Systemic Jumps in International Equity Returns

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Abstract

This paper examines the existence of systemic jumps in international equity returns. I propose and estimate a multivariate stochastic volatility jump-diffusion model which assumes that returns are affected by both systemic and idiosyncratic jumps. I apply the model to groups of major international equity indices and in all cases I find significant evidence of the existence of systemic jumps. In North American markets, the majority of jumps are systemic, whereas in European and Asian markets, although systemic jumps are significantly present, the majority of jumps are idiosyncratic. In all cases, the mean sizes of systemic jumps are significantly negative, while the mean sizes of idiosyncratic jumps are not significantly different from zero. Surprisingly, I find in all cases that the correlation coefficients between sizes of systemic jumps are not significantly different from zero.

Keywords: Systemic risk; Multivariate stochastic volatility jump-diffusion; Bayesian inference; Markov chain Monte Carlo.
JEL classifications: C13; C15.

1 Introduction

It is a well known fact that markets in different countries tend to experience large sudden losses—interpreted in this paper as large jumps in returns—simultaneously. This fact is for example supported by events such as the crash of Black Monday, 1987, the Asian financial crises in the late 90's, and the terrorist attacks of September 11, 2001. However, although such simultaneous jumps are empirically interesting and can be important for international diversification (see Das and Uppal (2004)), to my knowledge, no previous study has attempted to estimate in a plausible multivariate setting how frequent they actually are. In this paper, as a first attempt, I propose a multivariate model that allows

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for the simultaneous estimation of the intensities of both systemic (simultaneous across markets) and idiosyncratic (market specific) jumps in returns—an approach which sheds light from a new angle on the issue of downside dependence in international returns. To estimate the model, I develop a Markov chain Monte Carlo (MCMC) based estimation method and I apply the model to groups of major North American, European, and Asian equity indices.

This paper is related to a number of previous studies. On the empirical side, Longin and Solnik (2001) use extreme value theory to estimate correlations between extreme returns of pairs of countries. They find that the correlation between returns below (above) a certain threshold value increases (decreases) as the threshold value decreases (increases) and conclude that "the probability of having large losses simultaneously on two markets is much larger than what would be suggested under the assumption of multivariate normality". Ang and Chen (2002) develop a statistic to measure asymmetries in correlations and report similar results and studies on applications of copula functions to equity returns also tend to report similar results (see, e.g., Chollete et al. (2004) and Hu (2004)). Papers on financial contagion often find significant evidence that financial crises are transmitted across borders (see, e.g., De Bandt and Hartmann (2000), Forbes and Rigobon (2002), Bae et al. (2003), and Boyer et al. (2005)). Although these papers do not look at jumps in returns, the approach of Bae et al. (2003) is, to some extent, related to the approach of this paper. They study simultaneous extreme international returns using a multinomial logistic regression model and find, for example, that such returns are more frequent than assumptions of returns being multivariate normally or Student's t distributed or generated by a GARCH process would imply. As opposed to Bae et al. (2003), who arbitrarily specifies the "extreme" returns of an equity index as those that belong either to the five percent most negative or five percent most positive returns in the sample, I need not specify beforehand which returns in the sample are "extreme"—these are identified automatically in the estimation. Asgharian and Bengtsson (2004) focuses, as the present paper does, on jumps in returns. They estimate a univariate model on a number of international equity indices to infer the historical jump times of each index and find significant evidence of the existence of systemic jumps. The multivariate approach of this paper is, however, preferable over the univariate approach of Asgharian and Bengtsson (2004) for a couple reasons. Firstly, the intensity of systemic jumps can be estimated directly as a parameter in a multivariate model—as opposed to estimated ex-post from posterior jump probabilities. Secondly, using a multivariate model simplifies the inference about the estimated intensity of systemic jumps—in this paper because its significance can be determined directly from the posterior distribution.

On the more theoretical side, Das and Uppal (2004) look at effects of systemic jumps on international dynamic asset allocation. They derive optimal portfolio weight expressions for and estimate a multivariate model with systemic jumps based on the model of Merton (1976). They find that the cost from ignoring systemic jumps can be substantial for aggressive investors who hold highly leveraged portfolios. As is discussed below, however, their model which assumes constant volatility is not appropriate in an empirical study such as this paper. Ang and Bekaert (2002) look at effects of regime shifts on international dynamic assets allocation. They find evidence of a normal regime with low correlations and a more volatile regime with high correlations, although the cost from ignoring these regime shifts is only substantial when a conditionally risk free asset is available. Finally, Branger and Schlag (2004) look at implications of systemic and idiosyncratic jumps on the pricing of options on equity indices and options on individual stocks. They do not, however, estimate their model.

The proposed model in this paper—the multivariate stochastic volatility with systemic and idiosyncratic jumps (MSVSIJ) model—falls into the class of affine jump-diffusion models proposed by Duffie et al. (2000) and is a multivariate version of the stochastic volatility with independently arriving jumps (SVIJ) model of Eraker et al. (2003). The model assumes that each return process is affected by both a systemic jump process (with correlated jump sizes across countries) and an idiosyncratic jump process. The frequency of systemic (idiosyncratic) jumps is measured through the intensity of the systemic (idiosyncratic) jump process. The volatilities of returns are allowed to be time-varying with jumps since it is important to distinguish between returns that are actual jumps from returns that are large simply because volatility is high. Intuitively, if (shocks to) spot volatilities are positively correlated (which, it turns out, my empirical results show they are), volatility will tend to be high simultaneously in different markets. Because returns generally are positively correlated, this implies the presence of simultaneous "volatility-induced" large diffusive returns, which, if volatility is wrongly assumed to be constant, may be mistaken for systemic jumps. Furthermore, even if spot volatilities were uncorrelated, allowing volatility to be time-varying is important for the estimation of the intensities of idiosyncratic jumps and for answering the question of which type of jumps dominate; systemic or idiosyncratic? If volatility is assumed to be constant, estimated jump times will cluster to capture the time-varying nature of volatility (see Johannes and Polson (2003)), effectively making impossible the estimation of the intensities of idiosyncratic jumps.

The empirical results show significant evidence of the existence of systemic jumps. I find that, on average, systemic jumps arrive in the North American markets (the US and Canada) approximately one time per year, in the European markets (the UK, Germany, and France) approximately 0.3 times per year, and in the Asian markets (Japan and Hong Kong) approximately 0.9 times per year. In the North American markets, the majority of jumps are systemic. The intensity of idiosyncratic jumps in the US is insignificant and that of Canada is significant but smaller than the intensity of systemic jumps. In the European and Asian markets, on the other hand, the majority of jumps are idiosyncratic. The intensities of idiosyncratic jumps in these markets are all significant and relatively large compared to the respective intensities of systemic jumps. For all markets, the mean jump sizes of systemic jumps are significantly negative, whereas the mean jump sizes of idiosyncratic jumps are not significantly different from zero. This implies that markets tend to experience large sudden losses, but not large sudden gains, simultaneously. In addition, the mean jump sizes of systemic jumps are, in absolute terms, greater than the respective mean jump sizes of idiosyncratic jumps (which are positive for some countries). This implies that the negative skewness of international returns that cannot be attributed to leverage effects between returns and volatilities is more likely to be caused by the presence of systemic jumps than by the presence of idiosyncratic jumps. Finally, I find, somewhat surprisingly, that none of the correlation coefficients between the sizes of systemic jumps are significantly different from zero, which seemingly is at odds with the results of Longin and Solnik (2001). I argue that the most likely cause of this difference in results is that my approach and the approach of Longin and Solnik (2001)—which does not explicitly take into account the time-varying nature of volatility—identifies very different returns as ”simultaneous extreme negative returns”. This paper is concerned with the returns that can be interpreted as jumps, whereas their approach makes no distinction between returns that can be interpreted as jumps and simultaneous large volatility-induced diffusive returns.

The rest of the paper proceeds as follows: Section 2 develops and discusses the MSVSIJ model, Section 3 develops the MCMC based estimation method, Section 4 discusses the empirical results, and Section 5 concludes.

