Contagion and Interdependence in Financial Markets: A New Approach

Giampiero M. Gallo

Dipartimento di Statistica "G. Parenti", Università di Firenze

Via G. B. Morgagni, 59 - 50134 Firenze, Italy (*gallog@ds.unifi.it*)

Edoardo Otranto

Dipartimento di Economia, Impresa e Regolamentazione,

Università di Sassari, Via Torre Tonda - 07100 Sassari, Italy (*eotranto@uniss.it*)

Abstract

The concepts of *contagion* and *interdependence* are two ways to characterize the transmission mechanisms of shocks from one market to the others. In the case of contagion, the implicit assumption is one of dominance of a market on the others and the transmission takes the form of the diffusion of the events with a temporal lag, while in the presence of interdependence this behavior is in two directions, in the sense that each market can transmit or receive the shock. In this paper we exploit a new approach to capture these aspects, applying it to the Asian case.

KEY WORDS: Markov Switching, multiple chains, volatility, transmission mechanisms, co–movements

JEL classification: C32 C52 C53

1 Introduction

The diffusion of international investments and capital movements across borders has marked the evolution of financial markets and has changed the profile of correlations among assets denominated in different currencies and exchanged in geographically separated markets. A single market volatility reacts to innovations in other markets as a result of financial integration.

Mechanisms of transmission of shocks across variables in an econometric model have received a great deal of attention in the literature. We can think of the large number of papers dealing with structural VAR models where the analysis is based on some identification constraints imposed on the model structure and the effects can be traced from shocks to individual variables. A different stream of financial research has dealt with spillovers of volatility from one market to another, focussing on shocks to volatility in a GARCH framework (Engle *et al.*, 1990).

In recent times a new stream of research has originated from the study of some financial crises (Mexico, Russia, East Asia, Argentina among the most famous) trying to assess whether they originated in one region and spilled over to other regions or whether interdependence among regions better describes the dynamics of the variables of interest.

In discussing the presence and the extension of contagion effects, several authors have concentrated on different aspects, and hence different definitions of contagion: the World Bank site on Financial Crises[1] provides a broad definition of cross-country transmission of shocks which may take place during both "good" and "bad" times, whereas more restrictive definitions focus on the situation of crisis and the consequent increase in the level of interdependence across countries. From an empirical point of view, methodologies vary considerably: according to Pericoli and Sbracia (2003) the main areas of analysis span from Probit/Logit models (in the form of a dummy variable representing a crisis in one country in the model for another country where the dependent variable assumes value equal to one in correspondence to a crisis in that country), Leading Indicators (predictive value of variables linked to economic fundamentals), GARCH models (following Engle *et al.*, 1990), and correlation breakdowns (in the line of Forbes and Rigobon, 2002).

A further category of models which has received considerable attention relates to Markov Switching models (MS; introduced in the literature by Hamilton, 1989): so far the framework of MS models has been used because the data can identify the presence of sudden switches ruled by a Markov chain, be they in the mean equation as in Fratzscher (2003) or in the variance equation (a multivariate version of the SWARCH model of Hamilton and Susmel, 1994, is suggested by Edwards and Susmel, 2001 and 2003). In the latter case the idea of a sudden change in the volatility of stock returns or interest rates measured in a pair of countries and of their correlation is set in a model where low and high volatility

[1] http://www1.worldbank.org/economicpolicy/managing%20volatility/contagion/index.html

in each countries combined with one another identify the states of nature. The interesting feature of their approach is that one country is *ex ante* considered the *originator* of the crisis (dominant market) and the correlation coefficient is made dependent on the state of such originator country. Contagion is had when the correlation coefficients change significantly across states.

The existence of different regimes of volatility seems consistent with one of the various possible definitions of contagion, interdependence and comovement, in which *contagion* is seen as a situation in which a switch in regime of a dominating market leads to a change in regime in the dominated market (with a lag), whereas *interdependence* is seen as a situation in which a switch in regime of one of the markets leads a change in regime of the other markets (in other terms, the same market could be dominated and dominant); finally, *comovement* is represented by contemporaneous change in regimes. We adopt this kind of definition, studying the volatility with various Markov Switching models.

In order to model transmission mechanisms, and in view of the computational difficulties limiting the extent of the analysis proposed by Edwards and Susmel, we choose to adopt a framework in which rather than concentrating on switching second moments, we construct an observable measure of volatility and we apply the Markov Switching model directly to a multivariate model for the mean of the volatility proxies observed on different markets. A further reason for preferring a switching-in-the-mean approach is to make use of some techniques to detect the presence of regimes. The nonparametric Bayesian approach by Otranto and Gallo (2002) may be helpful in casting some light on the number of different states of nature exhibited by the series involved.

We will focus on the weekly range (highest recorded minus lowest recorded value, cf. Alizadeh et al., 1999), emphasizing the peculiar role played by the originator market relative to the dominated market by adopting a new version of the Markov Switching model called the Multi Chain MS model (MCMS, Otranto, 2003) where an in-built asymmetry is inserted by making the transition probability of each market dependent on the state of the other markets. This model is particularly appealing in this kind of analysis because, starting from a general formulation, various hypotheses corresponding to different situations (interdependence, contagion, independence) can be tested.

The idea of using dependent Markov chains in a switching framework is also used by Anas et al. (2004) to study transmission mechanisms for business cycle. In their framework, there is no a priori on the originator country and all combinations of states between time $t-1$ and t are examined and Granger causality testing used to assess the direction of the transmission. In our approach some constraints on the specification of the autoregressive coefficients and the covariance matrix is inserted for the sake of computational gain and interpretability of coefficients.

In the next 2 sections the multivariate models used and their interpretation will be explained; in section 4 the methodology exposed will be applied to analyze the characteristics of the Asian markets in the period 1993-2004, including the East Asian crisis of 1997. Concluding remarks follow.

