

# Hunting the Living Dead

## A “Peso Problem” in Corporate Liabilities Data

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### Abstract

Recent literature has pointed out that information asymmetries may be the reason for the poor performance of structural credit risk models to fit corporate bond data. It is well known in fact that these models lead to a strong understatement of the credit spread terms structure, particularly on the short maturity end. Possible explanations stem from strategic debt service behavior and, as discovered more recently, the problem of accounting transparency. This raises the possibility that some of these flaws could be reconducted to a sort of “peso problem”, i.e. that the market may ask for a premium in order to allow for a small probability that accounting data may actually be biased (Baglioni and Cherubini, 2005). In this paper we propose a modified version of the Duan (1994,2000) MLE approach to structural models estimation in order to allow for this “peso problem” effect. The model is estimated for the Parmalat case, one of the most famous cases of accounting opacity, using both equity and CDS data.

**Preliminary and Incomplete.**

## 1 Introduction

Structural models of credit risk are considered a very elegant approach to the evaluation of corporate liabilities. Elegance stems from the fact that prices are obtained from the analysis of the structure of the balance sheet of the obligor firm and the dynamics of its assets. The main advantage is that these models are full of economic information content, while the so-called “reduced form” models are only based on statistical assumptions concerning the probability distribution of default events and the recovery rate, that is the amount that the investor expects to recover in case of default. Of course, the richer economic content in structural models comes at the cost of a loss of flexibility with respect to reduced form models, and of a poorer fit to market data.

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Structural models are reconducted to the seminal paper by Merton (1974), even though the famous Black and Scholes (1973) model was already devoted to the evaluation of corporate liabilities, as was explicitly recognised in the title. The key idea in structural models is that corporate liabilities, such as equity and debt, are actually positions in options. So, equity can be considered a call option written on the assets of the firm with a strike price equal to the face value of debt and the credit risk component of corporate bonds can be thought of as short position in a put option with same underlying and strike, the so called default put option. A first typical flaw of structural models of credit risk is that predicted credit spread are much lower than market quotes for reasonable values of leverage and volatility of assets. Several answers have been proposed as possible solutions to this problem. Anderson and Sundaresan (1996) suggest that the owner of the firm may engage in a strategic rescheduling process to exploit the bankruptcy costs at the expense of bondholders. Along the same lines, Leland (1994) and Leland and Toft (1996) allow the owner of the firm to terminate the process in such a way as to optimize the value of equity, again at the expense of debt.

An alternative explanation for the failure of structural models to fit the data stems from the fact that the value of the firm is not directly observed and this lack of transparency may affect market prices. In this spirit Cherubini and Della Lunga (2001) propose a conservative assessment of the probability of default by using a default probability interval, in line with the MaxiMin-Expected-Utility framework in Gilboa and Schmeidler (1989). However, this approach is not able to account for another typical flaw of structural models: the strong understatement of credit spreads for short maturities. A typical credit spread term structure in the Merton model shows a hump and zero intercept. The latter feature is particularly disturbing and it is due to the main assumption on which the model was built, that is the representation of the value of the firm as an adapted diffusion process. The need to account for higher credit spreads for shorter maturities can be achieved either by allowing for a jump process in the value of the firm (Zhou, 2001), that is dropping the diffusion process assumption, or by relaxing the adapted process hypothesis. The latter route was first followed by Duffie and Lando (2001) (see Yu, 2005 for empirical evidence), who propose a model with endogenous bankruptcy in which the market is assumed to observe a noisy signal of the value of the firm at discrete times, namely when balance sheet reports are released. An approach in the same spirit is followed by Cherubini and Baglioni (2005) who account for the fact that the signal may not only be noisy due to measurement errors, but it may also be biased because of deliberate fraud. In their model rational Bayesian investors would account for this possibility by subtracting part of the value released from the report, and the value of corporate liabilities is a mixture of the values they can take both in the case of presence and absence of fraud.

The fact that the value of the firm is not directly observed has also given rise to a stream of literature on the estimation of the relevant parameters of the model from market data. Actually, the real peculiarity of this application of the option pricing model stems from the fact that the derivative products

(equity stock, for example) are traded on liquid markets and their prices can be observed in real time, while the underlying is neither traded or observed. So, the problem is not only to estimate implied volatility as in standard option pricing applications, but also the implied value of the underlying asset, the value of the firm in this case. In his pioneer work Duan (1994) proposed a ML procedure on transformed data to address this problem. Further elaborations on this subject were provided by Ericsson and Reneby (2003), Brockman and Turtle (2003), Duan, Gauthier and Simonato (2004), Bruche (2004). An alternative line of research has used a different approach, based on simultaneous calibration of the value of assets and their volatility on a non-linear two equation system of the value of equity and its volatility (Vassalou and Xing, 2002); this iterative method can be easily extended to account for default before maturity, leading to the standard KMV approach (Crosbie, 2002).

