Time Matters: How Default Resolution Times Impact Final Loss Rates*

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Abstract

The Loss Given Default (LGD) and the Default Resolution Time (DRT) are two important credit risk parameters which are treated on an isolated basis in academia so far. However, dependency between the two seems to be obvious as complex default resolutions may be accompanied with long DRTs and high LGDs. We propose a hierarchical Bayesian model with multiple random effects for joint estimation of DRTs and LGDs. Our approach explicitly takes unresolved loans into account and eliminates a potential resolution bias which arises when LGD distributions are estimated using resolved cases only, as is standard in common approaches. Using a European data set, we find strong positive dependencies between DRTs and LGDs. We show that neglecting unresolved cases leads to systematic underestimation of average LGDs on an out of sample perspective. Thus, our approach leads to a better out of sample fit than a pure LGD model.

Keywords: credit risk; default resolution time; loss given default; random effects; resolution bias

JEL classification: C23, G21, G33

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1 Introduction

The Default Resolution Time (DRT) and the Loss Given Default (LGD) of defaulted loan contracts are outcomes of the same random process – the resolution process. Thus, an interconnection of DRTs and LGDs is plausible as complex default resolutions might be accompanied with longer resolution processes and higher losses. To the best of our knowledge, no study exists so far which deeply examines the relation of these credit risk parameters. However, its consideration may be beneficial for generating accurate predictions. Right after default, DRTs and LGDs are unknown and need to be estimated. As resolution proceeds, loans might still be stuck in the resolution process. However, additional information in terms of the time a loan has already spent in the resolution process is available. This information is not included in traditional LGD models, but might improve LGD predictions – particularly, in the presence of an interconnection of DRTs and LGDs. Moreover, its consideration controls for the so called resolution bias. In the more recent time periods, only LGDs of loans with short DRTs are observable. Assuming a positive dependence of DRTs and LGDs which is indicated by economic intuition and supported by descriptive analyses (see Betz et al., 2016, 2017), these loans tend to exhibit lower LGDs. This leads to downward biased LGD estimates.

As a numerical illustration, assume three loan contracts which defaulted at the same time. The first loan is almost completely recovered in half a year after default ($\text{LGD}_1 = 7\%$ and $\text{DRT}_1 = 0.5$). The second loan generates a medium loss in the first two years after default ($\text{LGD}_2 = 21\%$ and $\text{DRT}_2 = 2.0$). Finally, the resolution of the third loan is terminated after four years causing a high loss ($\text{LGD}_3 = 42\%$ and $\text{DRT}_3 = 4.0$).[1] Considerable distortions arise if expected LGDs are estimated at varying points in time. Thus, the estimated expected LGD half a year after default amounts to 7% ($\widehat{\text{E}}[LGD_i \mid t = 0.5] = 7\%$) as only the LGD of the first loan is observable. The estimated expected LGD yields to 14% ($\widehat{\text{E}}[LGD_i \mid t = 2.0] = 14\%$) two years after default. An unbiased estimate of the expected LGD can foremost be made four years after default ($\widehat{\text{E}}[LGD_i \mid t = 4.0] = 23\%$) when all loans are resolved. The expected LGD

[1] The figures reflect the descriptive statistics in our data set (see Figure 1). Loans with DRTs $\in [0, 0.5]$ generate average LGDs of 7%, wheres, DRTs $\in (1.0, 2.0]$ (DRTs $\in (3.0, \dots]$) correspond to average LGDs of 21% (42%). LGDs are calculated based on discounted recovery payments (see Section 2.1).

is underestimated by 16 percentage points half a year after default and still by 9 percentage points two years after default. The inclusion of censored observations, i.e., unresolved cases, might improve the estimation. After the resolution of the second loan, it is known that $\text{DRT}_3 > 2.0$ and, thus, $\text{LGD}_3 > 21\%$ might be expected. This leads to the conclusion that $\widehat{\text{E}}[\text{LGD}_i \mid t = 2.0] > 16\% \left(= \frac{7\%+21\%+21\%}{3}\right)$ which might be used as lower limit for LGD prediction. Considering the dependence structure of DRTs and LGDs in more detail, the estimation might be further adjusted.

In recent years, the literature regarding LGD modeling has widened considerably. Comparative studies can be found in, e.g., Qi and Zhao (2011) and Loterman et al. (2012). However, literature considering *workout* LGDs is still limited. Most of the publications refer to *market-based* LGDs, whereby, the corresponding Recovery Rate (RR) is defined as ratio of the market price 90 days after default to the outstanding amount. Hence, market-based LGDs are only observable for traded securities such as bonds. Workout LGDs are based on actual recovery payments collected during the resolution process and, thus, usually applied for loans. The distribution of workout LGDs is more extreme compared to market-based LGDs and, typically, high probability masses at no loss ($\text{LGD} = 0$) and total loss ($\text{LGD} = 1$) arise (see, e.g., Krüger and Rösch, 2017; Betz et al., 2018). Thus, the consideration of the distributional form is essential for workout LGDs. Altman and Kalotay (2014) develop a Bayesian Finite Mixture Model (FMM) with a probabilistic substructure in terms of an ordered logit (OL) model to estimate the probability of the mixture components depending on explanatory variables. A frequentistic version of this model is presented by Kalotay and Altman (2017). The model of Altman and Kalotay (2014) and Kalotay and Altman (2017) is applied by Bijak and Thomas (2015) and enhanced by Betz et al. (2018). Calabrese (2014) estimates a mixture of Beta distributions, whereas, Krüger and Rösch (2017) apply quantile regression on the LGD distribution. The literature regarding DRTs is more sparse and mainly refers to the duration of Chapter 7 and Chapter 11 resolutions (see, e.g., Helwege, 1999; Partington et al., 2001; Bris et al., 2006; Denis and Rodgers, 2007; Wong et al., 2007). Betz et al. (2016) and Betz et al. (2017) analyze DRTs of defaulted loan contracts and descriptively find impacts of DRTs on LGDs. The interconnection of DRTs and LGDs is also indicated in the LGD literature. Dermine and Neto de Carvalho (2006) apply mortality analysis on a data set of defaulted bank loans, whereas, Gürtler and Hibbeln (2013) focus on the resolution bias. They suggest to restrict the data set to avoid biased estimates. However, LGD data is sparse so constraints might be unfavorable. The inclusion of the DRT into the LGD modeling framework might diminish the effect of the resolution bias without restrictions in the data set. Common ways to implement dependence structures of credit risk parameters are random effects. By this means, joint time patterns of these parameters are considered in the modeling context. Rösch and Scheule (2010), Bade et al. (2011), and Rösch and Scheule (2014) apply random effects to model the dependence of probabilities of default (PDs) and LGDs. Furthermore, Lee and Poon (2014) state that frailties, i.e., random effects in survival models, make more sigificant risk contributions than macroeconomic factors in a credit risk context.

Using a unique European data set provided by Global Credit Data (GDC), we develop a hierarchical Bayesian modeling approach for joint estimation of DRTs and LGDs combining a Finite Mixture Model (FMM) with a probabilistic substructure for the LGD and an Accelerated Failure Time (AFT) model for the DRT. Thereby, we allow for direct and indirect dependency structures. The first is attained by the inclusion of the DRT in the LGD model. This allows LGD predictions for censored observations, i.e., non-performing loans, within the modeling framework as final DRTs are estimated based on censored observations in the DRT model. The second refers to common time patterns of DRTs and LGDs. We implement two correlated random effects in the DRT model and the LGD model.

We contribute to the literature in three ways. First, we deeply examine the dependence structure of DRTs and LGDs allowing for a direct and an indirect channel and find impacts of DRTs on LGD distributions which are even more pronounced in boom and crisis periods. Thus, DRTs are longer (shorter) in crisis (boom) periods. In crisis periods, this burdens financial market liquidity as systematically more loans are stuck in the resolution process. On top of that, losses are systematically higher during such periods due to the stronger positive dependence. Second, we analyze and quantify the impacts of the resolution bias. We compare a pure (traditional) LGD model with the proposed hierarchical approach and find parameter distortions in the pure LGD model. These result in biased predictive LGD distributions on an out of sample and out of sample out of time perspective. Impacts of the resolution bias are diminished in the hierarchical approach leading to appropriate predictive LGD distributions. Neglecting unresolved cases leads to an underestimation of average LGDs on a out of sample perspective. Third, we are able to generate intuitive LGD predictions for non-performing loans. As final DRTs are estimated within the AFT model, these can be applied to generate final predictive LGD distributions for censored observations. These considerably outperform predictions based on a pure LGD model.

The remainder of this paper is structured as follows. Section 2 describes the data and introduces the hierarchical modeling framework. Results are presented in Section 3. In Section 4, the model is validated on an in sample and out of sample perspective. Section 5 concludes.

2 Data and Methods

2.1 Data

We use access to the unique loss data base of Global Credit Data (GCD). The data base includes detailed loss information on transaction basis of 53 member banks all around the world. In the data base, the LGD is determined by:

$$\mathrm{LGD}_i = 1 - \mathrm{RR}_i, \tag{1}$$

whereby, LGD_i is the loss rate of loan i and RR_i is the corresponding RR. The RR is calculated as the sum over the present values of all relevant transactions divided by the outstanding amount.[2]

We follow Höcht and Zagst (2010) and Höcht et al. (2011) and develop two selection criteria to eliminate loans with extraordinary payment structures. Both criteria relate all relevant transactions including charge-offs (which are not included in the LGD calculation) to the outstanding amount. The first criterion, to which we refer as *pre-*

[2] See Betz et al. (2018). More detailed information of the LGD calculation can be found in Betz et al. (2016).

resolution criterion, relates transactions arising pre resolution to the outstanding amount at default. We set the barriers of the pre-resolution criterion to $[90\%, 110\%]$ for resolved and $[-50\%, 400\%]$ for unresolved loans. In the second criterion, i.e., *post-resolution criterion*, transactions occurring post resolution are related to a fictional outstanding amount at resolution. The barriers are set to $[-10\%, 110\%]$ for the post-resolution criterion. The post-resolution criterion applies for resolved loans only. Subsequently, loans with abnormal low and high LGDs ($< -25\%$ and $> 125\%$) are eliminated.[3] We consider a subsample of defaulted European term loans and lines to small and medium sized enterprises (SMEs).[4] We further exclude loans which defaulted before 2004 and after 2016 (10.02% of subsample data). A subsample of 38,165 loans remains.

Figure 1 illustrates the interconnection of the two dependent variables, i.e., the DRT and the LGD. Therefore, the data set is divided into DRT buckets based on DRTs. The first bucket includes all loans with DRTs $\in [0, 0.5]$ years. The second bucket contrains all loans with DRTs $\in (0.5, 1.0]$ years, and so on (see x-axis of left panel and legend of right panel). In the left panel, box plots of LGDs divided by DRT buckets are displayed. The thick black lines mark the medians, whereas, the thick gray lines are the means. Considering the latter, average LGDs seem to linearly increase in the DRT buckets. To examine the origin of this increase, the right panel displays kernel density estimates for the DRT buckets. The LGD distribution of higher DRT buckets is shifted towards higher LGD values, i.e., probability masses of lower LGD values decrease and probability masses of higher LGD values increase. Thus, average values increase.

Table 1 summarizes the descriptive statistics of the dependent and independent variables. Figures are stated for all loans (resolved and unresolved cases) and for resolved and unresolved loans separately. The upper panel of the table includes descriptive statistics for the LGD and the DRT. For unresolved cases, incurred LGDs are applied. Incurrent LGDs are computed as the sum over the present values of all relevant transactions, which occurred up to the end of the observation period (end of 2016), divided

[3] Overall, 0.50% of resolved loans are eliminated due to the pre-resolution criterion and 0.19% due to the post-resolution criterion, whereas, 0.23% of unresolved loans are eliminated based on the pre-resolution criterion. Subsequently, 0.13% are sorted out due to abnormal low and high LGD values.

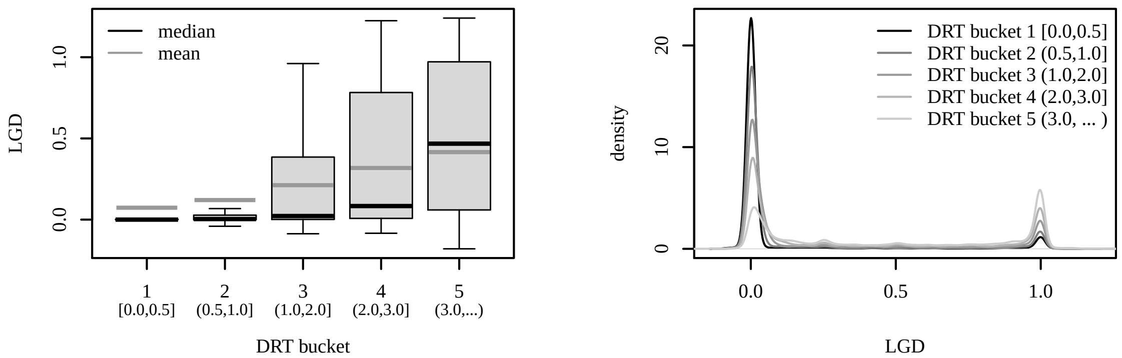
[4] We restrict the data base to reduce noise and generate a rather homogeneous sample. We consider the twelve most common European countries in the data base, i.e., Great Britain, Germany, Denmark, Portugal, Ireland, France, Finland, Sweden, Norway, Latvia, Estonia, and Poland.

Table 1: Descriptive statistics

		all	**resolved**	**unresolved**
n		38,165	35,272	2,893
	dependent variables			
LGD	mean	0.2534	0.2099	0.7839
	median	0.0133	0.0082	0.9780
	standard deviation	0.3810	0.3531	0.3017
DRT	mean	1.9882	1.7342	5.0839
	median	1.2621	1.1335	4.9090
	standard deviation	2.0756	1.7509	3.0147
	loan specific (metric)			
EAD	mean	533,118.89	516,582.05	734,739.20
	median	102,987.29	100,237.48	155,097.53
	standard deviation	3,624,711.52	3,610,978.60	3,782,983.43
	loan specific (categoric)			
Facility	*term loan*	62.00%	60.39%	81.68%
	line	38.00%	39.61%	18.32%
Protection	*no*	25.61%	26.03%	20.46%
	yes	74.39%	73.97%	79.54%
Industry	*non FIRE*	83.13%	82.13%	95.30%
	FIRE	16.87%	17.87%	4.70%
	macro variables			
Δ HPI	mean	-1.6966	-1.7765	-0.7229
	median	0.1662	0.1662	0.8360
	standard deviation	6.0221	5.9928	6.2878
VIX	mean	24.4762	24.3681	25.7947
	median	22.9249	22.6771	23.3451
	standard deviation	9.4105	9.4699	8.5465

Note: The table summarizes descriptive statistics for dependent and independent variables in the data set. For metric variables, means, medians, and standard deviations are stated. Proportions are presented for variables of categoric nature. The sample size is denoted by n. The abbreviation FIRE means *Finance*, *Insurance*, *Real Estate* and denotes corporations of this industries. The macro variable Δ HPI is the is the yoy percentage chance of the *House Price Index*, whereas, the VIX is the *Volatility Index*.