2 Multivariate MS

The presence of multiple regimes can be acknowledged using a popular model introduced by Hamilton (1990) where parameters are made dependent on a hidden state process ruled by a Markov chain: such a model, the multivariate Markov Switching Model (MS), considers an n-dimensional vector $\mathbf{y}_t \equiv (y_{1t}, \ldots, y_{nt})'$, which is assumed to follow a VAR(p) with time-varying parameters:

$$\begin{aligned} \mathbf{y}_t &= \mu(s_t) + \sum_{i=1}^{p} \mathbf{\Phi}_i(s_t)\mathbf{y}_{t-i} + \epsilon_t \\ \epsilon_t &\sim \mathcal{N}\left(\mathbf{0}, \mathbf{\Sigma}(s_t)\right) \end{aligned} \tag{1}$$

where the parameters for the mean equation μ and $\mathbf{\Phi}_i$, $i = 1, \ldots, p$, as well as the variances and covariances of the error terms ϵ_t in the matrix $\mathbf{\Sigma}$ all depend upon the state variable s_t which can assume a number q of regimes. The transition probability matrix $\mathbf{P}$ contains the probabilities of being in a generic state j at time t given that the state at time $t-1$ was i, namely, for a generic element

$$p_{ij} = Pr(s_t = j | s_{t-1} = i),\ i, j = 1, \ldots, q$$

By now, the properties of this model are well known and need not be discussed here: we refer to Hamilton (1994) for the estimation, filtering and smoothing procedures for this model. For this model it is crucial to keep in mind that all variables in the process $\mathbf{y}$ depend on the same state variable s_t, and as such they are subject to a common switching.

Such a model is of limited use in deciding whether there is contagion or interdependence, in that it may just signal the presence of common changes in regimes by the variables analyzed (hence interdependence always): it may therefore be misleading in cases in which there is not a common switch, but there exists a market originating such a comovement (asymmetric behavior and therefore contagion). In what follows, evidence for this model will be considered as evidence for common contemporaneous changes across markets and we call it comovement.

3 The Multi–Chain Markov Switching Model

The idea behind a Multi–Chain Markov switching model (MCMS), as suggested by Otranto (2003), is to consider a multivariate process in which the switching mechanism across regimes makes the state for one variable be dependent on the lagged states of all variables. This case could be considered as representative of the situation of interdependence, because the change in the state of each variable can be transmitted to all the others with a certain probability. As a special case, one can consider a process in which one variable is assumed to be dominant on the others and the switching dynamics intrinsically asymmetric: a particular state for one variable alters the probability of other variables to change states. This assumption is suitable to treat transmission mechanisms occurring in financial crises, but also to any relationship where a leading variable is present (in Otranto, 2005, new orders are assumed to be leading the turnover at the aggregate level) and can represent the case of contagion. Finally, the reciprocal dependence on the state of the other variables could turn out to be not significant, representing the case of independence among the markets.

To fix ideas, let us consider a bivariate case with two latent states for each variable: the dynamics of the two variables are thus subject to state dependence. The transition from one (multi–) state to another is ruled by a Markov chain obtained by letting the transition probabilities for one variable be a function of the (lagged) state of both variables and the transition probabilities for the other be a function of just its own lagged state.

In formal terms, as before, $\mathbf{y}_t$ is assumed to follow a VAR(p) process (note that $\mathbf{s}_t$ is now a vector):

$$\begin{aligned} \mathbf{y}_t &= \mu(\mathbf{s}_t) + \sum_{i=1}^{p} \mathbf{\Phi}_i(\mathbf{s}_t)\mathbf{y}_{t-i} + \epsilon_t \\ \epsilon_t &\sim \mathcal{N}\left(\mathbf{0}, \mathbf{\Sigma}(\mathbf{s}_t)\right) \end{aligned} \qquad (2)$$

where the parameters for the mean equation μ and $\mathbf{\Phi}_i$, $i = 1, \ldots, p$, as well as the variances and covariances of the error terms ϵ_t in the matrix $\mathbf{\Sigma}$ all depend upon the state vector $\mathbf{s}_t \equiv (s_{1t}, \ldots, s_{nt})'$ with s_{jt} representing the state associated with variable y_{jt}. Each state can assume a number q of regimes (in principle these could be different across states). The difference with respect to the classical multivariate MS models is that $y_{1,t}$ and $y_{2,t}$ depend on separate but related state variables.

To illustrate how the asymmetric behavior of the variables can be embedded in the model, let us consider the transition probability matrix $\mathbf{P}$ with generic element representing

$$\mathbf{P} = \{\Pr[\mathbf{s}_t | \mathbf{s}_{t-1}]\}.$$

If we consider, for simplicity, the case $n = q = 2$, the state vector $\mathbf{s}_t$ can assume four different values $\{(0,0), (0,1), (1,0), (1,1)\}$ and the matrix $\mathbf{P}$ is a 4×4 matrix. Let us suppose for the moment that the states s_{1t} and s_{2t}, conditional on (s_{1t-1}, s_{2t-1}), are independent, so that:

$$\Pr[s_{1t}, s_{2t} | s_{1t-1}, s_{2t-1}] = \Pr[s_{1t} | s_{1t-1}, s_{2t-1}] \Pr[s_{2t} | s_{1t-1}, s_{2t-1}] \tag{3}$$

In order to insert asymmetry of behavior in the model and assuming that the variable 2 has a dominant position, the right hand side of equation (3) can be parameterized with logistic functions to take into account the specific impact of the regimes assumed by the second state variable s_{2t}:

$$\begin{aligned}
\Pr(s_{1t} = h | s_{1t-1} = h, s_{2t-1}) &= \frac{\exp[\alpha_1(h,.) + \beta_1(h,1) s_{2t-1}]}{1 + \exp[\alpha_1(h,.) + \beta_1(h,1) s_{2t-1}]} \\
\Pr(s_{2t} = h | s_{1t-1}, s_{2t-1} = h) &= \frac{\exp[\alpha_2(.,h) + \beta_2(1,h) s_{1t-1}]}{1 + \exp[\alpha_2(.,h) + \beta_2(1,h) s_{1t-1}]} \\
\Pr(s_{jt} = k | s_{jt-1} = h, s_{it-1}) &= 1 - \Pr(s_{jt} = h | s_{jt-1} = h, s_{it-1}) \\
h, k = 0, 1; \quad h \neq k, & \qquad i, j = 1, 2; \quad i \neq j.
\end{aligned} \tag{4}$$

¿From (4), it is apparent that the state of the variable i at time $t-1$ influences the probability of variable j to stay in the same regime, and vice-versa.