2 Multivariate Stochastic Volatility Jump-Diffusion

My ambition is to specify a model that is rich enough to generate meaningful empirical results, but still simple enough to yield results that are easily interpreted and to retain econometrical tractability. I assume that (the natural logarithm of) the level of the equity index of country i at time t, S_{it}, where $i = 1, 2, ..., N$ and $t \geq 0$, in a meaningful way solves the stochastic differential equation

$$\begin{pmatrix} d\ln\left(S_{it}\right) \\ dV_{it} \end{pmatrix} = \begin{pmatrix} \mu_i \\ \varkappa_i\left(\vartheta_i - V_{it-}\right) \end{pmatrix} dt + \sqrt{V_{it-}} \begin{pmatrix} dW_{it}^Y \\ \sigma_i dW_{it}^V \end{pmatrix} + \begin{pmatrix} X_{it}^S dN_t^S + X_{it}^I dN_{it}^I \\ X_{it}^V dN_{it}^V \end{pmatrix}, \quad (1)$$

where V_{it} is the spot volatility of index i at time t, V_{it-} is equal to V_{it} if there is no jump at time t and equal to the V_{it} just before the discontinuity caused by the jump otherwise, W_{it}^Y and W_{it}^V are correlated standard Brownian motions, N_t^S, N_{it}^I, and N_{it}^V are independent poisson (jump) processes with constant intensities λ^S, λ_i^I, and λ_i^I, respectively, and X_{it}^S, X_{it}^I, and X_{it}^V are jump sizes. The poisson process N_t^S (N_{it}^I) captures systemic (idiosyncratic) jumps in returns and the intensity of systemic jumps, λ^S, can be interpreted as the the covariance between the increments of the compounded poisson processes $N_{it} = N_t^S + N_{it}^I$ and $N_{jt} = N_t^S + N_{jt}^I$, $i \neq j$, over a unit time interval. The vector of systemic jump sizes, $X_t^S = (X_{1t}^S, X_{2t}^S, ..., X_{Nt}^S)'$, is assumed to be multivariate normally distributed with mean vector μ^S and covariance matrix $\Sigma^S = \bar{\sigma}^S R^S \bar{\sigma}^S$, where $\bar{\sigma}^S$ is a diagonal matrix with diagonal equal to $\sigma^S = (\sigma_1^S, \sigma_2^S, ..., \sigma_N^S)$ and R^S is a correlation matrix, the jump size of the idiosyncratic jumps of index i, X_{it}^I, is assumed to be normally distributed with mean μ_i^I and standard deviation σ_i^I, and the jump size of the spot variance of index i, X_{it}^V, is assumed to be exponentially distributed with mean μ_i^V. To capture correlations between returns, between volatilities, and between returns and volatilities (so called leverage effects), I assume that the correlation matrix of $(dW_{1t}^Y, dW_{Nt}^Y, ..., dW_{Nt}^Y, dW_{1t}^V, dW_{2t}^V, ..., dW_{Nt}^V)'$ is constant and equal to $R = \{\rho_{ij}\}_{i,j=1}^{2N}, i \neq j$, which implies that covariances are time-varying only to the extent that volatility is time-varying. It should be noted that I place no restrictions on R besides that it must be a positive semi-definite correlation matrix.[1]

I refer to this model as the multivariate stochastic volatility with systemic and idiosyncratic jumps (MSVSIJ) model. The MSVSIJ model can be thought of as a straightforward multivariate extension of the stochastic volatility with independently arriving jumps (SVIJ) model of Eraker et al. (2003). The MSVSIJ model also nests multivariate extensions of other models. For example, with $\varkappa_i = \sigma_i = \lambda_i^V = 0$ and $V_{i0} = v_i$, it reduces to a multivariate version of Merton's (1976) model with both systemic and idiosyncratic jumps, and without jumps, it reduces to a multivariate version of Heston's (1993) model.

A few remarks about the MSVSIJ model are in place. Firstly, the assumption of constant jump intensities facilitates both the analysis and the estimation. Furthermore, the is no empirical evidence that jump intensities are time-varying (see, e.g., Chernov et al. (1999) and Andersen et al. (2002)). Secondly, the jumps in volatility are for simplicity assumed to be independent of each other and of jumps in returns. The alternative would be to specify that jumps in volatility always arrive simultaneously with jumps in returns. Such a specification would likely be easier to estimate (see Eraker et al. (2003)), but would also be more restrictive.[2] The purpose of the jumps in volatility is to allow for the

[1] An additional way to introduce dependencies between spot variances would be to specify the the drift term of $dV_t = (dV_{1t}, dV_{2t}, ..., dV_{Nt})'$ as $\bar{\varkappa}(\vartheta - V_{t-})$, where $\bar{\varkappa} = \{\varkappa_{ij}\}_{i,j=1}^N$ is an $N \times N$ matrix, $\vartheta = (\vartheta_1, \vartheta_2, ..., \vartheta_N)'$, and $V_{t-} = (V_{1t-}, V_{2t-}, ..., V_{Nt-})'$. This specification would be useful if the objective was to look also at volatility spillover effects. The specification above corresponds to assuming that $\bar{\varkappa}$ is diagonal with diagonal elements $\varkappa = (\varkappa_1, \varkappa_2, ..., \varkappa_N)'$.

[2] In the univariate case, Eraker et al. (2003) find that assuming independent jumps in returns and volatility performs better empirically than assuming simultaneous jumps in returns and volatility.

rapid changes in volatility reported by, for example, Bates (2000), Duffie et al. (2000), Pan (2002), and Eraker et al. (2003). Jumps in volatility improve the empirical fit of the model at a relatively low technical cost at the same time as they further prevent (estimated) jumps in returns from clustering. Although this paper focuses on jumps in returns, it would of course in theory be possible to separate also the jumps in volatility of each index into a systemic and an idiosyncratic component. However, because of the infrequent nature of jumps and since spot volatility is latent and relatively difficult to estimate, it is not clear if it would be possible to obtain meaningful estimates of the corresponding jump intensities in such a specification. Thirdly, an alternative specification for the multivariate stochastic volatility process would be to use a factor approach such as that of Jacquier et al. (1999) or Chib et al. (2005), which would be computationally less demanding for large values of N. For the values of N in this paper, however, the above specification (it turns out) is quite feasible for estimation purposes and it is therefore preferable—both because it is less restrictive and because it is more transparent. Finally, when N is large and the indices belong to different regions, it might be too restrictive and unrealistic to assume that there is only one *global* systemic jump process. A solution would be to introduce additional *regional* systemic jump processes which each affect different subsets of the indices. Generally, when a sufficient amount of historical data is available, it is not the jump intensities that are difficult to estimate, but rather the spot volatility path. For the time being, however, the case of truly large values of N is saved for future research. This paper concentrates on systemic jumps in groups consisting of a few major international equity indices.

3 Estimation Method

Building on the pioneering work of Jacquier et al. (1994) and Eraker et al. (2003), I use a Bayesian approach based on MCMC methods to estimate the MSVSIJ model. The estimation relies on an Euler discretization of equation (1) over a time interval Δ. Since the frequency of the data used later in the paper is daily ($\Delta = 1$), the discetizized model is

$$\begin{pmatrix} Y_{it+1} \\ V_{it+1} \end{pmatrix} = \begin{pmatrix} \mu_i \\ V_{it} + \alpha_i + \beta_i V_{it} \end{pmatrix} + \sqrt{V_{it}} \begin{pmatrix} \varepsilon^Y_{it+1} \\ \sigma_i \varepsilon^V_{it+1} \end{pmatrix} + \begin{pmatrix} X^S_{it+1} J^S_{t+1} + X^I_{it+1} J^I_{it+1} \\ X^V_{it+1} J^V_{it+1} \end{pmatrix} \tag{2}$$

where Y_{it} is the daily log return of index i at time t, J^S_t, J^I_{it}, and J^V_{it} are jump indicators which each take on the value 1 with probability λ^S, λ^I_i and λ^V_i, respectively, and the value 0 otherwise, ε^Y_{it} and ε^V_{it} are correlated standard normally distributed stochastic variables, and the drift parameters of the volatility process have been rewritten so that $\alpha_i = \varkappa_i \vartheta_i$ and $\beta_i = -\varkappa_i$. The distributions of the jump sizes remain the same.

The discretization of a continuous-time model can potentially introduce discretization biases in estimates. However, for the univariate case, Eraker et al. (2003) show that the biases are quite small when the discretization interval is daily, and there is no reason to believe that the situation is any different in a multivariate setting. Furthermore, although it provides a good foundation, the continuous-time specification is not critical for the purpose of this particular paper. The objective is to perform an empirical study of jumps and not to look at theoretical applications of the MSVSIJ model. Because of the infrequent nature of jumps relative to the daily frequency of the data, the discretization should not significantly affect the estimation of the jump intensities and the jump sizes.

The foundation of Bayesian inference is the joint distribution of parameters and latent variables conditional on the data. This distribution—the posterior distribution—is derived by applying Bayes rule as

$$p(V, J, X, \Theta | Y) \propto p(Y | V, J, X, \Theta) p(V, J, X | \Theta) p(\Theta), \tag{3}$$

where Y, V, J, and X are matrices containing $T \times 1$ vectors of historical time series of log returns, spot variances, jump times, and jump sizes, respectively, and $\Theta = (\Theta_1', \Theta_2', ..., \Theta_K')'$ is a vector of parameters consisting of K appropriately chosen blocks (sub-vectors) of parameters. The first term on the right hand side of equation (3) is the likelihood of the data, the second term, which is defined by the model assumptions, is the (prior) distribution of the latent variables conditional on the parameters, and the last term, which has to be specified independently of the data, is the prior distribution of the parameters. I follow Eraker et al. (2003) and choose very uninformative parameter priors on the parameters except possibly for the priors on the jump intensities and the variances of jumps in returns, which, in line with intuition, place low probabilities on jumps being small and frequent.[3] Bayesian point estimates are taken as the posterior expectations of the parameters and the latent variables.