Figure 1: Relation of DRT and LGD



Note: The figure illustrates the relation of DRTs and LGDs. The data is divided into DRT buckets based on the realized DRTs. Thus, the first bucket includes all loans with DRTs $\in [0, 0.5]$ years. The second bucket contains all loans with DRTs $\in (0.5, 1.0]$ years, and so on (see x-axis of left panel and legend of right panel). In the left panel, box plots of LGDs for the DRT buckets are displayed. Outliers are hidden. The thick black lines mark the medians, whereas, the thick gray lines are the means. In the right panel, kernel density estimates of LGDs for the DRT buckets are illustrated. The band width is fixed to 0.015 to ensure comparability.

by the outstanding amount. As the resolution process is not terminated, incurrent LGDs are higher than final LGDs. DRTs for unresolved cases are censored to the end of the observation period (end of 2016), e.g., for unresolved loans defaulted at the end of 2015, a censored DRT of one year is assigned. Censored DRTs are lower than final DRTs as the resolution process is not terminated. In the table, average values of LGDs and DRTs for unresolved cases are higher compared to resolved cases as unresolved cases are shaped by rather *bad* loans, i.e., loans exhibiting high DRTs and high LGDs. In the middle panels of the table, descriptive statistics of loan specific independent variables are stated. We use the EAD to control for the size of the loan. It is further distinguished between term loans and lines, if a loan is protected by collateral or guarantee or not, and if the debtor has Finance, Insurance, Real Estate (FIRE) industry affiliation. Reference categories in the subsequent models are written italic in the table.[5] The lower panel of the table contains descriptive statistics of the applied macro variables. The year-on-year (yoy) percentage change of weighted average real residential prices (Δ HPI) is employed as explanatory variable for the LGD, whereas, we use the VSTOXX Volatility Index (VIX) for the DRT.[6]

[5] The reference category is *term loan* for Facility, *no* for protection, and *non-FIRE* for industry.

[6] We tested further macro variables, e.g, the yoy percentage change of weighted average seasonally adjusted GDPs and the quarterly average yoy percentage change of weighted average equity indices.

Figure 2 illustrates the time patterns of average DRTs in the left panel and average LGDs in the right panel for resolved loans (thick black line) and all loans (resolved and unresolved loans, thin gray line). Regarding the latter, values for unresolved loans, i.e., censored observations, have to be calculated. Thus, DRTs are censored to the end of the observation period (end of 2016) and incurrent LGDs are considered for unresolved cases. Incurrent LGDs are computed as sum over the present values of all relevant transactions, which occurred up to the end of the observation period (end of 2016), divided by the outstanding amount. The relation of DRTs and LGDs (see

Figure 2: Time patterns of average DRTs and average LGDs

Note: The figure illustrates time patterns of average DRTs in the left panel and average LGDs in the right panel. The black lines display the average values for resolved loans, whereas, the gray lines are average values for all loans, i.e., resolved and unresolved cases. Thus, the latter include censored values. Means over the entire time period are illustrated by dotted lines.

Figure 1) might partly be driven by analogous time patterns. Both dependent variables sharply increase prior to the Global Financial Crisis (GFC, 2007 Q2) and reach the maximum during the climax of the GFC. The rebound in the aftermath of the crisis seems gradual. There are only minor deviations between resolved loans and all loans considering the average DRTs. The graph for all loans is slightly shifted upwards by the censored observations. Regarding average LGDs, this spread is severe particularly in the most recent time periods. This is mainly due to incurrent LGDs, i.e., LGDs based on transactions which occur up to the end of the observation period, in the averaging. Final LGDs will be lower. However, final LGDs of all loans will still lie above the black line (final LGDs of resolved loans). This mismatch is due to the resolution bias. Due to

However, Δ HPI and VIX exhibit the highest statistical evidence. Remaining results are available form the authors upon request.

the censoring, final LGDs are only observable for defaults with short DRTs in the more recent time periods. Due to the interconnection of DRTs and LGDs, these tend to be lower implying an underestimation of LGDs in the more recent time periods.

In this paper, we aim to analyze the effects of the resolution bias on an in sample and out of sample perspective. Therefore, we divide the data set as of Table 1 into subsamples. The first subsample serves as *estimation sample*. It includes all loans defaulted between 2004 Q1 and 2010 Q4. Thus, it comprises times of rather sound economic surrounding, the GFC, and parts of the rebound phase. As we aim to analyze effects of the resolution bias, we treat loans which are not resolved until 2010 Q4 as censored observations, i.e., unresolved loans. The second subsample, to which we refer to as *validation sample I*, includes the final observations to the censored observations as of the estimation sample. We apply validation sample I to perform an *out of sample validation* of LGDs. The third sample, i.e., *validation sample II*, includes all loans defaulted between 2011 Q1 and 2016 Q4. It is used to perform an *out of sample out of time validation* of LGDs. Table 2 summarizes the estimation sample and the validation samples. In the upper panel, the

Table 2: Estimation sample and validation samples

		estimation sample	**validation sample I** (*out of sample*)	**validation sample II** (*out of sample out of time*)
n	all	31,988	10,171	6,177
	resolved	21,817	8,447	5,008
	unresolved	10,171	1,724	1,169
	dependent variables			
average LGD	all	0.2586	0.4270	0.2267
	resolved	0.1801	0.3511	0.1017
	unresolved	0.4270	0.7987	0.7622
average DRT	all	1.5763	4.2566	1.0495
	resolved	1.1964	3.6851	0.7869
	unresolved	2.3911	7.0568	2.1743

Note: The table summarizes the applied samples. The number n, the average LGD, and the average DRT of all loans, resolved loans, and unresolved loans are presented for the estimation sample and the two validation samples. The models are estimated based on the estimation sample. This sample includes all loans defaulted between 2004 Q1 and 2010 Q4. Loans which are not resolved until 2010 Q4 are treated as censored observations, i.e., unresolved cases, in the estimation. Validation sample I contains the final observations of these unresolved cases. However, observations exist which are still censored at the end of 2016 (unresolved cases in validation sample I). In validation sample II, loans which defaulted between 2011 Q1 and 2016 Q4 are included. Thus, validation sample I is applied for the *out of sample* validation, whereas, the *out of sample out of time* validation is performed on validation sample II.

sample sizes are stated. Validation sample II consists of the 10,171 loans which are treated as unresolved cases in the estimation sample. Some of these loans (1,724) are still unresolved at the end of 2016. However, the proportion of unresolved loans is lower in validation sample I compared to the estimation sample. In the lower panel, average values of LGDs and DRTs are stated. These are rather similar comparing the estimation sample and validation sample II, however, considerably higher in validation sample I. This is due to the fact that validation sample I contains final observations to censored cases in the estimation sample, thus, observations with higher DRTs and higher LGDs.

2.2 Methods

This paper aims to compare a pure LGD model with a hierarchical approach combining a model for the DRT with a LGD model. For the LGD model, we adapt the model presented in Betz et al. (2018). In the hierarchical approach, this LGD model is combined with an *Accelerated Failure Time* (AFT) model for the DRT. Survival time models such as the AFT model bear the advantage of considering censored observations, i.e., unresolved loans. This can be applied to include censored observations in an LGD modeling context. By this means, we are able to diminish effects of the resolution bias. To investigate the direct dependence of DRTs on LGDs, the DRT serves as an explanatory variable in the LGD model. Two correlated random effects are included to study comovements of DRTs and LGDs in the time line (indirect dependence). In the following, we briefly review the LGD model of Betz et al. (2018) and discuss the extensions in the context of the hierarchical model.

LGD Model

A Normal Finite Mixture Model (FMM) combined with a probabilistic substructure in terms of an Ordered Logit (OL) model is applied to the loss rate L.[7] In FMMs, the dependent variable is assumed to be divided into a finite number of K latent classes. In each class k, L follows a normal distribution with parameters θ_k depending on the latent class k. Thus, the probability density function (PDF) of an FMM $g(L|\theta_1,\ldots,\theta_K)$

[7] We use the notation L for the LGD due to aesthetic reasons.

is the p_k weighted sum of the component PDFs $f_k(L|\theta_k)$:

$$g(L\,|\,\theta_1,\ldots,\theta_K) = \sum_{k=1}^{K} p_k\, f_k(L\,|\,\theta_k)\,. \tag{2}$$

To ensure the general properties of a PDF, i.e., $g(l) \geq 0$ for all $l \in \mathbb{R}$ and $\int_{-\infty}^{\infty} g(l) = 1$, $p_k \geq 0$ and $\sum_k p_k = 1$ must hold. Assuming conditional independence, the likelihood of a Normal FMM $\phi(L_1,\ldots,L_N\,|\,\mu,\sigma,p)$ is the product of the individual likelihood contributions:

$$\phi(L_1,\ldots,L_N\,|\,\mu,\sigma,p) = \frac{1}{(2\pi)^{\frac{N}{2}}} \prod_{i=1}^{N} \left(\sum_{k=1}^{K} \frac{p_k}{\sigma_k} \exp\left[-\frac{(L_i-\mu_k)^2}{2\sigma_k^2} \right] \right), \tag{3}$$

where, μ_k and σ_k are the parameters of the latent class k and N is the number of observations. To adapt data augmentation, the component weight p_k is replaced by an indicator variable d_{ik} which takes the value one if l_i is a random draw of component k and zero otherwise:

$$\phi(L_1,\ldots,L_N\,|\,\mu,\sigma,d) = \frac{1}{(2\pi)^{\frac{N}{2}}} \prod_{i=1}^{N} \left(\sum_{k=1}^{K} \frac{d_{ik}}{\sigma_k} \exp\left[-\frac{(L_i-\mu_k)^2}{2\sigma_k^2} \right] \right). \tag{4}$$

A probabilistic substructure is formulated to include covariates in the FMM. To rely on the classical formulation of the OL model, we define the component affiliation y_i:

$$y_i = k \quad \text{if } d_{ik} = 1\,, \tag{5}$$

where, d_{ik} is the indicator as of Equation (4). The component affiliation Y_i is categorically distributed and determined by the location of a metric latent variable Y_i^* to corresponding cut points c_k $(k \in \{1,\ldots,K-1\})$:

$$Y_i = \begin{cases} 1 & \text{if } Y_i^* \leq c_1 \\ 2 & \text{if } c_1 < Y_i^* \leq c_2 \\ \vdots & \\ K & \text{if } c_{K-1} < Y_i^*\,. \end{cases} \tag{6}$$

The latent variable Y_i^* follows a linear model:

$$Y_i^* = z_i \zeta + F_{t(i)} + e_i, \qquad e_i \sim \text{logistic}\,, \tag{7}$$

where, z_i is a $(1 \times J)$ vector of independent variables and ζ is the $(J \times 1)$ vector of coefficients. The term e_i describes the errors. A random effect $F_{t(i)}$ is introduced into the modeling framework to control for comovement in the time line. It originates from a Normal distribution with mean zero and standard deviation σ:

$$F_t \sim \mathrm{N}(0, \sigma)\,. \tag{8}$$

The time stamp $t(i)$ in Equation (7) indicates the default time t in quarters of loan i. Two loans i and i' which defaulted in the same quarter $(t(i) = t(i') = t)$ share the same realization of the random effect $(f_{t(i)} = f_{t(i')} = f_t)$. For $f_t > 0$ $(f_t < 0)$, both loans exhibit higher (lower) values of y_i^* and, thus, higher (lower) probabilities of high component affiliations y_i. Higher component affiliations y_i are accompanied with higher loss rates and vice versa. Thus, the random effect displays the comovement in time line, i.e., higher or lower average loss rates in specific default quarters which can not be explained by observable variables included in z_i.

Hierarchical Model

In the following, the hierarchical model for the joint estimation of DRTs and LGDs is discussed. We apply an AFT model for the DRT. Thus, the logarithm of the resolution time $\ln(T_i)$ can be expressed by a linear model:

$$\ln(T_i) = \beta_0 + x_i \beta + F_{t(i)}^T + s\,\varepsilon_i\,, \qquad \varepsilon_i \sim \text{negative Gumbel}\,, \tag{9}$$

where, x_i is a $(1 \times J_T)$ vector of independent variables, β is the $(J_T \times 1)$ vector of coefficients, and β_0 is the intercept. We assume the errors ε_i to follow a negative Gumbel distribution and, thus, the DRT to be Weibull distributed.[8] The term s is the scale parameter. A random effect $F_{t(i)}^T$ is introduced into the modeling framework to control for comovement in the time line. Equation (9) applies to non censored, i.e., final,

[8] We tested various distributional assumptions regarding the resolution time, e.g., log Normal, log Logistic, Exponential, and Weibull. The Weibull distribution seems to have the best fit.

observations. For censored observations, final observations are estimated within the Bayesian modeling framework. By this means, we are able to predict final DRTs for censored data points, i.e., unresolved loans.

In the hierarchical approach, the AFT model for the DRT is simultaneously estimated with the LGD model. As LGD model the Normal FMM with the probabilistic substructure is adapted, whereby, the logarithm of the DRT is included as explanatory variable to account for the direct dependence of DRTs on LGDs. Equation (7) modifies to:

$$\mathcal{Y}_i^* = z_i \gamma + \ln(T_i)\gamma_T + F_{t(i)}^L + \epsilon_i\,, \qquad \epsilon_i \sim \text{logistic}\,, \tag{10}$$

where, z_i is the $(1 \times J_L)$ vector of independent variables, γ is the $(J_L \times 1)$ vector of coefficients, and γ_T is the coefficient of the logarithm of the DRT. Again, a random effect $F_{t(i)}^L$ is introduced into the modeling framework to control for comovement in the time line.[9]

The random effects $F_{t(i)}^T$ as of Equation (9) and $F_{t(i)}^L$ as of Equation (10) originate from a bivariate normal distribution:

$$\begin{pmatrix} F_t^T \\ F_t^L \end{pmatrix} \sim \mathrm{N}_2\left(0_2, \Sigma\right)\,, \tag{11}$$

where, 0_2 is the two dimensional zero vector ($0_k = (\,0 \quad 0\,)^T$) and Σ is the (2×2) covariance matrix. Prior distributions are provided for the individual standard deviations (σ_T and σ_L) and the (2×2) correlation matrix Ω (see Appendix A):

$$\begin{aligned} \Sigma &= \mathrm{diag}\left(\sigma_T, \sigma_L\right)\, \Omega \, \mathrm{diag}\left(\sigma_T, \sigma_L\right) \\ &= \begin{pmatrix} \sigma_T^2 & \sigma_T \sigma_L \omega_{L,T} \\ \sigma_T \sigma_L \omega_{T,L} & \sigma_L^2 \end{pmatrix}, \end{aligned} \tag{12}$$

whereby, $\omega_{T,L} (= \omega_{L,T})$ is the correlation of F_t^T and F_t^L. By the inclusion of the random effects, we control for joint comovements of loss rates and resolution times in the time line. Two loans i and i' defaulted in the same quarter ($t(i) = t(i') = t$) share the

[9] Equation (2), (3), (4), (5), and (6) apply in analogy to Y_i*.

same realizations of the random effects ($f_{t(i)}^T = f_{t(i')}^T = f_t^T$ and $f_{t(i)}^L = f_{t(i')}^L = f_t^L$, however, $f_t^T \neq f_t^L$ in most of the cases). For $f_t^T > 0$ ($f_t^T < 0$), average DRTs are higher (lower). Assuming a positive correlation between the random effects and a positive parameter estimate of the logarithm of the DRT in the LGD model ($\gamma_T > 0$), the corresponding LGDs are effected through two channels:[10] Directly, as higher (lower) DRTs are inserted in the LGD model. Indirectly, as positive (negative) realizations of f_t^T tend to imply positive (negative) realizations of f_t^L due to the positive correlation. Thus, LGDs are even higher. However, negative realizations of f_t^L remain possible for a stochastic process as of Equation (11) which might reduce LGDs. Both scenarios are conceivable. Confronted with tense economic surrounding, financial institutions might decide to follow a wait-and-see strategy and relocate resolution efforts in the future. This might provide benefits and reduce the LGD ($f_t^L < 0$). However, LGDs might be further increased ($f_t^L > 0$) if financial institutions are forced to resolve defaulted loans at a certain point in time, e.g., if there is no further option to wait.

