, managerial effort can only be incentivized using costly inside equity. Hence, a LOLR renders managerial effort less attractive top banks, and so makes it more likely that banks give up on incentivizing maintenance effort.

8. Empirical Implications

This Section identifies some empirical implications of our analysis.

First, note that bank impatience ϕ appears in Program (P_1) as a negative coefficient on inside equity A. We can therefore think of ϕ as capturing the relative cost of retaining inside equity over accessing external debt. The most obvious manifestation of this cost is in the relative tax treatments of debt and equity: as the tax advantage of debt increases, so, too, does the impatience parameter ϕ. It follows immediately from Proposition 1 that the optimal grand contract calls for bank closure upon liquidity shock only when the tax advantage of corporate debt is sufficiently high. We know from Proposition 2 that banks achieve the optimal contract when renegotiation is possible by selecting correlated investments. The following hypothesis is therefore immediate:

Hypothesis 1. *When the tax advantage of debt increases, so too does the correlation of bank portfolio returns.*

Propositions 3 and 4 demonstrate that, when it is optimal to commit to close liquidity shocked banks, banks can achieve the optimal contract by writing protection on one another, or by lending to each other. These results therefore generate hypotheses 2 and 3.

Hypothesis 2. *When the tax advantage of debt increases, so too does the level of protection that banks sell on one another in the credit default swap market.*

Hypothesis 3. *When the tax advantage of debt increases, banks lend more to one another in the interbank market.*

Section 5 demonstrates that any policy measure that reduces the likelihood that banks are closed after a liquidity shock reduces incentives to select correlated investment portfolios. Proposition 5 demonstrates that capital requirements have this effect, and leads immediately to hypothesis 4.

Hypothesis 4. *The correlation between bank portfolio returns in countries that have high capital requirements is lower than the correlation between bank portfolio returns in countries with low capital requirements.*

Proposition 6 demonstrates that Lenders of Last Resort induce banks to select less correlated investments, and to hold more capital. But, as stated in Proposition 7, when the LOLR is combined with a bailout policy, this effect is reversed. Most advanced economies have both a Lender of Last Resort and were revealed by their actions during the 2008–09 financial crisis to operate a bailout policy. Hence, as a practical matter, it is difficult to establish whether the effects of Proposition 6 or of Proposition 7 dominate. We identify two ways that one could do so.

First, while measures intended to render bailouts illegal are arguably incredible, policies that reduce the need to inject funds into systemically shocked banking systems are not, and they do not affect the authorities' ability to provide Lender of Last Resort facilities. Several European countries reformed bank resolution procedures to reduce the likelihood of bailout after the crisis: hypothesis 5 spells out their consequences in our model.

Hypothesis 5. *Countries with better bank resolution procedures have higher bank capitalisation and lower return correlation between banks.*

Second, as in hypothesis 6, a country's fiscal base affects its ability to perform bailouts and, by extension, its systemic risk:

Hypothesis 6. *Countries with a weaker fiscal base should have lower return correlations across bank portfolios, and also higher bank capital levels.*

9. Conclusion

This paper demonstrates that systemic risk can arise as bankers' privately optimal response to incentive problems. Banks seek the cheapest means to commit themselves to reduce their exposure to liquidity shocks. They can do so either by ensuring that they close after a liquidity shock, or by retaining a sufficiently large inside equity exposure. A closure threat is more cost-effective when equity is sufficiently costly from a private perspective: for example, when there are substantial tax advantages to debt issuance. When contracts can be renegotiated, banks that wish to commit to close when they are liquidity shocked must ensure that they cannot access an alternative lender. They accomplish this by selecting correlated investments—in other words, by hard-wiring systemic risk into their portfolios. We demonstrate that this choice comes with a higher volume of bank CDS trades and of interbank lending.

The effect that we identify does not arise because banks seek bailout rents, although we are able to model this effect in our framework. Rather, systemic risk is a solution to a private contracting problem. Any regulatory device that controls systemic risk therefore constrains the contract space, and so imposes costs upon banks. Every regulatory response to systemic risk in our model involves higher capital requirements, either directly or as an endogenous consequence of a regulatory decision that prevents the closure of a liquidity-shocked bank.

Our paper speaks to some contemporary policy debates. In the wake of the 2008–09 financial crisis, international financial regulation started to impose formal liquidity requirements upon banks in addition to capital requirements. This trend is contrary to traditional analyses, in which a Lender of Last Resort ensures an adequate supply of liquidity to solvent institutions in stressed environments, and solvency is achieved through capital adequacy regulations. Our analysis suggests that the traditional perspective should not be abandoned: indeed, in our analysis, an effective Lender of Last Resort is sufficient to induce higher capital requirements, and so to reduce systemic risk. The incentive problems that motivate current regulations arise only when the Lender of Last Resort provides a net subsidy to the banking system. In short, our analysis suggests that a crude, and high, capital requirement remains the best protection against systemic risk.

