Asymmetric Periodic Models for High Frequency Data Analysis

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Abstract

This paper investigates the intraday and day-of-the-week seasonalities, as well as the non-linear dynamics of the two most traded American stock Index futures, i.e. the E-mini SP500 and the E-mini Nasdaq100, quoted on the Chicago Mercantile Exchange. The dataset covers both European and American trading hours. We employ the periodic autoregressive model together with the asymmetric periodic GARCH (PAR - APGARCH), with a flexible distribution given by the Skew-T, which allows for both conditional skewness and kurtosis. Differently from previous approaches, we directly model the strong seasonalities in the behavior of the conditional mean, volatility, skewness and kurtosis, and we avoid to use the common seasonal adjustment procedures, which can generate misleading inferences and blur the characteristics of the data (see Boswijk and Franses 1996, Osborn 2004).

JEL classification: $G10, G11, G12$.

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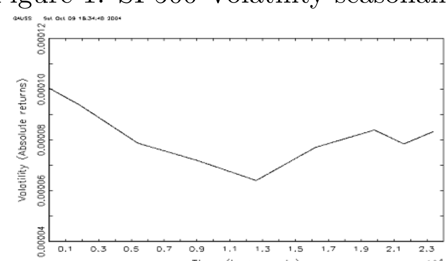
1 Introduction

The increased availability of high frequency financial data has determined a growing number of research studies examining the complex intraday return dynamics. We can identify at least three related but distinct literatures: the first one, analyses the *transmission of information across international financial markets*, that trade sequentially with little, if any, overlap in their trading hours. Early empirical papers include Engle, Ito and Lin(1990) and Becker, Finnerty and Friedman(1995), while more recent works are those of Connolly and Wang(2003), Wongswan(2003) and Ehrmann and Fratzscher(2003) who examine spillover effects between U.S. and other countries' financial markets. The second one examines the *links between asset prices and macroeconomic fundamentals as embodied in news announcements effects*: recent examples are Andersen and Bollerslev (1997), Balduzzi, Elton and Greene(2001), Hautsch and Hess(2002), Fair(2003), Andersen, Bollerslev, Diebold and Vega (2003,2004)[1]. The third strand of the intraday time-series-oriented literature has been concerned with the *role of information flow and other microstructure variables as determinants of intraday returns and volatilities*: seminal works are those of Bollerslev and Domowitz(1993), and Goodhart, Hall, Henry and Pesaran(1993). More recent works are Engle(2000), Bauwens and Giot(2002), Rydberg and Shephard(2002), Liesenfeld and Pohlmeier (2003) and Nolte(2004)[2].

When we start analyzing financial microdata, that is moving from a traditional "low frequency" dataset (usually daily or weekly), to a "high frequency" one (from hourly data to tick by tick data), we immediately realize that the analysis become more complex due to the strong intraday seasonalities of these variables, especially volatility and volume.

One of the main characteristics of the intraday market process is that traded volumes and prices volatility follow a "***U*** - shaped" path, (or to be more precise, an inverted ***J***): these two variables reach their highest values at the market opening, then go down and reach their lower point around the lunch hours; finally they rise again at the market closing.

Figure 1: SP500 Volatility seasonality



A similar pattern is present both in stock markets where there is a separate opening with a single multilateral auction (NYSE, Milan, for example), and in those without such

[1] See Fantazzini(2004), chapters 5 and 6, for a survey of this kind literature

[2] See Fantazzini(2004), chapter 7, for a survey of this literature

a system, such as the CBOT (Sheikh and Ronn, 1994) or the Toronto Stock Exchange (McInish and Wood, 1990), while it is not present in the FX market. Nevertheless, it must be recognized that a similar seasonal path for volatility and volume is not generalized to all financial markets: the London Stock Exchange, for instance (which, however, does not have a separated opening and closing like the NYSE and other European stock exchanges), presents a U – shape for volatility, while for traded volumes there is a double hump- shape path instead (Kleidon and Werner, 1994). If we consider the FX markets and use the frequency of intraday quotes data as a reasonable proxy for traded volumes, we can note that traded volumes don't show this U-shape path at all during the American trading hours, while the European and Asian markets show a rather weak temporal path (Demos and Goodhart, 1992).

To make matters more complex, numerous researches found weekend effects and other anomalies, too. In particular, the *day of the week effect* has been studied in a number of papers: French (1980), Hamon and Jacquillat (1990), to name a few. In these papers, Monday returns are found to be negative while the returns on Friday tended to be higher than the other days. Not only do the average returns on Monday tend to differ, Bessembinder and Hertzel (1993) show that returns on Mondays are positively correlated with those of Fridays while returns on Tuesdays are negatively correlated with those on Mondays. Additionally, there is evidence that the volatility vary with the day of the week, see Foster and Viswanathan (1990), Bollerslev and Ghysels(1996) and Franses and Paap (2000): the latter observe positive autocorrelation on Monday and day of the week variation in the persistence of volatility.

When working with high frequency data the problem of seasonalities becomes even more important and difficult to handle, because the entire form of both intra-daily and weekly patterns has to be taken into account, see Dacorogna et al. (1993, 2001), Breymann et al.(2003) and Dias and Embrechts (2004).

In order to deal with this reality, the usual approach consisted of deseasonalising the high frequency time series, first, then working with the resulting deseasonalised series. There are two main approaches for doing this: time transformation and volatility weighting by periodically varying weights. The former method has been developed by Olsen and Associates, and it consists in a transformation from physical time to an activity - related time scale, the so called ϑ - time scale. For more details see Dacorogna et al. (1993, 2001). The latter approach have been proposed by Andersen and Bollerslev (1997, 1998), Beltratti and Morana (1999), Martens et al. (2002), Andersen, Bollerslev, Diebold and Vega (2003,2004). In this framework the return is written as $r_t = \sigma_t s_t \varepsilon_t$ for the generic intraday return at day t: r_t represents the intraday 5 minutes return, σ_t the deseasonalised volatility, ε_t denotes an i.i.d. mean zero, unit variance error term, and s_t essentially represents the seasonal pattern.

However, the earlier approaches pose some problems: recent evidence by Osborn(2004), who extends the previous works by Franses(1995, 1996b) and Boswiik and Franses(1996), shows that *seasonal adjustment reduces but does not eliminate completely periodicity*, so that users of seasonally adjusted data cannot safely ignore the impact of seasonalities when estimating and testing their models. Moreover, she finds that a non-stationary periodically integrated process is converted into a process with a conventional unit root and induced periodic heteroscedasticity: therefore, *seasonal adjustment can blur or completely "annihilate"*[3] *some data properties.*

Secondly, in the periodic models used so far with financial daily data (Bollerslev and Ghysels 1996, Franses and Paap 2000), *positive and negative shocks have the same impact on the volatility.* A huge literature, starting from the seminal work of Black(1976) till the very recent work of Dias and Embrechts(2004) has observed instead, the existence of a negative correlation between the current return and the future volatility, and this leverage effect becomes even stronger when we consider intra-daily data.

Thirdly, *high frequency data present the well known characteristics of both skewness and fat tails.* Still, the normal or symmetric Student's T distribution functions are by far the most used distributions for the standardized residuals of ARMA-GARCH models.

As a response to the first problem, *we propose here to handle both intradaily and day-of-week effects on returns and variances with the periodic model proposed by Franses and Paap (2000)*, who employ a combination of models previously used separately. This combination includes a periodic autoregressive model (PAR) and a periodic generalized autoregressive conditional heteroskedasticity model (PGARCH) and represents a so-called *PAR-PGARCH model*, described in the next section. The PAR model which allows the autoregressive parameters to vary with the season, was considered in several empirical studies, see Bessembinder and Hertzel (1993) and Abraham and Ikenberry (1994). Nevertheless, in these models, the day-of-week correlation in returns is modelled using the autoregressive framework while volatility is held constant across the days of the week. The PGARCH model, instead, was first proposed by Bollerslev and Ghysels (1996) to cope with the problem of variable day to day volatility as well as of changing correlations between daily volatilities. However, the studies with periodic GARCH models that followed describe the returns by nonseasonal AR models. Still, as previously explained, both models mentioned need to be considered jointly, as neglecting periodic autoregressive behavior would spuriously suggest seasonal heteroskedasticity, Franses(1995) Osborn (1991, 2004) .

As a possible solution to the second problem, *we include a leverage effect parameter in the GARCH dynamics*, see for example Bollerslev et al. (1992), Ding et al. (1993) and Zivot and Wang (2003) . This extension wants to take into account the asymmetric contribution that negative innovations have on the volatility in some cases, that is negative shocks (bad

[3] Osborn (2004)

news) have a larger impact on volatility than positive shocks (good news).

In order to solve the third problem, *we resort to Hansen's Skew-T distribution*, which provide a more flexible model for real data than the traditional normal and symmetric Student's T ones.