Estimation

The parameters of the LGD model and hierarchical model are estimated via Bayesian inference. We use a Markov Chain Monte Carlo (MCMC) sampler to derive the posterior distributions of the parameters. The MCMC sampler generates samples by constructing reversible Markov chains. The equilibrium distribution corresponds to the target posterior distribution. The solution via MCMC sampling is necessary due to the model complexity of the LGD model and the hierarchical model, i.e., priors are partly non conjugate. Direct sampling from the posterior distributions is not possible as there is no analytical solution. The LGD model and the hierarchical model are sampled with two MCMC chains. Burn-in is set to 500. Posterior samples contain 25,000 iterations per chain with a thinning of 5. Metric dependent variables are standardized to ease convergence.[11]

Most of the model parameters are provided with weakly informative prior distributions. See Appendix A for detailed information on the Bayesian model specifications.

[10] These assumptions correspond to the empirical results (see Section 3).

[11] As MCMC sampler, we adapt Stan which is a Hamiltonian Monte Carlo (HMC) sampler. It overcomes some of the problems inherent in Gibbs sampling, e.g., regarding highly correlated posterior distributions.

Common convergence diagnostics can be found in Appendix B.

3 Empirical Results

In Bayesian inference, posterior distributions of parameters are assumed to be continuous. Thus, a single value of the posterior distribution has a probability of zero. On the contrary, one *true* parameter estimate is assigned in frequentistic terms. A null hypothesis is set up to reach a yes-or-no decision. Under the Bayesian approach, estimates are provided by posterior distributions which offer an intuitive consideration of parameter uncertainty. Thus, other concepts are indicated to quantify if results are in favor of an impact, i.e., if an impact is statistical evident. We apply two of them – *credible intervals* and *Bayes factors*.

Credible intervals, e.g., Highest Probability Density Intervals (HPDIs), specify intervals in the domain of the posterior distribution in which the unobservable parameter lies with a certain probability. The HPDI denotes the narrowest credible interval. If $0 \notin \text{HPDI}$, the domain of the posterior distribution is located in the positive (negative) value range indicating statistical evidence of the positive (negative) sign. Besides credible intervals, we apply Bayes factors to evaluate statistical evidence. Bayes factors are the relation of posterior odds to prior odds. Posterior odds are defined as the ratio of posterior probability masses under the null hypotheses and the alternative hypothesis. As we are interested in the evidence of the signs, posterior odds are derived as the ratio of posterior mass favoring the sign of the posterior mean to posterior mass of the opposite sign:

$$\text{posterior odds}_{\mathrm{E}[\theta]<0} = \frac{\mathbb{P}(\theta < 0 \mid \text{data})}{\mathbb{P}(\theta \geq 0 \mid \text{data})}$$
$$\text{posterior odds}_{\mathrm{E}[\theta]>0} = \frac{\mathbb{P}(\theta > 0 \mid \text{data})}{\mathbb{P}(\theta \leq 0 \mid \text{data})},$$

whereby, θ denotes an arbitrary parameter. Assuming a positive posterior mean ($\mathrm{E}[\theta] > 0$), $\text{posterior odds}_{\mathrm{E}[\theta]>0} = 3$ indicates that a positive impact is three times as likely as a negative impact. Prior odds are the corresponding ratio of the prior distribution. Assuming a symmetric prior distribution around zero, posterior odds are equivalent to the Bayes factor. A Bayes factor exceeding 3.2 is deemed as substantial evidence. Values above 10 are assigned with strong evidence, whereas, values above 100 are related to decisive evidence (see Kass and Raftery, 1995).

LGD Model

The LGD model is estimated based on the estimation sample (see Table 2). However, it offers no possibility to include censored observations, i.e., unresolved loans, in the estimation process. Thus, the 21,817 resolved cases are included, whereas, 10,171 unresolved defaults are neglected. As these unresolved loans tend to exhibit higher LGDs due to the resolution bias, parameter estimates might be distorted.

Table 3 summarizes the results of the LGD model. Parameters are stated in the first column, whereas, the second column presents posterior means. In the FMM within the LGD model, parameters of the outer components (μ_1 and σ_1 for the first component, μ_5 and σ_5 for the fifth component) are fixed to identify loans with no (LGD = 0) and total (LGD = 1) loss. The second and third component are located nearby the first component ($\mu_2 = 0.0067$ and $\mu_3 = 0.0290$) and have rather small standard deviations ($\sigma_2 = 0.0045$ and $\sigma_3 = 0.0249$), whereas, the forth component seems to cover the range in between the extremes of no and total loss ($\mu_4 = 0.5114$ and $\sigma_4 = 0.3364$). The posterior distributions of the cut points (c_k for $k \in \{1,2,3,4\}$) are not directly interpretable as they depend on the range of the latent variable (Y^*).

Component probabilities are derived based on the OL model within the LGD model. The parameter of the EAD (ζ_{EAD}) exhibits a negative posterior mean indicating a lower value of the latent variable (Y^*) for higher EADs and, thus, lower LGDs. This impact is characterized by decisive evidence as the posterior odds are tending to infinity (posterior $\text{odds}_{\text{E}[\zeta_{\text{EAD}}]<0} \rightarrow \infty$) and the HPDI ($\text{HPDI}_{\zeta_{\text{EAD}}} = [-0.14, -0.08]$) excludes zero. Reasons for the negative impact of the EAD might be found in higher resolution efforts and, thus, lower loss rates, for loans of major size. The posterior mean of lines (ζ_{Facility}) is positive. Thus, lines are characterized by higher LGDs compared to term loans. This positive influence is decisively evident (posterior $\text{odds}_{\text{E}[\zeta_{\text{Facility}}]>0} \rightarrow \infty$ and $0 \notin \text{HPDI}_{\zeta_{\text{Facility}}} = [0.15, 0.26]$). Protection ($\zeta_{\text{Protection}}$) exhibits a negative posterior mean with decisive evidence (posterior $\text{odds}_{\text{E}[\zeta_{\text{Protection}}]<0} \rightarrow \infty$ and $0 \notin \text{HPDI}_{\zeta_{\text{Protection}}} =$

Table 3: Results of the LGD model

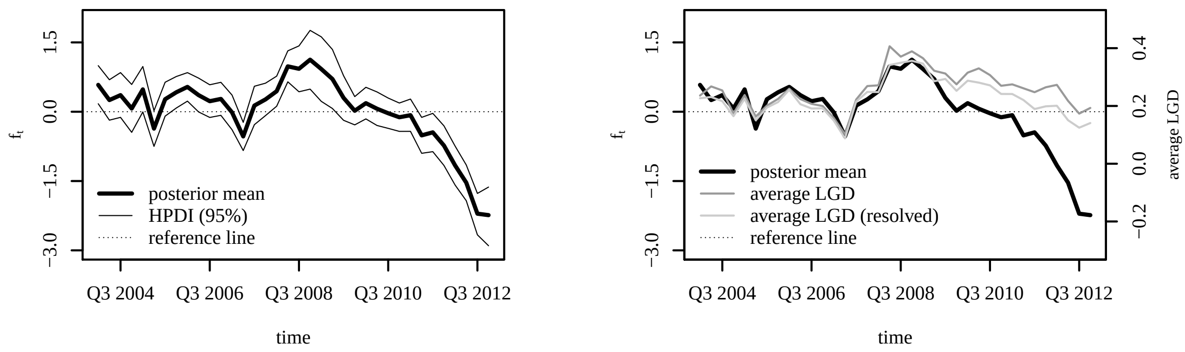
	posterior mean	HPDI (95%)		posterior odds	naive standard error	time series standard error
	LGD model					
μ_1	0.0000			*not estimated*		
μ_2	0.0067	0.0064	0.0070	∞	0.0000	0.0000
μ_3	0.0290	0.0277	0.0303	∞	0.0000	0.0000
μ_4	0.5114	0.5004	0.5229	∞	0.0000	0.0000
μ_5	1.0000			*not estimated*		
σ_1	0.0010			*not estimated*		
σ_2	0.0045	0.0042	0.0048	∞	0.0000	0.0000
σ_3	0.0249	0.0237	0.0261	∞	0.0000	0.0000
σ_4	0.3364	0.3295	0.3436	∞	0.0000	0.0000
σ_5	0.0010			*not estimated*		
c_1	-0.6959	-0.9773	-0.4082	∞	0.0006	0.0012
c_2	-0.0349	-0.3203	0.2510	1.4857	0.0006	0.0012
c_3	0.8952	0.6087	1.1777	∞	0.0006	0.0012
c_4	2.7509	2.4649	3.0421	∞	0.0007	0.0012
ζ_{EAD}	-0.1099	-0.1357	-0.0824	∞	0.0001	0.0001
ζ_{Facility}	0.2038	0.1495	0.2584	∞	0.0001	0.0001
$\zeta_{\text{Protection}}$	-0.4147	-0.4751	-0.3559	∞	0.0001	0.0001
ζ_{Industry}	-0.2355	-0.3000	-0.1683	∞	0.0002	0.0002
ζ_{HPI}	0.0590	-0.2183	0.3311	2.0243	0.0006	0.0010
	random effect					
σ	0.8191	0.6200	1.0329	∞	0.0005	0.0005

Note: The table summarizes the results of the LGD model. Parameters are stated in the first column. Categorical variables are included via dummy coding. The reference categories are term loan for facility, no for protection, and non FIRE for industry. The second column presents the posterior means. In the third and fourth column, lower and upper bounds of the corresponding HPDIs to a credibility level of 95% are displayed. The fifth column contains the posterior odds. Naive and time series standard errors are shown in the last two columns. Time series standard errors are calculated based on the effective chain length (N^*_{MCMC}) instead of the actual chain length (N_{MCMC}), whereby, $N^*_{\text{MCMC}} < N_{\text{MCMC}}$ holds for autocorrelated chains.

$[-0.48, -0.36]$). This indicates lower loss rates for protected loans which corresponds to the economic intuition. According to the negative sign of the industry FIRE (ζ_{Industry}), LGDs for loans of this industry affiliation are lower compared to other industries. This impact is decisively evident (posterior odds$_{\text{E}[\zeta_{\text{Industry}}]<0} \rightarrow \infty$ and $0 \notin \text{HPDI}_{\zeta_{\text{Industry}}} = [-0.30, -0.17]$). The applied macro variable, i.e., the HPI (ζ_{HPI}), exhibits a positive sign indicating higher LGDs for higher values of the HPI. This contradicts the economic intuition as a sound economic surrounding should be accompanied with lower loss rates. However, the positive sign is not statistical evident (posterior odds$_{\text{E}[\zeta_{\text{HPI}}]>0} = 2.02 < 3.2$ and $0 \in \text{HPDI}_{\zeta_{\text{HPI}}} = [-0.22, 0.33]$). The last row of the table summarizes the posterior distribution of the random effect parameter.[12]

Figure 3 illustrates the realizations of the random effect f_t in the LGD model. Higher realizations of the random effect ($f_t > 0$) indicate higher values of the latent variable Y^* for all loans defaulted in t and, thus, higher average LGDs in this quarter. The left

Figure 3: Random effect of the LGD model



Note: The figure illustrates the course of the random effect in the LGD model over time. In the left panel, the posterior means (thick line) and the HPDI (95%, thin lines) of the random effect realizations, i.e., f_t, are displayed. In the right panel, the random effect (black line) is contrasted with the time patterns of average LGDs for all loans (dark gray line) and for resolved loans (light gray line). Final and incurrent LGDs as of validation sample I are included in the averaging. The dotted lines mark zero and serves as a reference line.

panel of the figure presents the time patterns of f_t. The path of f_t seems to be related to the economic cycle. While the realizations of the random effect scatter around zero prior to the crisis, increased values occur since 2007 Q2. In the climax of the GFC, f_t reaches its maximum. The rebound in the aftermath of the crisis instates gradually. The right panel of the figure contrasts these time patterns of f_t to average LGDs in the time line as of Figure 2. Thus, the latter include observations which are not considered in the estimation.[13] Up to the more recent time periods, the random effect seems to mimic the path of average LGDs. The time series disperse afterwards, whereby, the spread further increases in the time line. This deviation might be attributed to the resolution bias. In the LGD model, unresolved cases are neglected due to the impossibility of observing final LGDs. Thus, observations are excluded which tend to have higher

[12] Results are similar to Betz et al. (2018).

[13] The LGD model is based on the estimation sample, whereas, the average LGDs include information of validation sample I, i.e., final LGD observations which are treated as unresolved cases in the estimation (see Table 2). We include final observations of unresolved cases to point out effects of the resolution bias.

LGDs. This distorts the estimated realizations of the random effect in the more recent time periods. The resolution bias and the associated distortion worsen in the time line, i.e., the distortion of f_t enlarges for higher t. Furthermore, a distortion of the random effect parameter σ has to be considered as the downward distortions in the random effect realizations might erroneously increase the underlying standard deviation of the random effect. We will come back to this later on (see subsequent paragraph).

Hierarchical Model

In analogy to the LGD model, the hierarchical approach is estimated based on the estimation sample (see Table 2). Due to DRT model in the hierarchical approach, it is possible to include censored observations, i.e., unresolved loans, in the estimation process. By this means, we are able to generate posterior predictive distributions for the DRT of unresolved cases and, thus, posterior predictive distributions for the LGD of unresolved loans. Furthermore, effects of the resolution bias as in the pure LGD model (see Figure 3) might be diminished.