Since the posterior distribution of equation (3) is extremely high-dimensional and not available in closed form, I develop an MCMC algorithm to evaluate it numerically. The foundation of MCMC algorithms is the insight of Hammersley and Clifford (1970) that a joint distribution can be separated into and completely characterized by a set of conditional distributions which are lower in dimension and more manageable; that is, for example, $p(V, J, X, \Theta | Y)$ is completely characterized by $p(V, J, X | Y, \Theta)$ and $p(\Theta | Y, V, J, X)$, and so forth. The algorithm iteratively makes draws from a set of conditional posterior distributions and produces a sequence of draws, $\{V^{(j)}, J^{(j)}, X^{(j)}, \Theta^{(j)}\}_{j=m}^{M}$,

[3] A potentially important difference between the priors chosen in this paper and those chosen by Eraker et al. (2003) is the prior placed on the jump intensities. I choose a prior that has approximately the same mean and standard deviation as the one chosen by Eraker et al. (2003), but puts more probability closer to zero which reduces the overestimation of small jump intensities.

which is a Markov chain with stationary distribution equal to the posterior distribution, where M is the total number of draws and m is the number of draws discarded as "burn in". Through the burn-in period the algorithm is given time to converge and for the influence of the starting values to disappear. The Bayesian point estimates are taken as the sample averages of the parameters and the latent variables in this Markov chain which is treated as a sample from the posterior distribution. With appropriate starting values $(V^{(0)}, J^{(0)}, X^{(0)}, \Theta^{(0)})$, the MCMC algorithm is to, for $j = 1, 2, ..., m, ..., M$, iteratively draw

Parameters

$$
\begin{aligned}
\Theta_1^{(j)} &\sim p(\Theta_1^{(j)} \mid Y, V^{(j-1)}, J^{(j-1)}, X^{V(j-1)}, \Theta_2^{(j-1)}, \Theta_3^{(j-1)}, ..., \Theta_K^{(j-1)}) \\
\Theta_2^{(j)} &\sim p(\Theta_2^{(j)} \mid Y, V^{(j-1)}, J^{(j-1)}, X^{V(j-1)}, \Theta_1^{(j)}, \Theta_3^{(j-1)}, ..., \Theta_{K-1}^{(j-1)}) \\
&\vdots \\
\Theta_K^{(j)} &\sim p(\Theta_K^{(j)} \mid Y, V^{(j-1)}, J^{(j-1)}, X^{V(j-1)}, \Theta_1^{(j)}, \Theta_2^{(j)}, ..., \Theta_{K-1}^{(j)})
\end{aligned}
$$

Jump times and jump sizes

for $t = 1, 2, ..., T$ and $i = 1, 2, ..., N$

$$
\begin{aligned}
J_t^{S(j)} &\sim p({J_t}^{S(j)} \mid Y, V_{t-1}^{(j-1)}, V_t^{(j-1)}, J_t^{I(j-1)}, J_t^{V(j-1)}, X_t^{V(j-1)}, \Theta^{(j)}) \\
X_t^{S(j)} &\sim p(X_t^{S(j)} \mid Y, V_{t-1}^{(j-1)}, V_t^{(j-1)}, J_t^{S(j)}, J_t^{I(j)}, J_t^{V(j-1)}, X_t^{I(j-1)}, X_t^{V(j-1)}, \Theta^{(j)}) \\
J_{it}^{I(j)} &\sim p({J_{it}}^{I(j)} \mid Y, V_{t-1}^{(j-1)}, V_t^{(j-1)}, J_t^{S(j)}, J_{(-i)t}^{I(j-1)}, J_t^{V(j-1)}, X_t^{V(j-1)}, \Theta^{(j)}) \\
X_{it}^{I(j)} &\sim p(X_{it}^{I(j)} \mid Y, V_{t-1}^{(j-1)}, V_t^{(j-1)}, J_t^{(j)}, J_t^{I(j)}, J_t^{V(j-1)}, X_t^{S(j)}, X_{(-i)t}^{I(j-1)}, X_t^{V(j-1)}, \Theta^{(j)}) \\
J_{it}^{V(j)} &\sim p({J_{it}}^{V(j)} \mid Y, V_{t-1}^{(j-1)}, V_t^{(j-1)}, J_t^{S(j)}, J_t^{I(j)}, J_{(-i)t}^{V(j-1)}, X_t^{V(j-1)}, \Theta^{(j)}) \\
X_{it}^{V(j)} &\sim p(X_{it}^{V(j)} \mid Y, V_{t-1}^{(j-1)}, V_t^{(j-1)}, J_t^{S(j)}, J_t^{I(j)}, J_t^{V(j)}, X_t^{S(j)}, X_t^{I(j)}, X_{(-i)t}^{V(j-1)}, \Theta^{(j)})
\end{aligned}
$$

Spot variances

$$
V_0^{(j)} \sim p(V_0^{(j)} \mid Y_1, V_1^{(j-1)}, J_1^{(j)}, X_1^{(j)}, \Theta^{(j)})
$$

for $t = 1, 2, ..., T - 1$

$$
\begin{aligned}
V_t^{(j)} &\sim p(V_t^{(j)} \mid Y_t, V_{t-1}^{(j)}, V_{t+1}^{(j-1)}, J_t^{(j)}, J_{t+1}^{(j)}, X_t^{(j)}, X_{t+1}^{(j)}, \Theta^{(j)}) \\
V_T^{(j)} &\sim p(V_T^{(j)} \mid Y_T, V_{T-1}^{(j)}, J_T^{(j)}, X_T^{(j)}, \Theta^{(j)})
\end{aligned}
$$

In the cases when the above conditional posterior distributions are standard distributions that are easily sampled from, the corresponding draws are usually referred to as Gibbs steps, and in the cases when they are non-standard distributions, the corresponding draws can be made by introducing Metropolis-Hastings steps. Details on the conditional posterior distributions used in the algorithm above can be found in Appendix A. For a more detailed discussion on MCMC methods, I refer to Johannes and Polson (2003) for a thorough and excellent discussion.

4 Empirical Results

In this section I discuss the empirical results. I start by estimating the MSVSIJ model on artificial (simulated) data sets in order to show that the MCMC algorithm delivers meaningful estimates. After that I estimate the model on groups of major international equity indices.

4.1 Artificial Data

For $N = 2$, I simulate ten independent artificial data sets, each with $T = 5,000$ realizations, using as data generating process the discrete-time model of equation (2). The average parameter estimates over all these ten data sets, together with the assumed true parameter values, are presented in Table 2.

The results shows that the MCMC algorithm is able to produce quite accurate estimates of the parameters (and the latent variables) of the MSVSIJ model. Naturally, the parameters of the latent processes pose the greatest challenge to estimate accurately; in particular the parameters of the jump processes since jumps are inherently rare events. The average estimates of the parameters of the spot variance processes are all close to their assumed true values, with the possible exception of $\varkappa$ which appears to be slightly upwards biased (Eraker et al. (2003) report similar results).[4] The average estimates of the elements of the correlation matrix of returns and spot variances are also very close to their respective assumed true values—even the correlation coefficient between the diffusive terms of the two latent spot variance processes. Although the estimation risk attached to the parameters of the jump processes—the jump intensities and parameters of the jump size distributions—is large compared to the estimation risk of the remaining parameters, the estimates are still quite accurate. In particular, the average estimate of the intensity of systemic jumps is extremely close to its assumed true value.

4.2 International Equity Indices

I restrict the analysis to groups of two or three major international equity indices at a time and to groups of indices that belong to the same regions. This avoids the problem that arises because markets in different regions have different and sometimes non-overlapping opening hours.[5] Related previous

[4] To examine what causes this bias, I have also estimated the model when the spot variance process is treated as observable. This simulation study shows that when the historical spot variances are known, all parameters (and remaining latent processes) are estimated with much greater precision and with no discernable biases present. This implies that the biases found here and in Eraker et al. (2003) are not discretization biases, but rather biases caused by the difficulty of estimating the latent spot variances.

[5] In addition, Asgharian and Bengtsson (2004) find evidence that pairs of countries that belong to the same region tend to jump simultaneously more often than countries that belong to different regions. Ideally, the MSVSIJ model should be modified if applied to groups of indices that belong to different regions. For example to take into account that jumps in the

work such as Longin and Solnik (2001) and Ang and Bekaert (2002) also restrict focus to small groups of major international markets.

Each equity index is the major index of its country of origin—the S&P 500 for the US, for instance—and the groups are (1) the US and Canada (North America), (2) the UK, France, and Germany (Europe), and (3) Japan and Hong Kong (Asia). The countries are chosen based on the weights assigned to them in the 2002 MSCI All Country World Index (ACWI) Free index. For reasons of clarity, I refer to each index by the name of its country of origin. The data sets are all obtained from the EcoWin database and they cover the period from February 1, 1985, to April 28, 2004. Descriptive statistics of the data is given in Table 1. For each of the three groups, the days that do not have return observations for all countries are eliminated from the samples.

The respective parameter estimates for each one of the three groups above are presented in Tables 3, 4, and 5. Since the focus is on the analysis of jumps in returns, I do not provide a complete general discussion of the parameter estimates. I simply note that in the cases where comparisons are meaningful, they are reasonably consistent with the parameter estimates of Eraker et al. (2003), Eraker (2004), and Asgharian and Bengtsson (2004) for similar but univariate models.

4.2.1 Jumps in North American Markets

The estimate of the intensity of systemic jumps in the US and Canada is significant and equal to 0.0039 (see Table 3).[6] This implies that, on average, systemic jumps arrive in the North American markets approximately one times per year, which is almost twice as often as implied by the estimate of Asgharian and Bengtsson (2004). The estimated intensity of systemic jumps combined with the estimated intensities of idiosyncratic jumps, which are equal to 0.0007 and 0.0024 for the US and Canada, respectively, imply that approximately 85 (62) percent of the jumps in the US (Canada) arrive simultaneously as jumps in Canada (the US). However, it is important to note that the intensities of idiosyncratic jumps are quite imprecisely estimated. In fact, judging from the respective posterior

US market can arrive both when the European markets are open and when the are closed, and so forth.

