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SYSTEMICALLY IMPORTANT BANKS: A PERMUTATION TEST APPROACH

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Abstract

According to the definition of Financial Stability Board (FSB), Systemically Important Banks (SIBs) are the banks "whose disorderly failure, because of their size, complexity and systemic interconnectedness, would cause significant disruption to the wider financial system and economic activity". The current methodology for their determination is based on balance-sheet variables and expert judgment. We use permutation tests to investigate the relevance of equity-based systemic risk measures in the SIBs choice. Restriction of the analysis to European Banks, for which full information is available, allows understanding the importance of equity-based systemic risk measures also for size, interconnectedness, substitutability/financial Institution Infrastructure, complexity and cross-jurisdictional Activity categories.

Keywords: Systemic risk, Particle Swarm Optimization, nonparametric combination.

1. Introduction and Objectives

The framework of Systemically Important Financial Institutions (SIFIs) was introduced by the Financial Stability Board (FSB) in October of 2010 as the institutions "whose disorderly failure, because of their size, complexity and systemic interconnectedness would cause significant disruption to the wider financial system and economic activity", FSB (2010). The current methodology to determine the Globally Systemically Important Banks (G-SIBs) is outlined by the Bank for International Settlements (BIS (2013)). In particular, the banks included in the analysis have to fulfill any of the following criteria:

- Banks that the Committee identifies as the 75 largest global banks, according to the leverage ratio exposure measure, based on Basel III and at the end of the financial year.
- Banks that were designated as G-SIBs in the previous year (unless supervisors agree

that there is a compelling reason to exclude them).

- Banks, with a score produced by the indicator-based measurement approach that exceeds a cut-off level set by the Committee.
- Banks that have been added to the sample by national supervisors using supervisory judgment (subject to certain criteria).

The regulator builds the selection process on proprietary annual data. The collection of the dataset is time-consuming and delays the publication of the selection. For example, the FSB published the last release based on end-2014 data only in November 2015. The European Banking Union provides full disclosure of the data used to define the European SIBs for the year 2014 (using data from 2013) and 2015 (using data from 2014). For a review of the literature on the G-SIBs and a critique of the methodology see Iwanicz-Drozdowska (2014) and Barth, Nolle, Li, and Brummer (2014). Bongini, Nieri, and Pelagatti (2015) discuss the financial impact of the SIFIs selection. In parallel, several papers proposed systemic risk measure based on stock returns. For a clear description of those measures refer to the recent survey Bisias, Flood, Lo, and Valavanis (2012). In this way, market participants are able to compute them at any moment due to their easy access to this type of data. Furthermore, Scholars use of them in ranking timely the systemic importance of single institutions. Recently, Giglio, Kelly, and Pruitt (2016) investigate their macroeconomic meaning and substantiated this practice. The study of their relationship with the variables used by the FSB is new in the literature to our knowledge. A bridge between those two set of measures could bring a more timely and accurate choice of the SIBs. All the above reasons justify the present investigation of the linkage among them. In particular, we consider as given the choice made by the FSB about the two groups of SIBs and non-SIBs. Combining statistics from permutations of several variables is a well-known possibility, see Pesarin and Salmaso (2010). We propose a combination based on the systemic risk measures to test the identity of the two FSB groups . Rejection of the null hypothesis implies the reproduction of the choice using equity information. So,we choose the combination weights by minimizing the p-value of the combination test. If the weights add up to one, one can interpret them as the measure relevance in the FSB choice. Also, we are able to associate a SIBs selection to each macro category using a heuristic. Then, we repeat the procedure for size, interconnectedness, substitutability, complexity and cross-jurisdictional Activity. Non-smoothness of the p-value as a function of weights requires a global optimization method. The parallelizable Particle Swarm Optimization (PSO) Kennedy (2010) is valid candidate. In fact, we exploit the multi-core architecture of the Sistema per Calcolo Scientifico di Ca' Foscari (SCSCF) for computations[1]. The paper is organized as follows. In Section 2 the methodology is presented, in Section 3 we show the results for the FSB selection and for macro categories driven selections, then in Section 4 we discuss the results and propose some possible extensions.