Hypothesis testing can be performed on the estimated model (2)–(4) in order to assess the relevance of the dependence structure assumed for the states and whether the presence of asymmetric effects in the dynamics of regimes is supported by the data. Statistical significance of all parameters in (4) will provide evidence in favor of the case of *interdependence*. If the coefficient $\beta_j(h,k)$=0, the state of the variable i at time $t-1$ influences the probability of variable j to stay in the same regime, but not vice-versa; this is evidence in favor of the dominant status of variable i or *contagion*. This property gives meaning to our envisaging contagion as a stable asymmetric relationship between markets and not necessarily related to the effects of single shocks. Finally, the non significance of all the coefficients $\beta_j(h,k)$ and $\beta_i(h,k)$ would show evidence for *independence* between markets.

In this way, the estimated probabilities in (4) will show the impact of the regime of variable i on the transition probabilities for variable j; moreover, we would expect the signs of coefficients $\beta_1(0,1)$ and $\beta_2(1,0)$ to be negative and those of coefficients $\beta_1(1,1)$ and $\beta_2(1,1)$ to be positive.

Disposing the estimated transition probabilities (3) in a matrix, with rows representing the multiple state at time $t-1$ and columns the multiple state at time t, it is possible to evaluate the most probable scenario (a particular combination of s_{1t} and s_{2t}) at time t, given a certain state at time $t-1$.

The properties of the model from a theoretical point of view coincide with those of a standard Markov switching model: estimation filtering and smoothing can be performed according to the procedures described by Hamilton (1990) and Kim (1994). It should be clear that in practice some restrictions will have to be imposed on the general model (2) in order to make it tractable from a computational point of view, also to retain interpretability of the results according to the specific application at hand.

4 Hong Kong's Role in Asian Markets

The Asian markets are a classical example for which there is a large debate to establish if the presence of common shocks could be interpreted as contagion or interdependence. For example, Forbes and Rigobon (2002) note that the shock originating from Hong Kong in October 1997 has not implied a significant increase in the correlation coefficients of the other main Asian markets: the conclusion reached is that the series analyzed cannot be considered as subject to a form of contagion from Hong Kong, but exhibit interdependence. As said in the previous sections, our definitions of contagion, interdependence, independence and comovements are related to the possible changes in regime detected by various kind of models.

4.1 The Presence of Regimes

We analyze the stock market indices of 5 Asian countries starting from daily data spanning a period between November 29, 1993 and April 26, 2004; the indices are the Hang Seng index (Hong Kong-HSI hereafter), the KOSPI index (South Korea-KS11), the KLSE composite index (Malaysia-KLSE), the Straits Times index (Singapore-STI), the Thailand SET index (Thailand, SETI). The proxy of the volatility is computed as the weekly range of the logarithm of the data (highest recorded minus lowest recorded value) and results in 544 observations.

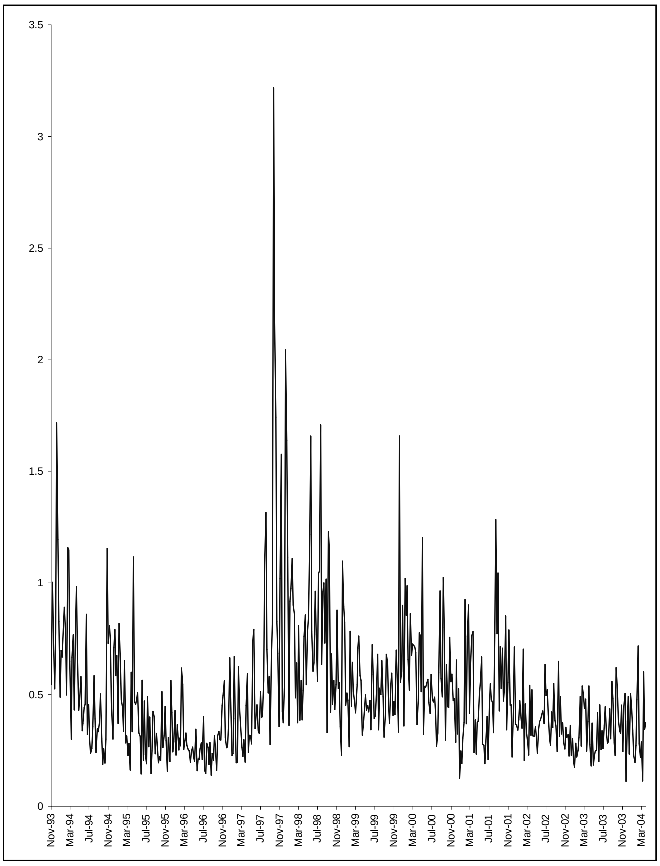

Figure 1: Hang Seng volatility

The proxy used delivers the HSI series shown in Figure 1; the East Asian crisis shows its most evident effect in the third week of October 1997, in which the volatility increased almost 300 percent. The dramatically high volatility of this period is a common feature of all the series analyzed (Figure 2), with a different degree of persistence and depth, but a similar general behavior. In particular the Korean market seems to suffer dramatically from the October crisis, increasing