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APPENDIX

Proof of Proposition 1

Let $\mathscr{C}_0$ be the set of grand contracts which satisfy all requirements except the Limited Liability constraint (8). We consider a relaxed version of problem (P) which does not require Limited Liability:

$$V_0 = \max_{K \in \mathscr{C}_0} \sum_i \mathbb{E}[v_i] + (1+\phi)(E - A_i) \tag{P0}$$

In this problem, it is immediate that, no matter how small η is, it can never be optimal to hold cash, so that we can impose $C_i = 0$. We can then write problem (P0) as follows:

$$\begin{aligned}
V_0 &= \max_{\substack{K \in \mathscr{C}_0 \\ s.t.\ C=0}} \left(\sum_i \mathbb{E}\left[(R-\omega_i)x_i\right] - \phi \sum_i A_i \right) + 2\left(E(1+\phi) - I\right) \\
&= \max_{\bar{x}} \max_{\substack{K \in \mathscr{C}_0 \\ s.t.\ C=0 \\ x=\bar{x}}} \left(\sum_i \mathbb{E}\left[(R-\omega_i)\bar{x}_i\right] - \phi \sum_i A_i \right) + 2\left(E(1+\phi) - I\right) \\
&= \max_{\bar{x}} \left(\sum_i \mathbb{E}\left[(R-\omega_i)\bar{x}_i\right] - \phi \min_{\substack{K \in \mathscr{C} \\ s.t.\ x=\bar{x} \\ C=0}} \sum_i A_i \right) + 2\left(E(1+\phi) - I\right) \\
&= \max_{\bar{x}} \left(\sum_i \mathbb{E}\left[(R-\omega_i)\bar{x}_i\right] - \phi \bar{A}(\bar{x}) \right) + 2\left(E(1+\phi) - I\right),
\end{aligned} \tag{26}$$

where

$$\bar{A}(\bar{x}) = \min_{\substack{K \in \mathscr{C}_0 \\ s.t.\ x=\bar{x} \\ C=0}} \sum_i A_i. \tag{27}$$

We establish the result in three steps. First, we solve the equity minimization problem (27). Second, we show that when $\phi > \phi^\star$, any solution to the relaxed problem must have $x_i = 1(\omega_i = 0)$, where $1(.)$ is the indicator function. Third, we show that the Limited Liability constraint is slack at the solution, so that this continuation policy is also optimal in problem (P).

Step 1: Solving the equity minimization problem (27). — We start by noting that *maximal monitoring*, with monitoring decisions given by $\bar{\mu}_i = \min\{\bar{x}_1, \bar{x}_2\}$, is optimal in problem (27). Take a pair $(K', \mu') \in \mathscr{C}$ which achieves the minimum in (27), and substitute $\bar{\mu}$ for μ'. The contract K' may no longer be feasible; the Monitoring Effort IC constraint (5) may be violated. However, if we can show that the substitution does not increase the *aggregate* date 2 rent required in any state, we can construct a contract $\bar{K}$ which is identical to K' up to a side payment between banks which redistributes rents to ensure (5) holds. Hence, it suffices to show that (summing the right-hand side of (5) before and after the change to obtain aggregate rents)

$$\begin{aligned}
\sum_i \bar{x}_i F - \mu'_i(F - f - M) &\geq \sum_i \left[\bar{x}_i F - min\{\bar{x}_1, \bar{x}_2\}(F - f - M)\right] \\
\Leftrightarrow (F - f - m) \cdot \sum_i \left[min\{\bar{x}_1, \bar{x}_2\} - \mu'_i\right] &\geq 0.
\end{aligned}$$

This is true by Assumption (A1).

Next, combining the Monitoring Incentive constraint (5) with Equation (14), and imposing maximal monitoring, we have

$$\begin{aligned}\sum_i A_i \;\; &\geq \;\; 2I - \sum_i \mathbb{E}[(R-\omega_i)\bar{x}_i] + \frac{B}{\Delta} \\ &\quad + \frac{1}{2}\sum_i \sum_{\omega:\omega_i=\rho} [\bar{x}_i(\omega)F - \min\{\bar{x}_1(\omega), \bar{x}_2(\omega)\}(F-f-M)]. \qquad (28)\end{aligned}$$

This gives a lower bound on $\sum_i A_i$, and we can verify that there is a feasible contract K which achieves the lower bound. Hence $\bar{A}(\bar{x})$ is equal to the right-hand side of (28).

Step 2: Optimal continuation decisions in the relaxed problem. — Using our characterization of $\bar{A}(\bar{x})$, we find that the optimal continuation decisions x in problem (26), which are also the optimal continuation decisions in the relaxed problem (P0), must maximize the function

$$H(\bar{x}) = (1+\phi)\sum_i \mathbb{E}[(R-\omega_i)\bar{x}_i] - \frac{\phi}{2}\sum_i \sum_{\omega':\omega'_i=\rho} \left[\bar{x}_i(\omega')F - \min\{\bar{x}_1(\omega'), \bar{x}_2(\omega')\}(F-f-M)\right].$$

We prove that it is optimal to set $x_i = 1(\omega_i = 0)$. First, we show that $\omega_i = \rho \Rightarrow x_i = 0$. Second, we show that $\omega_i = 0 \Rightarrow x_i = 1$. Together, they imply $x_i = \mathbb{1}(\omega_i = 0)$ as required.