The goal of this paper is to investigate the issue of empirical performance of structural models of credit risk. A first question is whether all information should be only found in equity prices, as it is implicitly assumed in most of the literature quoted above. In the same line as in Bruche (2004) we try to estimate structural models on both equity prices and the credit spread. Of course, the main reason why most of the attempts at estimating structural models use equity data is because it is well known, as we said before, that this approach is not borne out by credit spread data. The second question is then whether recent models on uncertain information are able to provide a better fit to credit spread data. In particular, we look for support to the idea of a “peso problem” in corporate securities data, as predicted in the Baglioni and Cherubini (2005) model quoted above: the market may assign some small probability to the event that the balance sheet reports of a firm, and its whole “investor relationship” style, could be misleading, and the firm is already bankrupt, in spite of any favorable report or analyst presentation. It is reasonable to expect that the market may require a premium for that. To explain the joke in the title, some firms and obligors may be “leaving dead” wandering around issuing debt, and when they are proved to be acutally dead they may bite investors to death. Examples are the Enron and Worldcome cases (to quote only the most famous) in the US and Parmalat in Europe. It is from the equity and CDS data of Parmalat, throughout two years before the final crisis, that we look for an answer to this question.

The plan of the paper is as follows. Section 2 reviews structural models, particularly those that will be used in the empirical work. In section 3 we review the MLE approach to the problem and we apply it to the different models and to the different corporate securities (equity and debt). In section 4 we present the data set used and the results obtained. Section 5 concludes.

## 2 Structural models

Let us describe the basic structure of a corporate securities model. At time  $t_0$  a firm is issuing debt to finance a project that will be completed at time  $T$ , when

it will be worth  $V(T)$ . It is assumed that the value of assets follows a geometric brownian motion

$$dV(t) = \mu V(t) dt + \sigma V(t) dz \quad (1)$$

with  $\mu$  and  $\sigma$  constant drift and diffusion parameters and  $dz$  a Wiener process. This means that  $V(T)$  is log-normally distributed. We also rule out estimation risk and model risk, assuming that the parameters of the process are common knowledge, and that the no-arbitrage condition holds. Then, both the volatility parameter  $\sigma$  and the market price of risk  $\lambda$  are assumed to be common knowledge, so that the drift of the assets is recovered from the usual no-arbitrage restriction  $\mu = r + \lambda\sigma$  where  $r$  is the instantaneous risk-free rate, assumed to be constant.

## 2.1 Default at maturity: Merton model

In the seminal paper by Merton (1974) default is assumed to be possible only at the end of the contract, when the value of assets  $V(T)$  is observed. Defaultability of debt is then represented by the non-linear pay-off at maturity  $D(T) = \min(D, V(T))$ , meaning that if the value of the firm is not sufficient to cover repayment of the debt, the creditors will be allowed, only then and not before, to take over the firm at no cost. The value of corporate debt can be decomposed as  $D(T) - \max(D - V(T), 0)$ , that is a default-free bond and a short position in a put option, written on the asset for a strike equal to the nominal value of debt: it is this short position, also called default put option, that measures the default risk in the price. The call option with same underlying and strike  $E(T) = \max(V(T) - D, 0)$  represents the value of equity. It may be verified that Modigliani-Miller theorem holds in this setting.

A clarifying note is in order about the notation. Merton rescaled all of the results in the model by the value of assets, leading to the definition of a key variable, called the quasi-debt-to-firm value ratio (or quasi-leverage) defined as

$$\gamma(t) = \frac{\exp(-r(T-t)D)}{V(t)} \quad (2)$$

Here we find more natural to rescale everything by the discounted value of debt (the quasi-value of debt), so that as the underlying asset (the value of the firm) we prefer to use  $v(t) \equiv 1/\gamma(t)$ : all prices expressed in lower case will be assumed to be rescaled in the same way.