By coupling the previous three ideas, we proposed here the *PAR - Asymmetric PGARCH model* (henceforth PAR-APGARCH) with a Skew-T density function: this approach enable us to model the complex intradaily and day-of-the-week seasonalities of high frequency financial data and consider their time-changing skewness and leptokurtosis as well.

The sequel of the paper is organized as follows. In Section 2 we present the PAR-APGARCH model, along with a discussion of its theoretical properties. In Section 3, we provide an outline of conditional asymmetric marginals, focusing on Hansen's Skew-T, while in Section 4 we present the diagnostics tests which will be used to assess the adequacy of our model. Section 5 presents the stock index futures dataset used in the analysis and report the empirical results given by the proposed methodology. Section 6 concludes.

2 Periodic modeling: the PAR - APGARCH model

This work employs the PAR - PGARCH model introduced in Franses and Paap (2000) to investigate the seasonality in intra-daily returns of two American futures, the E-mini SP500 and E-mini Nasdaq100. Nevertheless, and to complement the work of Franses and Paap (2000), we introduce a leverage effect in the specification of the conditional variance and we assume that the residuals can follow a more general distribution than the traditional normal one, like the Skew - T distribution.

Denoting by P_t the futures price at time t, the PAR(p) - APGARCH(1,1) model for the continuously compounded future returns $y_t = 100[log(P_t) - log(P_{t-1})]$, is given by

$$y_t = \mu_{id} + \sum_{l=1}^{p} \phi_{l,id} \; y_{t-1} + \varepsilon_t \tag{2.1}$$

$$\varepsilon_t = \eta_t \sqrt{h_t}, \;\; \eta_t \overset{i.i.d.}{\sim} f(0,1) \tag{2.2}$$

$$h_t = \omega_{id} + \alpha_{id}\varepsilon_{t-1}^2 + \gamma_{id}\varepsilon_{t-1}^2 D_{t-1} + \beta h_{t-1} \tag{2.3}$$

where $t = 1 \ldots T$, id denotes the season corresponding to the i intraday observation of the d day of the week, and $i = 1 \ldots n$, $d = 1, 2, 3, 4, 5$. Besides, $T = n \cdot 5 \cdot N$, where N is the number of weeks W, with $W = 1 \ldots N$, while $D_{t-1} = 1$ if $\varepsilon_{t-1} < 0$, and 0 otherwise.

As opposed to the usual AR-GARCH model, the parameters in a PAR-PGARCH model are allowed to vary with the hours and day of the week: this is why this specification is suitable for modelling seasonalities in the mean as well as volatility of intra-daily returns. The model can also capture the differences in the degree of predictability of stock returns and the persistence of variance across the days of the week. Following Bollerslev and

Ghysels (1996) and Franses and Paap (2000) we specify the PGARCH equation (2.3) with β *not* varying, as this turns out to be convenient for estimation purposes.
Furthermore, good news $\varepsilon_{t-1} > 0$ and bad news $\varepsilon_{t-1} < 0$, have differential effects on the conditional variance in this model: good news has an impact of α_{id}, while bad news has an impact of $\alpha_{id} + \gamma_{id}$. If $\gamma_{id} > 0$ we say that the leverage effect exists, while if $\gamma_{id} \neq 0$ the news impact is asymmetric.

2.1 Stationarity conditions for the conditional mean

The stationarity conditions for the mean process given in equation (2.1) can be derived by rewriting the PAR model as a vector autoregressive model. If we stack the $(n \cdot 5)$ intraday observations into a weekly observed vector $Y_W = (Y_{11,W} \ldots Y_{n1,W} \ldots Y_{n5,W})'$, where $Y_{id,W}$ is the intraday observation i on day d, in week W, with $W = 1, \ldots, N = T/(n \cdot 5)$, the PAR(p) model has the following alternative representation:

$$\mathbf{A}_0 \mathbf{Y}_W = \mu + \mathbf{A}_1 \mathbf{Y}_{W-1} + \mathbf{A}_2 \mathbf{Y}_{W-2} + \cdots + \mathbf{A}_m \mathbf{Y}_{W-m} + \varepsilon_W \tag{2.4}$$

where μ contains the stacked seasonal constants, ε_W is a vector white noise process containing the stacked ε_t variables and the number of lags m depends on p as follows: for $p \leq n \cdot 5$, $m = 1$, for $n \cdot 5 < p < 2 \cdot (n \cdot 5)$, $m = 2$ and so on.
The VAR model in (2.4), and hence the PAR model in (2.1), is stationary provided that the roots of the characteristic equation

$$|\mathbf{A}_0 \mathbf{z}^m - \mathbf{A}_1 \mathbf{z}^{m-1} - \mathbf{A}_2 \mathbf{z}^{m-2} - \cdots - \mathbf{A}_m| \tag{2.5}$$

lie inside the unit circle. When k solutions to (2.5) are on the unit circle, the $\mathbf{Y}_W$ process and also the y_t process, has k unit roots. We remark that the number of unit roots in y_t equals that in $\mathbf{Y}_W$. For a simple PAR(1) model that will be later used in the empirical analysis, $m = 1$, while the square matrices A_0, A_1 are given by

$$\underset{(n \cdot 5 \times n \cdot 5)}{A_0} = \begin{pmatrix} 1 & 0 & 0 & & \cdots & & & 0 & 0 \\ -\phi_{1,21} & 1 & 0 & & & & & & 0 \\ 0 & -\phi_{1,31} & 1 & & & & & & \\ 0 & 0 & -\phi_{1,41} & \ddots & & & & & \vdots \\ 0 & 0 & 0 & \ddots & & & & & \\ \vdots & \vdots & & \ddots & & & & & \\ & & & & & \ddots & 0 & 0 & 0 \\ & & & & & \ddots & 1 & 0 & 0 \\ 0 & & & & & \ddots & -\phi_{1,(n-1)5} & 1 & 0 \\ 0 & 0 & & & \cdots & & 0 & -\phi_{1,n5} & 1 \end{pmatrix}, \tag{2.6}$$

$$\underset{(n\cdot 5\times n\cdot 5)}{A_1} = \begin{pmatrix} 0 & 0 & \cdots & 0 & \phi_{1,11} \\ 0 & 0 & & 0 & 0 \\ 0 & & & & \\ \vdots & \vdots & \ddots & & \vdots \\ 0 & & & & 0 \\ 0 & 0 & \cdots & 0 & 0 \end{pmatrix} \tag{2.7}$$

It is straightforward to verify that the stationarity condition reduces to $\left(\prod_{id=1}^{n\cdot 5} \phi_{1,\,id}\right) < 1$.

2.2 Stationarity conditions for the conditional variance

The stationarity conditions for the variance process given in (2.3) can be obtained in a similar way[4]. Let's define the variable $z_t = \varepsilon_t^2 - h_t$ and write the equation (2.3) as a periodic ARMA process

$$\varepsilon_t^2 = \omega_{id} + (\alpha_{id} + \beta + \gamma_{id} D_{t-1})\varepsilon_{t-1}^2 + z_t - \beta z_{t-1} \tag{2.8}$$

Let $\xi_T^2 = (\varepsilon_{11,W}^2 \ldots \varepsilon_{n1,W}^2 \ldots \varepsilon_{n5,W}^2)'$, consists of the intradaily observations stacked in a weekly vector, that is $\varepsilon_{id,W}^2$ is the intraday observation i on day d, in week W, similarly to what we've done before; equation (2.8) can then be written as follows

$$\mathbf{\Lambda}_0 \xi_W^2 = \mathbf{\Omega} + \mathbf{\Lambda}_1 \xi_{W-1}^2 + \mathbf{B}_0 \mathbf{Z}_W + \mathbf{B}_1 \mathbf{Z}_{W-1} \tag{2.9}$$

where $\mathbf{\Omega} = (\omega_{11} \ldots \omega_{n1} \ldots \omega_{n5})'$, $\mathbf{Z}_W = (z_{11,W} \ldots z_{n1,W} \ldots z_{n5,W})'$, and where $\mathbf{\Lambda}_0$ and $\mathbf{B}_0$ are $(n \cdot 5 \times n \cdot 5)$ matrices similar to A_0 in (2.6), but instead of $-\phi_{1,id}$ we have $-\lambda_{id} = -(\alpha_{id} + \beta + \gamma_{id} D_{t-1})$, with $id = 1 \ldots n \cdot 5$, for $\mathbf{\Lambda}_0$, and $-\beta$ for $\mathbf{B}_0$. Alike, the $(n \cdot 5 \times n \cdot 5)$ matrices $\mathbf{\Lambda}_1$ and $\mathbf{B}_1$ are similar to A_1 in (2.7), but instead of $\phi_{1,11}$ we have $\lambda_{11} = (\alpha_{11} + \beta + \gamma_{11} D_{t-1})$ for $\mathbf{\Lambda}_1$, and $-\beta$ for $\mathbf{B}_1$. The vector process ξ_W^2 is stationary if the root of the following characteristic equation is inside the unit circle:

$$|\mathbf{\Lambda}_0 \mathbf{Z} - \mathbf{\Lambda}_1| = \left(\prod_{id=1}^{n\cdot 5} \lambda_{id} - 1\right) = \left(\prod_{id=1}^{n\cdot 5} (\alpha_{id} + \beta + \gamma_{id} D_{t-1}) - 1\right) \tag{2.10}$$

Furthermore, when one or more λ_{id} values are unequal to a single value λ, i.e. when $\lambda_{id} \neq \lambda$ for all id, but $\left(\prod_{id=1}^{n\cdot 5} \lambda_{id}\right) = 1$, the ε_t^2 process in (2.8) is said to be *Periodically Integrated of order 1* (PI(1))[5]. If $\lambda_{id} = \lambda = 1$, we have the usual Integrated GARCH process nested within the more general Periodically Integrated GARCH process.