2. Methodology

In this Section, we review the permutation and combination test methodology, following Pesarin and Salmaso (2010), introduce our optimization procedure and justify the use of Particle Swarm technique. Let be $I(\cdot)$ the indicator function, that is equal to 1 if the condition in parenthesis is satisfied and zero otherwise.

[1]This research used the SCSCF multiprocessor cluster system at University Ca' Foscari of Venice.

The baseline permutation test for a single cross sectional variable, X, can be summarized in the following steps:

1. The observed units, corresponding to different banks, are divided into two groups, g_1 and g_2,according to a given selection.
2. Given a cross sectional variable, X, observed on values x_i, compute a relevant statistic for each group; in our case we use the empirical distribution function $\hat{F}_k$ for group $k = 1, 2$. The maximum of the difference of the two functions $KS\left(\hat{F}_1, \hat{F}_2\right)$, that is a two-sample Kolmogorov-Smirnov statistic, will be our test statistic, and its observed value will be v_{obs}.
3. Exchange randomly the participants in the groups, retaining only their sizes. We randomly choose a permutation of the indices i, named π_b, $b = 1, \ldots, B$, obtaining two new groups g_1^b and g_2^b. Then, considering the exchangeability assumption of X and under the hypothesis of identical distribution for the two groups, $H_0 : F_1 = F_2$, the statistic $v^b = KS\left(\hat{F}_{g_1^b}, \hat{F}_{g_2^b}\right)$ would have the same distribution of v_{obs}.
4. Compute, according to Pesarin and Salmaso (2010), an approximated p-value by

$$P_B = \frac{1}{B}\sum_{b=1}^{B} 1\left(v_b \geq v_{obs}\right) \tag{1}$$

The combination of several partial permutation tests requires additional steps. After the definition of the size of each group, we apply the procedure outlined before to get the permutation distribution of each partial test statistic t_j, typically $t_{j,obs} = KS\left(\hat{F}_1^j, \hat{F}_2^j\right)$, $j = 1, \ldots, p$, where the empirical cumulative distribution functions, $\hat{F}_{g_k}^{(j)}$, $k = 1, 2$, refer to the observed values of X_j, furthermore we denote with t_j^b the partial test statistics computed on each permutation b of the two groups, with $b = 1, \ldots, B$. Then each dimension is transformed to an auxiliary variable related into the single p-values

$$\lambda_j = \frac{1}{B+1}\left(\frac{1}{2} + \sum_{b=1}^{B} 1\left(t_j^b \geq t_{j,obs}\right)\right) \tag{2}$$

that takes values strictly inside the unit interval. So, it may be defined in such a way that they can be aggregated in a single variable using a combination function. In our case, we use a Fisher omnibus function with the same weights of the indices $t = -\sum_{j=1}^{p} w_j \log\left(\lambda_j\right)$. In addition, we get the value of the statistics in each permutation, by

$$\lambda_j^b = \frac{1}{B+1}\left(\frac{1}{2} + \sum_{b=1}^{B} 1\left(t_j^r \geq t_j^b\right)\right) \tag{3}$$

in such a way to produce an approximated permutation distribution. This the procedure may be extended to the combined variable for each permutation, $t^b = -\sum_{j=1}^{p} w_j \log\left(\lambda_j^b\right)$. Given B random permutations, according to Pesarin and Salmaso (2010), we can obtain an approximated p-value by

$$P^B = \frac{1}{B}\sum_{b=1}^{B} 1\left(t_b \geq t\right) \tag{4}$$

and we can reject the global null hypothesis of equality the two groups $H_0 : F_1 = F_2$ at the α significance level if $P^B \leq \alpha$.

The lower is the value of P^B the higher is the significance of the test and the higher is the difference between the multivariate distribution of the two groups. For this is the reason we choose to optimize the weights in order to minimize the p-value.