[6]Because of the events of September 11, 2001, which caused jumps in market indices throughout the world (see Asgharian and Bengtsson (2004)), it is likely that the estimate of the intensity of systemic jumps is somewhat underestimated. The reason is that the US market never opened on September 11 and were closed until September 17 when they reopened and the effects of the events on the US equity market were realized. The Canadian market, on the other hand, was open on September 11 and therefore experienced the effect of the events on that very same day. Although the jumps caused by the events of September 11 were systemic in the sense that the same underlying event triggered jumps in both markets, since the jumps arrived on different dates (the data has not been manipulated to control for this) and, in particular, since since the observations from September 11 to September 16 have been eliminated from the sample, the intensity of systemic jumps is likely to be underestimated by one jump over the sample period; that is, by approximately $1/T = 0.0002$.

standard deviations alone, a naive conclusion would be to immediately conclude that they are insignificant. This conclusion is, however, not necessarily correct. The reason is that the conditional posterior distributions of the intensity parameters are very positively skewed. A better alternative is to look instead at the posterior confidence interval (credible set) of each estimate. These are not reported in the tables in order to reduce the sizes of the tables, but are given in the text where appropriate. The left limits of the single-sided 95 percent posterior confidence intervals of the intensities of idiosyncratic jumps for the US and Canada are equal to 0.0000 and 0.0004, respectively. So consequently, by this measure, the conclusion is that the intensity of idiosyncratic jumps in the US is not significantly greater than zero at the 95 percent level, while the one of Canada is. Some caution here is necessary, however. The reason for this is that for small jumps intensities, because of the prior distribution, a jump intensity estimate will be slightly upwards biased. Take for example the extreme case when $J_{it}^{I(j)} = 0, t = 1, 2, ..., T, j = 1, 2, ..., n$. In this case the posterior mean of λ_i^I, given the chosen prior distribution (see Appendix A) and the sample size, is equal to 0.0002, although the, in some sense, "correct" estimate is zero. For an estimated intensity to be referred to as significantly greater than zero (at the 95 percent level), I therefore require that the left limit of its single-sided 95 percent posterior confidence interval is greater than 0.0002. This limit for the above estimate of the intensity of systemic jumps is for example equal to 0.0020. So consequently, the conclusion that the intensity of idiosyncratic jumps of Canada is significant is in fact correct and it appears that major events and news releases—both of international and domestic nature—that trigger jumps in the US market also trigger jumps in the smaller Canadian market, while the opposite is not true.

Figure 1 shows the posterior probabilities of the historical jump times of systemic and idiosyncratic jumps. The posterior probabilities of systemic jumps are generally quite large, while the posterior probabilities of idiosyncratic jumps are generally very low for the US and less distinct than the posterior probabilities of systemic jumps for Canada.

The mean sizes of systemic jumps are significantly negative and are approximately equal to minus four times the unconditional volatilities of returns, which each can be calculated as the square-root of $\bar{V}_i = \vartheta_i + (\mu_i^V \lambda_i^V)/\varkappa_i$ (see Eraker et al. (2003) for a proof of this expression). The mean sizes of idiosyncratic jumps, on the other hand, are not significantly different from zero and smaller in absolute terms than the mean sizes of systemic jumps. This implies that the skewness of North American returns that can not be attributed to leverage effects is more likely to be caused by systemic jumps than by idiosyncratic jumps. To illustrate, when the returns of the US and Canada are filtered from the estimated systemic jump components by calculating $Y_{it} - \widehat{X_{it}^S J_t^S}, i = 1, 2, t = 1, 2, ..., T$, where

$$\widehat{X_{it}^S J_t^S} = \frac{1}{M - m} \sum_{j=m}^{M} X_{it}^{S(j)} J_t^{S(j)},$$

the (negative) sample skewness is reduced by 105 and 96 percent (to 0.1007 and -0.0429), respectively, relative to the values in Table 1. When, on the other hand, the returns are filtered from the estimated idiosyncratic jump components by calculating $Y_{it} - \widehat{X_{it}^I J_{it}^I}, i = 1, 2, t = 1, 2, ..., T$, where

$$\widehat{X_{it}^I J_{it}^I} = \frac{1}{M - m} \sum_{j=m}^{M} X_{it}^{I(j)} J_{it}^{I(j)},$$

the sample skewness is only reduced by 6 and 29 percent, respectively.

Surprisingly, the correlation coefficient between the sizes of systemic jumps is smaller than the correlation coefficient between diffusive shocks to returns and not significantly different from zero. This result is, at least at first glance, at odds with the findings of Longin and Solnik (2001). They find that simultaneous large negative returns display a higher degree of dependence than other returns—a pattern which would not be present if returns were simply jointly normally distributed (see their paper for a discussion on the asymptotic behavior of the correlation coefficient of extreme returns). The likely reason for this difference in results is that the approach of this paper and the approach of Longin and Solnik (2001) classifies very different observations as "simultaneous extreme negative returns". This paper is interested in returns that can be interpreted as systemic jumps and consequently cannot be explained as diffusive returns. These returns are economically important since they tend to catch the market by surprise given the current levels of spot volatilities. Longin and Solnik (2001), on the other hand, use coarser monthly data which can be expected to smooth out the effect of jumps and they also do not explicitly take into account the time-varying nature of volatility. Therefore, it is likely that their approach directs focus more on simultaneous returns that can be explained as positively correlated diffusive returns related to simultaneous high levels of spot volatility.

To illustrate this in a very informal manner, I begin by finding the days in the sample that are such that the return of the US belongs to the percentile of most negative returns of the US at the same time as and the return of Canada belongs to the percentile of most negative returns of Canada. I find that there are 23 such days and that the sample correlation between the returns on these days is equal to 0.8783.[7] This is, just as predicted by Longin and Solnik (2001), greater than the correlation coefficient

[7]For a data set consisting of 1,000,000 observations simulated from a bivariate normal distribution with correlation coefficient 0.6841, the corresponding correlation coefficient is equal to 0.2002, and for a data set consisting of 1,000,000 observations simulated from a bivariate student's t distribution with correlation coefficient 0.6841 and four degrees of freedom (which gives levels of kurtosis roughly equal to those in Table 1), it is equal to 0.6442.

for the entire sample. It turns out, however, that only 7 of the days identified above coincide with the 18 days in the sample that have the largest posterior probabilities of systemic jumps (the implied number of systemic jumps in the sample is equal to 18; that is, $4,756 \cdot 0.0039 \approx 18$). Furthermore, the correlation between the returns of the above implied days of systemic jumps is much smaller and equal to 0.4570, which is quite close to the estimated correlation between the jump sizes of systemic jumps.

Although the simple procedure above gives a hint on what is the likely cause of the difference in results, it is also interesting to examine if the MSVSIJ model can—through skewness and excess kurtosis caused by stochastic volatility—in fact accommodate patterns in correlations similar to the correlation asymmetries documented by Longin and Solnik (2001). For this purpose, I use equation (2) together with the parameter estimates in Table 3 to simulate artificial daily returns which are then compounded into monthly returns. The result of this simulation study is that the MSVSIJ model does not seem to be able to generate any such correlation patterns. Although correlation asymmetries between diffusive returns should not affect the estimation of the intensities and sizes of systemic jumps,[8] which is the focus of this paper, extensions of the MSVSIJ model to allow also for time-varying correlations can therefore potentially be relevant. The fact still remains, however, that simultaneous extreme negative returns that can be interpreted as systemic jumps are likely to be less correlated than simultaneous large negative diffusive returns—a feature of extreme international returns that has not previously been documented and that the approach of Longin and Solnik (2001) is unable to capture.

4.2.2 Jumps in European Markets

The estimate of the intensity of systemic jumps in the UK, Germany, and France is equal to 0.0012 (see Table 4) and significant; the left limit of its single-sided 95 percent posterior confidence interval is equal to 0.0004. This implies that, on average, systemic jumps arrive in the European markets approximately 0.3 times per year (or alternatively, once every 3.4 years) which is much less frequent than in the North American markets. To examine if this difference is simply an effect from looking at three indices simultaneously rather than two, I re-estimated the model when leaving out Germany.[9]

[8] This is for the simple reason that diffusive returns, no matter what is the correlation between them, can not explain the large returns identified as systemic (or idiosyncratic) jumps. Intuitively and trivially, a sample from a multivariate distribution with heavy tails can not be explained properly by a multivariate normal distribution just by letting the correlation coefficients vary since the unconditional marginal distributions would still be univariate normal distributions.

[9] The reason for leaving out Germany rater than France is that Asgharian and Bengtsson (2004) find that Germany is relatively unaffected by jumps in other countries.

The result was that the estimate remained unchanged which implies that systemic jumps are in fact much less common in European markets than in North American markets (this is also consistent with the results of Asgharian and Bengtsson (2004)). The estimated intensities of idiosyncratic jumps in the UK, Germany, and France are equal to 0.0036, 0.0045, and 0.0156, respectively, and significant; the left limits of their single-sided 95 percent posterior confidence intervals are equal to 0.0014, 0.0006, and 0.0016, respectively. In addition, they are all larger than the intensity of systemic jump, which implies that in contrast to the North American markets, the majority of jumps in European markets are idiosyncratic. Approximately, only 25, 20, and 7 percent of the jumps in the UK, Germany, and France, respectively, are systemic.