[4] We follow here Franses and Paap (2000)

[5] See Franses (1996, 2004) for a detailed overview

3 Conditional Marginal Modeling: Hansen's Skew - T

The marginal distributions that are commonly used to model the standardized residuals in equation (2.2) are the following three:

1. Normal;
2. T-student;
3. Skew-T student.

Two of the most common deviations from normality are fat tails and asymmetry. One distribution that is commonly used to allow for excess kurtosis is the Students-T and it has been generalized to allow for skewness by Hansen (1994). Despite other generalizations have been proposed, we chose this one due to its simplicity and its past success in modelling economic variables (Patton 2003, 2004).

Skew-T distribution: *Let S be a Skewed-T random variable with density function f(ν, λ) and mean zero and variance one by construction, in order to be a suitable model for the standardized residuals. The parameters υ, λ control the kurtosis and skewness of the variable. The functional form of the skew-t density is the following*:

$$Skew-T: \quad g(s,\upsilon,\lambda) = \begin{cases} bc\left(1+\frac{1}{\upsilon+2}\left(\frac{bs+a}{1-\lambda}\right)^2\right)^{-(\upsilon+1)/2} & for\ s \leq -\frac{a}{b} \\ bc\left(1+\frac{1}{\upsilon+2}\left(\frac{bs+a}{1+\lambda}\right)^2\right)^{-(\upsilon+1)/2} & for\ s \geq -\frac{a}{b} \end{cases} \tag{3.1}$$

$$c = \frac{\Gamma\left(\frac{\upsilon+1}{2}\right)}{\Gamma\left(\frac{\upsilon}{2}\right)\sqrt{\pi(\upsilon-2)}} \tag{3.2}$$

$$b = \sqrt{1+3\lambda^2-a^2} \tag{3.3}$$

$$a = 4\lambda c\left(\frac{\upsilon-2}{\upsilon-1}\right) \tag{3.4}$$

Proof: See Hansen (1994).

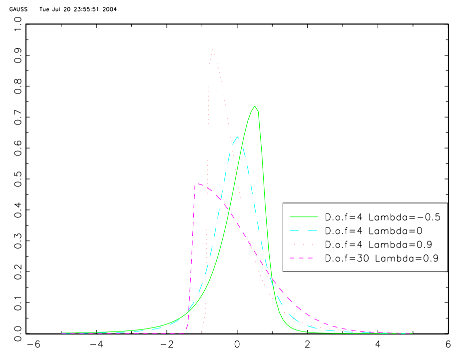
This density is defined for 2$< \upsilon < \infty$ and -1$< \lambda <$+1. Moreover, this density encompasses a large set of conventional densities, allowing us to use standard ML tests:

1. if λ=0, the Skew-t reduces to the traditional Student-T distribution.
2. If λ=0 and υ=∞ we have the normal density.

Similarly to Student's T, given the restriction $\upsilon > 2$, this distribution is well defined and its second moment exists, while skewness exists if $\upsilon > 3$, and kurtosis is defined if $\upsilon > 4$.

The parameter λ controls for skewness: If it is > 0, we have positive skewness, while if it is < 0 the distribution is negative skewed. Fig. 2 displays different densities obtained for various values of degrees of freedom υ and skewness parameter λ.

Figure 2: Density of Hansen's skew-t distribution



Skewness and Kurtosis for a r.v. $S \sim$ Skew-T $(\cdot|\ 0,\ 1,\ \upsilon, \lambda)$: *If $S \sim Skew-T$, the formulae for the third and fourth moments are the followings*:

$$\mathrm{E}\left[Z^3\right] = [m_3 - 3am_2 + 2a^3]/b^3 \tag{3.5}$$

$$\mathrm{E}\left[Z^4\right] = [m_4 - 4am_3 + 6a^2m_2 - 3a^4]/b^4 \tag{3.6}$$

where

$$m_2 = 1 + 3\lambda^2 \tag{3.7}$$

$$m_3 = 16c\lambda(1+\lambda^2)\frac{(\upsilon-2)^2}{(\upsilon-1)(\upsilon-3)} \qquad \text{if} \quad \upsilon > 3 \tag{3.8}$$

$$m_4 = 3\frac{\upsilon-2}{\upsilon-4}(1+10\lambda^2+5\lambda^4) \qquad \text{if} \quad \upsilon > 4 \tag{3.9}$$

Proof: Straightforward but tedious computations similar to Hansen (1994). See also Rockinger and Jondeau (2001)

Since S has zero mean and unit variance, we directly have that skewness $Sk[S] = \mathrm{E}\left[Z^3\right]$, and kurtosis $K[S] = \mathrm{E}\left[Z^4\right]$.
When working with the Skew-T distribution, we have to specify a dynamic model for the conditional degrees of freedom and conditional skewness, as well. We propose here a specification similar to Hansen(1994) and Rockinger and Joundeau (2001):

$$\upsilon_t = \upsilon_{id} \tag{3.10}$$

$$\lambda_t = \Lambda\left(\zeta + \delta_{id} \cdot \varepsilon_{t-1}\right) \tag{3.11}$$

where id denotes the i intraday observation of the d day of the week, with $i = 1, \ldots, n$, and $d = 1, 2, 3, 4, 5$, while $\Lambda(\cdot)$ is the modified logistic transformation, designed to keep the conditional skewness parameter λ_t in (-1, 1) at all times.
We avoid an autoregressive specification, in so far as it may lead to spuriously significant parameters (see Rockinger and Joundeau 2001, for a proof). Moreover, we tried different specifications, but with similar results and increased computational time. This is why we resort to this simple modeling.

4 Diagnostics and Periodic Hypothesis testing

Having found and estimated a satisfactory PAR-PGARCH model for stock returns, it is of interest to perform diagnostics on the residuals and test hypothesis on the parameters of this model.
As a first step we'll perform standard Box-Pierce tests for 25 lags of the standardized residuals by levels and squared, which will be denoted as $BP^1(25)$ and $BP^2(25)$.
Secondly, in order to refine the diagnostics about the quality of our estimates, we'll present an analysis of orthogonality conditions that should be satisfied by the residuals. The use of orthogonality conditions to test the specification of a model has been advocated by Newey (1985) and can be found in many subsequent contributions, e.g., Harvey and Siddique (1999), that we follow here. We group these orthogonality conditions in order to focus on the dependency of higher moments:

$$\mathrm{E}\left[\eta_t\right] = 0, \tag{4.1}$$

$$\mathrm{E}\left[\eta_t^2\right] - 1 = 0, \tag{4.2}$$

$$\mathrm{E}\left[\frac{(\eta_t - \mathrm{E}\left[\eta_t\right])^3}{(\mathrm{V}\left[\eta\right])^{3/2}}\right] = 0, \tag{4.3}$$

$$\mathrm{E}\left[\frac{(\eta_t - \mathrm{E}\left[\eta_t\right])^4}{(\mathrm{V}\left[\eta\right])^{2}}\right] - 3 = 0 \tag{4.4}$$

We'll present the means associated with conditions (4.1) - (4.4) and associated standard errors, and the Jarque-Bera test, which test conditions (4.3) - (4.4) jointly.
As a third step, we'll check for seasonalities in the mean or variance or both. In particular, we consider the following set of hypotheses for the mean process:

$$H_1 : \phi_{i,id} = \phi_i, \tag{4.5}$$

$$H_2 : \phi_{i,id} = \phi_i, \ \mu_{id} = \mu \tag{4.6}$$

Under H_1 the PAR model in (2.1) reduces to that presented in Bollerslev and Ghysels (1996), while under H_2 there is no periodic behavior in the mean at all. For the variance, process we have instead,

$$H_3 \quad : \quad \omega_{id} = \omega, \tag{4.7}$$

$$H_4 \quad : \quad \alpha_{id} = \alpha,\ \gamma_{id} = \gamma,\ \omega_{id} = \omega, \tag{4.8}$$

$$H_5 \quad : \quad \prod_{id=1}^{n\cdot 5} (\alpha_{id} + \beta + \gamma_{id} D_{t-1}) = 1 \tag{4.9}$$

Under hypotheses H_3 and H_4, the PGARCH model is reduced to a GARCH model with and without deterministic seasonal variation, respectively. When H_5 is not rejected, we have a periodically integrated GARCH, while if both H_3, H_4 and H_5 are not rejected the process is the standard non-periodic IGARCH process. Finally, we'll test the joint null hypothesis that the asymmetric coefficients are zero and the degrees of freedom are equal or greater than 30, to verify whether the residuals are normally distributed:

$$H_6 \quad : \quad \zeta = 0,\ \delta_{hd} = 0,\ \upsilon_{hd} \geq 30 \tag{4.10}$$

The aforementioned hypotheses will be tested on the estimated parameters for the American futures with the Wald test, as proposed by Franses and Paap(2000): we choose this strategy, since simulation evidence about IGARCH processes in Lumsdaine (1995) is in favor of the Wald type test statistic.