2.1. Particle Swarm Optimization

In order to test our procedure we consider an optimization problem in which the function that have to be optimized depends on the weights used in the Fisher omnibus function that combines several tests. Given the complexity of the optimization problem we consider the bio-inspired iterative metaheuristic called Particle Swarm Optimization (PSO) introduced by Kennedy and Eberhart (1995). This procedure, based on *swarm intelligence*, is a robust stochastic method for unconstrained optimization problem although it is possible to treat also constrained ones (Corazza et al. (2013)). The PSO algorithm exploits the concept of social intelligence and co-operation to mimic the intelligence that moves together individuals of the same species looking for food.

To this aim, each member of the swarm explores the search area reminding its best position reached so far, exchanging the information with the other ones. The whole swarm will converge to the best global position.

Each member of the swarm is a particle and represents a possible solution to the investigated optimization problem. The initialization of the particle placement is random as well as their initial velocity. The movement of each particle of the swarm depends on its position with respect to the best position. In the following, we give a description of the standard PSO procedure.

Let us consider the unconstrained optimization problem

$$\min_{x \in R^d} f(x)$$

where $f : R^d \to R$ is the objective function in the minimization problem. At the k-th iteration of the PSO algorithm, the following objects are associated to the j-th particle $\{j = 1, \ldots, M\}$:

- $x_j^k \in R^d$: position of the j-the particle at step k;
- $f(x_j^k) \in R$: fitness of the j-th particle at step k;
- $v_j^k \in R^d$: velocity of the j-the particle at step k;
- $p_j \in R^d$: best position visited so far by the j-th particle at step k.

Furthermore $pbest_j$ denotes the objective function in the position p_j of the j-th particle.

- Set $k = 0$. Randomly generate x_j^k and v_j^k.
- Set $k = 1$. Set $pbest_j = +\infty$ for all j and set $gbest = +\infty$.

The overall PSO algorithm is described in the following.

1. Evaluate $f(x_j^k)$ for all j.
2. If $f(x_j^k) < pbest_j$ then $p_j = x_j^k$ and $pbest_j = f(x_j^k)$ for all j. If $f(x_j^k) < gbest$ then $p_g = x_j^k$ and $gbest = f(x_j^k)$.
3. Update velocity and position for all j:

$$v_j^{k+1} = w^{k+1} v_j^k + U_1 \otimes (p_j - x_j^k) + U_2 \otimes (p_g - x_j^k)$$

$$x_j^{k+1} = x_j^k + v_j^{k+1}$$

   where $U_1, U_2 \in R^d$ such that each their dimension is a realization of uniform random variables over $[0, \theta_1]$ and $[0, \theta_2]$, respectively. The symbols $\otimes$ denotes the component-wise product and p_g is the best position of the j-th particle in a neighborhood. The value assumed by the inertia weight w^k will be discussed in the following.
4. If a convergence criterion is not satisfied then update $k = k + 1$ and go to step 2.

And the convergence criterion is either the maximum number of iterations or if there are no movements among particle.

The choice of the inertia weights w^k affects the convergence of the swarm. In literature, the parameters w^k, $k = 1, \ldots, K$ are generally proposed as linearly decreasing together with the number of steps, i.e.

$$w^k = w_{\max} + \frac{w_{\max} - w_{\min}}{K} \cdot k$$

where common values of $w_{\max}$ and $w_{\min}$ are respectively 0.9 and 0.4, while K is usually the maximum number of allowed steps.

One of the great advantages of the particle swarm optimization is that it is highly parallelizable. In fact, the update of particles characteristics is independently computable given the previous iteration. As already remarked the SCSCF cluster performed the computations. This allows us to use 200 hundred particles handled by 100 cores of the cluster. Instead one of the main drawbacks of particle swarm optimization is that is stochastic in nature. In our case also the permutation test is randomized doubling the source of stochasticity. Robustness checks that we report in the Appendix ensure reproducibility of the whole procedure. The use of 200 particles and 10000 random permutations suffices to get reliable results.