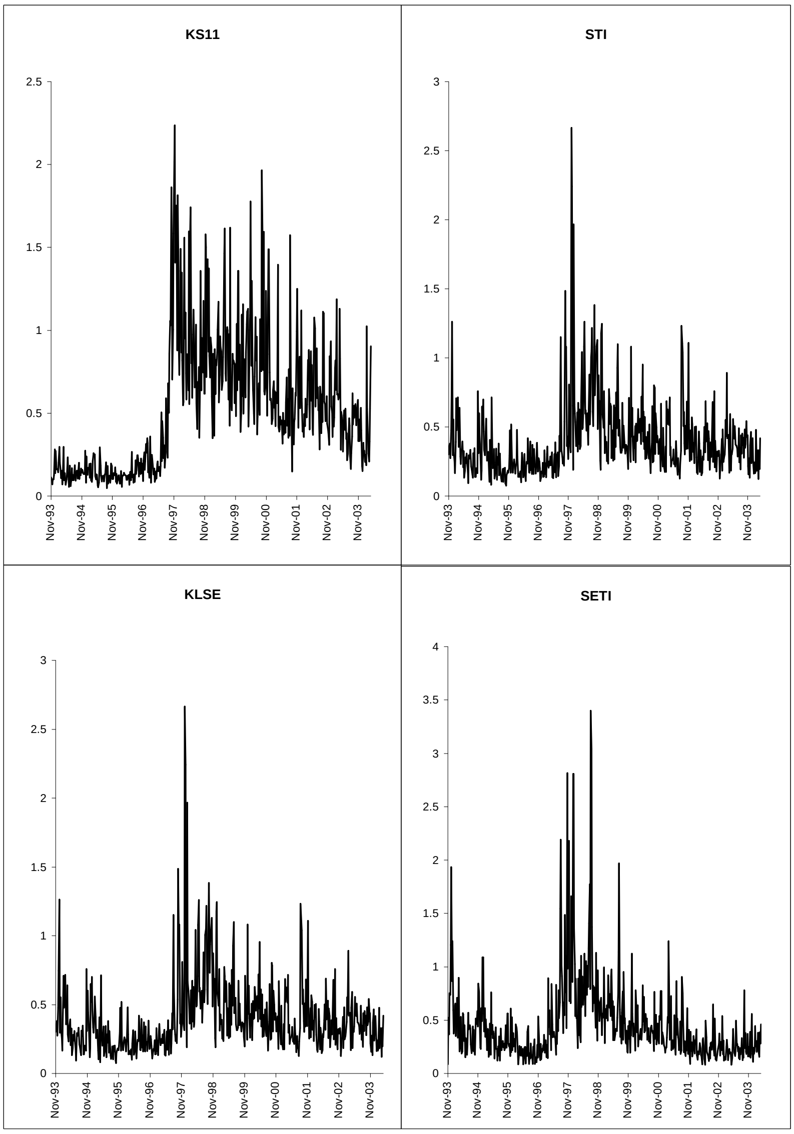

Figure 2: Volatility of Asian markets

Table 1: Empirical Posterior Distribution of the number of regimes

	1	2	3	4	5	6	7	8
HSI	0.000	0.000	0.872	0.118	0.008	0.002	0.000	0.000
KS11	0.000	0.000	0.768	0.200	0.024	0.008	0.000	0.000
STI	0.000	0.806	0.182	0.012	0.000	0.000	0.000	0.000
KLSE	0.000	0.000	0.850	0.124	0.024	0.002	0.000	0.000
SETI	0.000	0.018	0.420	0.494	0.052	0.016	0.000	0.000

its volatility by 81 percent and never really reverting to lower volatility levels in successive periods. Other series seem to absorb the shock, albeit gradually.

The existence of at least two regimes is clear observing these graphs, as is the presence of some sort of common feature, with possible lags. The presence of regimes can be detected by means of the nonparametric Bayesian procedure of Otranto and Gallo (2002). With this approach, using diffuse priors for the distributions of the number of regimes, we obtain the posterior distributions shown in Table 1.[2] The series HSI, KS11 and KLSE show a strong evidence in favor of three regimes, whereas STI seems to exhibit only two regimes. The case of SETI is not clear, since very similar probabilities are present for 3 and 4 states. At any rate, for each series the situation of no regimes (1 state) is ruled out by the Bayesian procedure; these results are therefore consistent with our approach.

4.2 The Empirical Results

The first step of our approach is to estimate a MCMS model without constraints on the parameters relative to the probability equations (4) and then to test some restrictions. We shall consider bivariate models keeping HSI as the second variable in all models and letting the data suggest which type of relationship it holds with other markets, given the great influence exerted by Hong Kong for the other the Asian economies.

Following this hypothesis, we estimate 4 separate MCMS models with 2x2 states; the value 0 represents the ordinary regime, the value 1 occurs in the turbulent regime. In addition, we consider the autoregressive parameters in (2) not to be state–dependent and the order p equal to 2; for these coefficients the usual

[2]We use the same priors of Otranto and Gallo (2002), with the hyperparameter A, regulating the prior probabilities of the number of regimes, equal 0.15.

stationarity constraints hold. Finally, we suppose a structure of the covariance matrix as:

$$\begin{bmatrix} \sigma_1^2(s_{1t},.) & \rho(s_{1t},s_{2t})\sigma_1(s_{1t},.)\sigma_2(.,s_{2t}) \\ \rho(s_{1t},s_{2t})\sigma_1(s_{1t},.)\sigma_2(.,s_{2t}) & \sigma_2^2(.,s_{2t}) \end{bmatrix}$$

In other terms, the variances of each variable (related to fourth moments of returns) depend only on the variable's own state, whereas the effect of the multi–state affects the correlation coefficient, that varies in [-1,1].

We allow the intercept of model (2) to vary with both the regimes of the two markets; in other terms, we will have four possible intercepts for each variable.