5 Empirical analysis

5.1 Data Description

We perform the analysis using intradaily returns on the following stock index futures: SP500 and NASDAQ100, both of them quoted on the Chicago Mercantile Exchange.
These two derivatives, besides being the most important stock index futures, have the characteristics of being quoted almost 24 hours a day: to be more precise, there is normal "pit" trading from 9.30 to 16.15 - NewYork time - (named as "Regular Trading Hour, RTH"), and an electronic session from 16.45 until 9.15 the day after ("Globex").
All indices contain the most liquid stocks from the corresponding markets and hence the problem of spurious autocorrelation induced by non-synchronous trading should not arise (see Lo and McKinley, 1990, for a model of non-synchronous trading).
We focus on a two year period starting in December, 2002 and ending in October, 2004, while the prices of the futures contracts used has been built with the futures contracts closest to expiration (*).
Following Bollerslev and Ghysels(1996), but mostly after long conversations with portfolio managers and futures traders working in London and Milan, we've decided to sample the high frequency data using the following scheme:

Intraday seasonalities	***DAY OF THE WEEK***	***TIME - SAMPLE (NewYork Time)***	***TYPE OF TRADING***
1		**03. 00 - 06. 15**	*European Morning trading:* - European managers rebalance their portfolio after weekend news - European Traders, scalpers start their week operating strategies - Europe news announcements
2		**06. 15 - 09. 30**	*European Afternoon trading:* - Europe Lunch break - U.S.A. news announcements
3	**Monday**	**09. 30 - 12. 45**	*American Morning trading:* - American managers rebalance their portfolio after weekend and European news - American Traders, scalpers start their week, daily operating strategies - European traders, scalpers close their intraday positions
4		**12. 45 - 16.00**	*American Afternoon trading:* - American Lunch break - American traders, scalpers close their intraday positions
5		**16. 00 - 03.00**	*Asian trading:* - Asian news announcements - Asian traders, scalpers react to American Monday closing prices

Intraday seasonalities	***DAY OF THE WEEK***	***TIME - SAMPLE (NewYork Time)***	***TYPE OF TRADING***
6		**03. 00 - 06. 15**	*European Morning trading:* - European managers rebalance their portfolio after Asian night news - European Traders, scalpers start their daily operating strategies - Europe news announcements
7		**06. 15 - 09. 30**	*European Afternoon trading:* - Europe Lunch break - U.S.A. news announcements
8	**Tuesday**	**09. 30 - 12. 45**	*American Morning trading:* - American managers rebalance their portfolio after Asian and European news - American Traders, scalpers start their daily operating strategies - European traders, scalpers close their intraday positions
9		**12. 45 - 16. 00**	*American Afternoon trading:* - American Lunch break - American traders, scalpers close their intraday positions
10		**16. 00 - 03. 00**	*Asian trading:* - Asian news announcements - Asian traders, scalpers react to American Tuesday closing prices

(*)The data were obtained from Computer Information Systems 101 Holly Ridge Monroe, LA 71203 U.S.A.

Intraday seasonalities	*DAY OF THE WEEK*	*TIME - SAMPLE (NewYork Time)*	*TYPE OF TRADING*
11		**03. 00 - 06. 15**	*European Morning trading:* - European managers rebalance their portfolio after Asian night news - European Traders, scalpers start their daily operating strategies - Europe news announcements
12		**06. 15 - 09. 30**	*European Afternoon trading:* - Europe Lunch break - U.S.A. news announcements
13	**Wednesday**	**09. 30 - 12. 45**	*American Morning trading:* - American managers rebalance their portfolio after Asian and European news - American Traders, scalpers start their daily operating strategies - European traders, scalpers close their intraday positions
14		**12. 45 - 16. 00**	*American Afternoon trading:* - American Lunch break - American traders, scalpers close their intraday positions
15		**16. 00 - 03. 00**	*Asian trading:* - Asian news announcements - Asian traders, scalpers react to American Wednesday closing prices

Intraday seasonalities	*DAY OF THE WEEK*	*TIME - SAMPLE (NewYork Time)*	*TYPE OF TRADING*
16		**03. 00 - 06. 15**	*European Morning trading:* - European managers rebalance their portfolio after Asian night news - European Traders, scalpers start their daily operating strategies - Europe news announcements
17		**06. 15 - 09. 30**	*European Afternoon trading:* - Europe Lunch break - U.S.A. news announcements
18	**Thursday**	**09. 30 - 12. 45**	*American Morning trading:* - American managers rebalance their portfolio after Asian and European news - American Traders, scalpers start their daily operating strategies - European traders, scalpers close their intraday positions
19		**12. 45 - 16. 00**	*American Afternoon trading:* - American Lunch break - American traders, scalpers close their intraday positions
20		**16. 00 - 03. 00**	*Asian trading:* - Asian news announcements - Asian traders, scalpers react to American Thursday closing prices - Asian managers rebalance their portfolio for the weekend market close

Intraday seasonalities	***DAY OF THE WEEK***	***TIME - SAMPLE (NewYork Time)***	***TYPE OF TRADING***
21	**Friday**	**03. 00 - 06. 15**	*European Morning trading:* - European managers rebalance their portfolio after Asian night news and for the weekend market close - European Traders, scalpers start their daily operating strategies - Europe news announcements
22		**06. 15 - 09. 30**	*European Afternoon trading:* - Europe Lunch break - U.S.A. news announcements
23		**09. 30 - 12. 45**	*American Morning trading:* - American managers rebalance their portfolio after Asian and European news and for the weekend market close - American Traders, scalpers start their daily operating strategies - European traders, scalpers close their intraday and week positions
24		**12. 45 - 16. 00**	*American Afternoon trading:* - American Lunch break - American traders, scalpers close their intraday and week positions
25		**16. 00 - 03.00(*) (*)next monday**	*Weekend break / Asian trading:* -Asian,European,American, weekend news - Asian traders, scalpers react to American Friday closing prices - Asian managers rebalance their portfolio after weekend news

A higher intraday sampling, at the single hour level for example, has been discarded for two reasons:

1. It would have involved a great number of parameters, with a model almost impossible to estimate;

2. All the financial operators contacted considered a higher sampling useless, since the 5 intraday regimes per day are completely sufficient to describe a single trading day. Besides, some of them even suggest us to consider five regimes for Monday and five regimes for all the remaining weekdays, only, for an overall number of 10 regimes. However, since this opinion was not generally accepted, we stick to the previous 25 seasons.

Table 1 reports descriptive statistics for continuously compounded intraday returns on the above mentioned index futures, estimated by using the previous sampling scheme :

Table 1: **Descriptive Statistics Intraday Futures Returns**

	SP500	*NASDAQ100*
# of obs.	2167	2167
Mean	0.0094	0.0145
Median	0.0000	0.0322
Maximum	3.2673	4.5300
Minimum	-2.1488	-2.8988
Std. Dev.	0.4304	0.6439
Skewness	0.2323	0.2371
Kurtosis	8.3575	7.3405
Jarque-Bera	2611.16	1721.43
BP^1 (25 lags)	48.01	32.60
BP^2 (25 lags)	339.69	250.69

The summary statistics are very similar across the two indices: The positive means and positive skewness highlight the falling prices and the subsequent rebound that characterize financial markets after the burst of the high-tech bubble. Besides, Jarque-Bera statistics are highly significant and point out the non-normality of the corresponding intra-day futures returns. Finally, the high BP statistics for the squared returns show the strong persistence in futures volatility.

5.2 Estimation results

The results obtained from the estimation of the various models for the NASDAQ100 and SP500 index futures are displayed in Tables 2 - 3, respectively. The maximization was carried out using both the Broyden-Fletcher-Goldfarb-Shanno (BFGS) and the BHHH algorithms in Gauss 6.0 .