Table 4 summarizes the results of the hierarchical model. Parameters are stated in the first column, whereas, the second column presents posterior means. Posterior distributions for the estimated component parameters (μ_k and σ_k for $k \in \{2,3,4\}$) and loan specific covariate parameters of the LGD model in the hierarchical approach (γ_{EAD}, γ_{Facility}, $\gamma_{\text{Protection}}$, and γ_{Industry}) are similar to their counterparts in the pure LGD model (see Table 3, μ_k and σ_k for $k \in \{2,3,4\}$ and ζ_{EAD}, ζ_{Facility}, $\zeta_{\text{Protection}}$, and ζ_{Industry}). A deviation arises for the parameter of the HPI (γ_{HPI}). In comparison the corresponding parameter in the pure LGD model (ζ_{HPI}) it exhibits an intuitively negative sign, thus, indicating lower LGDs in sound economic surroundings which is displayed by an increasing HPI. However, the parameter of the macro variable is still characterized by a lack of statistical evidence (posterior $\text{odds}_{\text{E}[\gamma_{\text{HPI}}]<0} = 1.18 < 3.2$ and $0 \in \text{HPDI}_{\gamma_{\text{HPI}}} = [-0.13, 0.12]$). The sign switch of γ_{HPI} compared to ζ_{HPI} might be due to the inclusion of the logarithmized DRT as explanatory variable in the LGD model of the hierarchical approach (γ_T) as further systematic variables, i.e., the VIX and the random effect of the DRT model, enter the LGD model through the DRT. The posterior mean of γ_T has a positive sign indicating higher LGDs for loans with higher DRTs. In Section 2.1 (see Figure 1), we determined this relation descriptively. The impact of the

Table 4: Results of the hierarchical model

	posterior mean	HPDI (95%)		posterior odds	naive standard error	time series standard error
	LGD model in the hierarchical approach					
μ_1	0.0000			*not estimated*		
μ_2	0.0064	0.0062	0.0067	∞	0.0000	0.0000
μ_3	0.0279	0.0268	0.0290	∞	0.0000	0.0000
μ_4	0.5033	0.4923	0.5144	∞	0.0000	0.0000
μ_5	1.0000			*not estimated*		
σ_1	0.0010			*not estimated*		
σ_2	0.0043	0.0040	0.0045	∞	0.0000	0.0000
σ_3	0.0234	0.0223	0.0244	∞	0.0000	0.0000
σ_4	0.3384	0.3314	0.3453	∞	0.0000	0.0000
σ_5	0.0010			*not estimated*		
c_1	-1.4391	-1.5803	-1.3004	∞	0.0003	0.0005
c_2	-0.5848	-0.7242	-0.4422	∞	0.0003	0.0006
c_3	0.5728	0.4306	0.7090	∞	0.0003	0.0005
c_4	2.6716	2.5262	2.8169	∞	0.0003	0.0005
γ_{EAD}	-0.1952	-0.2233	-0.1667	∞	0.0001	0.0001
γ_{Facility}	0.3259	0.2700	0.3840	∞	0.0001	0.0001
$\gamma_{\text{Protection}}$	-0.6291	-0.6932	-0.5676	∞	0.0001	0.0002
γ_{Industry}	-0.2736	-0.3437	-0.2036	∞	0.0002	0.0002
γ_{HPI}	-0.0061	-0.1287	0.1170	1.1847	0.0003	0.0005
γ_T	0.9996	0.9711	1.0280	∞	0.0001	0.0001
	DRT model in the hierarchical approach					
β_0	0.7341	0.6112	0.8521	∞	0.0003	0.0006
β_{EAD}	0.0512	0.0343	0.0678	∞	0.0000	0.0000
β_{Facility}	-0.0903	-0.1238	-0.0555	∞	0.0001	0.0001
$\beta_{\text{Protection}}$	0.1345	0.0981	0.1718	∞	0.0001	0.0001
β_{Industry}	-0.1555	-0.1954	-0.1141	∞	0.0001	0.0001
β_{VIX}	0.2731	0.1514	0.3946	7141.8571	0.0003	0.0004
s	0.8488	0.8395	0.8583	∞	0.0000	0.0000
	random effect					
σ_T	0.3424	0.2627	0.4327	∞	0.0002	0.0002
σ_L	0.3615	0.2696	0.4634	∞	0.0002	0.0003
$\omega_{T,L}$	0.1863	-0.1398	0.5031	6.3057	0.0007	0.0008

Note: The table summarizes the results of the hierarchical model. Parameters are stated in the first column. Categorical variables are included via dummy coding. The reference categories are term loan for facility, no for protection, and non FIRE for industry. The second column presents the posterior means. In the third and fourth column, lower and upper bounds of the corresponding HPDIs to a credibility level of 95% are displayed. The fifth column contains the posterior odds. Naive and time series standard errors are shown in the last two columns. Time series standard errors are calculated based on the effective chain length (N^*_{MCMC}) instead of the actual chain length (N_{MCMC}), whereby, $N^*_{\text{MCMC}} < N_{\text{MCMC}}$ holds for autocorrelated chains.

DRT is decisively evident (posterior odds$_{E[\gamma_T]>0} \rightarrow \infty$ and $0 \notin \text{HPDI}_{\gamma_T} = [0.97, 1.03]$).

In the DRT model of the hierarchical approach, loan specific covariates and a macro variable, i.e., the VIX, are included. The posterior mean of the EAD (β_{EAD}) exhibits a positive sign. Thus, loans of major size are accompanied with longer DRTs. This supports the thesis we stated in the previous paragraph. Financial institutions might undertake higher resolution efforts for loans of major size. This might increase the DRTs and simultaneously lower LGDs. Decisive evidence can be stated for the positive impact of the EAD in the DRT model (posterior odds$_{E[\beta_{\text{EAD}}]>0} \rightarrow \infty$ and $0 \notin \text{HPDI}_{\beta_{\text{EAD}}} = [0.03, 0.07]$). According to the negative posterior mean of lines (β_{Facility}), this facility type is accompanied with shorter DRTs compared to term loans. This impact is decisively evident (posterior odds$_{E[\beta_{\text{Facility}}]<0} \rightarrow \infty$ and $0 \notin \text{HPDI}_{\beta_{\text{Facility}}} = [-0.12, -0.06]$). In analogy to the EAD, the impact of facility is opposite in the LGD and DRT model of the hierarchical approach. While lines are characterized by shorter DRTs, they result in higher LGDs. Reasons may be found in divergent resolution efforts related to the size of the loan and its protection. The posterior mean of protection ($\beta_{\text{Protection}}$) exhibits a positive, decisively evident (posterior odds$_{E[\beta_{\text{Protection}}]<0} \rightarrow \infty$ and $0 \notin \text{HPDI}_{\beta_{\text{Protection}}} = [-0.69, -0.57]$), sign indicating longer DRTs for protected loans. The impact of protection is divergent among the models in the hierarchical approach ($\gamma_{\text{Protection}} < 0$ and $\beta_{\text{Protection}} > 0$). This might be due to the nature of protection itself. If loans are secured either by collateral or guarantees, efforts have to be taken to realize the protection value. This might extent DRTs, however, reduce LGDs when the protection value is realized. The industry affiliation FIRE (β_{Industry}) reveals a negative posterior mean, thus, it is connected to shorter DRTs. The sign is decisively evident (posterior odds$_{E[\beta_{\text{Industry}}]<0} \rightarrow \infty$ and $0 \notin \text{HPDI}_{\beta_{\text{Industry}}} = [-0.69, -0.57]$) and corresponds to the sign of the LGD model in the hierarchical approach ($\gamma_{\text{Industry}} < 0$ and $\beta_{\text{Industry}} < 0$). Resolution prospects in the FIRE industry might be limited compared to other industries due to less tangible assets. Thus, DRTs are short and LGDs high. To control for the impact of the macro economy, the VIX (β_{VIX}) is included in the DRT model of the hierarchical approach. Its posterior mean is positive and decisively evident (posterior odds$_{E[\beta_{\text{VIX}}]>0} = 7,141.86 > 100$ and $0 \notin \text{HPDI}_{\beta_{\text{VIX}}} = [0.15, 0.39]$). This entails longer DRTs in bad economic surroundings which corresponds to the economic intuition.

The parameters of the multivariate random effect as of Equation (11) are stated in the lower panel of Table 4. As the DRT is included in the LGD model of the hierarchical approach, the random effect of the DRT model (F_t^T) enters the LGD model. Thus, the aggregated systematic impact of the random effects on LGDs ($\mathcal{F}_t$) is the linear combination of $\gamma_T F_t^T$ and F_t^L :

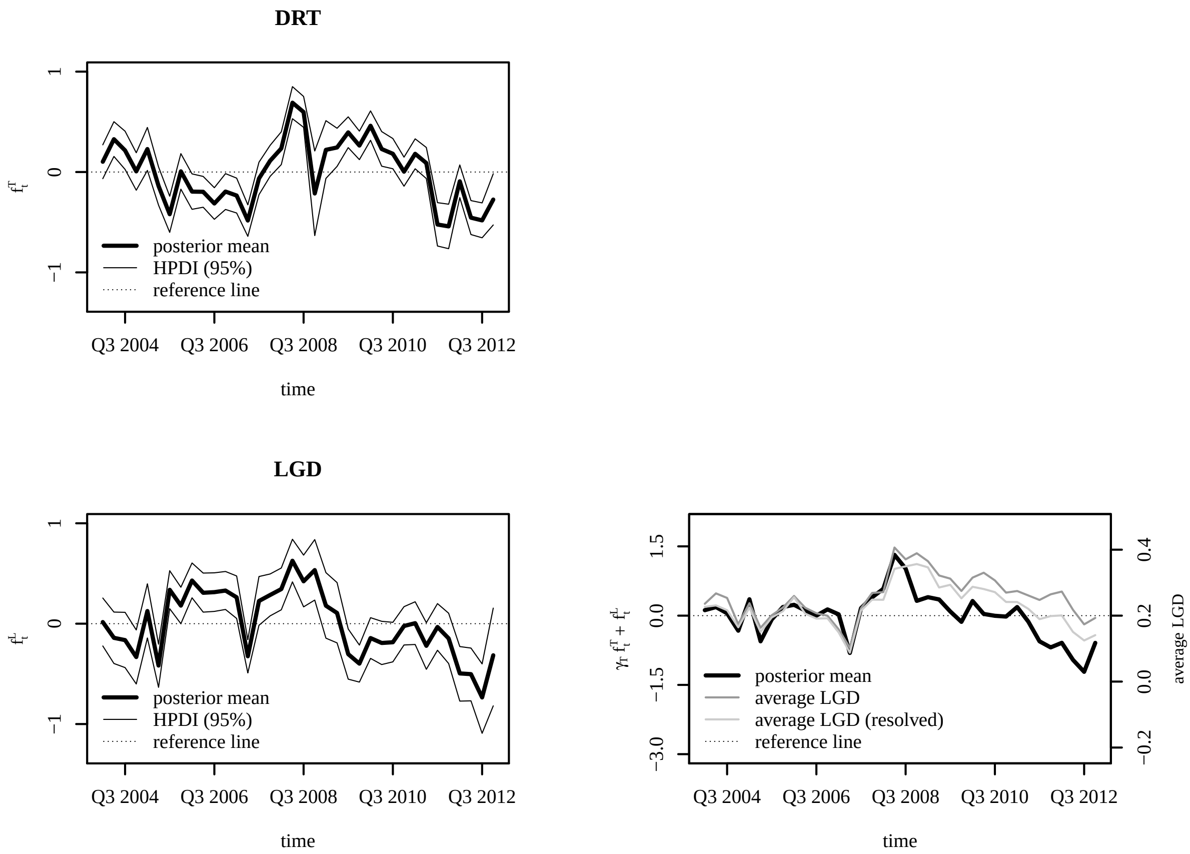
$$\begin{aligned} \mathcal{F}_t &= \gamma_T F_t^T + F_t^L \\ \sigma_{\mathcal{F}}^2 &= \gamma_T^2 \sigma_T^2 + \sigma_L^2 + 2\gamma_T \sigma_T \sigma_L \omega_{T,L}, \end{aligned} \tag{13}$$

whereby, $\sigma_{\mathcal{F}}^2$ is the variance of the aggregated systematic effect. Considering the results of Table 4, the standard deviation $\sigma_{\mathcal{F}}$ of $\mathcal{F}_t$ amounts to 0.54. This standard deviation is considerably smaller compared to the standard deviation of the random effect in the pure LGD model (see Table 3, $\sigma = 0.82$). As suspected in the previous paragraph, the estimated standard deviation of the random effect in the pure LGD model seems to be distorted due to the resolution bias. Neglecting censored observations, i.e., unresolved loans, leads to distorted realizations of the random effect (f_t) and, thus, subsequently to distorted parameters (σ).[14] However, the standard deviation of the random effect should be reduced, if additional explanatory variables are included in the model.

Figure 4 illustrates the realizations of the random effects of the DRT model f_t^T (upper left panel) and the LGD model f_t^L (lower left panel) in the hierarchical approach. Higher realizations of the random effect in the DRT model ($f_t^T > 0$) imply higher DRT for all loans defaulted in t, whereas, higher realizations of the random effect in the LGD model ($f_t^L > 0$) lead to higher values of the latent variable $\mathcal{Y}^*$ for all loans defaulted in t and, thus, to higher average LGDs in this quarter. Hence, DRTs impact LGDs through two channels (see Section 2.2). Directly, as higher DRTs are inserted in the LGD model. Indirectly, as positive realizations of f_t^T tend to imply positive realizations of f_t^L due to the positive correlation ($\omega_{T,L}$). However, the indirect channel might also weaken the impact of DRTs on LGDs as negative realizations of f_t^L are still possible. Considering the time patterns of the random effects as of Figure 4, four settings of the indirect channel are apparent. In the first setting prior to the GFC, $f_t^T < 0$ and

[14] Based on our data set, we find $\sigma > \sigma_{\mathcal{F}}$. However, $\sigma < \sigma_{\mathcal{F}}$ is conceivable if the most recent time period is characterized by crisis conditions. Considering the time period from 2004 Q1 to 2008 Q3, f_t would be lower during the crisis and presumable σ would be lower compared to $\sigma_{\mathcal{F}}$.

Figure 4: Random effect of the hierarchical model



Note: Note: The figure illustrates the course of the random effects in the hierarchical model over time. In the left panels, the posterior means (thick lines) and the HPDI (95%, thin lines) of the random effect realizations, i.e., f_t^T (DRT) and f_t^L (LGD), are displayed. In the right panel, the combined systematic effect on the LGDs according to the random effects of the hierarchical model ($\gamma_T f_t^T + f_t^L$, black line) is contrasted with the time patterns of average LGDs for all loans (dark gray line) and for resolved loans (light gray line). Final and incurrent LGDs as of validation sample I are included in the averaging. The dotted lines mark zero and serves as a reference line.

$f_t^L > 0$ are valid. Thus, average DRTs of loans defaulted in t are shorter. The positive realization of f_t^L, however, increases average LGDs. Resolutions of these loans at least partly take place during the crisis. This might depress recovery payments at the end of the resolution process and, thus, increase LGDs. The second setting in the climax of the GFC is characterized by positive realizations of both random effects ($f_t^T > 0$ and $f_t^L > 0$) indicating longer DRT and simultaneously higher LGDs of loans defaulted in t. In the third setting in the aftermath of the GFC, signs of the random effects are contrary ($f_t^T > 0$ and $f_t^L < 0$). Hence, average DRTs of loans defaulted in t are longer, whereas, average LGDs are lower. This might be due to the time delay as of the first setting. Analogously, parts of the recovery payments take place during the rebound period which favors recovery collection and decreases LGDs. The fourth setting is located in the most recent time period. The realizations of both random effects exhibit negative signs ($f_t^T < 0$ and $f_t^L < 0$) indicating shorter DRTs and simultaneously lower LGDs for loans defaulted in t. These settings illustrate the impacts of systematic effects in the resolution process. The positive correlation of the random effects ($\omega_{T,L}$) seems to be driven by extreme economic surroundings as synchronism appears in crises and boom periods. Furthermore, reasoning for the gradual rebound in the aftermath of the GFC can be provided (see Figure 2). While the random effect of the LGD model f_t^L indicates the rebound in the aftermath of the crisis (third setting), the random effect of the DRT model f_t^T remains on its high level. This might be due to the high stock of non-performing loans in the aftermath of the GFC which decelerated resolution proceedings. Average LGDs increase due to the direct channel.