3. Empirical results

In this section, we begin introducing the used data. Then we derive the selection associated with the macro categories. Finally, we report and comment our results for the optimal weights for 2013 and 2014.

3.1. G-SIB score evaluation and data

Regulators chose SIBs using a composite indicator that considers different aspects of systemic risk: Size, Interconnectedness, Substitutability, Complexity and Cross-Jurisdictional Activity. Those macro categories are themselves composite.The finest subdivision comprises a total of 12 variables detailed in Table 1 BIS (2014). Then a weighted sum of the indicators with the weights as in the last column of Table 1 represents the score.

Table 1: Indicators and relative score weights used by the Basel Committee for the evaluation of the Systemically Important Banks

Category	Indicator	Indicator weight
Size	Total exposures	1/5=20%
Interconnectedness	Intra-financial system assets	1/15= 6.66%
	Intra-financial system liabilities	1/15= 6.66%
	Securities outstanding	1/15= 6.66%
Substitutability/financial institution institution infrastructure	Payment activity	1/15= 6.66%
	Assets under custody	1/15= 6.66%
	Underwritten transactions in debt and equity markets	1/15= 6.66%
Complexity	National amount of OTC derivatives	1/15= 6.66%
	Trading and AFS securities	1/15= 6.66%
	Level 3 assets	1/15= 6.66%
Cross-jurisdictional activity	Cross-jurisdictional claims	1/10= 10%
	Cross-jurisdictional liabilities	1/10= 10%

The BIS reports balance sheets included in the SIBs 2014 evaluation sample. The extraction of the relevant indicators from them would be a nontrivial task. So we focus on the European sample whose variables are available on the EBA website. Here EBA publishes balance sheets variables in manageable *excel* format for 2013 and 2014. In 2013 within a pool of 36 European Banks, 13 of them were considered SIBs. In 2014 within a pool of 37, 14 of them were chosen as SIBs. To get alternative selection based on the 5 macro categories we proceed in the following way. We consider values of the category corresponding to the institutions selected by the regulator. We compute the minimum value among them. We include all the institutions with values higher than the minimum in the new selection. We report in Table 2 the numerosity and the percentage of institutions in the regulator sample still in the selection. As you can see selection sets obtained in this way always contain the regulator set. Unfortunately, not all the considered European banks are listed companies. Stock prices are available for 25 of them. We are able to compute systemic risk measures only for them. In the Appendix in Table 3 and 4 we report the names of the used institutions and the computation of the considered systemic risk measure. For a description of the Marginal Expected Shortfall (MES) we refer to Acharya, Pedersen, Philippon, and Richardson (2010). We did not use their SES measure because it includes also information about leverage. The Δ CoVaR is introduced and described in Adrian and Brunnermeier (2016). The remaining Interconnectedness measures come from Billio, Getmanski, Lo, and Pelizzon (2012). For a description of several other measures using also a different type of data refer to Bisias *et al.* (2012). We consider the opposite of MES and Δ CoVaR and the inverse of the closeness to obtain comonotonicity among the measures. With this convention, an higher measure corresponds to higher risk.