For the first part, suppose that there is a state ω' such that $\omega'_i = \rho$ and $x_i(\omega') > 0$. The function $H(\bar{x})$ is differentiable with respect to $\bar{x}_i(\omega')$ almost everywhere, and at every point of differentiability, i.e. where $x_i(\omega') \neq x_j(\omega')$,

$$\begin{aligned}\frac{\partial W(\bar{x})}{\partial \bar{x}_i(\omega')} &= (1+\phi)\mathbb{P}[\omega = \omega'](R-\rho) - \frac{\phi}{2} \times \begin{cases} F, & x_i(\omega') > x_j(\omega') \\ f+M, & x_i(\omega') < x_j(\omega') \end{cases} \\ &\leq (1+\phi)\mathbb{P}[\omega = \omega'](R-\rho) - \frac{\phi}{2}(f+M) \\ &\leq -\frac{\phi}{2}\left[f + M - \left(1+\frac{1}{\phi}\right)(R-\rho)\right],\end{aligned}$$

where the first inequality follows from Assumption (A1) and the second from $\mathbb{P}[\omega = \omega'] \leq 1/2$. Given $\phi > \phi^{\star}$, this is strictly negative, so that it would strictly increase the objective to set $x_i(\omega') = 0$, contradicting optimality.

For the second part, note that having imposed $x_i(\omega') = 0$ whenever $\omega'_i = \rho$, the remaining continuation decisions $x_i^0 = \{x_i(\omega')\}_{\omega':\omega_i'=0}$ maximize the function

$$H^0(\bar{x}_i^0) = (1+\phi)R \times \mathbb{E}[\bar{x}_i|\omega_i = 0],$$

which is trivially achieved by setting $x_i(\omega') = 1$ whenever $\omega_i' = 0$.

Step 3: Slackness of the Limited Liability constraint. — We use the characterization of $\bar{A}(\bar{x})$ and the optimal continuation decisions to arrive at the following maximized value of the relaxed problem:

$$V_0 = R - \phi\left(2I - R + \frac{B}{\Delta}\right) + 2(E(1+\phi) - I). \tag{29}$$

It is sufficient to construct a contract that achieves this value and satisfies limited liability. Set $x_i = 1(\omega_i = 0)$ and $A_i = \bar{A}(x)/2$. Choose an arbitrary correlation parameter β and set all interbank payments equal to zero: $B_i = b_{1i} = b_{2i} = 0$. Choose the initial deposit to satisfy the time 0 budget constraint: $D_i = I - A_i$, and specify a fixed repayment to depositors at time 2, which arrives if and only if bank i is not liquidity-shocked at time 1: $d_{1i} = 0$ and

$$d_{2i} = dx_i.$$

Finally, pick the repayment d such that depositors just break even: $d_i = 2D_i$. It is easy to verify that this contract achieves V_0 and satisfies all constraints including limited liability.

Proof of Lemma 1.

Consider and arbitrary grand contract K. We establish necessity and sufficiency of the proposed condition for renegotiation-proofness in the sense of Definition 3.

Step 1: Necessity (only if). — Suppose there exists a $\hat{x}$ and $\hat{\mu}$ satisfying (9) such that in some state ω, $S(x,\hat{x}) > 0$ and (20) is violated. Consider a renegotiation $\hat{K}$, which adapts the continuation policy $\hat{x}$ and the monitoring policy $\hat{\mu}$.

Let $\hat{d}_{1i} = d_{1i} - \omega_i(\hat{x}_i - x_i)$, and $\hat{d}_{2i} = d_{2i} + \omega_i(\hat{x}_i - x_i)$, so that investors cover any increased reinvestment requirements under $\hat{x}$ and get compensated at date 2. Then the resource constraint (3) is satisfied trivially.

We show that the contract K cannot be renegotiation proof by considering two cases. First, suppose $\hat{x}_1 F - \hat{\mu}_2(F-f) + \hat{\mu}_1 M > v_1$, so that bank 1's rents under the contract K are insufficient for its obligations under $\hat{K}$. Pick side payments $\hat{b}_{1i}, \hat{b}_{2i}$ which ensure that $\hat{v}_1 = \hat{x}_1 F - \hat{\mu}_2(F-f) + \hat{\mu}_1 M > v_1$. Summing (4) across i we have $\sum_i(\hat{v}_i - v_i) = S(x,\hat{x})$, and using the assumed violation of