The right panel of Figure 4 contrasts the aggregated systematic impact of the random effects ($\mathcal{F}_t$) to average LGDs in the time line. The latter include observations which are not considered in the estimation.[15] The aggregated systematic effect seems to mimic the path of average LGDs. However, slight dispersions are apparent in the more recent time periods. Reasons might be found in a less accurate estimation of the random effect realizations of the LGD model (f_t^L) in the more recent time periods. Although censored observations are included through the DRT model, unresolved loans do not directly enter the LGD model in the hierarchical approach. Comparing the dispersions of the hierarchical model with the pure LGD model (see Figure 3), improvements are apparent. While the spread extremely increases in the time line for the pure LGD model, the deviation is considerably less pronounced in the hierarchical approach. Thus, the hierarchical approach succeeds in reducing distortions due to the resolution bias.

4 Validation

The validation is conducted on an in sample, out of sample, and out of sample out of time perspective. As stated in Section 2.1 (see Table 2), the models are estimated

[15] The presentation corresponds to the right panel of Figure 3.

based on the estimation sample. In the *in sample* validation, the posterior predictive distributions based on the estimation sample are compared to the empirical distributions of completely resolved loans in the estimation sample. The *out of sample* validation examines the distributional fit for censored observations, i.e., loans which have defaulted till the end of the estimation period but are still unresolved. Thus, Posterior predictive distributions based on validation sample I are compared to the corresponding empirical distribution. The posterior predictive distributions are generated based on the estimated realizations of the random effect. In the *out of sample out of time* validation, loans which defaulted after the end of the estimation period are considered. As no random effect realizations are available for those loans, posterior predictive distributions are generated on the means of the random effects, i.e., zero, and compared to the corresponding empirical distribution.

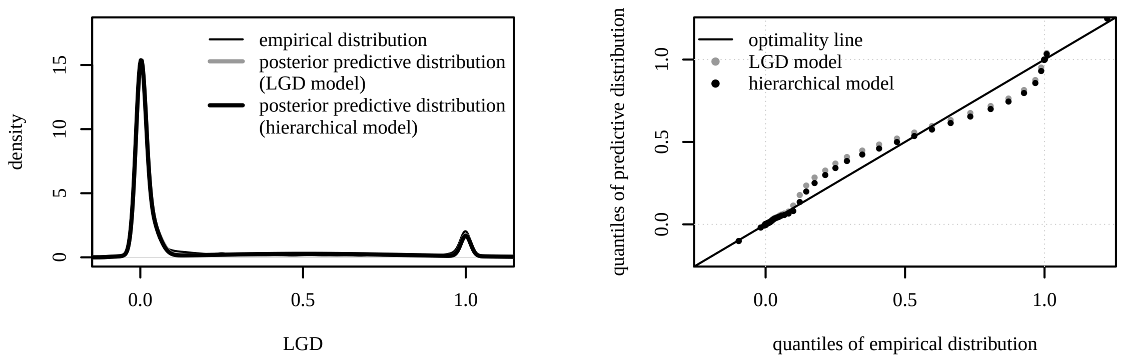
We adapt two graphical tools to evaluate the distributional fit of the models. First, kernel density estimates of the posterior predictive distributions are compared to kernel density estimates of the empirical data. The bandwidth is fixed to 0.015 to ensure comparability. So heights of the kernel density estimates are comparable despite ties. Second, quantile-quantile plots are applied. Hereby, the quantiles of the posterior predictive distributions are contrasted to the quantiles of the empirical distribution. In the case of optimality, i.e., if the distributions correspond to each other, the points of the quantile-quantile plot are on the bisector line. If the probability of low loss components is overestimated and the probability of high loss components is underestimated, the points are below the bisector line as the the quantiles of the posterior predictive distributions are smaller than the quantiles of the empirical distribution. This corresponds to an underestimation of average LGDs.

In sample

Figure 5 illustrates the *in sample* validation of the LGD model and the hierarchical model. In the left panel, kernel density estimates of the empirical distribution (thin black line), the posterior predictive distribution of the LGD model (thick gray line), and the posterior predictive distribution of the hierarchical model (thick black line) are presented. However, lines lie directly on top of each other, thus, the black line of the posterior predictive distribution of the hierarchical model overlays the remaining two.

To get a more detailed impression, the right panel of the figure illustrates quantile-

Figure 5: Validation (*in sample*)



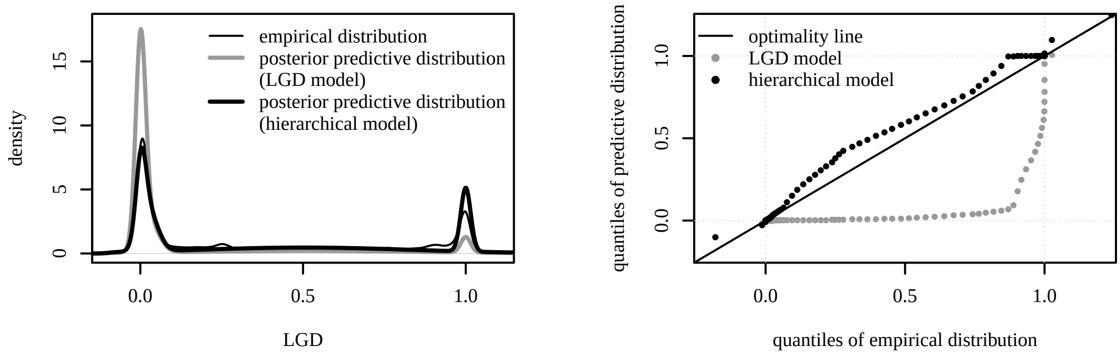
Note: The figure illustrates the *in sample* validation of the LGD model and the hierarchical model. In the left panel, the kernel density estimates of the empirical distribution (resolved loans as of estimation sample, thin black line), the posterior predictive distribution of the LGD model (thick gray line), and the posterior predictive distribution of the hierarchical model (thick black line) are displayed. The band width is fixed to 0.015 to ensure comparability. The kernel density estimates lie directly on top of each other. Thus, differences are not identifiable. In the right panel, the corresponding quantile-quantile plots are presented. The quantiles of the empirical distribution (x-axis) are plotted against the quantiles of the posterior predictive distributions (y-axis). The gray (black) dots mark the quantiles of the empirical distribution vs. the quantiles of the posterior predictive distribution of the LGD model (hierarchical model). The black line represents optimality.

quantile plots, whereby, the quantiles of the posterior predictive distributions are contrasted to the quantiles of the empirical distribution. The gray dots mark the LGD model, whereas, the hierarchical model is represented by black dots. The dots are near the optimality, i.e., bisector, line for both models. Thus, the in sample fit of the posterior predictive distributions is quite good with respect to the LGD model and the hierarchical model. This indicates that the applied FMM with five mixture components seems to deliver satisfactory results regarding the distributional fit.

Out of sample

Figure 6 illustrates the *out of sample* validation for the LGD model and the hierarchical model. The presentation corresponds to Figure 5 (in sample validation). The posterior predictive distribution of the LGD model (gray) seems to overestimate the probability mass of the low loss components, whereas, it underestimates the probability mass of the high loss components. In the left panel, its kernel density estimate lies above the kernel density estimate of the empirical distribution for no loss (LGD = 0) and below

Figure 6: Validation (*out of sample*)



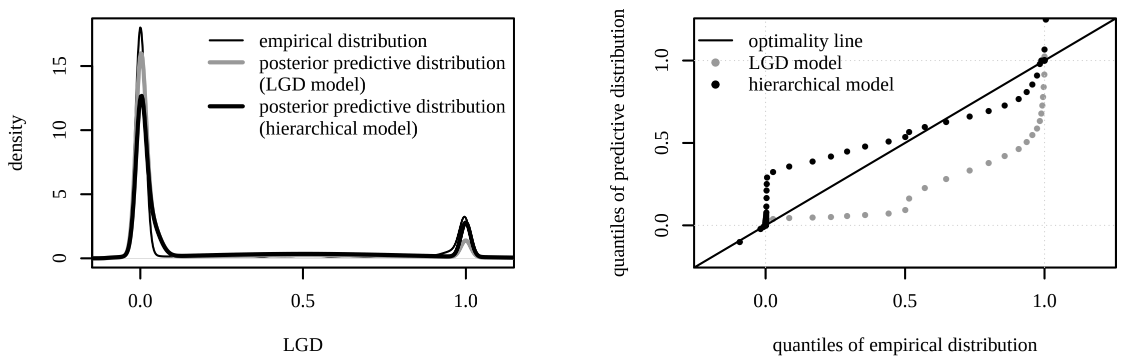
Note: The figure illustrates the *out of sample* validation of the LGD model and the hierarchical model. In the left panel, the kernel density estimates of the empirical distribution (resolved loans as of validation sample I, thin black line), the posterior predictive distribution of the LGD model (for resolved loans, thick gray line), and the posterior predictive distribution of the hierarchical model (for resolved loans, thick black line) are displayed. The band width is fixed to 0.015 to ensure comparability. In the right panel, the corresponding quantile-quantile plots are presented. The quantiles of the empirical distribution (x-axis) are plotted against the quantiles of the posterior predictive distributions (y-axis). The gray (black) dots mark the quantiles of the empirical distribution vs. the quantiles of the posterior predictive distribution of the LGD model (hierarchical model). The black line represents optimality.

for total loss (LGD = 1). This appears even clearer considering the quantile-quantile plots in the right panel. The dots are considerably below the optimality line indicating an underestimation of average LGDs. The underestimation is caused by the parameter distortion of the random effect due to the resolution bias. The random effect realizations f_t are characterized by a downward bias, thus, leading to downward biased estimates for Y_i and downward biased loss rates (see Section 3). In contrast, the distributional fit of the hierarchical model is good on an out of sample perspective. This is due to two reasons. First, the parameter distortions caused by the resolution bias are diminished (see Section 3). Second, the additional information of how long a loan is in resolution is utilized to improve predictions on an out of sample perspective. Final DRTs for censored observations, i.e., unresolved cases, are estimated within the hierarchical approach. These can be applied to generate predictions of final LGDs for unresolved loans.

Out of sample out of time

Figure 7 illustrates the *out of sample out of time* validation of the LGD model and the hierarchical model. The presentation corresponds to Figure 5. In analogy to the out of

Figure 7: Validation (*out of sample out of time*)



Note: The figure illustrates the *out of sample out of time* validation of the LGD model and the hierarchical model. In the left panel, the kernel density estimates of the empirical distribution (all loans as of validation sample II with incurrent LGDs, thin black line), the posterior predictive distribution of the LGD model (thick gray line), and the posterior predictive distribution of the hierarchical model (thick black line) are displayed. The band width is fixed to 0.015 to ensure comparability. In the right panel, the corresponding quantile-quantile plots are presented. The quantiles of the empirical distribution (x-axis) are plotted against the quantiles of the posterior predictive distributions (y-axis). The gray (black) dots mark the quantiles of the empirical distribution vs. the quantiles of the posterior predictive distribution of the LGD model (hierarchical model). The black line represents optimality.

sample validation, an overestimation of low loss components and an underestimation of high loss components arises for the LGD model. However, it is not as striking as in the out of sample validation (see Figure 6). This might be due to the use of the random effect in average terms – instead of the individual realizations f_t as in the out of sample validation – to generate the posterior predictive distribution. However, the poor distributional fit of the LGD model on the out of sample out of time perspective suggests that there are additional distortions beyond the realizations of the random effect and its standard deviation. These might be found in the cut points which represent the intercepts in an OL model.[16] In contrast to the LGD model, the distributional fit of the hierarchical model is quite good on an out of sample out of time perspective.

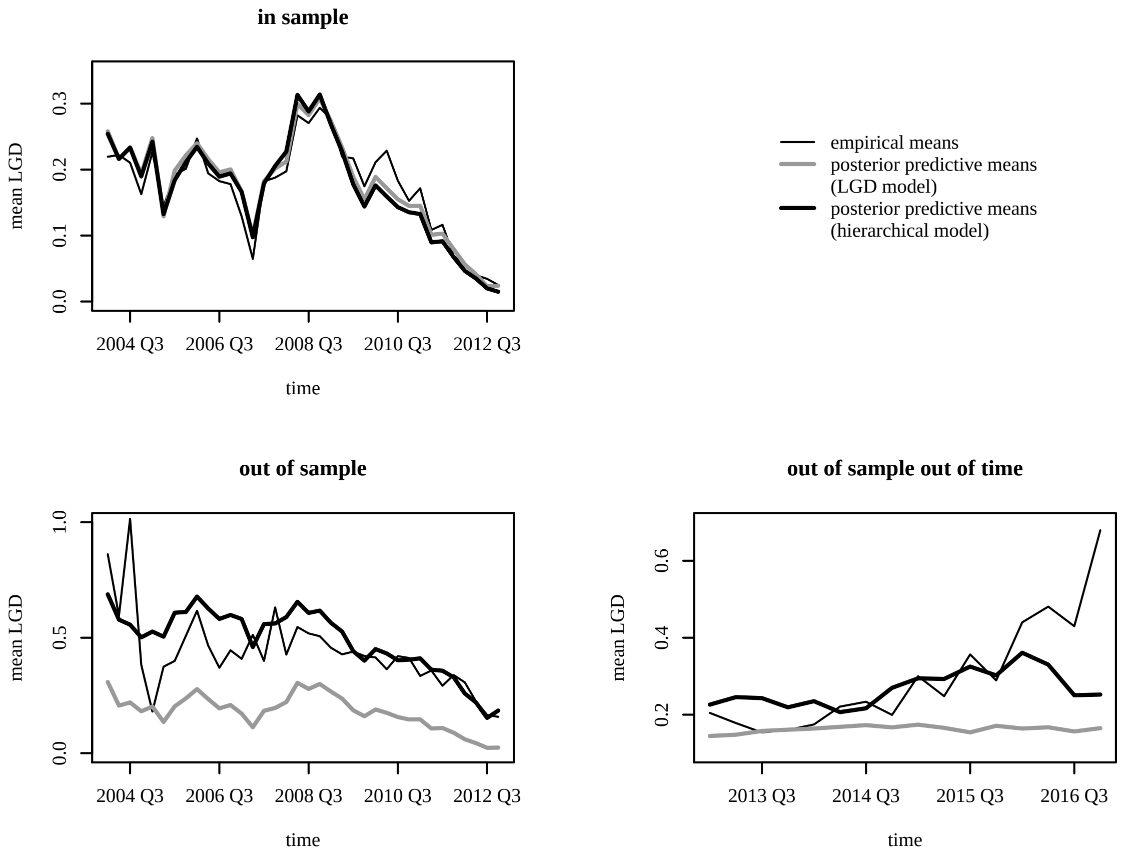
Validation in the time line

Thus far, the distributional fit of the LGD model and the hierarchical model are analyzed for the estimation sample and the validation samples. Figure 8 illustrates the time patterns of average LGD predictions based on the posterior predictive distributions

[16] The cut points of the LGD model and the hierarchical model are not directly comparable as the logarithm of the DRT is included as additional variable. By this means, the mean of the latent variable $\mathcal{Y}^*$ and, thereby, the level of the cut points are shifted.

for specific default quarters. The upper left panel contrast average LGDs (thin black

Figure 8: Validation in the time line



Note: The figure illustrates the validation in the time line. The means of the empirical distribution are displayed by a thin black line, whereas, the means of the posterior predictive distributions are marked by a thick gray line for the LGD model and a thick black line for the hierarchical model, respectively. In the upper panel, the *in sample* validation in the time line is presented (empirical means of resolved loans in estimation sample). The lower panels show the *out of sample* (empirical means of resolved loans in validation sample I) and *out of sample out of time* validation (empirical means of all loans in validation sample II with incurrent LGDs) in the time line.

line) to average LGD predictions based on the LGD model (thick gray line) and the hierarchical model (thick black line) on an *in sample* perspective. In analogy to Figure 5, a good in sample fit for both models can be stated. The lower left panel illustrates the time patterns of average LGDs and LGD predictions on an *out of sample* perspective. Although the relative progressions of the LGD predictions based on the LGD model and the hierarchical model are similar, the predictions based on the LGD model are downward biased. Thus, average LGDs are underestimated by the LGD model in almost all quarters in validation sample I. This is not the case considering the predictions of the hierarchical model. The noisy behavior of average LGDs at the beginning of the time period is due to a lack of data as most loans defaulted in these quarters are resolved by the end of 2010 and, thus, not included in validation sample I. The lower right panel illustrates the time patterns of average LGDs and LGD predictions on an *out of sample out of time* perspective. The predictions based on the LGD model seem to be constant through time as the random effect is set to its mean, i.e., zero, and the macro variable is the only remaining systematic factor. However, the latter is not statistically evident (see Table 3). Furthermore, LGD predictions based on the LGD model seem to be systematically too low. LGD predictions based on the hierarchical model better fit average LGDs. Deviations at the end of the time period might be attributed to the inclusion of incurrent LGDs for unresolved cases (see Figure 2). Final LGDs will be lower and adjust the line downwards. In addition, LGD predictions based on the hierarchal model display systematic movement as the statistically evident macro variable of the DRT model is enclosed in the LGD model of the hierarchical approach (see Table 4).