Due to the high number of parameters to estimate, we had to impose some initial restrictions in order to have an identified and parsimonious model. The criteria we used to achieve this goal are the followings:

1. *Financial managers suggestions*: For example, the highest asymmetric and nonlinear futures behaviors usually take place on Mondays and Fridays, while the other days follow a similar pattern.

2. *Minimize the scaled residuals and the scaled squared residuals*;

3. *Minimize the Schwartz Criterion*;

Other criteria can surely be suggested: however, it is beyond the scope of this paper to find the best criterion and can be a nice topic of future research, instead.

Table 2: **SP500: Skew-T distribution**

		Mean		***Volatility***				***Skewness***		***D.o.F***
Day of the week	***Intraday seasonality***	μ_{hd}	ϕ_{hd}	ω_{hd}	α_{hd}	γ_{hd}	β	ζ	δ_{hd}	υ_{hd}
Monday	1 = 3.00 − 6.15	/	−0.031 (0.076)	−0.151 (0.029)	/	/	0.976 (0.006)	−0.103 (0.048)	0.338 (0.204)	4.59 (1.11)
	2 = 6.15 − 9.30	0.041 (0.026)	/	/	/	/	0.976 (0.006)	−0.103 (0.048)	0.333 (0.198)	3.30 (0.51)
	3 = 9.30 − 12.45	/	−0.248 (0.074)	−0.051 (0.016)	/	0.078 (0.034)	0.976 (0.006)	−0.103 (0.048)	−0.031 (0.058)	3.05 (0.45)
	4 = 12.45 − 16.00	/	/	0.280 (0.070)	/	0.042 (0.029)	0.976 (0.006)	−0.103 (0.048)	−0.653 (0.317)	11.37 (4.78)
	5 = 16.00 − 03.00	/	/	−0.086 (0.069)	/	0.020 (0.008)	0.976 (0.006)	−0.103 (0.048)	−0.436 (0.111)	5.85 (1.43)
Tuesday	6 = 3.00 − 6.15	0.037 (0.019)	/	−0.177 (0.029)	/	/	0.976 (0.006)	−0.103 (0.048)	0.338 (0.204)	4.59 (1.11)
	7 = 6.15 − 9.30	/	/	/	/	/	0.976 (0.006)	−0.103 (0.048)	0.333 (0.198)	3.30 (0.51)
	8 = 9.30 − 12.45	/	/	/	0.110 (0.069)	0.078 (0.034)	0.976 (0.006)	−0.103 (0.048)	−0.031 (0.058)	3.05 (0.45)
	9 = 12.45 − 16.00	/	/	0.185 (0.037)	/	0.042 (0.029)	0.976 (0.006)	−0.103 (0.048)	−0.653 (0.317)	11.37 (4.78)
	10 = 16.00 - 03.00	/	0.194 (0.081)	/	/	0.020 (0.008)	0.976 (0.006)	−0.103 (0.048)	−0.436 (0.111)	5.85 (1.43)
Wednesday	11 = 3.00 − 6.15	/	/	0.167 (0.035)	/	/	0.976 (0.006)	−0.103 (0.048)	0.338 (0.204)	4.59 (1.11)
	12 = 6.15 − 9.30	/	/	/	/	/	0.976 (0.006)	−0.103 (0.048)	0.333 (0.198)	3.30 (0.51)
	13 = 9.30 − 12.45	/	/	/	/	0.078 (0.034)	0.976 (0.006)	−0.103 (0.048)	−0.031 (0.058)	3.05 (0.45)
	14 = 12.45 - 16.00	/	/	0.205 (0.025)	/	0.042 (0.029)	0.976 (0.006)	−0.103 (0.048)	−0.653 (0.317)	11.37 (4.78)
	15 = 16.00 - 03.00	/	/	/	/	0.020 (0.008)	0.976 (0.006)	−0.103 (0.048)	−0.436 (0.111)	5.85 (1.43)
Thursday	16 = 3.00 − 6.15	/	−0.061 (0.031)	−0.203 (0.023)	/	/	0.976 (0.006)	−0.103 (0.048)	0.338 (0.204)	4.59 (1.11)
	17 = 6.15 − 9.30	/	/	/	/	/	0.976 (0.006)	−0.103 (0.048)	0.333 (0.198)	3.30 (0.51)
	18 = 9.30 − 12.45	/	−0.184 (0.136)	0.054 (0.029)	/	0.078 (0.034)	0.976 (0.006)	−0.103 (0.048)	−0.031 (0.058)	3.05 (0.45)
	19 = 12.45 - 16.00	/	/	0.151 (0.040)	/	0.042 (0.029)	0.976 (0.006)	−0.103 (0.048)	−0.653 (0.317)	11.37 (4.78)
	20 = 16.00 - 03.00	/	/	/	/	0.020 (0.008)	0.976 (0.006)	−0.103 (0.048)	−0.436 (0.111)	5.85 (1.43)
Friday	21 = 3.00 − 6.15	/	−0.055 (0.046)	−0.190 (0.025)	/	/	0.976 (0.006)	−0.103 (0.048)	0.338 (0.204)	4.59 (1.11)
	22 = 6.15 − 9.30	/	−0.159 (0.080)	/	/	/	0.976 (0.006)	−0.103 (0.048)	0.333 (0.198)	3.30 (0.51)
	23 = 9.30 − 12.45	/	/	0.090 (0.043)	0.179 (0.085)	0.078 (0.034)	0.976 (0.006)	−0.103 (0.048)	−0.031 (0.058)	3.05 (0.45)
	24 = 12.45 - 16.00	−0.052 (0.042)	−0.349 (0.128)	0.110 (0.052)	/	0.042 (0.029)	0.976 (0.006)	−0.103 (0.048)	−0.653 (0.317)	11.37 (4.78)
	25 = 16.00 - 03.00	/	/	/	0.013 (0.008)	0.020 (0.008)	0.976 (0.006)	−0.103 (0.048)	−0.436 (0.111)	5.85 (1.43)
Log-Lik.:	-286.23									
Sch. crit.:	572.62									

We report parameters significant at the 10 % level, only.

Table 3: **Nasdaq100: Skew-T distribution**

		Mean		***Volatility***				***Skewness***		***D.o.F***
Day of the week	***Intraday seasonality***	μ_{hd}	ϕ_{hd}	ω_{hd}	α_{hd}	γ_{hd}	β	ζ	δ_{hd}	υ_{hd}
Monday	1 = 3.00 – 6.15	/	/	−0.402 (0.093)	/	/	0.993 (0.002)	−0.121 (0.057)	0.142 (0.117)	5.45 (1.11)
	2 = 6.15 – 9.30	/	0.084 (0.049)	−0.183 (0.059)	/	0.026 (0.015)	0.993 (0.002)	−0.121 (0.057)	0.195 (0.240)	3.89 (0.72)
	3 = 9.30 – 12.45	/	−0.154 (0.088)	/	/	/	0.993 (0.002)	−0.121 (0.057)	−0.447 (0.280)	4.51 (1.02)
	4 = 12.45 – 16.00	/	/	0.782 (0.157)	/	/	0.993 (0.002)	−0.121 (0.057)	0.248 (0.235)	17.73 (9.89)
	5 = 16.00 – 03.00	/	0.060 (0.044)	−0.193 (0.106)	/	/	0.993 (0.002)	−0.121 (0.057)	−0.180 (0.108)	4.57 (1.10)
Tuesday	6 = 3.00 – 6.15	0.086 (0.029)	/	−0.575 (0.074)	/	−0.007 (0.003)	0.993 (0.002)	−0.121 (0.057)	0.142 (0.117)	5.45 (1.11)
	7 = 6.15 – 9.30	/	−0.259 (0.090)	/	/	/	0.993 (0.002)	−0.121 (0.057)	0.195 (0.240)	3.89 (0.72)
	8 = 9.30 – 12.45	/	/	/	/	0.039 (0.021)	0.993 (0.002)	−0.121 (0.057)	−0.447 (0.280)	4.51 (1.02)
	9 = 12.45 – 16.00	/	/	0.636 (0.102)	/		0.993 (0.002)	−0.121 (0.057)	0.248 (0.235)	17.73 (9.89)
	10 = 16.00 - 03.00	/	0.091 (0.047)	/	/	/	0.993 (0.002)	−0.121 (0.057)	−0.180 (0.108)	4.57 (1.10)
Wednesday	11 = 3.00 – 6.15	/	/	−0.511 (0.109)	/	−0.007 (0.003)	0.993 (0.002)	−0.121 (0.057)	0.142 (0.117)	5.45 (1.11)
	12 = 6.15 – 9.30	/	/	0.100 (0.042)	/	/	0.993 (0.002)	−0.121 (0.057)	0.195 (0.240)	3.89 (0.72)
	13 = 9.30 – 12.45	/	−0.122 (0.063)	/	/	0.039 (0.021)	0.993 (0.003)	−0.121 (0.057)	−0.447 (0.280)	4.51 (1.02)
	14 = 12.45 - 16.00	/	/	0.568 (0.067)	/	/	0.993 (0.002)	−0.121 (0.057)	0.248 (0.235)	17.73 (9.89)
	15 = 16.00 - 03.00	/	/	/	/	/	0.993 (0.002)	−0.121 (0.057)	−0.180 (0.108)	4.57 (1.10)
Thursday	16 = 3.00 – 6.15	/	/	−0.503 (0.081)	/	−0.007 (0.002)	0.993 (0.002)	−0.121 (0.057)	0.142 (0.117)	5.45 (1.11)
	17 = 6.15 – 9.30	/	/	−0.067 (0.035)	/	/	0.993 (0.002)	−0.121 (0.057)	0.195 (0.240)	3.89 (0.72)
	18 = 9.30 – 12.45	/	−0.309 (0.146)	0.108 (0.049)	/	0.039 (0.021)	0.993 (0.002)	−0.121 (0.057)	−0.447 (0.280)	4.51 (1.02)
	19 = 12.45 - 16.00	/	/	0.494 (0.087)	/	/	0.993 (0.002)	−0.121 (0.057)	0.248 (0.235)	17.73 (9.89)
	20 = 16.00 - 03.00	/	/	/	/	/	0.993 (0.002)	−0.121 (0.057)	−0.180 (0.108)	4.57 (1.10)
Friday	21 = 3.00 – 6.15	/	/	−0.568 (0.076)	/	/	0.993 (0.002)	−0.121 (0.057)	0.142 (0.117)	5.45 (1.11)
	22 = 6.15 – 9.30	/	/	/	/	/	0.993 (0.002)	−0.121 (0.057)	0.195 (0.240)	3.89 (0.72)
	23 = 9.30 – 12.45	/	−0.270 (0.169)	0.149 (0.062)	/	/	0.993 (0.002)	−0.121 (0.057)	−0.447 (0.280)	4.51 (1.02)
	24 = 12.45 - 16.00	−0.186 (0.093)	−0.595 (0.146)	0.557 (0.162)	/	0.044 (0.019)	0.993 (0.002)	−0.121 (0.057)	0.248 (0.235)	17.73 (9.89)
	25 = 16.00 - 03.00	/	/	−0.139 (0.077)	0.010 (0.004)	/	0.993 (0.002)	−0.121 (0.057)	−0.180 (0.108)	4.57 (1.10)
Log-Lik.:	-710.88									
Sch. crit.:	1421.92									