Figure 2 shows the posterior probabilities of systemic and idiosyncratic jumps in the European markets. The few jump times of systemic jumps are quite distinct. The posterior probabilities of idiosyncratic jumps are relatively large for many dates, but generally less distinct than the jump times of systemic jumps.

Similar to the North American markets, the mean sizes of systemic jumps are significantly negative, while the mean sizes of idiosyncratic jumps are not significantly different from zero and smaller in absolute terms than the mean jump sizes of systemic jumps. This implies that also the skewness of European returns which can not be attributed to leverage effects is more likely to be caused by systemic jumps than by idiosyncratic jumps. When the returns of the UK, Germany, and France are filtered from the estimated systemic jump components, the (negative) sample skewness is reduced by 0.5007, 0.7827, and 0.7242 percent, respectively, relative to the values in Table 1. When, on the other hand, the returns are filtered from the estimated idiosyncratic jump components, the sample skewness is reduced by 31 percent for the UK and increased by 10 and 6 percent for Germany and France, respectively. A potential interpretation may be that events that have international impacts on asset prices carry predominately negative news, whereas events that only have domestic impacts on asset prices can carry both positive and negative news.

Another and important similarity with the North American markets is that none of the estimated correlation coefficients between the jump sizes of systemic jumps are significantly different from zero in this case either. In fact, the correlation between the jump size of the UK is negatively correlated with the jumps sizes of both Germany and France. It should be noted, however, that the intensity of systemic jumps implies that there are very few observations that these estimated correlations are based on.

4.2.3 Jumps in Asian Markets

The estimate of the intensity of systemic jumps in Japan and Hong Kong is equal to 0.0035 (see Table 5) and significant; the left limit of its single-sided 95 percent posterior confidence interval is equal to 0.0015. This implies that, on average, systemic jumps arrive in the Asian markets approximately 0.9 times every year (or alternatively, once every 1.1 years). This estimate is more similar to that of the North American markets than to that of the European markets. The estimates of the intensities of idiosyncratic jumps are equal to 0.0160 and 0.0072, respectively, are significant; the left limits of their single-sided 95 percent posterior confidence intervals are equal to 0.0034 and 0.0005, respectively. This implies that, similar to the European markets, the majority of jumps in the Asian markets are idiosyncratic. Approximately, only 18 (33) percent of the jumps in Japan (Hong Kong) arrive simultaneously as jumps in Hong Kong (Japan).

Figure 3 shows the posterior probabilities of systemic and idiosyncratic jumps of the Asian markets. The dates with high posterior probabilities of systemic jumps are quite distinct, which implies that the intensity of systemic jumps should be very accurately estimated. The posterior probabilities of idiosyncratic jumps, on the other hand, are not as distinct—especially for Hong Kong—and for Japan they appear to be slightly clustered. This implies that the assumption of the presence of idiosyncratic jumps (with constant jump intensities) in the Asian markets may be inappropriate, which affects the reliability of the estimated intensities of idiosyncratic jumps.

As in both the North American and the European markets, the mean sizes of systemic jumps are significantly negative, while the mean sizes of idiosyncratic jumps are not significantly different from zero and smaller in absolute terms than the mean jump sizes of systemic jumps. Although the sample skewness of the returns of Japan is not negative, filtering the returns of Japan from the estimated systemic jump component increases the (positive) sample skewness by 89 percent relative to the value in Table 1, while filtering the returns from the idiosyncratic jump component reduces it by 115 percent. For Hong Kong, filtering the returns from the systemic jump component decreases, similar to the North American and European countries, the (negative) sample skewness by 29 percent relative to the value in Table 1, while filtering the returns from the idiosyncratic jump component increases it by 4 percent.

Again, the estimated correlation coefficient between the sizes of systemic jumps is not significantly different from zero. This implies that the finding that the sizes of systemic jumps may be independent is not a feature that is only present in a few markets, but rather, it seems to be a global phenomena.

5 Conclusions

This paper studies the existence of systemic jumps in international equity returns—an issue that, to my knowledge, has not been properly addressed before. I propose a multivariate model which assumes that returns are affected by both systemic (simultaneous across markets) and idiosyncratic (market specific) jumps. To estimate the model—the stochastic volatility with systemic and idiosyncratic jumps (MSVSIJ) model—I develop a Markov chain Monte Carlo (MCMC) based estimation method and I apply the model to groups of major North American, European, and Asian equity indices.

I find significant evidence of the presence of systemic jumps in international equity index returns. The empirical results indicate that systemic jumps arrive in the North American markets (the US and Canada) approximately one time per year, in the European markets (the UK, Germany, and France) approximately 0.3 times per year, and in the Asian markets (Japan and Hong Kong) approximately 0.9 times per year. The empirical results also indicate that the majority of jumps in the North American markets are systemic. The intensity of idiosyncratic jumps in the US is not significant while that of Canada is significant but smaller than the intensity of systemic jumps. In contrast, the majority of jumps in the European and Asian markets are idiosyncratic. The intensities of idiosyncratic jumps in these markets are significant and larger than the respective intensities of systemic jumps.

I also analyze the distributions of the jump sizes. I find that the mean sizes of systemic jumps are significantly negative for all countries, while, on the other hand, none of the mean sizes of idiosyncratic jumps are significantly different from zero. This implies that counties tend to experience large sudden losses, but not large sudden gains, simultaneously and that the skewness of international equity returns that cannot be explained by leverage effects between returns and volatilities is more likely to be caused by systemic jumps than by idiosyncratic jumps. Somewhat surprisingly, I find that none of the correlation coefficients between the sizes of systemic jumps are significantly different from zero. This implies that, although I find negative downside dependence in the sense that markets tend to be affected by negative jumps simultaneously, returns when such jumps arrive are likely to be close to uncorrelated in terms of deviations from the mean jump sizes. This result differs from the results of Longin and Solnik (2001) who find that simultaneous extreme negative returns display a higher degree of dependence than other returns. I argue (and verify in an informal manner) that the most likely reason for this seeming difference in results is that the approach of this paper and their approach identifies very different sets of returns as "simultaneous extreme negative returns". The approach of this paper focuses on returns that can not be explained as diffusive returns but can be interpreted as systemic jumps. Focusing on such returns is economically relevant since they, as opposed to diffusive returns, tend to catch the market by surprise. The approach of Longin and Solnik (2001), on the other hand, is likely to put more focus on returns that can be explained as diffusive returns inflated by simultaneous high levels of volatility. Although the presence of asymmetries in correlations should not affect the estimation of the intensities of systemic jumps, an extension of the model proposed in this paper to allow also for time-varying correlations between diffusive returns is a potentially relevant challenge for future research.

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A Posterior Distribution

This appendix lists the priors and a subset of the conditional posteriors for the parameters and latent variables used in the MCMC algorithm of Section 3. For the sample sizes used in this paper, the MCMC algorithm is relatively fast if properly implemented. It should be noted, however, that if the MCMC algorithm is not implemented well, the computing time is greatly affected. It should also be noted that the chosen priors and blocks of parameters and latent variables are appropriate for the sample sizes in this paper and may have to be adjusted for larger values of N. In particular, the Metropolis-Hastings algorithms that are suggested may have to be adjusted; for instance by applying them to smaller sub-blocks of parameters and latent variables.

A.1 Likelihood

Equation (2) can be written in matrix form as

$$\begin{pmatrix} Y_{t+1} \\ V_{t+1} \end{pmatrix} = \begin{pmatrix} \mu \\ V_t + \alpha + \bar{V}_t \beta \end{pmatrix} + \begin{pmatrix} e^Y_{t+1} \\ e^V_{t+1} \end{pmatrix} + \begin{pmatrix} X^S_{t+1} J^S_{t+1} + \bar{J}^I_{t+1} X^I_{t+1} \\ \bar{J}^V_{it+1} X^V_{t+1} \end{pmatrix},$$

where μ, α and β are vectors of parameters, $e_t = (e^{Y\,\prime}_{t+1}, e^{V\,\prime}_{t+1})' \sim N(0, \Sigma_t)$, $\Sigma^{YY}_t = \bar{V}^{1/2}_{t-1} R^{YY} \bar{V}^{1/2}_{t-1}$, $\Sigma^{VV}_t = \bar{V}^{1/2}_{t-1} \bar{\sigma} R^{VV} \bar{\sigma} \bar{V}^{1/2}_{t-1}$, $\Sigma^{YV}_t = \bar{V}^{1/2}_{t-1} R^{YV} \bar{\sigma} \bar{V}^{1/2}_{t-1} = (\Sigma^{VY}_t)'$, $\bar{V}_t$, $\bar{J}^I_t$, $\bar{J}^V_t$, and $\bar{\sigma}$ are diagonal matrices with diagonal elements V_t, J^I_t, J^V_t, and $(\sigma_1, \sigma_2, ..., \sigma_N)'$, respectively, and

$$\Sigma_t = \begin{pmatrix} \Sigma^{YY}_t & \Sigma^{YV}_t \\ \Sigma^{VY}_t & \Sigma^{VV}_t \end{pmatrix},$$

$$R = \begin{pmatrix} R^{YY} & R^{YV} \\ R^{VY} & R^{VV} \end{pmatrix}.$$

The likelihood of the data is simply the product of T conditional multivariate normal distributions

$$p(Y|V, J, X, \Theta) \propto \prod_{t=1}^{T} |\Sigma^{Y|V}_t|^{-1} \exp\left\{ -\frac{1}{2} \left(e^Y_t - \varphi^{Y|V}_t e^V_t \right) \left(\Sigma^{Y|V}_t \right)^{-1} \left(e^Y_t - \varphi^{Y|V}_t e^Y_t \right) \right\},$$

where $\varphi^{Y|V}_t = \Sigma^{YV}_t \left(\Sigma^{VV}_t \right)^{-1}$ and $\Sigma^{Y|V}_t = \Sigma^{YY}_t - \varphi^{Y|V}_t \Sigma^{VY}_t$.