5 Conclusion

In this paper, we deeply examine the dependence structure of DRTs and LGDs using a hierarchical modeling framework. We find direct and indirect dependencies among the credit risk parameters. First, LGDs seem to be directly impacted by DRTs, i.e., longer resolution processes are accompanied with higher losses. Second, the parameters are characterized by common time patterns as correlation of the random effects in the individual models is positive. Due to the random nature of these effects, the dependence of DRTs and LGDs might be intensified or weakened in certain time periods. We find similar signs of the random effect realizations during the GFC and deviating signs pre and post crisis. Due to the consideration of direct dependency structures, we are able to generate intuitive LGD predictions for censored cases, i.e., non-performing loans. As final DRTs for unresolved loans are estimated within the DRT model of the hierarchical approach, these estimations can be used to directly generate final LGD predictions for these cases. LGD predictions based on the hierarchical approach, thereby, outperform predictions based on a pure LGD model.

Furthermore, effects of the resolution bias are diminished in the hierarchical approach. While the parameters and, thus, the realizations of the random effect are biased in a pure LGD model, these distortions are eliminated in the hierarchical approach. The parameter distortions due to the resolution bias have considerable impacts on the *out of sample* and *out of sample out of time* performance of pure LGD models. Out of sample, a pure LGD model generates average LGD predictions underestimating actual average LGDs by up to 25 percentage points for loans defaulted during the GFC (2008 Q1 to 2009 Q3). The hierarchical approach delivers sufficiently conservative predictions for loans defaulted in the crisis (up to 16 percentage points above actual average LGDs). Assuming ten loans with an EAD of 500,000 EUR defaulted in 2008 Q4, a pure LGD model underestimates losses due to these loans by round about 1,05 million EUR. Out of sample out of time, these effects are less pronounced, however, still remarkable. A pure LGD model constantly underestimates actual average LGDs in the time period from 2013 Q1 to 2015 Q4 by up to 20 percentage points, while the hierarchical approach delivers slightly conservative predictions in most time periods (between 3 percentage points below and 8 percentage points above actual LGDs).[17] Assuming ten loans with an EAD of 500,000 EUR defaulted in 2015 Q4, a pure LGD model underestimates losses due to these loans by round about 600,000 EUR.

Concluding, the consideration of the dependency structure of DRTs and LGDs and the thereto entailed resolution bias is essential to generate suitable LGD predictions. The presented hierarchical model prevents the need of additional data constraints and provides fruitful insights into the dependence structure of DRTs and LGDs.

[17] The subsequent time period is hard to interpret due to incurrent LGDs.

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A Bayesian Model Specification

The LGD model and the hierarchical model are estimated via Bayesian inference.[18] Thus, prior distributions have to be specified for every parameter in the models. Most of the prior distributions are characterized by an uninformative specification. If weakly informative priors are a provided, this is done to avoid convergence reasons.

LGD model

In the FMM, the number of latent classes is set to five ($K = 5$). As the LGD distribution is extremely bimodal and characterized by high probability masses at zero and one, we fix the parameters of the outer components ($k = 1$ and $k = 5$). The means are set to $\mu_1 = 0$ and $\mu_5 = 1$ with rather small standard deviations ($\sigma_1 = \sigma_5 = 0.001$) to identify loans with no and total loss. The remaining component parameters are provided with uninformative prior distributions:

$$
\begin{aligned}
\mu_2 &\sim \mathrm{N}\Big(\mu = 0.0 \, , \, \sigma = 1 \cdot 10^5 \Big) \\
\mu_3 &\sim \mathrm{N}\Big(\mu = 0.5 \, , \, \sigma = 1 \cdot 10^5 \Big) \\
\mu_4 &\sim \mathrm{N}\Big(\mu = 1.0 \, , \, \sigma = 1 \cdot 10^5 \Big) \\
\sigma_k &\sim \mathrm{N}\Big(\mu = 0.0 \, , \, \sigma = 1 \cdot 10^5 \Big) \, [\, 0 \, , \quad] \text{ for } k \in \{2, 3, 4\} \, ,
\end{aligned} \tag{A.1}
$$

where, the squared brackets indicate truncation. The prior distributions of the component means are Normal distributions with mean $\mu = 0.0$ for component 2, $\mu = 0.5$ for component 3, and $\mu = 1.0$ for component 4. The means of the components are ordered and, thus, truncated to the interval $[0, 1]$ due to the fixed outer components. The standard deviations of the prior distributions are set to a rather high value which corresponds to a uninformative prior specification. The component standard deviations are provided with an uninformative truncated Normal prior distribution. We forgo the conjugate prior distributions (inverse Gamma distribution for the variance) as convergence problems might arise for small values of σ_k.

The cut points c_k for $k \in \{1, \ldots K - 1\}$ of the OL model are restricted to be ordered

[18] The MCMC samples are drawn via the sampler Stan.

$(c_1 < \cdots < c_{K-1})$ and provided with uninformative prior distributions:

$$c_k \sim \mathrm{N}\Big(\, \mu = 0 \,,\; \sigma = 1 \cdot 10^5 \,\Big) \text{ for } k \in \{1, \ldots, K-1\}\,. \tag{A.2}$$

The prior distributions of the coefficients ζ_j for $j \in \{1, \ldots, J\}$ are uninformative Normal prior distributions with mean 0 and high standard deviations:

$$\zeta_j \sim \mathrm{N}\Big(\, \mu = 0 \,,\; \sigma = 1 \cdot 10^5 \,\Big) \text{ for } j \in \{1, \ldots, J\}\,. \tag{A.3}$$

The random effect F_t follows a normal distribution with mean zero and standard deviation σ. We provide the standard deviation of the random effect σ with uninformative prior distribution:

$$\sigma \sim \mathrm{Gamma}\,(\, 0.001 \,,\; 0.001 \,)\;. \tag{A.4}$$

Hierarchical model

In the hierarchical model, we adopt the prior distributions of the components as of Equation (A.1) and the cut points as of Equation (A.2). The coefficients of the DRT model β_j for $j \in \{1, \ldots, J_T\}$ and the LGD model γ_j for for $j \in \{1, \ldots, J_L\}$ are also provided with uninformative Normal prior distributions:

$$\beta_j \sim \mathrm{N}\Big(\, \mu = 0 \,,\; \sigma = 1 \cdot 10^5 \,\Big) \text{ for } j \in \{1, \ldots, J_T\} \tag{A.5}$$

$$\gamma_j \sim \mathrm{N}\Big(\, \mu = 0 \,,\; \sigma = 1 \cdot 10^5 \,\Big) \text{ for } j \in \{1, \ldots, J_L\}\,. \tag{A.6}$$

The random effects F_t^T and F_t^L in the hierarchical model follow a bivariate normal distribution, whereby, the mean vector corresponds to the two dimensional zero vector $(0_2 = (\,0 \quad 0\,)^T)$. We provide the correlation matrix Ω of the bivariate normal distribution with an uninformative LKJ prior distribution (see Stan Development Team, 2016) and the standard deviations of the random effects σ_T and σ_L with uninformative gamma distributions:

$$\Omega \sim \text{LKJ}(\ 1\ ) \tag{A.7}$$

$$\sigma_T \sim \text{Gamma}(\ 0.001\ ,\ 0.001\ ) \tag{A.8}$$

$$\sigma_L \sim \text{Gamma}(\ 0.001\ ,\ 0.001\ )\ . \tag{A.9}$$

The covariance matrix Σ of the bivariately distributed random effects might be calculated based on the correlation matrix Ω and the individual standard deviations σ_T and σ_L.

B Bayesian Convergence Diagnostics

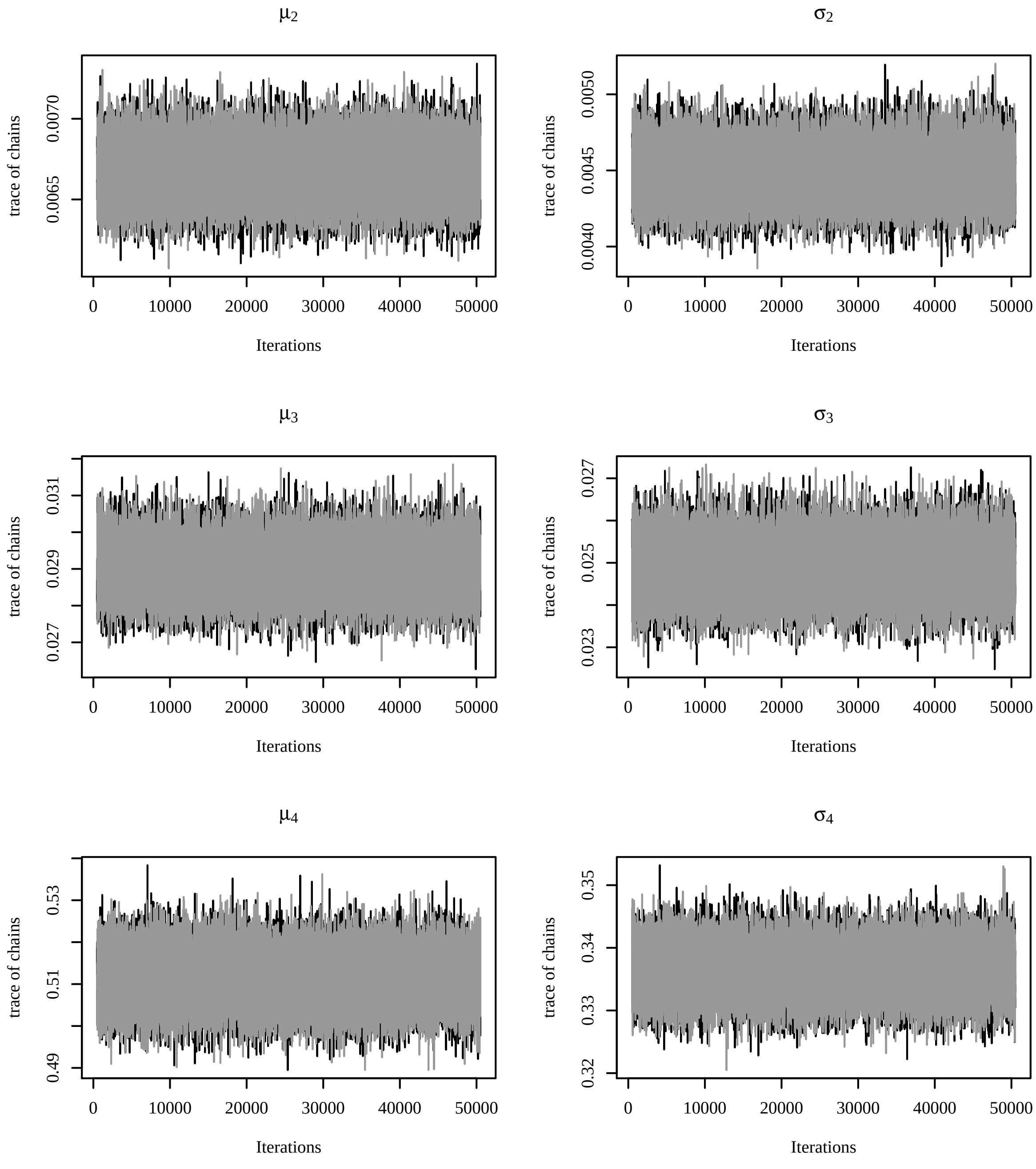
LGD model

Table B.1: Convergence diagnostics of LGD model

	Gelman-Rubin diagnostic		**Heidelberger-Welch** diagnostic		
	point estimate	upper confidence limits (95%)	stationary test	start	p value
μ_2	1.0000	1.0002	passed	1	0.3857
μ_3	1.0000	1.0000	passed	1	0.2169
μ_4	1.0000	1.0002	passed	1	0.2180
σ_2	1.0000	1.0000	passed	1	0.6438
σ_3	1.0000	1.0000	passed	1	0.3903
σ_4	1.0001	1.0008	passed	1	0.4836
c_1	1.0001	1.0005	passed	1	0.4102
c_2	1.0000	1.0003	passed	1	0.4826
c_3	1.0001	1.0005	passed	1	0.4600
c_4	1.0001	1.0005	passed	1	0.3650
ζ_{EAD}	1.0002	1.0013	passed	1	0.0595
ζ_{Facility}	1.0000	1.0001	passed	1	0.4377
$\zeta_{\text{Protection}}$	1.0000	1.0001	passed	1	0.6311
ζ_{Industry}	1.0002	1.0013	passed	1	0.4295
ζ_{HPI}	1.0000	1.0000	passed	1	0.3440
sigma	1.0000	1.0000	passed	1	0.9538