Table 4: **Diagnostic Tests: Skew - T Distribution (*)**

	SP500	*NASDAQ100*
BP^1 (*Std. res.*)	35.92 (0.072)	37.21 (0.055)
BP^2 (*Std. res. squared*)	34.97 (0.088)	29.50 (0.243)
$\mathrm{E}[\eta_t] = 0$	0.013 (0.521)	0.038 (0.062)
$\mathrm{E}[\eta_t^2]$ - 1 = 0	-0.109 (0.018)	-0.062 (0.166)
$\mathrm{E}\left[\frac{(\eta_t - \mathrm{E}[\eta_t])^3}{(\mathrm{V}[\eta])^{3/2}}\right] = 0$	0.165 (0.539)	0.340 (0.172)
$\mathrm{E}\left[\frac{(\eta_t - \mathrm{E}[\eta_t])^4}{(\mathrm{V}[\eta])^2}\right]$ -3 = 0	2.455 (0.209)	2.320 (0.187)
Jarque-Bera (*Std. res.*)	1362 (0.000)	847.8 (0.000)
$H_1: \quad \chi^2(7)(^{**}) \mid \chi^2(8)(^{***})$	23.24 (0.001)	38.38 (0.000)
$H_2: \quad \chi^2(9) \mid \chi^2(9)$	26.93 (0.001)	47.09 (0.000)
$H_3: \quad \chi^2(13) \mid \chi^2(16)$	132.6 (0.000)	123.4 (0.000)
$H_4: \quad \chi^2(17) \mid \chi^2(21)$	140.4 (0.000)	138.1 (0.000)
$H_5: \quad \chi^2(1) \mid \chi^2(1)$	2.161 (0.141)	0.959 (0.327)
$H_6: \quad \chi^2(11) \mid \chi^2(11)$	5285 (0.000)	1974 (0.000)

(*)P-values in brackets.(**) D.o.F. for SP500. (***) D.o.F. for Nasdaq100.

In Table 2 - 3 the estimated values of all relevant parameters from the equations (2.1) - (2.3) / (3.10) - (3.11) are presented[6], while Table 4-5 show the diagnostics tests previously presented in section 4. The main conclusions that can be drawn are the followings:

1. *Conditional Mean*: Focusing on the first parameter μ_{hd}, both index futures generated significant positive estimates on Tuesday European morning trading, and negative on Friday during the American afternoon trading. The SP500 future presented a positive constant on Monday European morning trading, too. These results did not come as a surprise to the contacted traders, since, according to them, "when the trick of negative Mondays and positive Friday come out during the '90s, financial operators reacted by behaving the other way round".

   As for the estimates of the first order correlation in futures returns, we notice that they are significantly positive on Monday American morning trading as well as on Tuesday Asian night trading (i.e. on the 10^{th} regime), similar to the findings of Bessembinder and Hertzel (1993) Abraham and Ikenberry (1994), and Franses and Paap (2000). The Nasdaq100 presented high negative autoregressive coefficients on American morning trading in all trading days, while the SP500 displayed similar negative coefficients, but smaller in absolute value, only on Mondays and at the end of the trading week. Financial traders suggested us that such an outcome is due to the more speculative nature of the Nasdaq100 future, mainly used by scalpers and

[6] For sake of brevity we do not report the results obtained when using the Normal distribution. However, they are available from the authors upon request

Table 5: **Diagnostic Tests: Normal Distribution**

	SP500	*NASDAQ100*
BP^1 (*Std. res.*)	37.92 (0.047)	41.60 (0.019)
BP^2 (*Std. res. squared*)	37.82 (0.048)	32.31 (0.149)
$\mathrm{E}[\eta_t] = 0$	0.015 (0.468)	0.039 (0.064)
$\mathrm{E}\left[\eta_t^2\right]$ - 1 = 0	−0.002 (0.948)	−0.002 (0.956)
$\mathrm{E}\left[\frac{(\eta_t - \mathrm{E}[\eta_t])^3}{(\mathrm{V}[\eta])^{3/2}}\right] = 0$	0.103 (0.611)	0.236 (0.118)
$\mathrm{E}\left[\frac{(\eta_t - \mathrm{E}[\eta_t])^4}{(\mathrm{V}[\eta])^2}\right]$ -3 = 0	2.173 (0.061)	1.40 (0.042)
Jarque-Bera (*Std. res.*)	437.7 (0.000)	184.7 (0.000)
$H_1 :\ \chi^2(7) \mid \chi^2(8)$	25.10 (0.001)	38.62 (0.001)
$H_2 :\ \chi^2(9) \mid \chi^2(9)$	26.90 (0.001)	51.01 (0.000)
$H_3 :\ \chi^2(13) \mid \chi^2(16)$	279.3 (0.000)	316.6 (0.000)
$H_4 :\ \chi^2(17) \mid \chi^2(21)$	328.9 (0.000)	357.5 (0.000)
$H_5 :\ \chi^2(1) \mid \chi^2(1)$	1.975 (0.160)	2.616 (0.106)
Log-Likelihood Normal model	-324.69	-736.61
Schwartz criterion Normal model	649.50	1473.36

intraday traders, with respect to the SP500 future, mainly used by fund managers to rebalance and cover their portfolios at the beginning and at the end of the week, respectively.

Besides, the differences among the considered regimes are statistically significant, as can be inferred from the results obtained in the Wald tests (H_1 and H_2, table 4-5)

2. *Conditional Variance*: We can observe that the estimates of both the deterministic component (ω_{hd}) and the delayed innovations (α_{hd} and γ_{hd}) are different from zero in many cases, while β is strongly different form zero for both the two index futures. Furthermore, the significance of the Wald tests (H_3 and H_4, table 4-5) show that the seasonal differences must be considered in their behavior. These results lead us to reject those models that assume constant volatility over days of the week. Note also that the negative estimates of ω_{hd} does not necessarily imply negative conditional variance as Franses and Paap (2000) show. Indeed, the estimated conditional variance of all indices never gets negative over the whole sample period we use. The H_5 null hypothesis cannot be rejected, so that the Periodically Integrated GARCH model yield an adequate description of the conditional variance of the two futures: this outcome confirms that of Franses and Paap (2000).