3.2. Systemic Measures contribution to Selections

Table 2: Systemic Measures contribution to Selections

2013				weights							
	SIBs partial	SIBs included	pval	MES	ΔCoVaR	in degree	out degree	in out degree	closeness	eigenvector centrality	PCA
Regulator Selection		n=14	0.002	0.001	0.572	0.023	0.001	0.378	0.023	0.001	0.001
Size	32	100%	0.1841	0.013	0.0000	0.0000	0.0000	0.9862	0.0004	0.0001	0.0004
Interconnectedness	26	100%	0.012	0.1320	0.3755	0.0000	0.0001	0.3450	0.0000	0.0000	0.1473
Substitutability/Financial Institution Infrastructure	24	100%	0.016	0.0000	0.7884	0.0016	0.0564	0.0050	0.1486	0.0000	0.0000
Complexity	32	100%	0.053	0.0000	0.2010	0.0000	0.0000	0.0000	0.0000	0.0117	0.7873
CrossJurisdictional Activity	30	100%	0.003	0.0016	0.3791	0.0973	0.0922	0.0000	0.0000	0.0247	0.4051
2014				weights							
	SIBs partial	SIBs included	pval	MES	ΔCoVaR	in degree	out degree	in out degree	closeness	eigenvector centrality	PCA
Regulator Selection		n=13	0.312	0.0000	0.0000	0.0000	0.0000	0.0211	0.0001	0.9782	0.0006
Size	19	100%	0.091	0.0005	0.5900	0.3546	0.0004	0.0000	0.0000	0.0000	0.0545
Interconnectedness	15	100%	0.190	0.0000	0.0000	0.9577	0.0000	0.0000	0.0422	0.0001	0.0000
Substitutability/Financial Institution Infrastructure	18	100%	0.015	0.0000	0.0002	0.9471	0.0024	0.0000	0.0018	0.0484	0.0000
Complexity	17	100%	0.020	0.0000	0.0000	0.4531	0.0001	0.0000	0.0009	0.5459	0.0000
CrossJurisdictional Activity	15	100%	0.231	0.0000	0.0079	0.0000	0.0000	0.0000	0.0002	0.0000	0.9919

Table 2 summarizes the main findings of the paper. For each SIB selection rows report the optimal p-value and the optimal weights in 2013 and 2014. In Section 2 we gave a plausible interpretation of the optimal weights. They represent the share of information about the selection associated with each measure. Lower p-values lead to a higher probability that measures with high weights are informative about the selection. In this regards, in 2013, ΔCoVar and the sum of in and out degrees seesm able to reproduce the choice of the regulators. A similar conclusion, given a p-value above the 30%, is improbable in 2014 even if the weight of eigenvector centrality is almost one. Analogous considerations derive from alternative analysis based on the categories. In fact, we get a coherent picture in 2013. In this year ΔCoVar has a weight over the 70% for substitutability. Also, it is largely participating in interconnectedness complexity and cross-jurisdictional activity. In + out degree is the only variable related to size and a relevant variable for interconnectedness. The role of PCA for complexity cross-jurisdictional activity and interconnectedness is also clear. For 2014 instead, the alternative selections elect in degree as the most relevant variable. This variable is almost the only one relevant for substitutability, it is also important for complexity and size. Instead, eigenvector centrality is informative for complexity. In addition, we discover that ΔCoVar is relevant for size.

4. Conclusions

We investigate the relationship between SIBs selections and equity-based systemic risk measures. The procedure proposed is completely nonparametric and requires only the hypothesis of exchangeability of variables. We optimize using particle swarms on the weights of a combination of permutation tests on the measures. The weights quantify how much the measure is informative about the selection. The optimal p-value measures how much the result is reliable. We consider regulator selections and selections based on macro categories. We found coherent results for the year 2013. All the selections consider ΔCoVar and the sum of in and out degrees as the most informative variables. For 2014 optimization on the regulator selection gave an unreliable result. But alternative selections points to in degree measure as the most informative. Although we consider our findings interesting we are obliged to point out the lack of regularity that they carry. Unfortunately, differences are already present in the numerosity of the selections associated with categories. In general, the requirement on single category seems much more stringent for 2014. Given the availability of only two years, it is difficult to understand where this variability comes from. The changes among the two years are quantitative and qualitative. Different categories are associated with the different systemic measure in different years. This hints maybe to a dynamical relationship among categories and systemic measures. It seems that the equity-based systemic risk measures are not able to track changes in the decision variables used by the regulator. In particular, for 2014, the measures are not informative about the interconnectedness category. In this light would be important to include also the measures proposed by Diebold and Yılmaz (2014). Unfortunately, with 25 series, the curse of dimensionality does not allow to estimate the VAR needed for computing the measures. The use of some penalization technique as in recent Demirer, Diebold, Liu, and Yilmaz (2015) could solve the issue. It could also lead to better Granger Causality Network on which measures of Billio *et al.* (2012) are based.