The value of equity and debt is recovered using the standard Black and Scholes formula

$$e(v(t_k), t_k) \equiv \frac{E(v(t_k), t_k)}{\exp(-r(T-t_k))D} = v(t_k)N(d_1) - N(d_2) \quad (3)$$

$$d(v(t_k), t_k) \equiv \frac{D(v(t_k), t_k)}{\exp(-r(T-t_k))D} = 1 + v(t_k)N(-d_1) - N(-d_2) \quad (4)$$

$$\begin{aligned}
d_1 &= \frac{\ln(v(t_k)) + \sigma^2(T-t)}{\sigma\sqrt{T-t}} \\
d_2 &= d_1 - \sigma\sqrt{T-t}
\end{aligned}$$

Notice that the value of debt can also be represented as

$$d(v(t_k), t_k) = 1 - [-v(t_k)N(-d_1) + N(-d_2)] \quad (5)$$

emphasising the nature of credit risk as a short position in a put option. It is the value of this put option that we will use in the empirical work.

## 2.2 Covenants: Black and Cox model

An important extension of the Merton model, particularly consistent with the assumption that some imperfect signals of the value of the firm can be observed before the maturity of debt, is the possibility that default could occur before that date. Black and Cox (1976) were the first to amend the model in this direction. The idea is that default may occur before maturity if some covenant written on debt is triggered. The covenant is typically referred to the relative size of the value of the firm with respect to the amount of debt. Following the Black and Cox approach, the covenant in its simplest form is represented as the inequality

$$\hat{v}(t_k) \leq \kappa \leq 1 \quad (6)$$

So, when the value of the firm is signalled to be too low with respect to the discounted value of debt, default is triggered. The presence of covenants, of course, reduces the default risk component of debt and the credit spreads. The reduction of risk depends on the value of parameter  $\kappa$ ; the model can be proved to converge to the standard Merton model as  $\kappa$  gets close to zero. If the value of the firm were perfectly observed in continuous time, as assumed in the original model, the covenant would tend to eliminate the risk of default as  $\kappa$  gets close to 1. Of course, given the parameter, the effect of default risk reduction also depends on the monitoring frequency and the information content of the signal.

A comment is in order on the differences with respect to the Merton model and the impact of our assumption of observing the signal at discrete times. If the covenant could be monitored in continuous time, the value of equity would be a call barrier option of the down-and-out type with zero rebate: that is, the option granted by the equity would cease to exist as soon as the default barrier were activated. In the case of continuous monitoring of the covenant the pricing formula for equity is readily available in the standard option pricing literature

$$\begin{aligned}
e(v(t_k), t_k; \kappa) &= e(v(t_k), t_k) \\
&\quad - \left[ v(t_k) \left( \frac{\kappa}{v(t_k)} \right)^{2\alpha} N(\xi) - \left( \frac{\kappa}{v(t_k)} \right)^{2\alpha-2} N\left(\xi - \sigma\sqrt{T-t_k}\right) \right]
\end{aligned} \quad (7)$$

$$\begin{aligned}\xi &= \frac{\ln(\kappa^2/v(t_k)) + \sigma^2(T-t)}{\sigma\sqrt{T-t}} \\ \alpha &= \frac{1}{2} + \frac{r}{\sigma^2}\end{aligned}$$

A more realistic assumption is instead that the barrier could be monitored, through the signal, only at discrete dates  $\{t_0, t_1, \dots, t_N\}$ . This makes equity a discrete barrier option in which typically the barrier is observed at fixed intervals of time, say every quarter or every semester. A closed form solution to this pricing problem was found by Heynen and Kat (1996). However, evaluation involves the computation of joint normal distributions in dimension  $N + 1$ , which is not available in closed form. For this reason, it may be useful to resort to approximations suggested in the literature. Broadie, Glasserman and Kou (1997) propose a strategy based on the displacement of the barrier in the formula above: so, denoting  $\tau$  the time interval between monitoring dates, they suggest

$$\begin{aligned}e(v(t_k), t_k; \kappa, \tau) &\simeq e(v(t_k), t_k; \tilde{\kappa}) \\ \tilde{\kappa} &\equiv \kappa \exp(-0.5826) \sigma \sqrt{\tau}\end{aligned}\tag{8}$$

### 2.3 A “peso problem” model: structural models with garbling

Recent literature has extended this model to account for the fact that the value of the firm, that is the underlying asset of the options involved in structural models (equity and the default put option), is not observed in continuous time. These models typically depart from standard literature in two ways. First, information on the value of assets is assumed to be available at discrete times  $\{t_0, t_1, \dots, t_N\}$ , that is when the balance sheet reports are issued. Second, the value of assets is not directly observed by the market, but it must be inferred from a garbled signal  $s(t_k)$ ,  $k = 0, 1, 2, \dots, N$ . Only at the final date  $T$ , or when a default event occurs, the value of the firm will be observed. Before that, the signal may be simply “noisy” due to imperfect observation (Duffie and Lando, 2001), or it may even be distorted because of fraudulent behavior by firm’s managers (Baglioni and Cherubini, 2005). Here in particular we focus on the latter possibility.