   In order to have an identified model, we initially impose that the leverage effect given by γ_{hd} is the same for Tuesdays, Wednesday and Thursday, while is different for Mondays and Fridays, so that wee can start with 15 seasons instead of 25. We did this following the suggestion of financial operators: we found out that, for the Nasdaq100 future, the leverage is significantly positive on American trading hours, particularly in the mornings; the SP500 showed, instead, a leverage effect from American morning trading till Asian night hours. Again, financial traders and managers stressed the different nature of the two futures: more speculative and short-term oriented the Nasdaq100, while more portfolio dedicated and long-term oriented the SP500.

3. *Conditional Skewness and Kurtosis*: To identify the model, we initially impose that the skewness persistence parameter δ_{hd} and the degrees of freedom υ_{hd} change with the intraday seasons but not with the day of the week, so that we have 5 regimes instead of 25. Tables 1-2 show similar results for both Nasdaq100 and SP500: negative shocks in European trading hours will determine negative skewness in the subsequent seasons, while negative shocks in American and Asian trading hours tend to increase skewness in the subsequent seasons, instead.

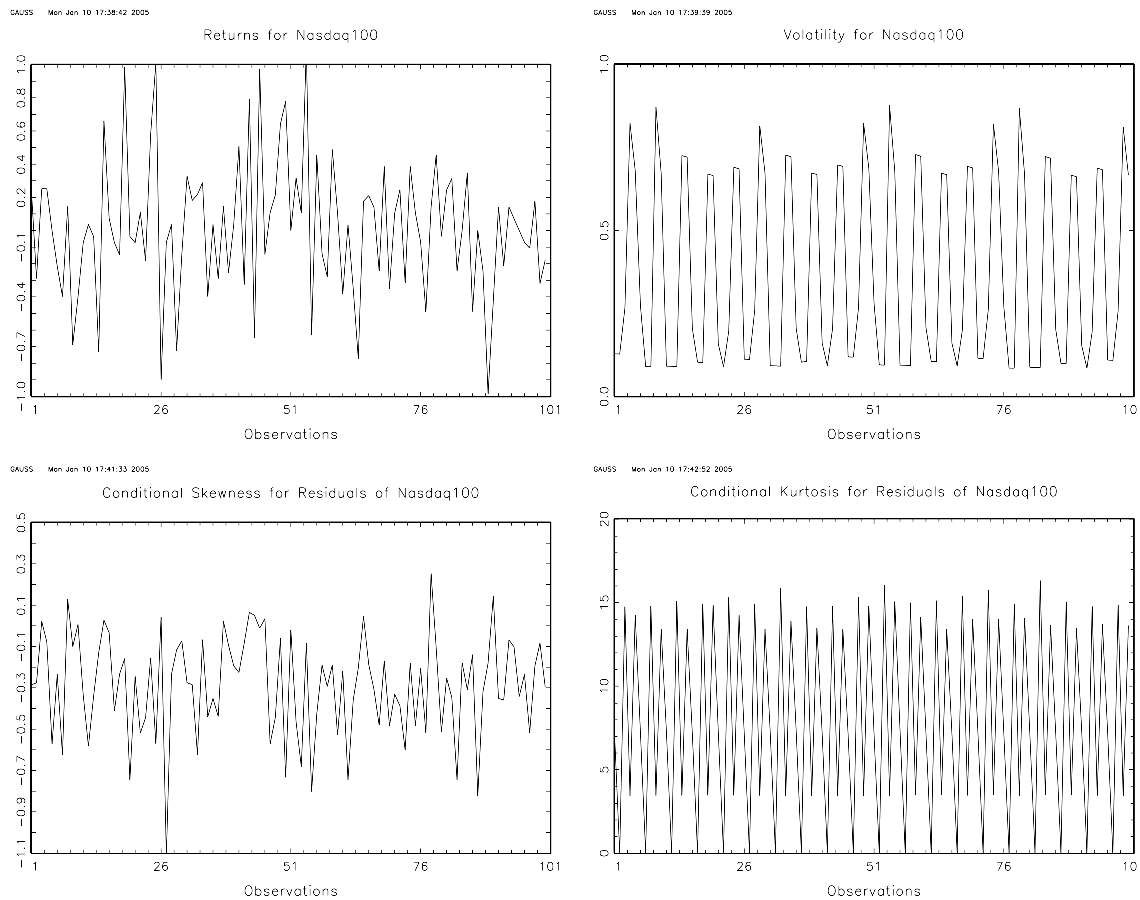
   The degrees of freedom are lowest in European afternoon trading and American morning trading, as expected, since it is in these two seasons, i.e. between 6.15 and 12.45 New-York time, that American macroeconomic announcements and most of trades take place.

   An interesting result emerged: the fourth moment ceases to exist for both index futures in the European afternoon trading between 6.15 and 9.30 NY time, and for the SP500, between 9.30 and 12.45, too, since the degrees of freedom are lower than 4. These results confirms similar findings obtained with daily data by Loretan and Philips (1994) and Jondeau and Rockinger(2003), who suggest that "the fourth moment of stock-index returns fails to exist due to some episodes where higher moments cease to exist" (Jondeau and Rockinger, 2003, p. 1722).

4. *Skew - T vs. Normal distribution*: When testing for autocorrelation up to 25 lags in the standardized residuals of the Skew-T model, the BP^1 and BP^2 statistics are statistically insignificant for both indices. This implies that all models have been properly specified and thus adequately describe the variation in the conditional mean and variance of stock returns. Besides, they have the desired properties of zero mean, unit variance 1 and not normal distribution. This does not hold for the residuals resulting from the Normal model, where the BP^1 statistics are statistically significant at the 5 % level for both indices: this result is not a surprise, since Newey and SteigerWald (1997) showed that the quasi-maximum likelihood estimators are not consistent in the presence of asymmetric distributions. Further evidence against the normal distribution is given by joint null hypothesis that the asymmetric coefficients are zero and the degrees of freedom are equal or greater than 30 (H_6, table 4): this test strongly reject this hypothesis.

Finally, we show the evolution of returns, volatility, skewness and kurtosis through time, with a time-window of 4 weeks (100 observations), to better highlight the seasonal behavior. The latter two where estimated using Eqs. (3.5) and (3.6) (when the 4^{th} moment ceases to exists, we set it to zero).

Figure 3: Nasdaq100: Returns, Volatility, Skewness and Kurtosis



6 Conclusions

This paper investigated the existence of intraday and day-of-the-week seasonalities in the returns, volatility, skewness and kurtosis of the two stock index futures SP500 and NASDAQ100. We employed the Periodic Auto-Regressive model together with the Asymmetric Periodically Integrated GARCH (PAR - APIGARCH), where we introduce a leverage effect in the GARCH dynamics. Moreover, we used the Skew-T distribution instead of the traditional Normal distribution , to have a more flexible modelling which allow for conditional Skewness and Kurtosis. This solution also enable us to avoid the asymptotic bias that quasi-maximum likelihood estimators show in the presence of asymmetric distributions.

The empirical results highlight the seasonal behavior in all four moments and the different type of traders that work with the two futures: mainly scalpers and intraday traders for the Nasdaq100, funds managers for the SP500.

We find some interesting regularities in the mean and we attempted to explain them with the help of professional traders and managers. It is worth emphasizing that predictability and seasonality of stock returns found in this work need not imply market inefficiency. Although our results can be useful in the real-world investment process, they do not imply that profitable trading strategies yielding superior returns when adjusted for transaction cost exist.

A further investigation into the economic significance of stock returns predictability and seasonality on futures markets is therefore called for.

Besides, there are other lines for future research: for example, how to achieve estimation parsimony when dealing with a high number of seasons and/or high autoregressive order is surely one of the most interesting and compelling topic of periodic modelling. In a similar fashion, a multivariate extension can imply a wealth of parameters to estimate and a dramatic loss of efficiency.

References

Abraham, A., and D. L. Ikenberry (1994): "The individual investor and the weekend effect," *Journal of Financial and Quantitative Analysis*, 29, 263–278.

Andersen, T. G., and T. Bollerslev (1997a): "Heterogeneous Information Arrivals and Return Volatility Dynamics: Uncovering the Long-Run in High-Frequency Returns," *Journal of Finance*, 52, 975–1005.

——— (1997b): "Intraday Periodicity and Volatility Persistence in Financial Markets," *Journal of Empirical Finance*, 4, 115–158.

——— (1998a): "Answering the Skeptics: Yes, Standard Volatility Models Do Provide Accurate Forecasts," *International Economic Review*, 39, 885–905.

——— (1998b): "Deutsche Mark-Dollar Volatility: Intraday Activity Patterns, Macroeconomic Announcements, and Longer Run Dependencies," *Journal of Finance*, 53, 219–265.

Andersen, T. G., T. Bollerslev, and F. X. Diebold (2003): "Some Like it Smooth, and Some Like it Rough: Untangling Continuous and Jump Components in Measuring, Modeling, and Forecasting Asset Return Volatility," Working Paper, Kellogg School of Management - Northwestern University, Duke University, University of Pennsylvania, NBER.