(20) this yields

$$\begin{aligned}\sum_i(\hat{v}_i - v_i) &= \hat{x}_1 F - \hat{\mu}_2(F-f) + \hat{\mu}_1 M - v_1 + \hat{v}_2 - v_2 \\ &\geq \sum_i \max\{\hat{x}_i F - \hat{\mu}_j(F-f) + \hat{\mu}_i M - v_i, 0\} \\ &= \hat{x}_1 F - \hat{\mu}_2(F-f) + \hat{\mu}_1 M - v_1 + \max\{\hat{x}_2 F - \hat{\mu}_1(F-f) + \hat{\mu}_2 M - v_2, 0\},\end{aligned}$$

which is equivalent to $\hat{v}_2 - v_2 \geq \max\{\hat{x}_2 F - \hat{\mu}_1(F-f) + \hat{\mu}_2 M - v_2, 0\}$. Thus the incentive constraint (5) holds for both banks. Moreover, $\hat{v}_2 \geq v_2$ and $\hat{v}_1 > v_1$, which establishes that K is not renegotiation proof.

Second, suppose $\hat{x}_1 F - \hat{\mu}_2(F-f) + \hat{\mu}_1 M \leq v_1$ and pick side payments such that $\hat{v}_1 = v_1$. We have $\sum_i(\hat{v}_i - v_i) = \hat{v}_2 - v_2 = S(x,\hat{x})$, so that the violation of (20) implies $\hat{v}_2 \geq \hat{x}_2 F - \hat{\mu}_1(F-f) + \hat{\mu}_2 M$. Thus the incentive constraint (5) again holds for both banks. Moreover, since $S(x,\hat{x}) > 0$, $\hat{v}_2 > v_2$, which establishes that K is not renegotiation proof.

Step 2: Sufficiency (if). — Suppose K is not renegotiation proof, so there exists a renegotiation $\hat{K}$ such that (3), (9) and (5) hold, and $\hat{d}_{1i} + \hat{d}_{2i} \geq d_{1i} + d_{2i}$ and $\hat{v}_i \geq v_i$ for all i, with at least one strict inequality in some state ω. Consider the continuation decision $\hat{x}$ implied by $\hat{K}$. Summing (4) gives

$$S(x,\hat{x}) = \sum_i [(\hat{d}_{1i} + \hat{d}_{2i}) - (d_{1i} + d_{2i}) + (\hat{v}_i - v_i)] > 0$$

Moreover, (5) and $\hat{v}_i \geq v_i$ imply that

$$\sum_i \max\{\hat{x}_i F - \hat{\mu}_j(F-f) + \hat{\mu}_i M - v_i, 0\} \leq \sum_i (\hat{v}_i - v_i) \leq S(x,\hat{x})$$

which establishes that (20) is violated, as required.

Proof of Lemma 2.

Fix a contract K' satisfying the conditions in Corollary 1. For any continuation plan $(\hat{x},\hat{\mu})$, define the (state-contingent) function

$$J(\hat{x},\hat{\mu}) = \sum_i \max\{\hat{x}_i F - \hat{\mu}_j(F-f) + \hat{\mu}_i M - v'_i, 0\} - S\left(x',\hat{x}\right)$$

where x' and v'_i are as determined by K'. By Lemma 1, the contract K' is RP if and only if in all states, $J(\hat{x},\hat{\mu}) \leq 0$ implies $S(x',\hat{x}) \leq 0$. We show that this is the case if and only if Condition (21) and the parametric restriction (22) are both satisfied.

Step 1: Necessity (only if). — Given a violation of either condition, we find a continuation plan $(\hat{x},\hat{\mu})$ such that in some state, $J(\hat{x},\hat{\mu}) \leq 0$ and $S(x',\hat{x}) > 0$.

For the first condition, note that by incentive compatibility (5), $v_2'(\rho,0) \geq F$. Suppose $v_2'(\rho,0) > F$. The argument for $v_1'(0,\rho)$ is analogous. Consider state $(\rho,0)$. By Corollary 1, the contract K' has bank 1 closing and bank 2 continuing at full scale. Take a continuation plan $(\hat{x},\hat{\mu})$ which reopens a small fraction ε of bank 1 and has bank 2 monitoring this fraction, while bank 1 does not monitor: $\hat{x}_1 = \hat{\mu}_2 = \varepsilon > 0$, $\hat{x}_2 = 1$, $\hat{\mu}_1 = 0$. The surplus created is $S(x',\hat{x}) = \varepsilon(R-\rho) > 0$, and for small enough ε, using Assumption (A3) and $v_2' > F$,

$$J(\hat{x},\hat{\mu}) = \varepsilon(f-(R-\rho)) \leq 0.$$

For the second condition, suppose $f + M^2/(F-f) \leq R-\rho$. Consider state $(\rho,0)$ again. Take a plan which fully reopens bank 1 and has full monitoring by bank 2: $\hat{x}_1 = \hat{x}_2 = 1$, $\hat{\mu}_2 = 1$. The surplus created is $S(x',\hat{x}) = R-\rho > 0$. Using $v_2'(\rho,0) \geq F$,

$$\begin{aligned} J(\hat{x},\hat{\mu}) &= f + \hat{\mu}_1 M + \max\{F - \hat{\mu}_1(F-f) + M - v_2', 0\} - (R-\rho) \\ &\leq f + \hat{\mu}_1 M + \max\{M - \hat{\mu}_1(F-f), 0\} - (R-\rho). \end{aligned}$$

Now set $\hat{\mu}_1$ just big enough to make the max term equal to zero: $\hat{\mu}_1 = M/(F-f)$. Then we obtain $J(\hat{x},\hat{\mu}) \leq 0$ by assumption.