To give a more clear description of the idea, consider the possibility that a signal  $s(t_k) > 1$  be issued, even though the true state is  $v(t_k) < 1$ , to hide a possible state of financial distress at the advantage of management careers, or even the entrepreneur private wealth (as in the Parmalat case). It may also happen that signal  $s(t_k) < 1$  be issued, while in the true state it is  $v(t_k) > 1$ , for example to solicitate debt rescheduling. In the Baglioni and Cherubini approach Bayesian rational investors accounting for this possibility update their prior probability evaluation leading to price reactions smaller than those predicted by the full information model. In particular, their model shows that the price

can be obtained as a mixture of different states, under which the full information value of the securities is computed.

While referring the reader to the original work for the description of the structure of signal used and the consequent derivation of the model, we want to focus here on its intuitive content. Suppose a signal  $s(t_k) = h > 1$  is released. Conditional on the true value being actually greater than debt, the signal is assumed to be precise. However, we assume that some probability is given to the event that the report is false, and the true state is  $v(t_k) \leq 1$  instead. This probability is indeed recovered as a posterior probability from a Bayesian updating process.

As a result, the value of equity following the release of the news is

$$\widehat{e}(v(t_k), t_k | h) = f(v(t_k) > 1 | h) e(h, t_k) + (1 - f(v(t_k) > 1 | h)) \bar{e}(v(t_k) \leq 1, t_k) \quad (9)$$

where  $f(v(t_k) > 1 | h)$  is the probability that the value of asset is actually greater than debt (or the default boundary), so that the report is truthful.  $\bar{e}(v(t_k) \leq 1, t_k)$  represents instead the expected value of equity, conditional on the report being false: actually, this value would be zero in a standard structural model with covenants such as the Black and Cox model described above, but it needs not be zero in the presence of default at maturity or bankruptcy costs and debt rescheduling possibility. The same holds for the value of debt, which gives

$$\widehat{d}(v(t_k), t_k | h) = f(v(t_k) > 1 | h) d(h, t_k) + (1 - f(v(t_k) > 1 | h)) \bar{d}(v(t_k) \leq 1, t_k) \quad (10)$$

Notice that now  $\bar{d}(v(t_k) \leq 1, t_k)$  represents a proxy for the recovery rate implied in the price: if the report were suddenly discovered to be false and the firm were in default, the value of debt would actually boil down to the recovery rate.

Finally, the same holds for the value of the firm. Remember in fact that by Modigliani-Miller theorem we have  $e(h, t_k) + d(h, t_k) = h$  and  $\bar{e}(v(t_k) \leq 1, t_k) + \bar{d}(v(t_k) \leq 1, t_k) = \bar{v}(v(t_k) \leq 1, t_k)$  so that we may compute

$$\widehat{v}(v(t_k), t_k | h) = f(v(t_k) > 1 | h) h + (1 - f(v(t_k) > 1 | h)) \bar{v}(v(t_k) \leq 1, t_k) \quad (11)$$

Notice that, as both equity and debt are worth less in the bad state than in the good one, the effect of garbling is to prevent equity and debt from reacting completely to the announcement of a value  $v(t_k) = h$ . There is always a small probability that the good signal be deceptive, so that the worse scenario does actually take place. This possibility endows the model with the usual peculiarity of raising the credit spread curve particularly in the short end, as in the Duffie and Lando approach.

### 3 Maximum likelihood estimation

Maximum likelihood estimation of the dynamics of the underlying asset in cases where it cannot be directly observed on the market was first proposed by Duan (1994,2000). Let us focus on our specific problem. To begin with the simple case, assume that a sample of observations on the asset value of the firm was directly observed on the market at discrete dates  $\{t_1, t_2, \dots, t_N\}$ . If its dynamics is that of a geometric brownian motion

$$dV(t) = \mu V(t) dt + \sigma V(t) dz \quad (12)$$

we know that the conditional distribution is

$$\ln \left( \frac{V(t_i)}{V(t_{i-1})} \right) \sim N \left( (\mu - 1/2\sigma^2) \delta_i, \sigma^2 \delta_i \right) \quad (13)$$

with  $\delta_i = t_i - t_{i-1}$ ,  $i = 2 \dots N$ . The log-likelihood in this case would then be

$$\begin{aligned} L_V(V(t_i), i = 1, 2 \dots N, \mu, \sigma) &= -\frac{N-1}{2} \ln(2\pi) - \frac{N-1}{2} \ln \sigma^2 - \sum_{i=2}^N \ln V(t_i) \\ &\quad - \frac{1}{2} \sum_{i=2}^N \left[ \ln \left( \frac{V(t_i)}{V(t_{i-1})} \right) - \mu \delta_i \right]^2 \end{aligned} \quad (14)$$