After the estimation of the models, we would like to test some proposition to evaluate the presence of dependence on the state of the other variable, in particular the case of contagion or independence. The testable propositions are:

1. No dependence of the intercept of y_1 on the state of y_2:
   H_0: $\mu_1(0,0)$=$\mu_1(0,1)$ and $\mu_1(1,0)$=$\mu_1(1,1)$;
2. No dependence of the intercept of y_2 on the state of y_1:
   H_0:$\mu_2(0,0)$=$\mu_2(1,0)$ and $\mu_2(0,1)$=$\mu_2(1,1)$
3. y_2 does not Granger cause y_1:
   H_0:ϕ^1_{12}=ϕ^2_{12}=0
4. y_1 does not Granger cause y_2:
   H_0:ϕ^1_{21}=ϕ^2_{21}=0
5. No dependence of the correlation on the state of y_2:
   H_0:$\rho(0,0)$=$\rho(0,1)$ and $\rho(1,0)$=$\rho(1,1)$
6. No dependence of the correlation on the state of y_1:
   H_0:$\rho(0,0)$=$\rho(1,0)$ and $\rho(0,1)$=$\rho(1,1)$
7. No contagion from y_2 to y_1:
   H_0:$\beta_1(0,1)$=$\beta_1(1,1)$=0
8. No contagion from y_1 to y_2:
   H_0:$\beta_2(1,0)$=$\beta_2(1,1)$=0

Table 2: p-values from Wald Tests on MCMS models

Null Hypothesis	p-values			
	KS11/HSI	STI/HSI	KLSE/HSI	SETI/HSI
$\mu_1(0,0)$=$\mu_1(0,1)$; $\mu_1(1,0)$=$\mu_1(1,1)$	0.000	0.000	0.000	0.000
$\mu_2(0,0)$=$\mu_2(1,0)$; $\mu_2(0,1)$=$\mu_2(1,1)$	0.000	0.000	0.000	0.000
ϕ^1_{12}=ϕ^2_{12}=0	0.000	0.000	0.000	0.000
ϕ^1_{21}=ϕ^2_{21}=0	0.004	0.000	0.000	0.036
$\rho(0,0)$=$\rho(0,1)$; $\rho(1,0)$=$\rho(1,1)$	0.981	0.000	0.000	0.000
$\rho(0,0)$=$\rho(1,0)$; $\rho(0,1)$=$\rho(1,1)$	0.705	0.000	0.000	0.000
$\beta_1(0,1)$=$\beta_1(1,1)$=0	0.003	0.044	0.003	0.000
$\beta_2(1,0)$=$\beta_2(1,1)$=0	0.445	0.365	0.002	0.275
$\beta_1(0,1)$=$\beta_1(1,1)$=$\beta_2(1,0)$=$\beta_2(1,1)$=0	0.010	0.066	0.000	0.000
Comovement	0.000	0.025	0.000	0.000

9. No contagion and no interdependence:

   H_0:$\beta_1(0,1)$=$\beta_1(1,1)$=$\beta_2(1,0)$=$\beta_2(1,1)$=0

10. Comovement between y_1 and y_2

    $\alpha_1(0,.) = \alpha_2(.,0)$; $\alpha_1(0,.) + \beta_1(0,1) + \alpha_2(.,1) = 0$; $\alpha_1(.,1) + \alpha_2(.,0) + \beta_2(1,0) = 0$;$\alpha_1(1,.) + \beta_1(1,1) = \alpha_2(.,1) + \beta_2(1,1)$.

The last hypothesis is not intuitive because the MMS model is not nested into the MCMS model because it corresponds to the case in which $s_{1t} = s_{2t}$ for each t, so the rows and the columns of the transition probabilities matrix of the independent MCMS model corresponding to $s_{1t} \neq s_{2t}$ can not be constrained to obtain the transition probabilities matrix of the MS model, which has a lower dimension. The set of constraints sub 10 above derives from the idea that the comovement implies a similar succession of the regime along the time, which is the case in the MMS model. The analytical way to obtain these constraints is developed in the appendix at the end of the paper.

All these hypotheses can be tested by means of classical Wald statistics and are consistent with the idea of Granger causality for MS VAR models proposed by Warne (2000).

In Table 2 we show the p-values associated with the test statistics for the ten

Table 3: STI/HSI: Loss functions for MCMS and MS models

	MSE	MAE	$[LE]^2$	$\|LE\|$
MS 2 states	12.776	3.346	0.450	0.756
MS 3 states	**12.734**	**3.337**	**0.447**	**0.751**
MCMS	12.936	3.369	0.458	0.763

hypotheses above; as a result, the parameters of (2) show some form of dependence between the couples of series; only the correlation between KS11 and HSI seems not be dependent on the four multiple states. The case of interdependence seems satisfied only by the couple KLSE–HSI, whereas Hong Kong market has a contagion effect on the Korean and Thailand markets. The Singapore case is a puzzling one: in fact, the hypothesis of no contagion from HSI is accepted at 99% significance level, but rejected at 95%; the same is valid for the hypothesis of comovement. Anyway, the joint test to verify the case of no contagion and no interdependence of the markets fail to reject this hypothesis, so we can limit the analysis to the cases of comovement (represented by a bivariate MS model) or independence (represented by a bivariate MCMS model with independent s_{1t} and s_{2t}).

To gather further empirical evidence we can compare the MCMS independent model with proper bivariate MS models. In view of the results in Table 1 where we assessed that STI seems to possess two regimes and HSI three, we will estimate two MS models, one with two regimes and one with three. The comparison is carried out by showing in-sample goodness of fit performance using the Mean Square Error (MSE) and Mean Absolute Error (MAE) or their equivalents for the variables expressed in logs ($[LE]^2$ and $|LE|$ respectively, following Hamilton and Susmel's, 1994, notation). The results are shown in Table 3 (the boldface figures indicate the best performance).

The MS model with 3 states clearly performs better than the others, with the MS model with 2 states coming in as second: we interpret this to be evidence in favor of the comovement between STI and HSI.

We show the estimation of the selected models for each pair of markets in Tables 4, 5, 6, 7, reporting in the last rows the p-values relative to the Jarque-Bera test (JB), the Ljung-Box test (LB(10)) and the Ljung-Box test on squared residuals (LBS(10)), both calculated with 10 lags.