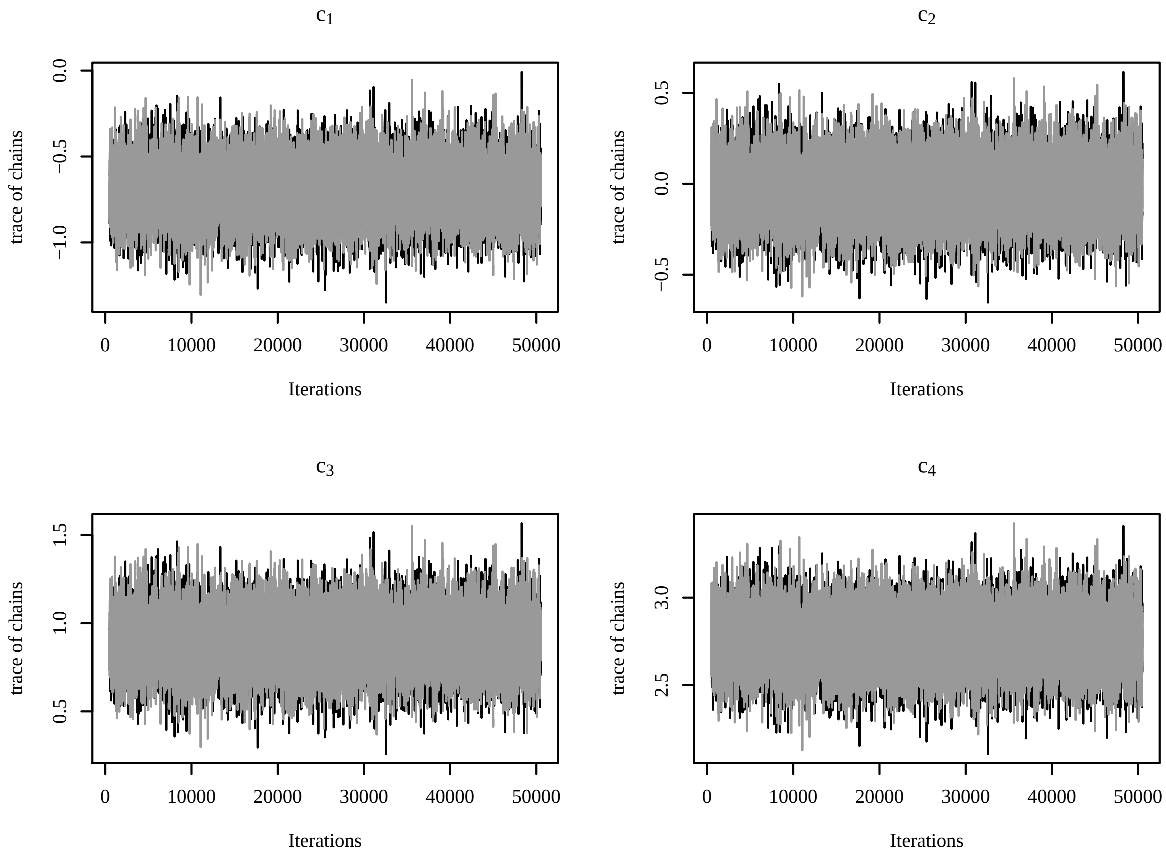
Note: The table summarizes the convergence diagnostics of the LGD model. Parameters are stated in the first column. In the second and third column, the Gelman-Rubin diagnostics are displayed. The *potential reduction factor* and its upper confidence limit are calculated. Convergence is diagnosed if chains have "forgotten" their initial values, i.e., for upper limit close to one (see Gelman and Rubin, 1992). A rule of thumb assumes 1.1 as critical value. The Gelman-Rubin diagnostic examines the length of burn-in. In the last three columns, the Heidelberger-Welch diagnostics are presented. The two chains are combined to calculate a criterion of relative accuracy for the posterior means. The frequentistic stationarity test adopts the Cramer-von-Mises statistic to test the null hypothesis of a stationary process in the chains (see Heidelberger and Welch, 1981, 1983). The Heidelberger-Welch diagnostic examines the length of chains.

Figure B.1: Trace plots of the component parameters (LGD model)



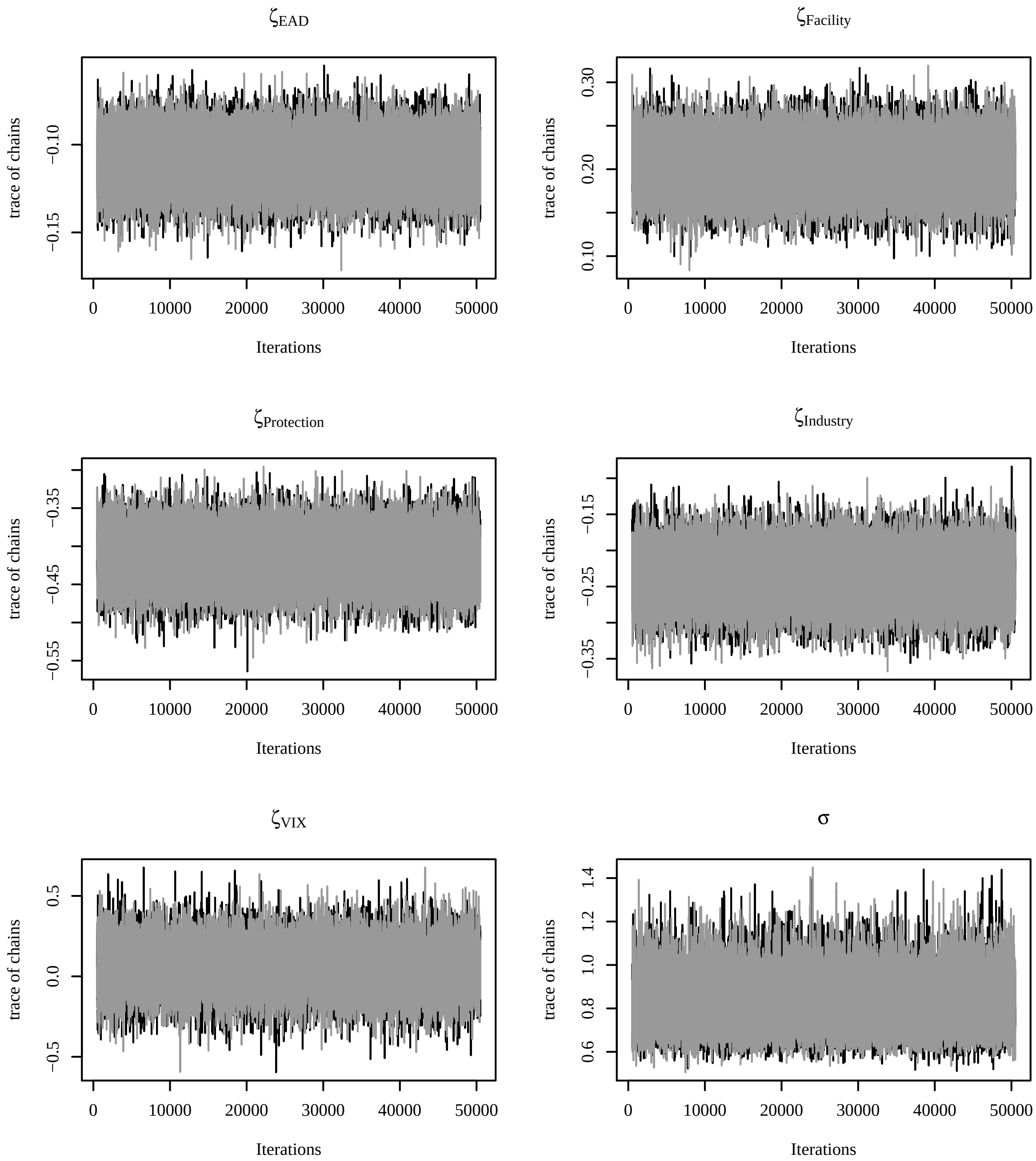
Note: The figure illustrates the trace of MCMC chains for the component parameters (μ_k and σ_k for $k \in \{2, 3, 4\}$) in the LGD model. The first chain is displayed in black, the second in gray.

Figure B.2: Trace plots of the cut points (LGD model)



Note: The figure illustrates the trace of MCMC chains for the cut points (c_k for $k \in \{1, 2, 3, 4\}$) in the LGD model. The first chain is displayed in black, the second in gray.

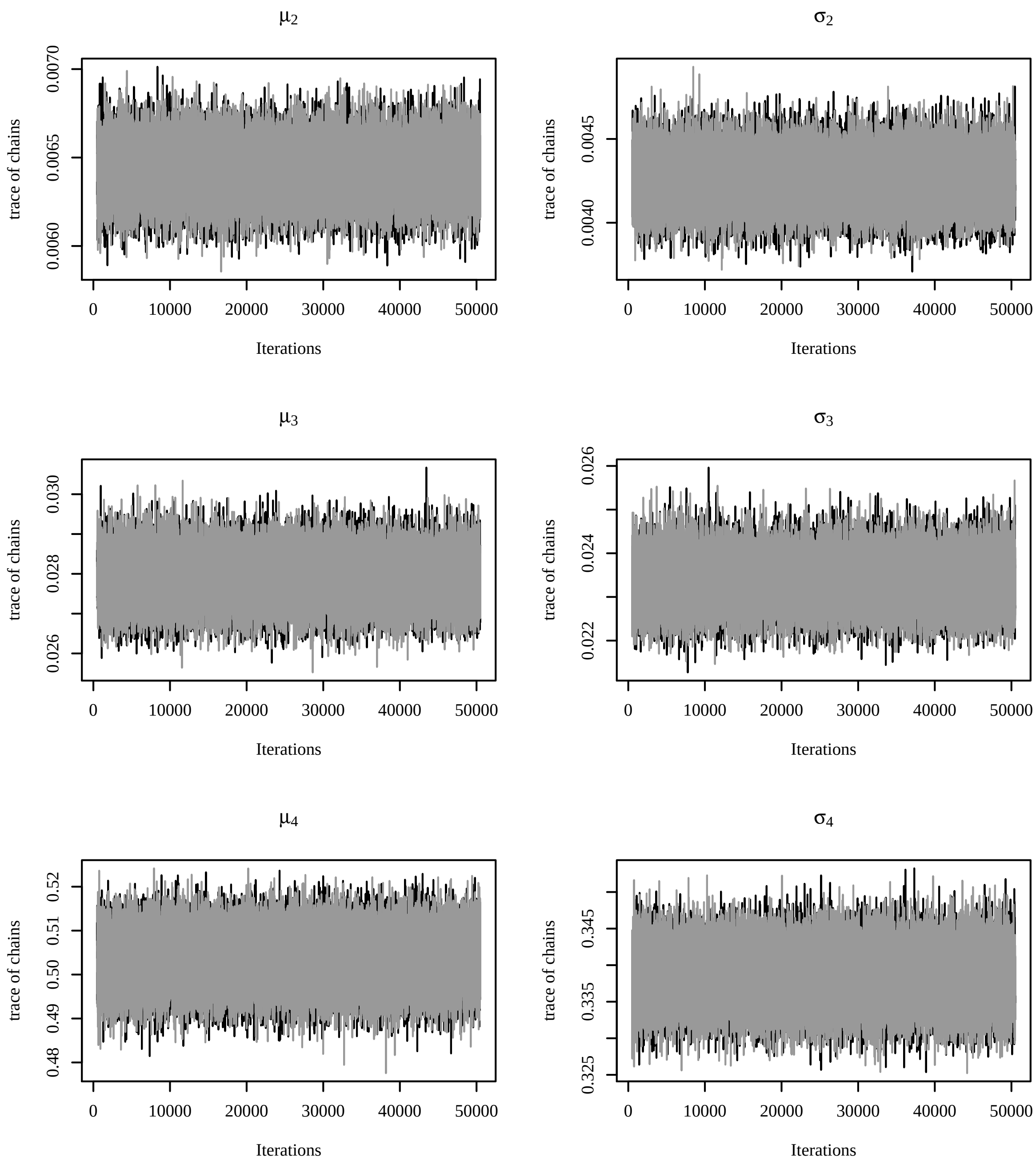
Figure B.3: Trace plots of the covariate parameters ζ_j and the random effect parameter σ (LGD model)



Note: The figure illustrates the trace of MCMC chains for the covariate parameters (ζ_j for $j \in$ {EAD, Facility, Protection, Industry, HPI}) and the parameter of the random effect (σ) in the LGD model. The first chain is displayed in black, the second in gray.

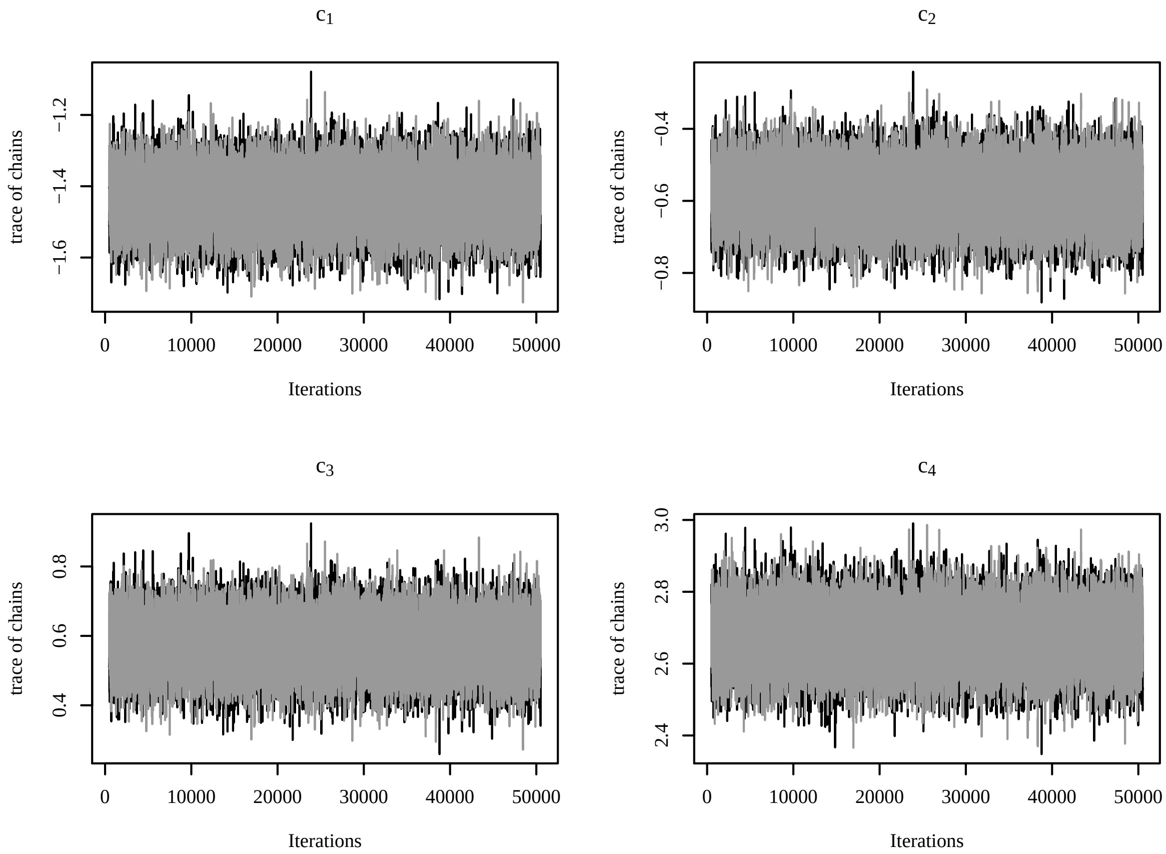
Hierarchical model

Figure B.4: Trace plots of the component parameters (hierarchical model)