Step 2: Sufficiency (if). — We show that under the proposed conditions, $J(\hat{x},\hat{\mu}) \leq 0$ implies $S(x',\hat{x}) \leq 0$ in each state.

State $(0,0)$. The contract has both banks continuing, so that $S(x',\hat{x}) \leq 0$ for all $\hat{x}$.

State (ρ,ρ). The contract has both banks closing, so that $S(x',\hat{x}) = (R-\rho)\sum_i \hat{x}_i$. Moreover, $v_i = 0$ for all i, so that

$$\begin{aligned} J(\hat{x},\hat{\mu}) &= \sum_i \hat{x}_i(F-(R-\rho)) - \hat{\mu}_j(F-f) + \hat{\mu}_i M \\ &= \sum_i \hat{x}_i(F-(R-\rho)) - \hat{\mu}_i(F-f-M) \\ &\geq \sum_i \hat{x}_i(f+M-(R-\rho)) \geq 0, \end{aligned}$$

where the first inequality uses $\hat{\mu}_i \leq \hat{x}_i$ and the second uses Assumption (A2). This implies that $J(\hat{x},\hat{\mu}) \leq 0$ only if $\hat{\mu}_i = \hat{x}_i$ and $\hat{x}_i = 0$, i.e. when nothing changes compared to K. But in this case, $S(x',\hat{x}) = 0$.

State $(\rho,0)$. The contract has bank 1 closing and bank 2 continuing, so that $S(x',\hat{x}) = (R-$

$\rho)\hat{x}_1 - R(1-\hat{x}_2)$. Moreover, $v_1' = 0$ and $v_2' = F$ by assumption, so that

$$\begin{aligned} J(\hat{x},\hat{\mu}) &= \hat{x}_1 F - \hat{\mu}_2 (F-f) + \hat{\mu}_1 M + \max\{\hat{\mu}_2 M - \hat{\mu}_1 (F-f) - (1-\hat{x}_2) F, 0\} \\ & \quad -(R-\rho)\hat{x}_1 + R(1-\hat{x}_2). \end{aligned}$$

We show that $J(\hat{x},\hat{\mu}) \leq 0$ only if $\hat{\mu}_i = 0$, $\hat{x}_1 = 0$ and $\hat{x}_2 = 1$, i.e. when nothing changes compared to K'. This is sufficient because in this case, $S(x,\hat{x}) = 0$. The argument for state $(0,\rho)$ is analogous.

It is without loss to focus on choices with maximal monitoring by bank 2, and monitoring by bank 1 chosen just big enough to make the max term in J equal to zero:

$$\begin{aligned} \hat{\mu}_2 &= \min\{\hat{x}_1, \hat{x}_2\}; \\ \hat{\mu}_1 &= \max\left\{0, \frac{\hat{\mu}_2 M - (1-\hat{x}_2) F}{F-f}\right\}. \end{aligned} \tag{30}$$

To see this, note that choices satisfying (30) give strictly lower J than others. More precisely, for any given $\hat{x}$, setting $\hat{\mu}$ as in (30) is the unique minimiser of J: Take an arbitrary $(\hat{x},\hat{\mu})$. If $\hat{\mu}_1$ is above the level in (30), reducing it by ε unit lowers J by $M\varepsilon > 0$. If $\hat{\mu}_1$ is below it, increasing it by ε lowers J by $(F-f-M)\varepsilon > 0$. If $\hat{\mu}_2$ is set below the suggested maximal quantity, increasing it by ε lowers J by at least$(F-f-M)\varepsilon > 0$.

Let $J^0(\hat{x})$ denote J when evaluated at $\hat{x}$ and the levels of $\hat{\mu}$ in (30). To complete our argument, we must show that $J^0(\hat{x}) \leq 0$ only if $\hat{x}_1 = 0$ and $\hat{x}_2 = 1$. We do so by considering the four cases implied by (30):

1. $\hat{x}_1 \leq \hat{x}_2$ and $\hat{x}_1 M - (1-\hat{x}_2)F \leq 0$.
2. $\hat{x}_1 \geq \hat{x}_2$ and $\hat{x}_2 M - (1-\hat{x}_2)F \leq 0$.
3. $\hat{x}_1 \leq \hat{x}_2$ and $\hat{x}_1 M - (1-\hat{x}_2)F > 0$.
4. $\hat{x}_1 \geq \hat{x}_2$ and $\hat{x}_2 M - (1-\hat{x}_2)F > 0$

Case 1: $J^0(\hat{x}) = \hat{x}_1[F-(R-\rho)] + (1-\hat{x}_2)R$. This is strictly positive unless $\hat{x}_1 = 0$ and $\hat{x}_2 = 1$.