The residuals of HSI exhibit non normality in all cases, but this outcome af-

Table 4: Estimated parameters of the MCMS Model for Korea/Hong Kong (standard errors in parentheses)

Switching coefficients - Constant Term							
Korea Equation				Hong Kong Equation			
$\mu_1(0,0)$	$\mu_1(0,1)$	$\mu_1(1,0)$	$\mu_1(1,1)$	$\mu_2(0,0)$	$\mu_2(1,0)$	$\mu_2(0,1)$	$\mu_2(1,1)$
0.714	1.307	3.446	8.222	1.760	2.336	5.260	6.103
(0.064)	(0.210)	(0.181)	(0.264)	(0.104)	(0.381)	(0.367)	(0.426)
Autoregressive Terms							
Korea Equation				Hong Kong Equation			
ϕ_{11}^1	ϕ_{12}^1	ϕ_{11}^2	ϕ_{12}^2	ϕ_{21}^1	ϕ_{22}^1	ϕ_{21}^2	ϕ_{22}^2
0.284	-0.020	0.197	-0.000	0.028	0.158	-0.000	0.178
(0.007)	(0.007)	(0.006)	(0.008)	(0.009)	(0.013)	(0.004)	(0.012)
Switching coefficients - Standard deviations				Switching coefficients - Correlation Terms			
Korea Equation		Hong Kong Equation					
$\sigma_1(0,.)$	$\sigma_1(1,.)$	$\sigma_2(.,0)$	$\sigma_2(.,1)$	$\rho(0,0)$	$\rho(0,1)$	$\rho(1,0)$	$\rho(1,1)$
0.412	1.580	0.675	2.446	0.000	0.000	0.037	0.039
(0.012)	(0.041)	(0.016)	(0.050)	(0.050)	(0.079)	(0.064)	(0.051)
Probability parameters							
Korea Equation				Hong Kong Equation			
$\alpha_1(0,.)$	$\beta_1(0,1)$	$\alpha_1(1,.)$	$\beta_1(1,1)$	$\alpha_2(.,0)$		$\alpha_2(.,1)$	
1.614	-1.203	1.053	0.000	1.119		0.012	
(0.246)	(0.362)	(0.221)	(0.357)	(0.166)		(0.188)	
p-values of test statistics							
Korea				Hong Kong			
JB	LB(10)	LBS(10)		JB	LB(10)	LBS(10)	
0.241	0.360	0.040		0.000	0.306	0.311	

Table 5: Estimated parameters of the MS–3 states Model for Singapore/Hong Kong (standard errors in parentheses)

Switching coefficients - Constant Term					
Singapore Equation			Hong Kong Equation		
$\mu_1(1)$	$\mu_1(2)$	$\mu_1(3)$	$\mu_2(1)$	$\mu_2(2)$	$\mu_2(3)$
1.298	2.737	6.801	1.445	3.623	6.847
(0.056)	(0.071)	(0.357)	(0.063)	(0.092)	(0.427)

Autoregressive Terms							
Singapore Equation				Hong Kong Equation			
ϕ_{11}^1	ϕ_{12}^1	ϕ_{11}^2	ϕ_{12}^2	ϕ_{21}^1	ϕ_{22}^1	ϕ_{21}^2	ϕ_{22}^2
0.209	-0.007	0.156	-0.000	0.121	0.145	-0.004	0.163
(0.014)	(0.015)	(0.013)	(0.011)	(0.016)	(0.015)	(0.014)	(0.012)

Switching coefficients - Standard deviations						Switching coefficients Correlation Terms		
Singapore Equation			Hong Kong Equation					
$\sigma_1(1)$	$\sigma_1(2)$	$\sigma_1(3)$	$\sigma_2(1)$	$\sigma_2(2)$	$\sigma_2(3)$	ρ_1	$\rho(2)$	$\rho(3)$
0.528	0.845	2.932	0.539	1.045	3.370	0.108	0.000	0.456
(0.019)	(0.031)	(0.082)	(0.016)	(0.040)	(0.096)	(0.041)	(0.057)	(0.058)

Transition Probabilities					
p_{11}	p_{12}	p_{21}	p_{22}	p_{31}	p_{32}
0.521	0.410	0.459	0.464	0.307	0.195
(0.039)	(0.040)	(0.044)	(0.047)	(0.065)	(0.064)

p-values of test statistics					
Singapore			Hong Kong		
JB	LB(10)	LBS(10)	JB	LB(10)	LBS(10)
0.058	0.013	0.003	0.000	0.129	0.938

Table 6: Estimated parameters of the MCMS Model for Malaysia/Hong Kong (standard errors in parentheses)

Switching coefficients - Constant Term							
Malaysia Equation				Hong Kong Equation			
$\mu_1(0,0)$	$\mu_1(0,1)$	$\mu_1(1,0)$	$\mu_1(1,1)$	$\mu_2(0,0)$	$\mu_2(1,0)$	$\mu_2(0,1)$	$\mu_2(1,1)$
1.261	3.523	8.084	8.084	2.105	2.210	3.428	8.683
(0.051)	(1.095)	(1.091)	(1.156)	(0.081)	(0.233)	(0.190)	(0.320)
Autoregressive Terms							
Malaysia Equation				Hong Kong Equation			
ϕ_{11}^1	ϕ_{12}^1	ϕ_{11}^2	ϕ_{12}^2	ϕ_{21}^1	ϕ_{22}^1	ϕ_{21}^2	ϕ_{22}^2
0.183	0.025	0.126	0.000	0.066	0.195	0.009	0.135
(0.009)	(0.010)	(0.007)	(0.010)	(0.011)	(0.016)	(0.013)	(0.017)
Switching coefficients - Standard deviations				Switching coefficients - Correlation Terms			
Malaysia Equation		Hong Kong Equation					
$\sigma_1(0,.)$	$\sigma_1(1,.)$	$\sigma_2(.,0)$	$\sigma_2(.,1)$	$\rho_{0,0}$	$\rho(0,1)$	$\rho(1,0)$	$\rho(1,1)$
0.523	3.951	0.879	2.009	0.139	0.000	0.581	0.085
(0.013)	(0.222)	(0.020)	(0.030)	(0.037)	(0.055)	(0.052)	(0.056)
Probability parameters							
Malaysia Equation				Hong Kong Equation			
$\alpha_1(0,.)$	$\beta_1(0,1)$	$\alpha_1(1,.)$	$\beta_1(1,1)$	$\alpha_2(.,0)$	$\beta_2(1,0)$	$\alpha_2(.,1)$	$\beta_2(1,1)$
2.507	-1.149	-1.077	0.950	0.963	-1.219	-0.238	0.801
(0.270)	(0.365)	(0.480)	(0.616)	(0.147)	(0.403)	(0.184)	(0.458)
p-values of test statistics							
Malaysia				Hong Kong			
JB	LB(10)	LBS(10)		JB	LB(10)	LBS(10)	
0.004	0.725	0.104		0.000	0.708	0.162	