Notice that we could impose further economic restrictions in the model. The no-arbitrage condition requires in fact that  $\mu = r + \lambda\sigma$ . This restriction could be important in a panel data estimation of the model, but even in our case could give an idea of a plausible range of values for the drift: this is very useful mainly because it is well known that one of the flaws of these model is represented by the imprecise estimation of the drift term. Of course, the likelihood could be written in the same way for the value of the firm rescaled in terms of discounted value of debt,  $v(t_i)$ , as in the analysis before: one has only to keep in mind that the restriction implied by no-arbitrage in this case would be  $\mu = \lambda\sigma$ .

Let us now get to the heart of the matter and assume that the value of the firm is not observed directly, but can only be evaluated through a transformation  $g(v(t_i), t_i)$ . Denote by  $\Delta_g$  the partial derivative of the function with respect to variable  $v(t_i)$ . It may be proved that in this case the likelihood can be written as

$$\begin{aligned} L(g(v(t_i), v(t_i)), i = 1, 2 \dots N, \mu, \sigma) &= -\frac{N-1}{2} \ln(2\pi) - \frac{N-1}{2} \ln \sigma^2 \quad (15) \\ &\quad - \sum_{i=2}^N \ln v^*(t_i, \sigma) - \sum_{i=2}^N \ln |\Delta_g(t_i, \sigma)| \\ &\quad - \frac{1}{2} \sum_{i=2}^N \left[ \ln \left( \frac{v^*(t_i, \sigma)}{v^*(t_{i-1}, \sigma)} \right) - \mu \delta_i \right]^2 \end{aligned}$$

where  $v^*(t_i, \sigma)$  is the value of assets implied in the value of  $g(v(t_i), t_i)$  given that the volatility is  $\sigma$ . In the typical application we would set  $g(v(t_i), t_i) = e(v(t_i), t_i)$  and the values used to estimate the model are equity prices. In this case, sticking to the standard Merton model for simplicity, we have  $\Delta_g = N(d_1)$ . Nowadays for many firms an appraisal of the credit risk is also available from very liquid markets, such as some credit derivatives market. From CDS quotes, for example, it is possible to estimate the market value of debt and the corresponding default put option. In this case, if one observe an estimate of  $g(v(t_i), t_i) = d(v(t_i), t_i)$  he can compute  $\Delta_g = N(-d_1)$ . Using a shorthand notation one could then write

$$\begin{aligned}
L(g(v(t_i), v(t_i)), i = 1, 2 \dots N, \mu, \sigma) &= -\frac{N-1}{2} \ln(2\pi) - \frac{N-1}{2} \ln \sigma^2 & (16) \\
&- \sum_{i=2}^N \ln v^*(t_i, \sigma) - \sum_{i=2}^N \ln N(\mathbf{1}d_1(t_i, \sigma)) \\
&- \frac{1}{2} \sum_{i=2}^N \left[ \ln \left( \frac{v^*(t_i, \sigma)}{v^*(t_{i-1}, \sigma)} \right) - \mu \delta_i \right]^2
\end{aligned}$$

where  $\mathbf{1}$  is an indicator function taking value 1 if the data observed is equity and - 1 if the data debt.

Finally, the ‘‘peso problem’’ effect that was addressed above would imply

$$\widehat{\Delta}_g(v^*(t_i), t_i) = fN(\mathbf{1}d_1(t_i, v^*(t_i))) + (1-f)\overline{N}(\mathbf{1}d_1(t_i, v(t_i)) \leq 1) \quad (17)$$

where  $\overline{N}(\mathbf{1}d_1(t_i, v(t_i)) \leq 1)$  is the integral of the derivative over the region  $v(t_i) \leq 1$ .