Analogously refinement of MES (Brownlees and Engle (2010), Acharya, Engle, and Richardson (2012)) and CoVaR (Girardi and Ergün (2013)) based on Multivariate GARCH were proposed. Also, completely new equity-based measures are emerging from the literature. All those refinements and new measure should be in the future included in this kind of analysis. The quest for better tracking measures is the next move along this line of research of which this preliminary analysis represents the first step.

5. Appendix

5.1. Systemic Risk Measures

In the following we report the names, the Bloomberg ticker and the associated systemic risk measures used in our analysis.

Table 3: Equity based systemic risk measures for European Banks based on stock returns in the year 2013. Transformations are applied in such a way that higher measure correspond to higher risk

	-MES	-Δ CoVaR	in degree	out degree	in+out degree	1/closeness	eigenvector centrality	PCA
Banca Monte dei Paschi di Siena SpA (BMPS IM)	0.0286	0.0057	1	1	2	0.429	0.381	0.040
Barclays PLC (BARC LN)	0.0268	0.0099	2	0	2	0.037	0.000	0.011
Banco Bilbao Vizcaya Argentaria SA (BBVA SM)	0.0248	0.0134	0	0	0	0.037	0.000	0.011
Bankia (BKIA SM)	0.0927	0.0016	3	1	4	1.000	0.127	0.628
BNP Paribas SA (BNP FP)	0.0358	0.0134	1	0	1	0.037	0.000	0.012
Commerzbank AG (CBK GR)	0.0302	0.0058	1	0	1	0.037	0.000	0.032
Credit Agricole SA (ACA FP)	0.0343	0.0106	1	1	2	1.000	0.000	0.017
Danske Bank A/S (DANSKE DC)	0.0175	0.0072	0	1	1	1.000	0.000	0.010
Deutsche Bank AG (DBK GR)	0.0217	0.0065	1	1	2	1.000	0.127	0.009
DNB ASA (DNB NO)	0.0291	0.0072	4	1	5	0.667	0.127	0.012
Erste Group Bank AG (EBS AV)	0.0405	0.0077	1	0	1	0.037	0.000	0.019
Svenska Handelsbanken AB (SHBA SS)	0.0234	0.0116	2	2	4	0.455	0.127	0.008
HSBC Holdings PLC (HSBA LN)	0.0183	0.0088	0	1	1	0.353	0.127	0.005
ING Groep NV (INGA NA)	0.0358	0.0087	0	1	1	0.429	0.381	0.015
Intesa Sanpaolo SpA (ISP IM)	0.0365	0.0075	0	3	3	0.464	0.508	0.019
KBC Groep NV (KBC BB)	0.0319	0.0063	0	2	2	0.571	0.127	0.018
La Caixa Bank SA (CABK SM)	0.0193	0.0058	1	1	2	0.667	0.127	0.019
Lloyds Banking Group PLC (LLOY LN)	0.0153	0.0070	1	2	3	0.556	0.127	0.010
Nordea Bank AB (NDA SS)	0.0227	0.0093	0	0	0	0.037	0.000	0.008
Royal Bank of Scotland Group PLC (RBS LN)	0.0279	0.0091	0	2	2	0.345	0.381	0.020
Banco Santander SA (SAN SM)	0.0274	0.0123	0	0	0	0.037	0.000	0.011
Skandinaviska Enskilda Banken AB (SEBA SS)	0.0267	0.0093	2	1	3	0.500	0.127	0.008
Societe Generale SA (GLE FP)	0.0385	0.0081	1	0	1	0.037	0.000	0.019
Standard Chartered PLC (STAN LN)	0.0174	0.0099	3	5	8	0.667	0.381	0.009
Swedbank AB (SWEDA SS)	0.0284	0.0116	2	0	2	0.037	0.000	0.011
UniCredit S.p.A. (UCG IM)	0.0382	0.0119	0	1	1	0.500	0.127	0.021