Table 7: Estimated parameters of the MCMS Model for Thailand/Hong Kong (standard errors in parentheses)

Switching coefficients - Constant Term							
Thailand Equation				Hong Kong Equation			
$\mu_1(0,0)$	$\mu_1(0,1)$	$\mu_1(1,0)$	$\mu_1(1,1)$	$\mu_2(0,0)$	$\mu_2(1,0)$	$\mu_2(0,1)$	$\mu_2(1,1)$
0.490	1.695	3.599	5.910	2.061	2.061	3.153	7.200
(0.029)	(0.299)	(0.293)	(0.407)	(0.081)	(0.273)	(0.241)	(0.349)
Autoregressive Terms							
Thailand Equation				Hong Kong Equation			
ϕ_{11}^1	ϕ_{12}^1	ϕ_{11}^2	ϕ_{12}^2	ϕ_{21}^1	ϕ_{22}^1	ϕ_{21}^2	ϕ_{22}^2
0.357	-0.010	0.148	-0.000	0.038	0.230	-0.000	0.146
(0.005)	(0.006)	(0.005)	(0.003)	(0.013)	(0.014)	(0.012)	(0.013)
Switching coefficients - Standard deviations				Switching coefficients - Correlation Terms			
Thailand Equation		Hong Kong Equation					
$\sigma_1(0,.)$	$\sigma_1(1,.)$	$\sigma_2(.,0)$	$\sigma_2(.,1)$	$\rho_{0,0}$	$\rho(0,1)$	$\rho(1,0)$	$\rho(1,1)$
0.261	2.898	0.933	2.154	0.000	0.000	0.546	0.044
(0.007)	(0.047)	(0.022)	(0.036)	(0.043)	(0.073)	(0.044)	(0.054)
Probability parameters							
Thailand Equation				Hong Kong Equation			
$\alpha_1(0,.)$	$\beta_1(0,1)$	$\alpha_1(1,.)$	$\beta_1(1,1)$	$\alpha_2(.,0)$		$\alpha_2(.,1)$	
1.544	-1.261	0.199	0.468	0.889		-0.166	
(0.213)	(0.311)	(0.202)	(0.383)	(0.136)		(0.178)	
p-values of test statistics							
Thailand				Hong Kong			
JB	LB(10)	LBS(10)		JB	LB(10)	LBS(10)	
0.000	0.089	0.087		0.000	0.027	0.806	

fects most volatility models, given the presence of a few exceptional values in the series (see Figures 1 and 2).

It is interesting to note that many correlation coefficients are equal to zero; we have estimated a bivariate VAR model on the four pairs of variables and the residuals are strongly correlated; it seems that if one does not consider the possible presence of regimes may end up with significant correlations between residuals. We ran a few Monte Carlo experiments on this aspect and the results (not reported here for the sake of space, but available upon request) show that when MS and MCMS models with uncorrelated disturbances are simulated and then estimated by a VAR the residuals are cross-correlated.

We can note that the signs of the parameters of the logistic functions are consistent with our expectations. In the KS11/HSI case the state of HSI has influence only on the probabilities of low volatility. For the STI/HSI case, the switch to state 3 from state 1 or 2 is not likely, whereas the change from state 3 to state 1 is more likely than the change to state 2; we can interpret the state 1 as the case of low volatility and the state 2 as the high volatility one, whereas the state 3, given its infrequent occurrence accompanied by low persistence, can be considered as an extremely turbulent state. All the intercepts of the MCMS models exhibit a gradual change from the (0,0) to the (1,1) state (note that in the Malaysia case - Table 6 - the intercept does not change between (1,0) and (1,1)).

5 Concluding Remarks

In this paper we propose a new model, based on correlated Markov chains, to represent the case of interdependence among financial markets, with the case of contagion and independence markets as particular cases. The fact that the two last cases are nested in the more general model provides the possibility to test statistically the various scenarios. The case of comovement among variables, represented by a classical Markov Switching model is not represented as a case which is nested in the MCMS model. We derived a separate test for common dynamics of the two state variables and we produced some further evidence of comovement between Singapore and Hong Kong in the period at hand by resorting to the evaluation of a number of loss functions. The analysis on comovements may be extended by applying some nonparametric test in order to evaluate the equality of the smoothed probabilities of the states s_{1t} and s_{2t}, but this was not pursued here.

The estimation of a bivariate model is forced by the difficulty of increasing the number of variables in the model without stumbling into the usual numerical problems encountered in Markov Switching models with higher number of regimes. A n-variate model with k states per variable would have a transition matrix of order k^n, which is rapidly intractable (flat likelihood function) for even moderate numbers of n or k above 2. There is therefore a trade-off between the depth of the economic interpretation which one would have available if more than two markets were to be compared and the numerical difficulties which accompany such an effort.

The definition of contagion, interdependence, comovement and independence are consistent with large part of the literature, but it is different in terms of statistical instruments utilized. For example, Forbes and Rigobon (2002) base their analysis only on the behavior of the correlation coefficients, and on a significant increase changing from a state of low to another of high volatility (with the periods of low and high volatility established a priori). In our approach, the periods of high and low volatility are selected by the model itself and the effect of the correlation is captured from the presence of regimes.

Appendix

In this Appendix we demonstrate that testing the null of comovement against the hypothesis of MCMS model is equivalent to verifying a set of linear restrictions on the MCMS model.

The case of comovement corresponds to the case in which the state of y_{1t} and y_{2t} is the same for each t; this situation can justify the employ of a classical MS model. The MS model is not nested into the MCMS model because of the different number of states, so that the classical tests based on the likelihood can not be applied; this representation allows to avoid this situation.