A.2 Priors

Whenever possible, I choose conditionally conjugate priors. A conjugate prior is a distribution under which the prior and the corresponding conditional posterior is the same type of distribution but with different hyperparameters. Also, as much as possible, parameters and latent variables that can be suspected to be correlated are collected into blocks. The priors are: $\gamma = (\mu', \alpha', \beta')' \sim N(a, A), a =$

$0\iota_{3N}, A = 25I_N$, $\sigma_i \sim IW(B,b), b = 2.4, B = 0.1$, $\rho_{ij} \sim U(-1,1), i \neq j$, $\lambda^S \sim \beta(o,O)$, $\lambda_i^k \sim \beta(o,O), k = I, V, o = 1, O = 25$, $\mu^S \sim N(d,D), d = 0\iota_N, D = 100I_N$, $\Sigma^S \sim IW(E,e), e = 10\iota_N, E = 40I_N$, $\mu_i^I \sim N(f,F), f = 0, F = 100$, $(\sigma_i^I)^2 \sim IW(G,g), g = 10, G = 40$, $\mu_i^V \sim IW(H,h), h = 10, H = 20$, where ι_N denotes a $N \times 1$ vector of ones and I_N denotes an $N \times N$ identity matrix.

A.3 Conditional Posteriors

All conditional posteriors, except the ones for R, $\sigma_i, i = 1,2,...,N$, and $V_t, t = 1,2,...,T$, are conditionally conjugate and straightforward to derive. For example, it can be showed that the conditional posterior of γ,

$$p(\gamma|Y,V,J,X,\Theta_{(-\gamma)}) \propto p(Y,V|J,X,\Theta)p(\gamma) = \left(\prod_{t=1}^{T} p(Y_t,V_t|V_{t-1},J_t,X_t,\Theta)\right)p(\gamma),$$

is a multivariate normal distribution, $N(a^*,A^*)$, with parameters

$$\begin{aligned} a^* &= A^*\left(\sum_{t=1}^{T} W_t{}'R^{-1}Q_t + A^{-1}a\right), \\ A^* &= \left(\sum_{t=1}^{T} W_t{}'R^{-1}W_t + A^{-1}\right)^{-1}, \end{aligned}$$

and

$$\begin{aligned} Q_t &= \left(\frac{Y_{1t}-X_{1t}^S J_t^S - X_{1t}^I J_{1t}^I}{\sqrt{V_{1t-1}}}, \ldots, \frac{Y_{Nt}-X_{Nt}^S J_t^S - X_{Nt}^I J_{Nt}^I}{\sqrt{V_{Nt-1}}}, \frac{V_{1t}-V_{1t-1}-X_{1t}^V J_{1t}^V}{\sqrt{\sigma_1^2 V_{1t-1}}}, \ldots, \frac{V_{Nt}-V_{Nt-1}-X_{Nt}^V J_{Nt}^V}{\sqrt{\sigma_N^2 V_{Nt-1}}} \right)', \\ W_t &= \begin{pmatrix} \bar{V}_{t-1}^{-\frac{1}{2}} & \iota_N\iota_N' & \iota_N\iota_N' \\ \iota_N\iota_N' & \left(\bar{\sigma}^2\bar{V}_{t-1}\right)^{-\frac{1}{2}} & \bar{V}_{t-1}\left(\bar{\sigma}^2\bar{V}_{t-1}\right)^{-\frac{1}{2}} \end{pmatrix}; \end{aligned}$$

that the conditional posterior of J_t^S, $t = 1,2,...,T$, is a Bernoulli distribution, $Ber(\lambda_t^{S*})$, with Bernoulli probability λ_t^{S*} given by

$$\begin{aligned} & p(J_t^S = 1|Y_t,V_t,V_{t-1},J_t^I,J_t^V,X_t,\Theta) \propto p(Y_t|V_t,V_{t-1},J_t^S = 1,J_t^I,J_t^V,X_t,\Theta)p(J_t^S = 1|\Theta) \\ & \propto \lambda^S \exp\left\{-\frac{1}{2}\sum_{t=1}^{T}\left(Y_t - \mu - X_t^S - \bar{J}_t^I X_t^I - \varphi_t^{Y|V}e_t^V\right)'\left(\Sigma_t^{Y|V}\right)^{-1}\left(Y_t - \mu - X_t^S - \bar{J}_t^I X_t^I - \varphi_t^{Y|V}e_t^V\right)\right\}, \end{aligned}$$

and

$$\begin{aligned} & p(J_t^S = 0|Y_t,V_t,V_{t-1},J_t^I,J_t^V,X_t,\Theta) \propto p(Y_t|V_t,V_{t-1},J_t^S = 0,J_t^I,J_t^V,X_t,\Theta)p(J_t^S = 0|\Theta) \\ & \propto \left(1-\lambda^S\right) \exp\left\{-\frac{1}{2}\sum_{t=1}^{T}\left(Y_t - \mu - \bar{J}_t^I X_t^I - \varphi_t^{Y|V}e_t^V\right)'\left(\Sigma_t^{Y|V}\right)^{-1}\left(Y_t - \mu - \bar{J}_t^I X_t^I - \varphi_t^{Y|V}e_t^V\right)\right\}; \end{aligned}$$

and that the conditional posterior of λ^S,

$$p(\lambda^S|Y, V, J, X, \Theta_{(-\lambda^S)}) \propto p(Y|V, J, X, \Theta)p(\lambda^S),$$

is a beta distribution, $\beta(o^*, O^*)$, with parameters

$$\begin{aligned} o^* &= o + \sum_{t=1}^{T} J_t^S, \\ O^* &= O + T - \sum_{t=1}^{T} J_t^S. \end{aligned}$$

The conditional posterior of R is given by

$$\begin{aligned} p(R|Y, V, J, X, \Theta_{(-R)}) &\propto p(Y, V|J, X, \Theta)p(R) \\ &\propto |R|^{-\frac{T}{2}} \exp\left\{-\frac{1}{2}\sum_{t=1}^{T} u_t{}' R^{-1} u_t\right\} \mathbf{1}_{|R|>0} \prod_{k,\ell=1,k\neq\ell}^{2N} \mathbf{1}_{|\rho_{k\ell}|<1}. \end{aligned}$$

where $u_t = Q_t - W_t\gamma$. This is a highly non-standard distribution, and to sample from it, I use a random walk Metropolis-Hastings algorithm. I draw the error terms in the algorithm from a re-scaled multivariate Student's t distribution with non-diagonal correlation matrix (chosen based on simulation experiments using the inverted Wishart distribution) and standard deviations (also chosen based on simulation experiments using the inverted Wishart distribution) which have been are adjusted so that the acceptance ratio is approximately in the range 0.2-0.3. It should be noted that it generally helps to stabilize the algorithm by replacing $\mathbf{1}_{|R|>0}$ with $\mathbf{1}_{|R|>\delta}$, where $\delta > 0$ is some small number. Alternatively, the method of Chan et al. (2004) can be used.

The conditional posterior of σ_i^2 is given by

$$\begin{aligned} & p(\sigma_i^2|Y, V, J, X, \Theta_{(-\sigma_i^2)}) \propto p(V_i|Y, V_{(-V_i)}, J, X, \Theta)p(\sigma_i^2) \\ & \propto (\sigma_i^2)^{T/2} \exp\left\{-\frac{1}{2}\sum_{t=1}^{T} \frac{\left(V_{it} - V_{it-1} - \alpha_i - \beta_i V_{it-1} - X_{it}^V J_{it}^V - \varphi_t^{V_i|YV_{(-i)}} e_{(-V_i)t}\right)^2}{\Sigma_t^{V_i|YV_{(-i)}}}\right\} \\ & \times (\sigma_i^2)^{\frac{b+2}{2}} \exp\left\{-\frac{1}{2}\frac{B}{\sigma_i^2}\right\}, \end{aligned}$$

where $e_{(-V_i)t} = (e_{1t}, e_{2t}, ..., e_{N+i-1t}, e_{N+i+1t}, ..., e_{2Nt})'$, $\varphi_t^{V_i|YV_{(-i)}} = \Sigma_t^{V_i,YV_{(-i)}}(\Sigma_t^{YV_{(-i)}})^{-1}$, $\Sigma_t^{V_i|YV_{(-i)}} = \sigma_i^2 V_{it-1} - \varphi_t^{V_i|YV_{(-i)}}\Sigma_t^{YV_{(-i)},V_i}$, and $\Sigma_t^{V_i,YV_{(-i)}} = (\Sigma_t^{YV_{(-i)},V_i})'$ denotes the row of covariances of V_{it} with $Y_{it}, i = 1, ..., N$ and $V_{jt}, j = 1, ..., i-1, i+1, ...N$. This is a non-standard distribution because of the correlations between spot variances and between spot variances and returns. However, it is easy to see that a special case arises when V_{it} is uncorrelated to all other volatilities and returns; that is, when $\varphi_t^{V_i|YV_{(-i)}} = 0$ and $\Sigma_t^{V_i|YV_{(-i)}} = \sigma_i^2 V_{it-1}$. In this case the posterior of σ_i^2 reduces to an inverted Wishart distribution which is easy to sample from. For this reason, I use an Independence Metropolis-Hastings algorithm to sample from σ_i^2 where the proposal is drawn from an inverted Wishart distribution, $IW(b^*, B^*)$, with parameters

$$\begin{aligned} b^* &= b + T, \\ B^* &= B + \sum_{t=1}^{T} \frac{\left(V_{it} - V_{it-1} - \alpha_i - \beta_i V_{it-1} - X_{it}^V J_{it}^V\right)^2}{V_{it-1}}. \end{aligned}$$