Case 2: Using $\hat{x}_2 \leq \hat{x}_1$ and $\hat{x}_2 \leq F/(F+M)$,

$$\begin{aligned} J^0(\hat{x}) &= \hat{x}_1[F-(R-\rho)] - \hat{x}_2[F-f+R] + R \\ &\geq -\hat{x}_2[2R-f-\rho] + R \\ &\geq R - \frac{F}{F+M}[2R-f-\rho] \\ &\propto RM - (R-\rho-f)F, \end{aligned}$$

which is strictly positive since $R > F > 0$ and $M > R-\rho-f \geq 0$ by Assumptions (A2) and (A3).

Case 3: We have

$$J^0(\hat{x}) = \hat{x}_1 \left[f + \frac{M^2}{F-f} - (R-\rho) \right] - \hat{x}_2 \left[R - \frac{FM}{F-f} \right].$$

The factor on $\hat{x}_1$ is positive by assumption, the factor on $-\hat{x}_2$ is positive since $R > F > 0$ and $F - f > M > 0$ imply that $R(F-f) > FM$. Hence, using $\hat{x}_2 \leq 1$ and $\hat{x}_1 > (1-\hat{x}_2)F/M$, we obtain $J^0(\hat{x}) > 0$.

Case 4: Using $\hat{x}_1 \geq \hat{x}_2$,

$$\begin{aligned} J^0(\hat{x}) &= \hat{x}_1[F-(R-\rho)] - \hat{x}_2 \left[F - f + R - \frac{M(F+M)}{F-f} \right] + R - \frac{FM}{F-f} \\ &\geq \hat{x}_2 \left[f + \rho - 2R + \frac{M(F+M)}{F-f} \right] + R - \frac{FM}{F-f}. \end{aligned}$$

If the term in square brackets is strictly positive, then using $\hat{x}_2 > F/(F+M)$, $J^0(\hat{x}) > R - \frac{F}{F+M}[2R - f - \rho] > 0$ by the argument of case 2. If the term is negative, then using $\hat{x}_2 \leq 1$, $J^0(\hat{x}) \geq f + \frac{M^2}{F-f} - (R-\rho) > 0$ by assumption.

Proof of Proposition 2

Suppose $\phi > \phi^*$, and that (22) holds. First, we show that there exists a renegotiation-proof contract which achieves the maximized value V of problem (P). Any optimal contract in problem (P^{RP}) therefore achieves this value. Second, we show that it cannot be optimal to set $\beta < \beta^*$.

Step 1: A renegotiation-proof contract achieves V . — Consider the following grand contract: Choose x_i, C_i and A_i as in Corollary 1, with inside equity contributions chosen symmetrically,

$$A_i = I - \frac{1}{2}\left(R - \frac{B}{\Delta} \right) \text{ for all } i.$$

Impose maximal monitoring with $\mu_i = \min\{x_i, x_j\}$. The budget constraint (2) yields

$$D_i = I - A_i = \frac{1}{2}\left(R - \frac{B}{\Delta} \right).$$

If the remaining choices can be made such that all constraints in Definition 2 and Condition (21) in Lemma 2 are satisfied, then the contract achieves V and is renegotiation-proof and we are done.

Make all side payments zero: $B = b_{1i} = b_{2i} = 0$, and write $d_i = d_{1i} + d_{2i}$. We still need to specify the payouts to depositors d_i for all i in all states, and the correlation parameter β. Satisfying

Condition (21) requires that (using Equation (4) for v_i),

$$d_1(0,\rho) = R - F = d_2(\rho,0).$$

Moreover, set the payouts from liquidity-shocked banks, which are closed, to zero:

$$d_1(\rho,0) = d_2(0,\rho) = d_i = (\rho,\rho) = 0.$$

Choose the payouts in state $(0,0)$ to satisfy the Individual Rationality constraint (7), which yields

$$\begin{aligned} D_i &= \frac{1}{2}\left(R - \frac{B}{\Delta}\right) = \mathbb{E}[d_i] \\ &= \frac{\beta}{2} d_i(0,0) + \frac{1-\beta}{2}(R-F) \\ \Leftrightarrow d_i(0,0) &= \beta^{-1}\left[R - \frac{B}{\Delta} - (1-\beta)(R-F)\right]. \end{aligned}$$

It is easy to see that these choices satisfy all relevant constraints for the right choice of β. In particular, they satisfy the Limited Liability requirement (8) as long as $\beta \geq \beta^*$.