Recalling Hamilton (1994), a Markov chain can be represented as an AR(1) process:

$$\xi_{t+1} = \mathbf{P}'\xi_t + \mathbf{v}_{t+1},$$

where ξ_t is a vector containing 1 in correspondence of the state at time t, $\mathbf{P}$ is the transition probabilities matrix and $\mathbf{v}_t$ is a vector innovation with zero mean. In our case, the multiple states are (0,0), (0,1), (1,0), (1,1) and, for example, when $\xi_t = [0, 0, 1, 0]'$, means that the multiple state at time t is (1,0).

The conditional expectation of ξ_{t+1} is:

$$E(\xi_{t+1}|\xi_t) = \mathbf{P}'\xi_t.$$

If we are interested to the behavior of the single regimes s_{1t} and s_{2t}, respectively represented by the vectors $\xi^*_{\mathbf{t}}$ and $\xi^{**}_{\mathbf{t}}$, their expected values are given respectively by the 2x1 vectors:

$$\begin{aligned} E(\xi_{t+1}^{*}|\xi_t^{*}) &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \mathbf{P}'\xi_t \\ E(\xi_{t+1}^{**}|\xi_t^{**}) &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \mathbf{P}'\xi_t \end{aligned}$$

To verify the idea of comovement, as expressed in our paper, we can verify the equality of the two previous vectors, or:

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \mathbf{P}' = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \mathbf{P}'. \tag{5}$$

It is easy to verify that the two rows provide equal constraints; this is logical, because if it is verified that the first element of ξ_t^* is equal to the first element of ξ_t^{**} for each t, automatically the second elements of the two vectors will be equal, being the complement with respect to 1 of the first corresponding element. Let us indicate with $p(ij|wz)$ the probability $\Pr[s_{1t} = i, s_{2t} = j|s_{1t-1} = w, s_{2t-1} = z]$; then, the $\mathbf{P}$ matrix is:

$$\begin{bmatrix} p(00|00) & p(01|00) & p(10|00) & p(11|00) \\ p(00|01) & p(01|01) & p(10|01) & p(11|01) \\ p(00|10) & p(01|10) & p(10|10) & p(11|10) \\ p(00|11) & p(01|11) & p(10|11) & p(11|11) \end{bmatrix}$$

Developing the first (or the second) equation of (5), the four constraints to be verified are:

$$\begin{aligned} \Pr[s_{1t} = 0, s_{2t} = 1|s_{1t-1} = 0, s_{2t-1} = 0] &= \Pr[s_{1t} = 1, s_{2t} = 0|s_{1t-1} = 0, s_{2t-1} = 0] \\ \Pr[s_{1t} = 0, s_{2t} = 1|s_{1t-1} = 0, s_{2t-1} = 1] &= \Pr[s_{1t} = 1, s_{2t} = 0|s_{1t-1} = 0, s_{2t-1} = 1] \\ \Pr[s_{1t} = 0, s_{2t} = 1|s_{1t-1} = 1, s_{2t-1} = 0] &= \Pr[s_{1t} = 1, s_{2t} = 0|s_{1t-1} = 1, s_{2t-1} = 0] \\ \Pr[s_{1t} = 0, s_{2t} = 1|s_{1t-1} = 1, s_{2t-1} = 1] &= \Pr[s_{1t} = 1, s_{2t} = 0|s_{1t-1} = 1, s_{2t-1} = 1] \end{aligned} \tag{6}$$

Recalling the hypothesis of conditional independence (3) and the parameterization (4), we obtain that (6) corresponds to the four nonlinear constraints:

$$\frac{\exp[\alpha_1(0,.)]}{1+\exp[\alpha_1(0,.)]}\frac{1}{1+\exp[\alpha_2(.,0)]} = \frac{1}{1+\exp[\alpha_1(0,.)]}\frac{\exp[\alpha_2(.,0)]}{1+\exp[\alpha_2(.,0)]}$$

$$\frac{\exp[\alpha_1(0,.)+\beta_1(0,1)]}{1+\exp[\alpha_1(0,.)+\beta_1(0,1)]}\frac{\exp[\alpha_2(.,1)]}{1+\exp[\alpha_2(.,1)]} = \frac{1}{1+\exp[\alpha_1(0,.)+\beta_1(0,1)]}\frac{1}{1+\exp[\alpha_2(.,0)]}$$

$$\frac{1}{1+\exp[\alpha_1(.,1)]}\frac{1}{1+\exp[\alpha_2(.,0)+\beta_2(1,0)]} = \frac{\exp[\alpha_1(1,.)]}{1+\exp[\alpha_1(1,.)]}\frac{\exp[\alpha_2(.,0)+\beta_2(1,0)]}{1+\exp[\alpha_2(.,0)+\beta_2(1,0)]}$$

$$\frac{1}{1+\exp[\alpha_1(1,.)+\beta_1(1,1)]}\frac{\exp[\alpha_2(.,1)+\beta_2(1,1)]}{1+\exp[\alpha_2(.,1)+\beta_2(1,1)]} = \frac{\exp[\alpha_1(1,.)+\beta_1(1,1)]}{1+\exp[\alpha_1(1,.)+\beta_1(1,1)]}\frac{1}{1+\exp[\alpha_2(.,1)+\beta_2(1,1)]}$$

After simple algebra, the previous nonlinear relationships among the probabilities parameters are equivalent to the following linear restrictions:

$$\begin{aligned} \alpha_1(0,.) &= \alpha_2(.,0) \\ \alpha_1(0,.)+\beta_1(0,1)+\alpha_2(.,1) &= 0 \\ \alpha_1(.,1)+\alpha_2(.,0)+\beta_2(1,0) &= 0 \\ \alpha_1(1,.)+\beta_1(1,1) &= \alpha_2(.,1)+\beta_2(1,1) \end{aligned}$$

In the simultaneous presence of these four constraints occur we can think of common dynamics for the state variables and hence of comovement.

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