## 4 An application to the Parmalat case

In this section we apply the methodology above to estimate the value of Parmalat assets and their volatility, allowing for the market to give some probability to the event of fraud. Actually, in the wake of the scandal in December 2003 the debate was focussing on the fact that Parmalat had been traditionally following an investor relationship policy particularly opaque. It is then interesting to address the question whether this lack of transparency was actually priced by the market. Furthermore, Parmalat was a listed company, and its stock had been included in the Italian blue-chips index, so that information about it was processed in a very liquid market. Finally, as one of the most relevant obligors, Parmalat was among those ‘‘names’’ on which credit risk was actively traded on the CDS market. Obviously, the latter is more a ‘‘wholesale’’ market than that of equity. It is then interesting to check whether the two markets did share the same information. This is all the more relevant also in view of the lively

discussion on the question whether banks were actually aware of the financial distress situation of Parmalat.

## 4.1 The data

We collected data on equity and 5-year CDS mid-quote for the years 2002-2003. The data is depicted in figure 1. The data show the story of the two “hot” years of Parmalat. Both the markets record the confidence crisis in February 2003, when a new bond issue by Parmalat was hastily withdrawn from the market, and when analysts reports were highlighting the mystery of Parmalat liquidity endowment. After a period in which the situation calmed down, so that both the stock prices and the CDS quotes move back toward their standard values, the new final crisis outbroke at the end of November. On December 8, Parmalat defaulted on a 150 million euro bond. The day after, Standard and Poors downgraded Parmalat to junk status. Finally, on December 19 the story of default turned into one of outright fraud when Bank of America announced that 3.9 million dollars of liquidity that Parmalat claimed to have deposited in that band, were not simply there.

From this data, using the quantity of stock and the value of debt reported in the balance sheet, we computed the time series of our data of interest. As we have no information concerning the term structure of debt, we assume it to be all on the five year maturity, and we evaluate its market value by discounting it by the 5-year swap rate increased by the CDS spread. The idea behind this assumption is that actually one could have actually swapped the whole credit risk on the Parmalat debt at the cost of that spread. This is clearly an approximation, both because not all the debt was actually on the five year maturity, and because there is evidence that this arbitrage relationship is not perfect at all. In figure 2 we plot the value of quasi-debt, the estimated value of debt, their difference, that is the default put option and the value of equity capital.

## 4.2 Estimates and results

The strategy we follow is to estimate drift and volatility of the market value of Parmalat by the maximum likelihood procedure with transformed data discussed above, assuming two different models: i) the standard Merton model, ii) the Merton model with garbling. The analysis is carried out both on a subsample stopped at the default date (December 8) and on the whole sample reaching the end of December. In the model with garbling, the probability of fraud is estimated along with the drift and volatility of the stochastic process. The model is estimated on equity data and then checked checked against the value of CDS spreads: this will enable to check whether the market for “protection” on the name Parmalat was quoting prices that were actually more expensive than those implied by stock prices.

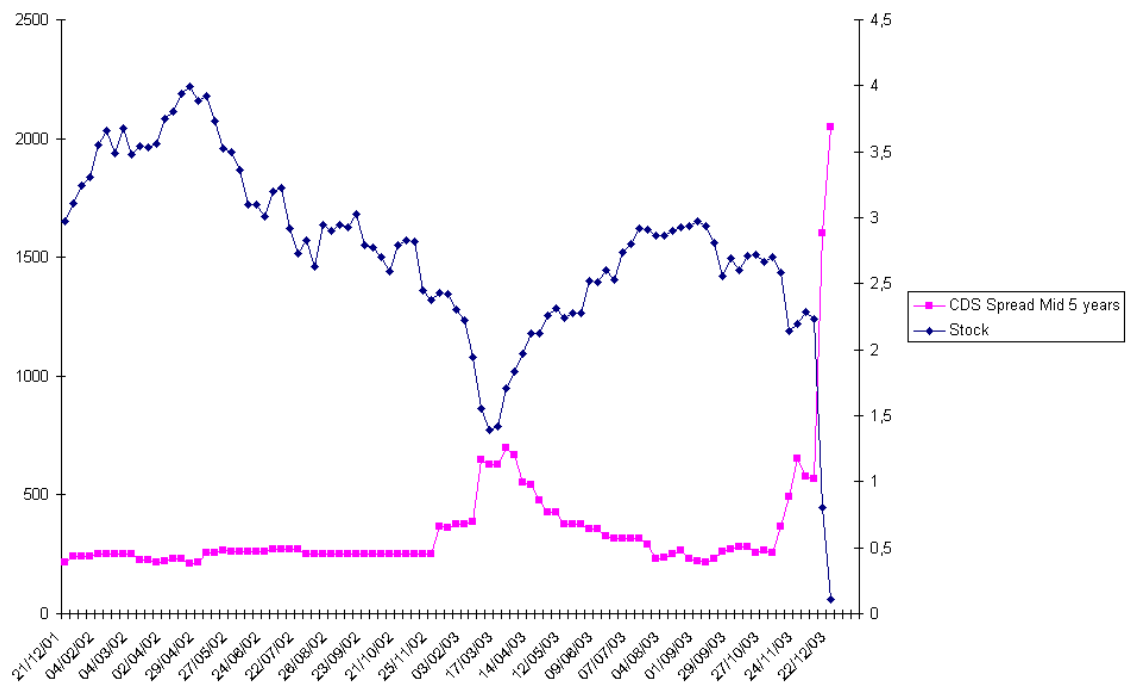


Figure 1:



Figure 2:

Table. Parmalat firm value: MLE estimates			
Sample	Drift	Volatility	$f(v(t_k) > 1   h)$
08/12/2003	-0.0099651 (0.0549867)	0.0995914 (0.0079003)	—
08/12/2003	-0.016538 (0.0719129)	0.11472252 (0.0315613)	0.80006038 (0.3077854)
22/12/2003	-0.172803 (0.146902)	0.18192321 (0.0367712)	0.9999998 (0.2660050)

The estimates show that fraud probability plays an important role in the specification of the model. In the subsample, the stock price dynamics implies a 20% probability of fraud. If the estimate is instead carried out on the whole sample, this probability is found to be zero because the default event did actually take place. The estimate of volatility of the assets is similar in both the estimates of the subsample, around 10-11%: this is much lower, and more realistic than the estimates one would obtain by using the value of the firm computed adding equity and the appraised market value of debt. The estimate of drift is not statistically significant as it is the usual to find in this kind of estimates.

The estimates show a typical case of “peso problem”: before the default event on December 8, the market was assigning some probability to the event that such default could occur. What is actually surprising is that this 20% probability is substantial with respect to that one would expect in a “peso problem” case. In line with this effect, instead, the probability disappears as soon as the event is included in the sample. But more surprises have yet to come. The biggest one is in figure 3. Here we compare the credit risk premium implied by the CDS quotes, and reported in terms of difference between the value of quasi-debt and its estimated market value, and the premium predicted by the two models estimated. The picture shows that the standard Merton model largely underestimates the value of the credit risk premium, as it is usual in structural model applications. It looks like there would have been unexploited arbitrage opportunities in the CDS market versus the equity market, or else that the two markets have been segmented, carrying different information. Allowing for a “peso problem” in the estimates reverses this evidence, and these arbitrage opportunities disappear: in most of the sample the credit risk premium implied in equity prices is even higher than that implied by CDS quotes.

Another striking result emerges from figure 4, in which we depict the implied value of the firm obtained from the estimates. The value of Parmalat is the linear combination of a higher state value and the lower state in the event of mis-reporting and fraud. It is really surprising that the value of the firm in the lower state is actually almost constant around a value which is 20% of the value of quasi-debt: this figure is actually very close to that reportedly found in the Italian market.