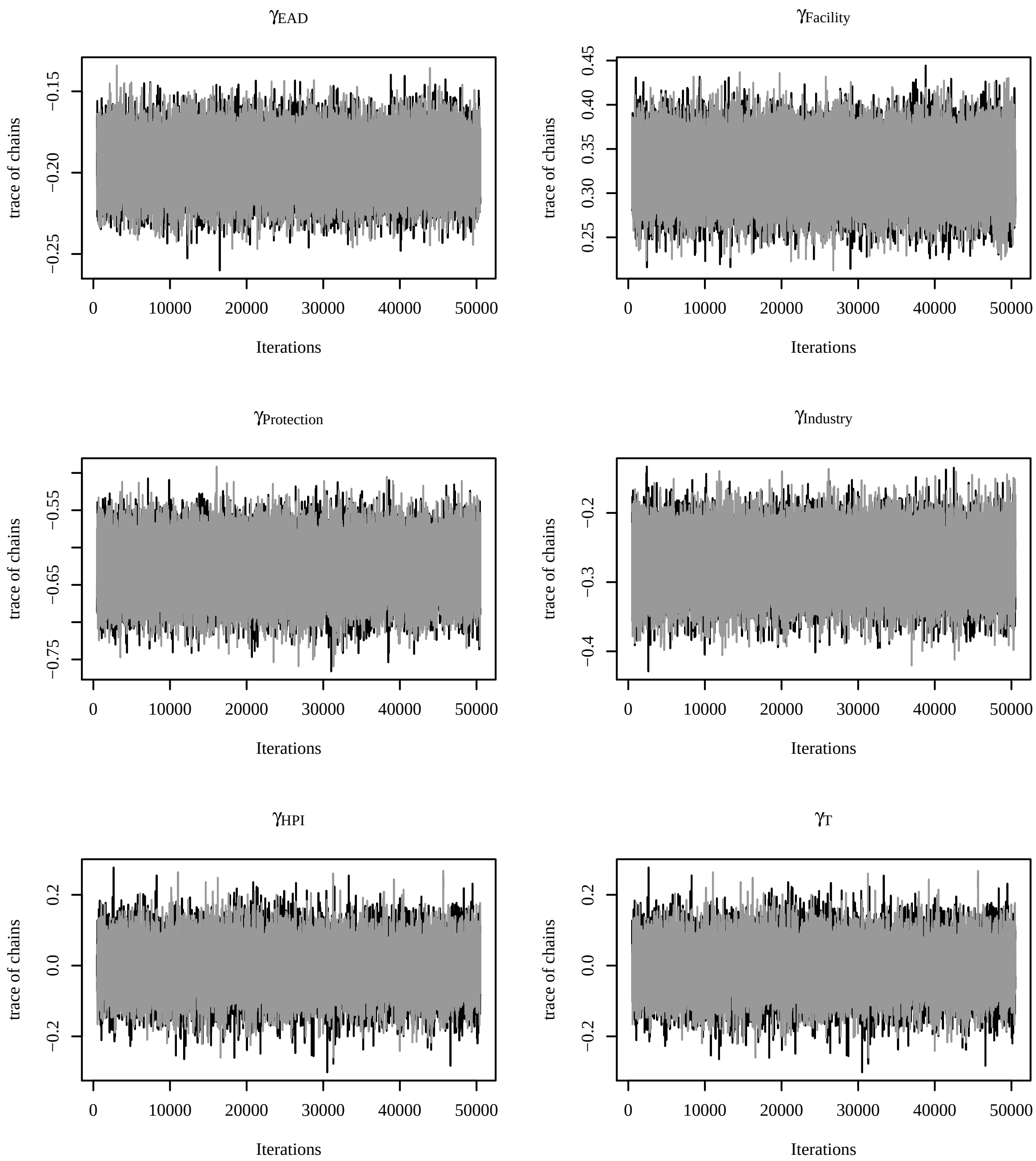
Note: The figure illustrates the trace of MCMC chains for the component parameters (μ_k and σ_k for $k \in \{2, 3, 4\}$) in the hierarchical approach. The first chain is displayed in black, the second in gray.

Figure B.5: Trace plots of the cut points (hierarchical model)



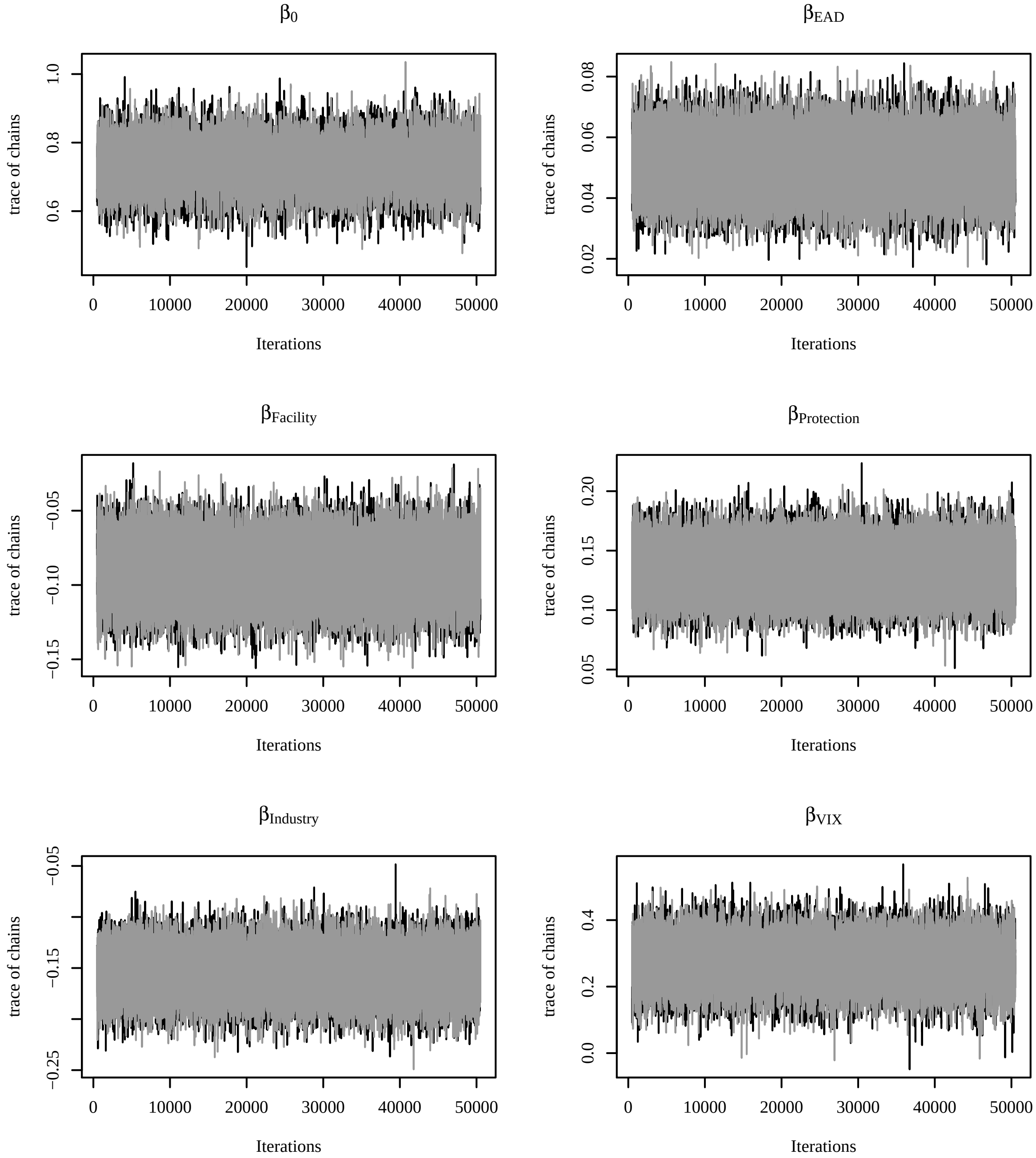
Note: The figure illustrates the trace of MCMC chains for the cut points (c_k for $k \in \{1, 2, 3, 4\}$) in the hierarchical approach. The first chain is displayed in black, the second in gray.

Figure B.6: Trace plots of the covariate parameters γ_j and impact parameter γ_T (hierarchical model)



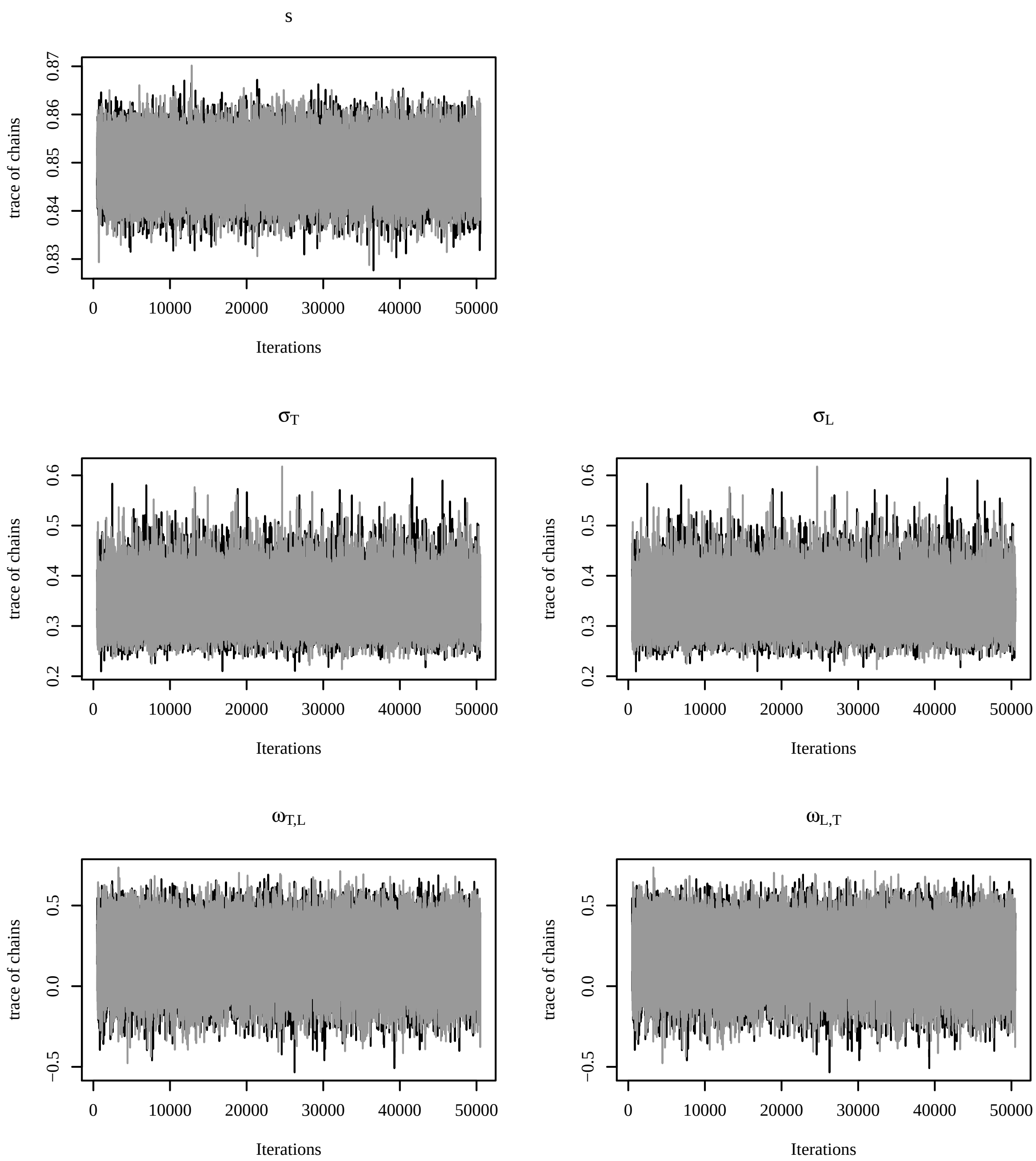
Note: The figure illustrates the trace of MCMC chains for the covariate parameters (γ_j for $j \in$ {EAD, Facility, Protection, Industry, HPI}) and the impact parameter of the DRT (γ_T) of the LGD model in the hierarchical approach. The first chain is displayed in black, the second in gray.

Figure B.7: Trace plots of the intercept β_0 covariate parameters β_j (hierarchical model)



Note: The figure illustrates the trace of MCMC chains for the intercept (β_0) and the covariate parameters (β_j for $j \in$ {EAD, Facility, Protection, Industry, VIX}) of the DRT model in the hierarchical approach. The first chain is displayed in black, the second in gray.

Figure B.8: Trace plots of the standard error s and the random effect parameters σ_T, σ_L, $\omega_{T,L}$, and $\omega_{L,T}$ (hierarchical model)



Note: The figure illustrates the trace of MCMC chains for the standard error (s) of the DRT model and the random effect parameters (σ_T, σ_L, $\omega_{T,L}$, and $\omega_{L,T}$) in the hierarchical approach. The first chain is displayed in black, the second in gray.

Table B.2: Convergence diagnostics of hierarchical model

	Gelman-Rubin diagnostic		**Heidelberger-Welch** diagnostic		
	point estimate	upper confidence limits (95%)	stationary test	start	p value
μ_2	1.0000	1.0000	passed	1	0.17944
μ_3	1.0000	1.0002	passed	1	0.87543
μ_4	1.0000	1.0001	passed	1	0.44269
σ_2	1.0000	1.0000	passed	1	0.69836
σ_3	1.0002	1.0003	passed	1	0.54110
σ_4	1.0002	1.0008	passed	1	0.89167
c_1	1.0000	1.0000	passed	1	0.50214
c_2	1.0000	1.0000	passed	1	0.74251
c_3	1.0000	1.0002	passed	1	0.83016
c_4	1.0001	1.0004	passed	1	0.87461
γ_{EAD}	1.0000	1.0000	passed	1	0.61537
γ_{Facility}	1.0000	1.0001	passed	1	0.72393
$\gamma_{\text{Protection}}$	1.0001	1.0009	passed	1	0.20917
γ_{Industry}	1.0000	1.0001	passed	1	0.62439
γ_{HPI}	1.0004	1.0005	passed	1	0.27974
γ_T	1.0000	1.0000	passed	5001	0.23503
β_0	1.0001	1.0004	passed	1	0.32351
β_{EAD}	1.0000	1.0000	passed	1	0.70406
β_{Facility}	1.0000	1.0000	passed	1	0.38782
$\beta_{\text{Protection}}$	1.0000	1.0001	passed	1	0.42642
β_{Industry}	1.0001	1.0007	passed	1	0.23275
β_{VIX}	1.0003	1.0015	passed	10001	0.21704
s	1.0000	1.0001	passed	1	0.46664
σ_T	1.0000	1.0002	passed	1	0.99576
σ_L	1.0000	1.0000	passed	1	0.18011
$\omega_{T,L}$	1.0000	1.0003	passed	1	0.47216

Note: The table summarizes the convergence diagnostics of the hierarchical model. Parameters are stated in the first column. In the second and third column, the Gelman-Rubin diagnostics are displayed. The *potential reduction factor* and its upper confidence limit are calculated. Convergence is diagnosed if chains have "forgotten" their initial values, i.e., for upper limit close to one (see Gelman and Rubin, 1992). A rule of thumb assumes 1.1 as critical value. The Gelman-Rubin diagnostic examines the length of burn-in. In the last three columns, the Heidelberger-Welch diagnostics are presented. The two chains are combined to calculate a criterion of relative accuracy for the posterior means. The frequentistic stationarity test adopts the Cramer-von-Mises statistic to test the null hypothesis of a stationary process in the chains (see Heidelberger and Welch, 1981, 1983). The Heidelberger-Welch diagnostic examines the length of chains.