Andersen, T. G., T. Bollerslev, F. X. Diebold, and C. Vega (2003): "Micro Effects of Macro Announcements: Real-Time Price Discovery in Foreign Exchange," *American Economic Review*, 93(1), 38–62.

——— (2004): "Real Time Price Discovery in Stock , Bond and Foreign Exchange Markets," working paper.

Balduzzi, P., E. Elton, and T. C. Green (1998): "Economic News and the Yield Curve: Evidence from the U.S. Treasury Market," Salomon Center Working Paper S-98-5.

——— (2001): "Economic News and the Yield Curve: Evidence from the U.S. Treasury Market," *Journal of Financial and Quantitative Analysis*, 36.

Bauwens, L., and P. Giot (2002): "Asymmetric ACD models: introducing price information in ACD models," Core Discussion Paper 9844, (now in Empirical Economics , 28/4, 709-731, 2003).

Becker, K. G., J. E. Finnerty, and J. Friedman (1995): "Economic News and Equity Market Linkages between the U.S. and U.K.," *Journal of Banking and Finance*, 19, 1191–1210.

Beltratti, A., and C. Morana (1999): "Computing value at risk with high frequency data,," *Empirical Finance*, 6(5), 431–455.

Bessembinder, H., and M. Hertzel (1993): "Return autocorrelations around nontrading days," *Review of Financial studies*, 6, 155–189.

Black, F. (1976): "Studies of Stock Price Volatility Changes," *Proceedings of the American Statistical Association, Business and Economic Statistics Section*, pp. 177–181.

Bollerslev, T., R. Y. Chou, and K. F. Kroner (1992): "ARCH Modelling in Finance: A Review of the Theory and Empirical Evidence," *Journal of Econometrics*, 52, 5–59.

Bollerslev, T., and I. Domowitz (1993): "Trading patterns and prices in the Interbank Foreign Exchange Market," *Journal Of Finance*, 48, 1421–1443.

Bollerslev, T., and E. Ghysels (1996): "Periodic autoregressive conditional heteroscedasticity," *Journal of Business and Economic Statistics*, 14, 139–151.

Boswijk, H., and P. Franses (1996): "Unit Root in Periodic Autoregressions," *Journal of Time Series Analysis*, 17, 221–245.

Breymann, W., A. Dias, and P. Embrechts (2003): "Dependence structures for multivariate high-frequency data in finance," *Quantitative Finance*, 3, 1–14.

Connolly, R., and A. Wang (2003): "International Equity Market Comovements: Economic Fundamentals or Contagion?," *Pacific Basin Finance Journal*, 11, 23–43.

Dacorogna, M. M., U. A. Muller, R. J. Nagler, R. B. Olsen, and O. V. Pictet (1993): "A Geographical Model for the Daily and Weekly Seasonal Volatility in the Foreign Exchange Market," *Journal of International Money and Finance*, 12, 413–438.

Demos, F., and C. Goodhart (1992): "The interaction between the frequency of market quotations , spread and volatility in the foreign exchange markets," Discussion Paper 152, LSE Financial Markets Group Discussion Paper.

Dias, A., and E. P. (2003): "Dynamic copula models for multivariate high-frequency data in finance," Department of Mathematics ETH Zurich , working paper.

——— (2004): "Dynamic copula models for multivariate high-frequency data in finance," Department of Mathematics ETH Zurich , working paper.

Ehrmann, M., and M. Fratzscher (2003): "Interdependence betwen the Euro Area and the U.S.: What Role for EMU," Manuscript, European Central Bank, Frankfurt.

Embrechts, P., W. Breymann, and A. Dias (2003): "Dependence structures for multivariate high-frequency data in finance," *Quantitative Finance*, 3, 1–14.

Engle, R. F. (2000): "The Econometrics of Ultra-High-Frequency Data," *Econometrica*, 68, 1, 1–22.

Engle, R. F., T. Ito, and W.-L. Lin (1990): "Meteor Showers or Heat Waves? Heteroskedastic Intra-Daily Volatility in the Foreign Exchange Market," *Econometrica*, 58(3), 525–542.

Fair, R. (2003): "Shock Effects on Stocks , Bonds and Exchange Rates," *Journal of International Money and Finance*, 21, 307–341.

Fantazzini, D. (2004): *Financial Markets Microstructure and High Frequency Data: Theoretical issues, Stylized Facts and Econometric Tools*. Digital University Press.

Foster, S., and S. Viswanathan (1990): "A theory of the interday variations in volume, variance, and trading costs in security markets," *Review of financial studies*, 3, 593–624.

Franses, P. (1995): "The effects of seasonal adjusting a periodic autoregressive process," *Computational Statistics and Data Analysis*, 19, 683–704.

——— (1996a): *Periodicity and Stochastic Trends in Economic Time Series*. Oxford University Press.

——— (1996b): "Recent Advances in modeling seasonality," *Journal of Economic Surveys*, 10, 299–345.

Franses, P., and R. Paap (2000): "Modelling day-of-the-week seasonality in the SP 500 index," *Applied Financial Economics*, 10, 483–488.

French, K. (1980): "Stock returns and the Week-end effect," *Journal of Financial Economics*, 8(1), 55–70.

Goodhart, C. A., S. Hall, S. B. Henry, and B. Pesaran (1993): "News Effects in a High-Frequency model of the Sterling Dollar Exchange Rate," *Journal of Applied Econometrics*, 8, 1–13.

Hamon, J., and B. Jacquillat (1990): "Saisonnalité dans la semaine et la séance à la bourse de Paris," cahier de recherche du CEREG, n 9007, université Paris-Dauphine.

Hansen, B. E. (1994): "Autoregressive Conditional Density Estimation," *International Economic Review*, 35(3), 705–730.

Harvey, C., and A. Siddique (1994): "Autoregressive Conditional Skewness," *Journal of Financial and Quantitative Analysis*, 34(4), 465–487.

Hautsch, N., and D. Hess (2002): "The Processing of Non-Anticipated information in Financila Markets: Analysing the Impact of the Surprises in the employment Report," *European Finance Review*, 6, 133–161.

Jondeau, E., and M. Rockingera (2003): "Conditional volatility, skewness, and kurtosis: existence, persistence, and comovements," *Journal of Economic Dynamics and Control*, 27, 1699–1737.

Kleidon, and Werner (1994): Discussion paperDiscussion Paper London School of Economics.

Liesenfeld, R., and W. Pohlmeier (2003): "A Dynamic Integer Count Data Model for Financial Transaction Prices," University of Konstanz, CoFE Working Paper.

Martens, M., Y. Chang, and S. J. Taylor (2002): "A comparison of seasonal adjustment methods when forecasting intraday volatility,," *J. Financ. Res.*, 15(2), 283–299.

McInish, T., and R. Wood (1990): "A transaction data analysis of the variability of common stocks returns during 1980 - 1984," *Journal of Banking and Finance,*, 14, 99 – 112.

Michael M. Dacorogna, Ramazan Gencay, U. M. R. B. O., and O. V. Pictet (2001): *An Introduction to High-Frequency Finance*. Academic Press.

Newey, W. (1985): "Generalized method of moments specification testing," *Journal of Econometrics*, 29, 229–256.

Nolte, I. (2004): "Modelling a Multivariate Transaction Process," University of Konstanz, CoFE Working Paper.

Osborn, D. R. (1991): "The implications of periodically varying coefficients for seasonal time series processes.," *Journal of Econometrics*, 48, 373–384.

Osborn, D. R., and T. Castro (2004): "The consequences of seasonal adjustment for periodic autoregessive processes," *Econometrics Journal*, 7, 307–321.

Patton, A. (2003): "Modeling Time-Varying Exchange Rate Dependence Using the Conditional Copula," Working Paper 01-09, Department of Economics, University of California, SanDiego.

——— (2004): "On the Importance of Skewness and Asymmetric Dependence in Stock Returns for Asset Allocation," *Journal of Financial Econometrics, forthcoming.*

Rockinger, M., and E. Jondeaua (????): "Conditional dependency of 5nancial series: an application of copulas," Working paper, Banque de France, NER82.

Rydberg, T., and N. Shephard (2002): "Dynamics of Trade-by-Trade Price Movements: Decomposition and Models," *Journal of Financial Econometrics*, 1, 2–25.

Sheikh, and Ronn (1994): "A Characterization of the Daily and Intraday Behaviour of Returns on Options," *Journal of Finance*, 49, 557 – 580.

Wongswan, J. (2003): "Trasmission of Information across International Equity Markets," International Finance Discussion Paper 759, Federal Reserve Board.

Zhuanxin Ding, C. W. J. G., and R. F. Engle (1993): "A Long Memory Property of Stock Market Returns and a New Model," *Journal of Empirical Finance*, 1, 83–106.

Zivot, E., and J. Wang (2003): *Modeling Financial Time Series with S-Plus.* Springer, New York.