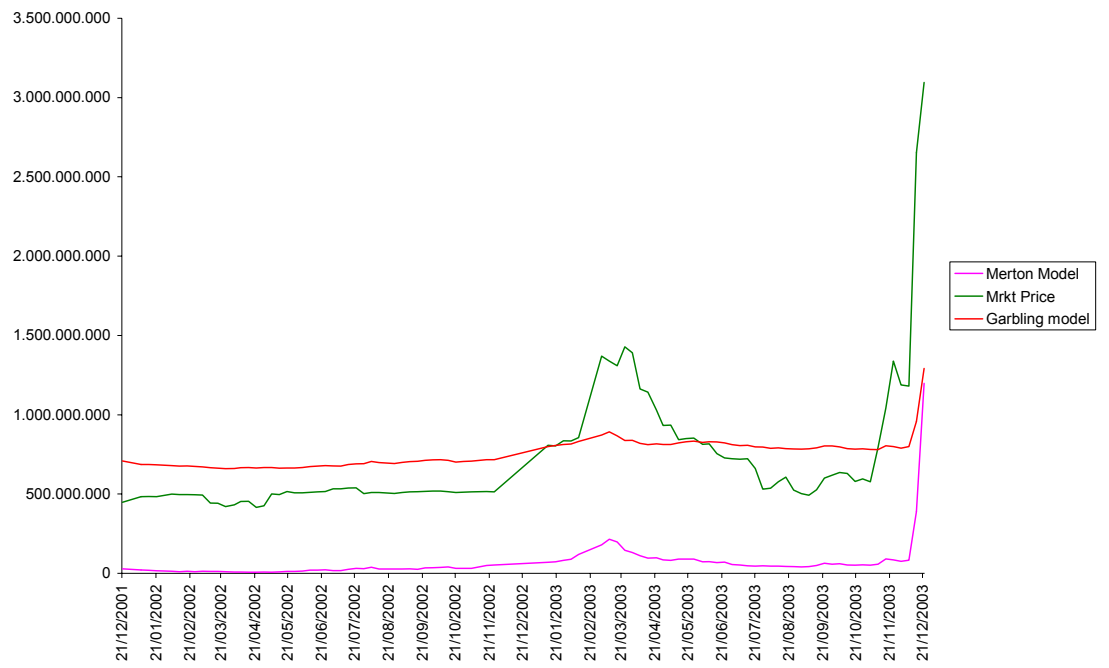


Figure 3:

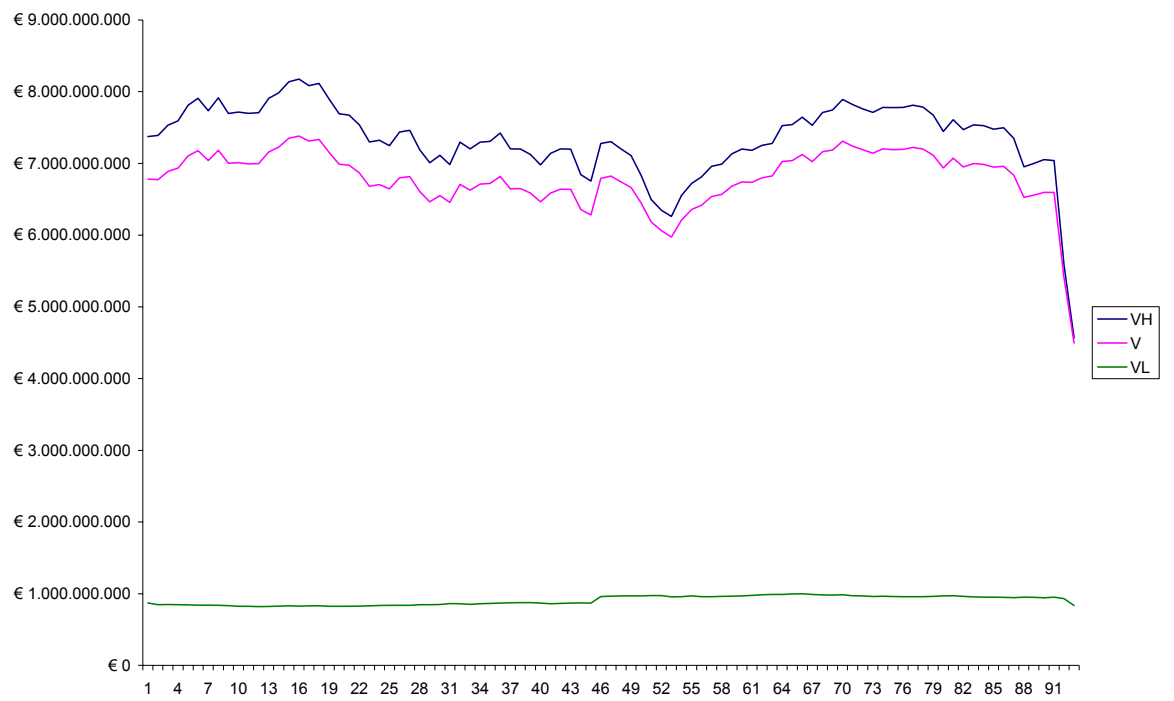


Figure 4:

## 5 Conclusions

In this paper we show that accounting for mis-reporting may actually reconcile structural models with market data. The effect of mis-reporting is modelled as a “peso problem”, that is the market allows for a small probability that a catastrophic event, such as a fraud case, could actually take place. We show how to amend Duan (1994,2000) maximum likelihood estimation on transformed data to allow for such problem and we apply it to the Parmalat case, one of the most famous fraud events ever. The results strongly support the “peso problem” hypothesis. The market was assigning a 20% probability to a scenario in which the firm was already in a default state. Confirming this result, we find that while CDS appear to be largely overvalued once the standard Merton model is estimated on equity prices, such mis-pricing disappears if we allow for fraud probability. Furthermore, computing the value of the firm under such catastrophic event yield a value which is about constant around 20% of the quasi-debt value, a figure which is very much consistent with the recovery rate value which is typically reported for the Italian market.

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