Finally, the conditional posterior of V_t is given by

$$\begin{aligned} & p(V_t | Y_t, Y_{t+1}, V_{t-1}, V_{t+1}, J_t^S, J_t^I, J_t^V, X_t^S, X_t^I, X_t^V, J_{t+1}^S, J_{t+1}^I, J_{t+1}^V, X_{t+1}^S, X_{t+1}^I, X_{t+1}^V, \Theta) \\ & \propto\ p(Y_t | V_t, V_{t-1}, J_t^S, J_t^I, J_t^V, X_t^S, X_t^I, X_t^V, \Theta) p(Y_{t+1} | V_{t+1}, V_t, J_{t+1}^S, J_{t+1}^I, J_{t+1}^V, X_{t+1}^S, X_{t+1}^I, X_{t+1}^V, \Theta) \\ & \times p(V_t | V_{t-1}, J_t^S, J_t^I, J_t^V, X_t^S, X_t^I, X_t^V, \Theta) p(V_{t+1} | V_t, J_{t+1}^S, J_{t+1}^I, J_{t+1}^V, X_{t+1}^S, X_{t+1}^I, X_{t+1}^V, \Theta) \\ & \propto\ \exp\left\{ -\frac{1}{2} \sum_{\tau=0}^{1} \left(\left(e_{t+\tau}^Y - \varphi_{t+\tau}^{Y|V} e_{t+\tau}^V \right)' \left(\Sigma_{t+\tau}^{Y|V} \right)^{-1} \left(e_{t+\tau}^Y - \varphi_{t+\tau}^{Y|V} e_{t+\tau}^V \right) \right.\right. \\ & \left.\left. +\ e_{t+\tau}^{V\ \prime} \left(\Sigma_{t+\tau}^V \right)^{-1} e_{t+\tau}^V \right) \right\} |\Sigma_{t+1}^{Y|V}|^{-\frac{1}{2}} |\Sigma_{t+1}^{VV}|^{-\frac{1}{2}}. \end{aligned}$$

Since for each t, this distribution is very non-standard, I use a random walk Metropolis Hastings algorithm to simulate from it. I draw the error terms in the algorithm from a re-scaled multivariate Student's t distribution with diagonal correlation matrix, carefully having adjusted the standard deviations to obtain acceptance ratios that are in the range 0.2-0.5. Special attention has to be given to V_0 (the pre-sample spot volatility vector) and to V_T.

B Tables

Table 1: Descriptive Statistics

Descriptive statistics of the country equity indices included in the empirical study. The data consists of daily percentage log returns for each country equity index from February 1, 1985, to April 28, 2004. The mean and standard deviations have been annualized through multiplication by 252 and $\sqrt{252}$, respectively.

	Mean	Std. Dev.	Skewness	Kurtosis	Min.	Max.
US	9.5004	17.4651	-2.0522	46.2513	-22.8997	9.0952
Canada	5.9976	13.8426	-1.1929	20.3630	-11.7948	8.6459
UK	6.6276	15.0760	-0.9387	14.6655	-11.9142	5.6976
Germany	8.2404	23.4879	-0.4566	8.5149	-13.7099	7.5527
France	8.6940	21.3861	-0.2830	7.0183	-9.8945	7.9658
Japan	0.0000	22.5116	0.0318	8.1657	-12.6558	12.4301
Hong Kong	11.5416	27.9264	-3.2788	76.4680	-40.5422	17.2471

Table 2: Parameter Estimates for Simulated Data

MSVSIJ model pverage parameter estimates over ten artificially generated data sets together with the assumed true parameter values. The MCMC algorithms were each run for $M = 100,000$ iterations out of which the first $m = 20,000$ iterations were discarded as burn in. Given in parenthesis are the respective root mean squared errors (RMSE). For ease of readability, only the element below the diagonal of R and R^S are reported.

μ	$\varkappa$	ϑ	σ
		Estimated	
0.0525 (0.0157)	0.0391 (0.0105)	0.5093 (0.0521)	0.1091 (0.0171)
0.1012 (0.0143)	0.0355 (0.0108)	0.4905 (0.0694)	0.1077 (0.0124)
		True	
0.0500	0.0300	0.5000	0.1000
0.1000	0.0300	0.5000	0.1000
μ^S	σ^S	μ^I	σ^I
		Estimated	
-3.4908 (1.4271)	3.8555 (0.6370)	-2.0695 (1.3111)	2.1576 (0.4269)
-3.2793 (1.4241)	3.7339 (0.8764)	-1.5096 (2.2546)	2.1806 (0.4952)
		True	
-3.0000	3.5000	-2.0000	2.5000
-3.0000	3.5000	-2.0000	2.5000
μ^V	λ^S	λ^I	λ^V
		Estimated	
1.2140 (0.3619)	0.0051 (0.0020)	0.0076 (0.0049)	0.0120 (0.0064)
1.1631 (0.2706)		0.0062 (0.0033)	0.0118 (0.0060)
		True	
1.0000	0.0050	0.0050	0.0100
1.0000		0.0050	0.0100
	R		R^S
		Estimated	
0.5073 (0.0115)			0.3413 (0.3919)
-0.4750 (0.0706)	-0.2622 (0.1102)		
-0.2267 (0.1011)	-0.4822 (0.0882)	0.4620 (0.1544)	
		True	
0.5000			0.5000
-0.5000	-0.2500		
-0.2500	-0.5000	0.5000	

Table 3: Parameter Estimates for the US and Canada

MSVSIJ model parameter estimates for the US and Canada. The historical returns are measured in percents on daily basis and cover the period from February 1, 1985, to April 28, 2004; resulting in a sample consisting of $T = 4756$ times $N = 2$ observations. Each parameter is estimated by its sample mean in the posterior sample generated by the MCMC algorithm, and given in parenthesis are the corresponding sample posterior standard deviations. The MCMC algorithm was run for $M = 200,000$ iterations, out of which the first $m = 20,000$ iterations were discarded as burn in. For ease of readability, only the element below the diagonals of R and R^S are reported.

μ	$\varkappa$	ϑ	σ
0.0408 (0.0122)	0.0183 (0.0031)	0.9619 (0.1038)	0.1257 (0.0109)
0.0452 (0.0089)	0.0264 (0.0034)	0.4596 (0.0500)	0.1056 (0.0076)
μ^S	σ^S	μ^I	σ^I
-4.2368 (1.2809)	3.2494 (0.8998)	0.4941 (5.0831)	2.1647 (0.5608)
-3.2869 (1.0053)	2.5463 (0.4781)	-1.7696 (1.4448)	2.3127 (0.5326)
μ^V	λ^S	λ^I	λ^V
2.2666 (0.8704)	0.0039 (0.0013)	0.0007 (0.0009)	0.0012 (0.0009)
1.7218 (0.6153)		0.0024 (0.0017)	0.0044 (0.0022)
	R		R^S
0.6612 (0.0154)			0.4928 (0.2368)
-0.5365 (0.0661)	-0.2864 (0.0747)		
-0.4046 (0.0636)	-0.2114 (0.0728)	0.8969 (0.0263)	

Table 4: Parameter Estimates for the UK, Germany, and France

MSVSIJ model parameter estimates for the UK, Germany, and France. The historical returns are measured in percents on daily basis and cover the period from February 1, 1985, to April 28, 2004; resulting in a sample consisting of $T = 4728$ times $N = 3$ observations. Each parameter is estimated by its sample mean in the posterior sample generated by the MCMC algorithm, and given in parenthesis are the corresponding sample posterior standard deviations. The MCMC algorithm was run for $M = 200,000$ iterations, out of which the first $m = 20,000$ iterations were discarded as burn in. For ease of readability, R is separated into its sub-matrices R^{YY}, R^{VV}, and R^{YV}, and only the element below the diagonals of these matrices and R^S are reported.