Step 2: Setting $\beta < \beta^*$ is not optimal. — Suppose to that a grand contract achieves value V and is renegotiation-proof. Corollary 1 gives x_i, C_i and A_i, and Condition (21) in Lemma 2 must be satisfied. Moreover, the Individual Rationality constraint (7) must bind to achieve optimality. Summing this constraint across i and using the definition of v_i in (4), we obtain

$$\begin{aligned} \sum_i \mathbb{E}\left[v_i'\right] &= \frac{\beta}{2}(v_1'(0,0) + v_2'(0,0)) + \frac{1-\beta}{2}(v_1'(0,\rho) + v_2'(\rho,0)) \\ &= \frac{\beta}{2}(v_1'(0,0) + v_2'(0,0)) + (1-\beta)F = \frac{B}{\Delta}. \end{aligned}$$

Moreover, substituting the date 1 budget (3) into the limited liability requirement (8) for state $(0,0)$ and summing across i gives $v_1'(0,0) + v_2'(0,0) \leq 2R$. Substituting this into the last equation gives $\beta \geq \beta^\star$.

Proof of Proposition 3

The deposit and CDS agreements imply a grand contract. There are no cash holdings ($C_i = 0$) or interbank payments ($B_i = b_{1i} = b_{2i} = 0$). The total payment from depositors to banks at date 0 is $D_i = D + P$, and the expected repayment from bank i to its depositors is

$$\mathbb{E}[d_{1i} + d_{2i}] = \frac{1}{2}(d + (1-\beta)n)$$

No payments are made when a bank suffers a liquidity shock and closes. In order to implement the optimal contract, the date 0 budget (1) and the individual rationality constraint (7) must be satisfied, and the maintenance effort incentive constraint (6) must bind. Combining, we obtain

$$\begin{aligned}\frac{1}{2}(d+(1-\beta)n) &= D+P \\ &= I-A \\ &= \frac{R}{2}-\frac{B}{2\Delta}.\end{aligned}$$

For renegotiation-proofness, we need $v_1(0,\rho) = R-d-n = F$ (the analogue for bank 2 follows by symmetry). Thus we need to specify d and n to solve

$$\begin{aligned}d+(1-\beta)n &= R-\frac{B}{\Delta} \\ d+n &= R-F.\end{aligned}$$

The solution is

$$\begin{aligned}n &= \frac{B/\Delta-F}{\beta} \\ d &= R-F-\frac{B/\Delta-F}{\beta},\end{aligned}$$

and it is easy to see that $d>0$ if and only if $\beta>\beta^{\star}$.

Proof of Proposition 4

The steps are analogous to the proof of Proposition 3. There are no cash holdings or interbank payments. We now have $D_i = D$ and

$$\mathbb{E}[d_{1i}+d_{2i}] = \frac{1}{2}(d+(1-\beta)b),$$

where the term in $\tilde{b}$ term arises from the state where bank i is in default and bank j is solvent. Combining the date 0 budget, individual rationality and IC, we have

$$(d+(1-\beta)b) = R-\frac{B}{\Delta},$$

and renegotiation-proofness requires $v_1(0,\rho) = R-d-b = F$. Solving yields

$$\begin{aligned}n &= \frac{B/\Delta-F}{\beta} \\ b &= R-F-\frac{B/\Delta-F}{\beta},\end{aligned}$$

and we have $b > 0$ if and only if $\beta > \beta^{\star}$.

Proof of Proposition 5

As in Proposition 1, we start by considering the relaxed problem (P0), adding the capital constraint $A_i \geq A_{\min}$. First, we show that in any solution to the relaxed problem, banks always continue with $x = 1$. Second, we show that continuation is optimal in the full problem (P^{RP}) as long as η is small. Third, we show that any β can be part of an optimal contract.

Step 1: Optimal continuation in the relaxed problem. — We split the relaxed problem into two stages as in Equations (26) and (27). The first step is equity minimization given a continuation decision $\bar{x}$. It is easy to show that the minimal equity requirement per bank, $\bar{A}(\bar{x})/2$ satisfies

$$\frac{1}{2}\bar{A}(\bar{x}) \leq \bar{A}(1) = \frac{B}{2\Delta} + f + M - \left(R - \frac{\rho}{2} - I\right) \leq A_{min},$$

using the assumed lower bound on A_{min}. The optimal continuation decision solves (26), and therefore satisfies

$$\begin{aligned} x &= \arg\max_{\bar{x}} \sum_i \mathbb{E}[(R-\omega_i)\bar{x}_i] - \phi \min\{\bar{A}(\bar{x}), 2A_{min}\} \\ &= \arg\max_{\bar{x}} \sum_i \mathbb{E}[(R-\omega_i)\bar{x}_i] - 2\phi A_{min}. \end{aligned}$$

Since the second term is independent of $\bar{x}$, the problem reduces to maximizing the first term, which is the NPV of projects. This is achieved by setting $x = 1$.

Step 2: Optimal continuation in problem (P^{RP}). — It is always possible to find a contract with $x = 1$ which satisfies limited liability by introducing cash holdings $C_i = \rho$. The cost of cash holdings is then $\eta\rho$, which vanishes as $\eta \to 0$. Therefore, for sufficiently small η, it remains optimal in the full problem (P^{RP}) to set $x = 1$. The renegotiation-proofness constraint does not bind, since continuation decisions are ex post efficient. Hence, the solution of the problems with and without renegotiation are identical.