Table 4: Equity based systemic risk measures for European Banks based on stock returns in the year 2014. Transformations are applied in such a way that higher measure correspond to higher risk

	-MES	-ΔCoVaR	in degree	out degree	in + out degree	1/closeness	eigenvector centrality	PCA
Banca Monte dei Paschi di Siena SpA (BMPS IM)	0.0308	0.0059	0	0	0	0.037	0.000	0.261
Barclays PLC (BARC LN)	0.0253	0.0096	1	0	1	0.037	0.000	0.030
Banco Bilbao Vizcaya Argentaria SA (BBVA SM)	0.0271	0.0132	1	9	10	0.600	0.672	0.026
Bankia (BKIA SM)	0.0255	0.0108	0	1	1	1.000	0.000	0.042
BNP Paribas SA (BNP FP)	0.0283	0.0133	3	3	6	0.545	0.000	0.024
Commerzbank AG (CBK GR)	0.0275	0.0071	3	0	3	0.037	0.000	0.046
Credit Agricole SA (ACA FP)	0.0267	0.0105	2	5	7	0.333	0.081	0.036
Danske Bank A/S (DANSKE DC)	0.0192	0.0099	11	0	11	0.037	0.000	0.016
Deutsche Bank AG (DBK GR)	0.0250	0.0127	1	1	2	1.000	0.000	0.024
DNB ASA (DNB NO)	0.0243	0.0095	3	2	5	0.750	0.000	0.025
Erste Group Bank AG (EBS AV)	0.0308	0.0101	1	7	8	0.512	0.416	0.057
Svenska Handelsbanken AB (SHBA SS)	0.0154	0.0114	3	0	3	0.037	0.000	0.013
HSBC Holdings PLC (HSBA LN)	0.0182	0.0105	7	1	8	0.500	0.000	0.012
ING Groep NV (INGA NA)	0.0322	0.0131	1	1	2	1.000	0.000	0.031
Intesa Sanpaolo SpA (ISP IM)	0.0297	0.0136	3	9	12	0.553	0.503	0.047
KBC Groep NV (KBC BB)	0.0303	0.0082	1	3	4	0.362	0.156	0.035
La Caixa Bank SA (CABK SM)	0.0265	0.0128	0	1	1	1.000	0.000	0.037
Lloyds Banking Group PLC (LLOY LN)	0.0238	0.0138	2	1	3	1.000	0.000	0.021
Nordea Bank AB (NDA SS)	0.0198	0.0158	3	0	3	0.037	0.000	0.020
Royal Bank of Scotland Group PLC (RBS LN)	0.0239	0.0072	2	0	2	0.037	0.000	0.036
Banco Santander SA (SAN SM)	0.0259	0.0133	4	8	12	0.477	0.302	0.021
Skandinaviska Enskilda Banken AB (SEBA SS)	0.0217	0.0115	1	0	1	0.037	0.000	0.016
Societe Generale SA (GLE FP)	0.0349	0.0099	1	1	2	0.385	0.000	0.035
Standard Chartered PLC (STAN LN)	0.0184	0.0108	0	1	1	0.500	0.000	0.020
Swedbank AB (SWEDA SS)	0.0162	0.0122	1	2	3	0.750	0.000	0.015
UniCredit S.p.A. (UCG IM)	0.0349	0.0126	3	2	5	0.500	0.000	0.052

5.2. Robustness

In Table 5, we present some robustness check for the entire procedure. We considered 20 replications of the procedure with 200 hundred particles and 10000 random permutations in four settings:

1. Random start: random starting points of the swarm and random swarm update

2. Equal weights: start with the swarm concentrated in the center of the unit hypercube and random swarm update. This corresponds to giving equal weights to all the measures.