μ	$\varkappa$	ϑ	σ	μ^S	σ^S	μ^I
0.0267 (0.0109)	0.0252 (0.0034)	0.7025 (0.0498)	0.1019 (0.0063)	-5.3918 (1.6150)	2.6895 (0.6175)	0.0115 (0.9877)
0.0514 (0.0160)	0.0233 (0.0035)	1.6957 (0.1611)	0.1790 (0.0124)	-9.6906 (1.6449)	2.4563 (0.7166)	2.2258 (1.6170)
0.0275 (0.0164)	0.0304 (0.0042)	1.4712 (0.1213)	0.1609 (0.0117)	-6.9161 (1.6673)	2.2784 (0.5874)	0.1198 (0.7550)
σ^I	μ^V	λ^S	λ^I	λ^V	R^S	
3.1986 (0.7344)	2.5609 (1.1290)	0.0012 (0.0005)	0.0036 (0.0017)	0.0007 (0.0005)	-0.1500 (0.3069)	
2.0590 (0.4209)	2.4494 (1.0181)		0.0045 (0.0046)	0.0018 (0.0012)	0.0648 (0.3428)	0.0598 (0.3059)
1.5975 (0.3388)	2.5696 (1.0102)		0.0156 (0.0136)	0.0025 (0.0015)		
R^{YY}		R^{VV}		R^{YV}		
0.5663 (0.0119)		0.7709 (0.0413)		-0.4432 (0.0547)	-0.4700 (0.0504)	-0.3258 (0.0490)
0.6708 (0.0105)	0.6614 (0.0108)	0.7280 (0.0488)	0.8194 (0.0347)	-0.3112 (0.0515)	-0.4014 (0.0464)	-0.3625 (0.0525)
				-0.3567 (0.0543)	-0.3793 (0.0659)	-0.4208 (0.0563)

Table 5: Parameter Estimates for Japan and Hong Kong

MSVSIJ model parameter estimates for Japan and Hong Kong. The historical returns are measured in percents on daily basis and cover the period from February 1, 1985, to April 28, 2004; resulting in a sample consisting of $T = 4532$ times $N = 2$ observations. Each parameter is estimated by its sample mean in the posterior sample generated by the MCMC algorithm, and given in parenthesis are the corresponding sample posterior standard deviations. The MCMC algorithm was run for $M = 200,000$ iterations, out of which the first $m = 20,000$ iterations were discarded as burn in. For ease of readability, only the element below the diagonals of R and R^S are reported.

μ	$\varkappa$	ϑ	σ
0.0263 (0.0152)	0.0241 (0.0031)	0.9701 (0.1864)	0.1458 (0.0159)
0.0820 (0.0183)	0.0622 (0.0046)	1.2778 (0.1056)	0.1966 (0.0253)
μ^S	σ^S	μ^I	σ^I
-3.3649 (1.0813)	2.2851 (0.5094)	0.7377 (0.9367)	2.0217 (0.3647)
-6.6823 (1.8788)	3.1869 (0.9859)	0.8943 (1.5068)	2.0301 (0.5049)
μ^V	λ^S	λ^I	λ^V
1.4903 (0.4964)	0.0035 (0.0017)	0.0160 (0.0097)	0.0133 (0.0062)
10.5538 (2.7472)		0.0072 (0.0073)	0.0083 (0.0024)
	R		R^S
0.3207 (0.0152)			-0.0049 (0.3226)
-0.6366 (0.0732)	-0.2200 (0.0731)		
-0.2260 (0.0695)	-0.3410 (0.0612)	0.3522 (0.1023)	

C Figures

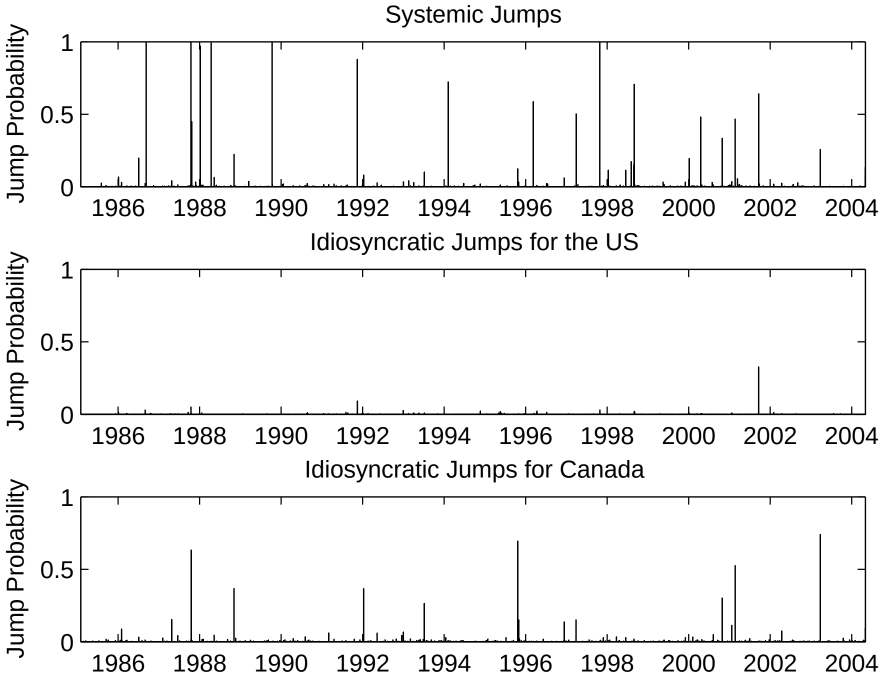

Figure 1: Posterior jump probabilities of the latent historical jump times of systemic and idiosyncratic jumps for the US and Canada.

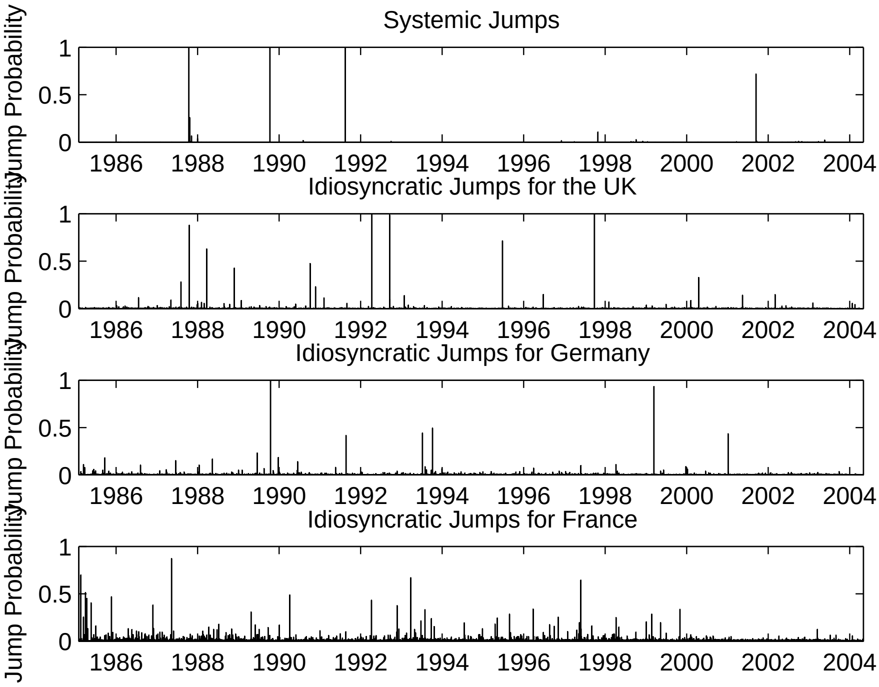

Figure 2: Posterior jump probabilities of the latent historical jump times of systemic and idiosyncratic jumps for the UK, Germany, and France.

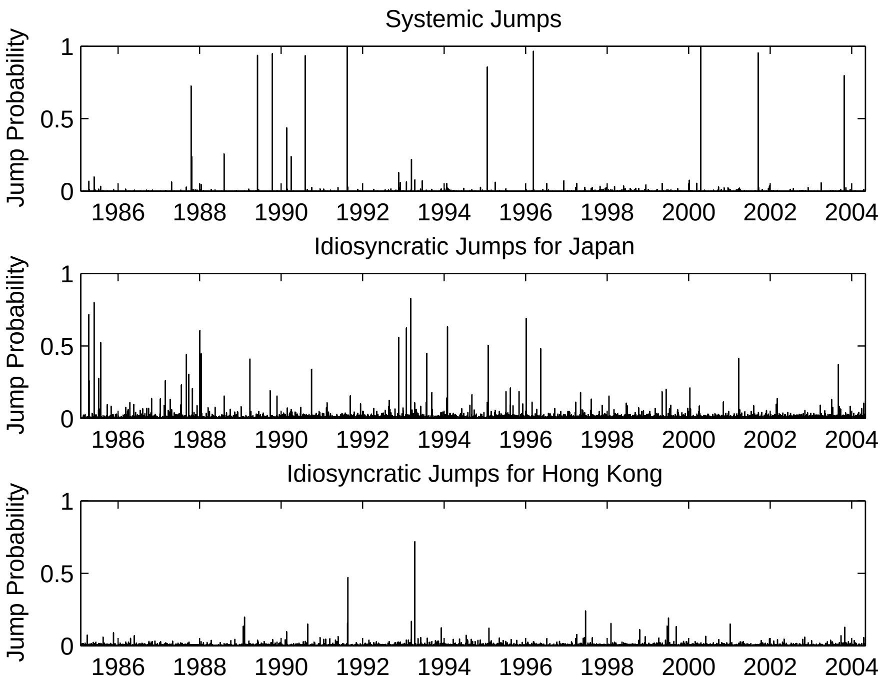

Figure 3: Posterior jump probabilities of the latent historical jump times of systemic and idiosyncratic jumps for Japan and Hong Kong.