Step 3: Indifference between all $\beta \in (0,1)$. — Consider an optimal contract in problem (P^{RP}) with correlation parameter β. It is possible to change the correlation parameter to any $\beta' \in (0,1)$ and re-design continuation values v_i such that the same value is achieved without violating any of the relevant constraints. In particular, the renegotiation-proofness constraint will not be violated since continuation decisions are ex post efficient.

Proof of Proposition 6

We show that with a lender of last resort, any renegotiation-proof contract has banks continuing in all states with $x = 1$. Therefore, with a lender of last resort, the optimal contract in problem (P^{RP}) has $x = 1$. Given this result, a parallel argument to the proof of Proposition 5 implies that banks are indifferent between all $\beta \in (0,1)$.

Take a contract with $x_i(\omega') < 1$ for some i, ω'. Consider a candidate renegotiated contract which fully re-opens bank i in that state, i.e. $\hat{x}_i(\omega') = 1$ and $\hat{x} = x$ otherwise. Suppose the continuation is funded (if funding is required) by a LOLR loans of $\hat{L}_i = L_i + \omega_i'(1 - x_i(\omega'))$ and also monitored by the LOLR with $\hat{\lambda}_i = \lambda_i + 1 - x_i(\omega')$. The renegotiated contract releases surplus

$$S(x,\hat{x}) = (R - \omega_i')(1 - x_i(\omega')) > 0,$$

and increases the aggregate rent required by at most

$$f(1 - x_i(\omega')) \leq (R - \rho)(1 - x_i(\omega')) \leq S(x,\hat{x})$$

using assumption (A3). The surplus released exceeds the added rent requirement. Therefore, by a parallel argument to Lemma 1, the original contract cannot be renegotiation proof.

To complete the proof, we show that banks will choose the proposed amount of inside equity. A parallel argument to the proof of Proposition 1 yields a 'maximal monitoring priniple': It is always (weakly) optimal to have all banks monitored by the lender of last resort, and not by each other. It follows that the minimal equity required with $x = 1$ satisfies

$$\bar{A}(1) = \frac{B}{\Delta} + 2f - 2\left(R - \frac{\rho}{2} - I\right)$$

as required.

Proof of Proposition 7

With a bailout S, the contracting problem (P^{RP}) becomes

$$V^{RP} = \max_{K \in \mathscr{C}^{RP}} \sum_i \mathbb{E}[v_i] + (1+\phi)(E - A_i) + \beta S \tag{31}$$

By the argument of Proposition 2, the maximum of the first two terms can be achieved with any $\beta \geq \beta^*$. Hence, it is optimal to set $\beta = 1$. We complete the proof with two further steps. First, we show that above a certain threshold $S^\star$, it is optimal to cease exerting maintenance effort. Second, we repeat the argument with a lender of last resort and show that the threshold S^* is lower in this case, as long as ϕ is large enough.

Step 1: Optimality of no maintenance effort. — If banks continue to make maintenance effort at date 0, then the optimal contract is as before, with $\beta = 1$. Banks' maximized per-bank value with effort, denoted V_e is the sum of the value in (29) and the expected value of state support:

$$V_e = \frac{R}{2} - I - \phi\left[\frac{B}{2\Delta} - \left(\frac{R}{2} - I\right)\right] + S + E(1+\phi).$$

If banks do not make effort at date 0, then the date 0 incentive compatibility constraint is slack, so that no inside equity or closure threat is required. It is optimal to set $A_i = 0$ and $x = 1$. Each bank's payoff is then

$$V_{ne} = R - \left(\frac{1}{2} + \Delta\right)\rho - I + S\left[1 + (2-\beta)2\Delta\right] + E(1+\phi).$$

Since $V_{ne} - V_e$ is strictly increasing in S, a large enough S implies that $V_{ne} > V_e$ and it is optimal not to make maintenance effort.

Step 2: Lender of last resort. — With a lender of last resort, the proof of Proposition 6 implies that banks always continue with $x = 1$.

If banks continue to make effort at date 0, the optimal contract is as in Proposition 6, and the maximized value is

$$\hat{V}_e = R - \frac{\rho}{2} - I - \phi\left[\frac{B}{2\Delta} + f - \left(R - \frac{\rho}{2} - I\right)\right] + S + E(1+\phi).$$

If banks do not make effort, the value is V_{ne} as before. For sufficiently large ϕ, we have that $V_{ne} - \hat{V}_e > V_{ne} - V_e$, and

$$\frac{d}{dS}\left(V_{ne} - \hat{V}_e\right) = \frac{d}{dS}\left(V_{ne} - V_e\right) > 0,$$

implying that the threshold S for which $V_{ne} > \hat{V}_e$ lies below the threshold S^* in the last step, which completes the proof.