3. Best Partial Test: start with the swarm concentrated on one edge of the unit hypercube with the i-th weight one for the measure with minimum λ_i and zero for the others and random swarm update. This is done to check if the procedure goes beyond the intuitive solution of choosing only the measure with the most significant partial test.

4. PSO const seed: the starting point of the swarm and the random number used for the updates are kept constant during the 20 replications

We report the mean and the standard deviation of optimal p-values and the mean and standard deviation (in parentheses) of the weights. The set up are ranked in terms of ascending standard deviation of the p-values and ascending maximum standard deviation among the weights.

Differences among the four set up are not so relevant and we reach a precision on p-values below the fourth decimal digit and around the second digit for weights.

Finally, we remark that we also conducted robustness checks substituting the coefficient of variation to the Kolmogorov-Smirnov statistic in the computation of the test. The results with the coefficient of variation showed much more variability and we opted for the more robust Kolmogorov-Smirnov statistic.

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Table 5: Performances based on Kolmogorov-Smirnov test statistic with $B = 10000$ randomized permutations

Year 2013									
p-value	mean	s.d							
Best partial test	0.0018	0.0004							
PSO const.seed	0.0018	0.0005							
Equal weitghs	0.0019	0.0005							
Random start	0.0019	0.0006							
Weights mean on $n = 20$	MES	*Delta*CoVaR	in degree	out degree	in outs degree	closeness	eigenvector centrality	PCA	max s.d.
Equal weitghs	0.002	0.605	0.011	0.001	0.358	0.019	0.002	0.001	0.044
sd	*(0.007)*	*(0.044)*	*(0.017)*	*(0.002)*	*(0.040)*	*(0.040)*	*(0.004)*	*(0.003)*	
PSO const.seed	0.001	0.589	0.018	0.002	0.370	0.018	0.002	0.000	0.049
sd	*(0.001)*	*(0.042)*	*(0.028)*	*(0.003)*	*(0.049)*	*(0.038)*	*(0.003)*	*(0.001)*	*(0.049)*
Best partial test	0.002	0.589	0.012	0.001	0.384	0.009	0.001	0.000	0.052
sd	*(0.006)*	*(0.052)*	*(0.026)*	*(0.001)*	*(0.035)*	*(0.028)*	*(0.003)*	*(0.001)*	*(0.052)*
Random start	0.001	0.572	0.023	0.001	0.378	0.023	0.001	0.001	0.057
sd	*(0.003)*	*(0.05)7*	*(0.041)*	*(0.003)*	*(0.051)*	*(0.038)*	*(0.001)*	*(0.001)*	*(0.057)*

Year 2014									
pval	mean	s.d							
Random start	0.3120	0.0038							
PSO const.seed	0.3148	0.0055							
Equal weitghs	0.3143	0.0056							
Best partial test	0.3133	0.0061							
Weights mean on $n = 20$	MES	*Delta*CoVaR	in degree	out degree	in outs degree	closeness	eigenvector centrality	PCA	max s.d.
Random start	0.0000	0.0000	0.0000	0.0000	0.0211	0.0001	0.9782	0.0006	0.0071
sd	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0069)*	*(0.0002)*	*(0.0071)*	*(0.0007)*	
Equal weitghs	0.0000	0.0000	0.0000	0.0000	0.0224	0.0011	0.9761	0.0004	0.0078
sd	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0078)*	*(0.0039)*	*(0.0062)*	*(0.0005)*	*(0.0078)*
PSO const.seed	0.0000	0.0000	0.0000	0.0000	0.0187	0.0022	0.9787	0.0004	0.0083
sd	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0083)*	*(0.0064)*	*(0.0056)*	*(0.0005)*	*(0.0083)*
Best partial test	0.0000	0.0000	0.0000	0.0000	0.0204	0.0011	0.9783	0.0002	0.0104
sd	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0104)*	*(0.0042)*	*(0.0093)*	*(0.0002*	*(0.0104)*

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